The Advancement of Neutrino Transport for Muonic Matter in Binary Neutron Star Merger

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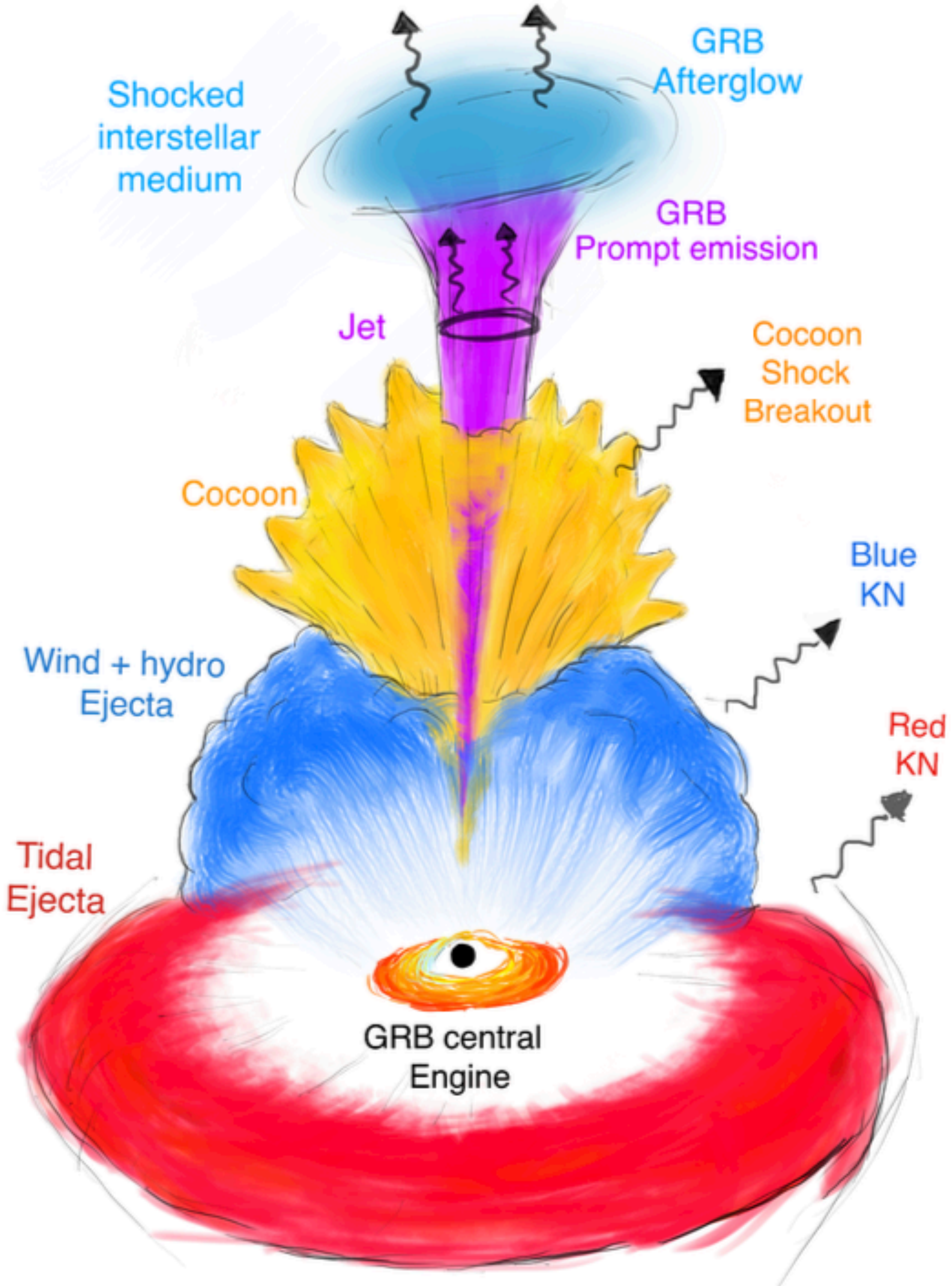
Collective Neutrino Oscillations in Supernovae and Neutron Star Mergers

Kilonova (KN) from BNS mergers (GW170817)

- Disk spiral-wave wind / Secular ejecta dominates:
  *long-term HMNS simulation [Radice’s talk]
- Ejecta properties highly depend on neutrino radiation and microphysics, also the neutrino schemes [Just’s talk]
- R-process nucleosysthesis [Gabriel, Radice, Just, Qiu’s talks]



Ascenzi+ 2020

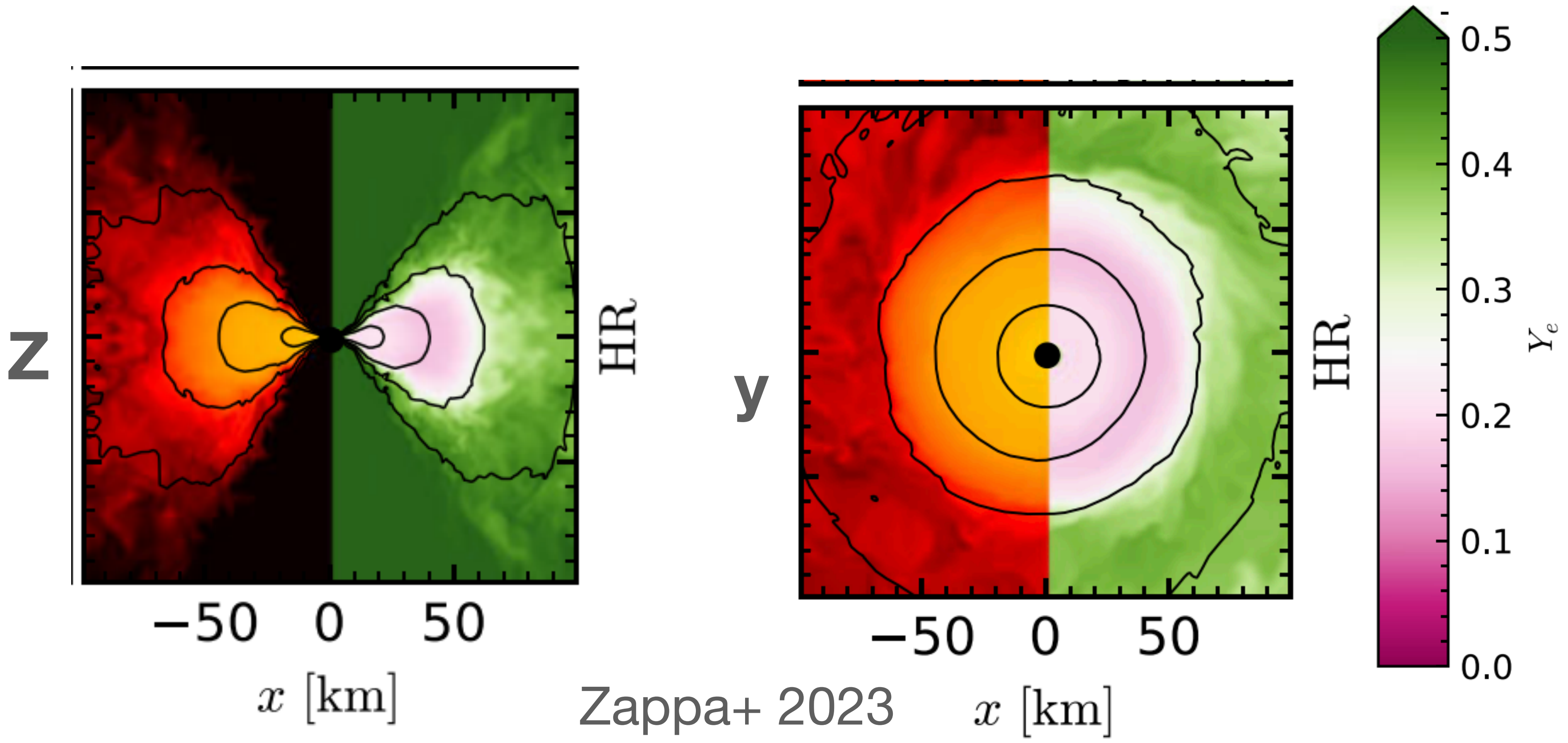
Red component :

=> Neutron-rich ejecta (low Y_p) => heavy elements

Blue component:

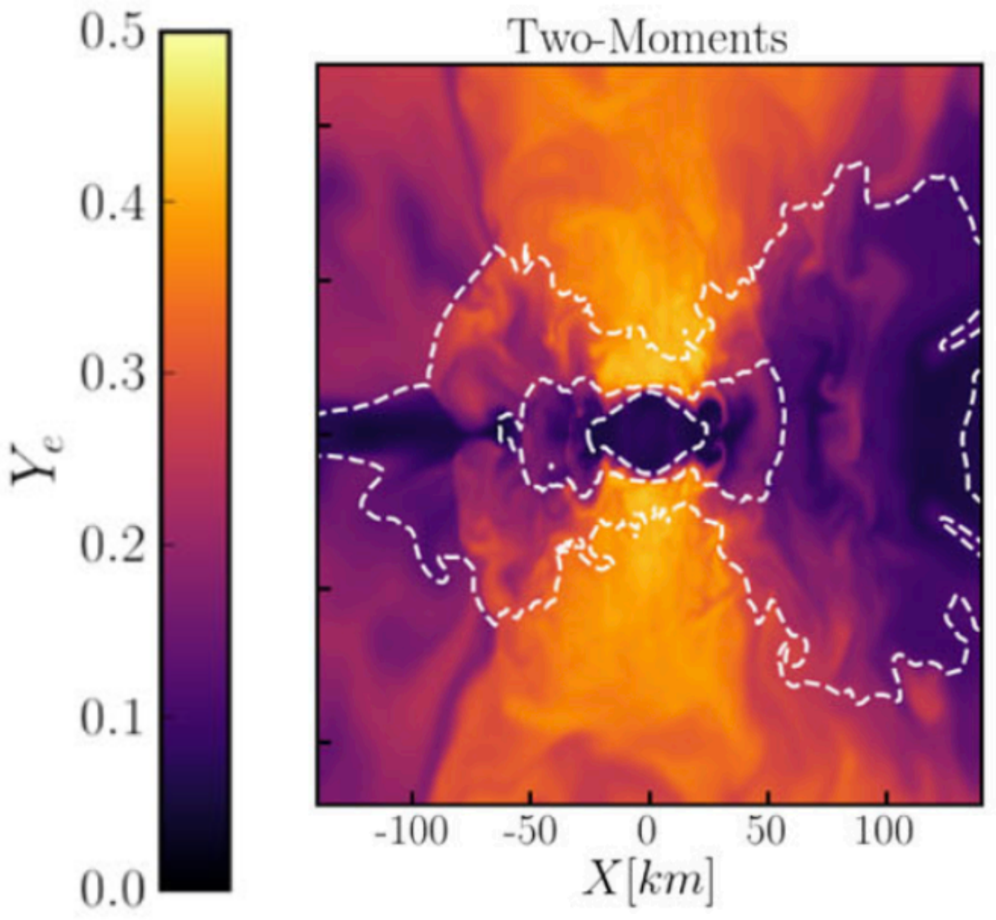
=> Proton-rich ejecta (high Y_p) => light elements

Current state-of-the-art of BNS mergers with neutrino radiation

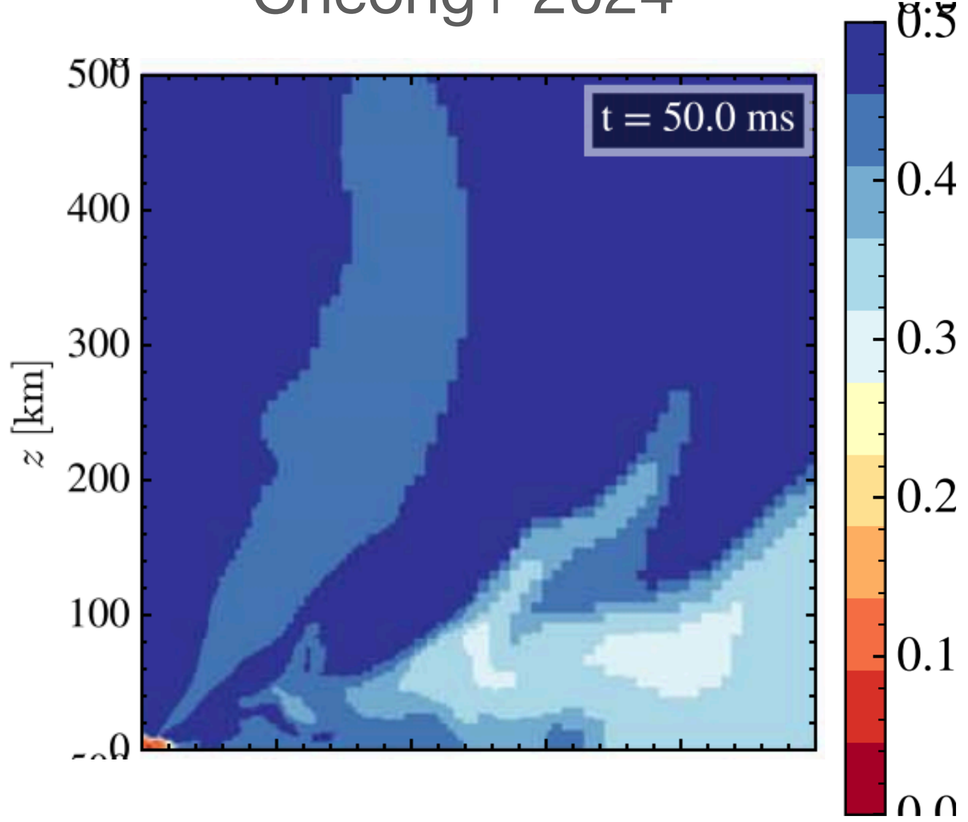


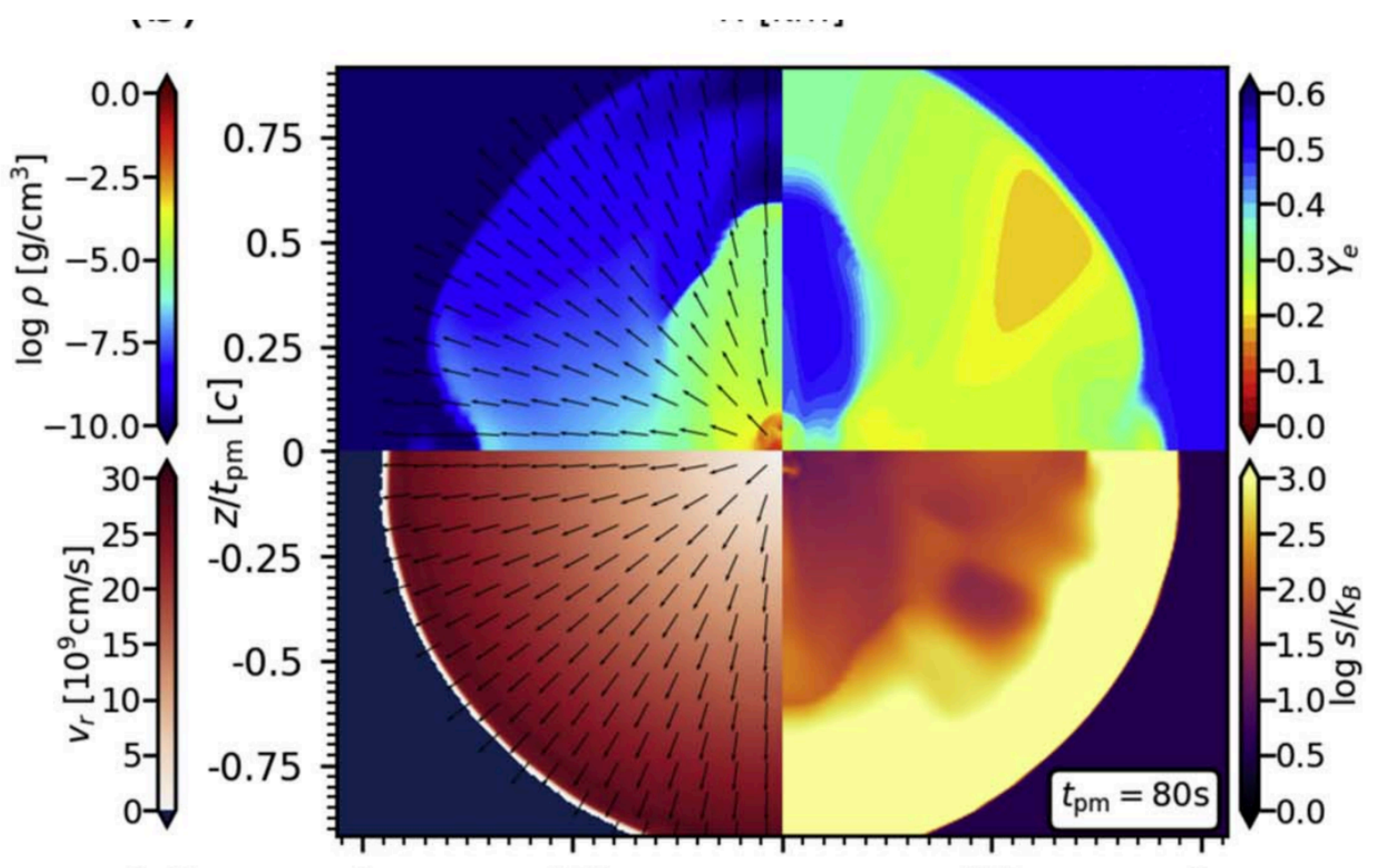
Zappa+ 2023

Foucart+ 2020



Cheong+ 2024





Just+ 2023

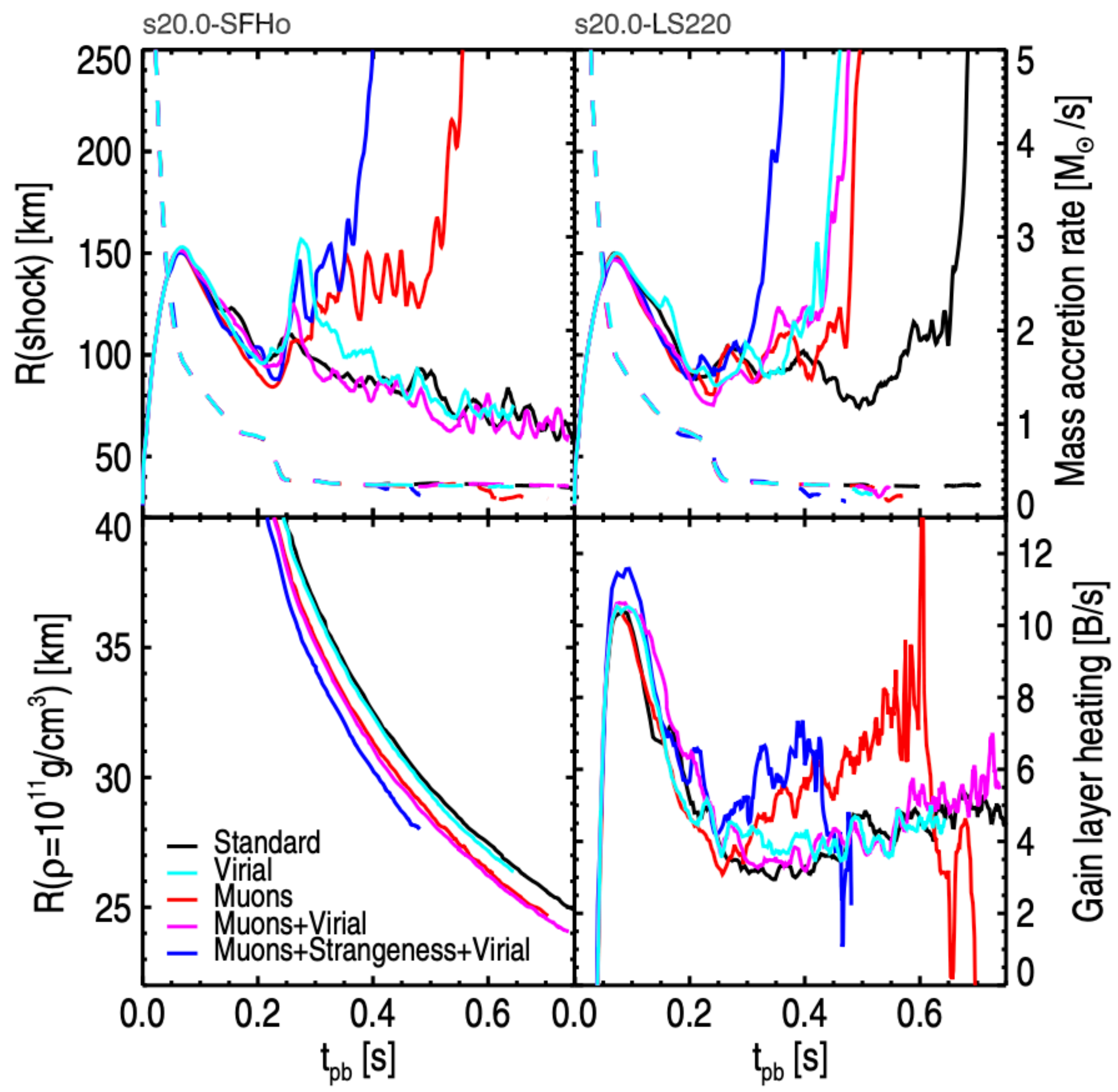
Leads to very fast electronization in early stage of post-merger

$Y_e \uparrow \sim\ 0.3 - 0.4$ very quickly

Are BNS community doing already gd enough neutrino microphysics?

Role of muons in CCSNe

Most of simulations with neutrino scheme:
3-species $(\nu_e, \bar{\nu}_e, \nu_x)$ **–** $\nu_x \in [\nu_\mu, \bar{\nu}_\mu, \nu_\tau, \bar{\nu}_\tau]$
∵ Too high rest-mass: $m_\mu \sim 105\ \mathrm{MeV}$



[Bollig+ PRL 2017]

- Spectral-M1 with muons:
  Easier explosion due to stronger contraction of the soften EOS [Bollig+ PRL 2017]

- **Full boltzmann + muons → modify neutrino signatures**
  [Guo+2020, Fischer+2020]

What abt in muons BNS merger?
Let’s simulate one!

Extension to muonic sims. Based FIL-M1 [Musolino+2023, Most+2019]

- **GRHD:** $\nabla_\mu(\rho u^\mu) = 0, \quad \cancel{\nabla_\mu F^{\mu\nu} = \mathcal{J}^\nu}, \quad \nabla_\mu T^{\mu\nu} = 0, \quad \cancel{\nabla_\mu {}^*F^{\mu\nu}} = 0$
- Einstein Gravity (BSSN-Z4): $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$
- Neutrino transport ≈ Grey M1 Scheme [Musolino+2023]
  3-species $(\nu_e, \bar{\nu}_e, \nu_x) \rightarrow$ 5-species $(\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \nu_x)$
- Nuclear EOS + e^-, e^+, μ^-, μ^+:
  $P(\rho, T, Y_e) \rightarrow P(\rho, T, Y_e, Y_\mu)$
- **Conservation of electronic/muonic lepton numbers**

$\nabla_\alpha(\rho Y_e u^\alpha) = -m_b(\mathcal{N}_{\nu_e} - \mathcal{N}_{\bar{\nu}_e})$ **Conservation of electronic lepton #**

$\nabla_\alpha(\rho Y_\mu u^\alpha) = -m_b(\mathcal{N}_{\nu_\mu} - \mathcal{N}_{\bar{\nu}_\mu})$ **Conservation of muonic lepton #**

- **Neutrino physics:** **~~Oscillation~~**
  Conventional [O'connor 2015, Radice+ 2022, Musolino+ 2023]
  → Developed an advanced neutrino library — `Weakhub` [Ng+ 2024]

Full kinematics Neutrino opacity: [Fischer's talk]

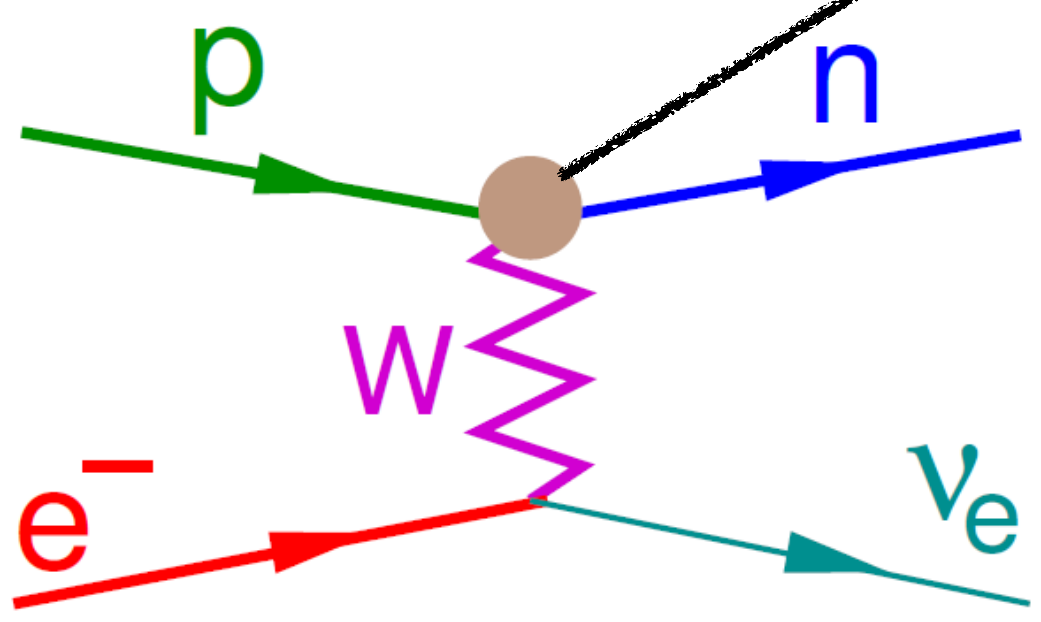
- Full kinematics calculation instead of elastic approx. for β-processes [Bollig+ 2017, Fischer+ 2020, Guo+ 2020]
  - High rest-mass of muons **=>** 4-D phase space integral in an effective 2D approach
  - Strong interaction correction, Weak magnetism, pseudoscalar, Form factor
  - **Code tested with Guo+ 2020 results (see appendix)**
  - **Emissivity :**

$$\nu_e + n \leftrightarrow p + e^- \quad \nu_\mu + n \leftrightarrow p + \mu^-$$

$$\bar{\nu}_e + p \leftrightarrow n + e^+ \quad \bar{\nu}_\mu + p \leftrightarrow n + \mu^+$$

$$Q\left(E_{\nu_e}\right) = 2\int \frac{d^3\boldsymbol{p}_p^*}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}_e}{(2\pi)^3} \int \frac{d^3\boldsymbol{p}_n^*}{(2\pi)^3} \frac{\left\langle |\mathscr{M}|^2 \right\rangle}{16 E_e E_p^* E_{\nu_e} E_n^*}$$
$$\times (2\pi)^4 \delta^{(4)}\left(p_e + p_p - p_{\nu_e} - p_n\right)$$
$$\times f_p \left(1 - f_{\nu_e}\right)\left(1 - f_n\right),$$

Where
$$\langle|\mathcal{M}|^2\rangle = \langle|\mathcal{M}|^2\rangle_{VV} \pm \langle|\mathcal{M}|^2\rangle_{VA} + \langle|\mathcal{M}|^2\rangle_{AA} + \langle|\mathcal{M}|^2\rangle_{VF} \pm \langle|\mathcal{M}|^2\rangle_{AF} + \langle|\mathcal{M}|^2\rangle_{FF} + \langle|\mathcal{M}|^2\rangle_{AP} + \langle|\mathcal{M}|^2\rangle_{PP},$$



