Fast Neutrino-Flavor Conversion with Attenuation and Global Lepton Gradient

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in prep.

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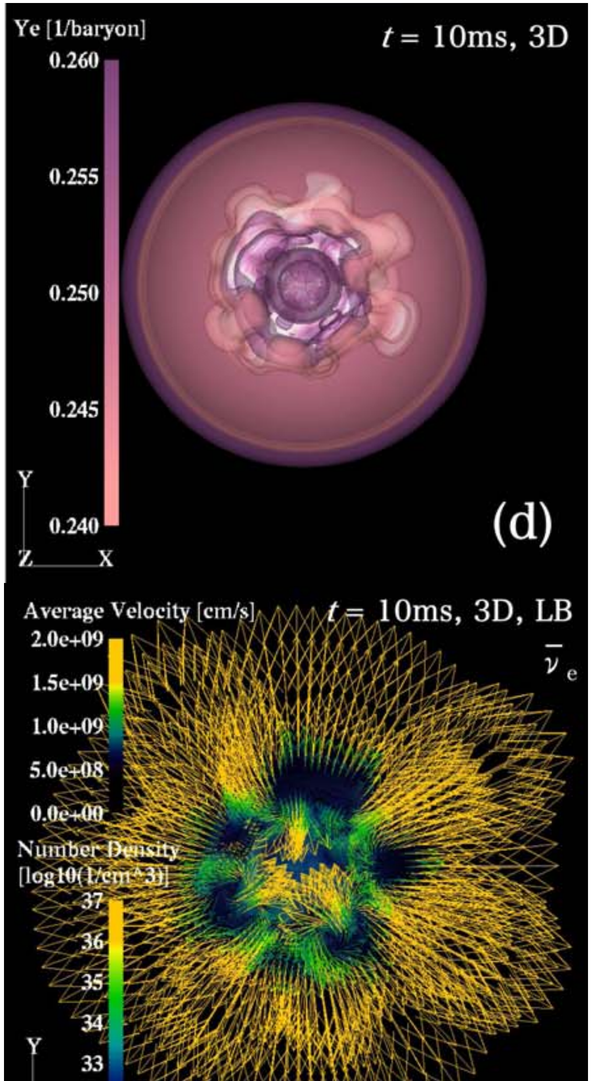
Collective Neutrino Oscillations in Supernovae and Neutron Star Mergers

Mar. 23-27, 2026 @Academia Sinica, Taipei

Neutrino Physics in CCSN & BNSM

Core-collapse supernova (CCSN)



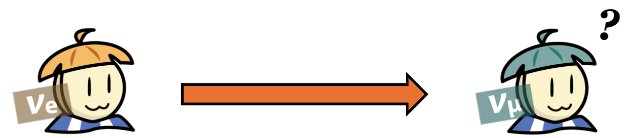
Iwakami+ ApJ '20

Neutrinos: coupling weakly with matters

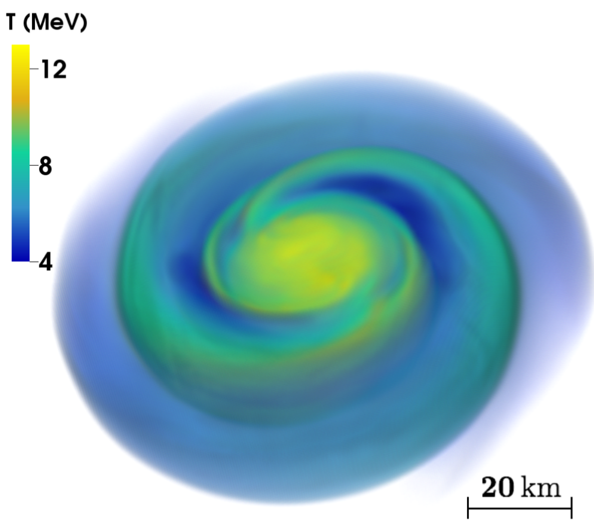
→ As a ***mediator*** for thermal energy/momenta/leptons

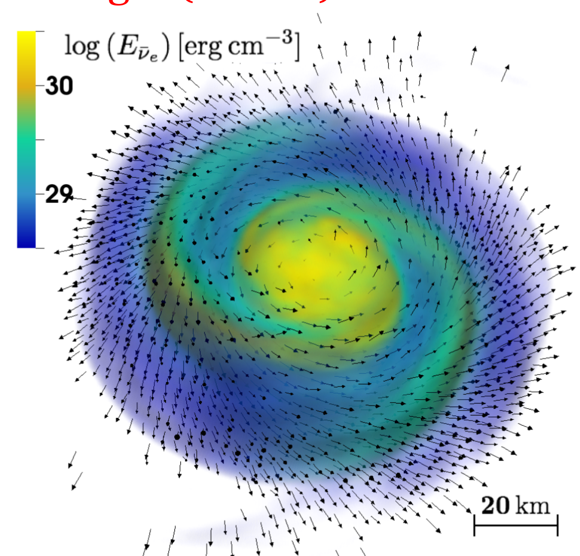
→ *One of the uncertainties is **"Neutrino Oscillation"***

higgstan.com



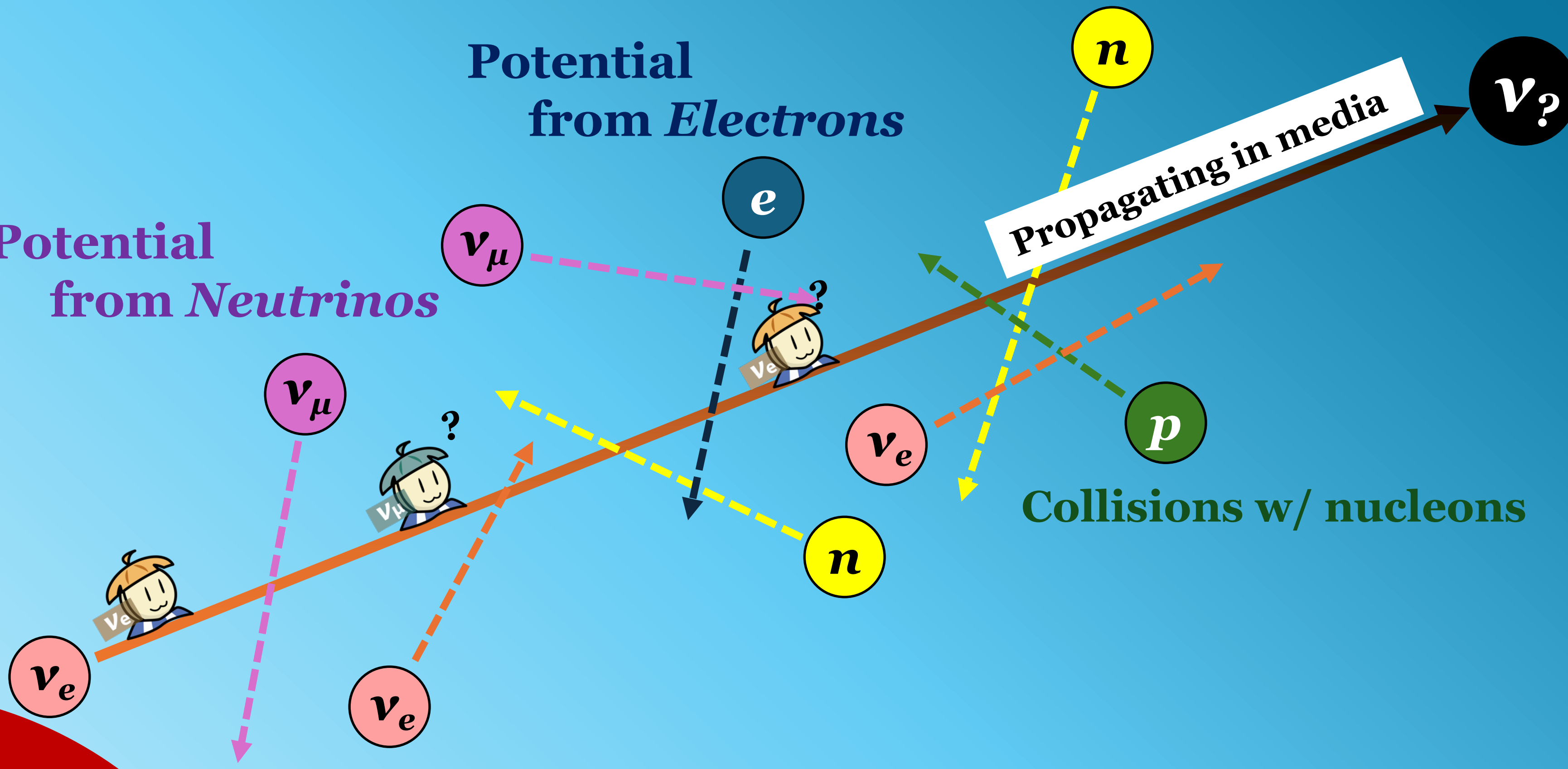
Binary Neutron Star Merger (BNSM)

