

Slow flavor conversion of neutrinos in core- collapse supernovae

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*Collective Neutrino Oscillations in Supernovae and Neutron Star Mergers,
March 23-27, Academia Sinica, Taipei, Taiwan*



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UNIVERSITY OF CALIFORNIA

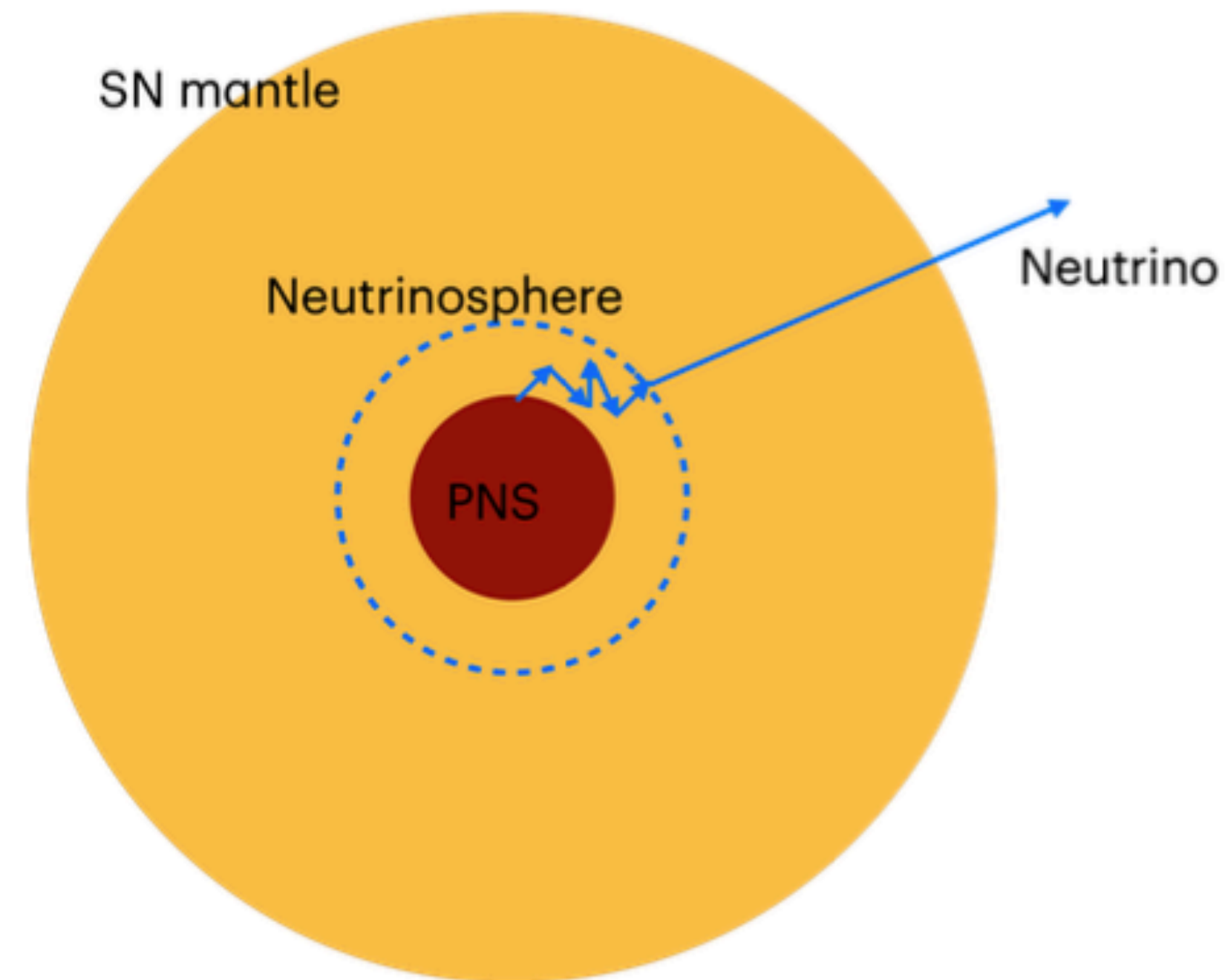


UC San Diego



(Unknown artist, spotted near Taipei 101)

Neutrinos from core-collapse SNe



- **Energy scale:** 99% of the released energy ($\sim 10^{53}$ erg) is emitted in the form of ν and $\bar{\nu}$ of all flavors, with $E \sim 15$ MeV
- **Time scale:** ~ 10 s
- **Expected:** 1-3 SN/century in our galaxy ($d \approx 10$ kpc)

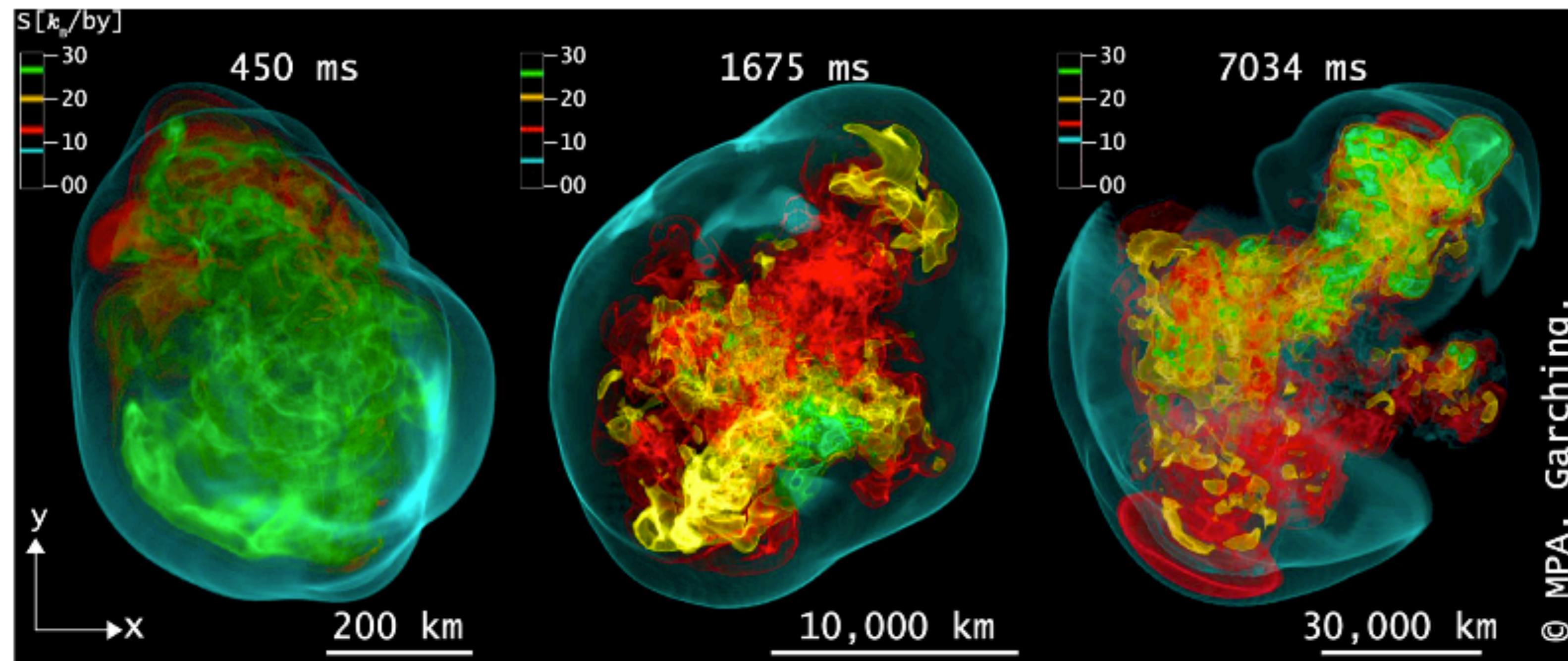
(To compare, the Sun will radiate $\sim 10^{51}$ erg over 10 billion years).

CCSNe are the most efficient neutrino factories in the Universe (10^{58} ν 's)

SN simulations

Most of our knowledge about SN explosions is based on complex 3D simulations:

1. Account for the effect of convection and turbulence
2. Account for neutrino transport in 3D. No detailed treatment of flavor conversion (yet)

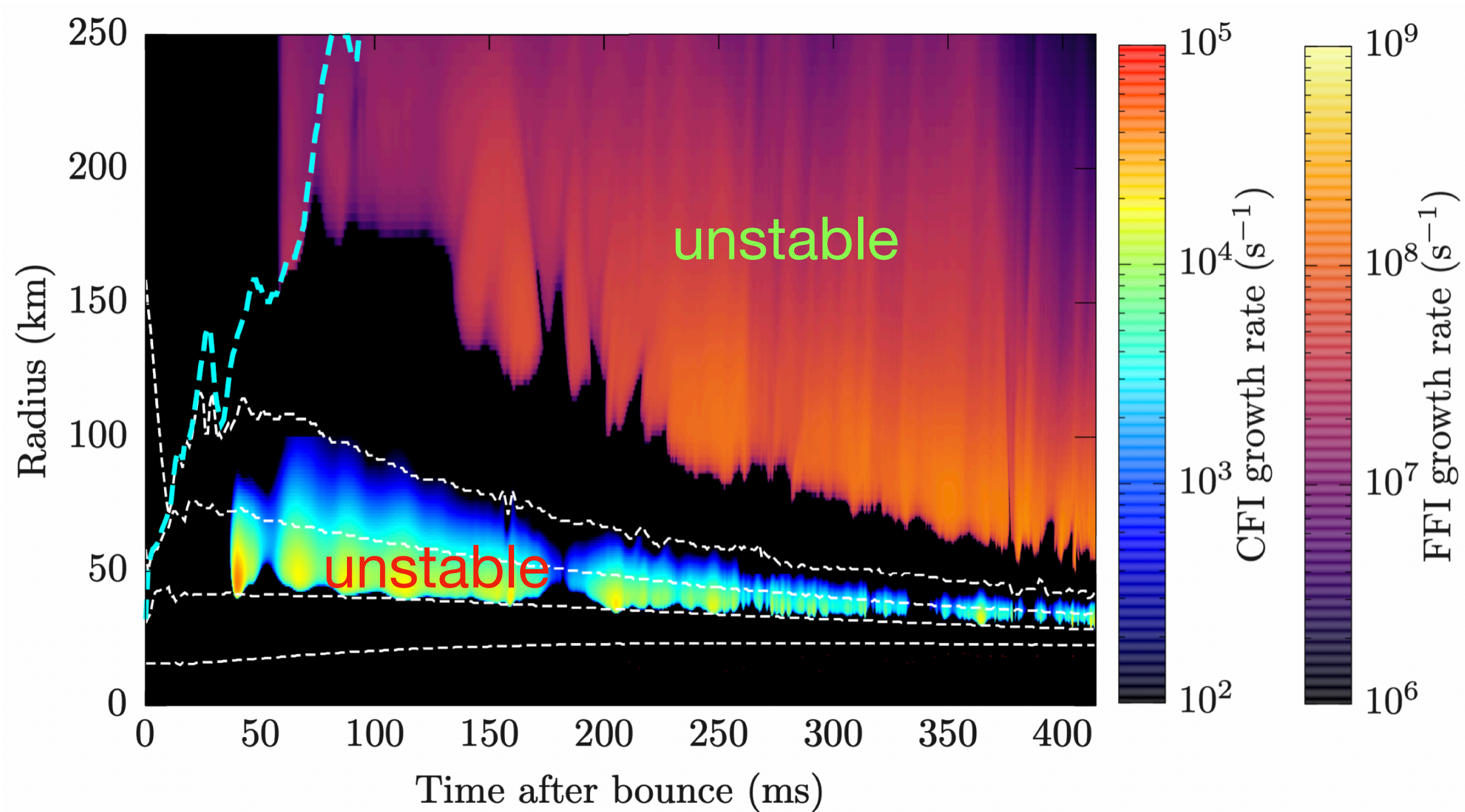


R. Bollig et al, [2010.10506](https://arxiv.org/abs/2010.10506)

Flavor oscillations are expected

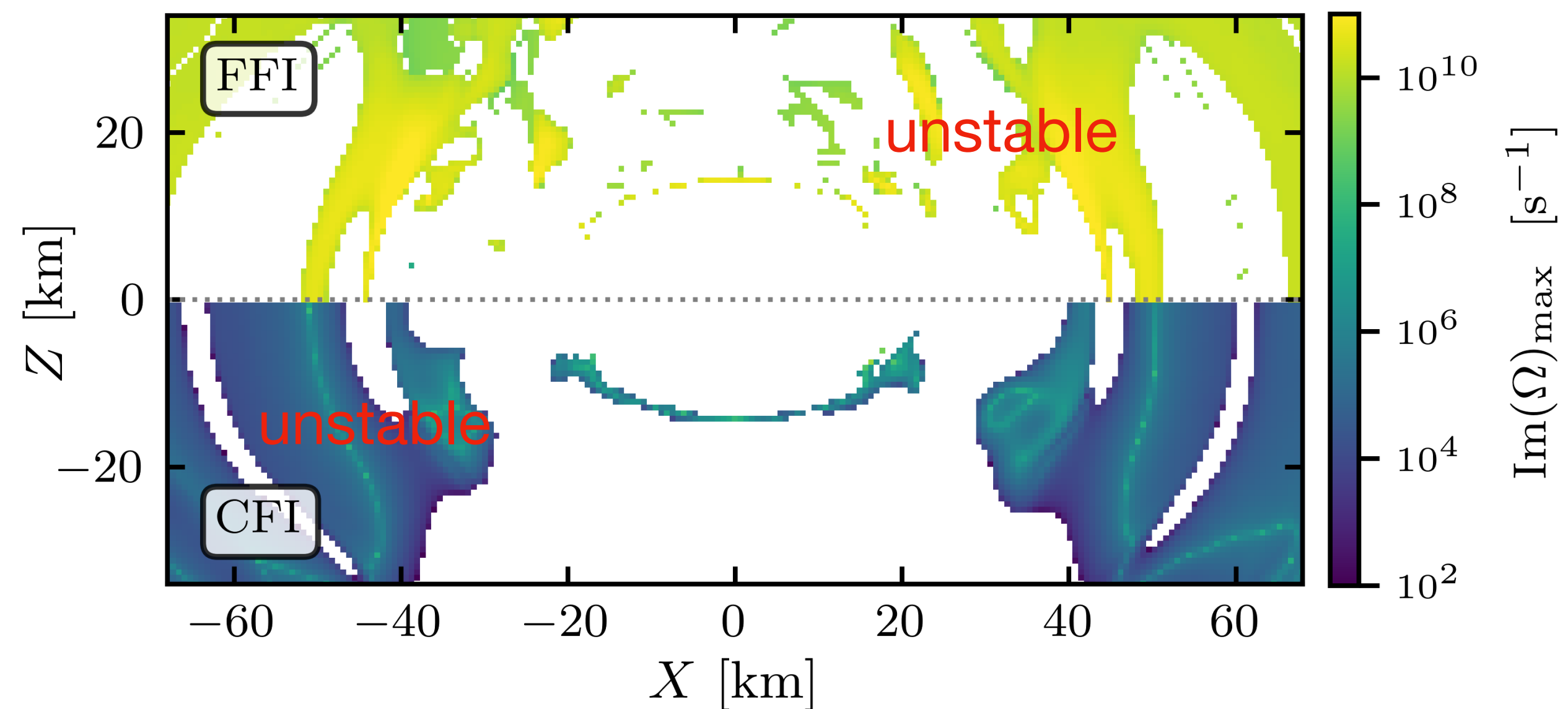
- Look for *flavor instabilities* in SN and NS merger simulations
- Linear stability analysis \rightarrow unstable solution \rightarrow flavor oscillations expected