(*) Strong interaction corrections at dense matter for nucleon propagator

Weakhub, Ng+ 2024

Initial setups:

$|\vec{B}| = 0$

- Cold Neutron star with $t \to \infty$: Cold electronic & muonic beta equilibrium conditions

$$0 = \mu_e + \mu_p - \mu_n \ , \quad 0 = \mu_\mu + \mu_p - \mu_n$$

- Charge Neutrality $Y_p = Y_e + Y_\mu$
- finest $\Delta x = 0.18 \ \mathrm{M}_\odot$
- **Full 3D cartesian**
- Total mass: 2.5 $M_\odot$, q = 1, SFHo and DD2

$\nu_e + n \leftrightarrow p + e^-$	Electronic Beta processes
$\bar{\nu}_e + p \leftrightarrow n + e^+$	
$\nu + N \leftrightarrow \nu + N$	elastic scattering
$\nu + A \leftrightarrow \nu + A$	
$\gamma^*_{\mathrm{T}} \leftrightarrow \nu + \bar{\nu}$	Plasma process
$e^- + e^+ \leftrightarrow \nu + \bar{\nu}$	electron-positron pair annihilation
$N + N \leftrightarrow N + N + \nu + \bar{\nu}$	Nucleon-Nucleon bremsstrahlung

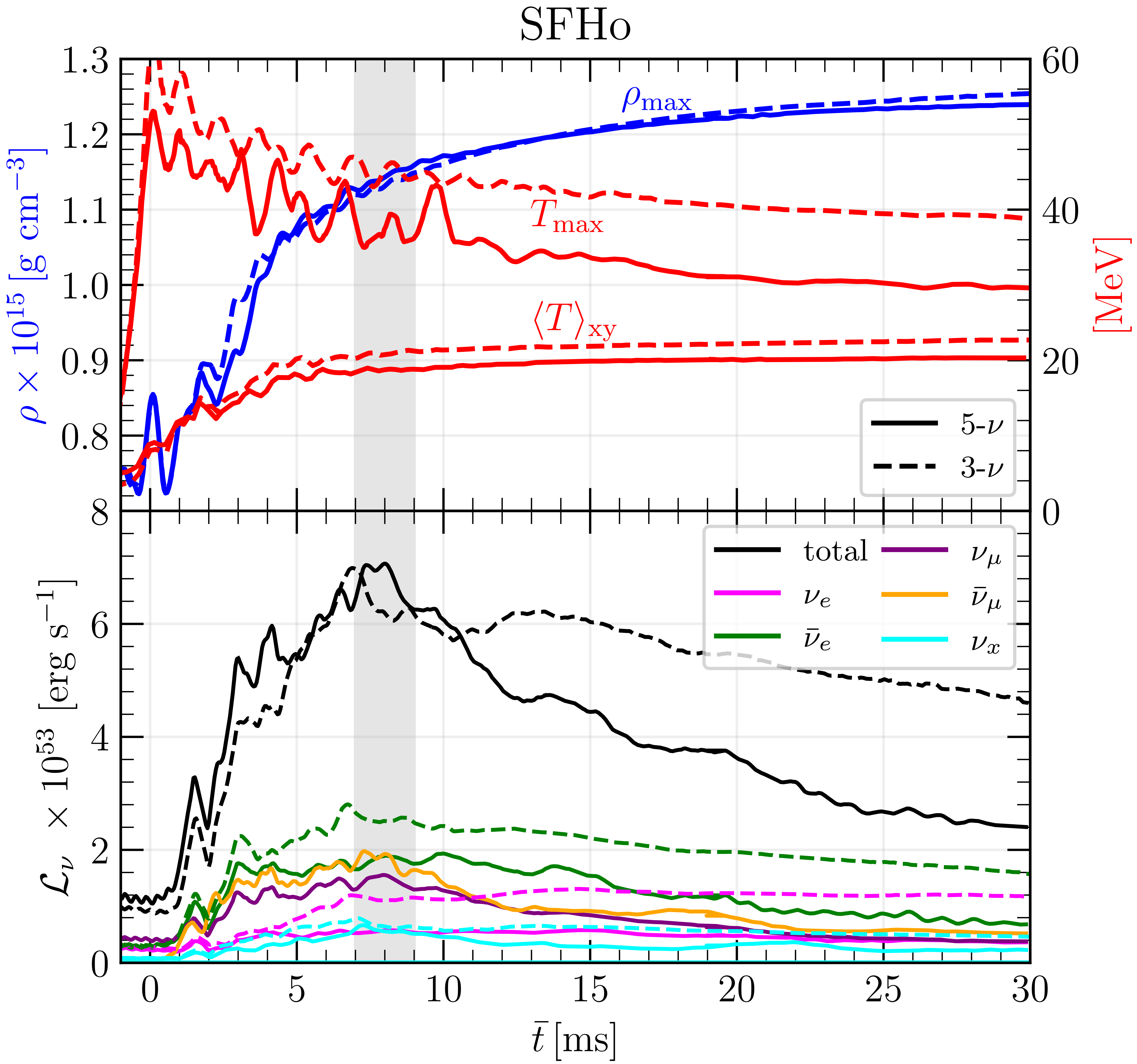
$5 - \nu$: 5-neutrino species ($npe\mu$-matter) $- \ \nu_x \in [\nu_\tau, \bar{\nu}_\tau]$

$\nu_\mu + n \leftrightarrow p + \mu^-$	Muonic Beta processes
$\bar{\nu}_\mu + p \leftrightarrow n + \mu^+$	

Spectral opacity → energy-averaging

Density/Temperature/Neutrino signals

$\bar{t}$ is postmerger time



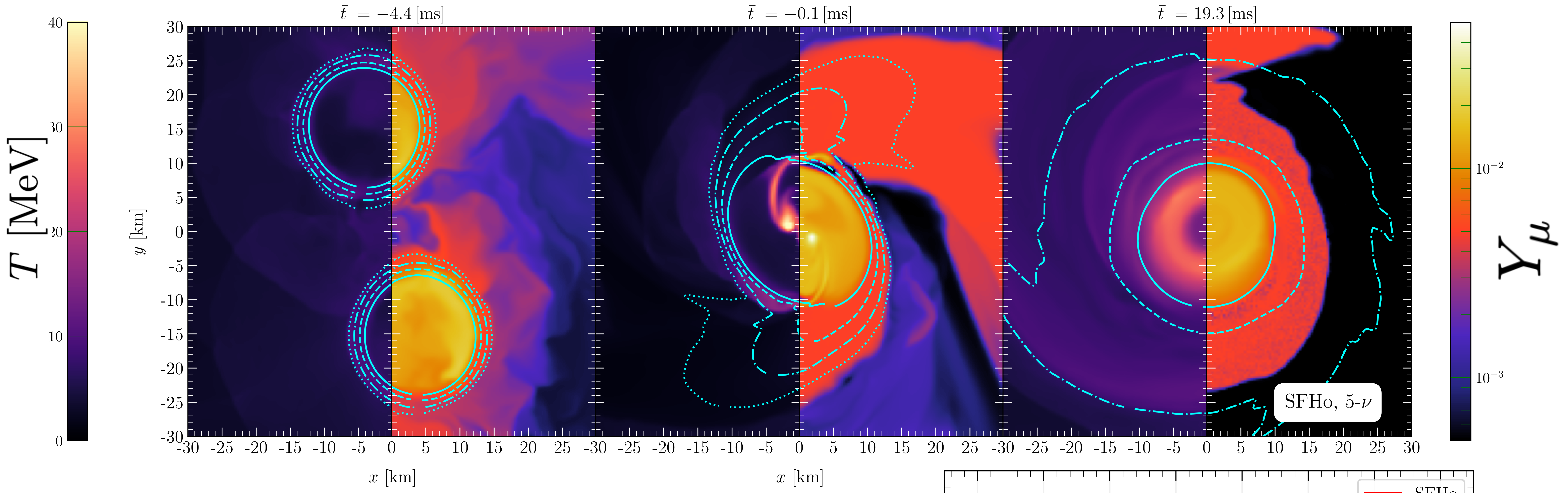
$5-\nu$ **Stronger Cooling !!!**

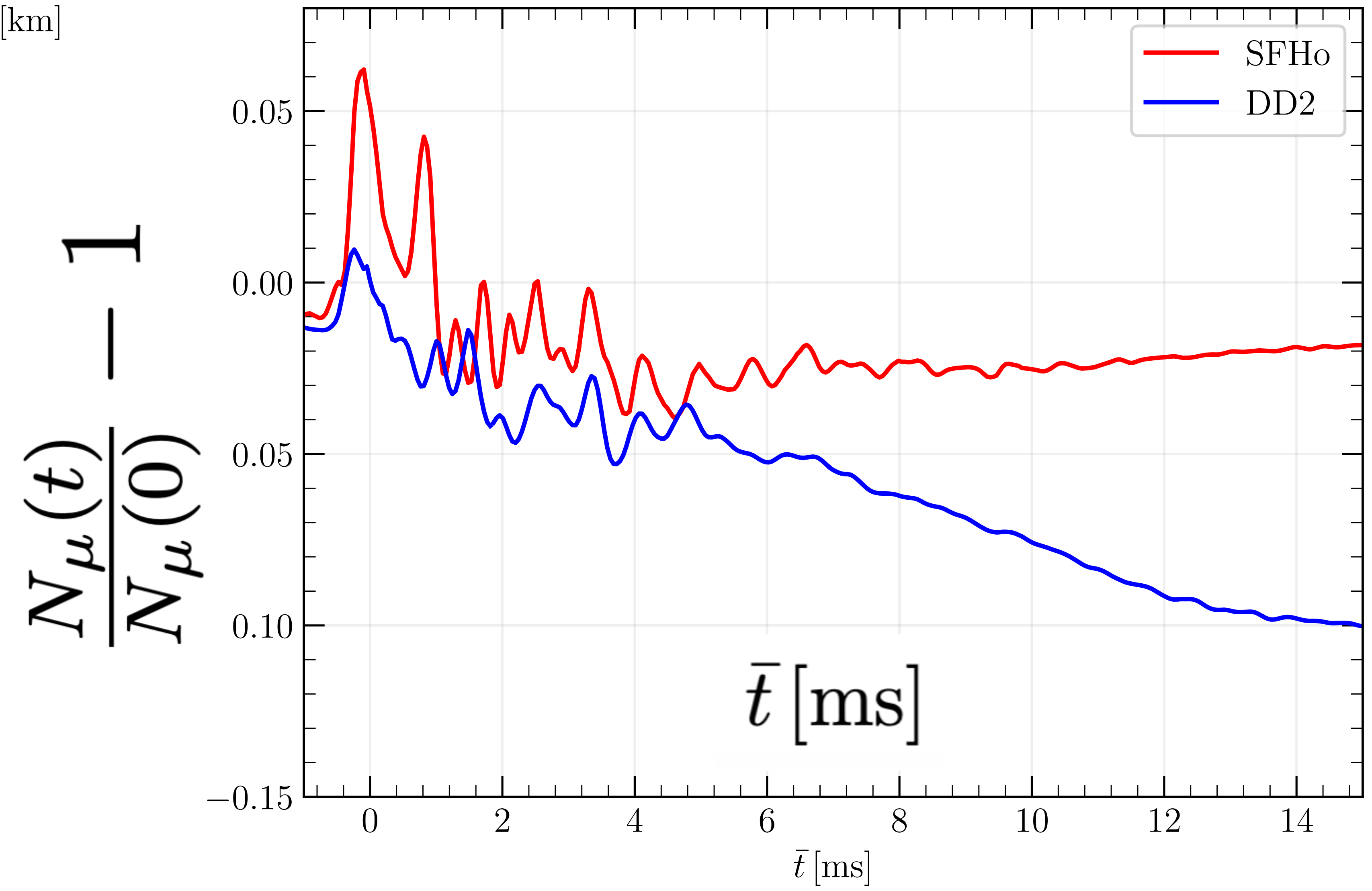
- Muonic processes redirect energy budget reserved for electronization
- During Merger, **Creation of $\mu^- - \mu^+$ pairs** consume electronic degeneracy energy
- $L_{\bar{\nu}_\mu} > L_{\nu_\mu}$ (Net muon lepton number pumps up ⇒ Muonization)
- Muonic β-processes ⇒ Muonization at merger ⇒ $Y_\mu \uparrow \uparrow$ Stronger emission for total ν_x

Muonization or De-muonization?

$5-\nu$

Inspiral, $Y_{\mu,\max} \approx 0.022$ **Merger,** $Y_{\mu,\max} \approx 0.05$ **Post-merger,** $Y_{\mu,\max} \approx 0.018$



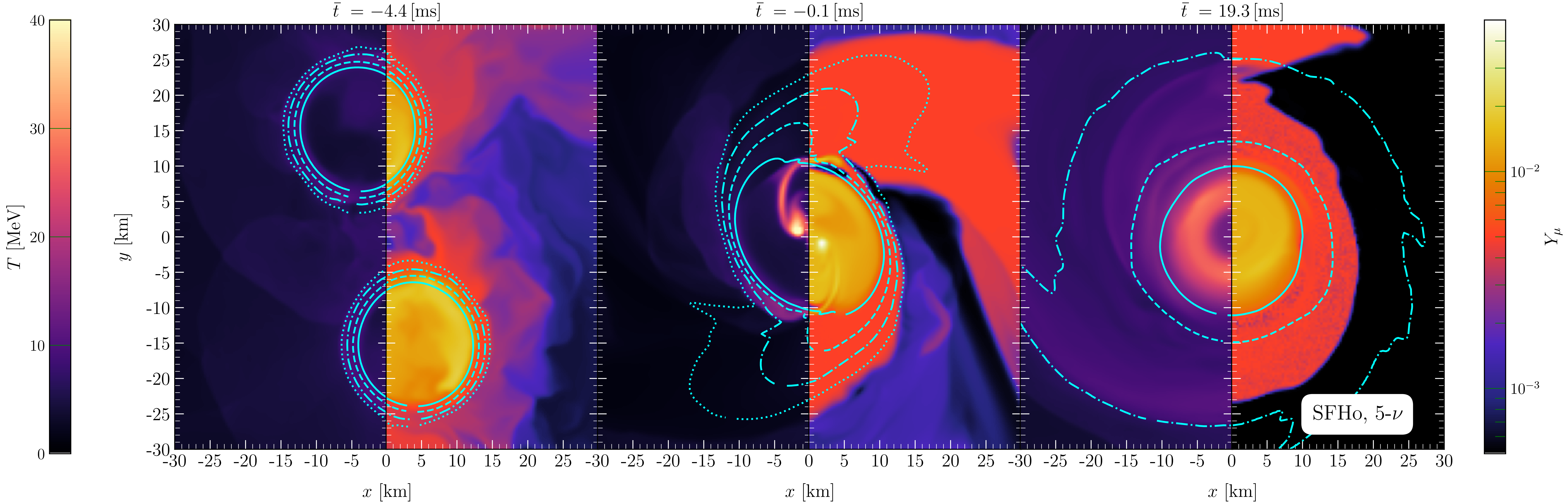


- Muon number

$$N_\mu := \frac{1}{m_b} \int_{\mathcal{V}} \sqrt{\gamma} \rho W Y_\mu d^3x,$$

- Muonization a lot when matter heating up / denser
- De-muonization in Disk

Inspiral, $Y_{\mu,\max} \approx 0.022$ **Merger,** $Y_{\mu,\max} \approx 0.04$ **Post-merger,** $Y_{\mu,\max} \approx 0.018$

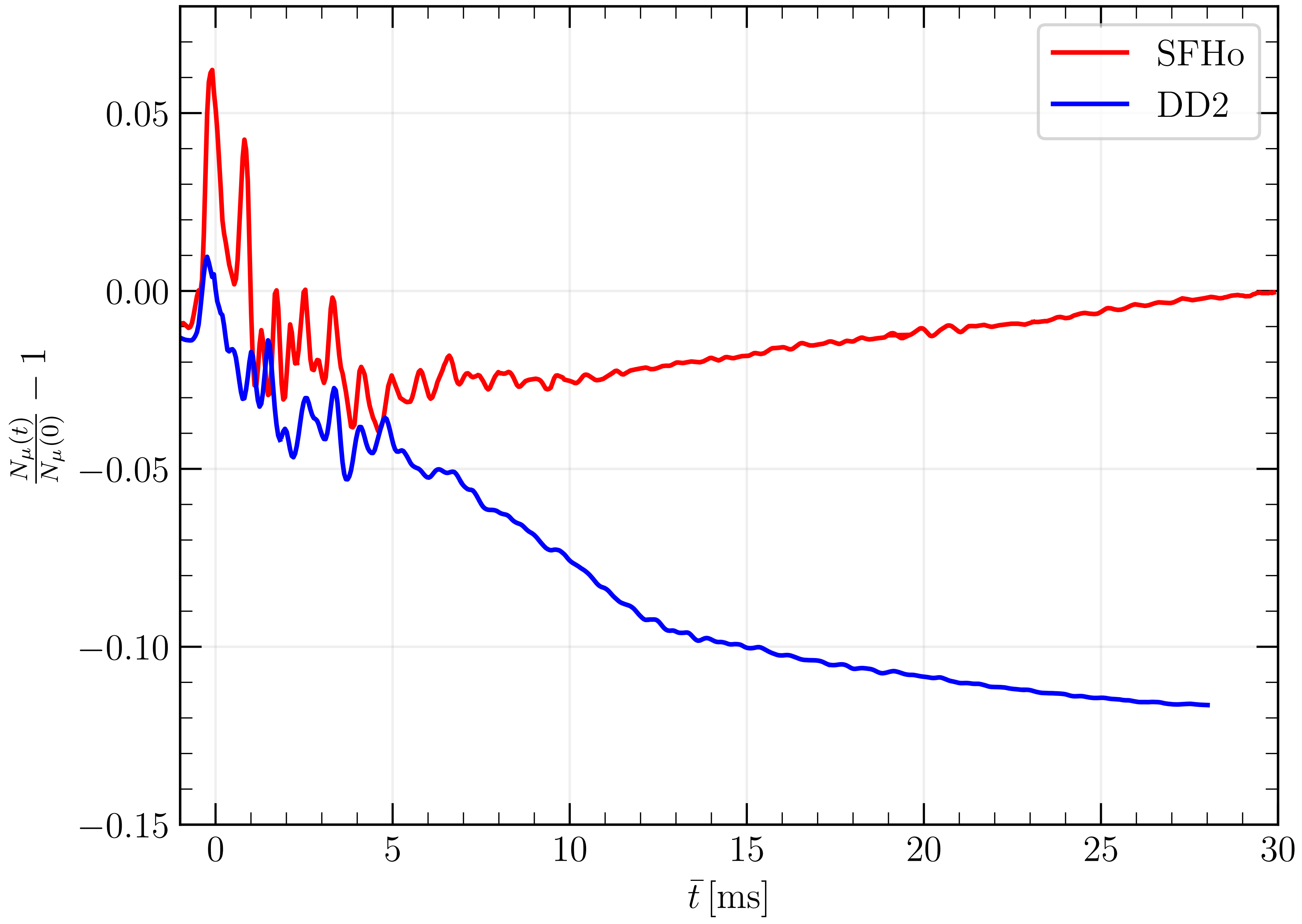


Muons only love hot and dense place!

Role of muons in Difference between CCSN and BNS

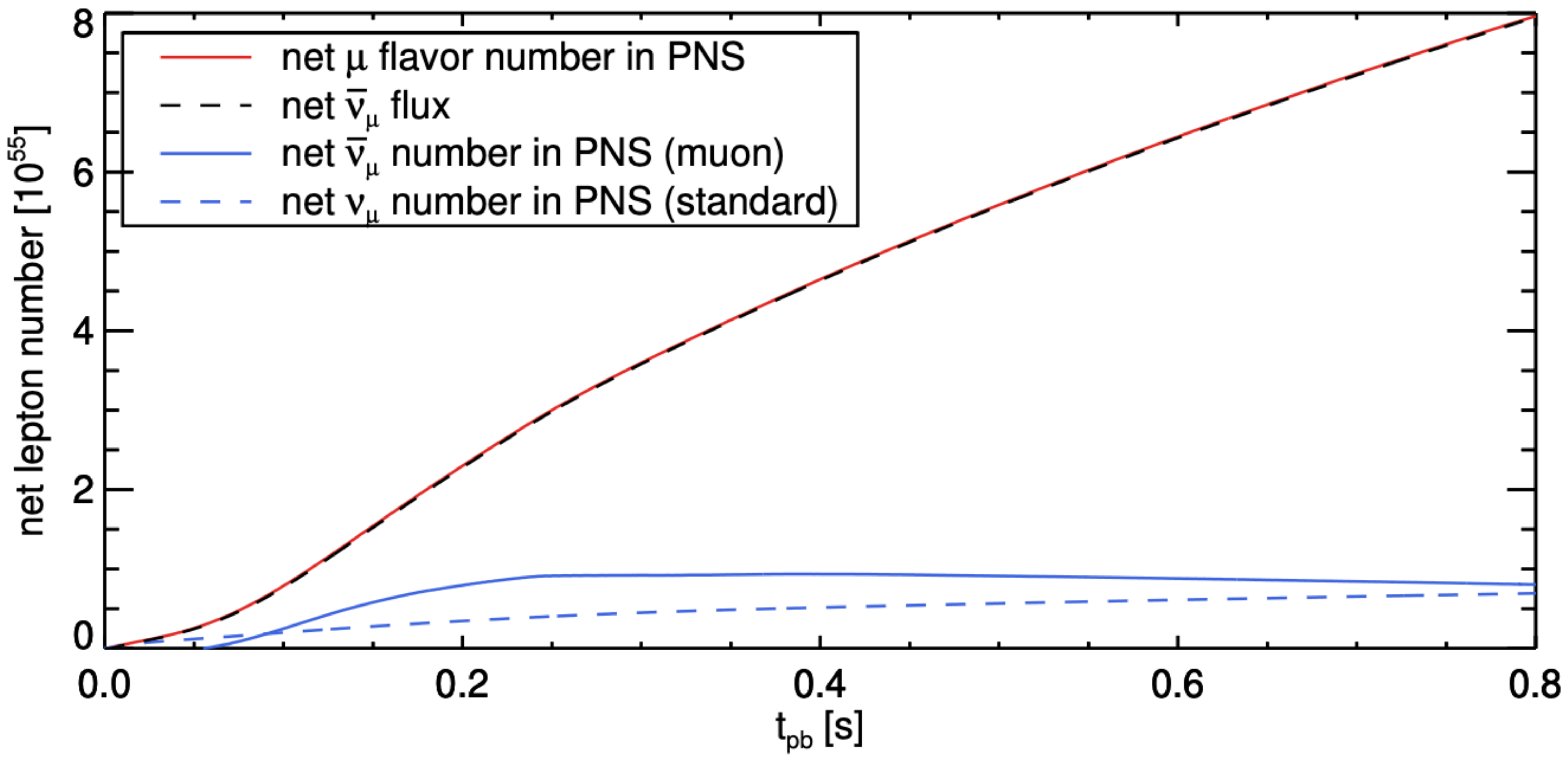
BNS

- Finite Initial Y_μ
- Rapid muonization at merger
- More muons ($Y_\mu \sim 10^{-2} - 0.05$) due to higher ρ, T



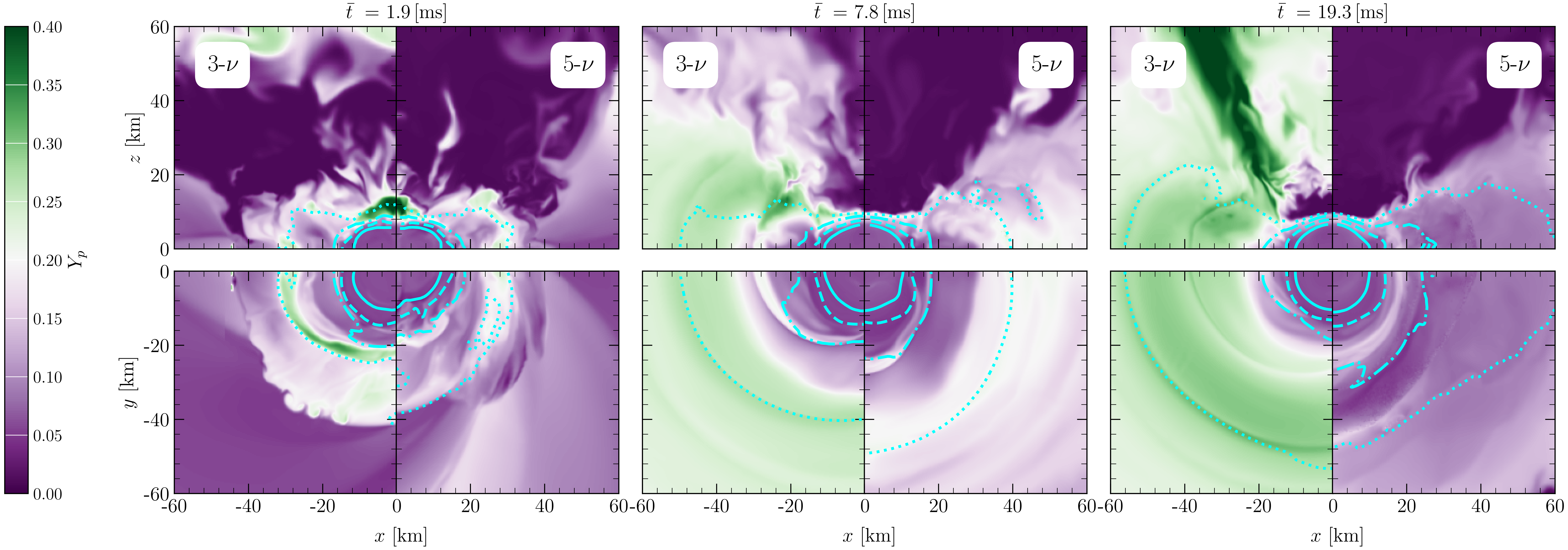
CCSNe

- Initial $Y_\mu \sim 0$
- Few hundred ms of accumulation of muons (Slower muonization)
- less muons ($Y_u \sim 10^{-3} - 0.01$)



Bollig thesis

$5-\nu$: Weaker Electronization than $3-\nu$



- **Energy budget for muon processes and muons**
- **Colder remnant:**
  weaker emission of $\bar{\nu}_e, \nu_e$; weaker reabsorption of ν_e
- Muonic processes compete with electronic processes
  ∵ **demuonization (** $n+\nu_\mu \leftarrow p+\mu^-$ **&.** $n+\mu^+ \leftarrow p+\bar{\nu}_\mu$**)**

Electronization

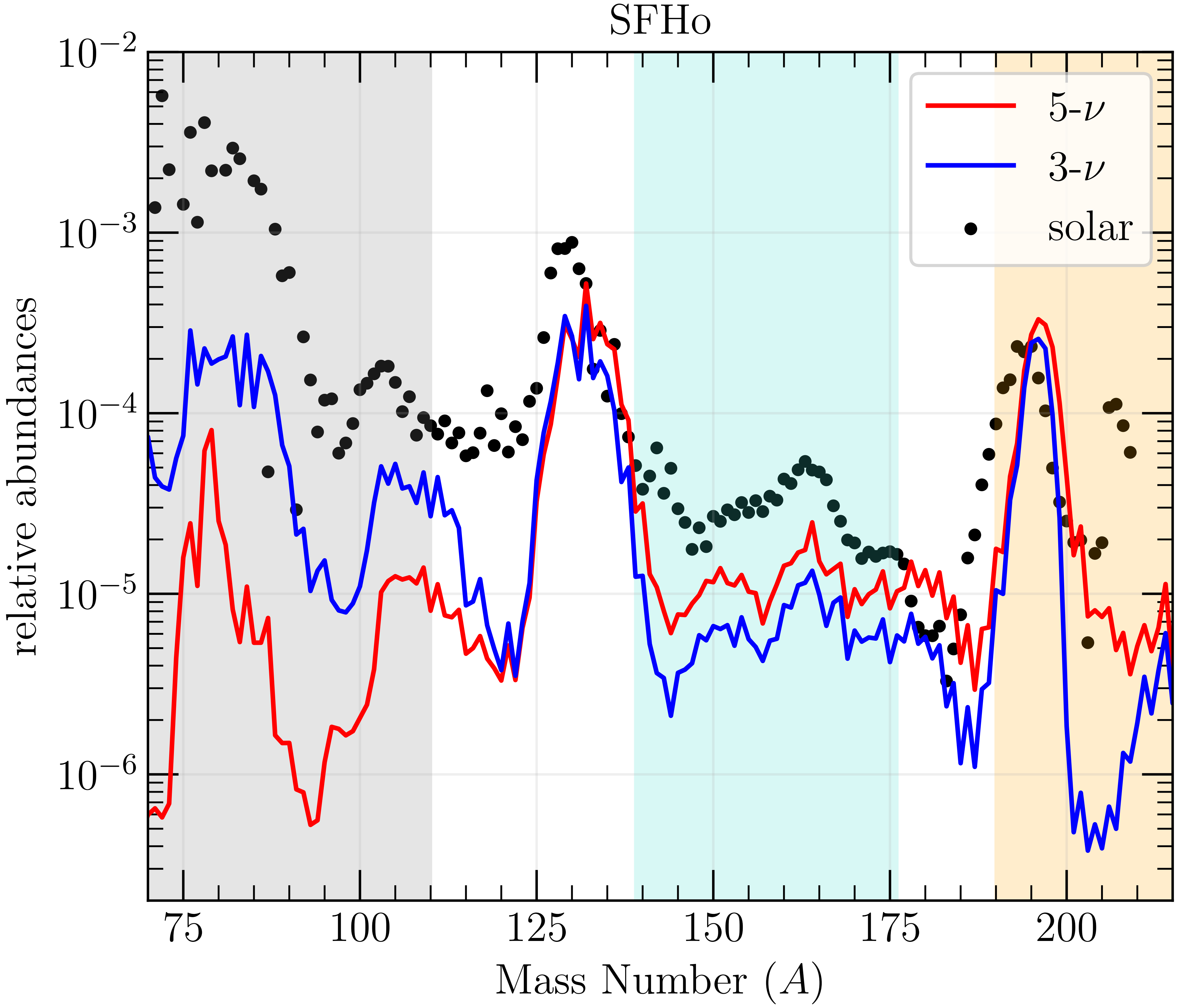
$\nu_e + n \rightarrow p + e^-$

$\bar{\nu}_e + p \leftarrow n + e^+$

r-process nucleosysthesis

- Ejecta => Nuclear Network: **Skynet**
- **More neutron-rich ejecta in $5-\nu$:**
  - Two times Lanthanides $(139 < A < 176)$
  - Ten times elements with $190 < A < 215$
  - +18% actinides $(232 < A < 238)$
  - Lower synthesis of light elements

Ng+ ApjL 2025



Muonic dof largely change perspective for the synthesis of heavy elements of BNS merger!

