

Fast Neutrino Flavor Conversion: Role of Matter Inhomogeneity

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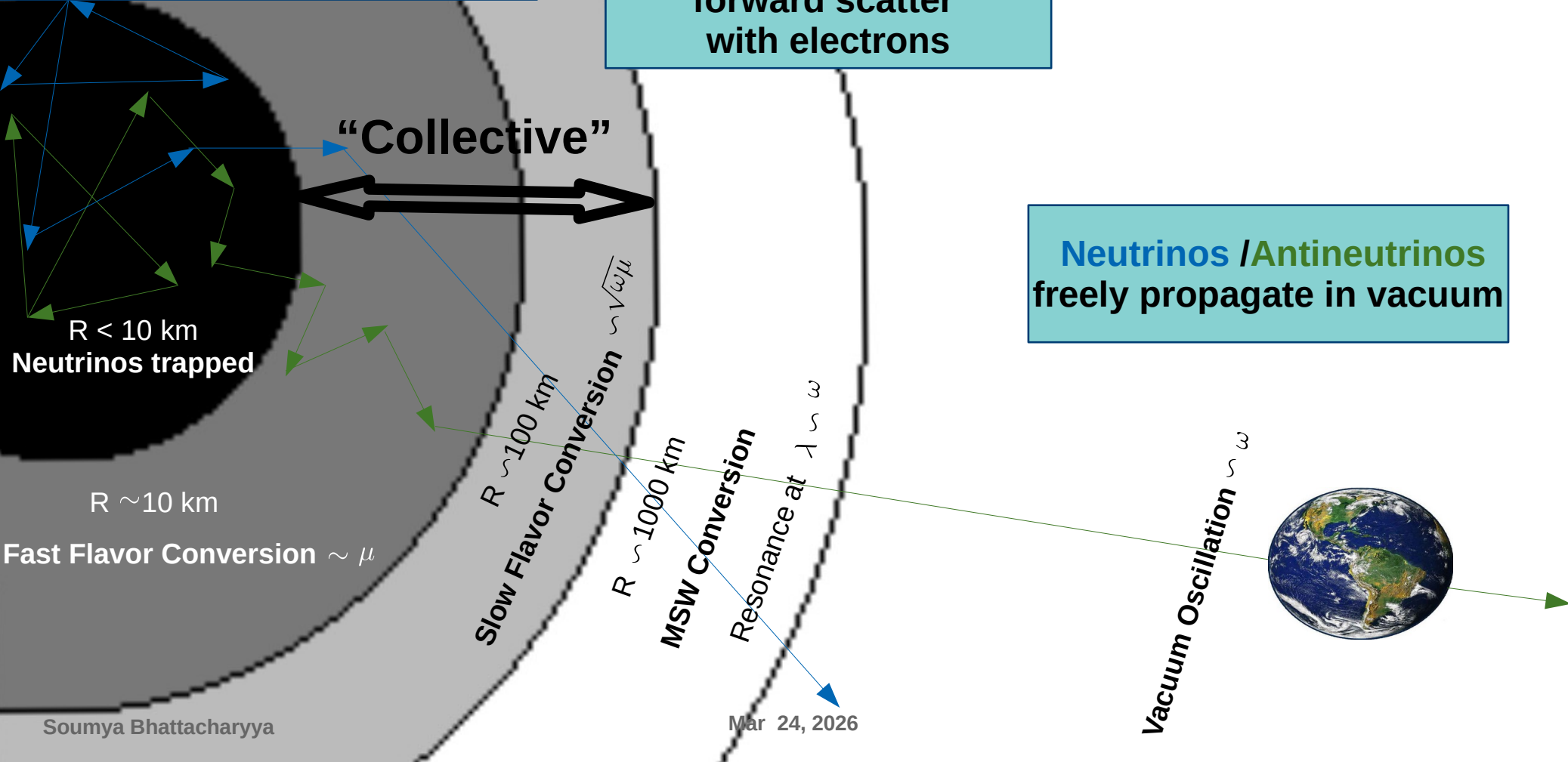
Content of The Talk

- Fast Flavor Conversion (FFC) : A Review
- Inhomogeneous Matter Potential : Why is it Important?
- Effect of Inhomogeneous Matter Potential on FFC : *a) Initial Evolution*
c) Final State
- Inhomogeneous Matter and FFC: A Realistic Supernova Perspective
- How to incorporate this effect in SN simulation
- Take Away Message

Neutrinos /Antineutrinos
forward scatter with each other

Neutrinos /Antineutrinos
forward scatter
with electrons

Neutrinos /Antineutrinos
freely propagate in vacuum



FFC : Flavor Evolution Equation

Ensemble of neutrinos $\rightarrow$ density matrix $\rightarrow \rho_{\vec{p}}[\vec{x}, t] \rightarrow \begin{pmatrix} \rho_{\vec{p}}^{ee} & \rho_{\vec{p}}^{ex} \\ \rho_{\vec{p}}^{xe} & \rho_{\vec{p}}^{xx} \end{pmatrix}$

$$(\partial_t + \vec{p} \cdot \nabla_{\vec{x}}) \rho_{\vec{p}} = \underbrace{\mu \int \frac{d^3 \vec{q}}{(2\pi)^3} (1 - \cos \theta_{\vec{p}\vec{q}}) \left[\rho_{\vec{q}} - \bar{\rho}_{\vec{q}}, \rho_{\vec{p}} \right]}_{\text{Neutrino Self-interaction}}$$

Coupling Nonlinearity

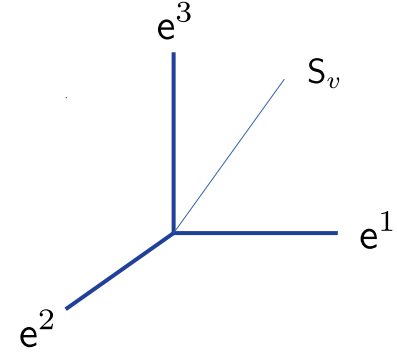
$$\mu = \sqrt{2} G_F n_\nu$$

FFC : Review

Phase Space : 1 space + 1 vel + 1 time dimension (Two Flavors)