SN simulation



R. Akaho et al, [2311.11272](#)

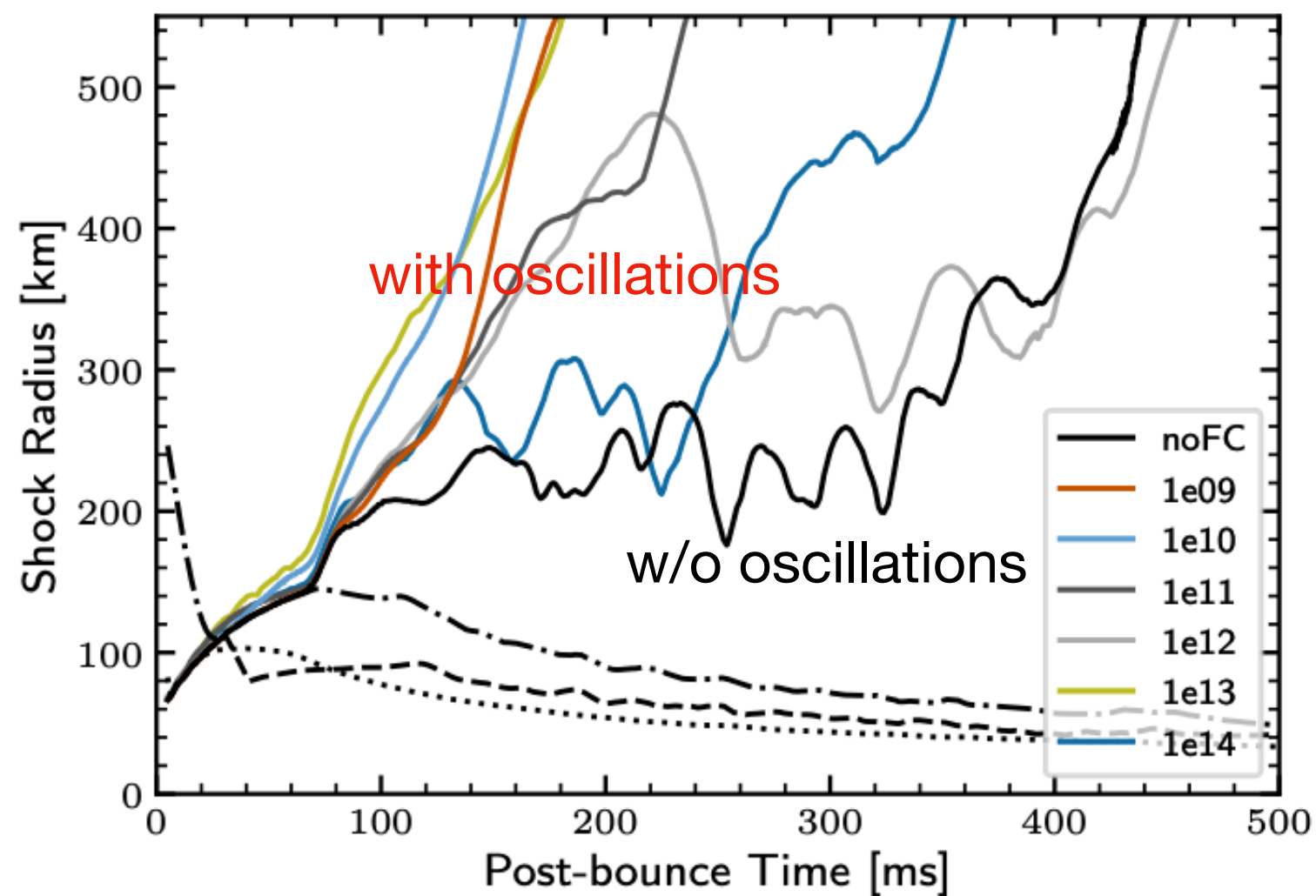
NS merger simulation



J. Froustey, [2311.11968](#)

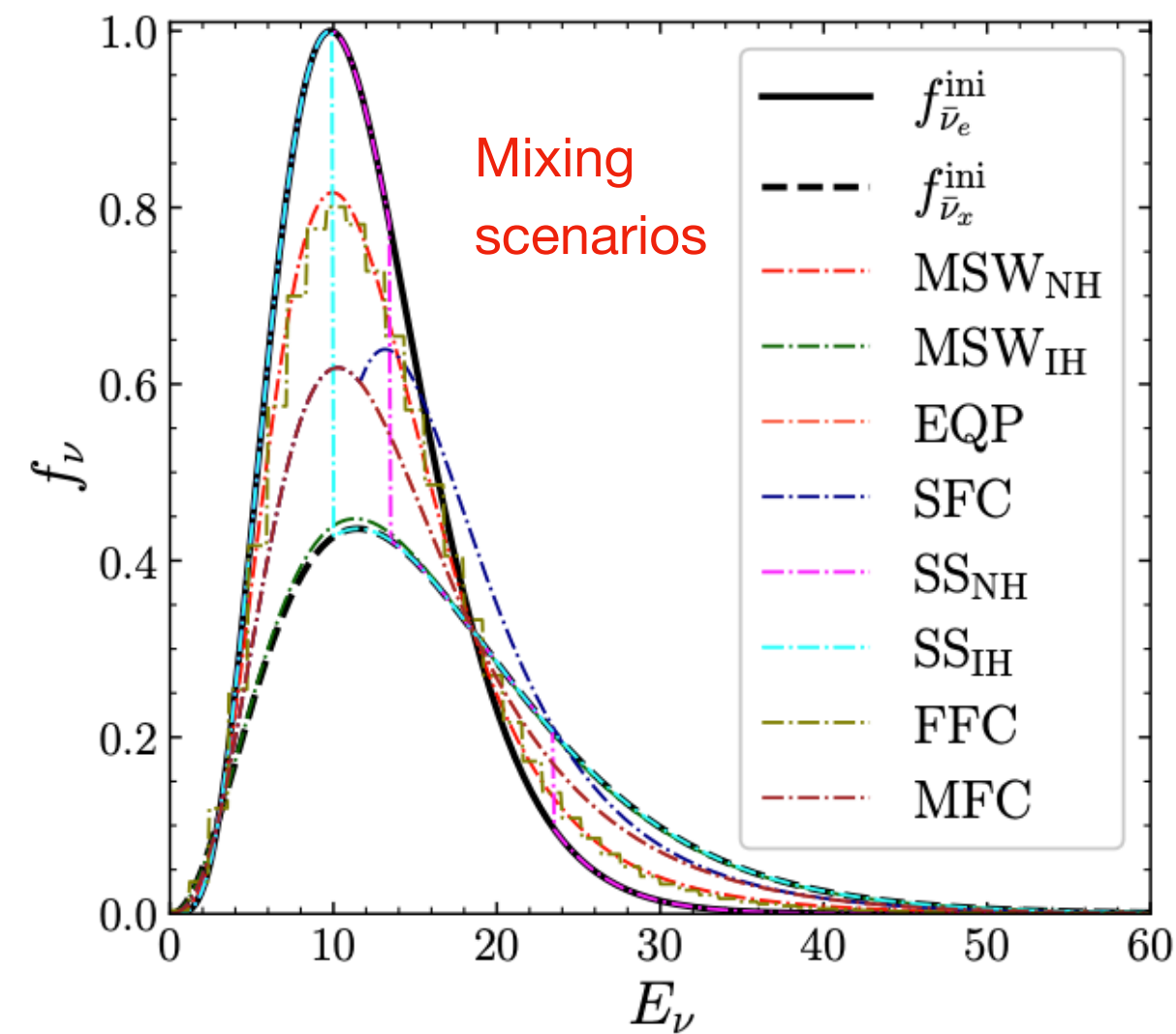
Potential impact of neutrino oscillations

Supernova dynamics



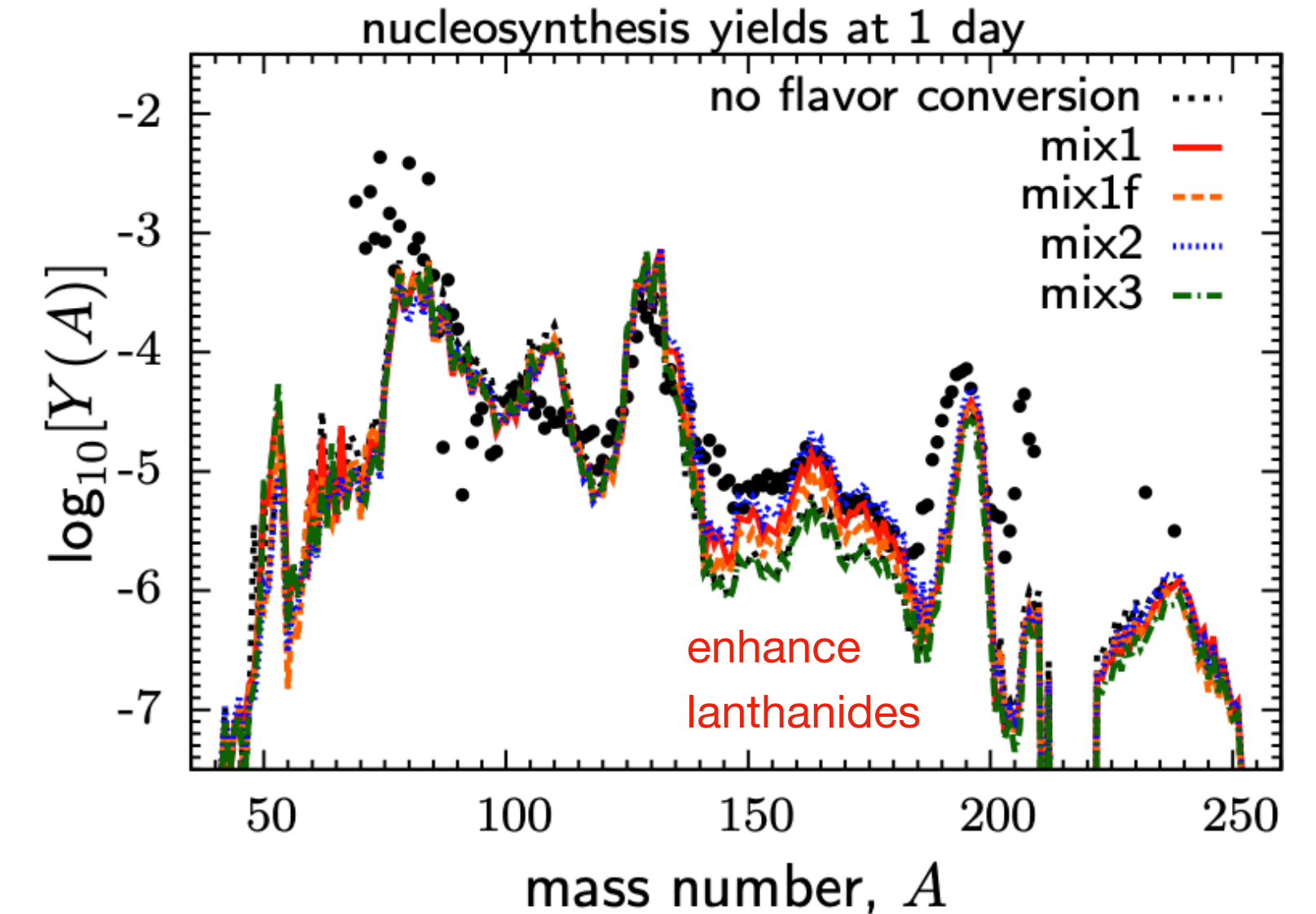
J. Ehring et al, [2305.11207](#)

ν energy spectra



S. Abbar et al, [2401.10851](#)

r-process nucleosynthesis



O. Just et al, [2203.16559](#)

People are estimating the influence of oscillations on:

- **Proton-rich nucleosynthesis** in SN: νp -process (A. Friedland, IPG et al, [2508.02055](#))
- **Gravitational-wave emission** in SN (J. Ehring et al, [2412.02750](#))

Current challenges

Supernova dynamics

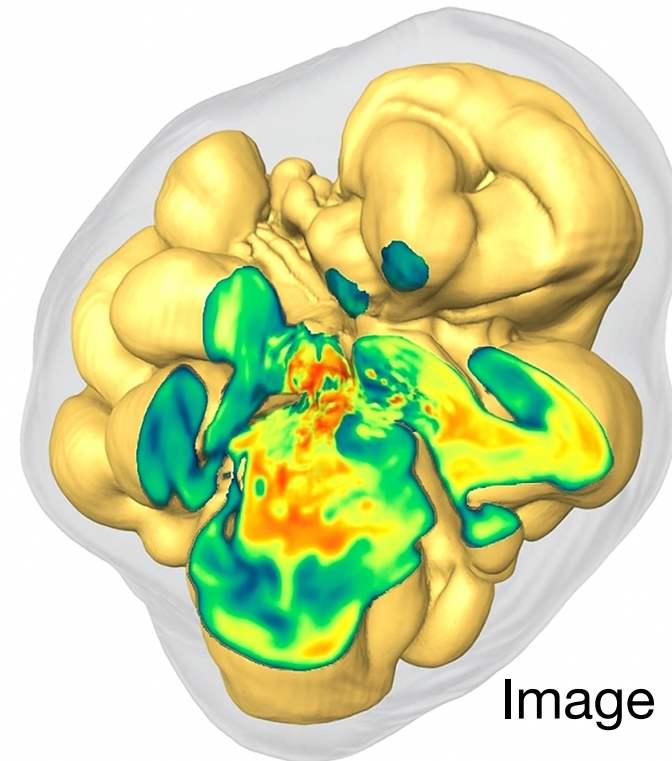
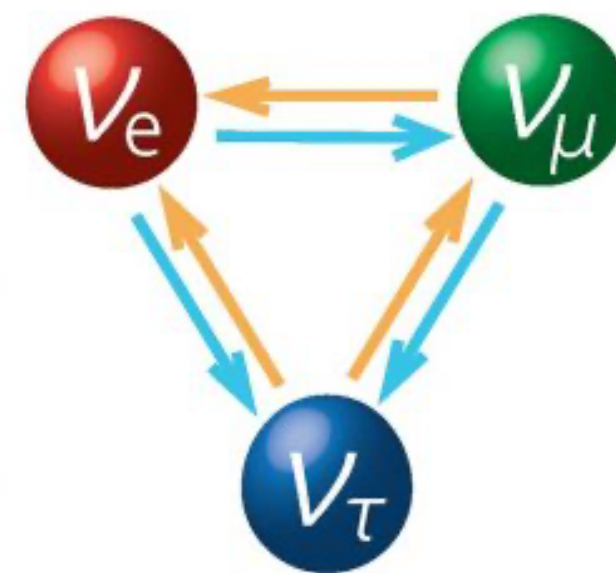


Image credits: T. Janka

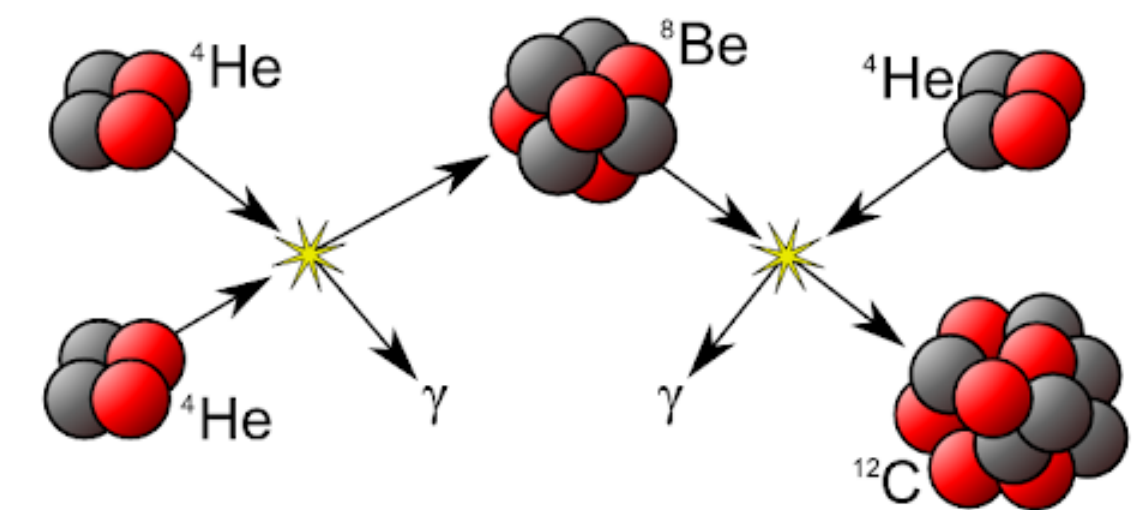
- Major uncertainty: neutrino oscillations near the core¹
- The “oscillation problem”:
simulations can’t resolve oscillations

Neutrino collective oscillations



- Quantum many-body problem
- Mean-field: 7D problem (\vec{x}, \vec{p}, t)
- Very short time scales²: \sim ns

Heavy-element nucleosynthesis



- Sensitive to p/n ratio (neutrinos!)
- Properties of rare isotopes unknown
- Efficient p-rich nucleosynthesis?^{3,4}

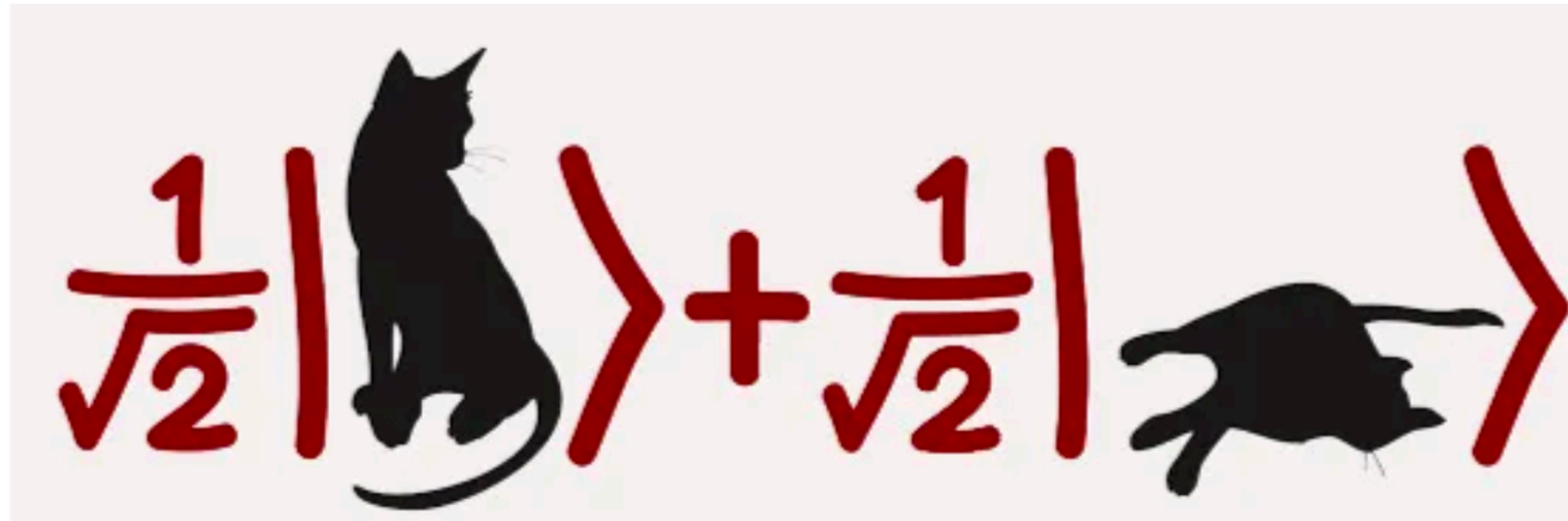
¹ Long-Term Multidimensional Models of Core-Collapse Supernovae: Progress and Challenges, Annual Reviews, Hans-Thomas Janka, 2502.14836

² New Developments in Flavor Evolution of a Dense Neutrino Gas, Tamborra et al, Ann. Rev. Nucl. Part. Sci. 71 (2021) 165-188

³ The p-process in supernovae, Woosley & Howard, Astrophysical Journal Supplement Series, vol. 36, Feb. 1978, p. 285-304.