Missing physics (I)

- Energy-dependent interactions / scheme (For inelastic kernels)

Bolling+, Fischer+, Guo+

Lepton flavour exchange

$\nu_\mu + e^- \rightleftarrows \nu_e + \mu^-$

$\bar{\nu}_\mu + e^+ \rightleftarrows \bar{\nu}_e + \mu^+$

Lepton flavour conversion

$\bar{\nu}_e + e^- \rightleftarrows \bar{\nu}_\mu + \mu^-$

$\nu_e + e^+ \rightleftarrows \nu_\mu + \mu^+$

Inverse lepton decay

$\bar{\nu}_e + e^- + \nu_\mu \leftrightarrow \mu^-$

$\nu_e + e^+ + \bar{\nu}_\mu \leftrightarrow \mu^+$

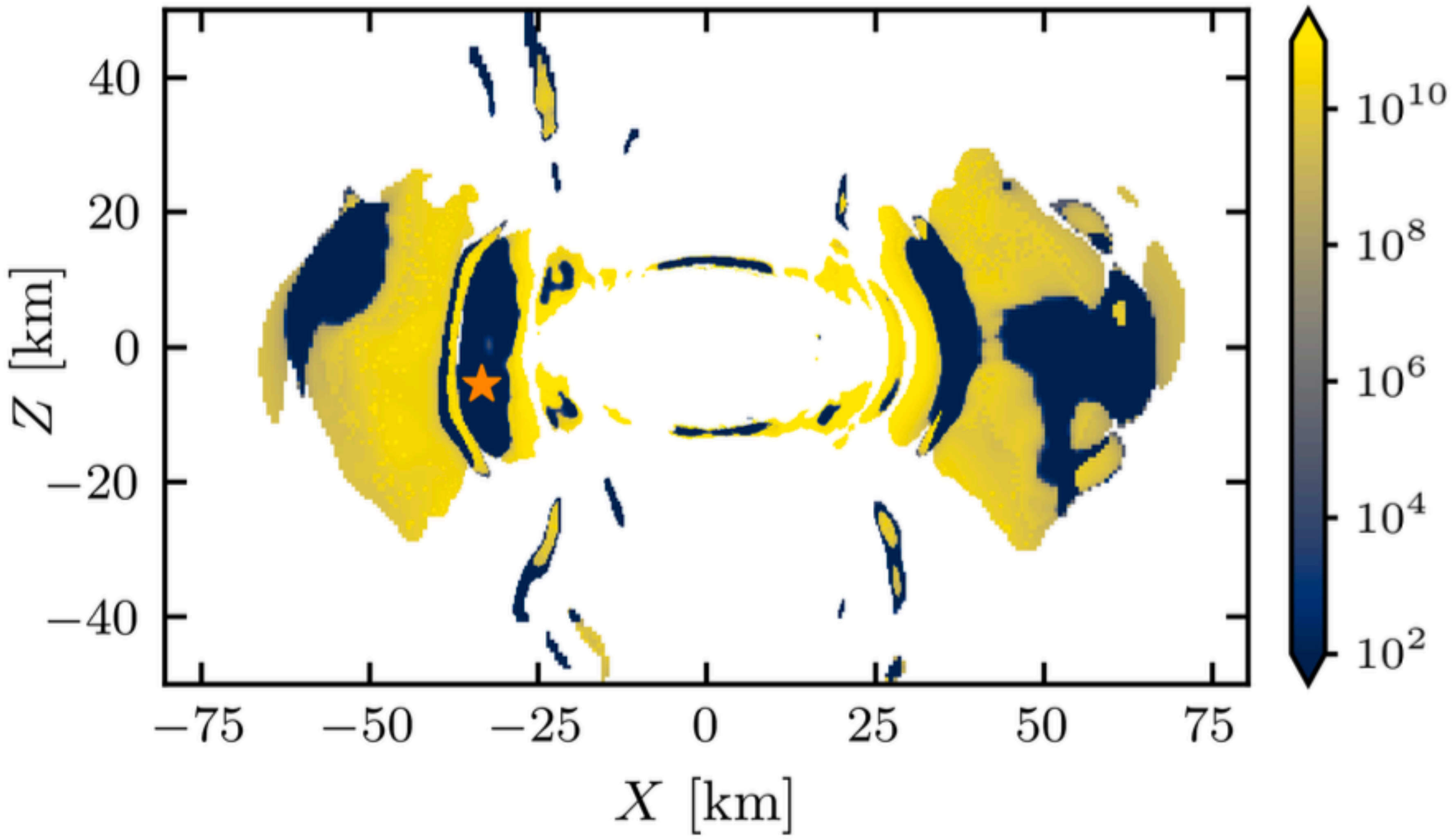
- Inverse lepton decay dominates low-energy neutrinos [Guo+2020]
- Magnetic field, neutrino-oscillation, etc…
- NOT Long simulation enough!

Not a final picture, but may suggest this muon dof potentially change a lot of our r-process/KN knowledge!

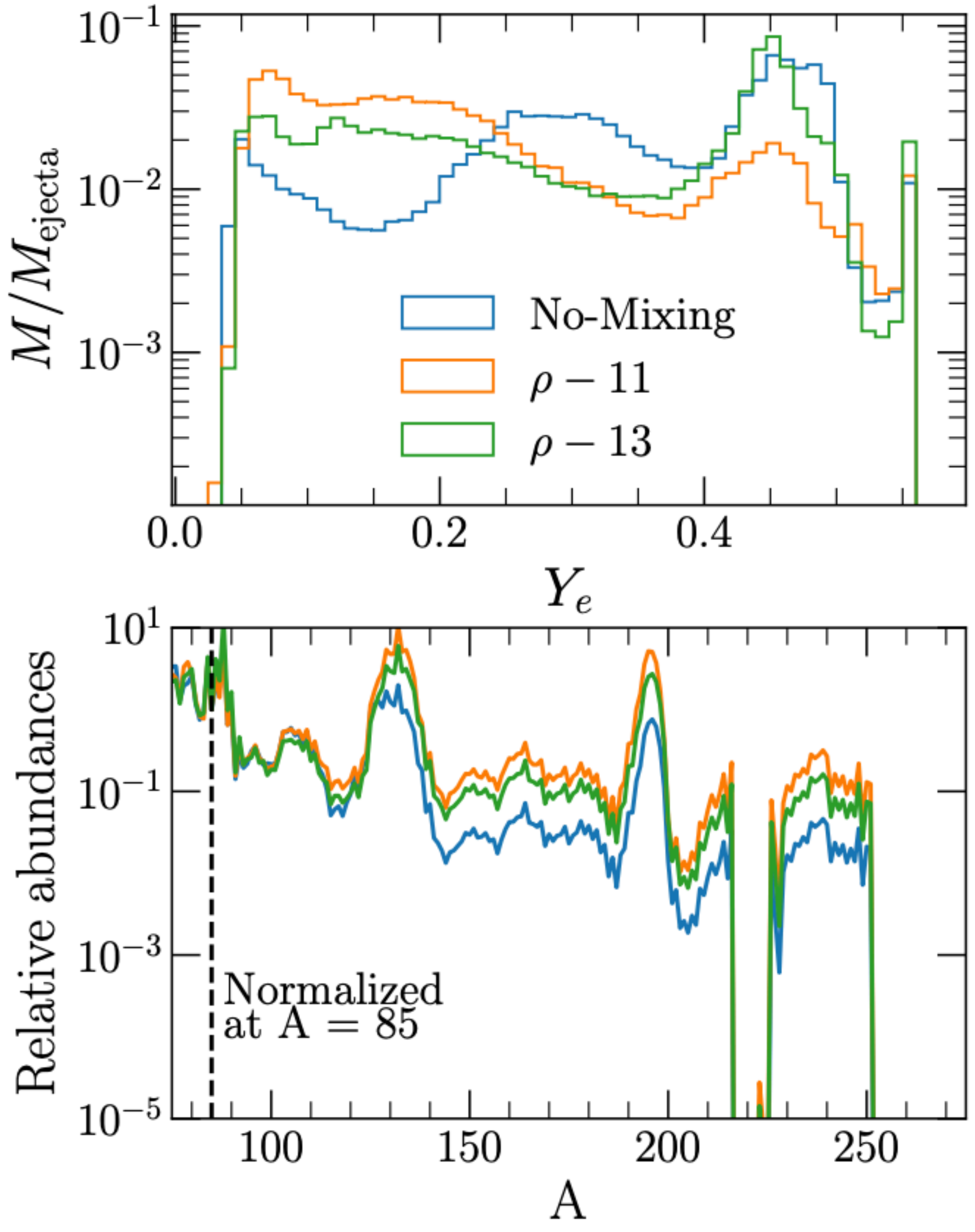
Extra:
Collisional flavor instability (CFI) in muonic BNS remnant

Harry Ho Yin Ng, Jiabao Liu, Arthur Offermans,
Shoichi Yamada, Luciano Rezzolla, in prep.

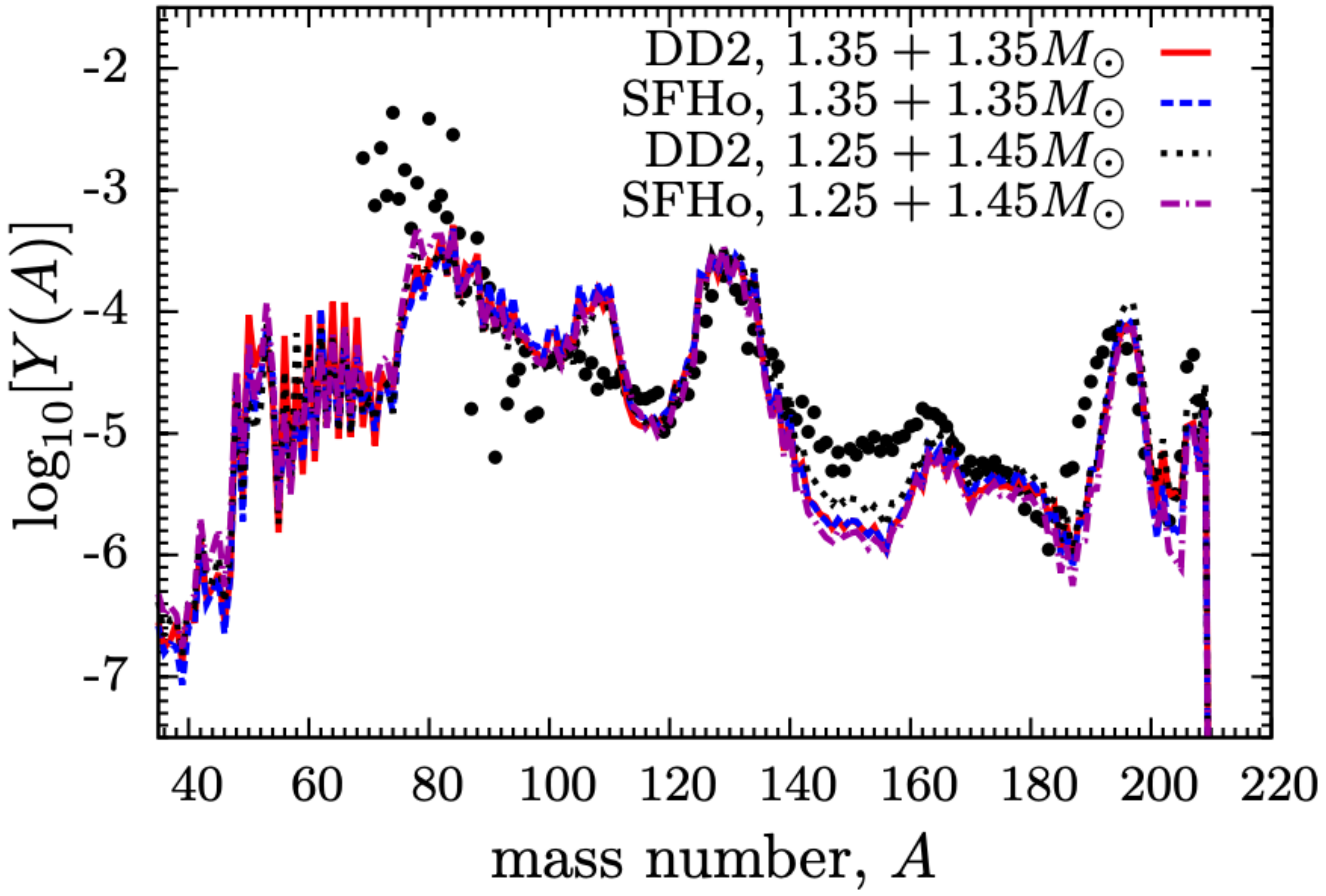
Flavor instabilities in mergers



Froustey+ 2026



Qiu+ 2025



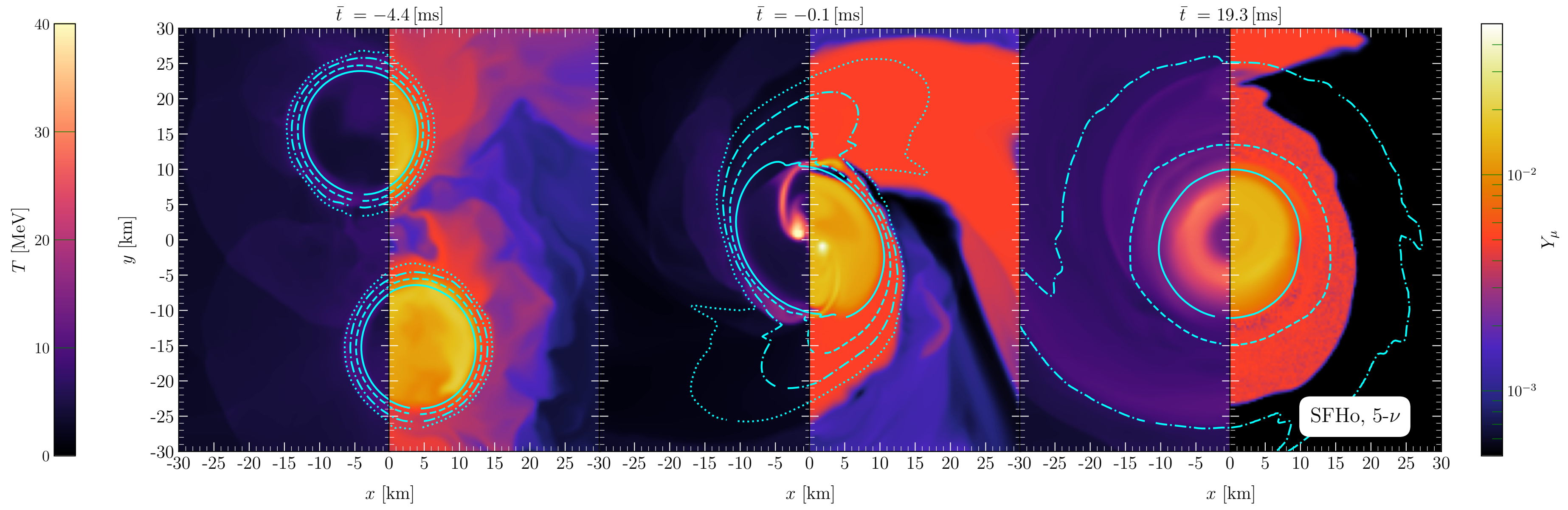
George, Wu, Tamborra, Janka+ 2020

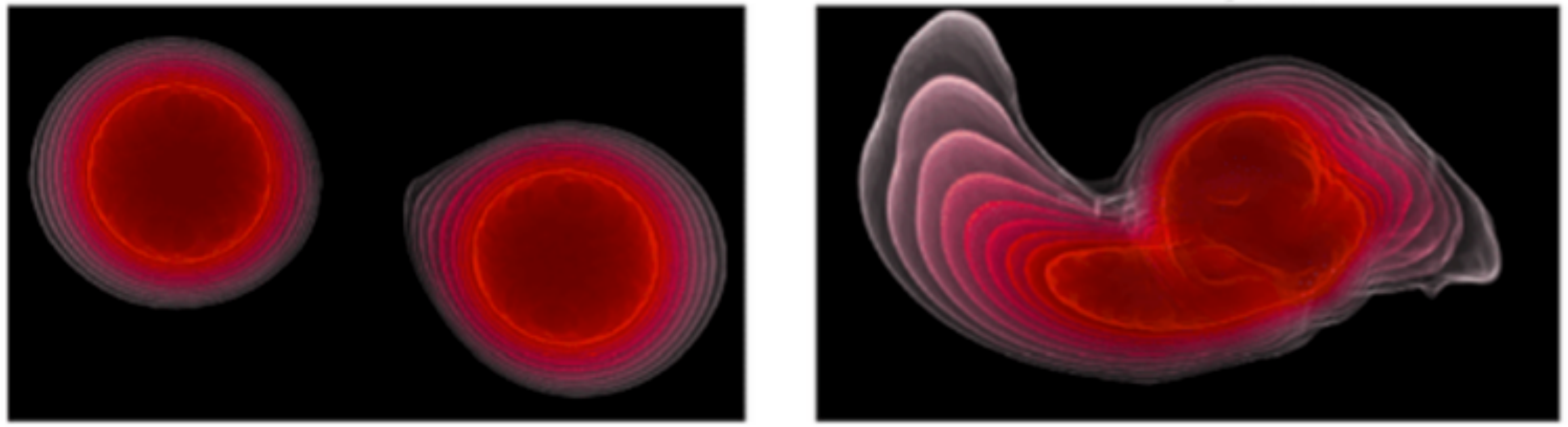
- O. Just+ 2023, Xiong+2023, Wu+2017, Qiu+ 2025, Johns+2022, Padilla-Gay+ 2021, Sajad+2024, ….

Covered a lot of interesting Flavor conversion topics

How muon dof change the Flavor instability? (CFI)

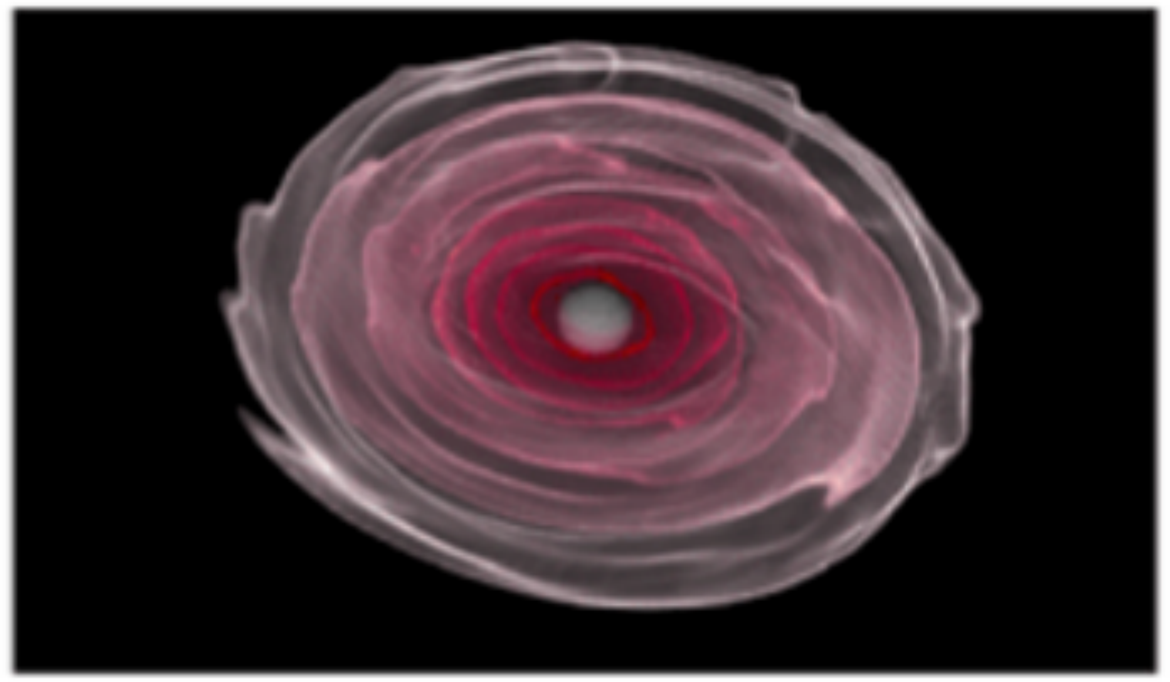
Hybrid approach [Ng+ 2024b]:





ϕ − Averaging

Handoff after
> 25ms of merger



***FIL-M1*: BSSN-Z4 + grey M1**

3D-Cartesian

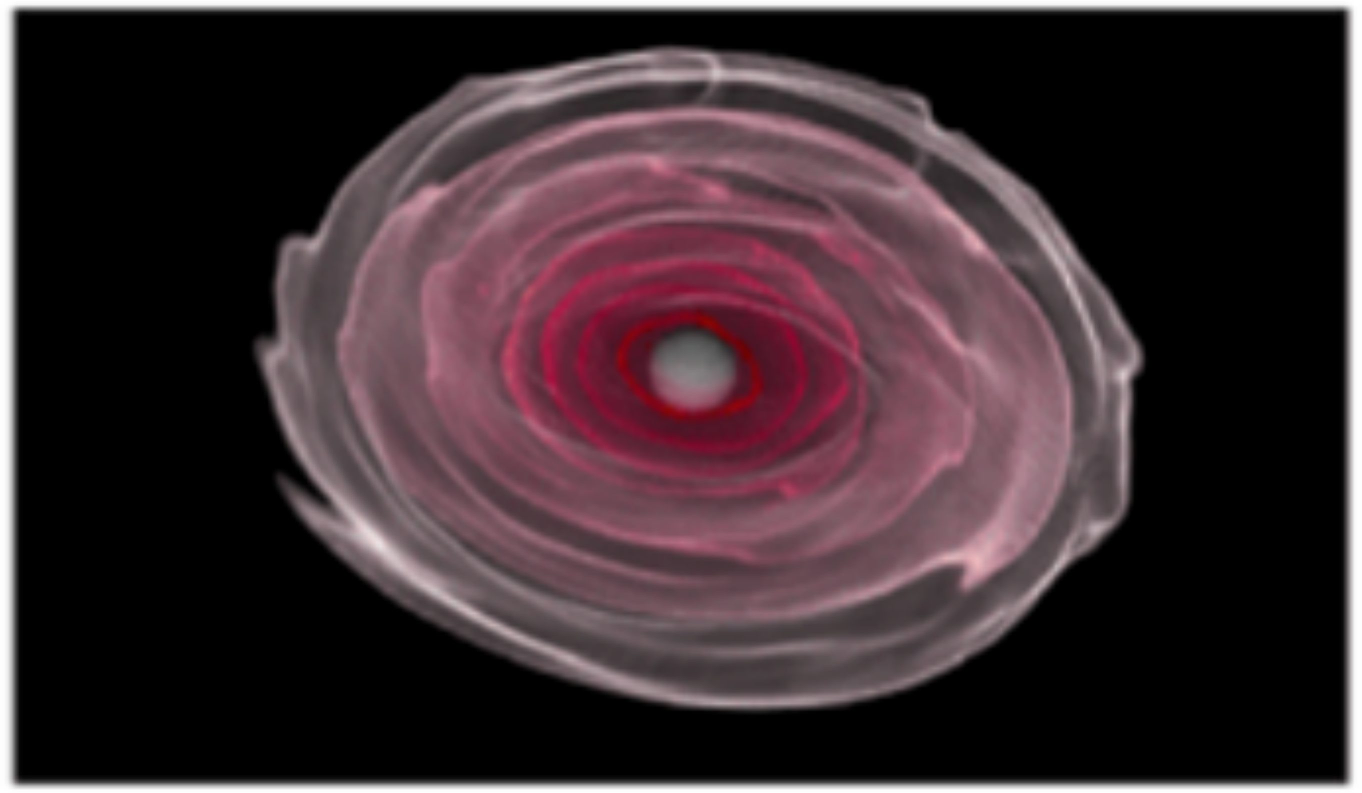
$5 - \nu$

***GMUNU*: xCFC**
(Conformal flatness condition) +
energy-dependent M1
[Cheong, Ng+ 2023, Ng, Cheong+ 2024a]

2D-Cylind

Extended to $6 - \nu$ + Muonic

Linear instability analysis for CFI [Liu+ in prep.]