Two Flavors :
$$\rho_{E,v} = \frac{\text{Tr}[\rho_{E,v}]}{2} \mathbb{I}_{2 \times 2} + \frac{g_{E,v}}{2} S_v \cdot \sigma$$

$$g_{E,v} = \begin{cases} f_{\nu_e}(E, v) - f_{\nu_\mu}(E, v), & E > 0 \\ -f_{\bar{\nu}_e}(E, v) - f_{\bar{\nu}_\mu}(E, v), & E < 0 \end{cases}$$



ELN distribution :
$$G_v = \sqrt{2} G_F \int_0^\infty \frac{dE E^2}{2\pi^2} [f_{\nu_e}(E, v) - f_{\bar{\nu}_e}(E, v)]$$

1 dimensional box of length L with periodic boundary conditions

Neutrinos emitted from each point in the box with velocity

Solve this equation :
$$(\partial_t + v \partial_x) S_v = \mu \int_{-1}^1 dv' G_{v'} (1 - vv') S_{v'} \times S_v$$

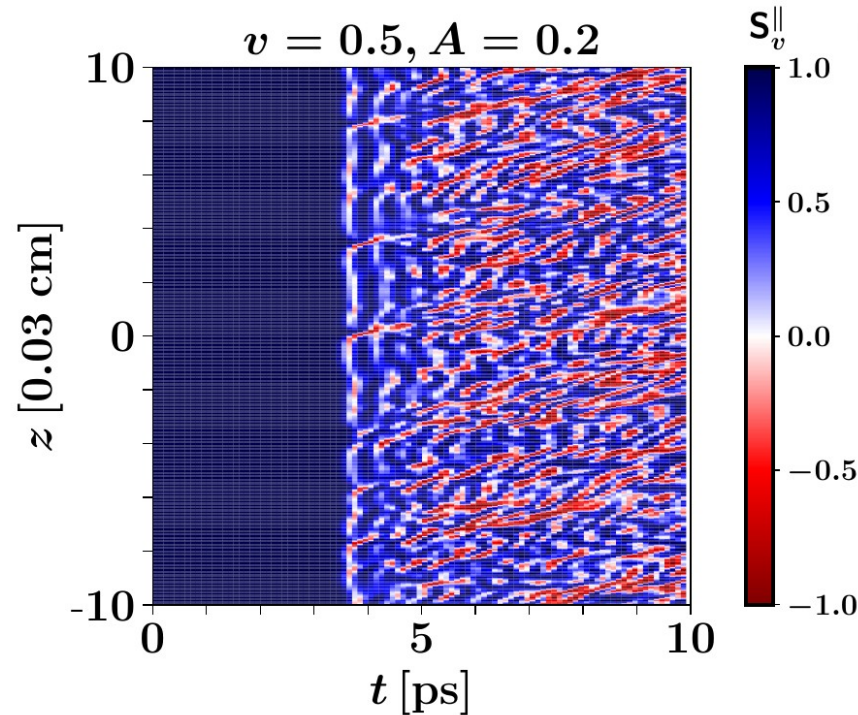
Fast Flavor Conversion : Review

Survival probability starts at 1
Oscillates coherently for few cycles
Finally reaches a thermalized state

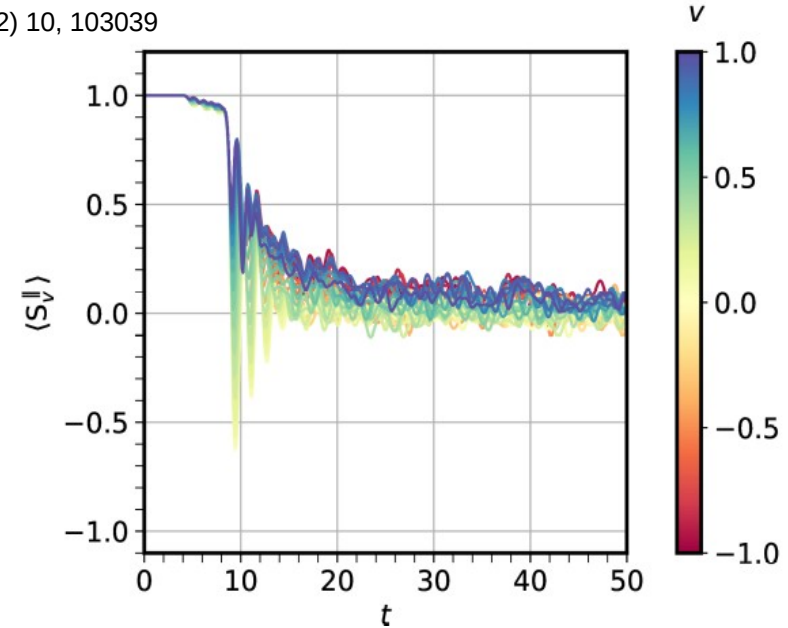
“Coarse-graining”



System shows irreversibility in time
The bloch vector shows tendency to flip over
Flavor depolarization occurs