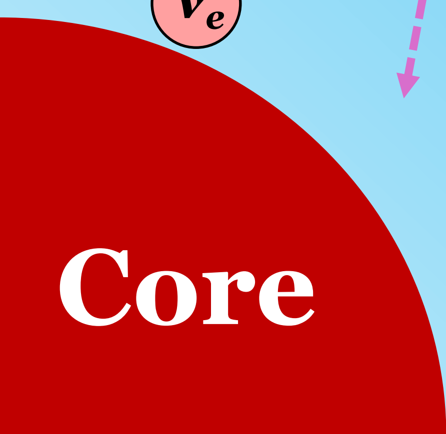




Foucart+ PRD '16

Sea of Leptons & Nucleons





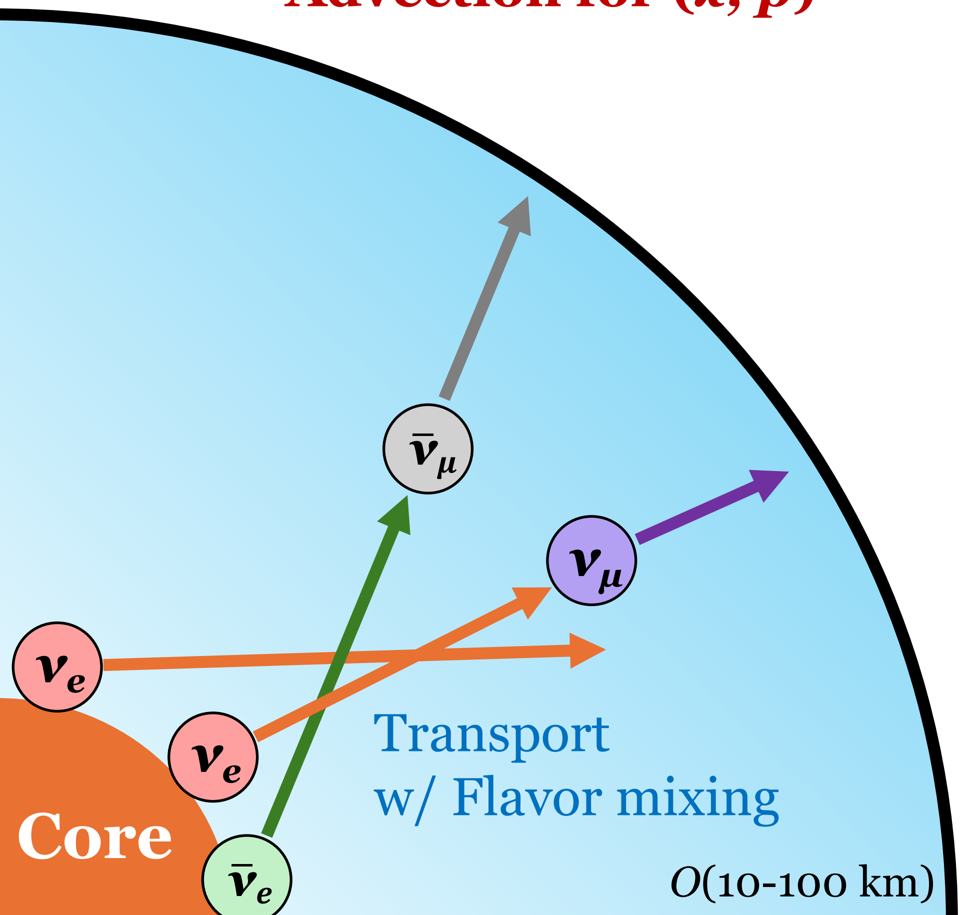
Neutrino Self-interaction ⇨ **Transport + Non-linear Flavor Conversion**

Pantaleone PLB '92

Quantum Kinetic Neutrino Transport

Beyond Boltzmann eq.:

$$\underbrace{\left(p^{\mu}\frac{\partial}{\partial x^{\mu}} - \Gamma^{j}_{\mu\nu}p^{\mu}p^{\nu}\frac{\partial}{\partial p^{j}}\right)\rho}_{\text{Advection for } (x,\,p)} = -\mathrm{i}\left[\mathcal{H}_{\rm osc},\ \rho\right] + \underbrace{\mathcal{C}[\rho]}_{\text{Collisions}}$$



Quantum Kinetic Neutrino Transport

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Advection for (*x*, *p*)

Neutrino oscillation

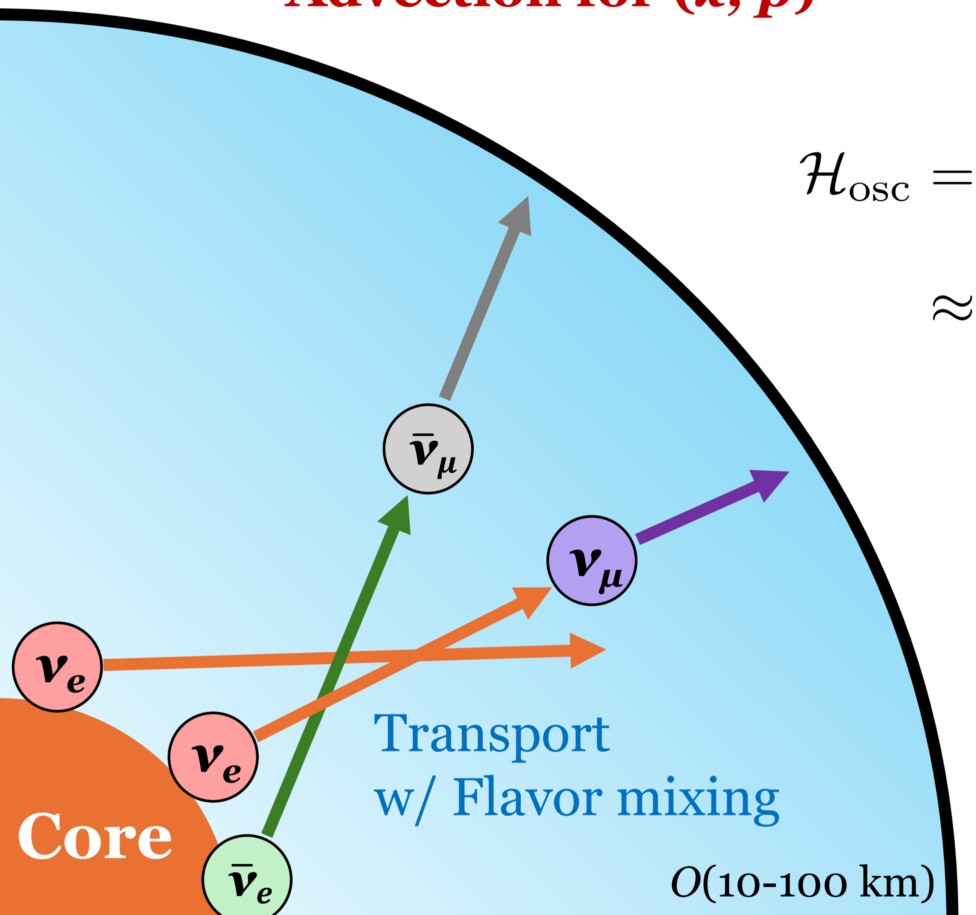
$$\mathcal{H}_{\mathrm{osc}} = \mathcal{H}_{\mathrm{vac}} + \mathcal{H}_{\mathrm{mat}} + \mathcal{H}_{\nu\nu}$$

$$\approx \frac{M^2}{2E_\nu} + \underbrace{\sqrt{2}G_{\mathrm{F}}\,n_e}_{\text{Electrons}} + \underbrace{\sqrt{2}G_{\mathrm{F}}\,n_\nu}_{\text{Neutrinos}}$$

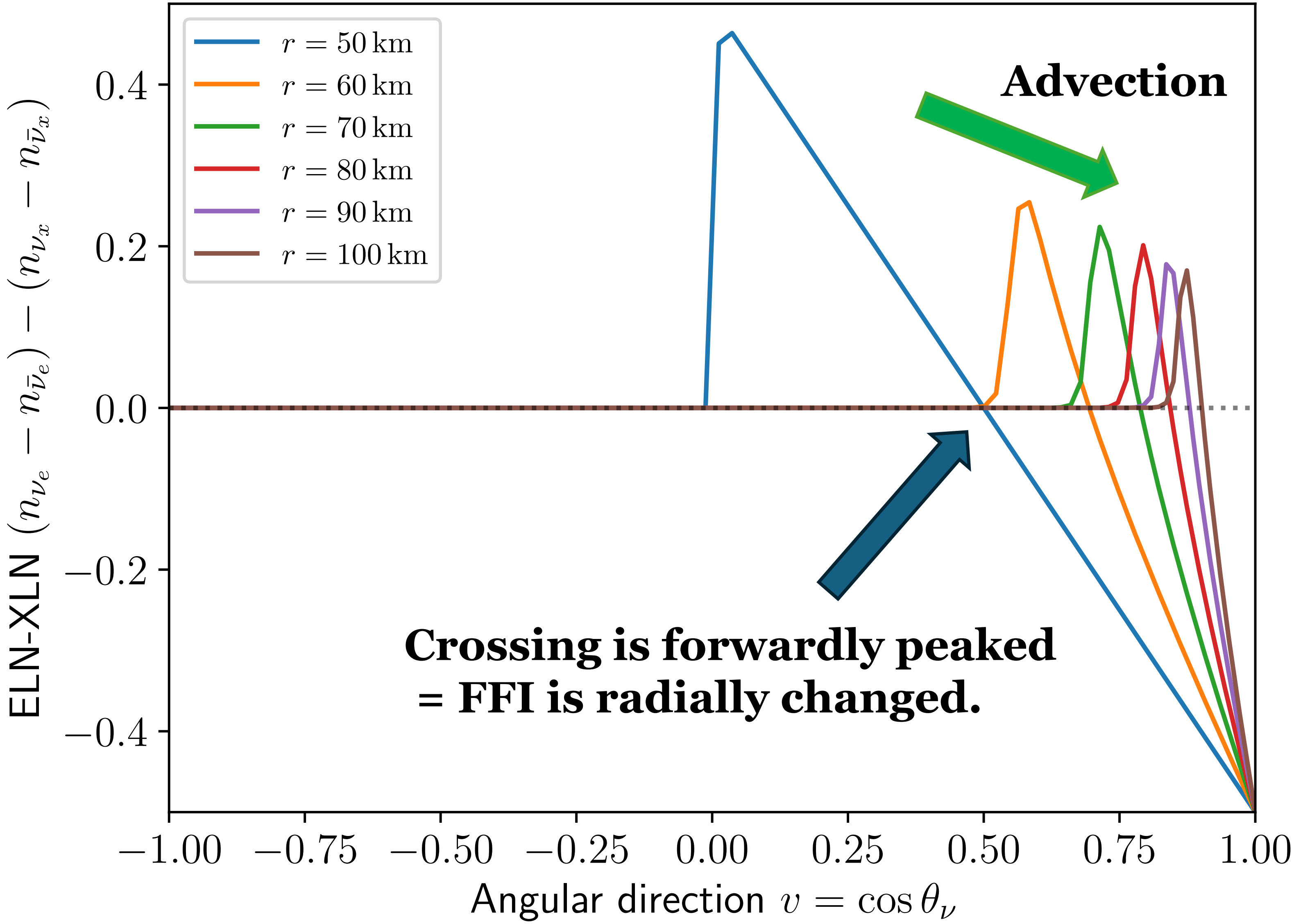
Electrons ***Neutrinos***

Radial profile

= *Refractive effects radially change.*

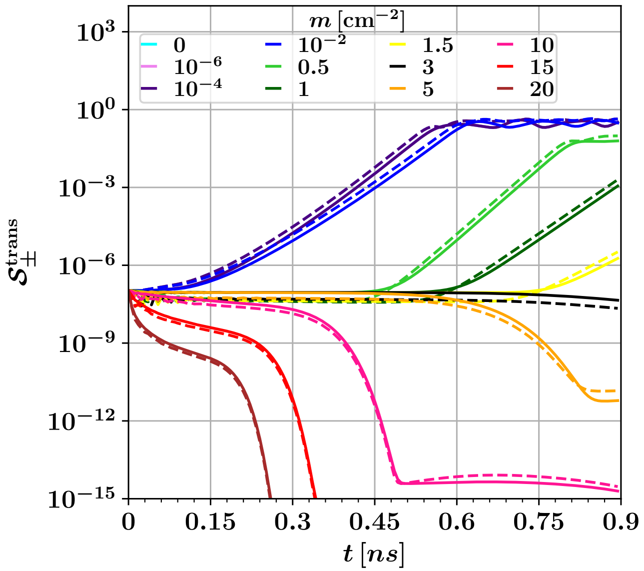


Global Effect on Neutrino Background



Global Effect on Matter Background

Bhattacharyya+ '25



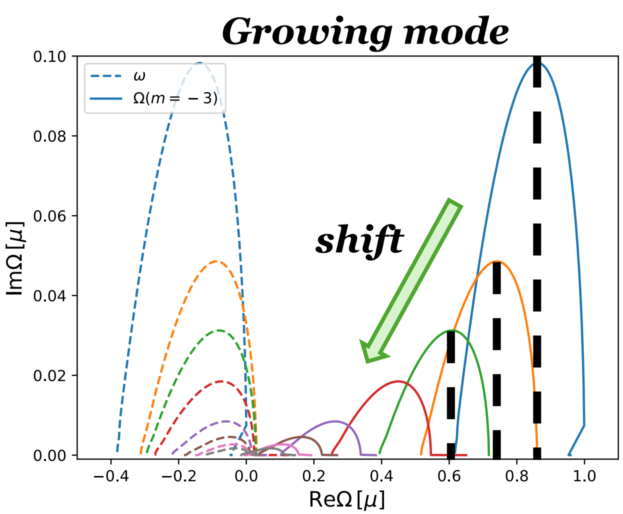
$$\mathcal{H}_{\rm osc} = \mathcal{H}_{\rm vac} + \mathcal{H}_{\rm mat} + \mathcal{H}_{\nu\nu}$$