⁴ vp-process in core-collapse supernovae: imprints of general relativistic effects, JCAP 02 (2026) 067

Flavor oscillations in dense media


$$\frac{1}{\sqrt{2}} |\text{cat}\rangle + \frac{1}{\sqrt{2}} |\text{dog}\rangle$$

$$a_e(t) |\nu_e\rangle + a_\mu(t) |\nu_\mu\rangle$$

Neutrino-neutrino interaction

- Neutrinos also constitute a background for *other* neutrinos
- $\nu\nu$ interactions have off-diagonal elements in flavor basis¹

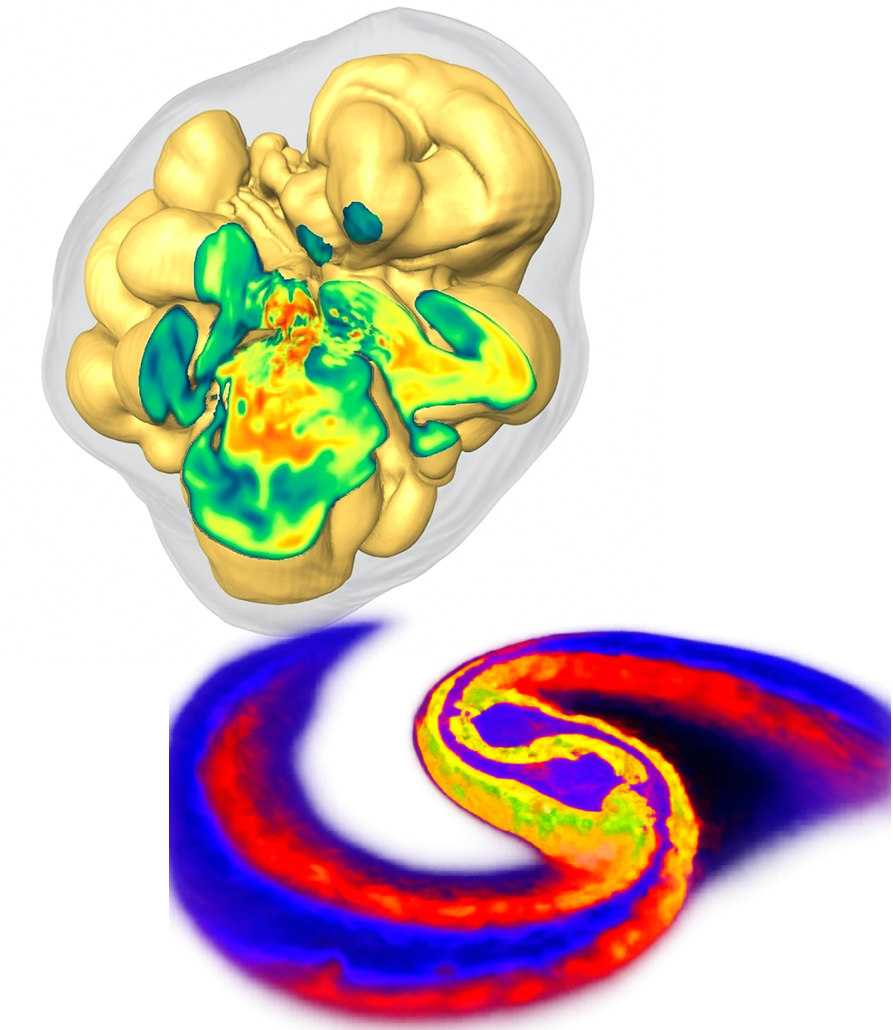
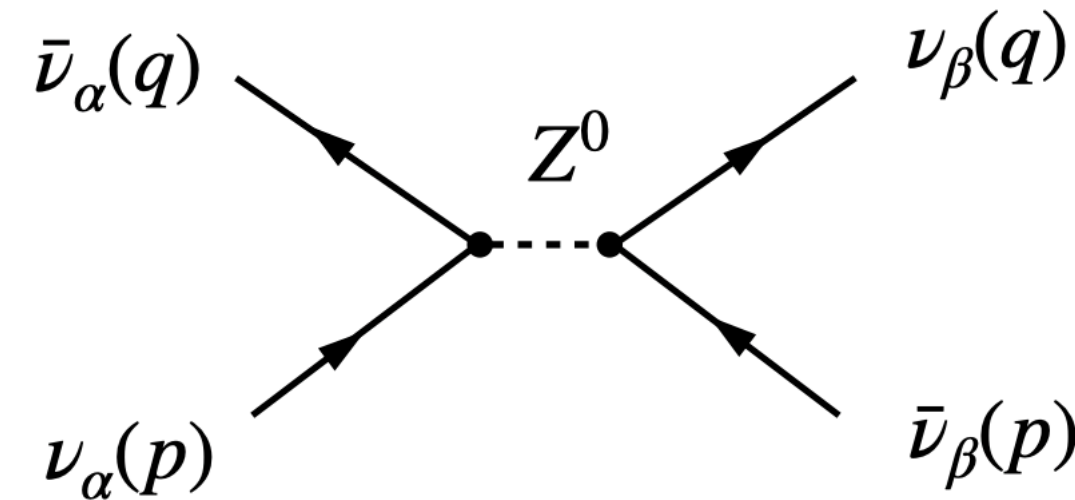
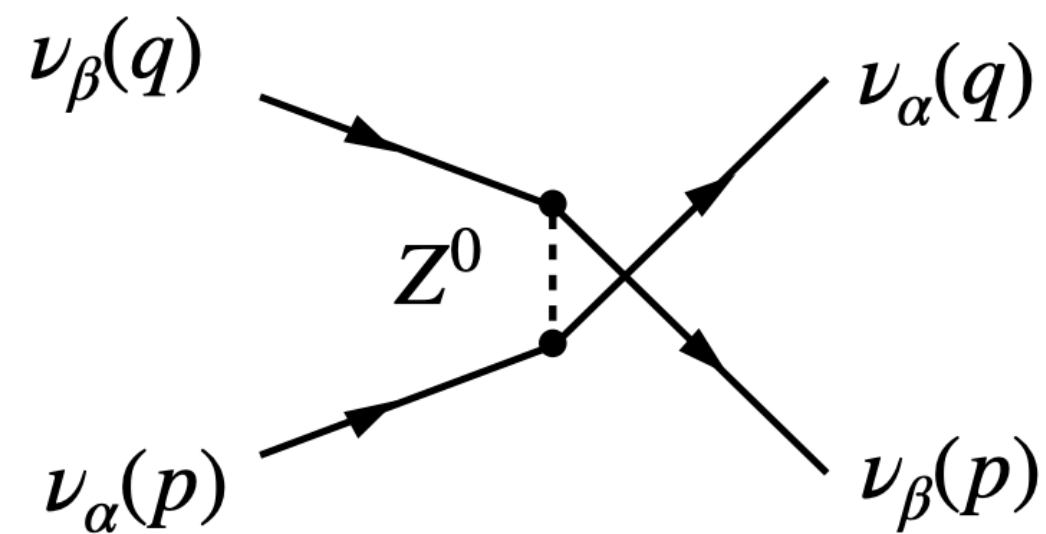


Image credits: T. Janka (upper) and S. Rosswog (lower)

Neutrino-neutrino coherent forward scattering



¹Neutrino oscillations at high densities, Pantaleone, Physics Letters B 287 (1992) 128-132

Neutrino-neutrino interaction

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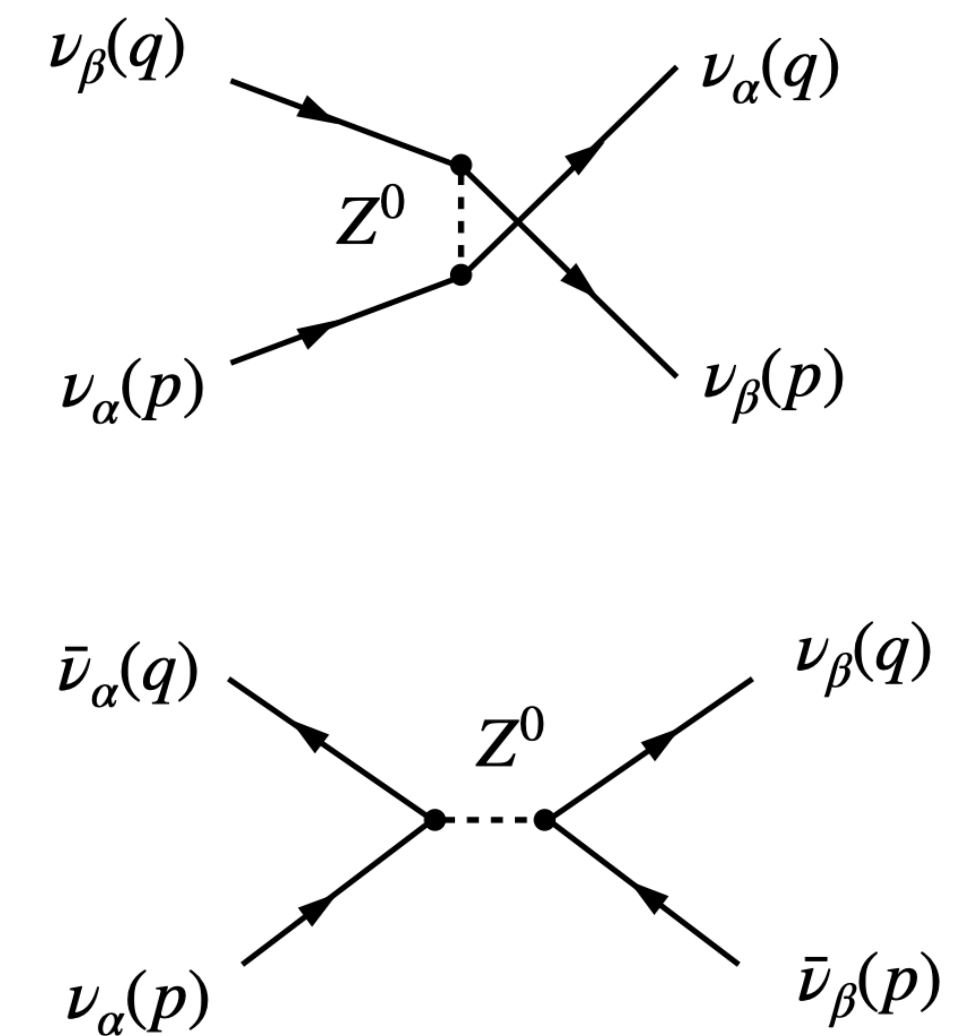
Momentum of test neutrino

$$H_{\nu\nu}(\vec{p}) = \sqrt{2}G_F n_\nu \int d\vec{q} [\rho(\vec{q}) - \bar{\rho}(\vec{q})] \left(1 - \frac{\vec{p} \cdot \vec{q}}{|\vec{p}||\vec{q}|}\right)$$

Neutrino interaction strength

Momentum of background neutrino

Density matrices (Flavor content of the medium)



- *Non-linear* behavior
- $\nu\nu$ interaction induces *collective oscillations*²⁻⁵

¹Neutrino oscillations at high densities, Pantaleone, Physics Letters B 287 (1992) 128-132

²Collective Neutrino Oscillations, Duan et al, Ann. Rev. Nucl. Part. Sci. 60:569 (2010)

³Supernova Neutrinos: Production, Oscillations and Detection, Mirizzi et al, Rivista del Nuovo Cimento Vol. 39, N. 1-2 (2016)

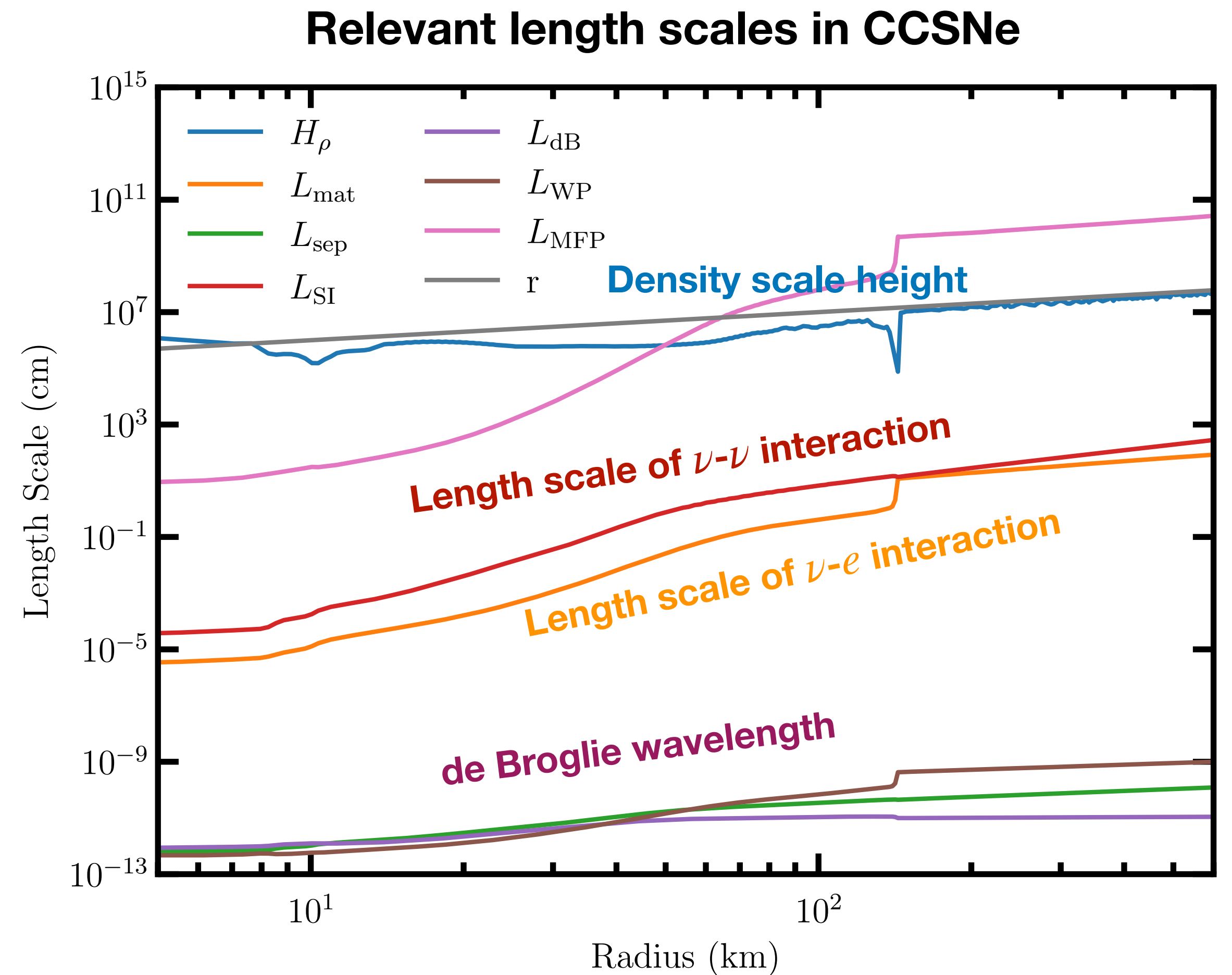
⁴Collective neutrino flavor conversion: Recent developments, Chakraborty et al, 1602.02766

⁵New Developments in Flavor Evolution of a Dense Neutrino Gas, Tamborra et al, Ann. Rev. Nucl. Part. Sci. 71 (2021) 165-188

Pedagogical resource: P. Dedin+, Mod.Phys.Lett.A 37 (2022) 08, 2250048

The oscillation problem

- Neutrinos experience a potential on **length scales much shorter** than the **macroscopic scale**
- Hydro simulations with this resolution are not possible
- The “**oscillation problem**” is to figure out how to implement flavor mixing without fully resolving it



Quantum kinetic equations

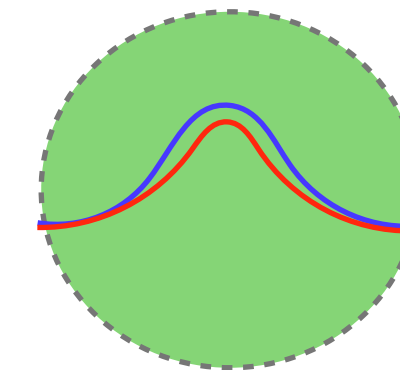
- Neutrinos with different momenta evolve collectively
- The (mean-field) equations of motion:

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}_x\right)\rho(\vec{x}, \vec{p}, t) =$$

$$\left(\frac{\partial}{\partial t} + \underbrace{\vec{v} \cdot \vec{\nabla}_x}_{\text{advection}}\right)\bar{\rho}(\vec{x}, \vec{p}, t) =$$

Spatial variations

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{ex}^* & \rho_{xx} \end{pmatrix}$$



$$\alpha|\nu_e\rangle + \beta|\nu_x\rangle$$

Superposition of two states

Quantum kinetic equations

- Neutrinos with different momenta evolve collectively
- The (mean-field) equations of motion:

$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}_x \right) \rho(\vec{x}, \vec{p}, t) = -i \left[H(\vec{x}, \vec{p}, t), \rho(\vec{x}, \vec{p}, t) \right]$$

$$\left(\frac{\partial}{\partial t} + \underbrace{\vec{v} \cdot \vec{\nabla}_x}_{\text{advection}} \right) \bar{\rho}(\vec{x}, \vec{p}, t) = -i \underbrace{\left[\bar{H}(\vec{x}, \vec{p}, t), \bar{\rho}(\vec{x}, \vec{p}, t) \right]}_{\text{refraction}}$$

Spatial variations

Coherent forward scattering

Quantum kinetic equations

- Neutrinos with different momenta evolve collectively
- The (mean-field) equations of motion:

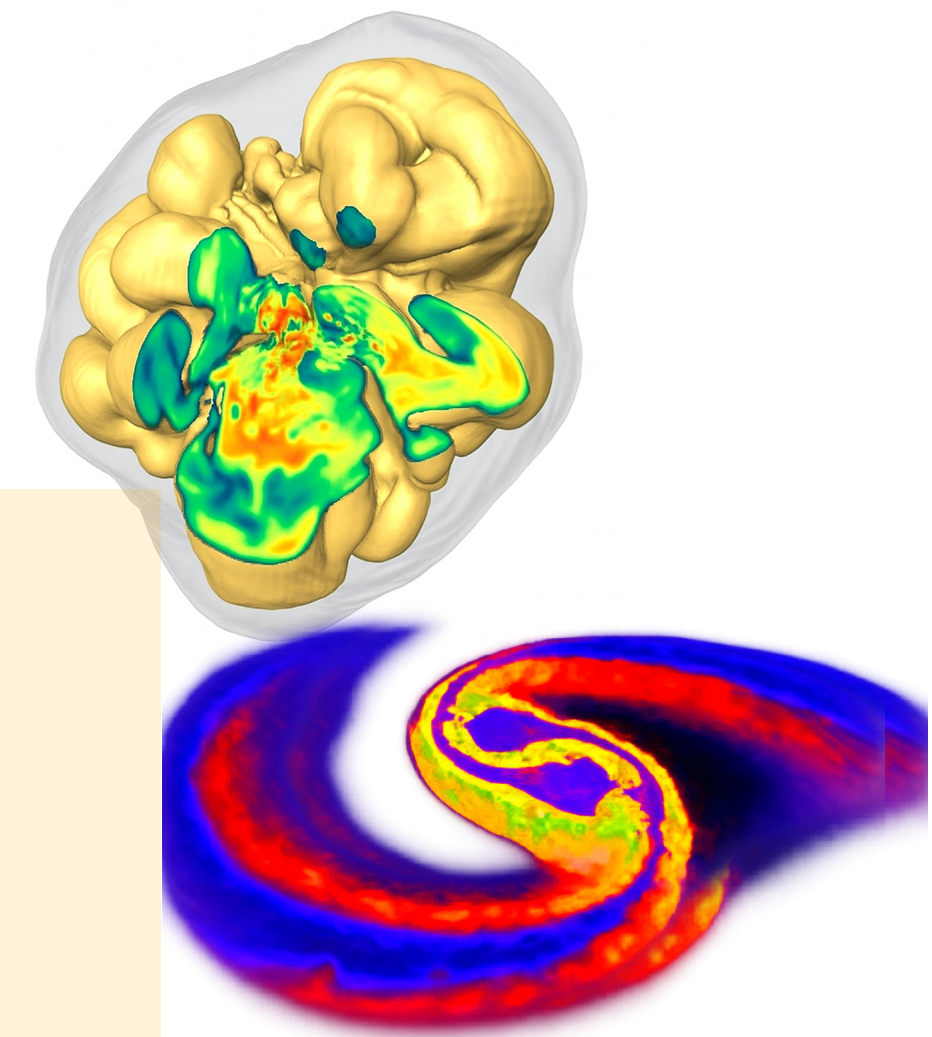
$$\left(\frac{\partial}{\partial t} + \vec{v} \cdot \vec{\nabla}_x \right) \rho(\vec{x}, \vec{p}, t) = -i \left[H(\vec{x}, \vec{p}, t), \rho(\vec{x}, \vec{p}, t) \right] + \mathcal{C}(\rho(\vec{x}, \vec{p}, t), \bar{\rho}(\vec{x}, \vec{p}, t))$$

$$\left(\frac{\partial}{\partial t} + \underbrace{\vec{v} \cdot \vec{\nabla}_x}_{\text{advection}} \right) \bar{\rho}(\vec{x}, \vec{p}, t) = -i \left[\underbrace{\bar{H}(\vec{x}, \vec{p}, t)}_{\text{refraction}}, \bar{\rho}(\vec{x}, \vec{p}, t) \right] + \underbrace{\bar{\mathcal{C}}(\rho(\vec{x}, \vec{p}, t), \bar{\rho}(\vec{x}, \vec{p}, t))}_{\text{collisions}}$$

Spatial variations

Coherent forward scattering

Non-forward scattering

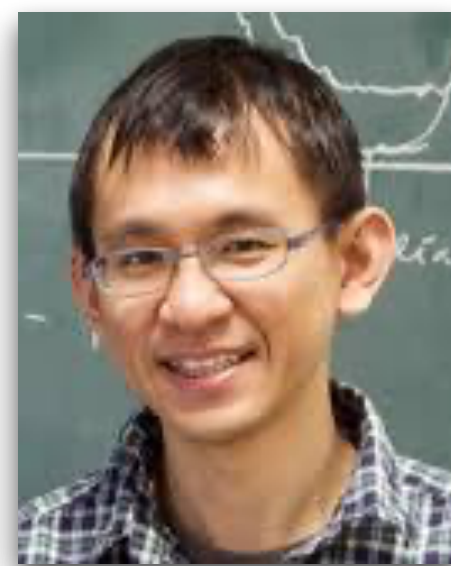


Flavor Equilibration of Supernova Neutrinos: Exploring the Dynamics of Slow Modes

arXiv: 2505.11588 (Phys. Rev. D 112, 043039)

In collaboration with:

Heng-Hao Chen (Jordan), Sajad Abbar, Meng-Ru Wu, and Zewei Xiong



Hamiltonian of the system

- Vacuum Hamiltonian**

$$H_{\text{vac}} = \frac{\omega}{2} \begin{pmatrix} -\cos 2\theta_V & \sin 2\theta_V \\ \sin 2\theta_V & \cos 2\theta_V \end{pmatrix}$$

$$\omega = \frac{\Delta m^2}{2E}, \text{ matter-suppressed } \theta_V = 10^{-5} \text{ rad}$$

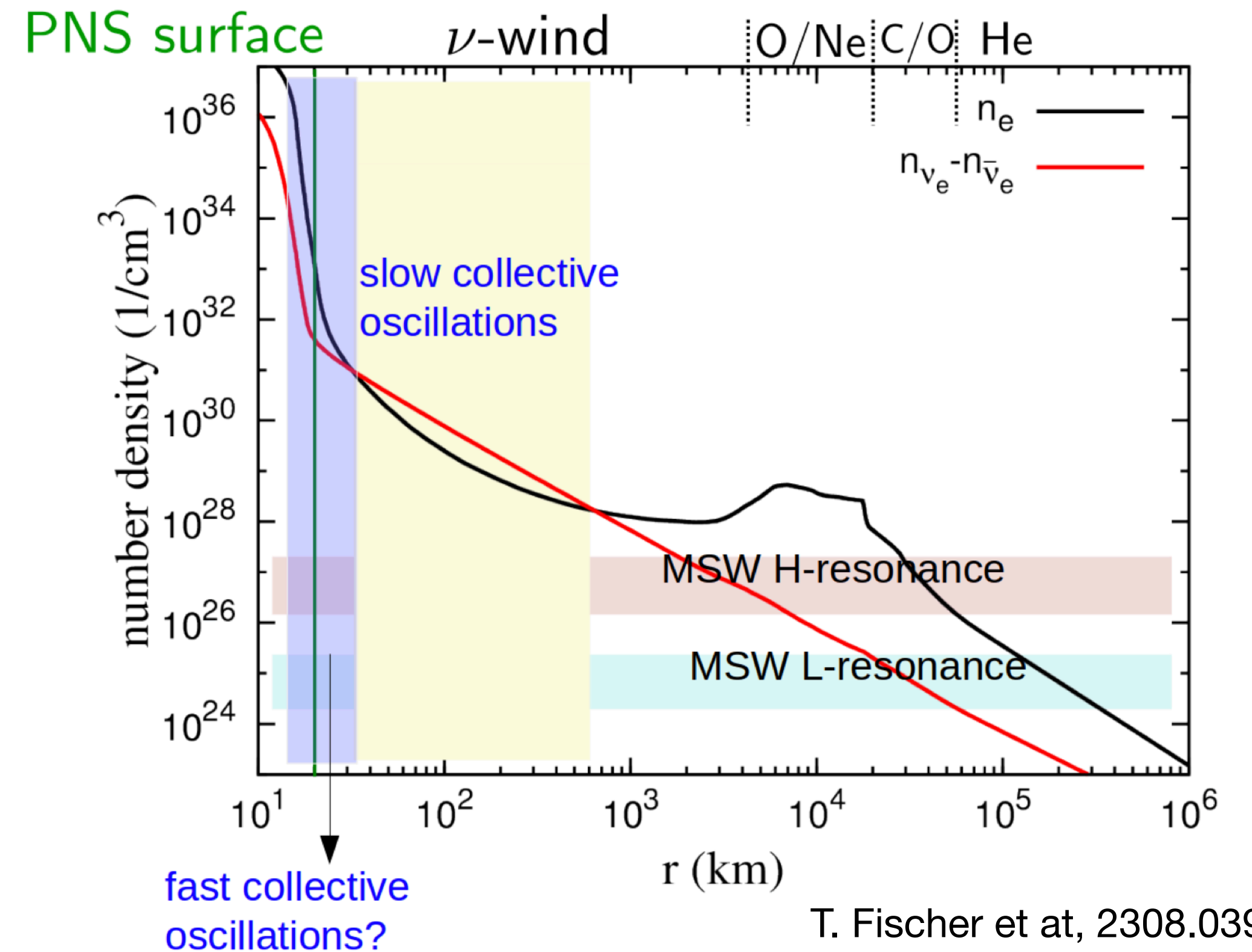
(single-energy $E = 10 \text{ MeV} \rightarrow \omega \simeq 0.6 \text{ km}^{-1}$)

- Neutrino-neutrino forward scattering**

$$H_{\nu\nu} = \mu \int dv'_z [\rho(t, z, v'_z) - \bar{\rho}(t, z, v'_z)] (1 - v_z v'_z)$$

$\mu \equiv \sqrt{2} G_F n_{\nu_e}$: self-interaction potential

(assume $n_{\nu_e} \simeq 10^{29} \text{ cm}^{-3} \rightarrow \mu \simeq 64.2 \text{ km}^{-1}$, ratio $\omega/\mu \approx 0.01$)



Neutrino gas in a box

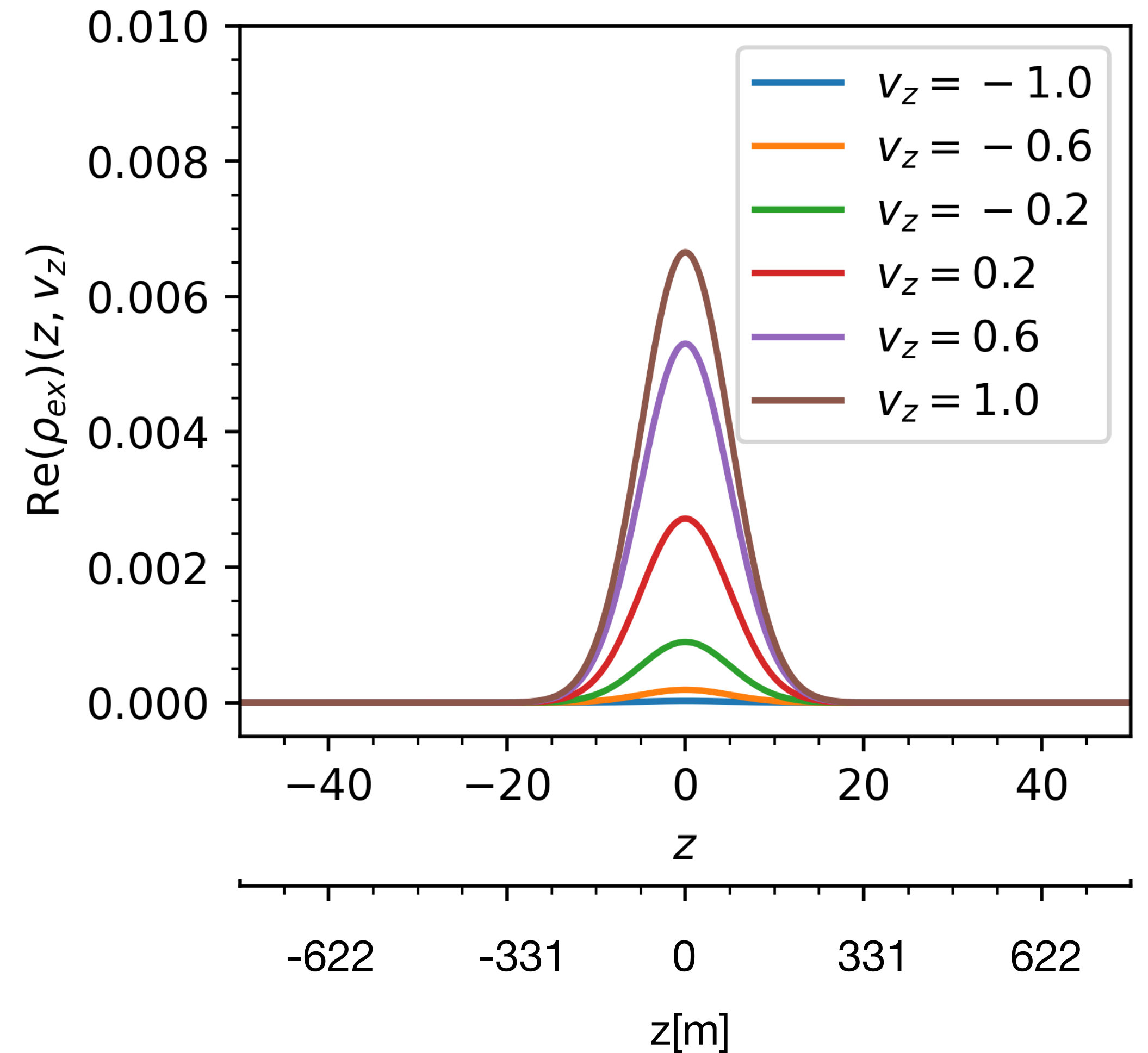
Initial configuration in space

- Box size: $L = 100[\mu^{-1}] \simeq 1.5$ km
- Periodic boundary (local quasi-equilibrium)
- Initial z -dependence of seed

$$\rho_{ex}(t = 0) \propto e^{-z^2/50} \text{ (i.e. width } \simeq 77 \text{ m)}$$