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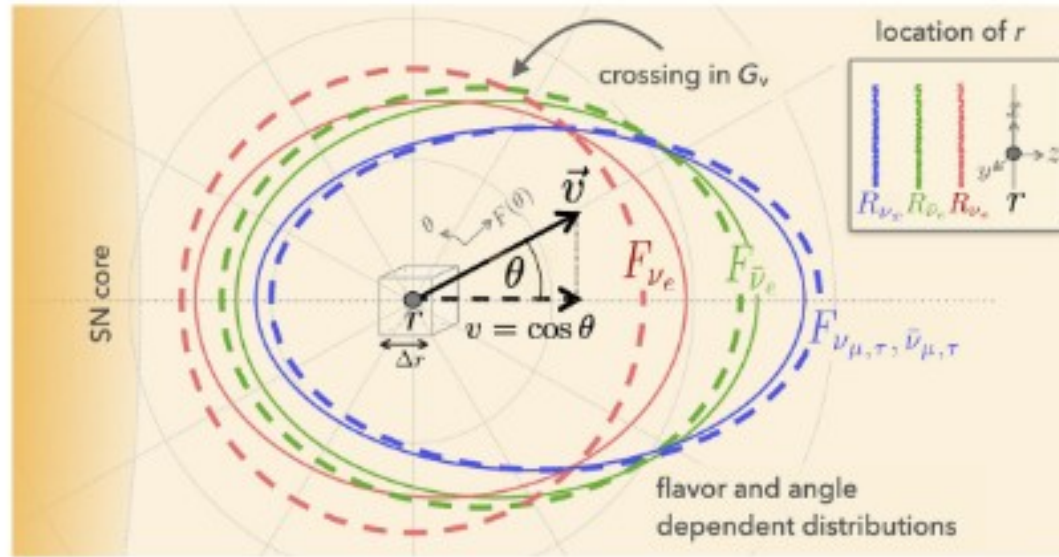
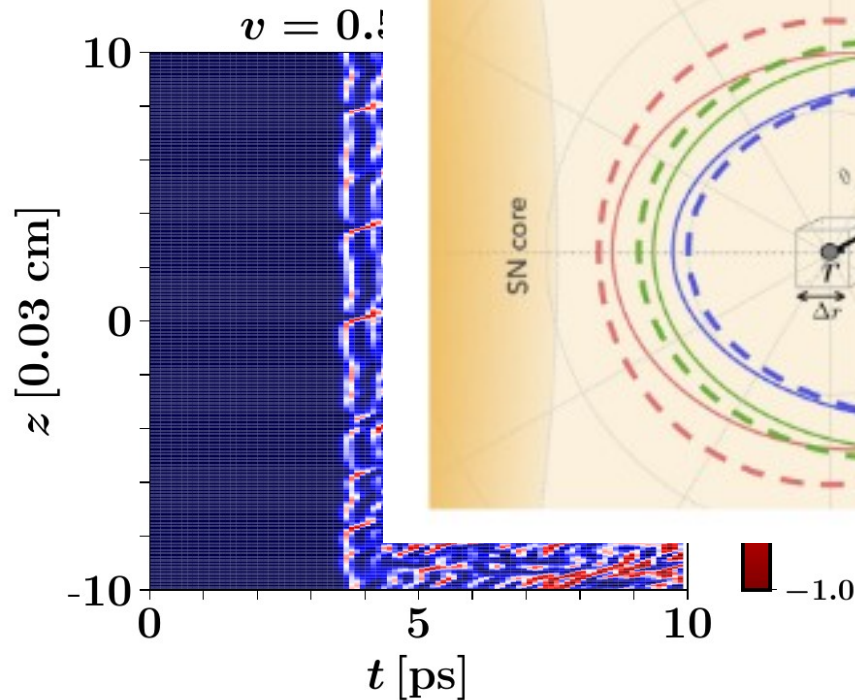
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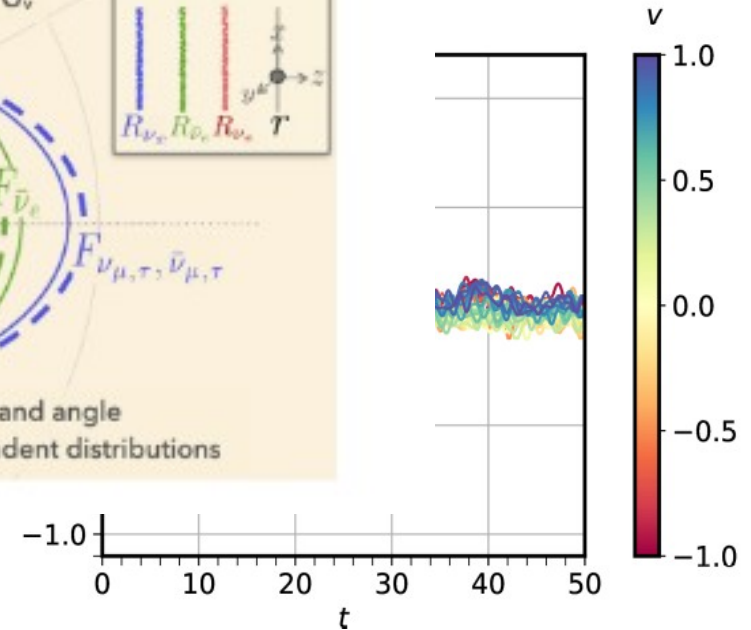
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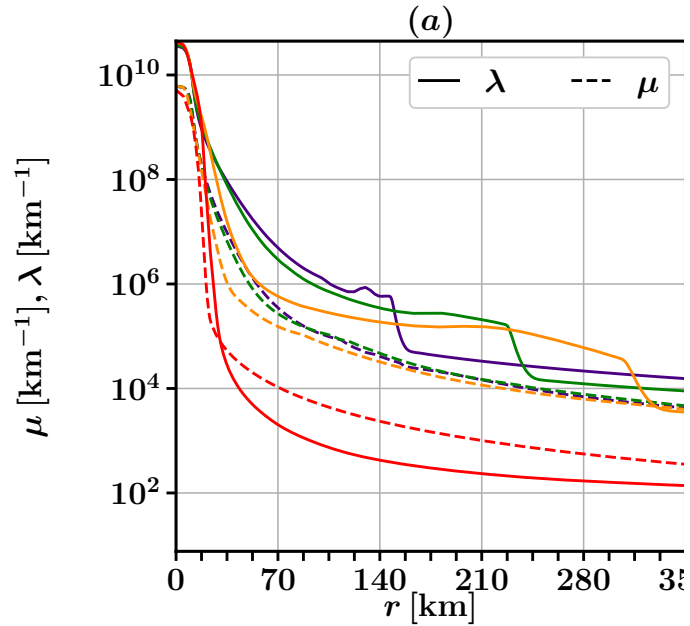
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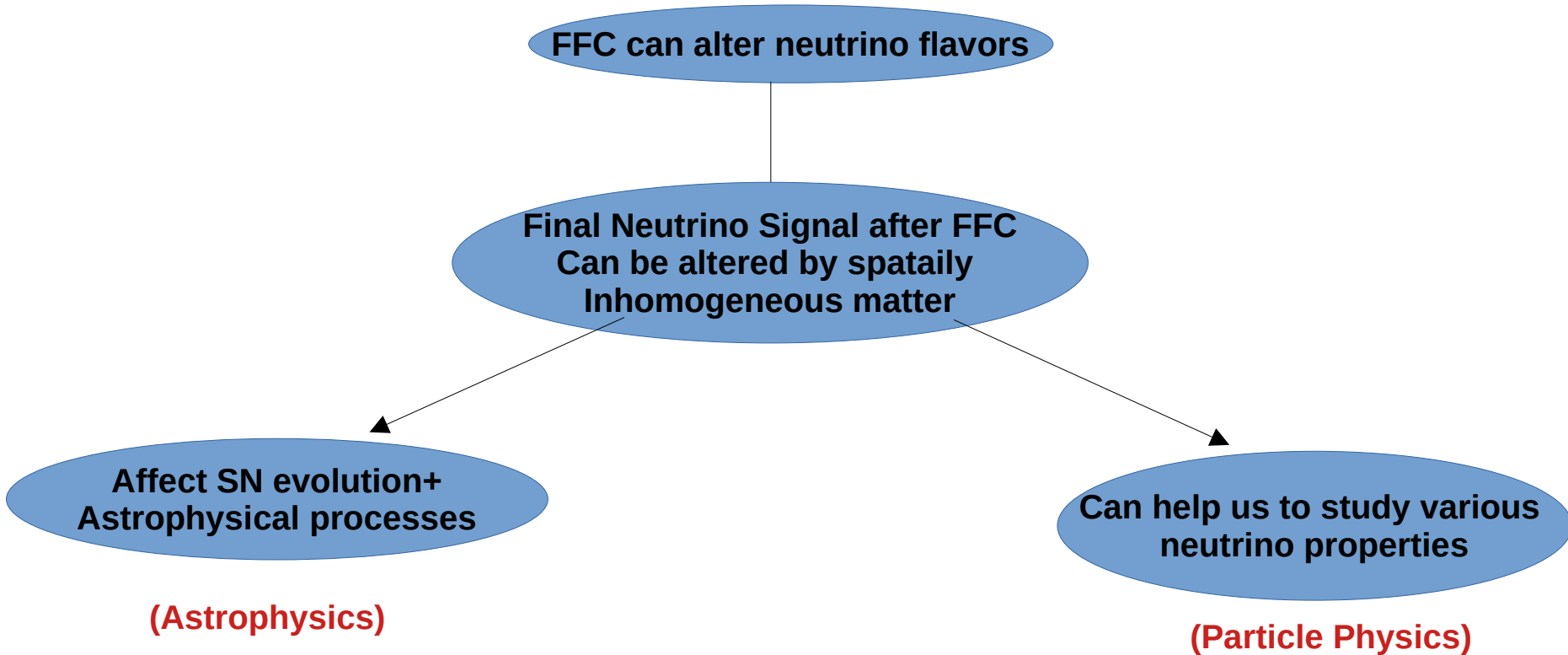
Fast Flavor Conversion : Spatailly inhomogenous matter