GMUNU: xCFC
(Conformal flatness condition) +
energy-dependent M1
[Cheong, Ng+ 2023, Ng, Cheong+ 2024a]

2D-Cylind

Extended to $6-\nu$ + Muonic

Freeze matter background,
Run M1 to relax neutrinos for 5 ms
[Nagakura+ 2025]

Construct an angle dependent $f(E,\mu)$
assuming axial symmetry, M1 closure
[Froutesy, Kneller+ 2026]

- Improved analysis [Liu+ in prep.]
- Combined advantages from Xiong+ 2025, Liu+2023
- Energy-dependent [Wang+ 2025]
- Avoid divergence at resonance-CFI

npe-matter, 4-species

$\nu_x/\bar{\nu}_x$ suppresses the R-CFI,

$\because$ higher extend of thermal eqm [Nagakura+ 2025]

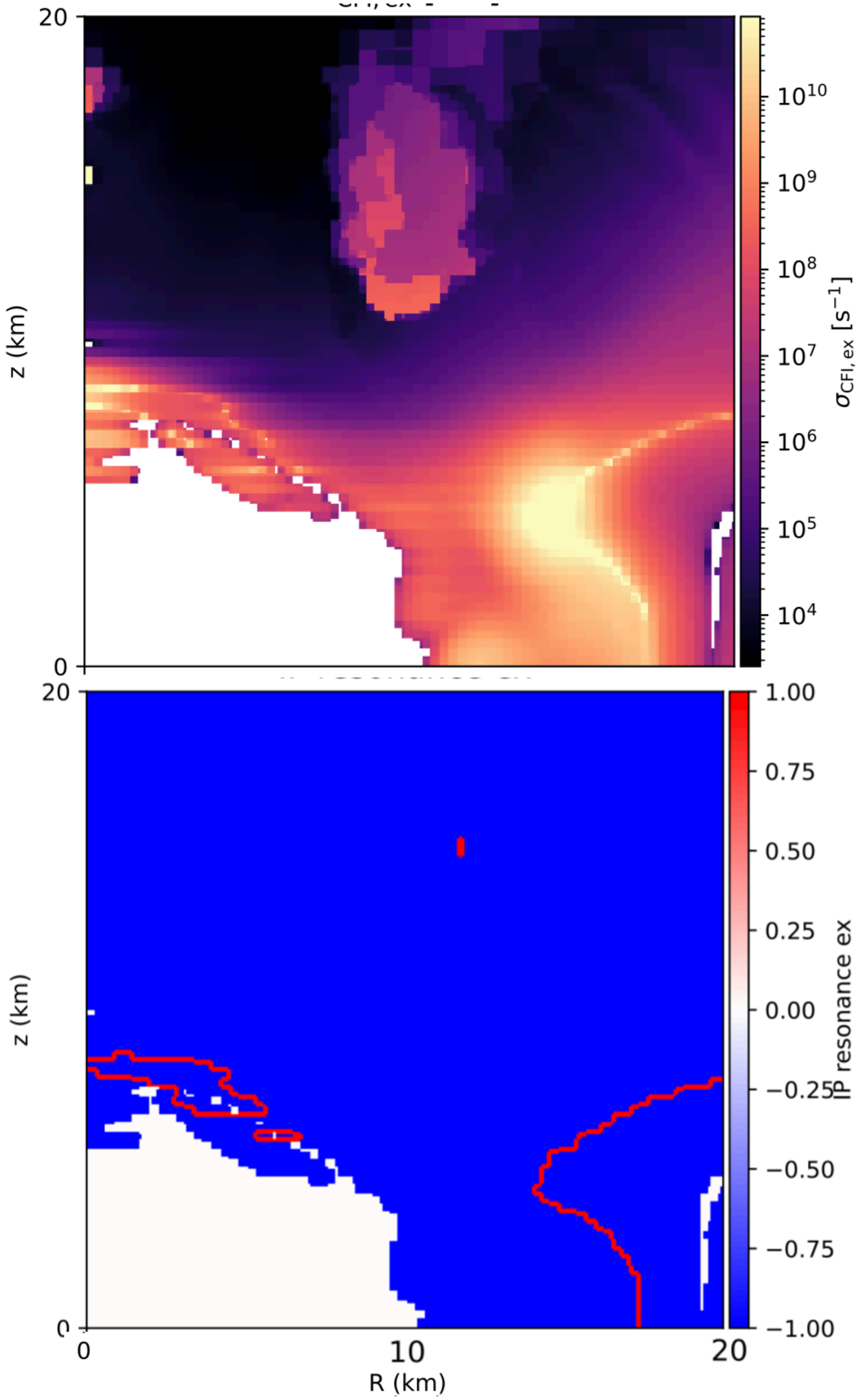
1: Resonant-CFI (R-CFI)

-1: non-Resonant-CFI (NR-CFI)

0: CF-stable

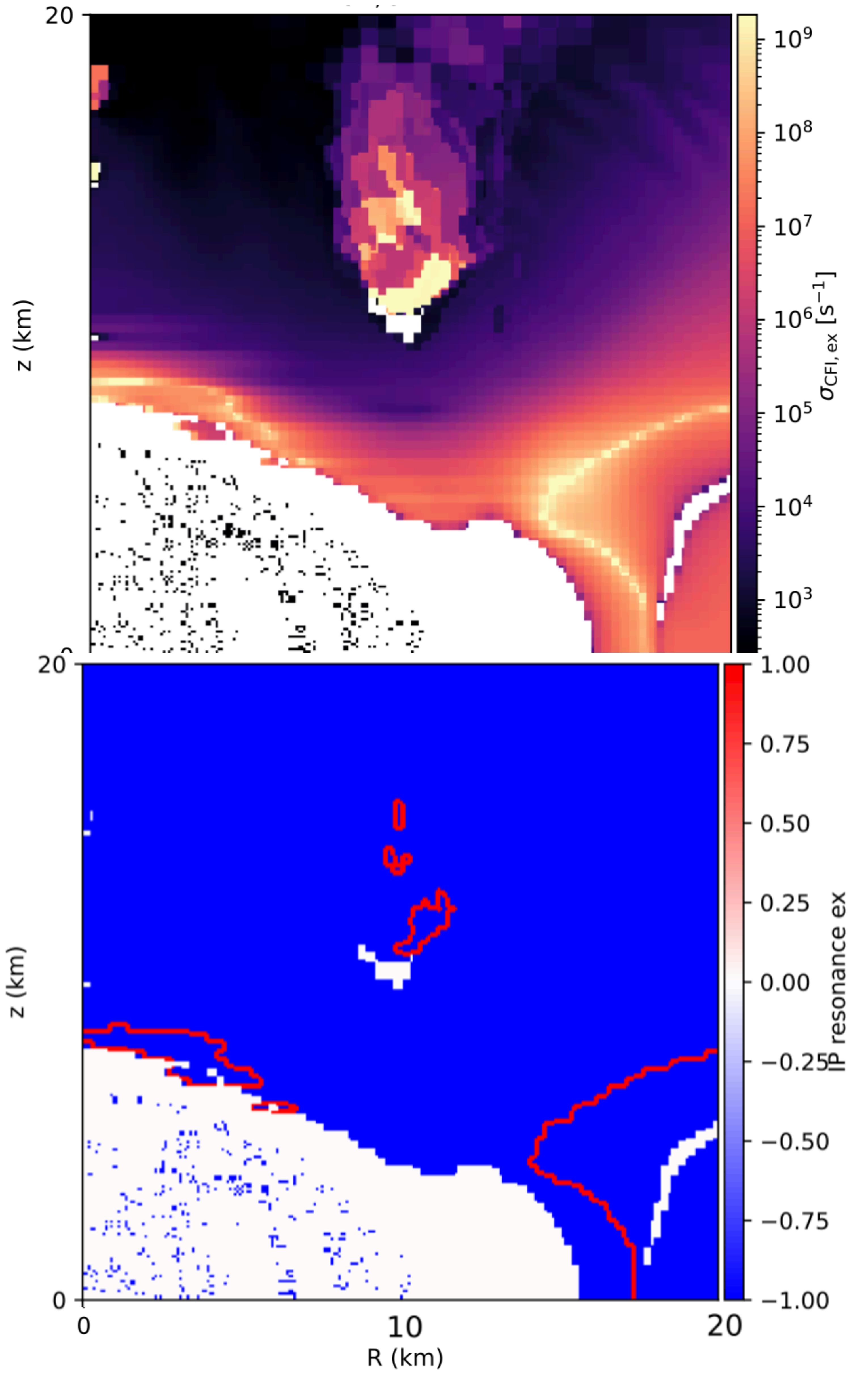
Without Pair processes for ν_x

ex sector: $\sigma_{\rm CFI}$ $[s^{-1}]$



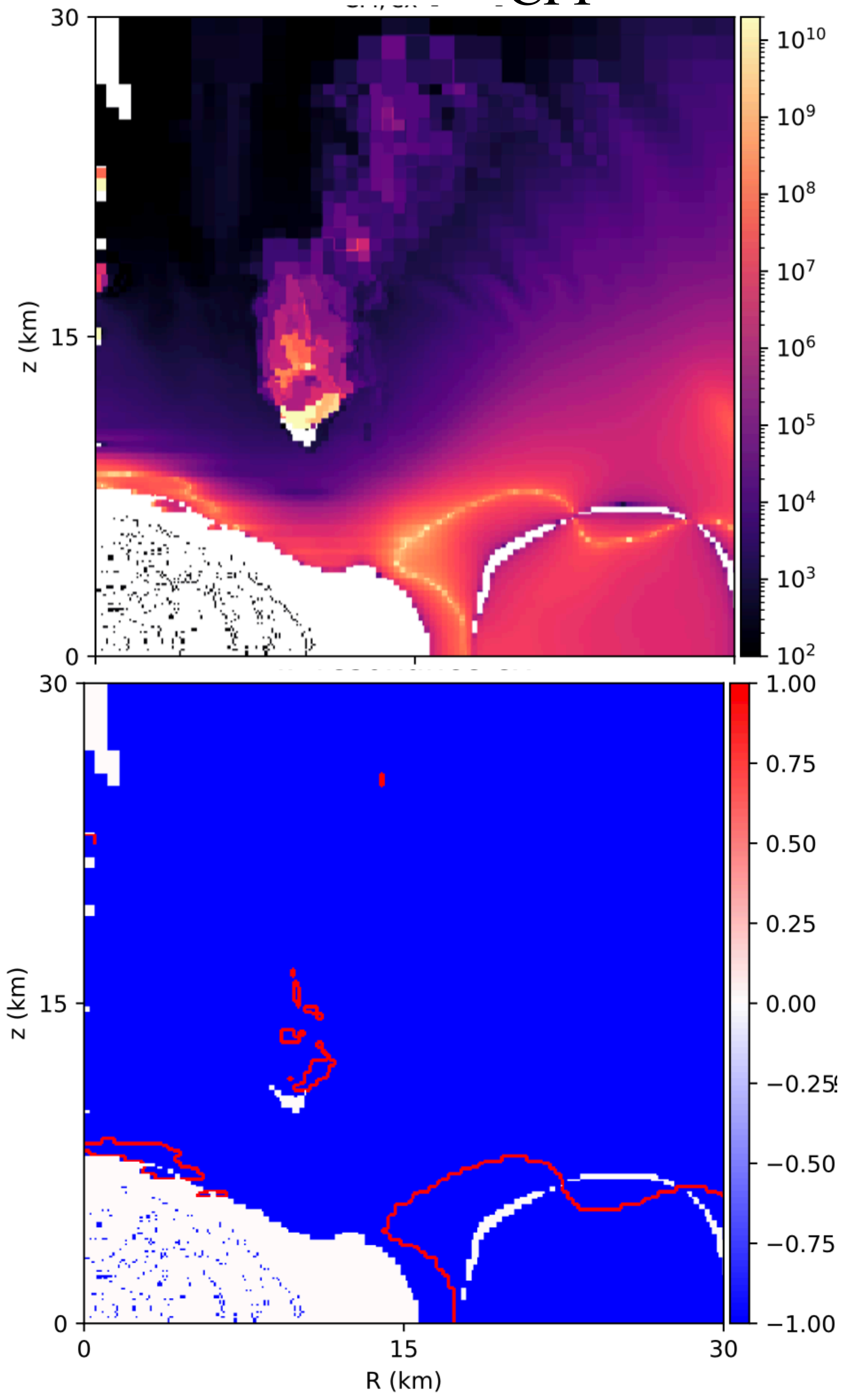
With Pair processes for ν_x

ex sector: $\sigma_{\rm CFI}$ $[s^{-1}]$



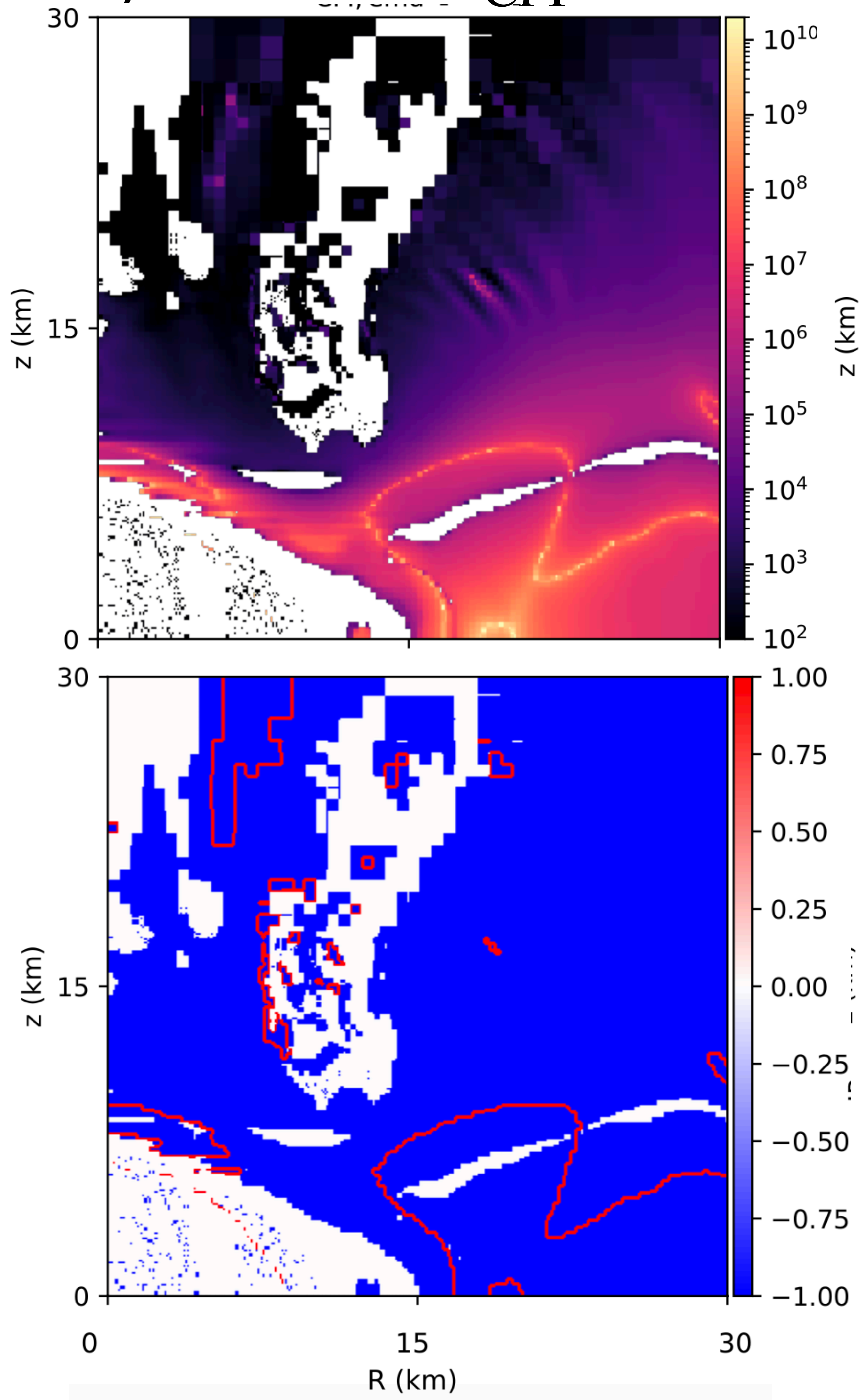
npe-matter, 4-species

ex sector: $\sigma_{\mathrm{CFI}}\ [s^{-1}]$

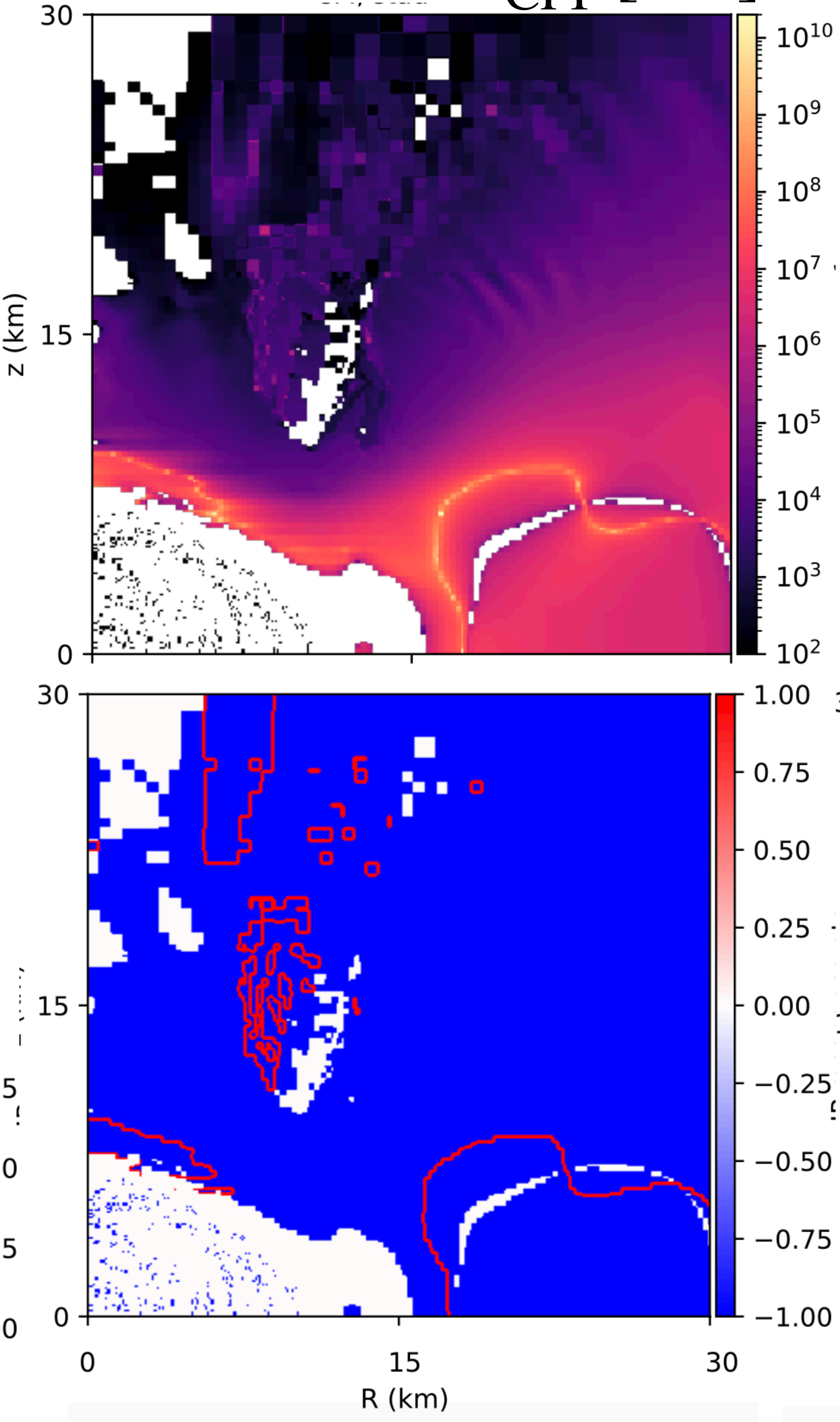


$npe\mu$ — matter, 6-species, 3-flavors

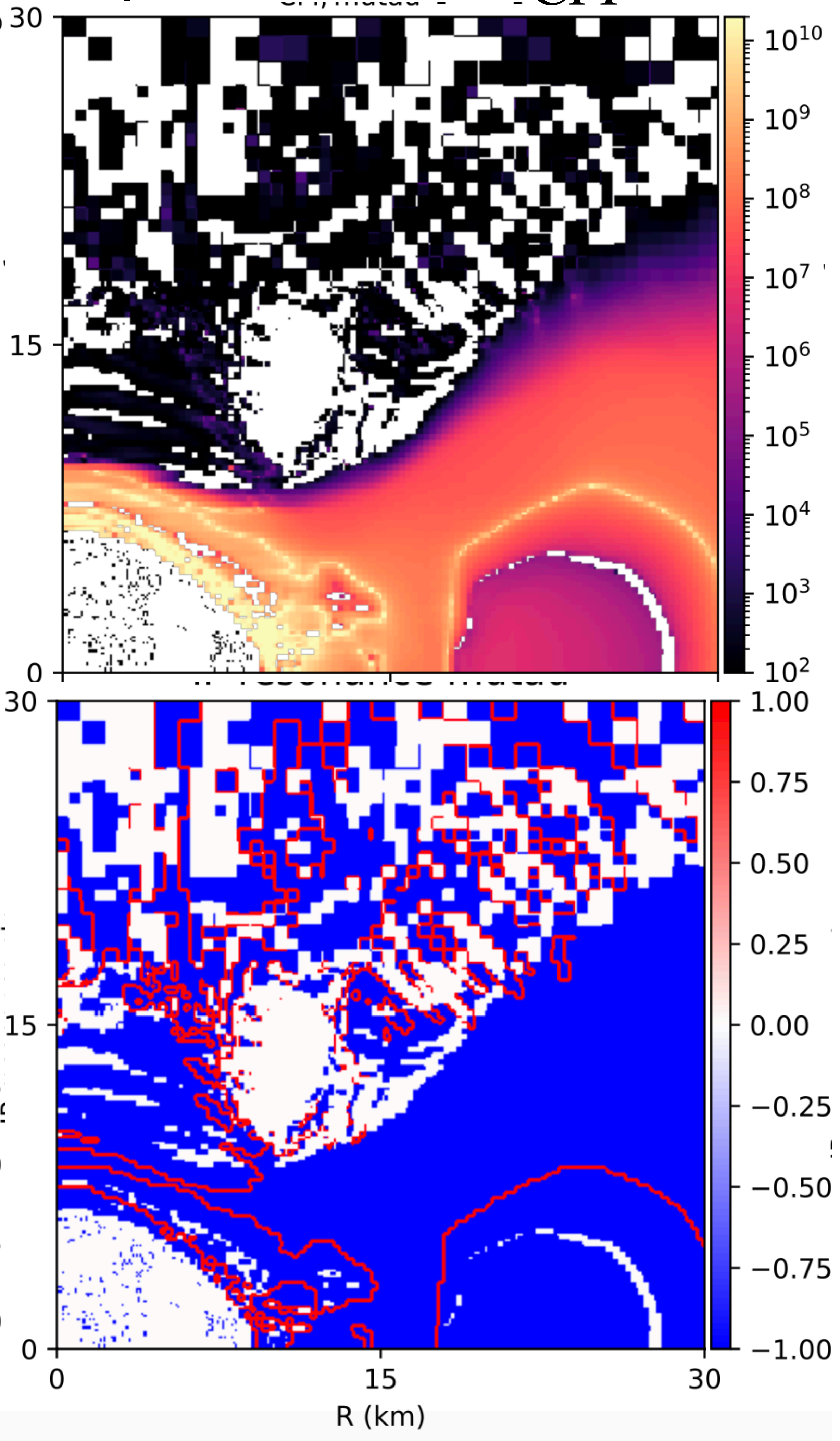
$e\mu$ sector: $\sigma_{\mathrm{CFI}}\ [s^{-1}]$



$e\tau$ sector: $\sigma_{\mathrm{CFI}}\ [s^{-1}]$

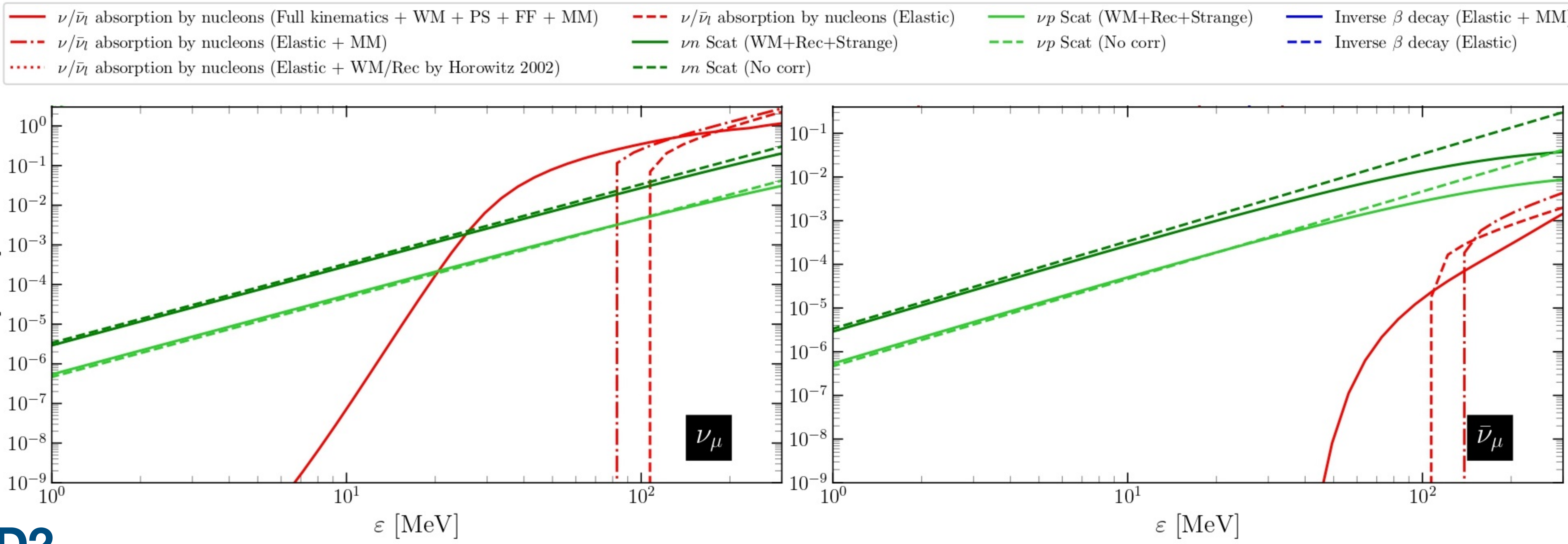


$\mu\tau$ sector: $\sigma_{\mathrm{CFI}}\ [s^{-1}]$



$\mu\tau$ sector is dominant and crazy resonance!