$$\approx \frac{M^2}{2E_\nu} + \underline{\sqrt{2}G_{\rm F}\, n_e} + \underline{\sqrt{2}G_{\rm F}\, n_\nu}$$

Electrons ***Neutrinos***

Bhattacharyya+ (2504.11316) reported

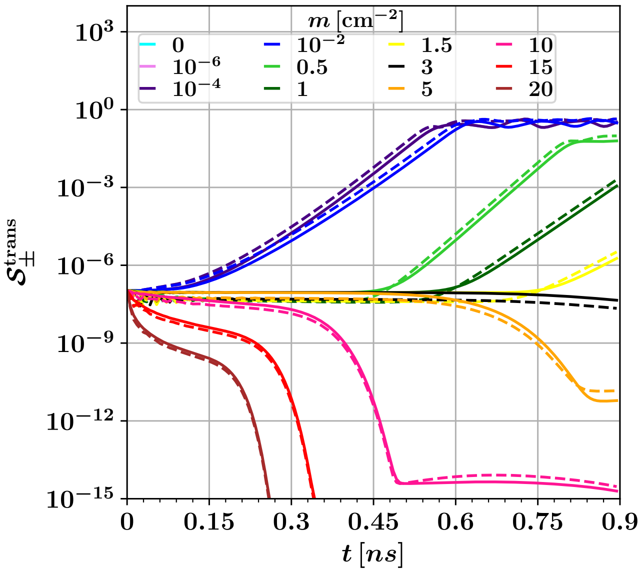
"Inhomogeneous matter can suppress the growth of flavor instability."



Competition between growth time vs. matter inhomogeneity

Global Effect on Matter Background

Bhattacharyya+ '25



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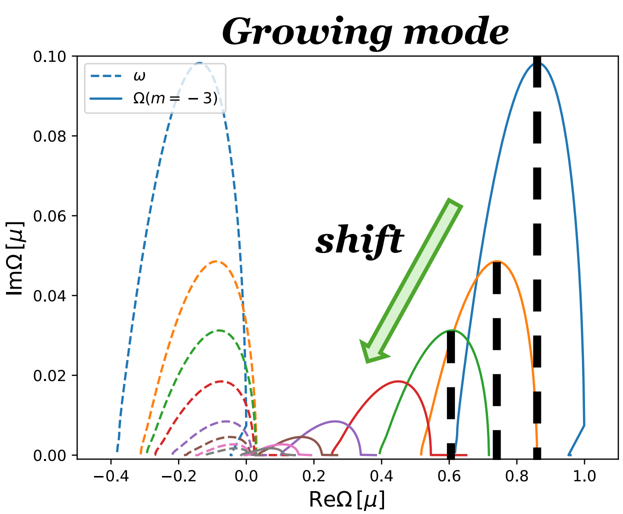


→ For global QKE w/ Dirichlet?

Global geometry forms

the radial gradient

for Background Leptons (e, ν).

Attenuation Technology for Global

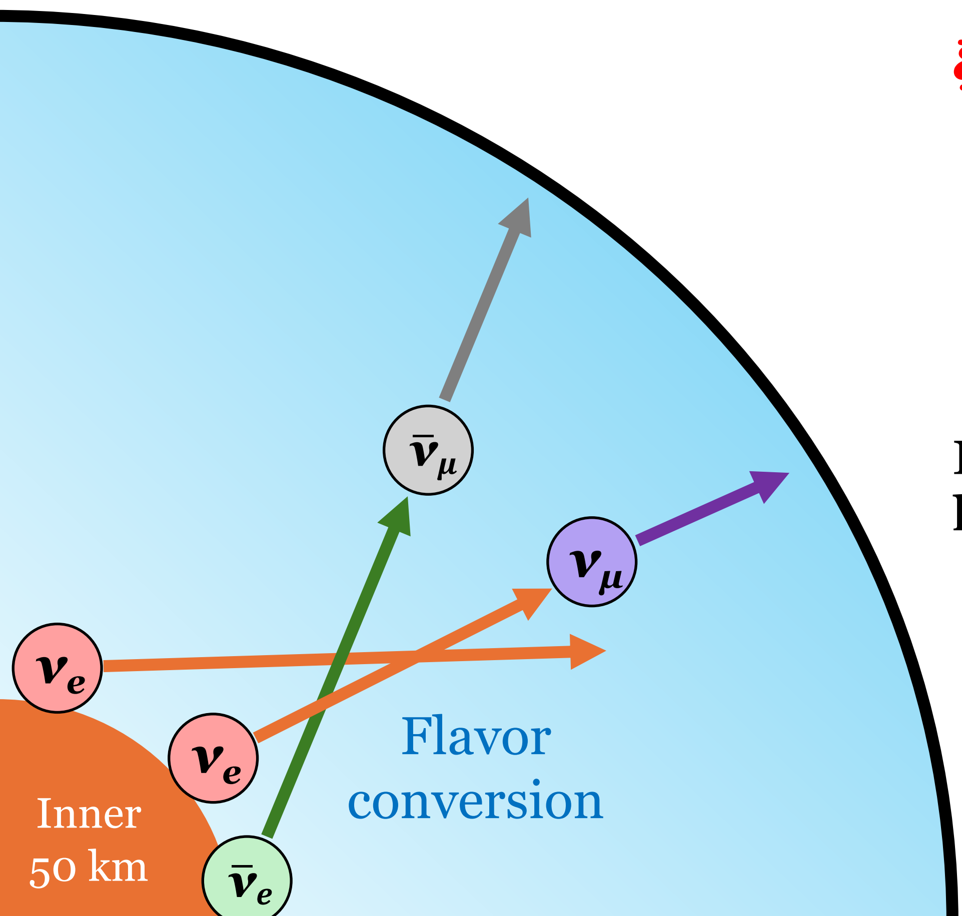
Quantum Kinetic Equation in 1D spherical symmetry ($r; E_\nu, \theta_\nu$):

$$\left(p^\mu \frac{\partial}{\partial x^\mu} - \Gamma^j_{\mu\nu} p^\mu p^\nu \frac{\partial}{\partial p^j}\right)\rho = -\mathrm{i}\,\xi\,[\mathcal{H}_{\rm osc},\ \rho]$$

$$0 \leq \xi \leq 1$$

ξ is an ***attenuation*** parameter handling oscillation scales.

Nagakura & MZ PRL '22
Xiong+ PRD '23
etc.



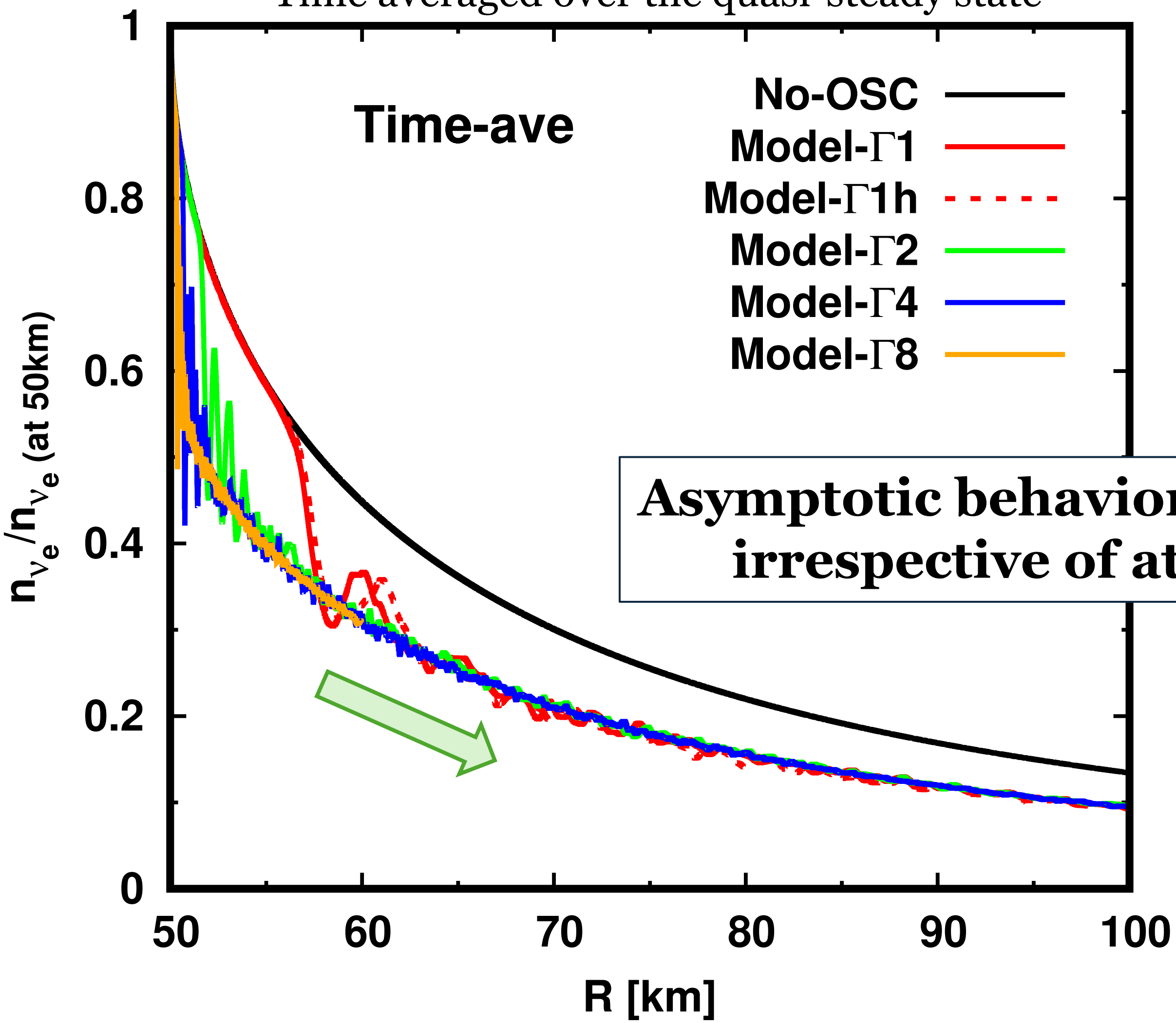
If the attenuated scales keep the hierarchy, it is acceptable.