$$\lambda = \sqrt{2}G_F n_e$$

$$\lambda \gg \mu, \frac{d\lambda}{dr} \gg \frac{d\mu}{dr}$$

Why Studying it important ?



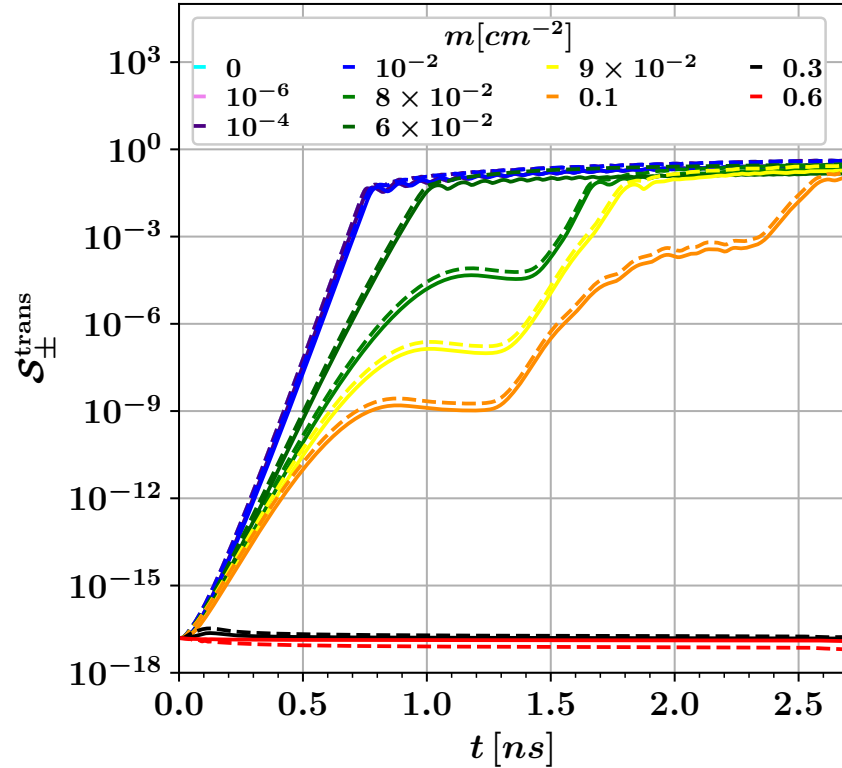
EOM

$$(\partial_t + \partial_z)\mathbf{S}_+ = (\lambda - 2\mu\alpha\mathbf{S}_-) \times \mathbf{S}_+,$$

$$(\partial_t - \partial_z)\mathbf{S}_- = (\lambda + 2\mu\mathbf{S}_+) \times \mathbf{S}_-.$$

$$\mathcal{S}_{\pm}^{\text{trans}}[t] = \frac{1}{\Delta z} \int_{-\frac{\Delta z}{2}}^{\frac{\Delta z}{2}} dz |\mathbf{S}_{\pm}^{\perp}[z, t]|$$

$$\lambda = m(z + 90) \text{ cm}^{-1}$$



Effect of Inhomogeneous Matter potential : Initial Evolution

$$Q_{\pm}[k, t] = \int dz, e^{i(\lambda[z]t - kz)} (\mathbf{S}_{\pm}[z, t] \cdot (\hat{\mathbf{e}}_1 - i\hat{\mathbf{e}}_2))$$

↓ $S_{\pm}^{\parallel}[z, t] = 1$ approximation

$$Q_{\pm}[k, t] \sim Q_{\pm}[k, 0] e^{-i\mu(1-\alpha)t - i \int_0^t \kappa[t] dt}$$