Point	ρ	T	Y_e	Y_μ	μ_e	μ_μ	μ_n	μ_p	X_n	X_p	X_H	$X_{\rm light}$	A	Z	$U_n - U_p$	m_n^*	m_p^*
III	5.47 $\times 10^{14}$	28.5	0.066	0.0001	154	4.57	1100	898	0.934	0.0663	0.00	0.00	0.00	0.00	28.6	344	343



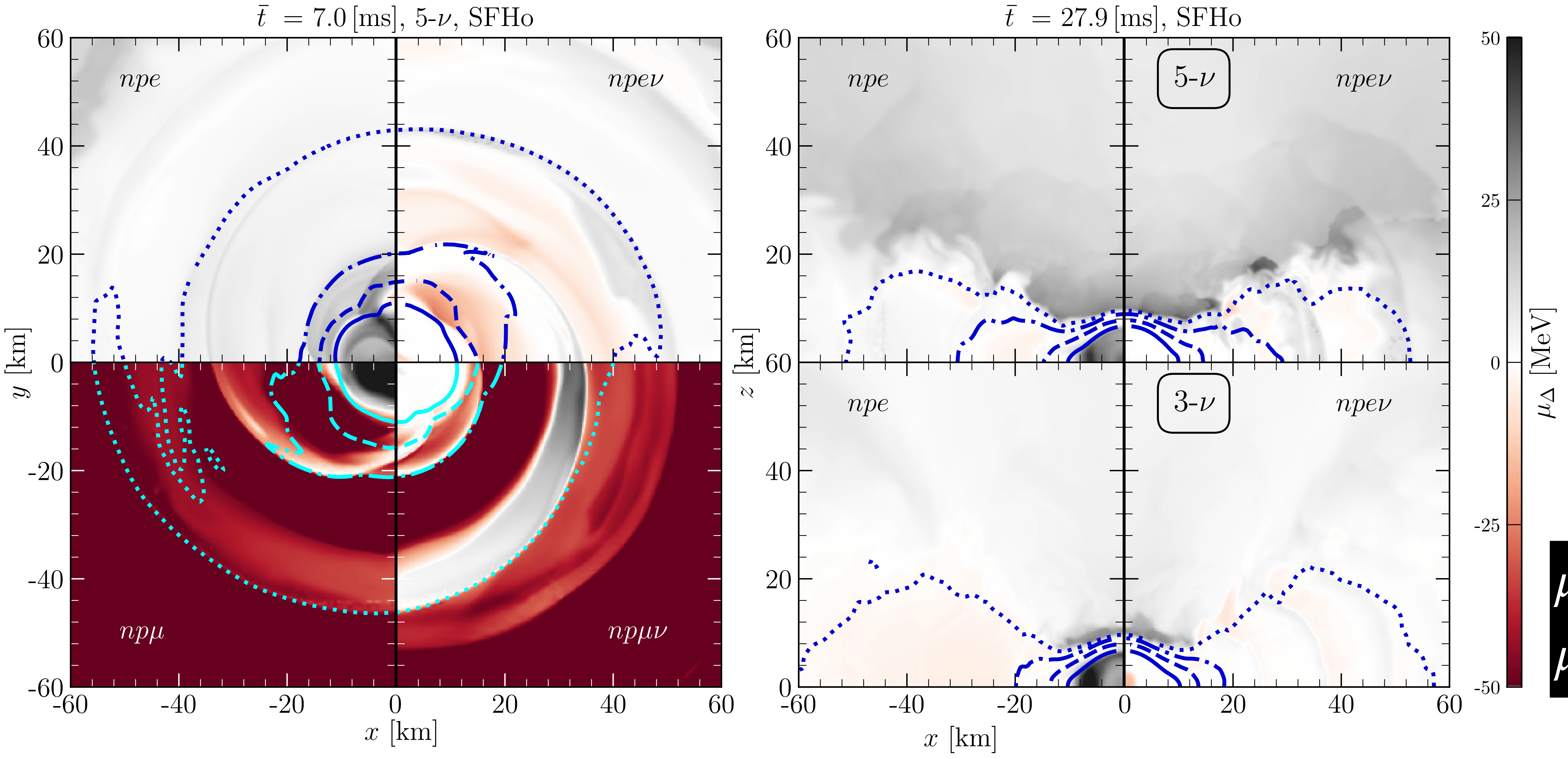
DD2

$$\boxed{\Gamma_{a,\nu_\mu} - \Gamma_{a,\bar{\nu}_\mu} \text{ is huge } \gg \Gamma_{a,\nu_e} - \Gamma_{a,\bar{\nu}_e}}$$

But Hartree-Fock method reduces the difference! [Fischer's talk]

Muonic dof Changed trapped neutrino hierarchy

Contour lines:
$10^{14}, 10^{13}, 10^{12}, 10^{11}$ g cm^{-3}



$$\mu_\Delta^{npl} := \mu_n - \mu_p - \mu_l$$
$$\mu_\Delta^{npl\nu} := \mu_n - \mu_p - \mu_l - \mu_{\bar{\nu}_l}$$

- **Trapped neutrino properties:**
  - **Hierarchy:** $\mu_{\bar{\nu}_e} > \mu_x > \mu_{\nu_e}$ **($3-\nu$ case)** **VS.** $\mu_{\bar{\nu}_\mu} > \mu_{\bar{\nu}_e} > \mu_{\nu_x} > \mu_{\nu_e} > \mu_{\nu_\mu}$ **($5-\nu$ case)**

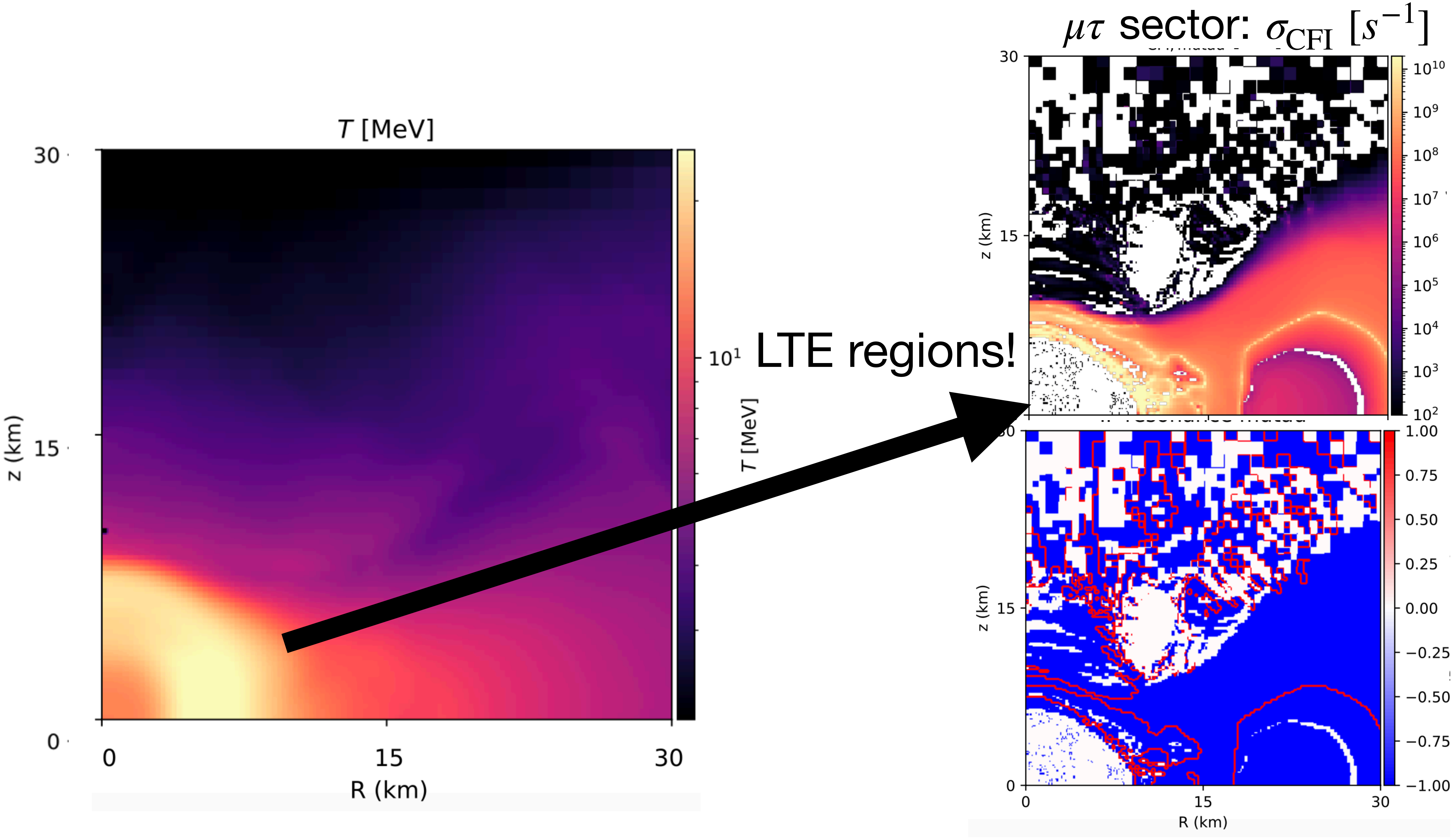
Linearized QKE language:

$$g_{\alpha\beta,0}(E) = \int \frac{d\mu}{2}\, g_{\alpha\beta}(E,\mu), \qquad \bar{g}_{\alpha\beta,0}(E) = \int \frac{d\mu}{2}\, \bar{g}_{\alpha\beta}(E,\mu).$$

$$g(\Gamma) = f_{\nu_\mu} - f_{\nu_\tau} \qquad g(\Gamma) = f_{\bar{\nu}_\mu} - f_{\bar{\nu}_\tau}$$

- **However,** $f_{\nu_\tau} \simeq f_{\bar{\nu}_\tau}$
- **Trapped neu Hierarchy:** $f_{\bar{\nu}_\mu} > f_{\bar{\nu}_e} > f_{\nu_\tau} = f_{\bar{\nu}_\tau} > f_{\nu_e} > f_{\nu_\mu}$
- **Also,** Γ_{ν_μ} **very different from** $\Gamma_{\bar{\nu}_\mu}$
- **Highest growth rate in** $\mu\tau -$ **sector**

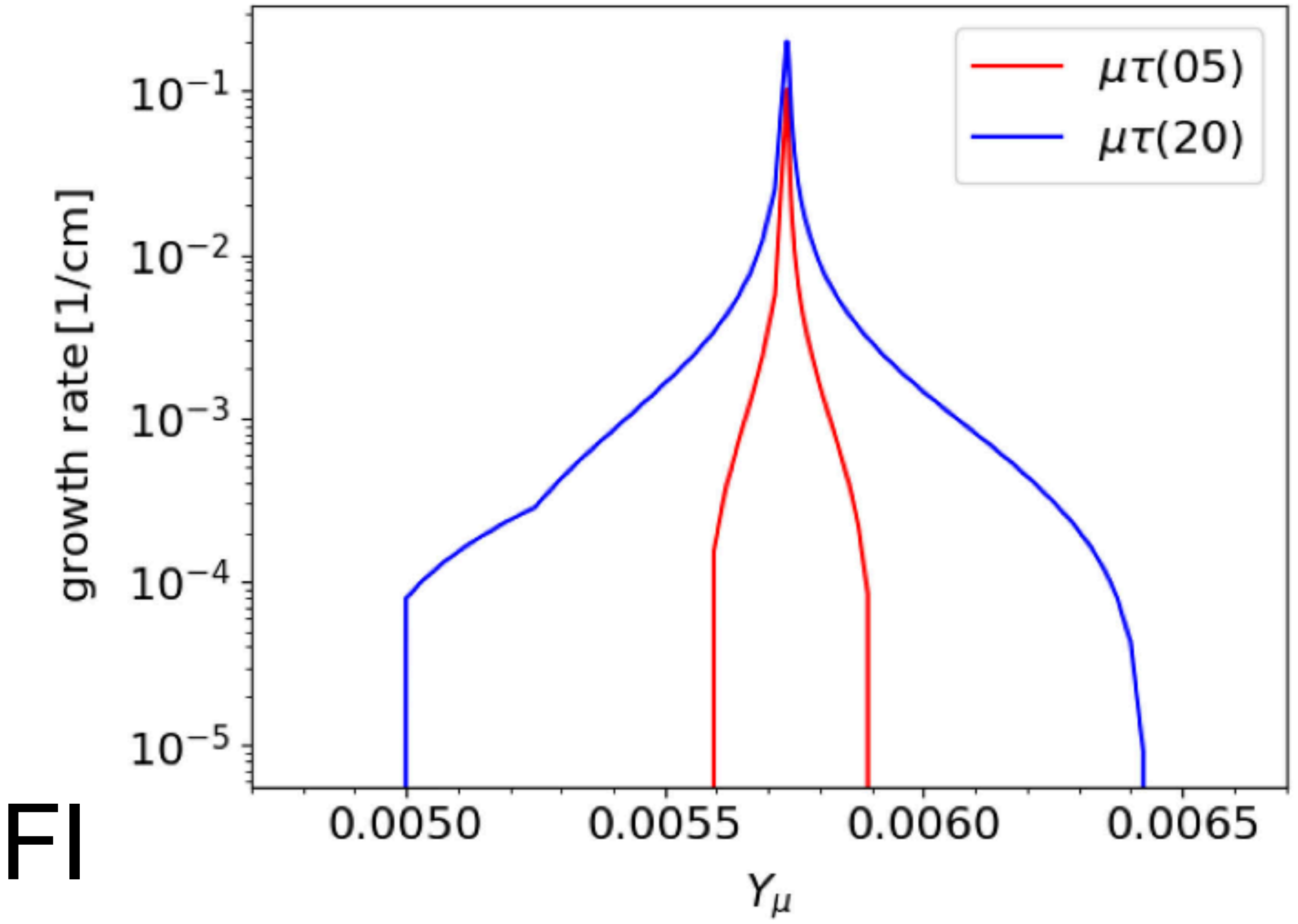
Thermo-eqm characterized by temperature gradient?



Temperature threshold treatment for mixing scheme?

CFI Nightmare in BNS postmerger

- **CCSNe: Low rate in NR-CFI ($\sim 200\ [\mathrm{s}^{-1}]$), only need to take care of R-CFI [Wang's talk]**



[Liu+ 2024]

- $\sigma_{\mathrm{CFI}}^{\mathrm{NR}} \lesssim \sigma_{\mathrm{FFI}}$
- $\sigma_{\mathrm{CFI}}^{\mathrm{R}} \gtrsim \sigma_{\mathrm{FFI}}\ [\because \sigma_{\mathrm{CFI}}^{\mathrm{R}} \sim (10^2 - 10^4)\ \sigma_{\mathrm{CFI}}^{\mathrm{NR}}]$
- Simulation is extremely hard to capture the peak of R-CFI
- Extremely hard to capture sign change between n_{ν_e} and $n_{\bar{\nu}_e}$ [Glas+ 2020]

CFI + muons in HMNS → Necessity + Urgency for CFI asymptotic states……

Missing physics (II)

- Weakhub also have inelastic kernels, e.g.

Lepton flavour exchange

$\nu_\mu + e^- \rightleftarrows \nu_e + \mu^-$

$\bar{\nu}_\mu + e^+ \rightleftarrows \bar{\nu}_e + \mu^+$

Lepton flavour conversion

$\bar{\nu}_e + e^- \rightleftarrows \bar{\nu}_\mu + \mu^-$

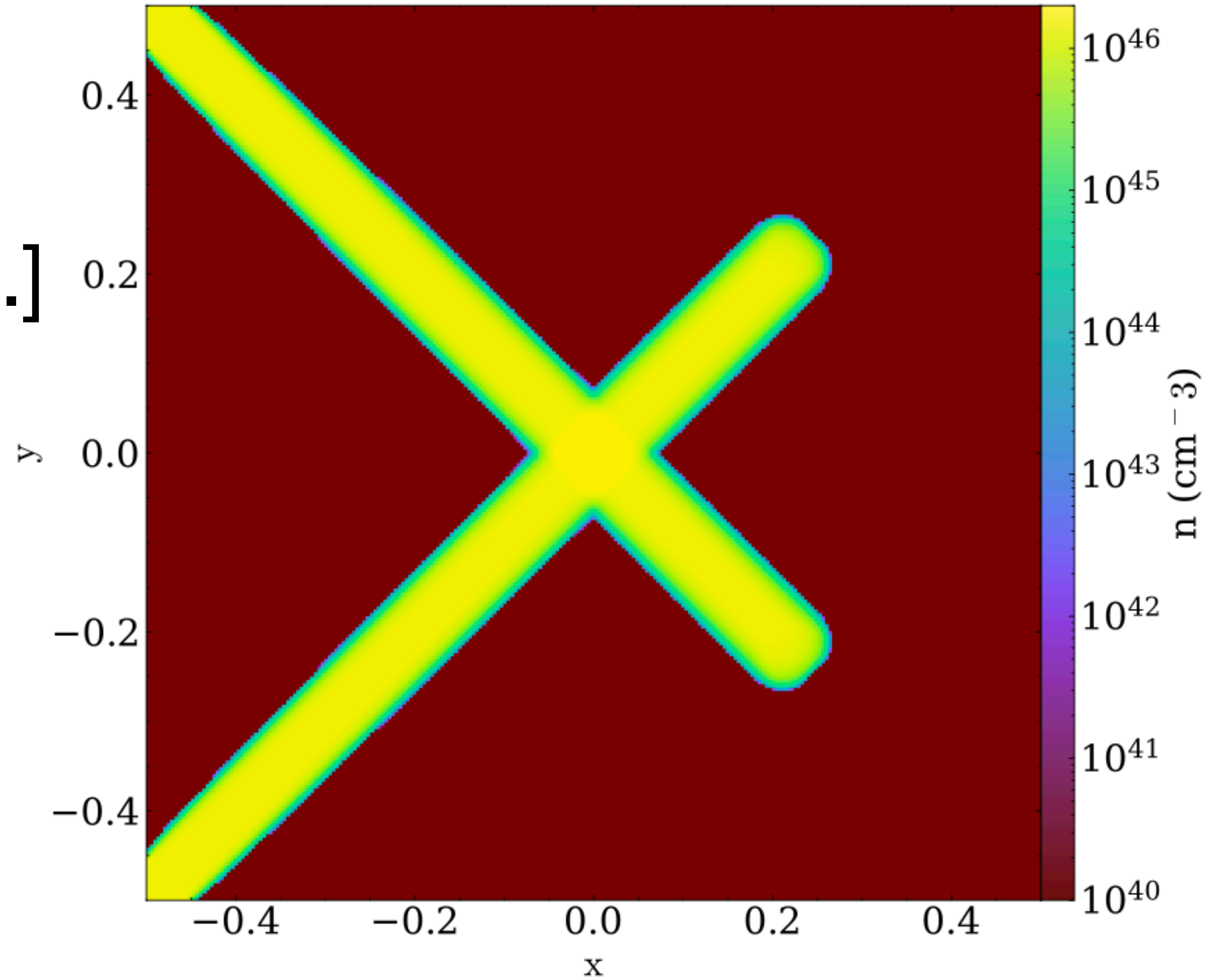
$\nu_e + e^+ \rightleftarrows \nu_\mu + \mu^+$

Inverse lepton decay

$\bar{\nu}_e + e^- + \nu_\mu \leftrightarrow \mu^-$

$\nu_e + e^+ + \bar{\nu}_\mu \leftrightarrow \mu^+$

- Muonic FFI analysis with full 6-species GR boltzmann solver [Offermans, **Ng**, Cheong, Mezzacappa, Li; in prep.]



Gmunu − Boltzmann

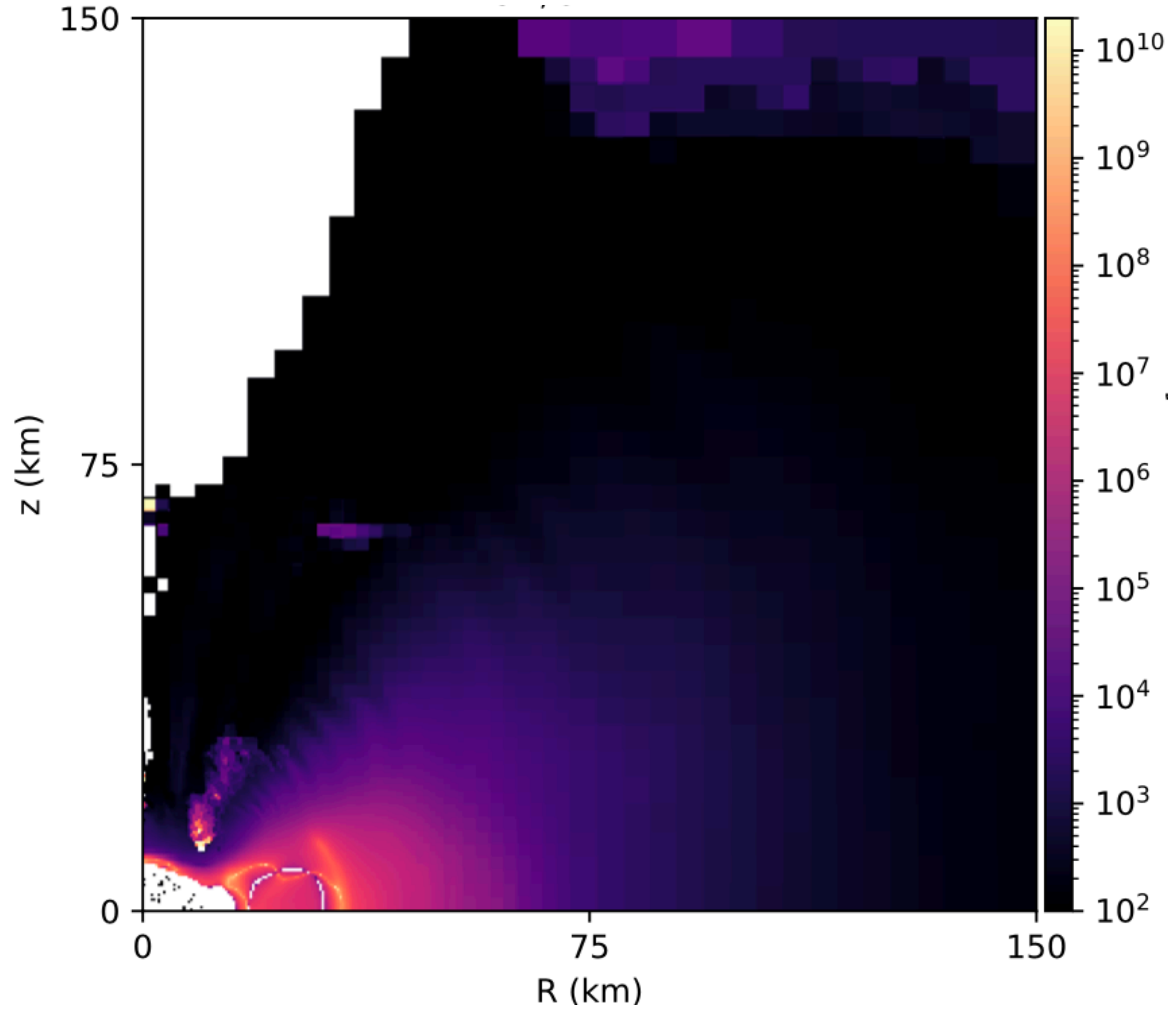
Take-away:

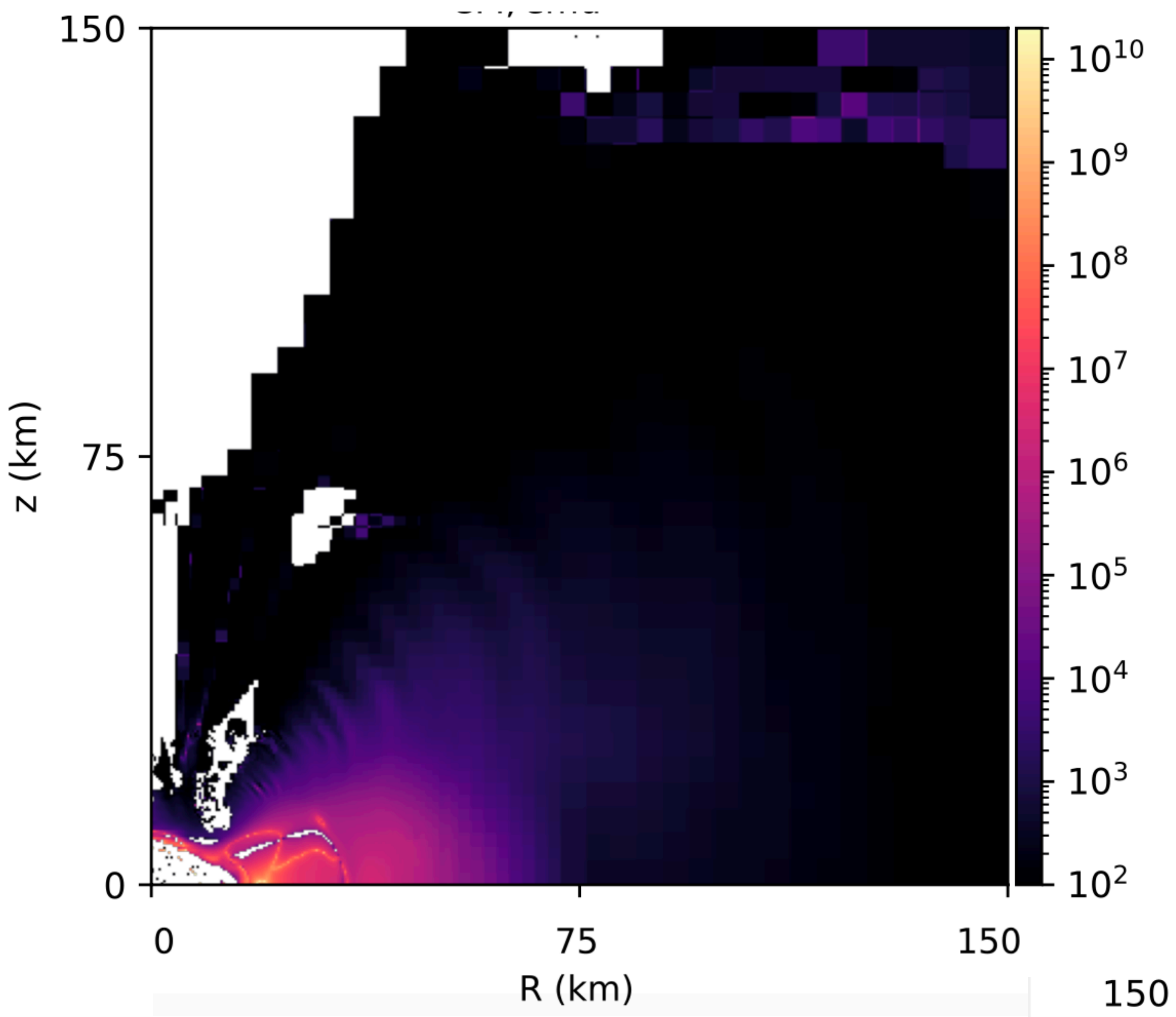
- Muonic => opens up a necessity of BNS merger modelling and r-process / Kilonova interpretation
- Suppress and delay the electronization => less proton-rich ejecta
- Flavor oscillation: Muonic degree of freedom **for** $e\mu$, $e\tau$, $\mu\tau$ **sectors**