- Initial size of inhomogeneity

$$\frac{\rho_{ex}(z, v_z, t = 0)}{\rho_{ee}(z, v_z, t = 0)} < 10^{-2}$$



Neutrino gas in a box

Initial velocity profile

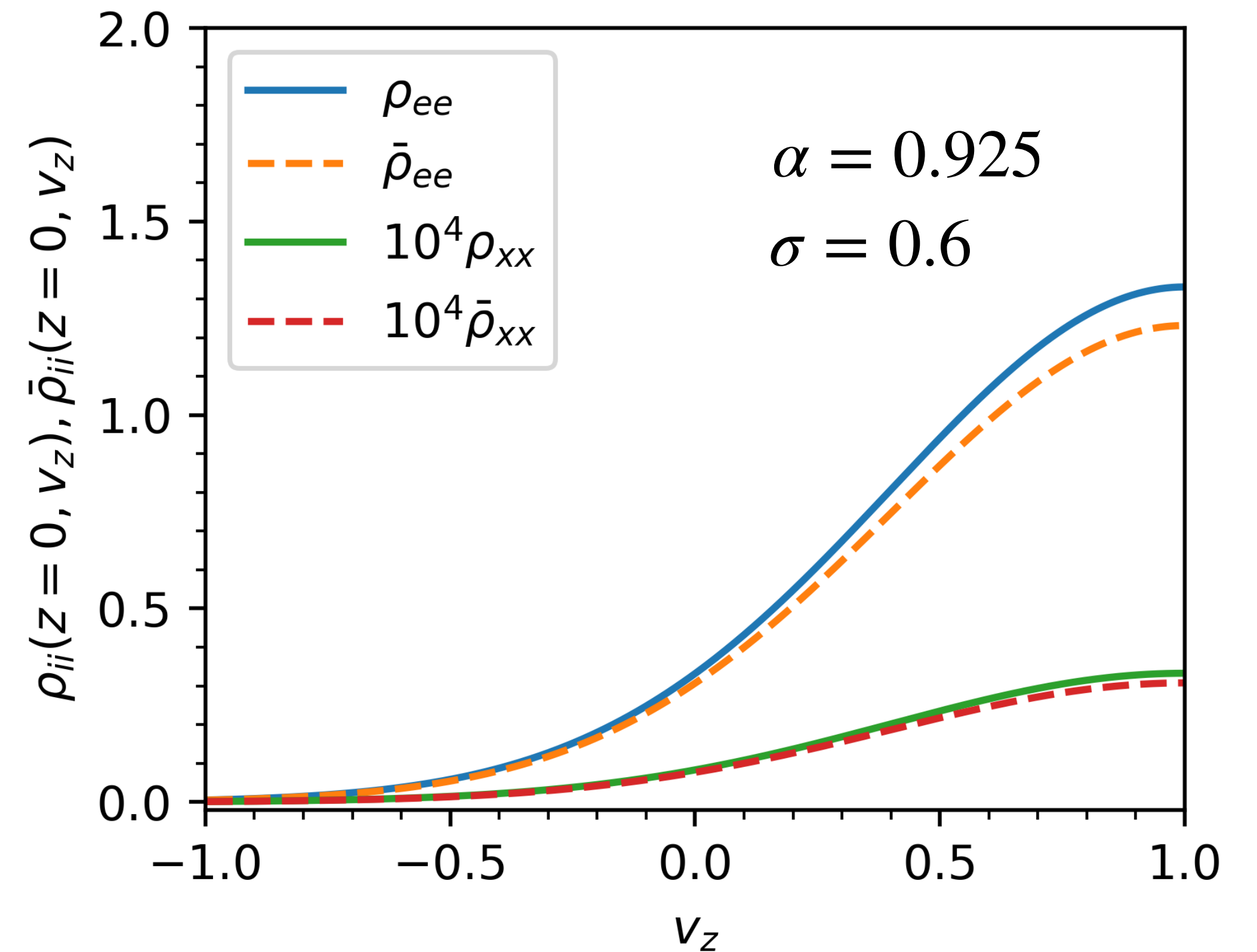
- Velocity bins within $-1 < v_z/c < 1$
- Forward-peaked angular profile (no ELN-XLN crossings)

$$g(v_z, \sigma) \propto e^{-(v_z-1)^2/(2\sigma^2)}$$

- Antineutrino-neutrino number density ratio

$$\alpha \equiv n_{\bar{\nu}_e} / n_{\nu_e}$$

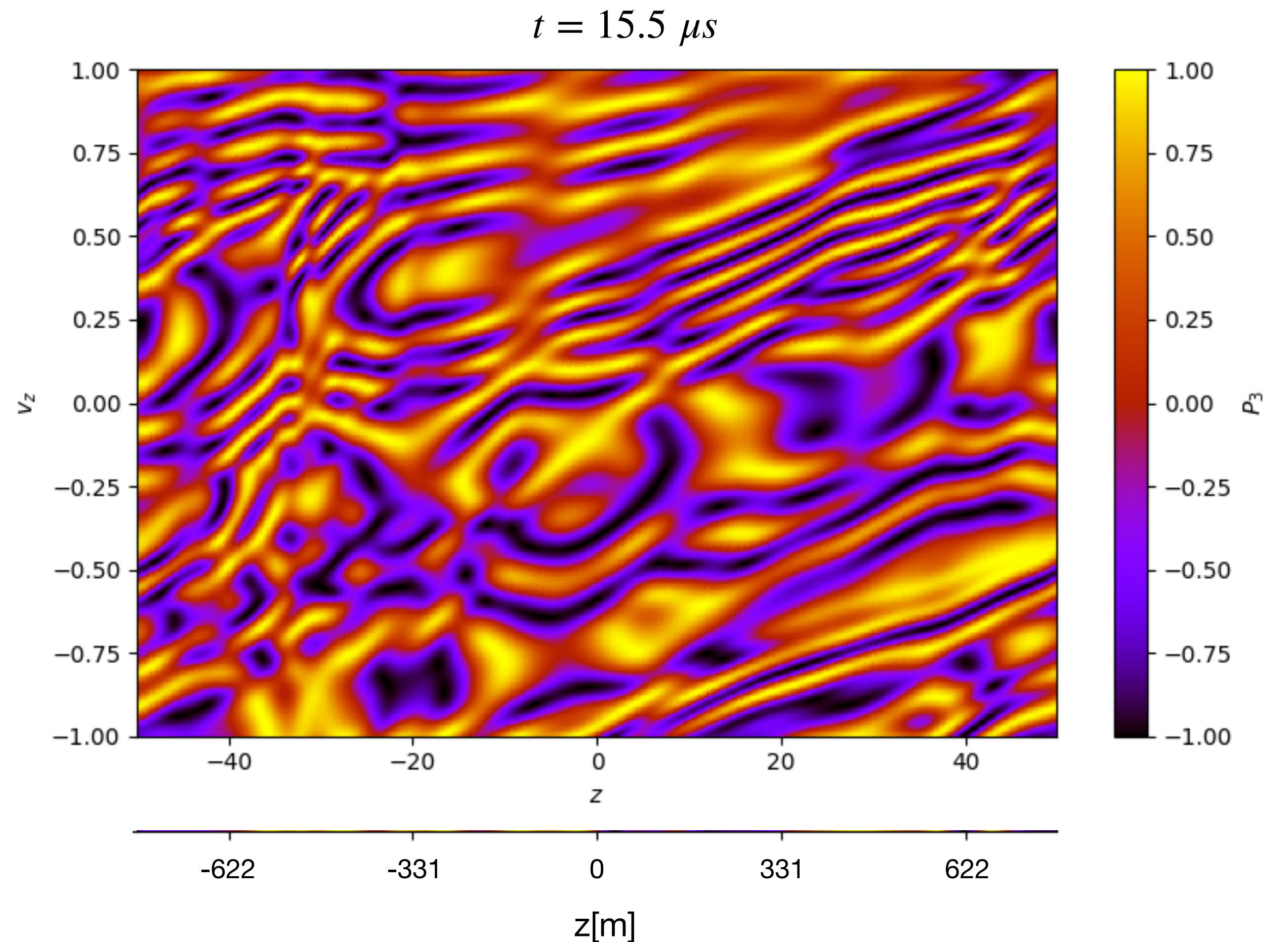
- Effectively no $\nu_x, \bar{\nu}_x$ flavors



Neutrino gas in a box

Method

- Solve nonlinear PDEs with COSE ν [M. George et al, 2203.12866]
- Discretization z (1000 bins)
- Discretization v_z (512 bins)
- Advection: Finite difference method
- Time evolution: RK4



Early dynamics

Linear stability analysis

- Early dynamics ($t \ll 1/\omega$) described by ρ_{ex}

$$\rho = \begin{pmatrix} \rho_{ee}(t_0) - \Delta(t) & \rho_{ex}(t) \\ \rho_{ex}^*(t) & \rho_{xx}(t_0) + \Delta(t) \end{pmatrix}$$

Diagonal terms evolve as $\Delta \sim \mathcal{O}(\rho_{ex}^2)$

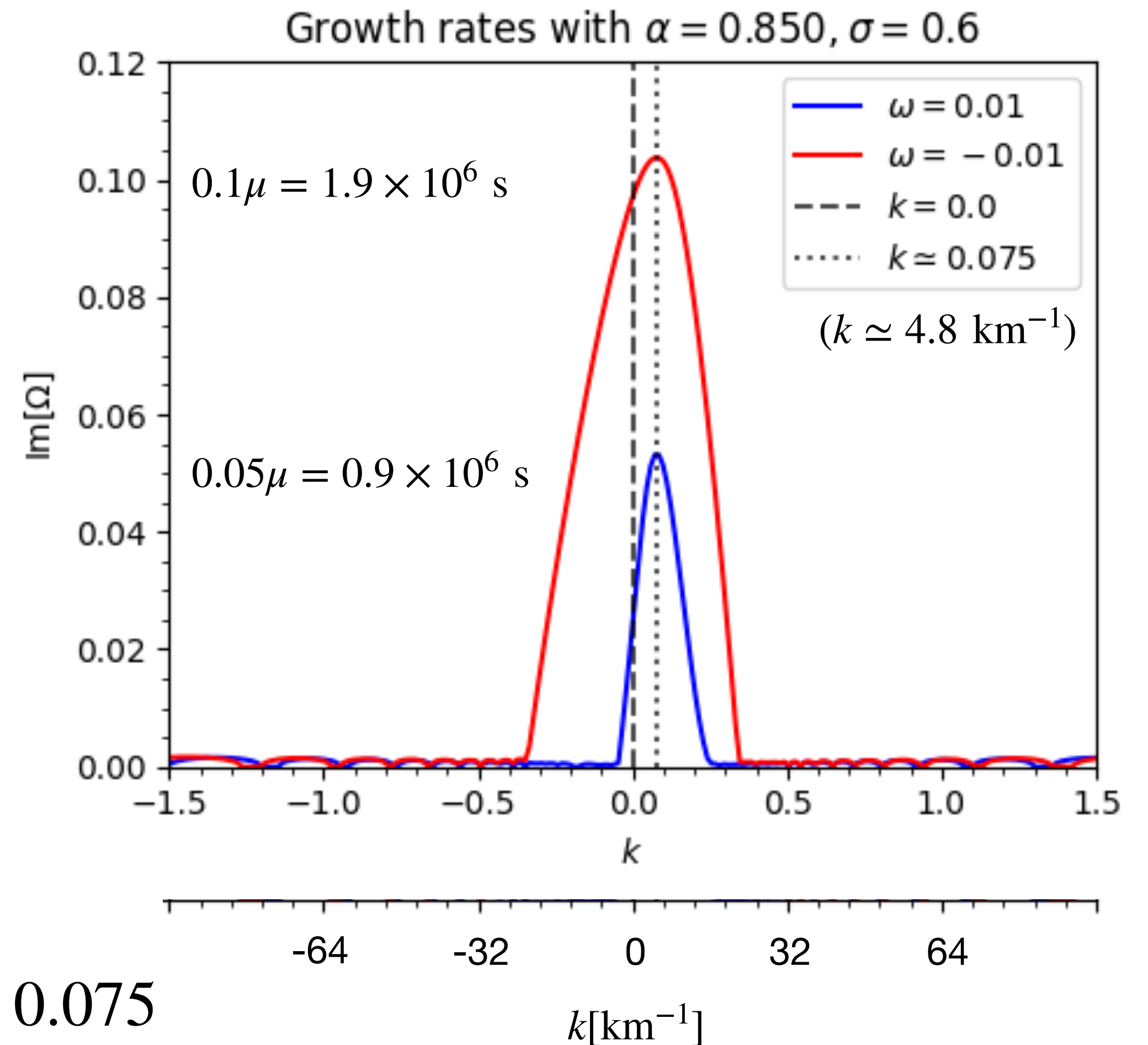
- Dynamics of off-diagonal terms

$$\rho_{ex}(v_z) \propto \sum_i C_j e^{-i(\Omega_j t - k_j z)}$$

$\text{Im}[\Omega](k)$: timescale, box size and resolution required

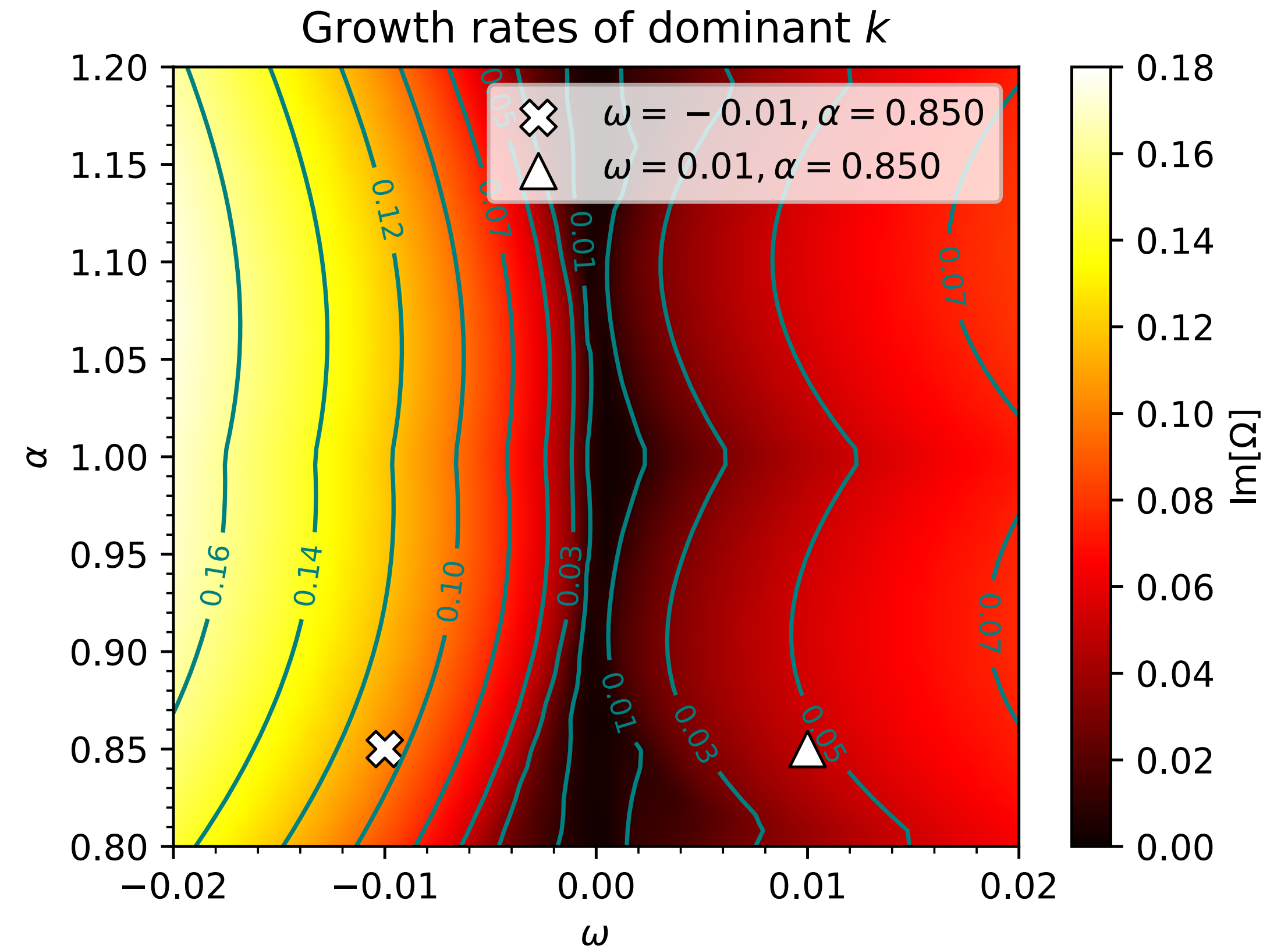
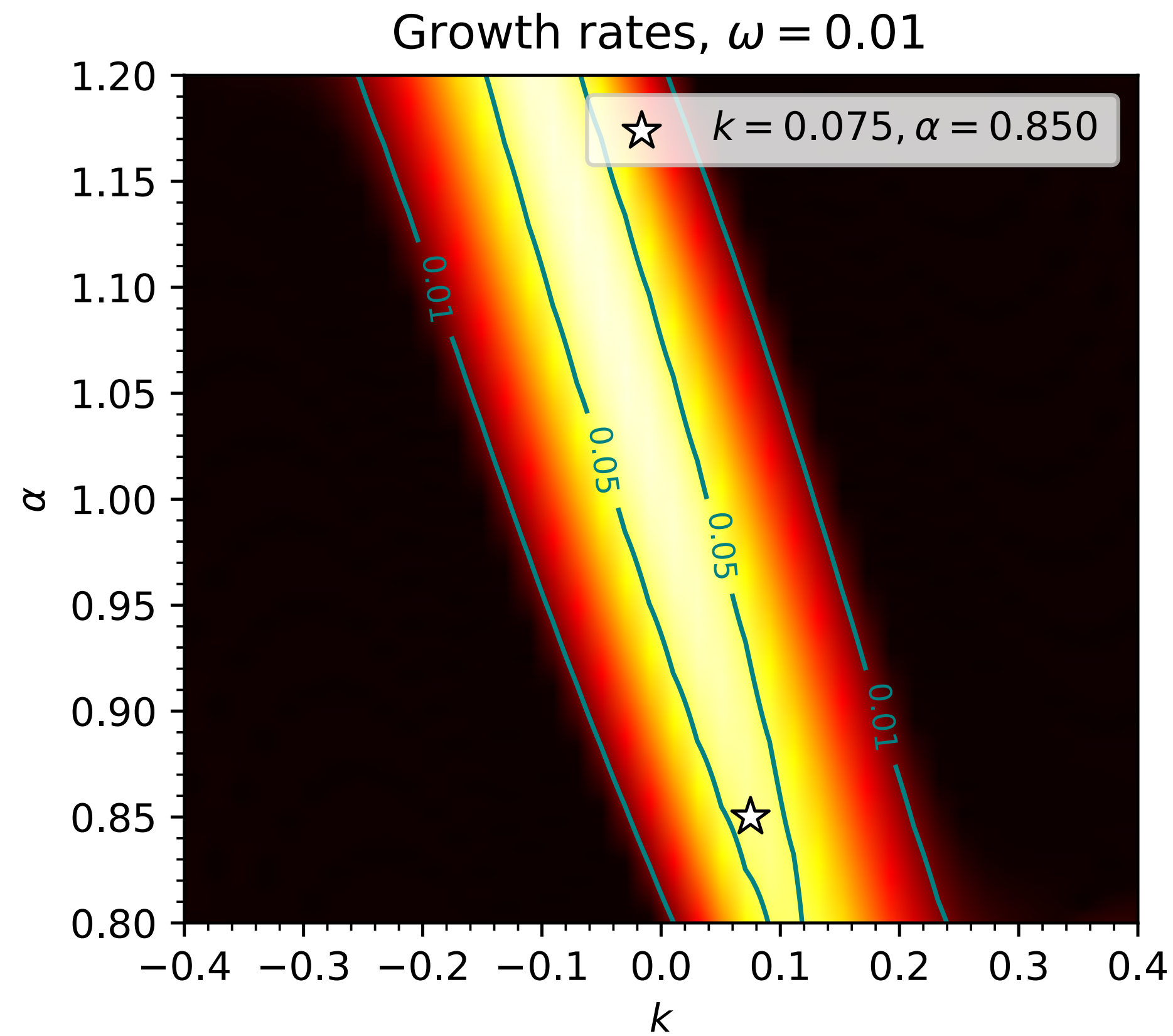
$$\text{“Zero mode”}: K = k - \int dv v (\rho_{ee} - \bar{\rho}_{ee}) = 0 \rightarrow k \simeq 0.075$$