↓

$$\text{Im} \left[\int_0^t \kappa[t] dt \right] = \frac{(-k_{\text{eff}}[t] + \mu(1 + \alpha)) \text{Im}[\kappa[t]] + 4\alpha\mu^2 \theta[t]}{2m}$$

$$\kappa[t] = \sqrt{k_{\text{eff}}^2 - 2(1 + \alpha)\mu k_{\text{eff}} + \mu^2(1 - \alpha)^2}$$

$$k_{\text{eff}} = k - mt$$

$$\theta[t] = \begin{cases} \tan^{-1} \left(\frac{\text{Im}[\kappa[t]]}{k_{\text{eff}}[t] - \mu(1 + \alpha)} \right), & \text{if } k_{\text{eff}}[t] > \mu(1 + \alpha), \\ \tan^{-1} \left(\frac{\text{Im}[\kappa[t]]}{k_{\text{eff}}[t] - \mu(1 + \alpha)} \right) + \pi, & \text{if } k_{\text{eff}}[t] < \mu(1 + \alpha). \end{cases}$$

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Effect of Inhomogeneous Matter potential : Initial Evolution

$$\kappa[t] \xrightarrow{k \rightarrow k + mt} \kappa[t]$$

$$k_{\text{eff}}[t] \xrightarrow{k \rightarrow k + mt} k_{\text{eff}}[t]$$

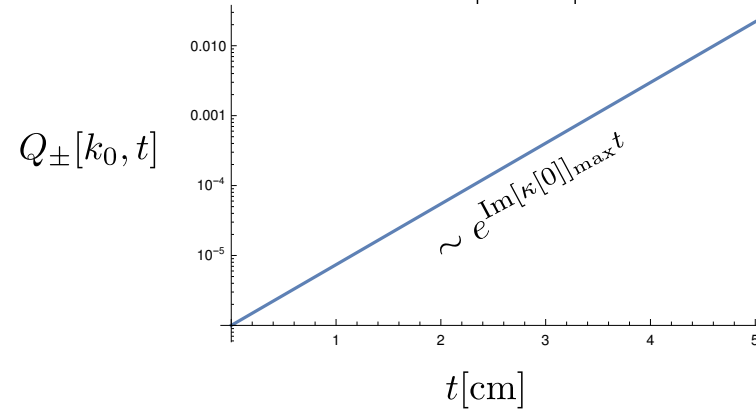
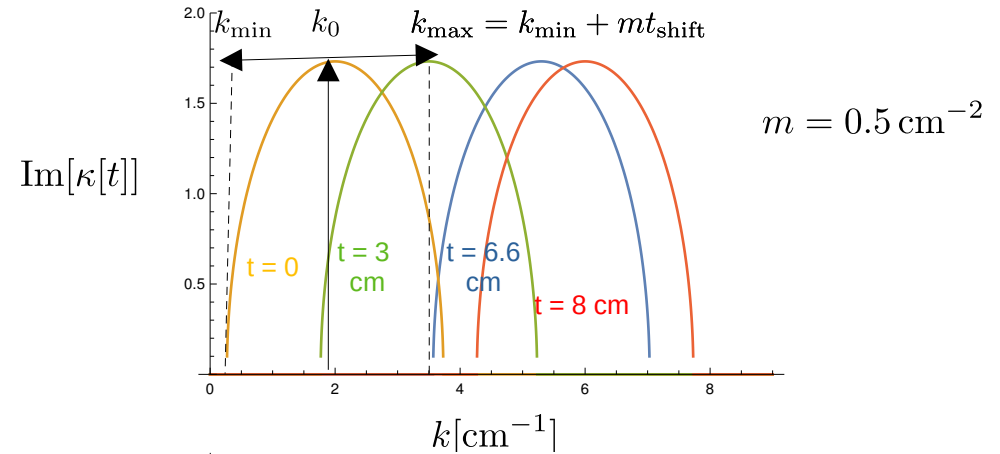
$$r_s \sim \frac{1}{t_{\text{shift}}} = \frac{m}{k_{\text{max}} - k_{\text{min}}}$$

Shift rate of the
intsabikity map

$$\text{Im}[\kappa[0]]_{\text{max}}$$

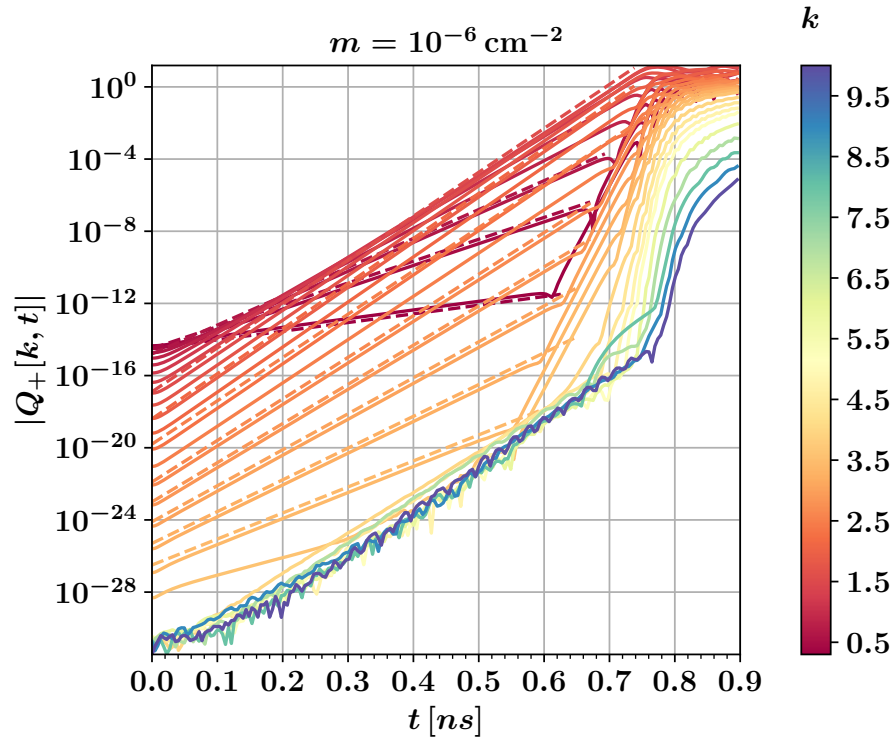
Growth rate of the maximum
growing fourier mode

$$\gamma = \frac{\text{Im}[\kappa[0]]_{\text{max}}}{r_s} \begin{cases} \gamma \gg 1 & \text{Flavor Conversion, Minimum effect} \\ \gamma \sim O(1) & \text{(Critical Value)} \\ \gamma \ll 1 & \text{No Flavor Conversion, Maximum effect} \end{cases}$$