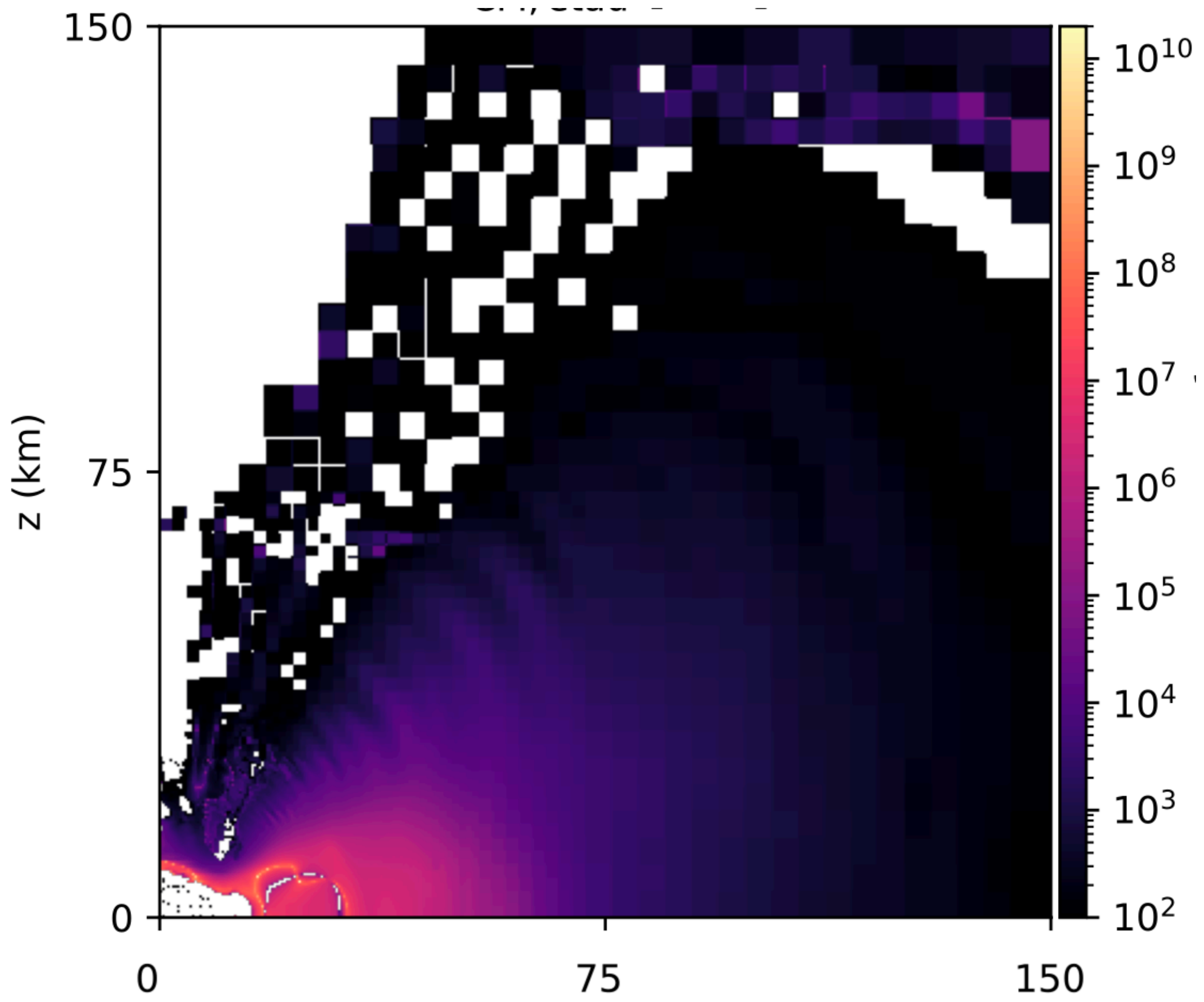
Backup:
More CFI details

Wide range of CFI for $\mu\tau$-sector in muonic matter

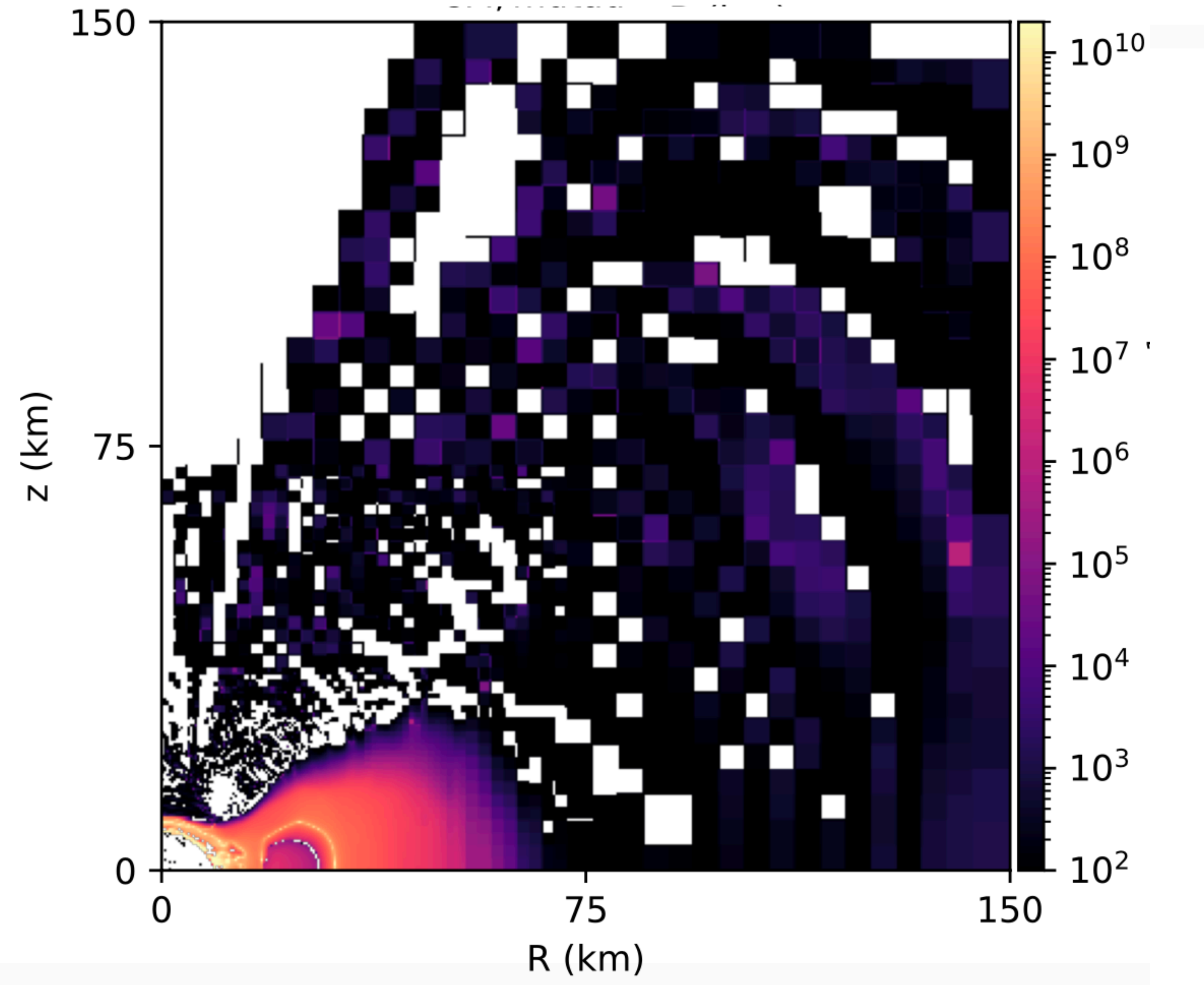
npe-matter, 4-species







$npe\mu$ — matter, 6-species, 3-flavors



Backup: Prelinmary [Liu+ in prep.], avoiding divergence for R-CFI

$$g_{ex,0}(E) = \int \frac{d\mu}{2} g_{ex}(E,\mu), \quad \bar{g}_{ex,0}(E) = \int \frac{d\mu}{2} \bar{g}_{ex}(E,\mu)$$

$$\begin{aligned} g_+ &= \max\left(g_{ex,0}, 0\right), \quad g_- = \max\left(-g_{ex,0}, 0\right) \\ \bar{g}_+ &= \max\left(\bar{g}_{ex,0}, 0\right), \quad \bar{g}_- = \max\left(-\bar{g}_{ex,0}, 0\right) \end{aligned}$$

$$g(\Gamma) \equiv f_{\nu_e}(\Gamma) - f_{\nu_x}(\Gamma), \qquad \bar{g}(\Gamma) \equiv f_{\bar{\nu}_e}(\Gamma) - f_{\bar{\nu}_x}(\Gamma).$$

$$n_+ \equiv \sqrt{2} G_F \int \frac{dE\, E^2}{2\pi^2}(g_+ + \bar{g}_-) \qquad \Gamma_+ \equiv \frac{\int \frac{dE\, E^2}{2\pi^2}(g_+ \Gamma_{\rm coh} + \bar{g}_- \bar{\Gamma}_{\rm coh})}{\int \frac{dE\, E^2}{2\pi^2}(g_+ + \bar{g}_-)} \qquad \Gamma_- \equiv \frac{\int \frac{dE\, E^2}{2\pi^2}(g_- \Gamma_{\rm coh} + \bar{g}_+ \bar{\Gamma}_{\rm coh})}{\int \frac{dE\, E^2}{2\pi^2}(g_- + \bar{g}_+)}.$$

$$n_- \equiv \sqrt{2} G_F \int \frac{dE\, E^2}{2\pi^2}(g_- + \bar{g}_+).$$

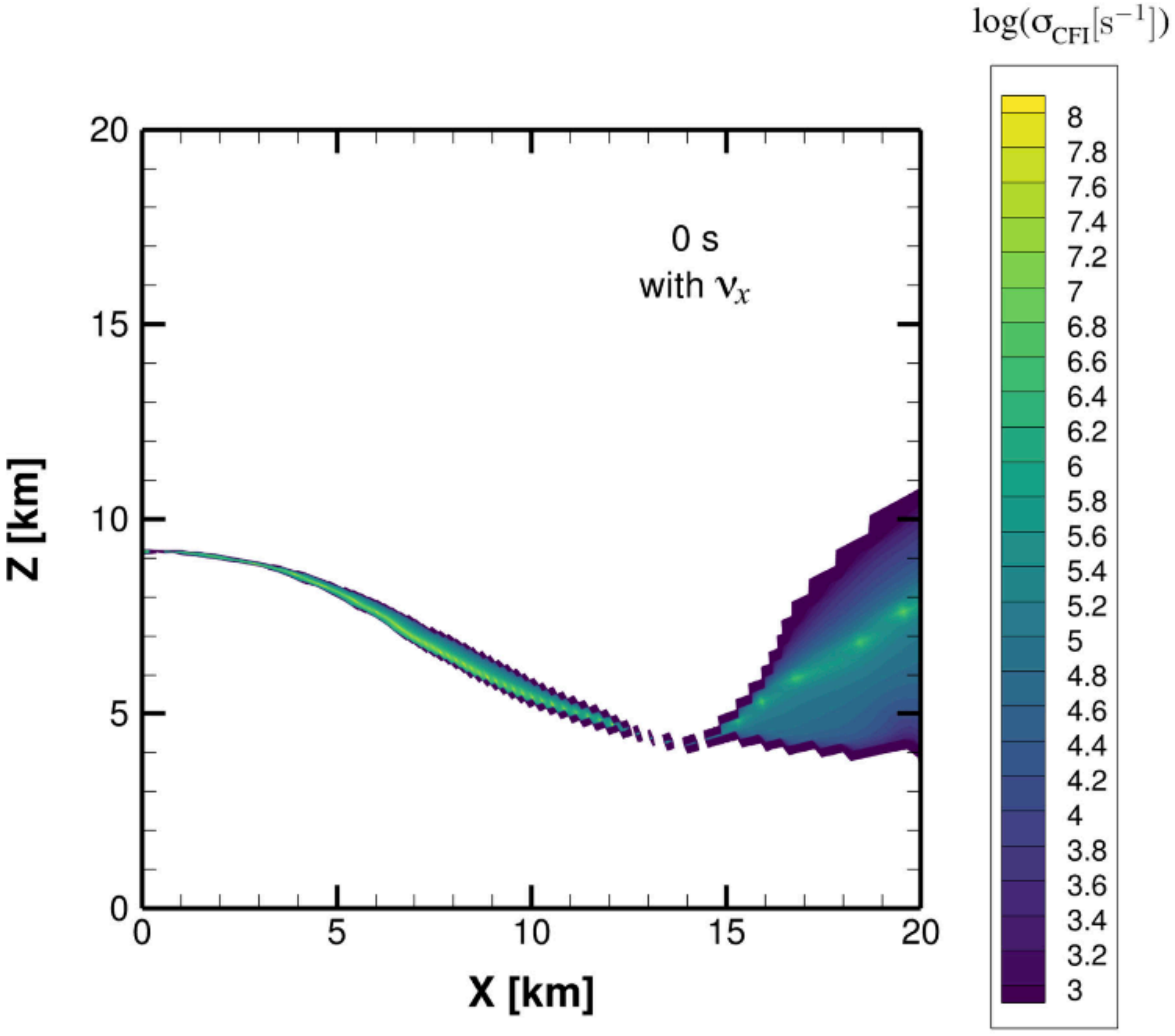
$$G \equiv \frac{n_+ + n_-}{2}, \qquad A \equiv \frac{n_+ - n_-}{2}, \qquad \gamma \equiv \frac{\Gamma_+ + \Gamma_-}{2}, \qquad \alpha \equiv \frac{\Gamma_+ - \Gamma_-}{2}.$$

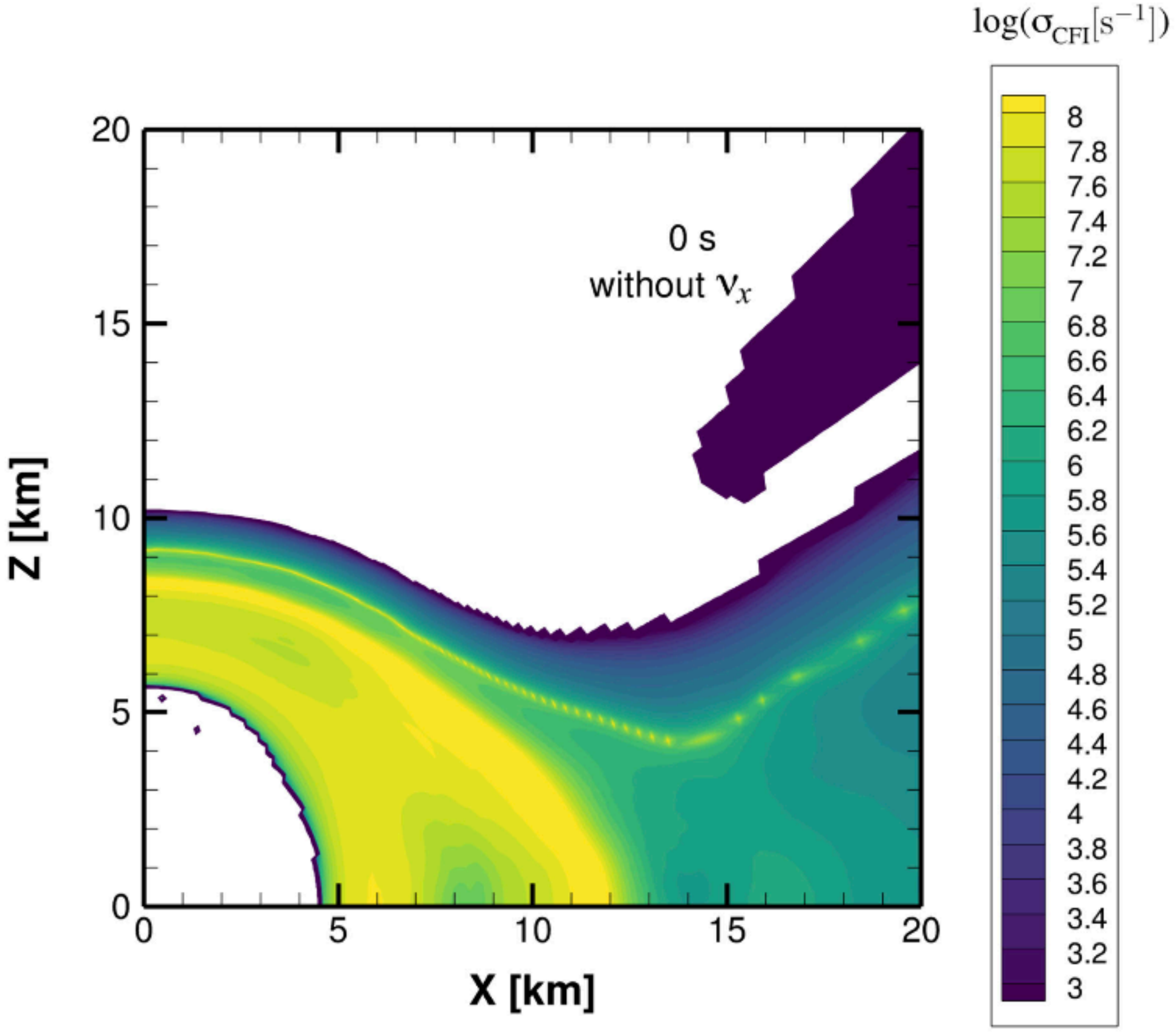
IP-branch: $\Omega = -A - i\gamma \pm \sqrt{A^2 - \alpha^2 + 2iG\alpha},$

IB-branch: $\Omega = \frac{A}{3} - i\gamma \pm \sqrt{\left(\frac{A}{3}\right)^2 - \alpha^2 - \frac{2}{3} iG\alpha}.$

Nagakura+ 2025

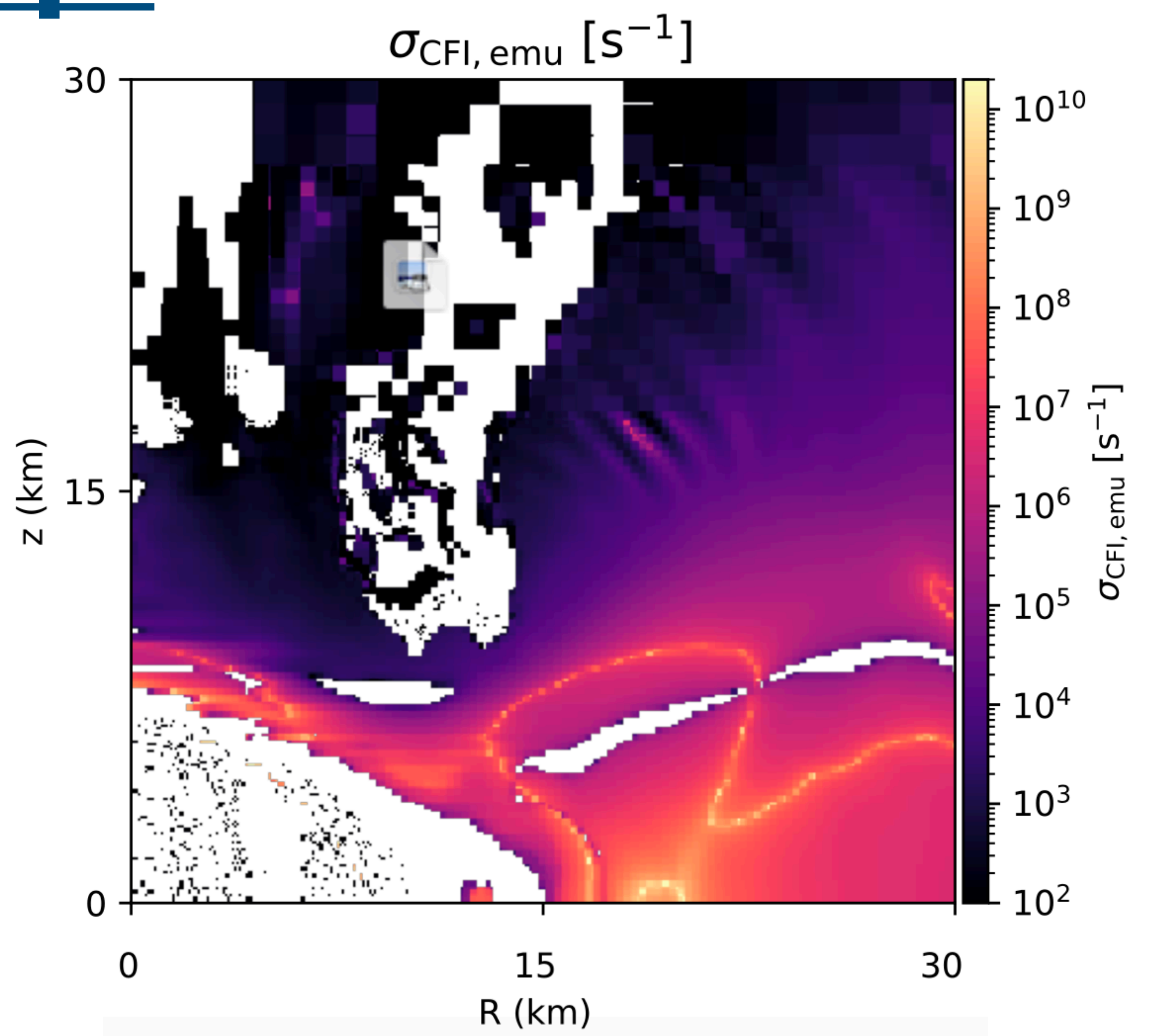
Npe: ν_x emission suppresses the R-CFI, due to higher extend of thermal eqm

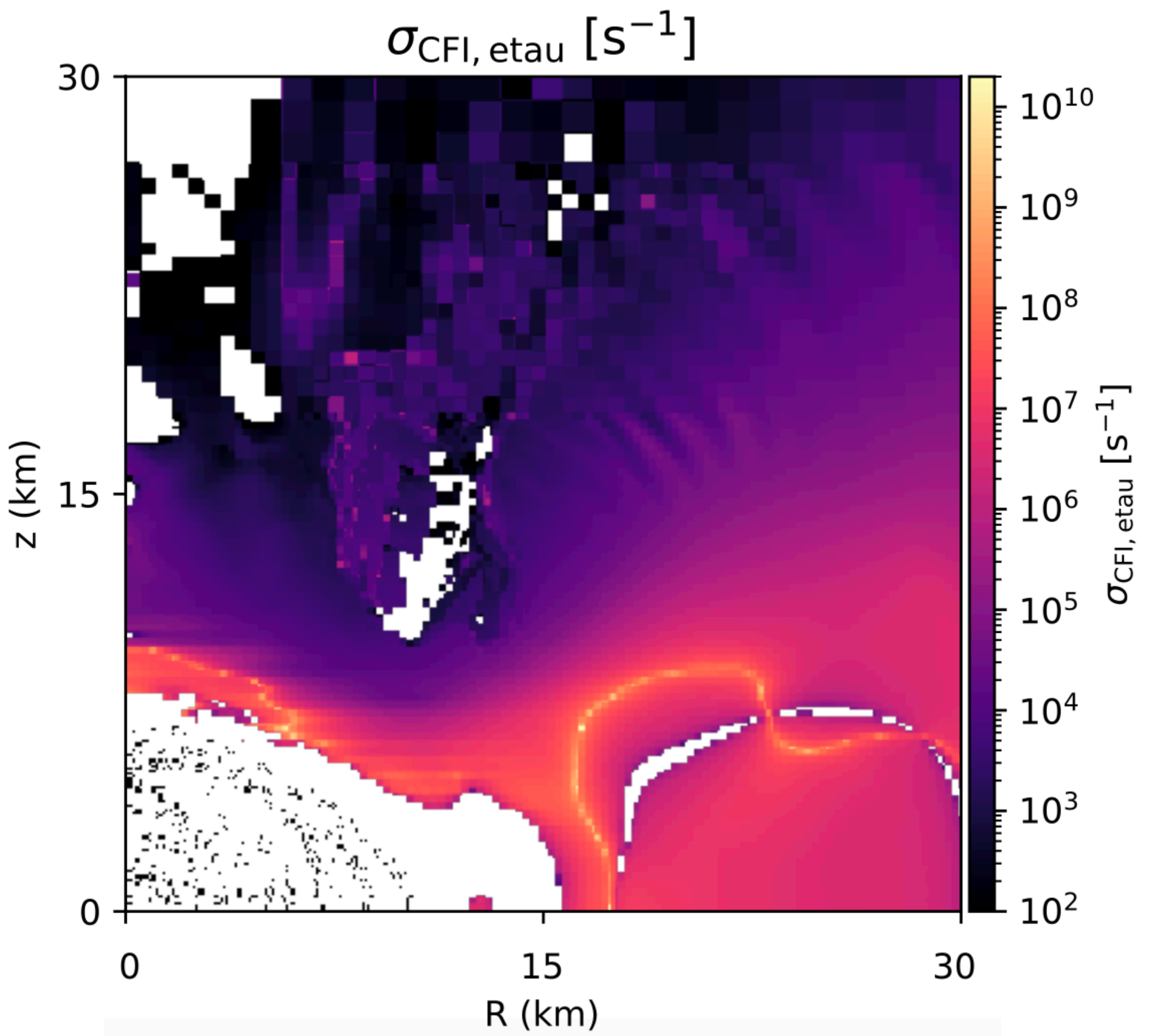


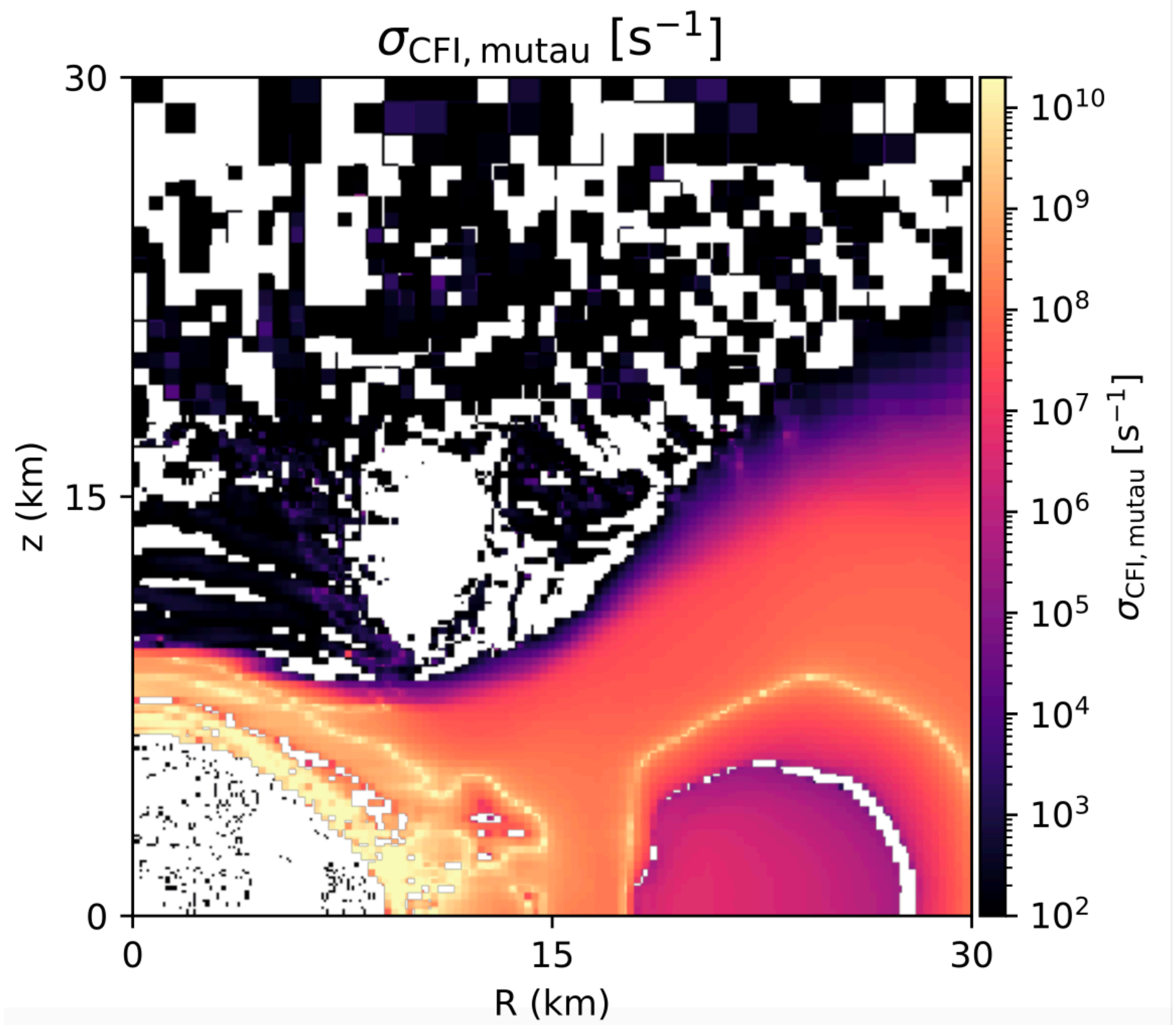


Backup:

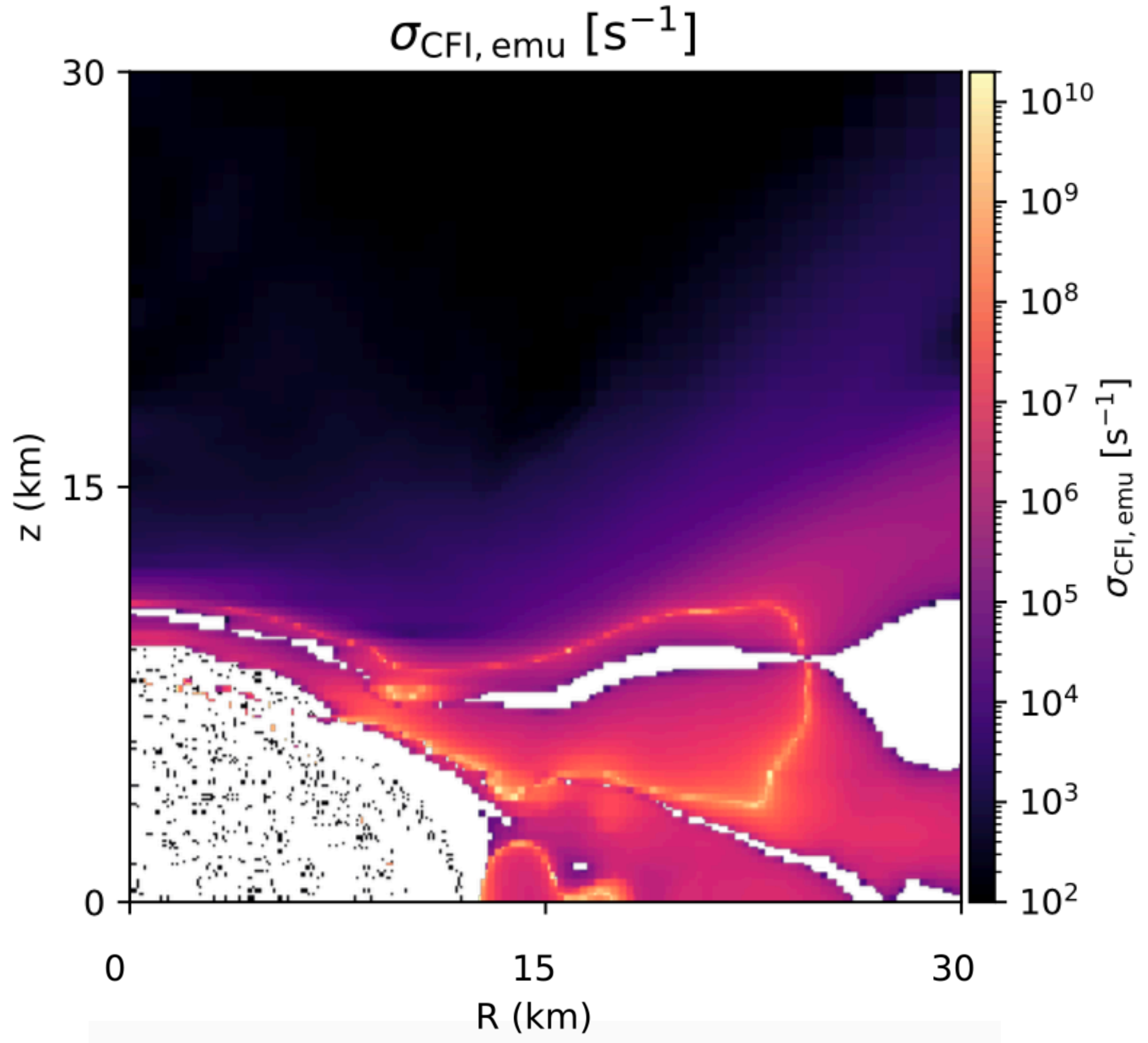
SFHo

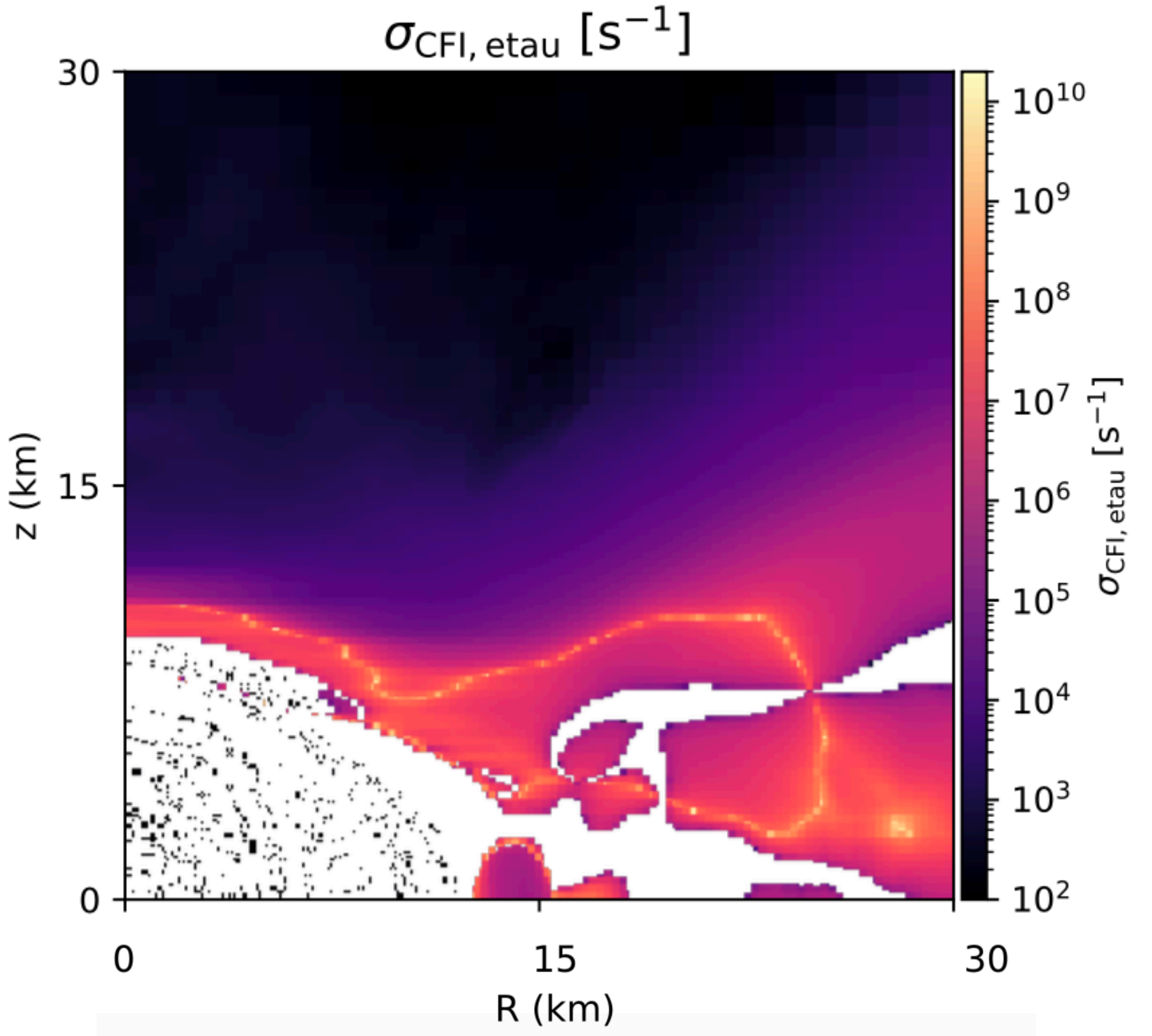


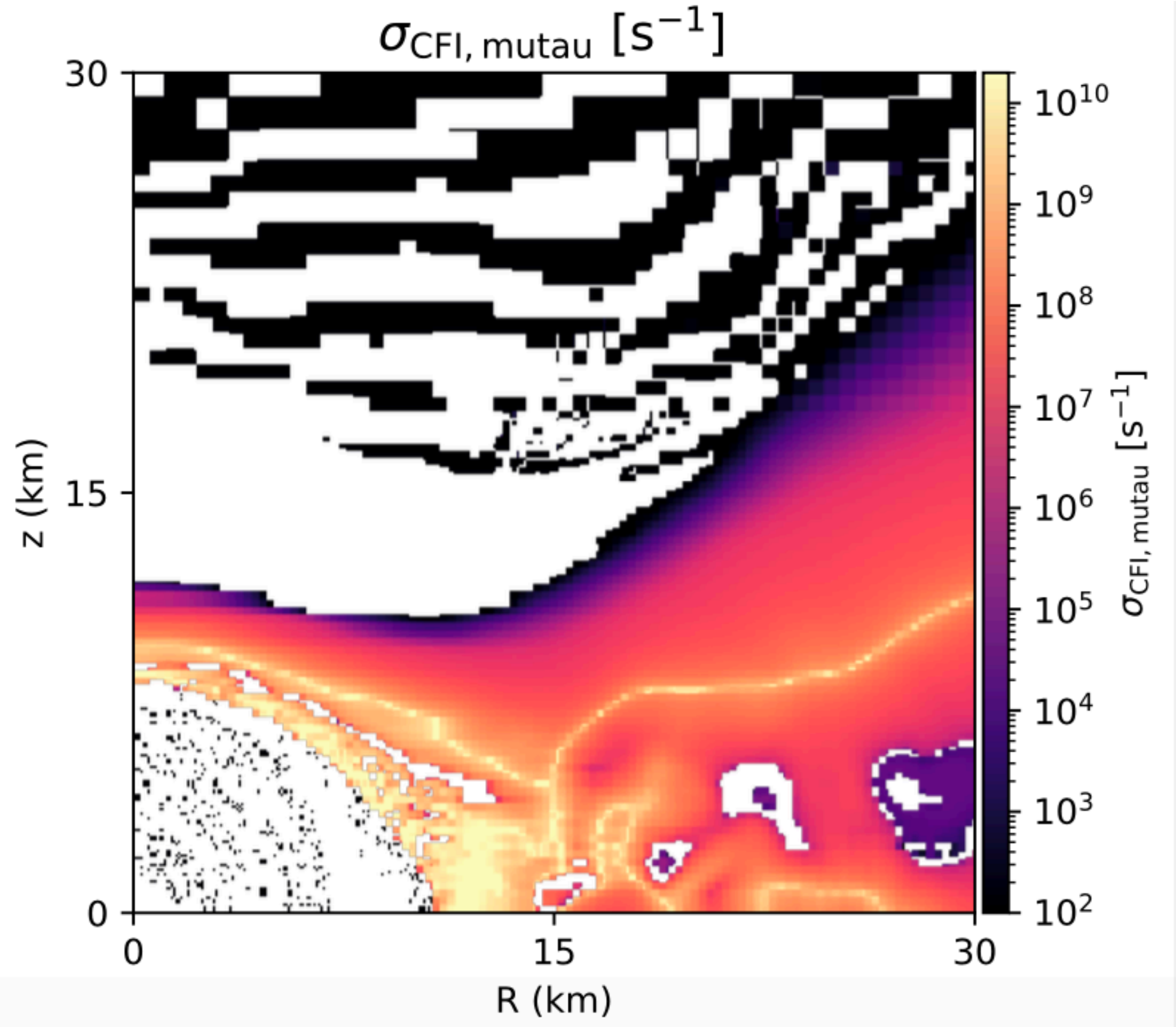




DD2







Backup:
Handoff details and accuracy

Backup: EFE after 3+1 decomposition

$$ds^2 = -(\alpha^2 - \beta_i\beta^i)dt^2 + 2\beta_i dx^i dt + \gamma_{ij}dx^i dx^j \, .$$

$$\partial_t\gamma_{ij} = -2\alpha K_{ij} + \nabla_i\beta_j + \nabla_j\beta_i$$

$$\partial_t K_{ij} = -\nabla_i\nabla_j\alpha + \alpha\left(R_{ij} + KK_{ij} - 2K_{ik}K^k_j\right) + \beta^k\nabla_k K_{ij}$$
$$+K_{ik}\nabla_j\beta^k + K_{jk}\nabla_i\beta^k - 4\pi\alpha\left[2S_{ij} - \gamma_{ij}\left(S^k_k - U\right)\right],$$

Free-evolution Eqs (Hyperbolic sector)

↓

+ Constraint violation damping + Moving puncture gauge (stronger Hyperbolicity) + etc..

↓

Baumgarte-Shapiro-Shibata-Nakamura (BSSN), Z4, Z4c, CCZ4, ...

$$\nabla_i\left(K^{ij} - \gamma^{ij}K\right) = 8\pi S^i,$$ Momentum constraint

$$R + K^2 - K_{ij}K^{ij} = 16\pi U,$$ Hamiltonian constraint

Constraint Eqs (Elliptic sector)

↓

+ Maximal slicing gauge: $K = \gamma^{ij}K_{ij} = 0$

+ $\gamma_{ij} = \psi^4\tilde{\gamma}_{ij} \approx \psi^4 f_{ij}$, Conformal flatness condition (CFC) => WAVELESS

+ etc...

↓

Extended CFC scheme (xCFC) [Cordero-Carrión+ 2009, Cheong2021]

$$\hat{\Delta}X^i + \frac{1}{3}\hat{\nabla}^i\left(\widehat{\nabla}_j X^j\right) = 8\pi f^{ij}\tilde{S}_j,$$

$$\hat{A}^{ij} \approx \hat{\nabla}^i X^j + \widehat{\nabla}^j X^i - \frac{2}{3}\widehat{\nabla}_k X^k f^{ij},$$

$$\hat{\Delta}\psi = -2\pi\psi^{-1}\tilde{E} - \frac{1}{8}\psi^{-7}f_{ik}f_{jl}\hat{A}^{kl}\hat{A}^{ij}\, .$$

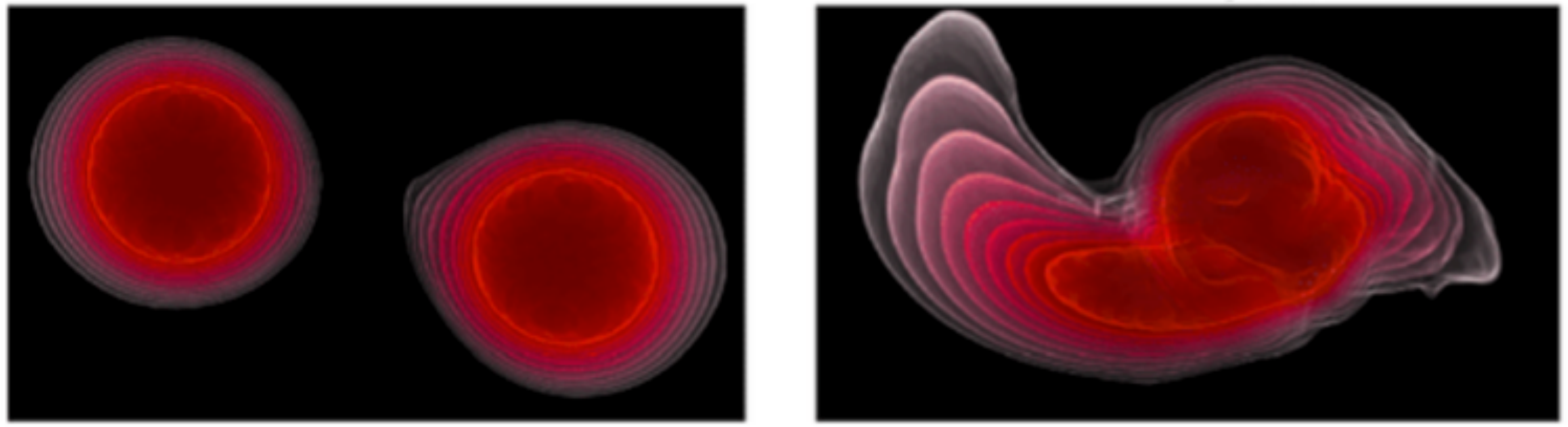
$$\hat{\Delta}(\alpha\psi) = (\alpha\psi)\left[2\pi\psi^{-2}(\tilde{E} + 2\tilde{S}) + \frac{7}{8}\psi^{-8}f_{ik}f_{jl}\hat{A}^{kl}\hat{A}^{ij}\right],$$

$$\hat{\Delta}\beta^i + \frac{1}{3}\widehat{\nabla}^i\left(\widehat{\nabla}_j\beta^j\right) = 16\pi\alpha\psi^{-6}f^{ij}\tilde{S}_j + 2\hat{A}^{ij}\widehat{\nabla}_j(\psi^{-6}\alpha),$$

Backup: Hybrid approach:

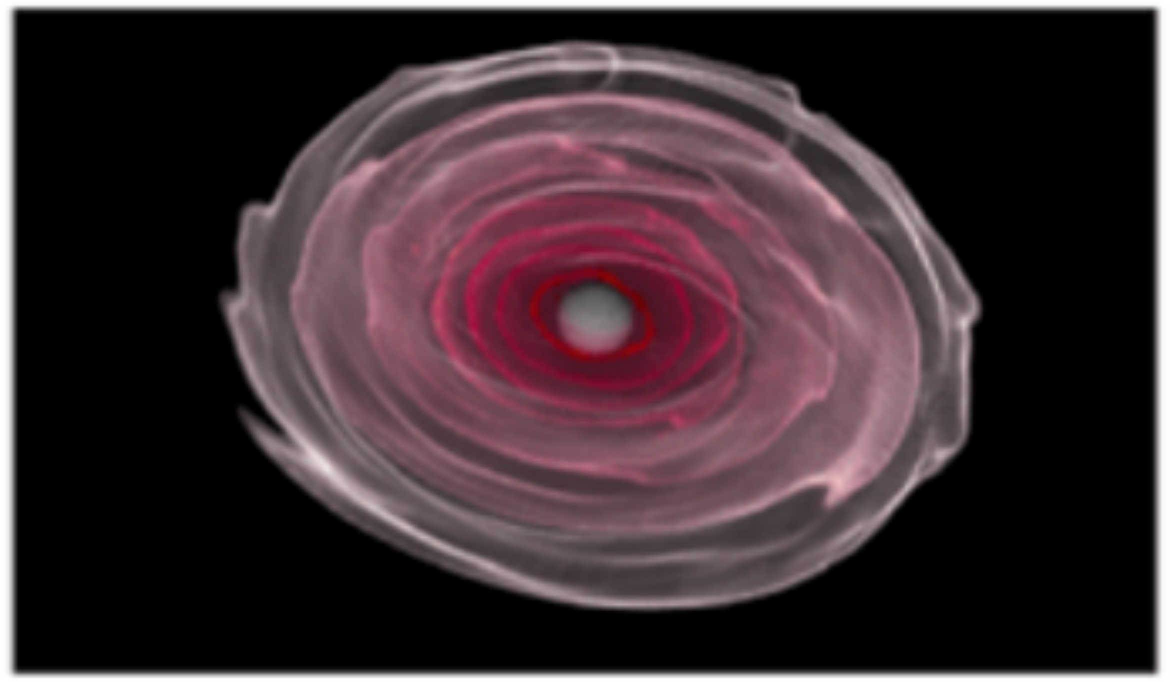
$\gamma_{ij} = \psi^4 \tilde{\gamma}_{ij} \approx \psi^4 f_{ij},$ Conformal flatness condition (CFC) => WAVELESS

Extended CFC scheme (xCFC) [Cordero-Carrión+ 2009, Cheong+ 2021, Ng+ 2024]



BSSN-Z4

Handoff after
> 15ms of merger

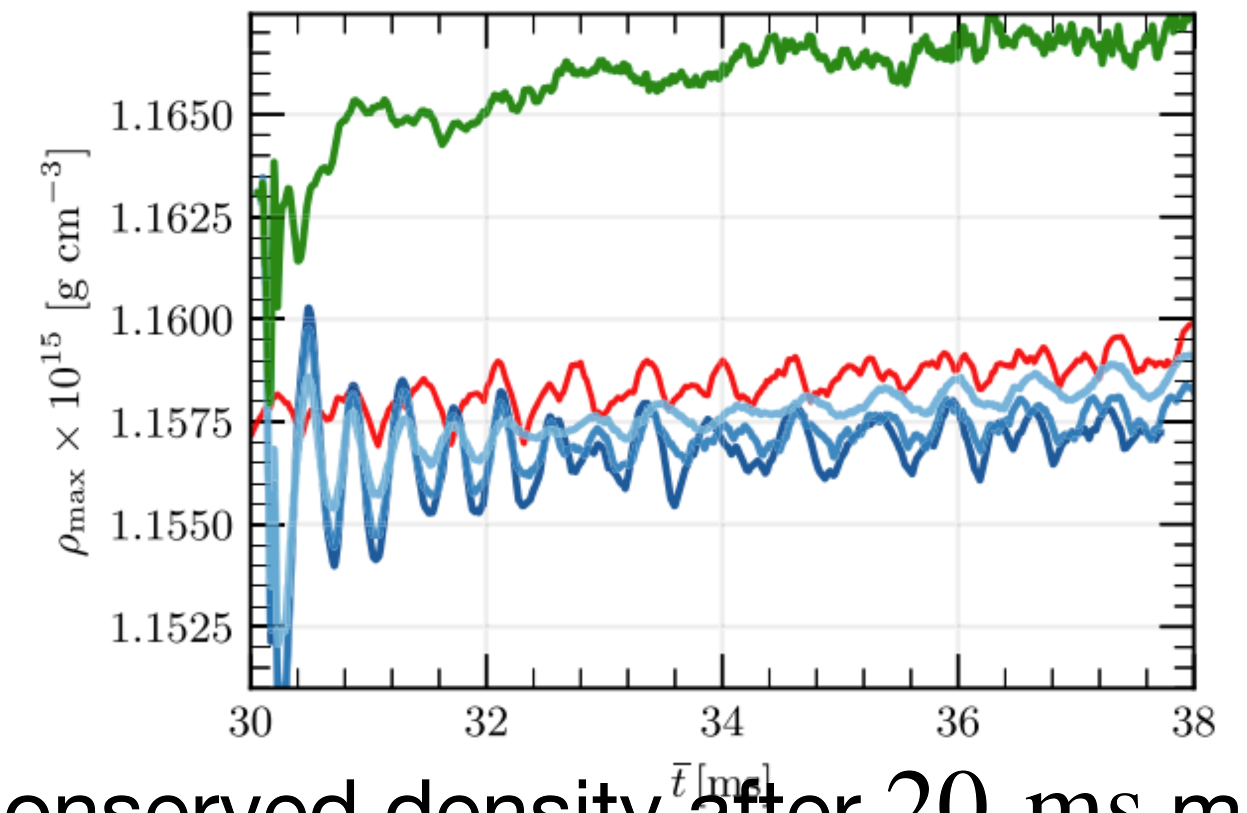


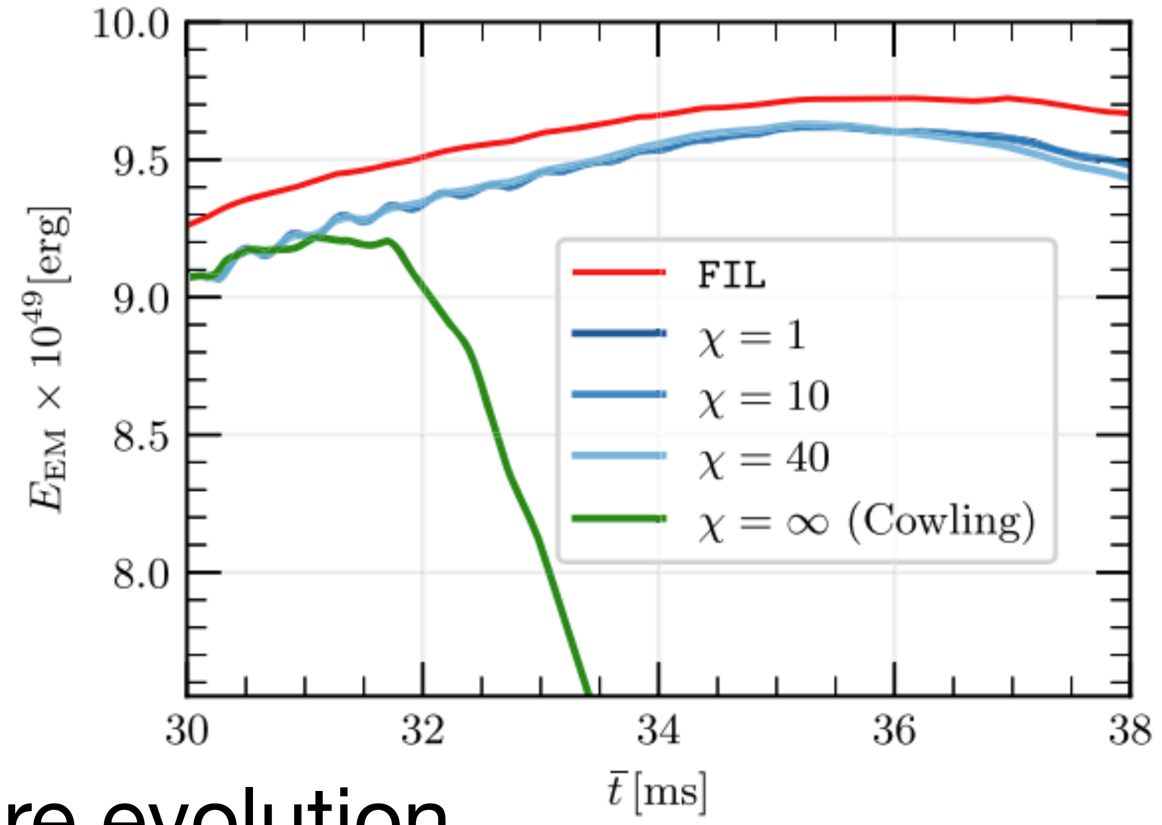
xCFC

Why we can do this?