Outer boundary r ~ 100 km

Successful Attenuation Technology

Attenuation Convergence in Spherically Symmetric Models

Time averaged over the quasi-steady state



Asymptotic behaviors are the same irrespective of attenuation!

Nagakura & MZ PRL '22

+ Background Matter Profile

Background matter profile: Radially declines w/ power index *m*.

$$\lambda(r) = \lambda_0 \left(\frac{r}{R_{\text{in}}} \right)^m$$

$$\lambda = \sqrt{2} G_F \, n_e$$
$$\mu = \sqrt{2} G_F \, n_{\nu_e}$$
$$\mu_0 = \mu(R_{\text{in}})$$

R_{in}: inner boundary **50 km**

$$n_{\nu_e} = 6 \times 10^{32} \text{g/cm}^3$$

Parameter setups

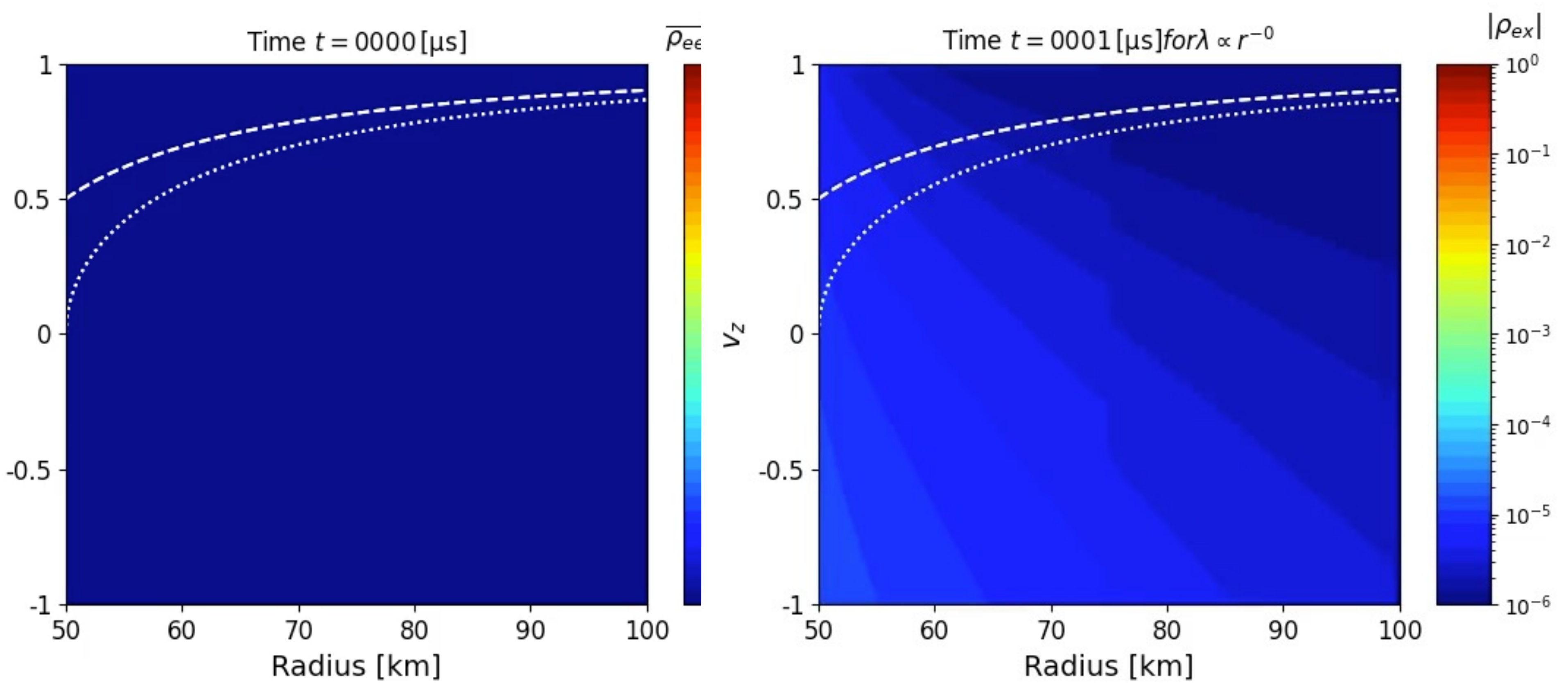
Attenuation: ξ	λ_o	***m***	***Domain*** ΔR **[km]**
10^{-4} (strong)	**1** μ_o	0	**50**
		-1	
		-3	
	4 μ_o	0	
		-1	
		-3	
10^{-2} (weak)	**1** μ_o	0	**0.5**
		-3	
	20 μ_o	0	
		-3	
	30 μ_o	0	
		-3	

Strong Attenuation: $\xi = 10^{-4}$

Strong attenuation & Flat Matter Profile:
Flavor conversion can occur.

$$\xi = 10^{-4}$$

$$\lambda(r) = \mu_0 \left(\frac{r}{R_{\rm in}}\right)^0 \propto r^0$$

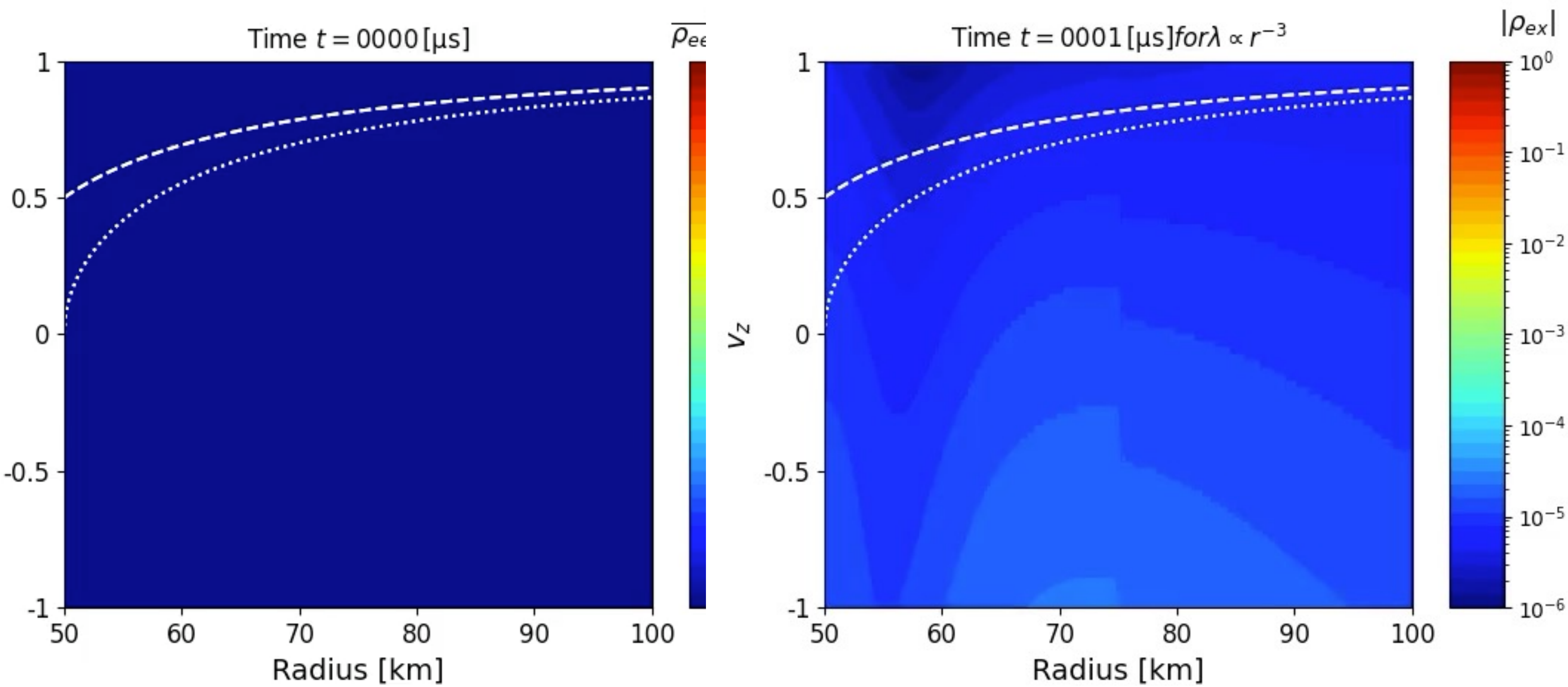


Strong Attenuation: $\xi = 10^{-4}$

Strong attenuation & Steep Matter Gradient:
Flavor conversion can be suppressed.