Effect of Inhomogeneous Matter potential : Initial Evolution

“Small” m

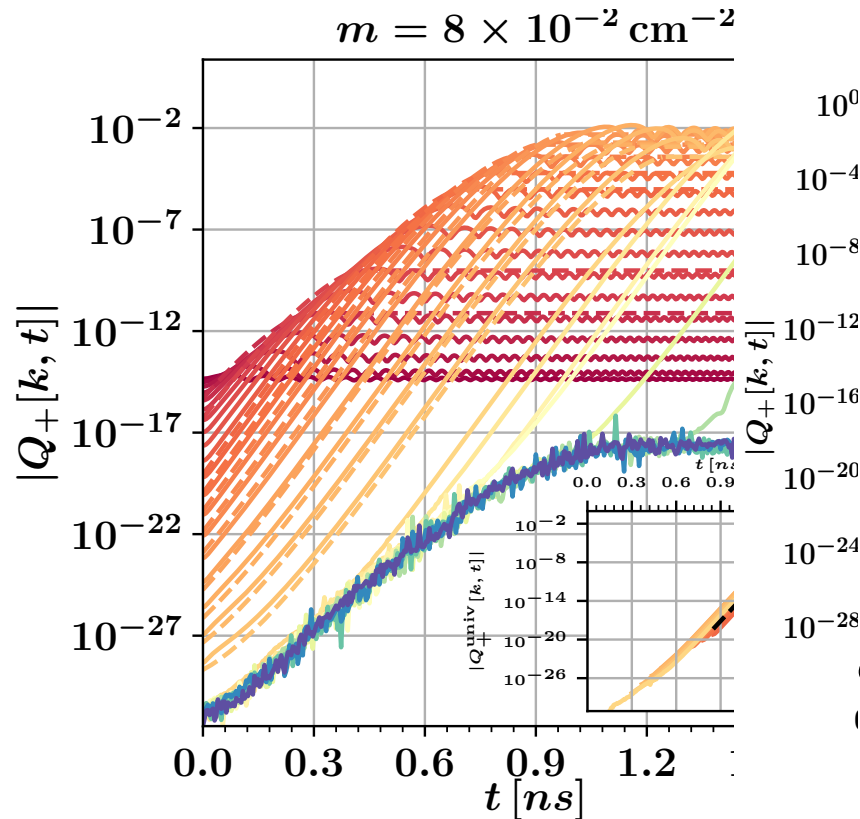


$$\gamma \sim 3 \times 10^6, r_s = 19 \times 10^{-6} \text{ ns}^{-1} \ll \beta = 58 \text{ ns}^{-1}$$

- No shift of the instability map
- Independent evolution of each Fourier mode
- Similar to the case without matter potential

Effect of Inhomogeneous Matter potential : Initial Evolution

“Intermediate” m



$$\gamma \sim 37.5, r_s = 0.03\beta, \beta = 58 \text{ ns}^{-1}$$

$$k > k_0 = 2 \text{ cm}^{-1}$$

Unstable, with
Similar growth
rates

$$k < k_0 = 2 \text{ cm}^{-1}$$

Stable

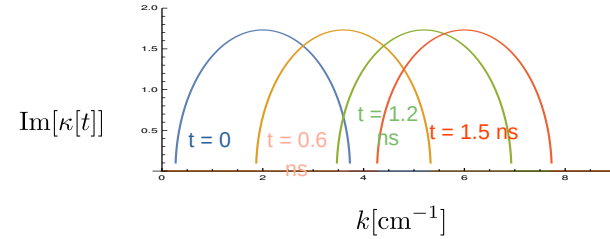
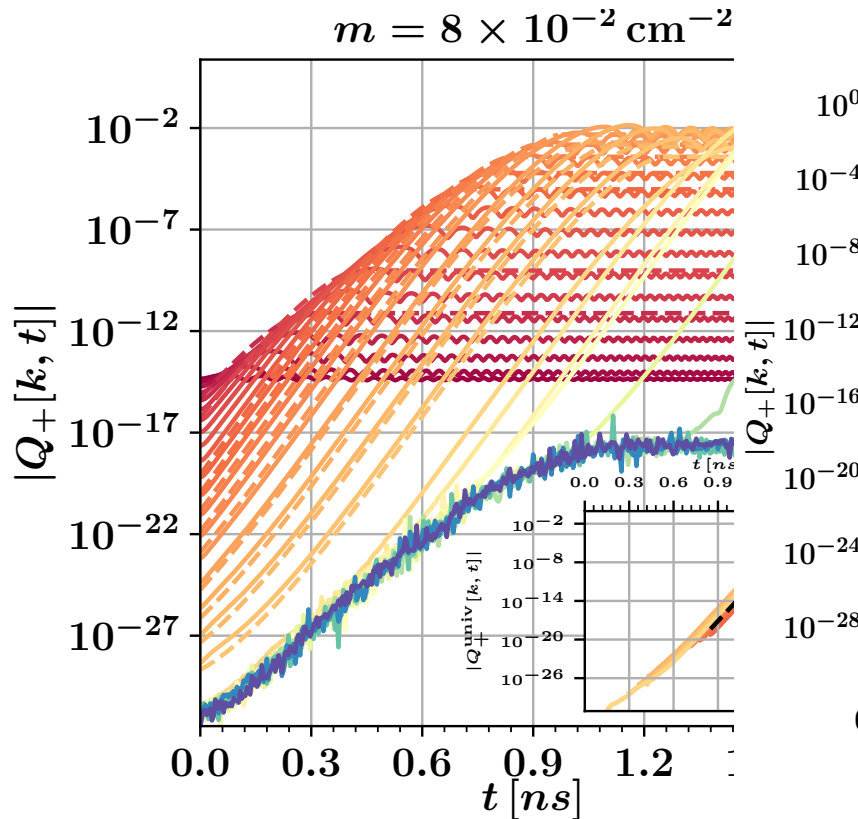
$$k \rightarrow k + mt \quad \text{symmetry}$$

$$Q[k + \delta k, t + \frac{\delta k}{m}] = Q[k, t] = Q^{\text{univ}}[k, t], \delta k = 4.4 - k, k > 2 \text{ cm}^{-1}$$