(If isotropic: $k \simeq 0$)



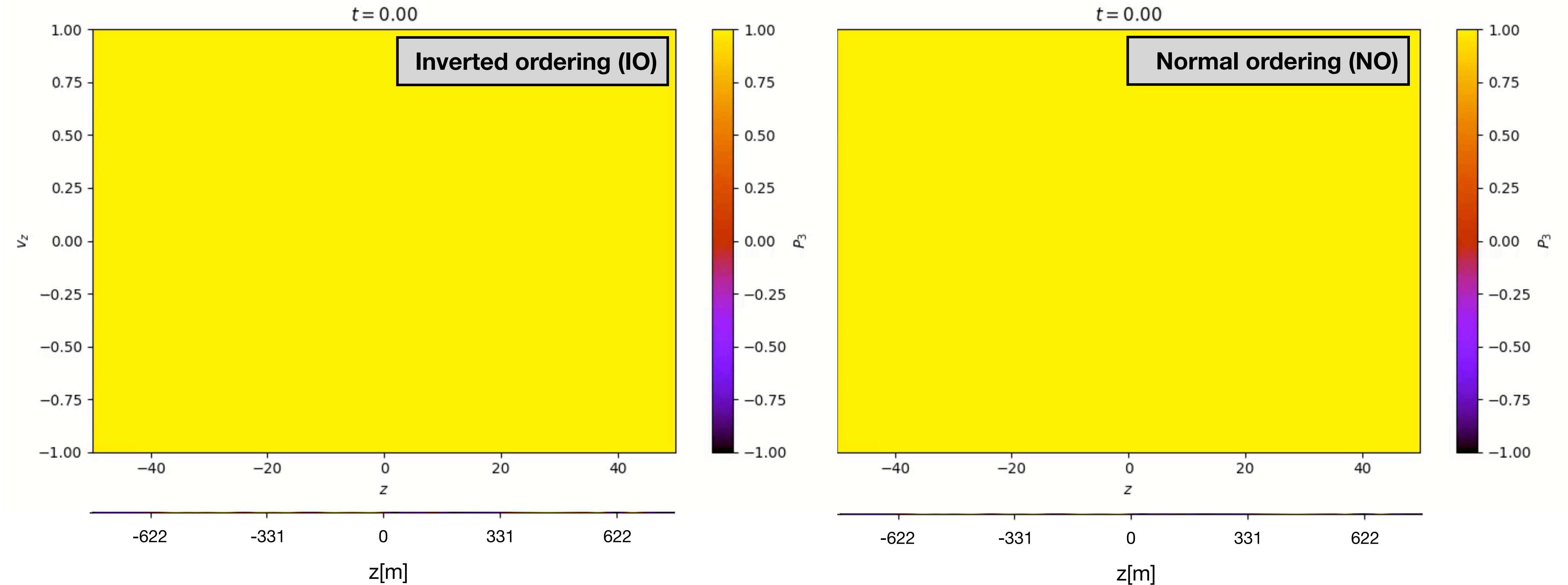
Early dynamics

Linear stability analysis



Flavor evolution: coarse-grained study

Final $t \simeq 52 \mu s$



Flavor evolution: coarse-grained study

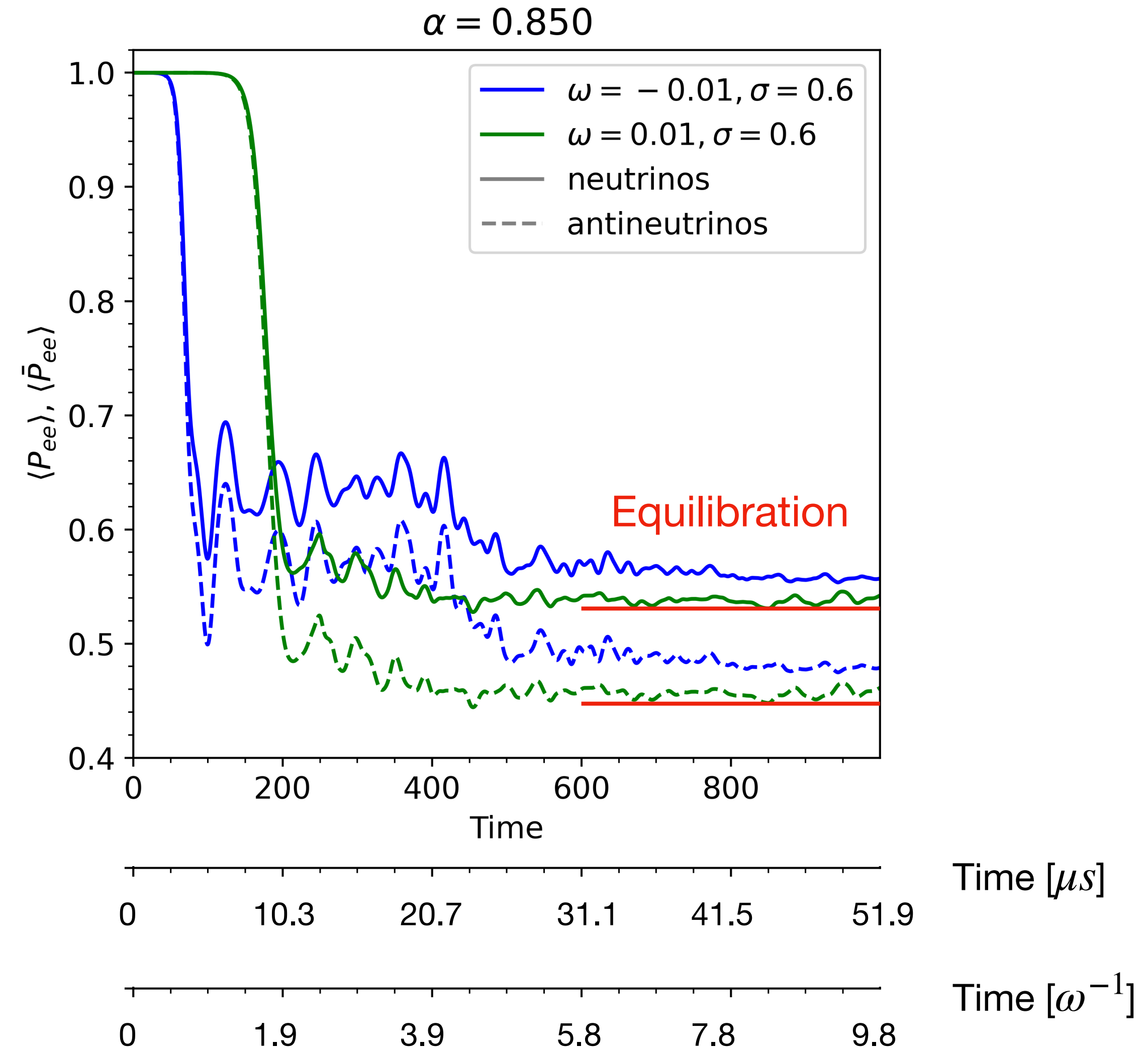
- Coarse-grained probability

$$\langle P \rangle_{z, v_z}(t) = \frac{\int dz dv_z \rho_{ee}(z, v_z, t)}{\int dz dv_z \rho_{ee}(z, v_z, t=0)}$$

- Flavor equilibration reached within the timescale $\sim 6\omega^{-1}$ (i.e. $31 \mu\text{s}$)
- Conservation of lepton number requires relation between probabilities

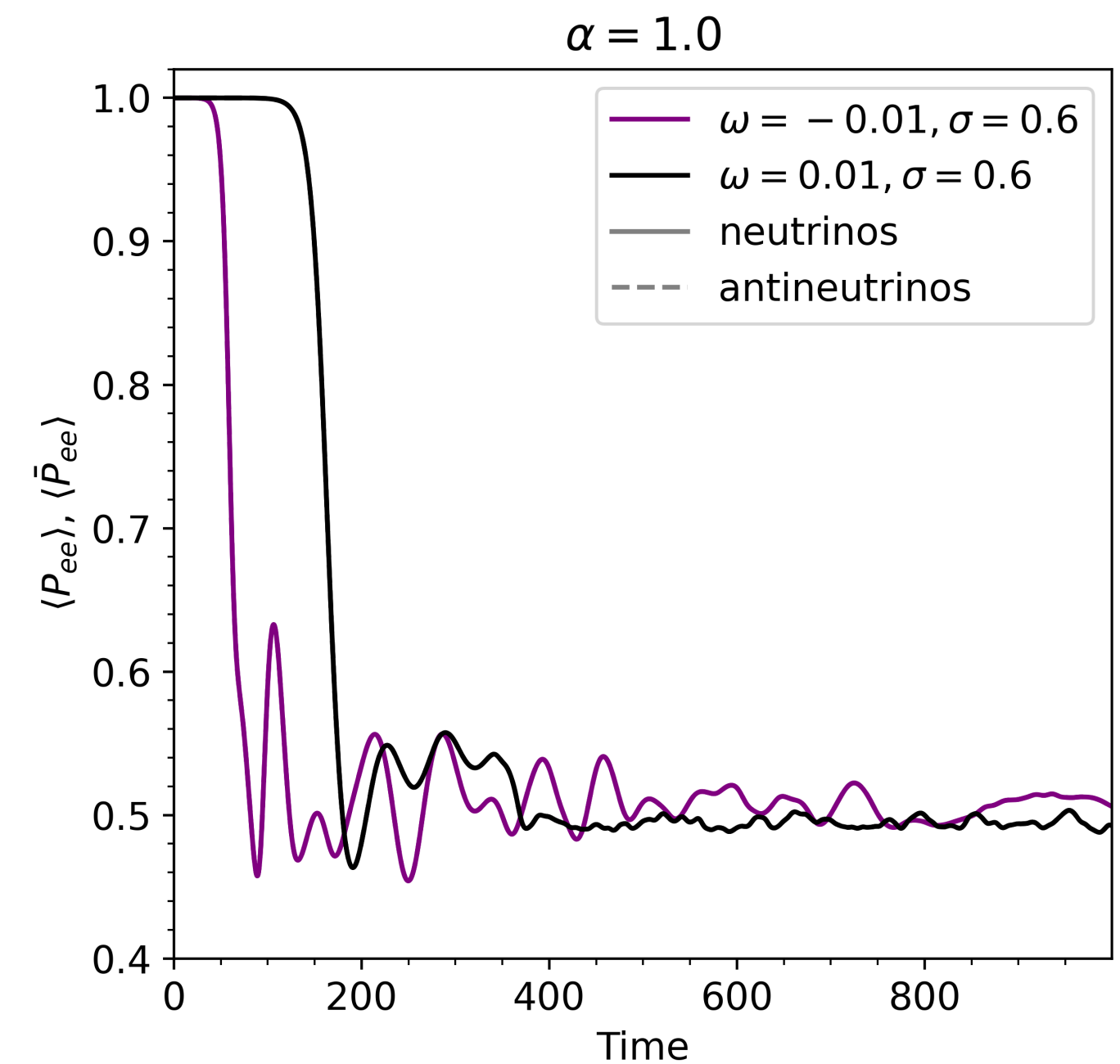
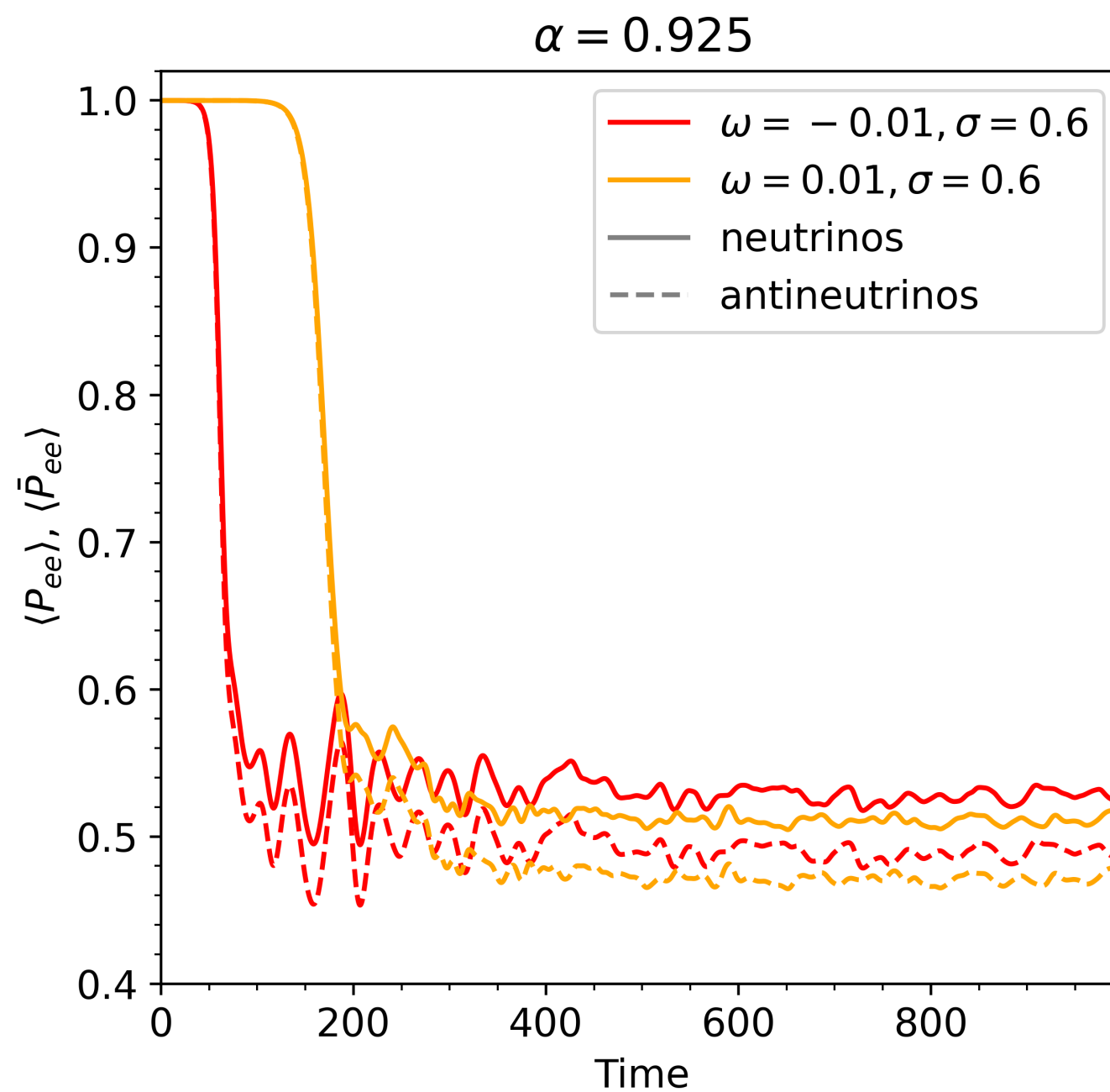
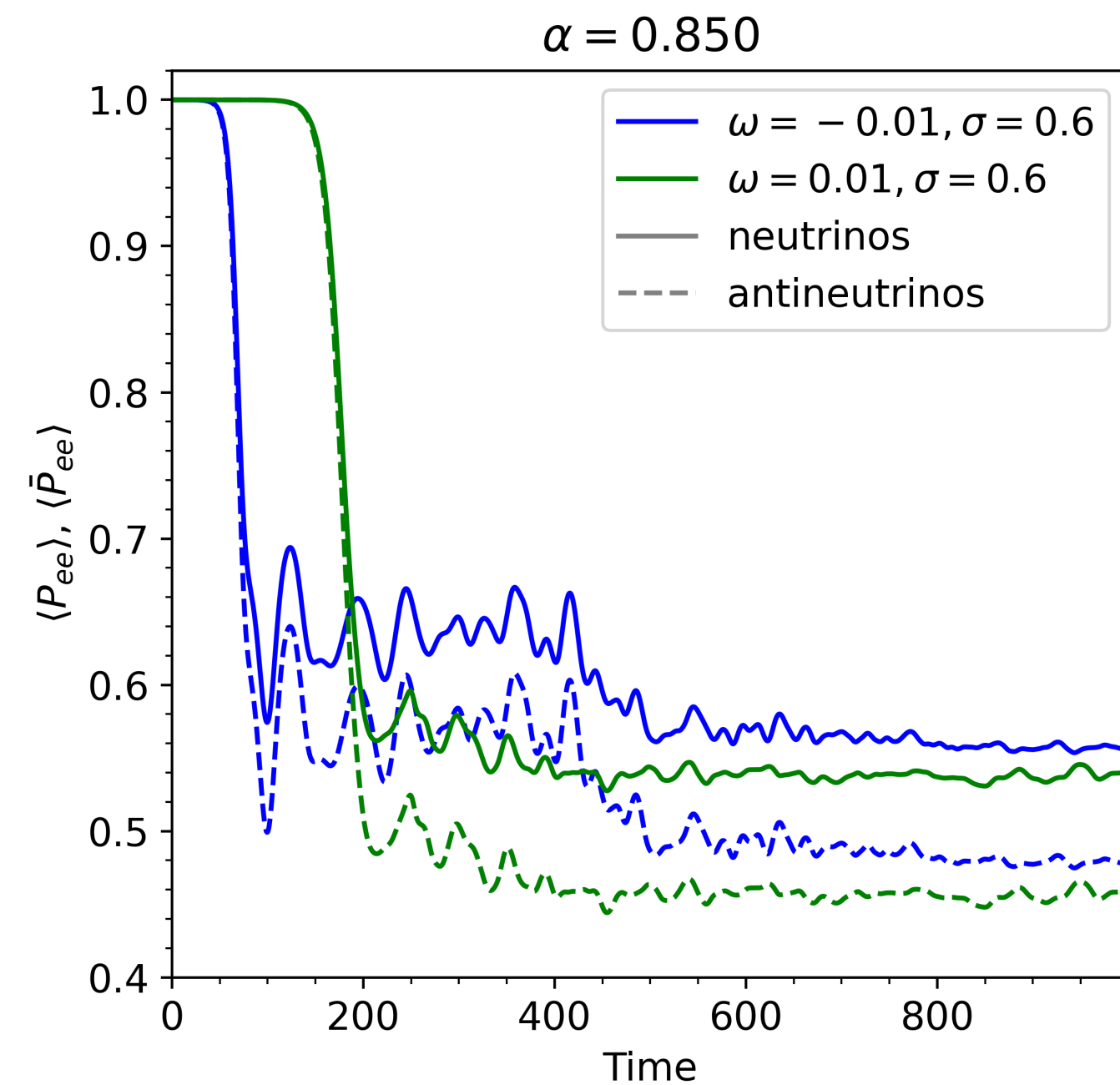
$$1 - \langle P \rangle_{z, v_z} \sim \alpha(1 - \langle \bar{P} \rangle_{z, v_z})$$