$$\xi = 10^{-4}$$

$$\lambda(r) = \mu_0 \left(\frac{r}{R_{\text{in}}} \right)^{-3} \propto r^{-3}$$



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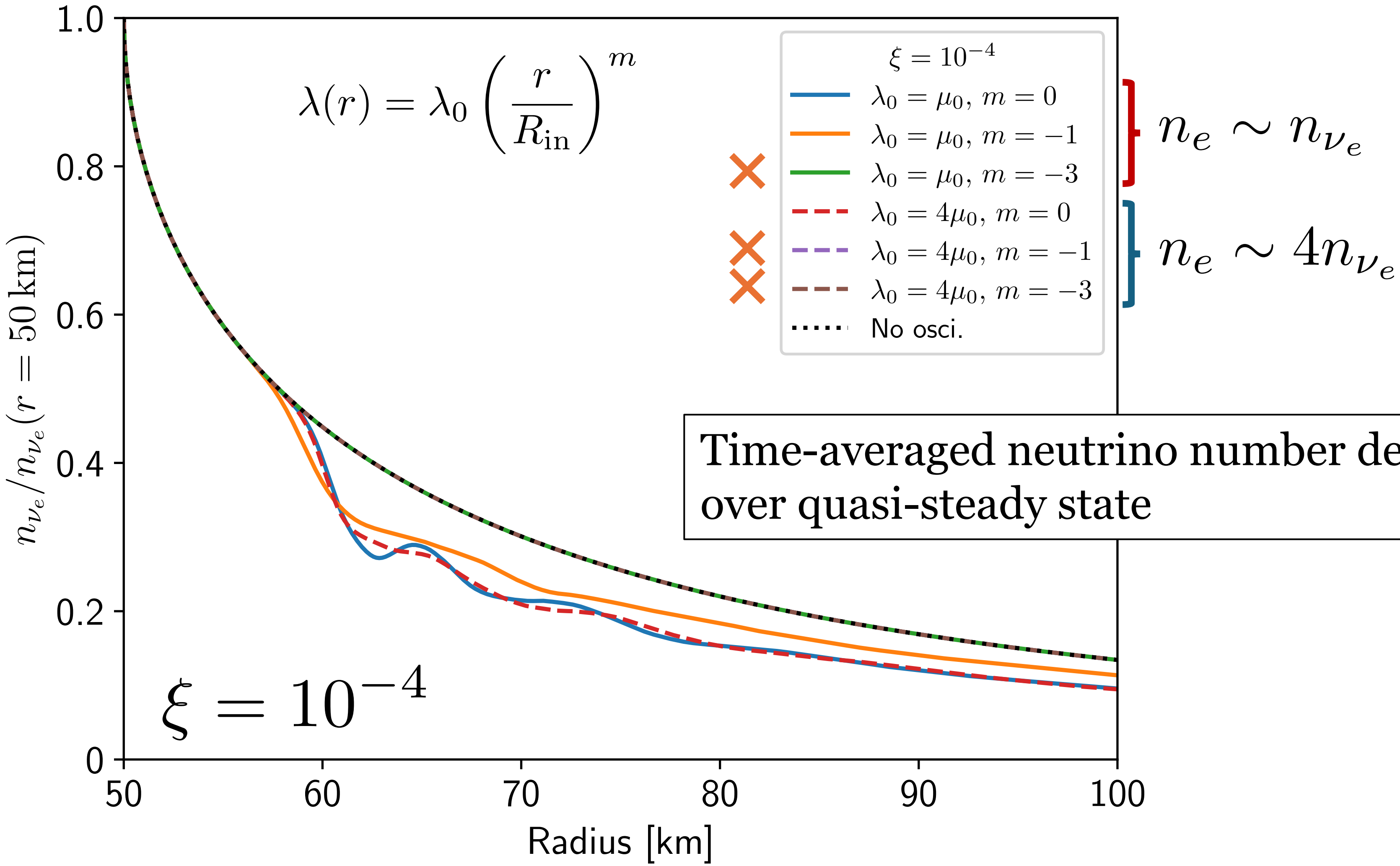


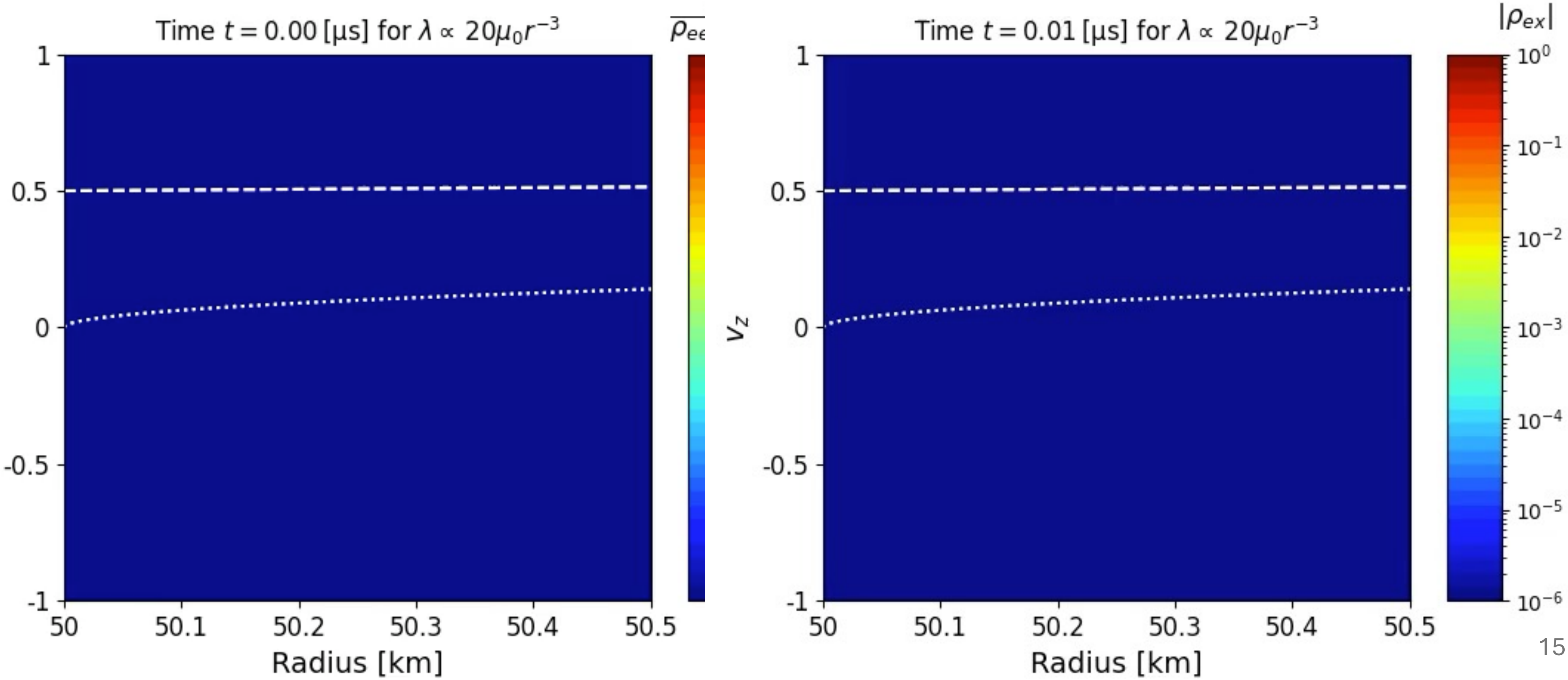
Time-averaged neutrino number density over quasi-steady state

Weak Attenuation: $\xi = 10^{-2}$

Weak attenuation & Steep Matter Gradient:
Flavor conversion can occur.

$$\xi = 10^{-2}$$

$$\lambda(r) = 20\mu_0 \left(\frac{r}{R_{\rm in}}\right)^{-3} \propto r^{-3}$$

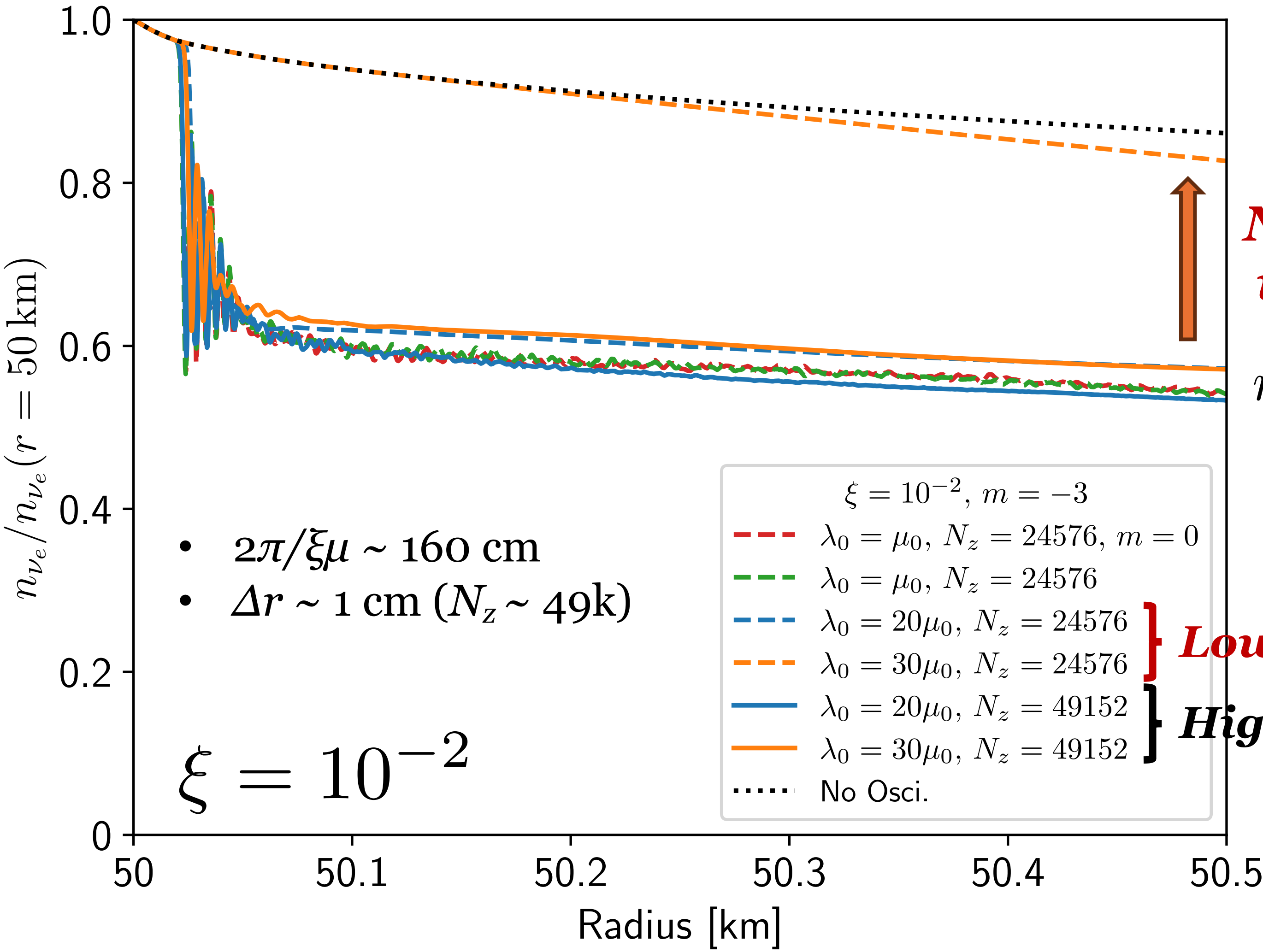


Weak Attenuation: $\xi = 10^{-2}$

Weak attenuation & Steep Matter Gradient:

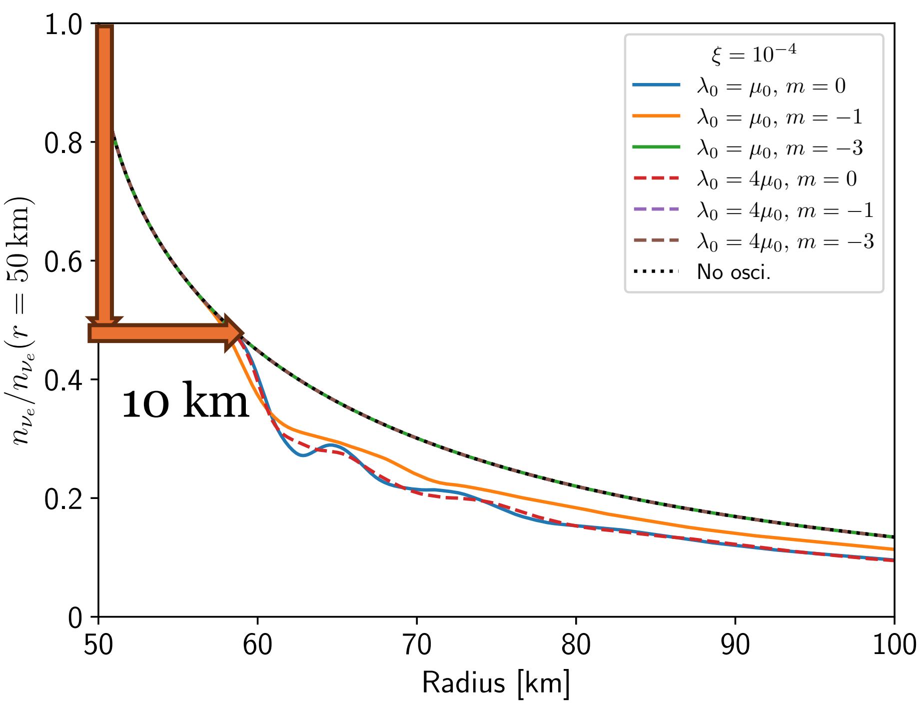
Flavor conversion can occur.

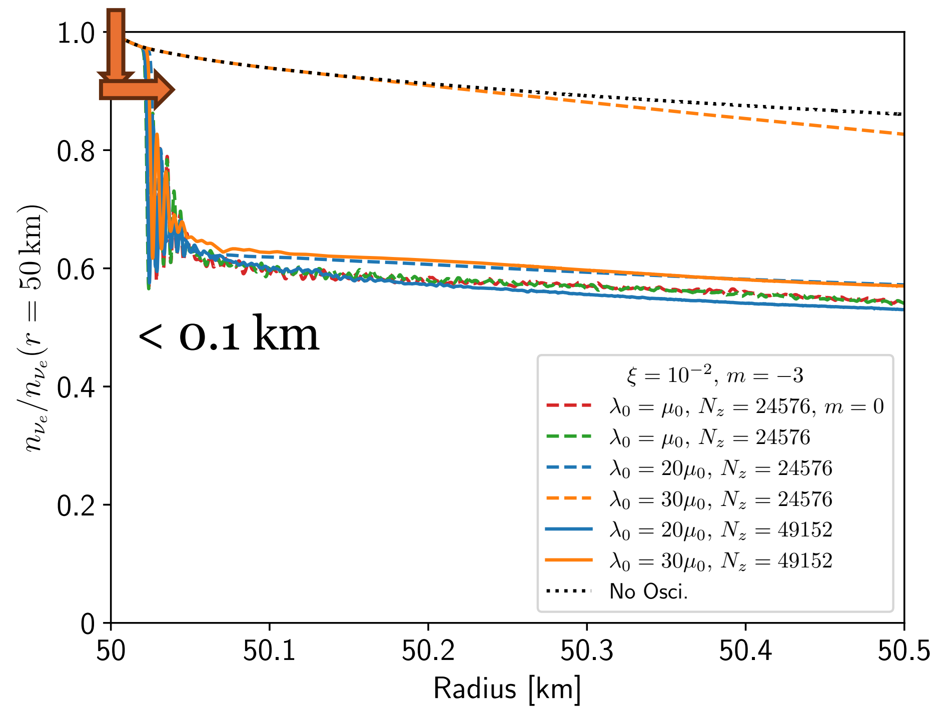
But a higher resolution is required to resolve shorter matter oscillations. $n_e \gg n_{\nu_e}$



Attenuation vs. Matter Gradient

Strong attenuation extends the time delay until FFC occurs.
→ Background significantly changes during it.
→ Attenuated Hamiltonian feels the variation heavily.





$\xi = 10^{-4}$

$\xi = 10^{-2}$

Growing mode vs. Background Shift

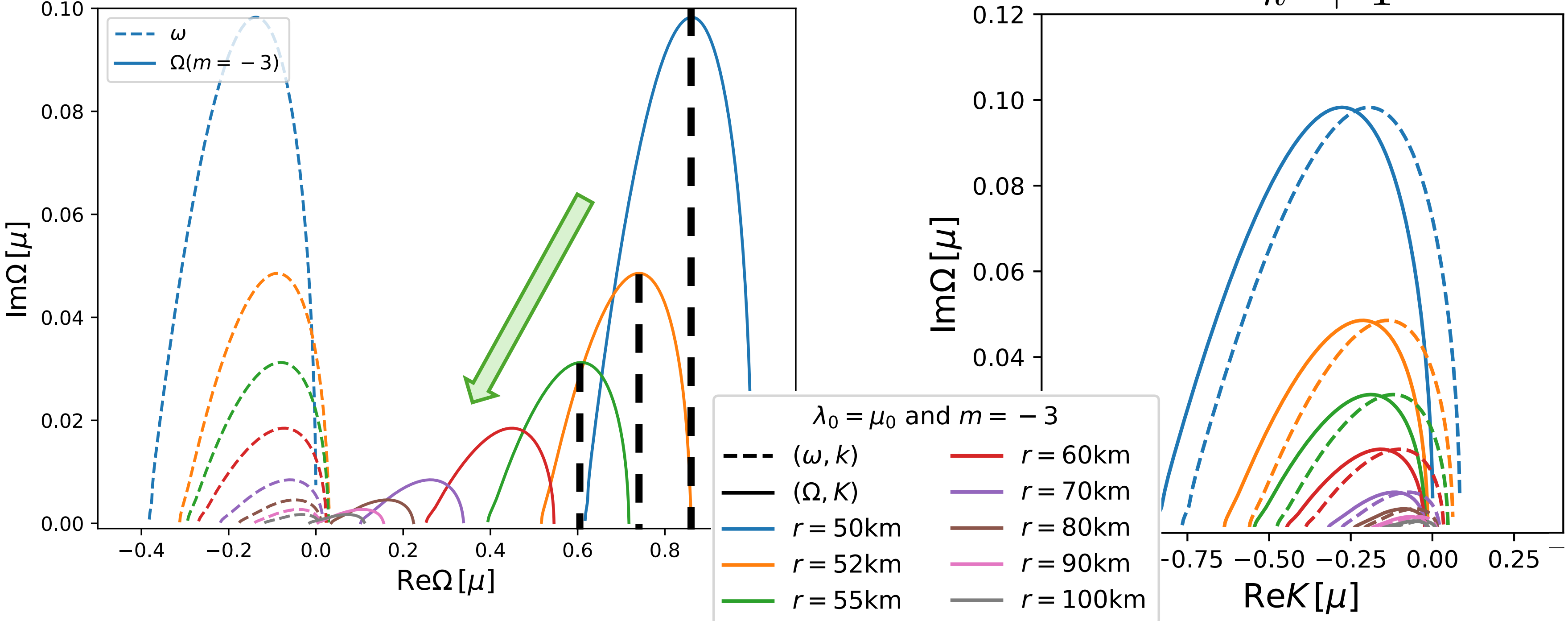
Focus on the radial profile of ***local*** unstable branches.

$$\rho_{ex} \approx \tilde{Q} \exp\left[\mathrm{i}(\Omega t - \boldsymbol{K} \cdot \boldsymbol{x})\right]$$

Radial variation of the background appears.

$$\Omega = \omega + \Lambda^0 + \Phi^0$$

$$K^j = k^j + \Lambda^j + \Phi^j$$
$$\sim k^j + \Phi^j$$



Growing mode vs. Background Shift

Refraction from the background varies with propagation.
→ Even initially unstable modes become stable.

Focus on the condition that flavor waves can **adiabatically grow.**

Competition

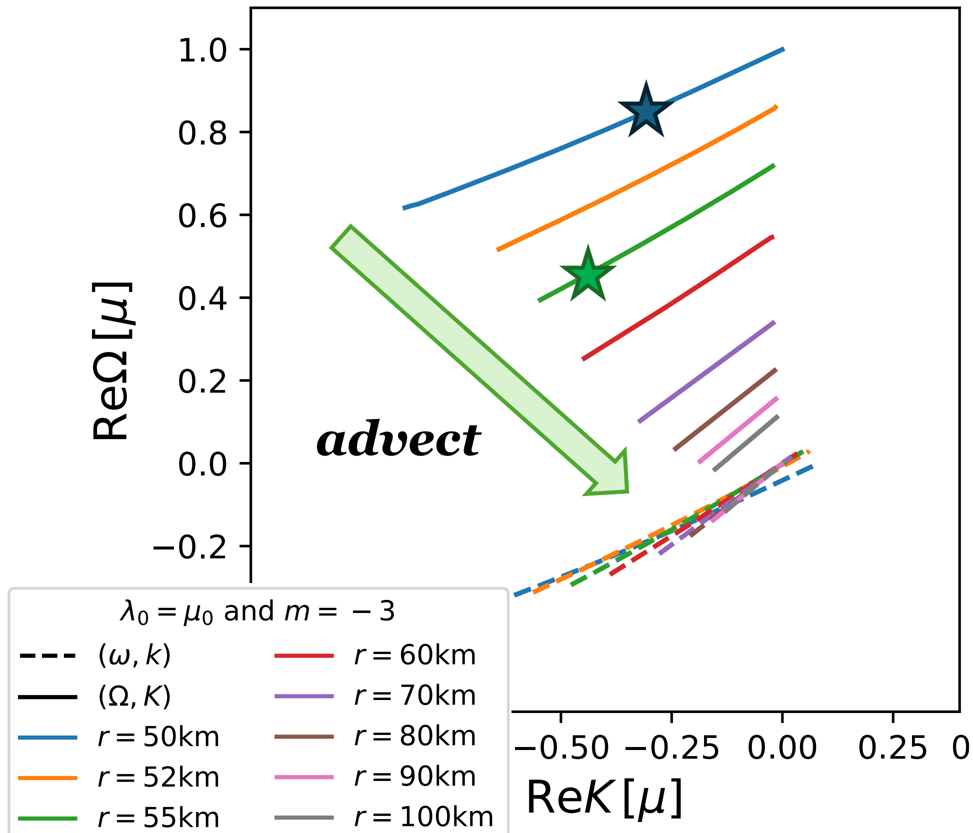
Growing scale
vs.
Refraction-changing scale

but sensitive to attenuation

⬇

Radial variation
of Adiabaticity ϵ_{ad}

Flavor mode



Adiabaticity for Flavor Wave

Adiabaticity at each radius:

$$\epsilon_{\rm ad}(r) = \frac{\left| \left[\partial_K {\rm Re}\Omega(r)\right] \left[{\rm d}_r {\rm Re}\Omega(r)\right] \right|}{\left[{\rm Im}\Omega(r)\right]^2}$$

$$\propto \xi^{-1}$$

Competition

Electron & Neutrino gradient (Less attenuated)