$k > 2 \text{ cm}^{-1}$ modes overlap

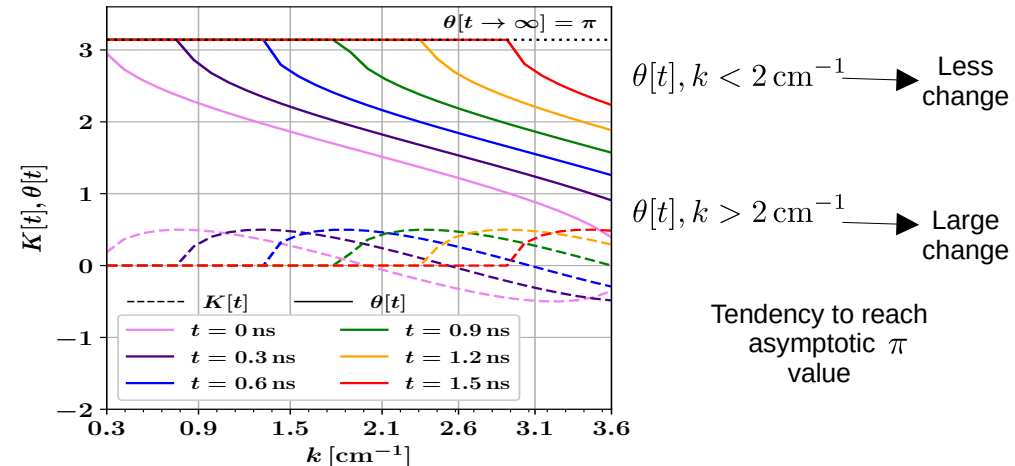
Effect of Inhomogeneous Matter potential : Initial Evolution

“Intermediate” m



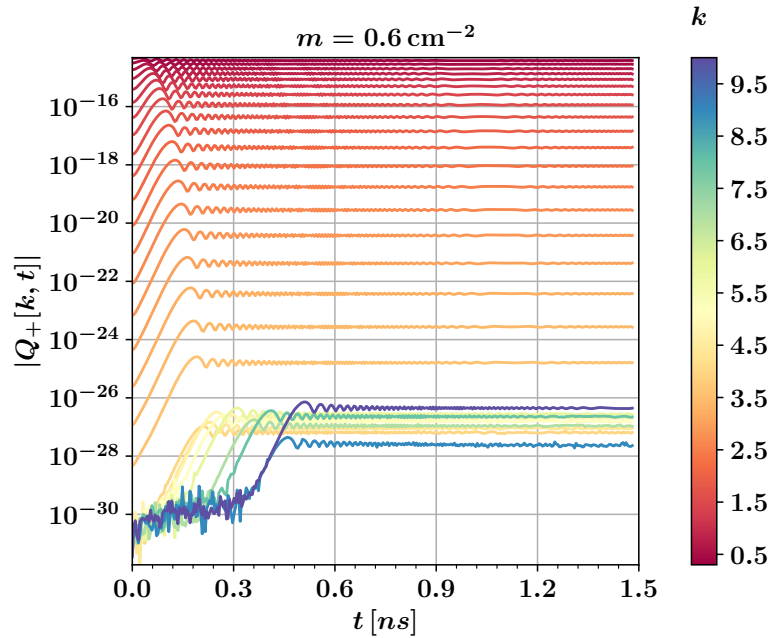
$$\text{Im} \left[\int_0^t \kappa[t] dt \right] = \frac{4\alpha\mu^2}{2m} (\theta[t] + K[t]), K[t] \ll \theta[t]$$

$$|Q_{\pm}[t]| \sim |Q_{\pm}[0]| e^{\frac{4\alpha\mu^2\theta[t]}{2m}}.$$



Effect of Inhomogeneous Matter potential : Initial Evolution

“Large” m

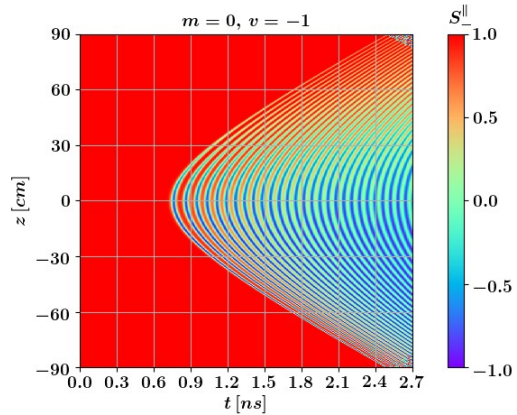


$$\gamma = 0.6, \beta = 58 \text{ ns}^{-1}, r_s = 95 \text{ ns}^{-1}$$

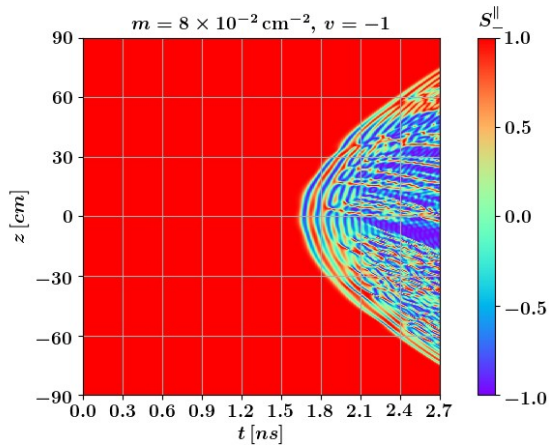
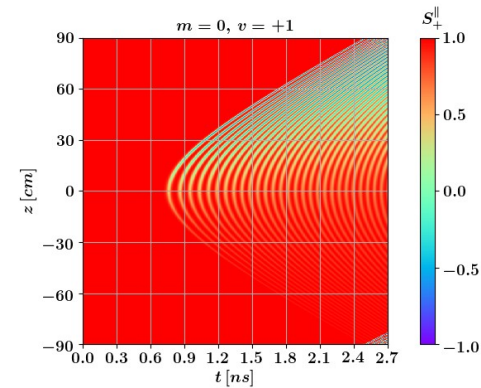
FFC completely suppressed !!