(remember $\alpha = n_{\bar{\nu}_e} / n_{\nu_e}$)



Flavor evolution: coarse-grained study

- Coarse-grained probability



$$1 - \langle P \rangle_{z, \nu_z} \sim \alpha (1 - \langle \bar{P} \rangle_{z, \nu_z})$$

(remember $\alpha = n_{\bar{\nu}_e} / n_{\nu_e}$)

0 10.3 20.7 31.1 41.5 51.9 Time [μs]

0 1.9 3.9 5.8 7.8 9.8 Time [ω^{-1}]

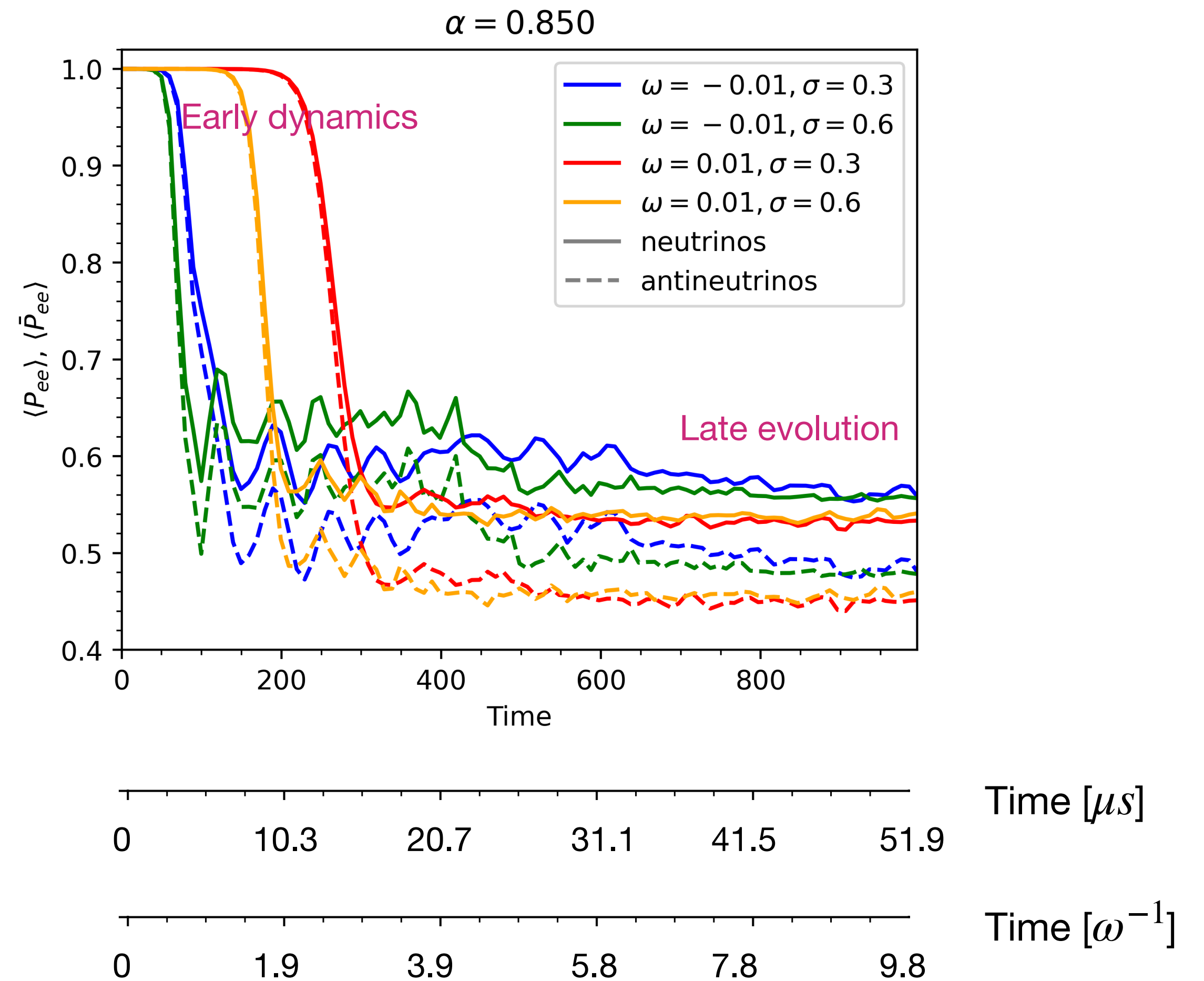
Flavor evolution: coarse-grained study

Early dynamics

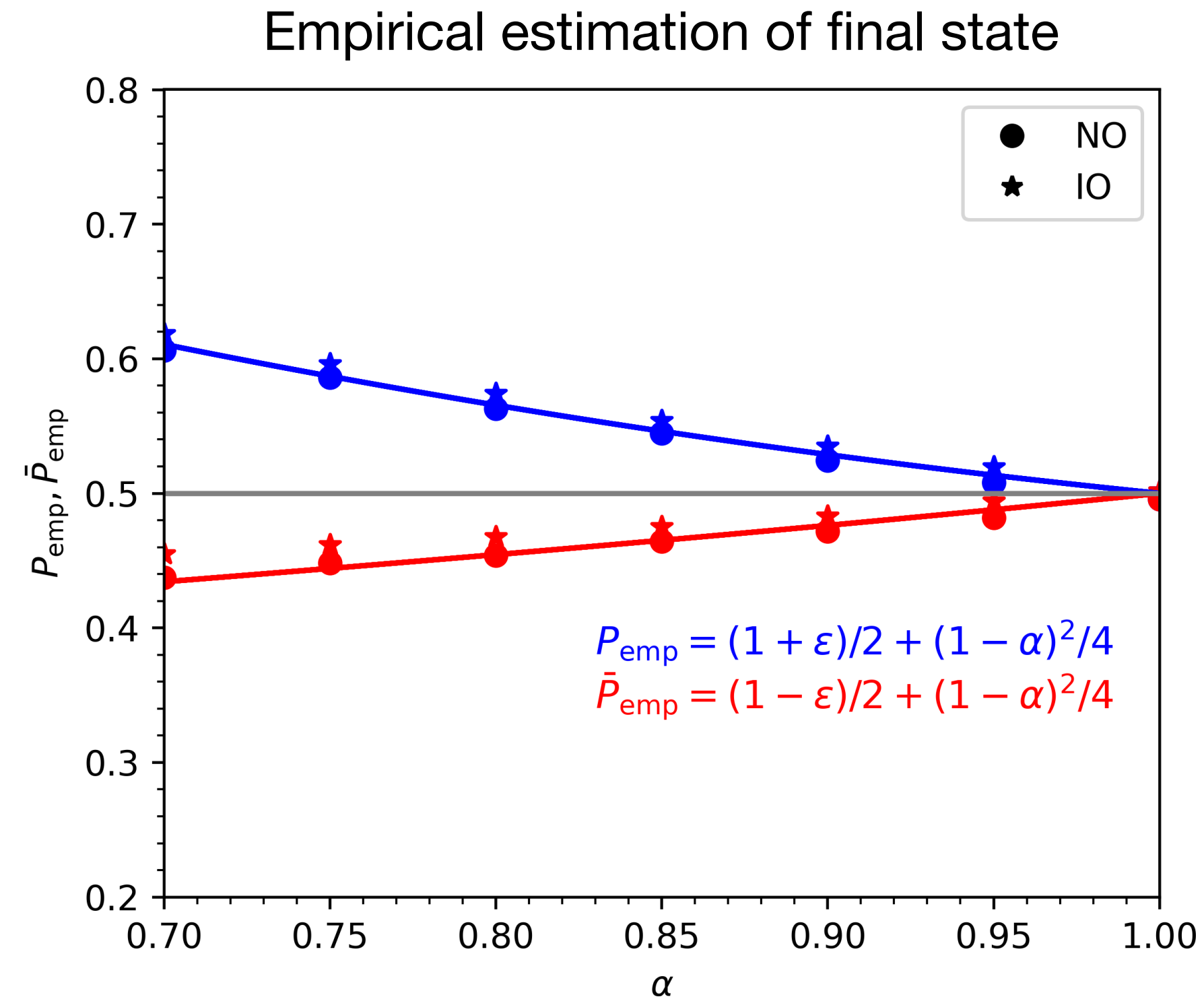
- Depends on angular distributions
- Faster growth rate in IO than NO

Late evolution

- Doesn't depend on angular distributions
- Marginally depends on mass ordering (next slides)

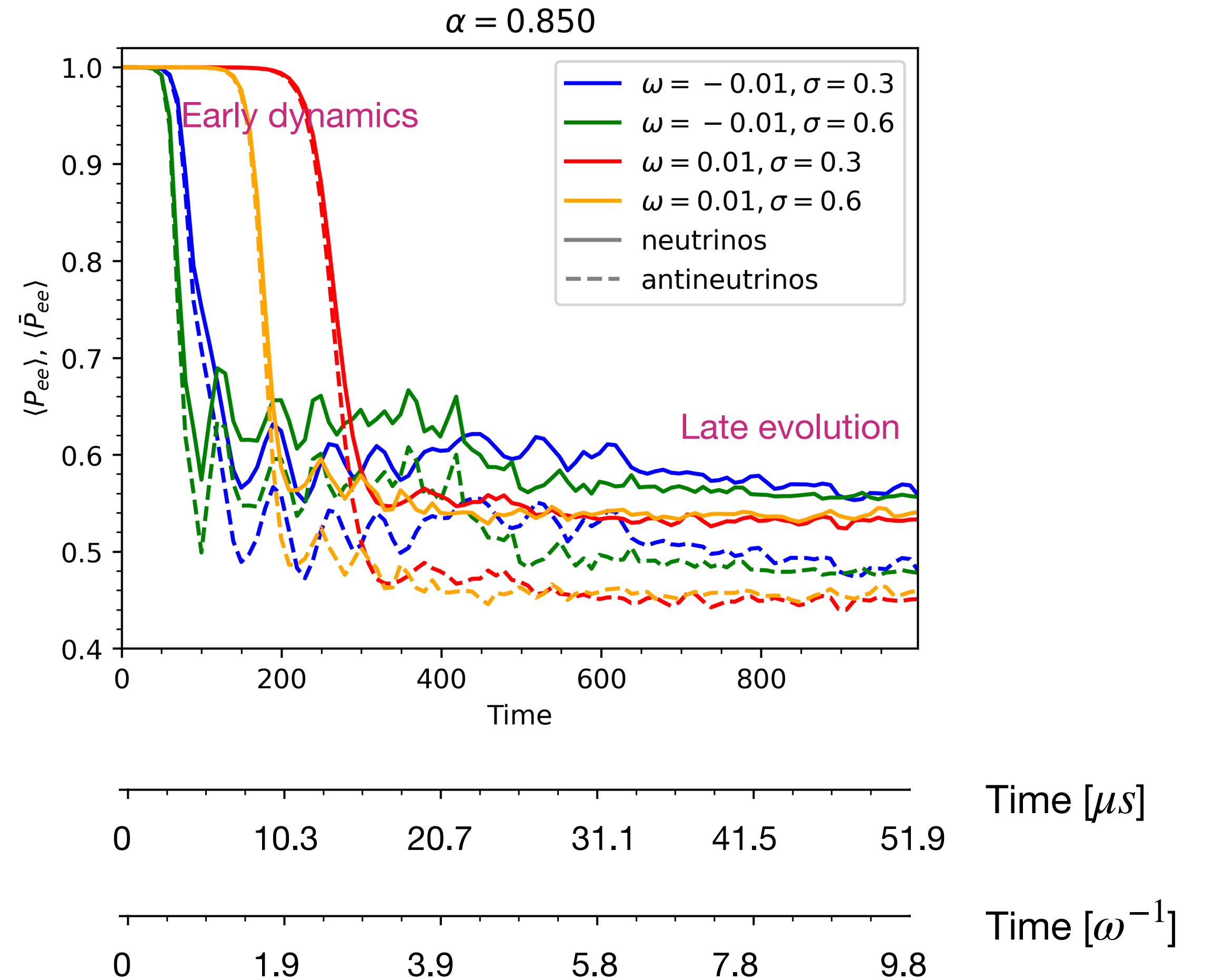


Flavor evolution: coarse-grained study



ratio between lepton and particle number (see Fiorillo et al 2412.02747)