Growth rate

The condition that the flavor wave can adiabatically grow:

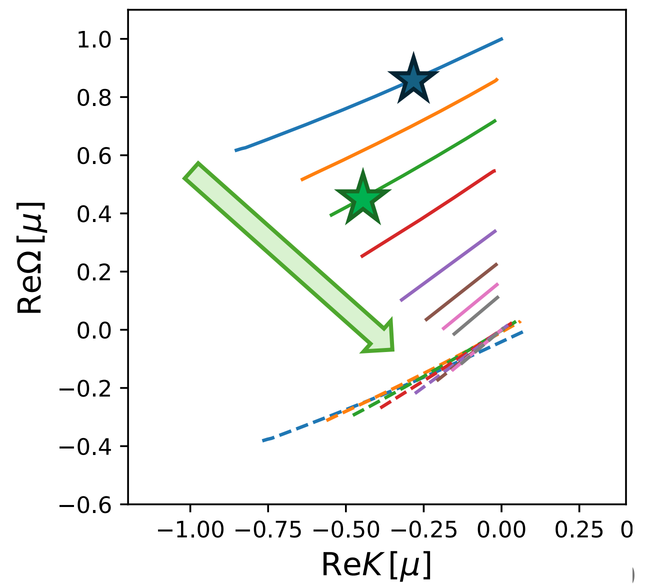
$$\tau_{\rm inst} \ll \tau_{\rm shift}.$$

Time scale (e-folding time) of FFI: $\tau_{\rm inst} \sim ({\rm Im}\Omega)^{-1}$

Refraction-changing scale: $\tau_{\rm shift} \sim \left[D_t\left(\mathcal{DR}\right) \cdot \tau_{\rm inst}\right]^{-1}$

Representative direction: group velocity v_g

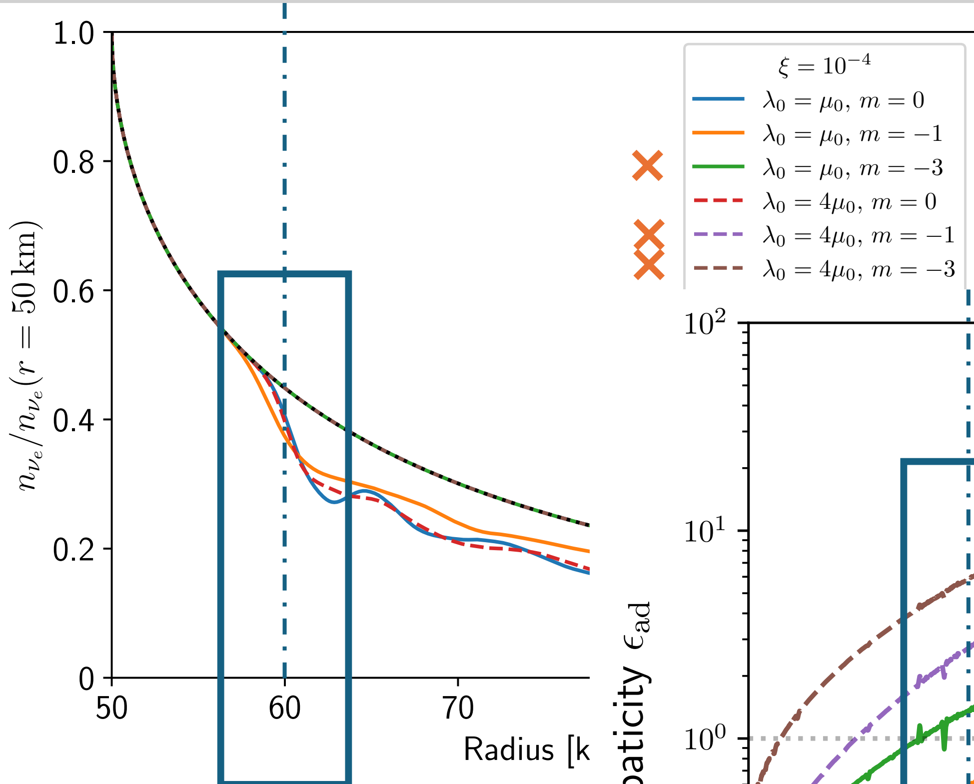
$$D_t \to v_g \partial_r = \left[\partial_K {\rm Re}\Omega\right] \partial_r$$