Slight initial growth due to finite r_s
corresponds to timescale of $O(1/r_s)$

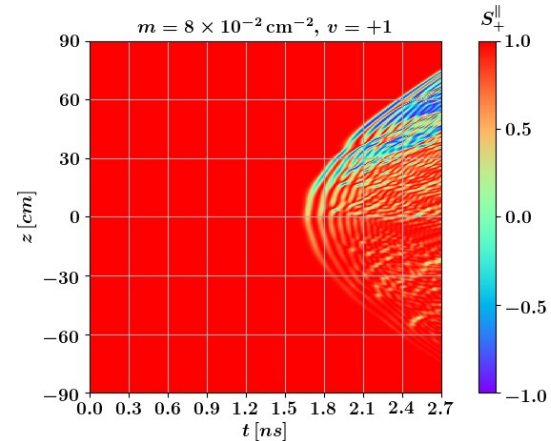
Effect of Inhomogeneous Matter potential : Nonlinear Evolution



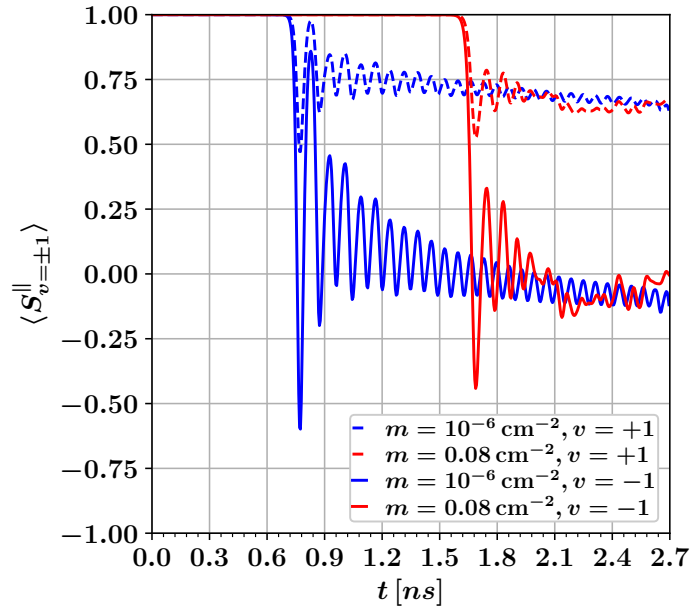
**Coherent
Wavelike
Behavior !!**



**No Coherent
Behavior !!
Small Scale
Structures !!
(Due to shift to high
fourier modes)**



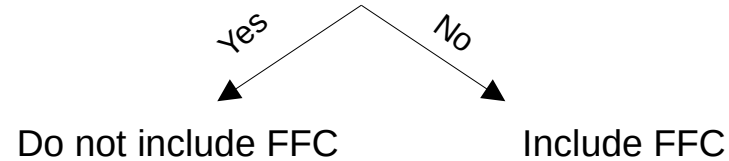
Incorporating the effect in SN simulation



Final Observable Signal after coarse-graining becomes independent of m !!

$$\gamma \sim O(1) \longrightarrow m_{\text{crit}} \sim O(4\mu^2\alpha) \quad (\text{Determined through initial conditions})$$

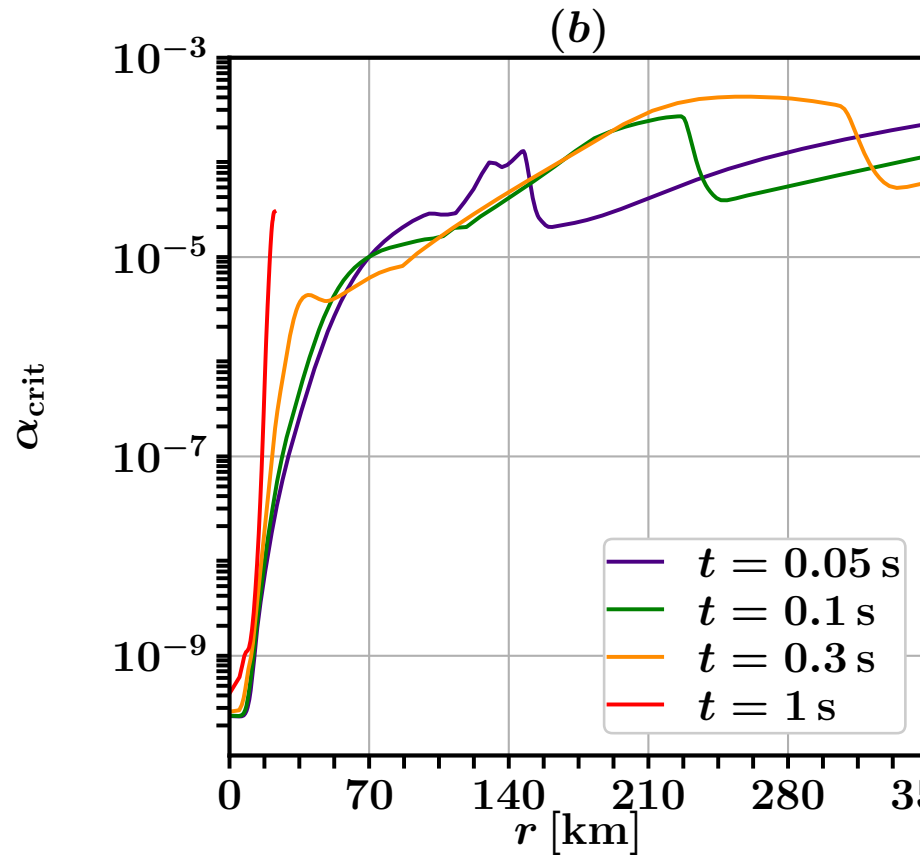
Check each pixel of a star if $m > m_{\text{crit}}$



$$F_{\nu_e, \nu_\mu}^{\text{fin}} = (1 - f_{\vec{p}}^D) F_{\nu_e, \nu_\mu}^{\text{ini}} + f_{\vec{p}}^D F_{\nu_\mu, \nu_e}^{\text{ini}}$$

Same as Flavor depolarization

A Realistic Scenario



$\alpha < \alpha_{crit}$ $\longrightarrow$ Matter Suppression

$\alpha > \alpha_{crit}$ $\longrightarrow$ No Matter Suppression