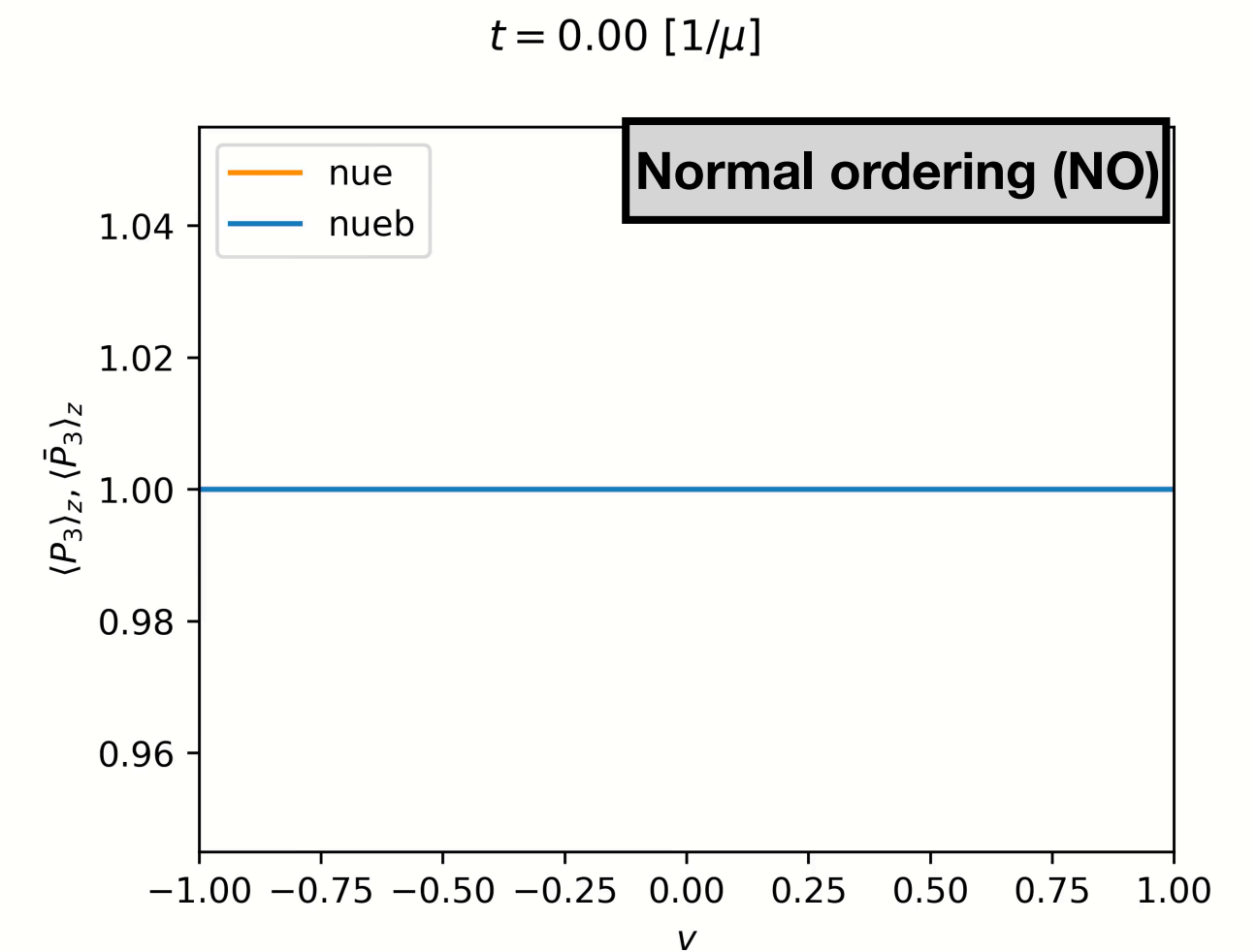
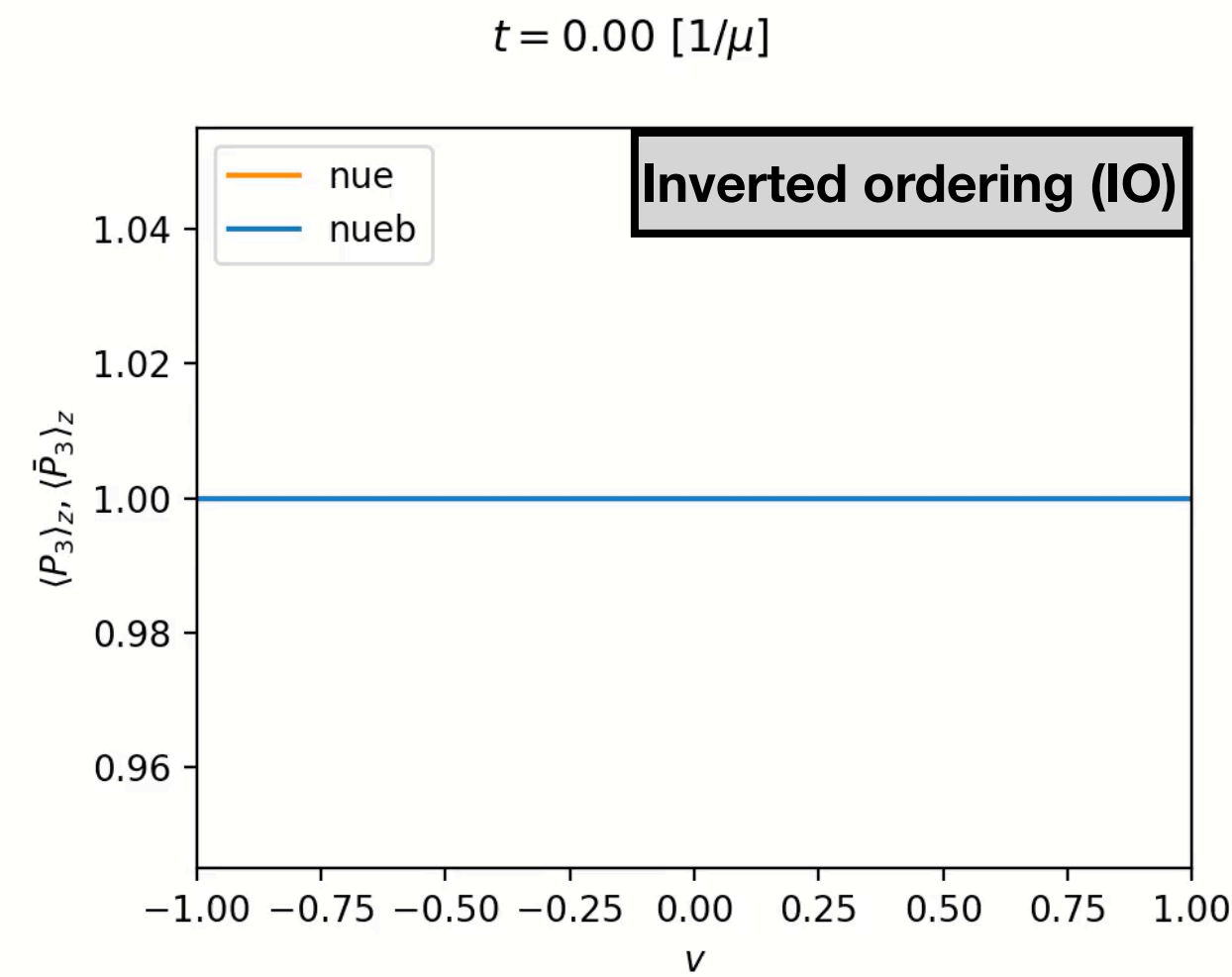
$$\epsilon \equiv |1 - \alpha| / (1 + \alpha)$$



Flavor evolution: coarse-grained study

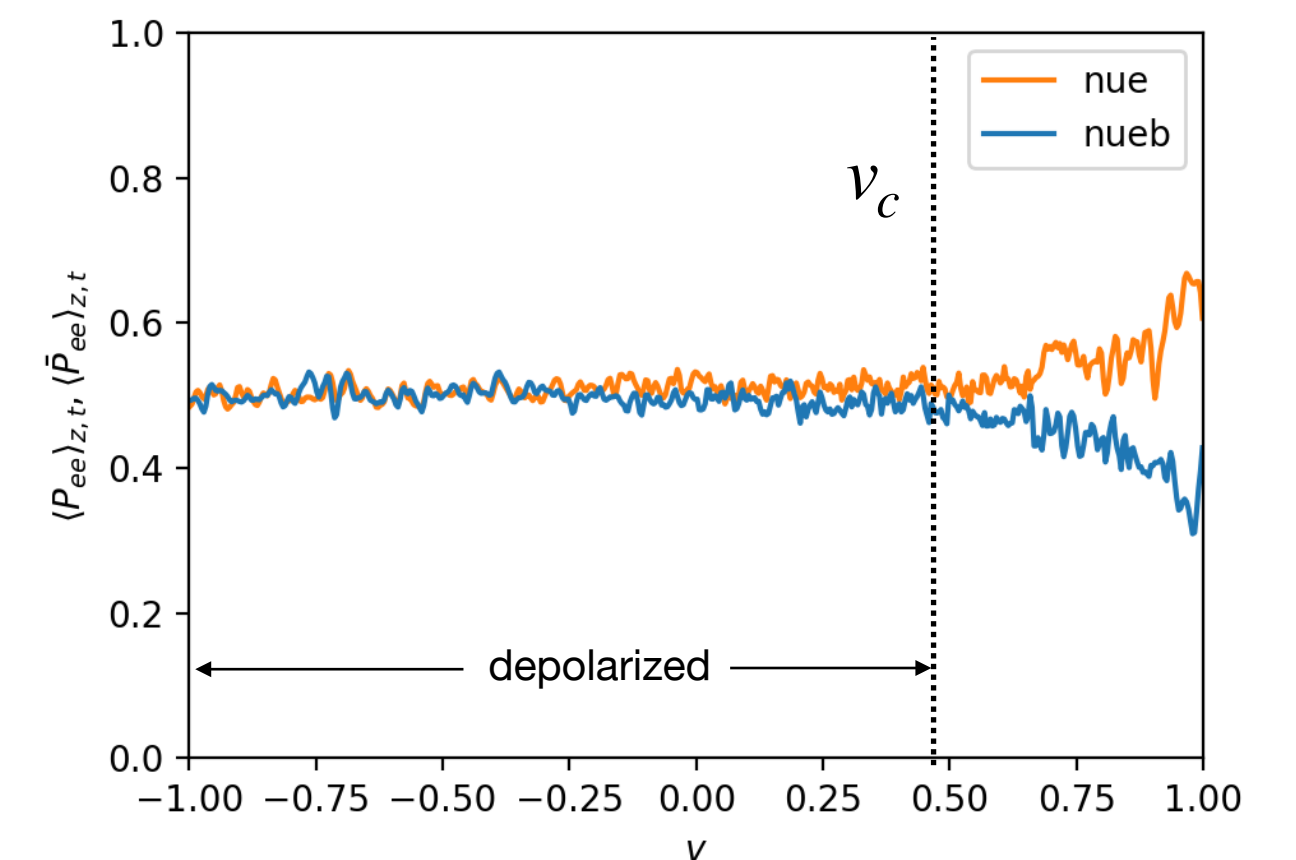
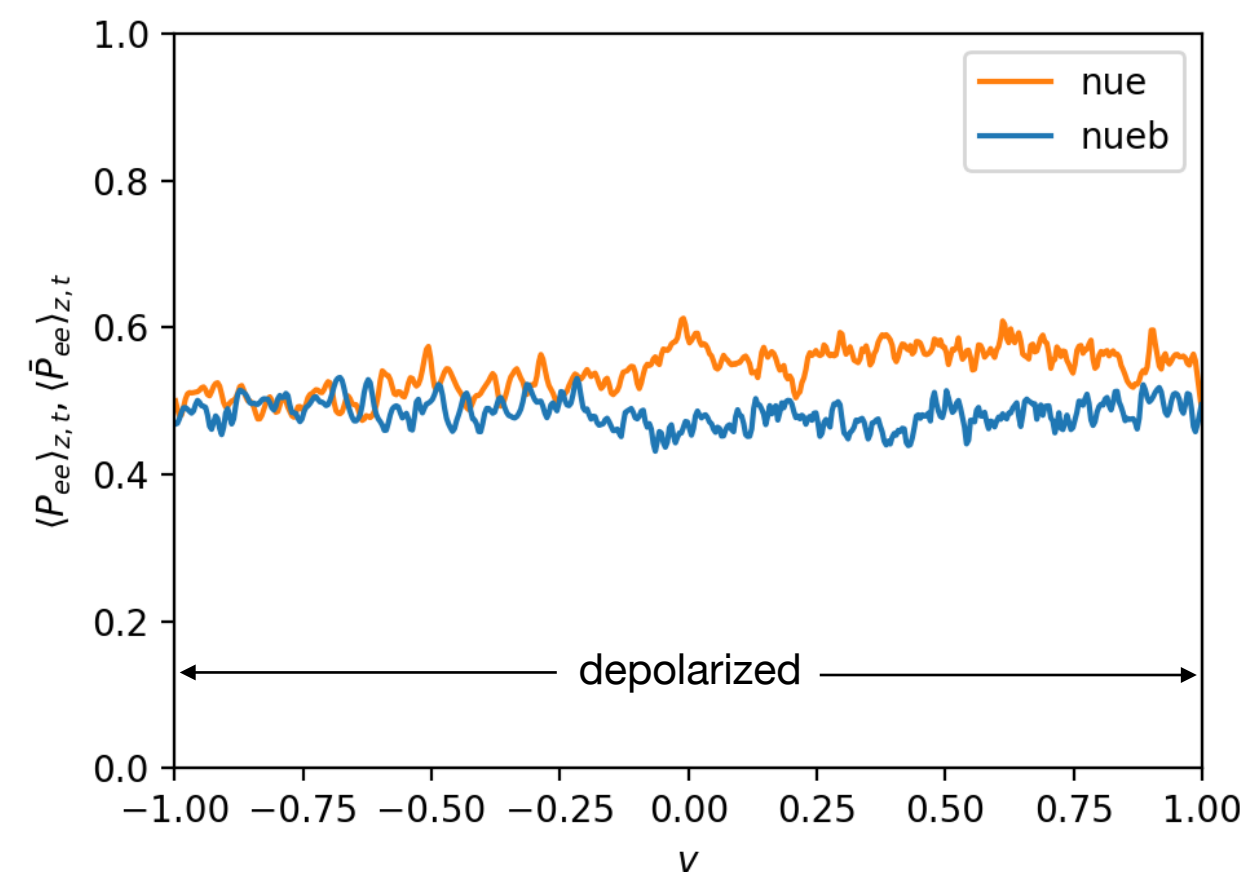
The time-averaged angular distributions depends on the mass ordering:

- In IO, deviation from flavor equilibration depends less on the velocity.
- In NO, for $\nu < \nu_c$: **completely depolarized**.
- In NO, for $\nu > \nu_c$: the deviation from flavor equilibration becomes more **pronounced***



Time-averaged $\langle \bar{P} \rangle$

Time-averaged $\langle P \rangle$

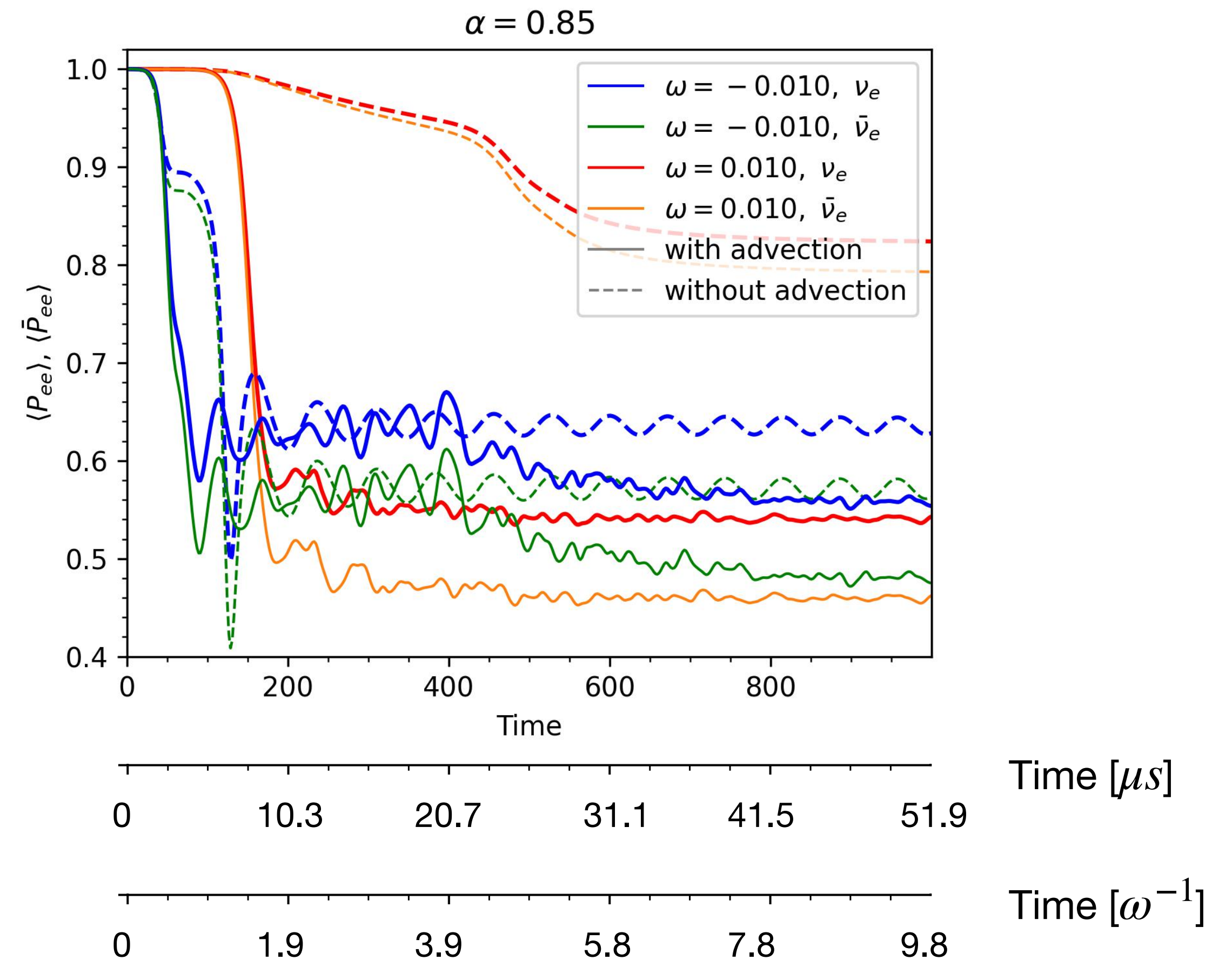


* ν_c can be explained by looking at the eigenfunctions from the LSA

Flavor evolution: coarse-grained study

Role of neutrino advection

- It strongly influences the quasi-steady state
- Normal ordering more severely affected (remember $\omega > 0$ case dominated by non homogeneous modes)



Main takeaways

- Dense neutrino gases in astrophysical environments exhibit **nonlinear collective flavor evolution** that cannot be resolved in global simulations
- Quantum kinetic calculations show that unstable systems very often evolve toward **flavor equipartition**
- Quasi-steady states **can be estimated** based on the ratio of number densities
- **Multi-energy** calculations within the same framework are under investigation