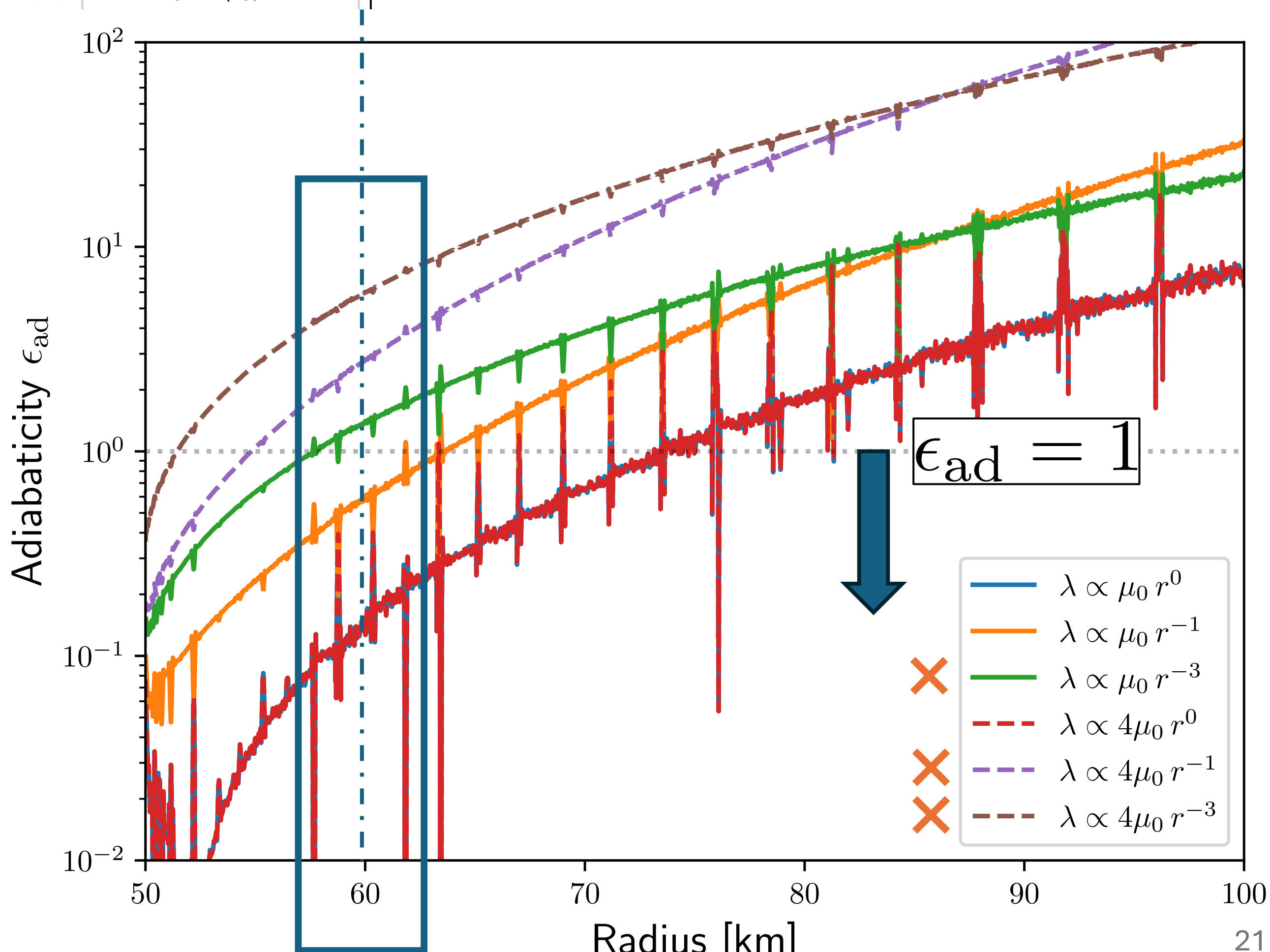


Adiabaticity for Strong Attenuation

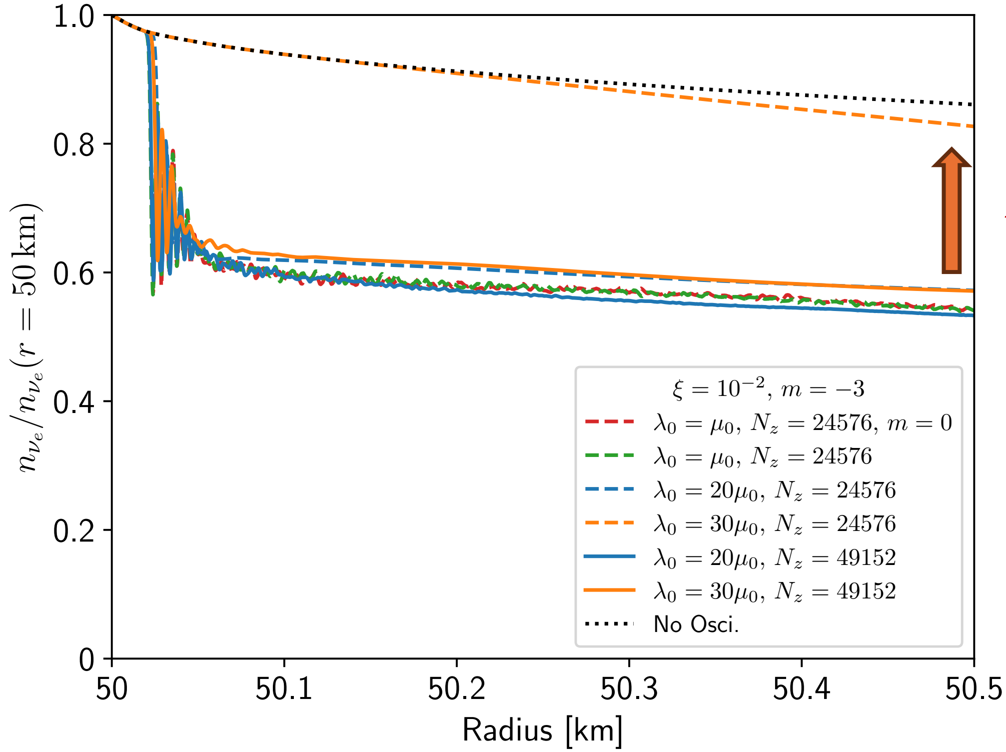
Strong attenuation

$\xi = 10^{-4}$





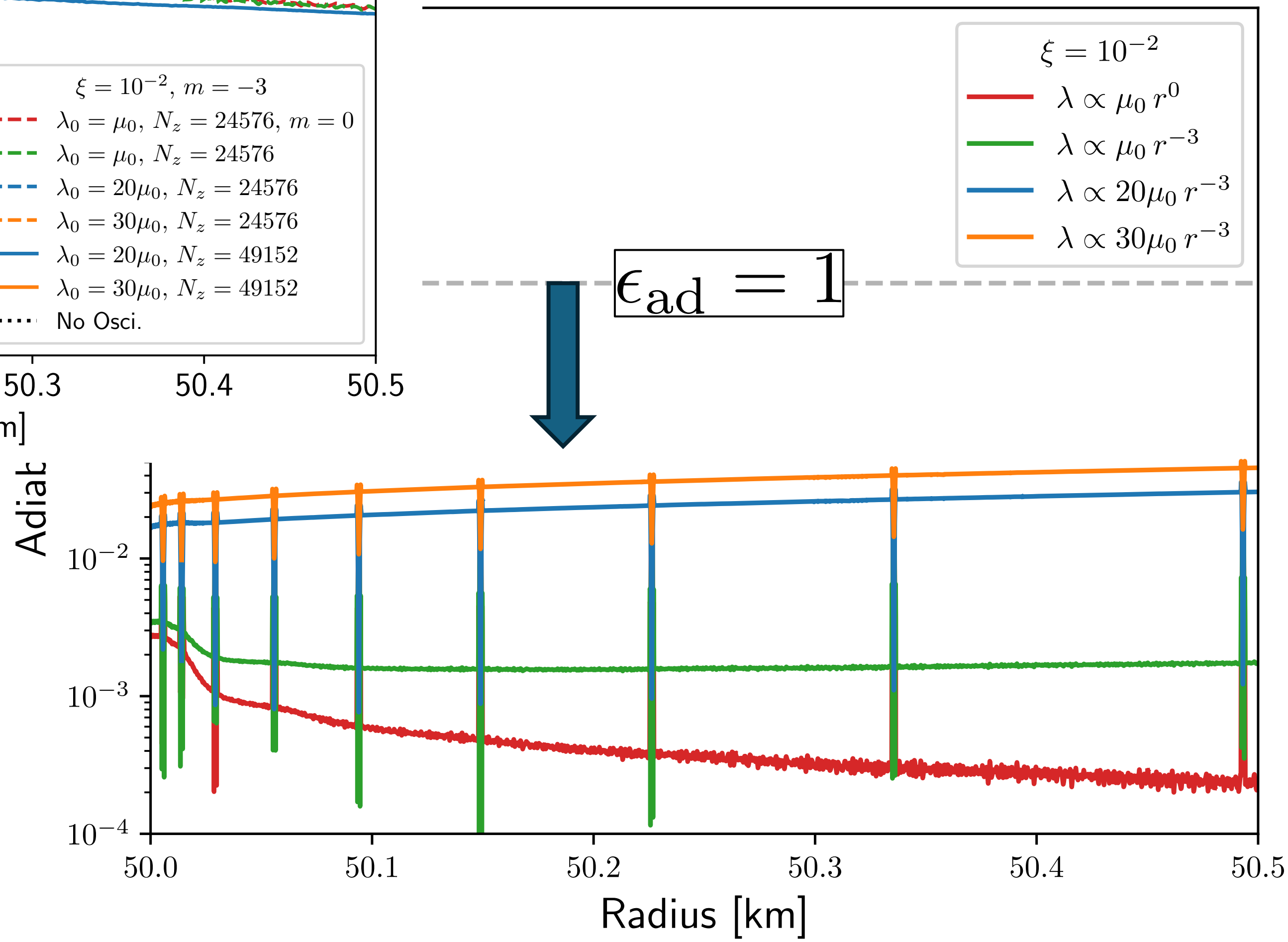
Adiabaticity for Weak Attenuation



Weak attenuation

$\xi = 10^{-2}$

Not resolved for lower reso.

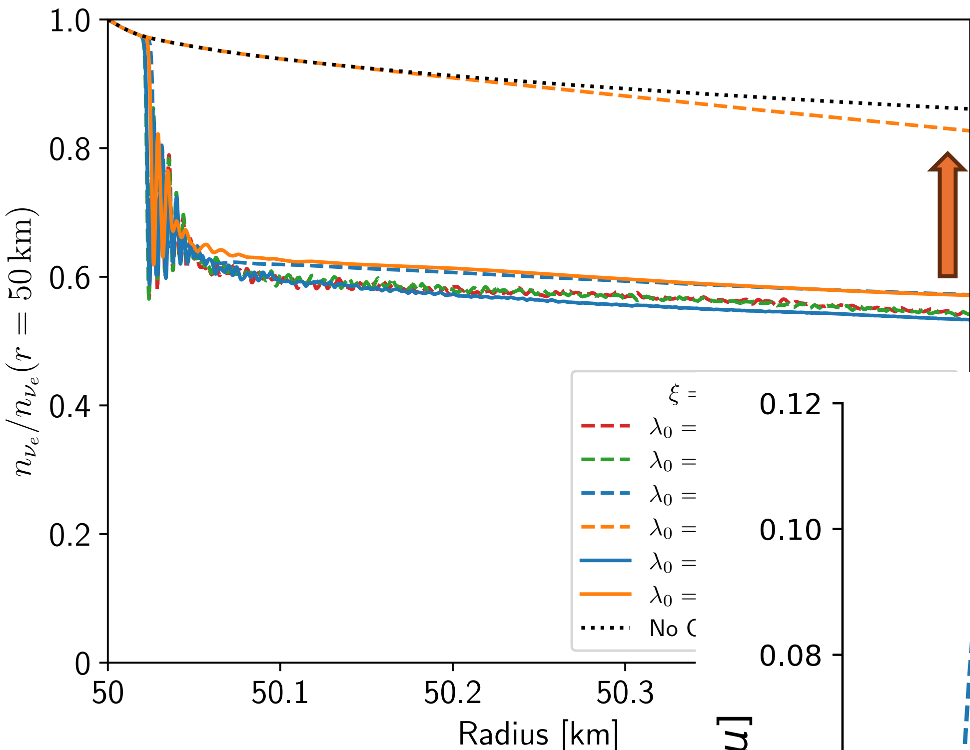


$\epsilon_{\rm ad} \ll 1$

Resolution for Finite Matter Density

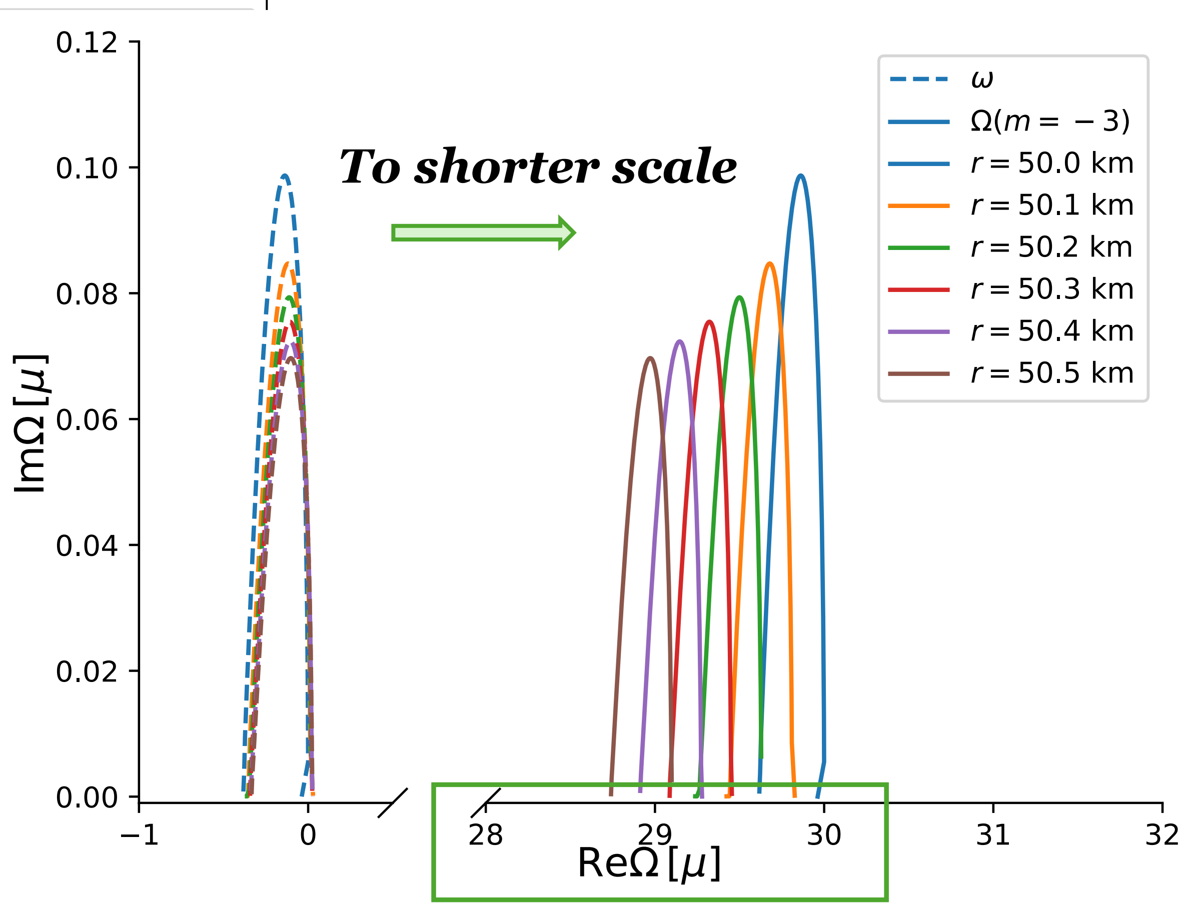
Weak attenuation

$$\xi = 10^{-2}$$



Not resolved for lower reso.

To shorter scale



Need to resolve much shorter scales in the unstable branches.

Summary & Conclusions

- Fast Flavor instability
  vs. Global background variation
  vs. Attenuation

- There are some **caveats as numerical issues**.
  1. ***Attenuation***
     1. It overestimates the changes in the background matter/neutrinos during the exponential growth.
  2. ***Resolution***
     1. It could not capture the shifted unstable branch due to significant matter density.

They could confuse us about whether the suppression is true or artificial.

- Be careful in calculating the global QKE for FFI, and particularly CFI and SFI, which have relatively longer oscillation scales.