- 20 - 30 ms after merger: $\tilde{\gamma}_{ij} - f_{ij} \approx 0.02 - 0.05$ [Fujibayashi+ 2017]
- GW emission not significant
- Quasi-Axisymmetric remnant, no matter mass ratio / EOS / spins [Hanauske+ 2017, Fujibayashi+ 2017, Dudi+ 2022]

Backup: Results:





- $\sim 1\,\%$ difference in rotational profiles and maximum conserved density after 20 ms more evolution

- **In 3D: 3-4 times faster than full GR results**
  - **Timestep no need to follow speed of light (∵ WAVELSS!)**
  - **xCFC => 40 timesteps to update once ($\sim 0\,\%$ cost from Gravity solver), but BSSN => Three times updates per timestep ($\sim 40\,\%$ cost from Grav. Solver)**

$$\chi = \frac{\Delta t_{\rm metric}}{\Delta t_{\rm hydro}}$$

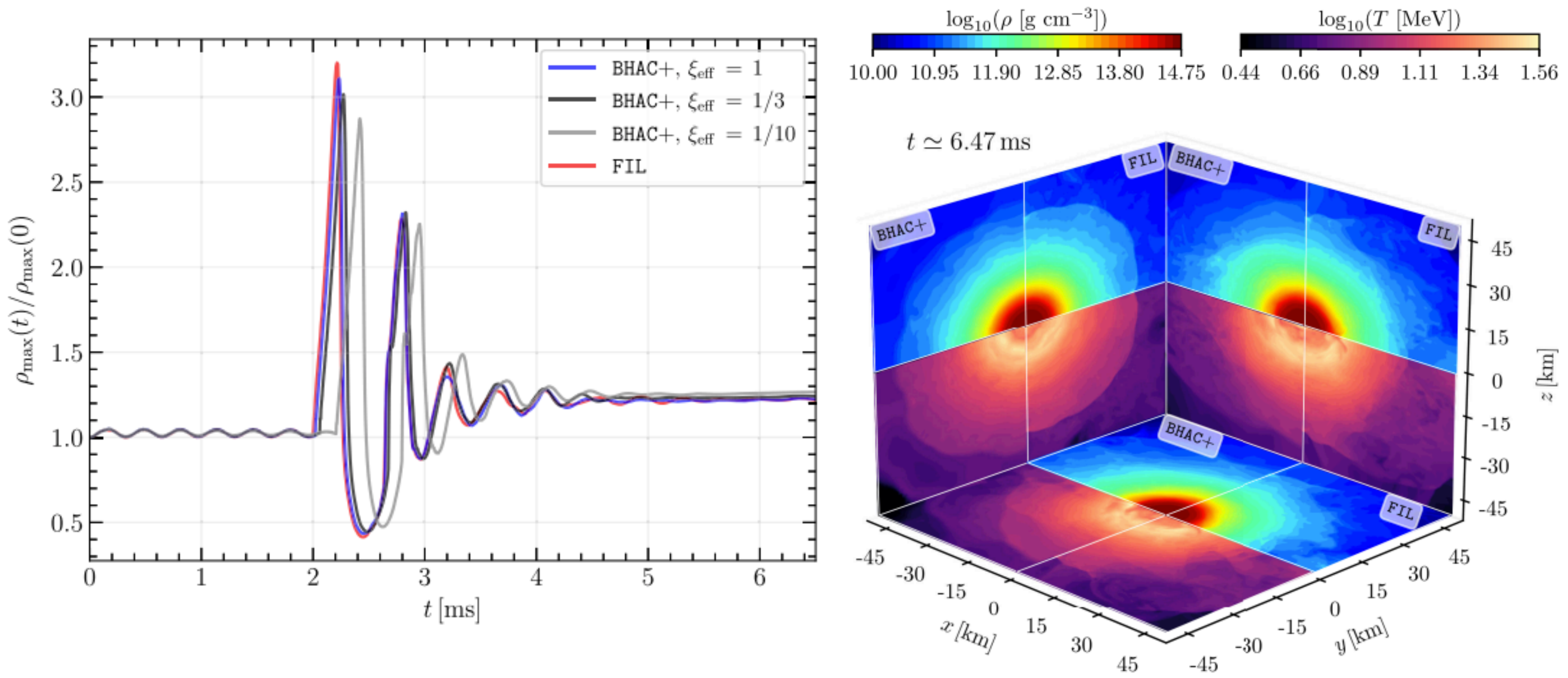
- Do it in 2D (Cylindrical), $< 3\,\%$ difference, but ~ 300 times faster than 3D Full GR code (∵ No need Cartoon in 2D)

Postmerger phase error sources mainly from MHD schemes / neutrino radiation schemes !

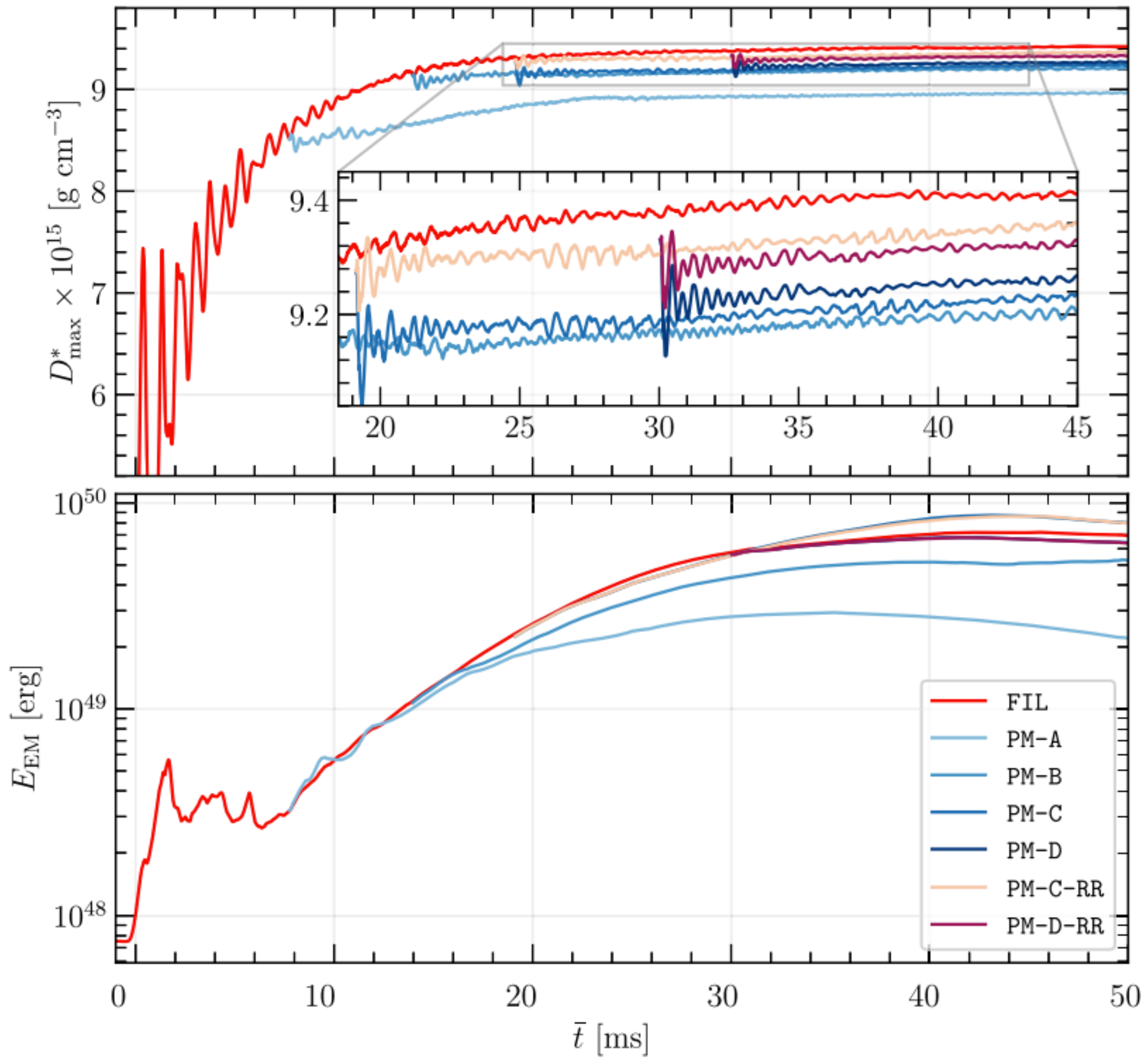
Backup: xCFC vs Full GR (BSSN-Z4)

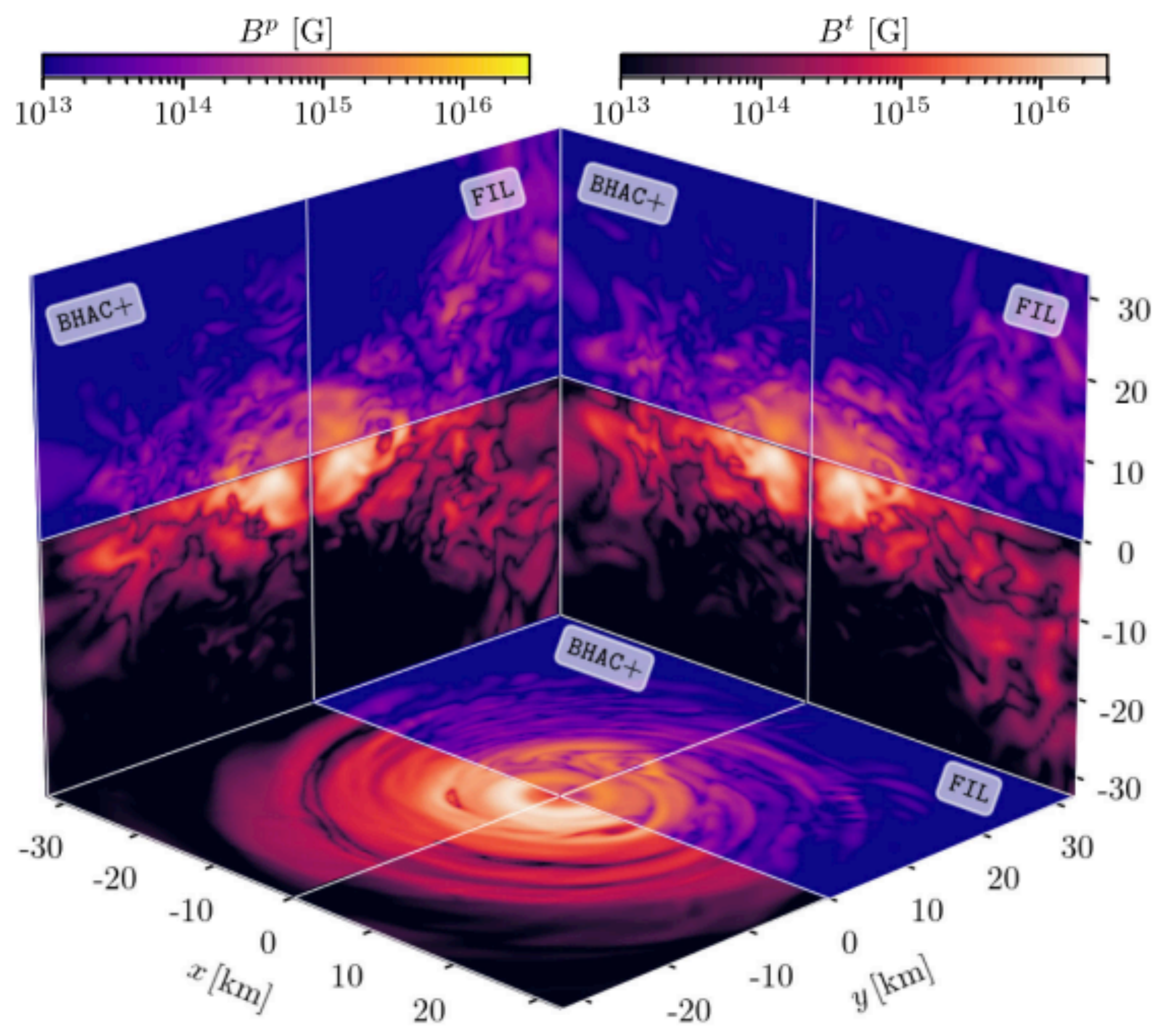
`BHAC+` vs `FIL` (Most+ 2019)

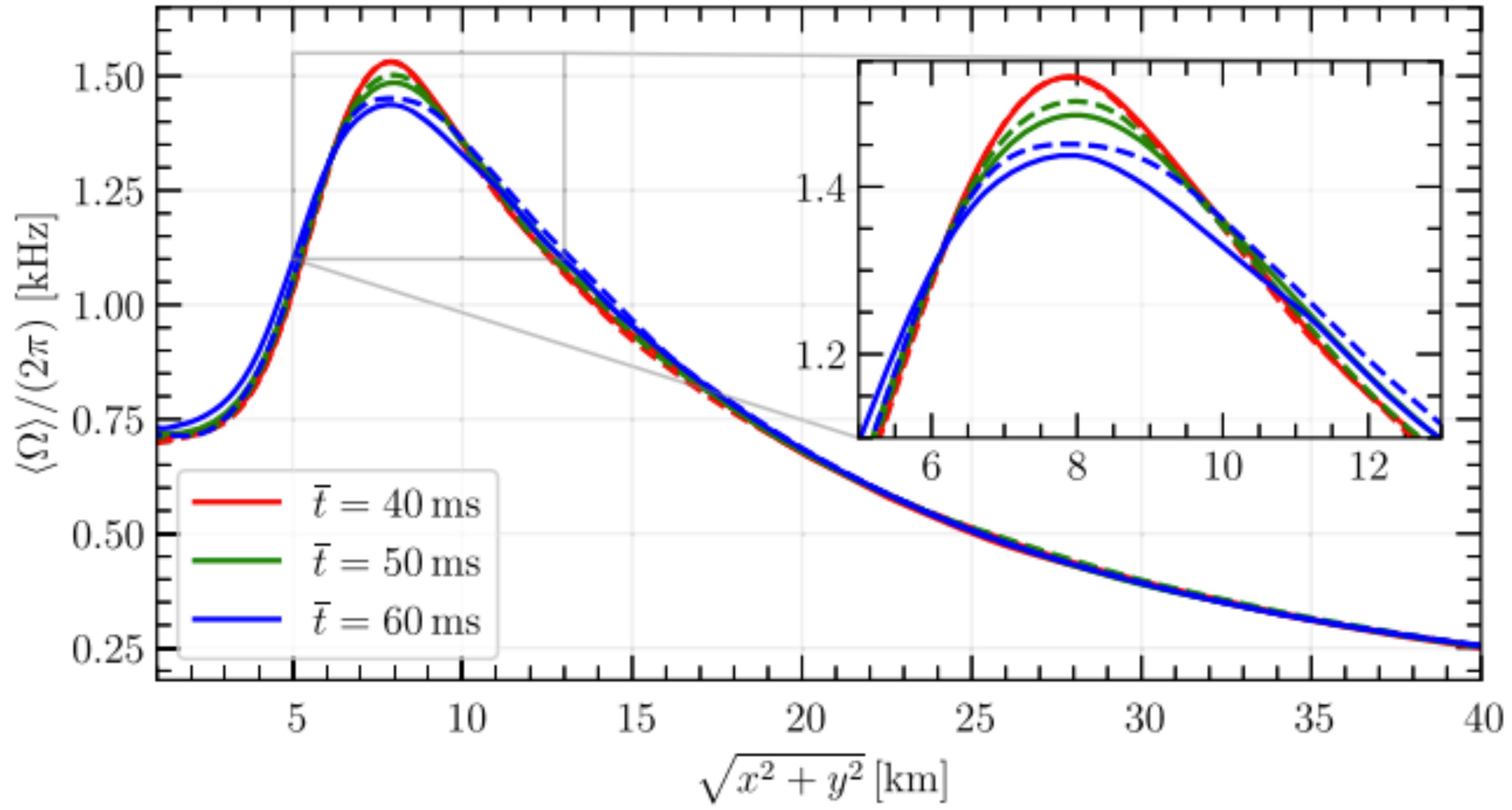
Head-on of non-magnetized BNS (3D-Cart) from beginning



Magnetized BNS Postmerger from Handoff ($M_{\rm tot} = 2.6\ M_\odot$)





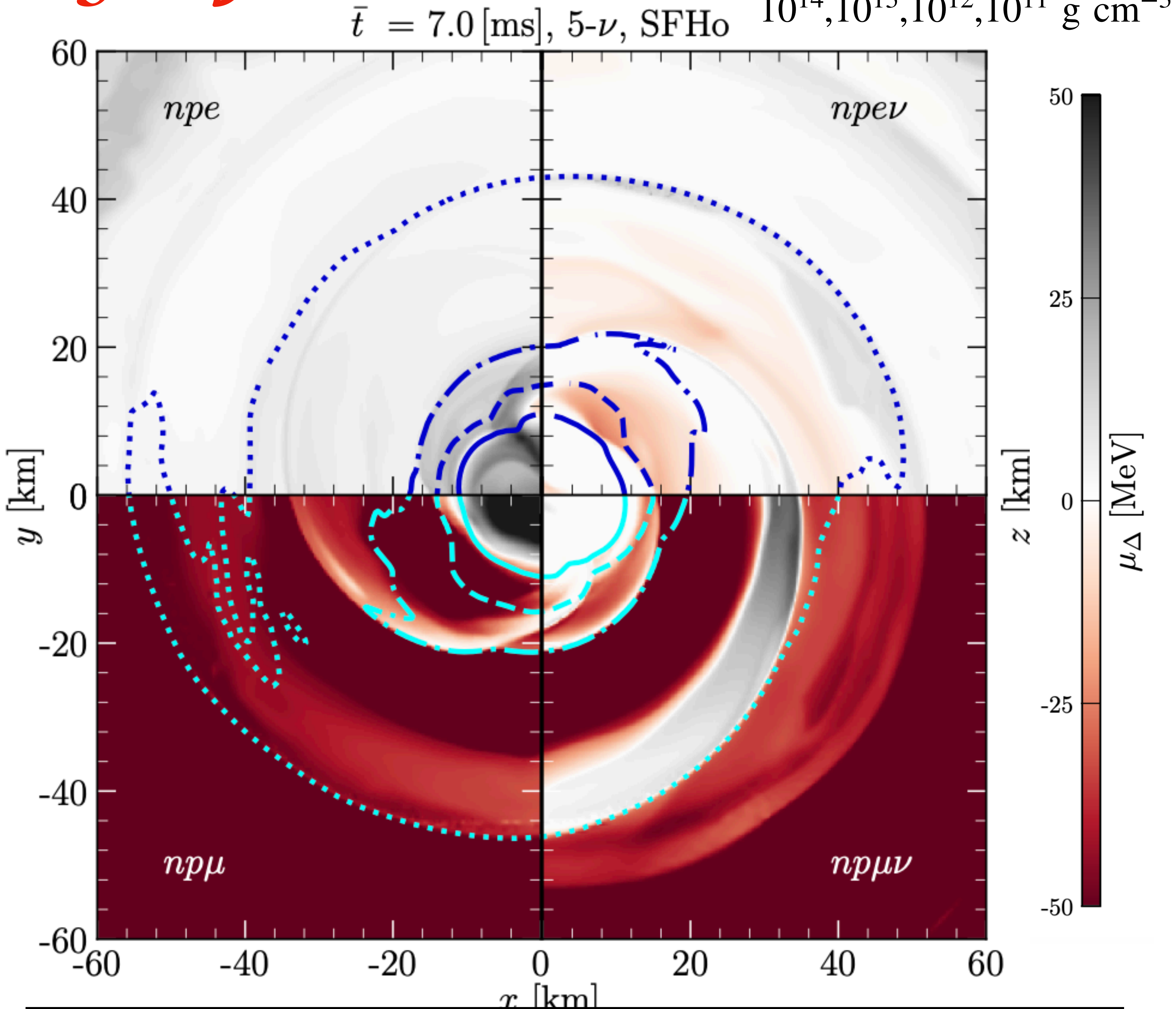


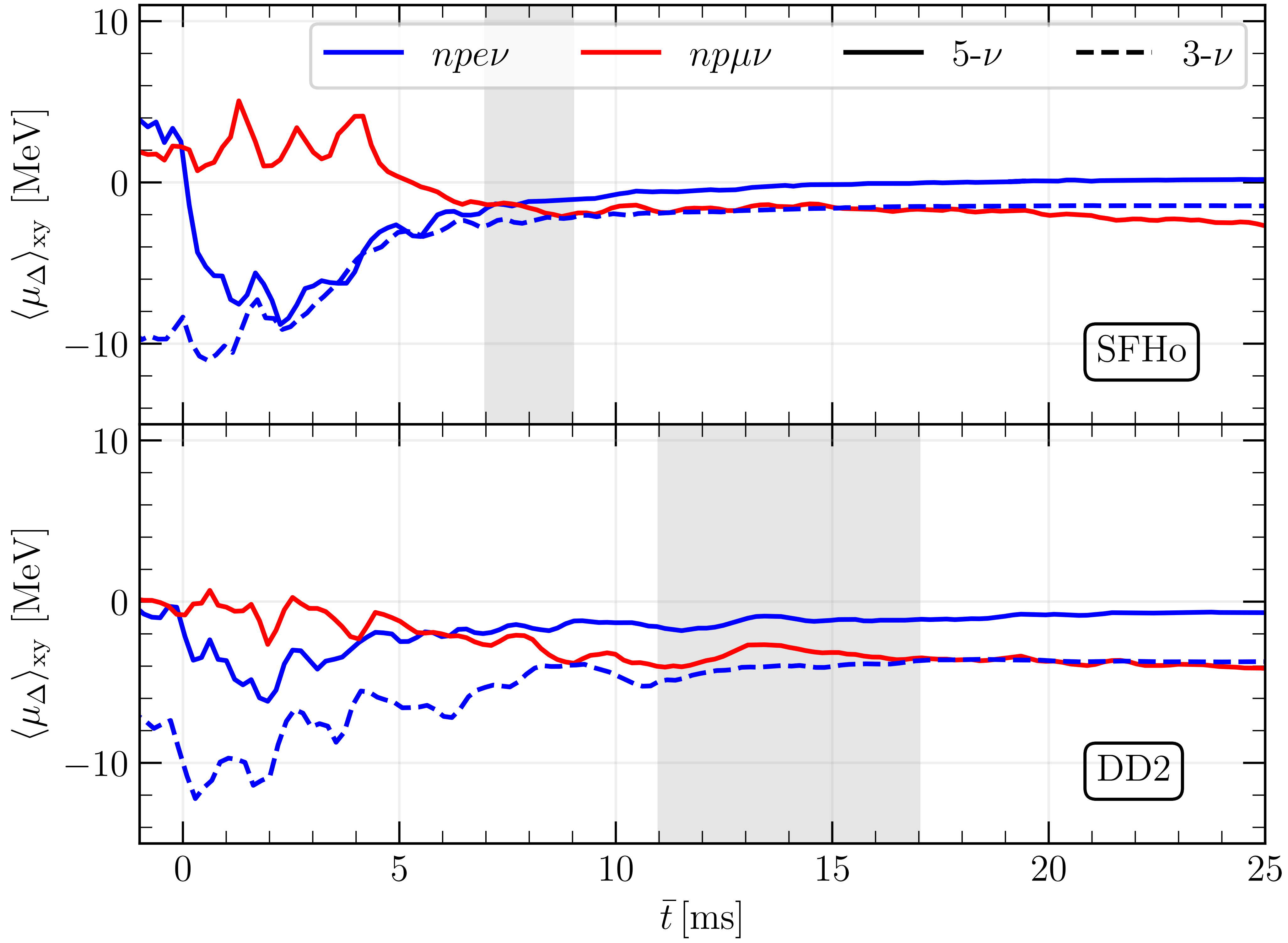
Backup: Muonic eqm

Backup: Weak equilibrium and trapped neutrinos

$5 - \nu$

Contour lines:
$10^{14}, 10^{13}, 10^{12}, 10^{11}$ g cm^{-3}





$\mu_\Delta^{npl} := \mu_n - \mu_p - \mu_l$ (npl-equilibrium)

$\mu_\Delta^{npl\nu} := \mu_n - \mu_p - \mu_l - \mu_{\bar{\nu}_l}$

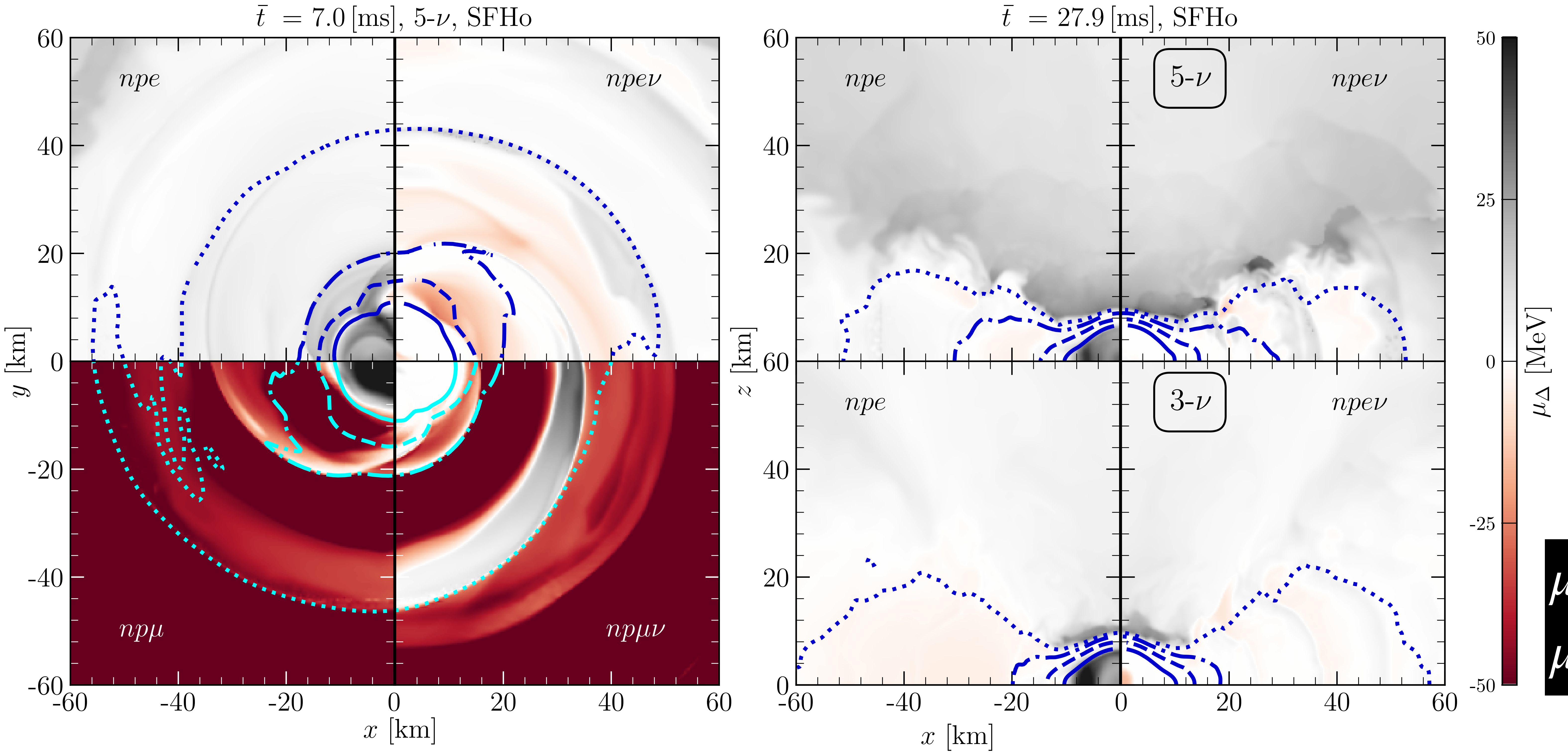
($npl\nu$-equilibrium / trapped neu equilibrium)

$l = e, \mu$: electronic / muonic

- Trapped neu generated: matter energy —> radiative energy but still stay there
  => Cool the remnant as well!
- $3 - \nu$ **case: attains** $npl\nu$**-eqm in few ms after merger [Espino+ PRL 2024]**
- $5 - \nu$ **case:** round 7 - 9 ms (11 - 17 ms) reaching both $npe\nu$ and $np\mu\nu$ equilibria
  See also left figure: **inner region of remnant**
- Outer region:
  For $\mu_\Delta^{np\mu} < 0$, system does not want muons be there =>
  de-muonization ($p + \nu_\mu \to \mu^- + n$ **&.** $\mu^+ + p \to n + \bar{\nu}_\mu$ **)**

Changed trapped neutrino hierarchy

Contour lines:
$10^{14}, 10^{13}, 10^{12}, 10^{11}$ g cm^{-3}



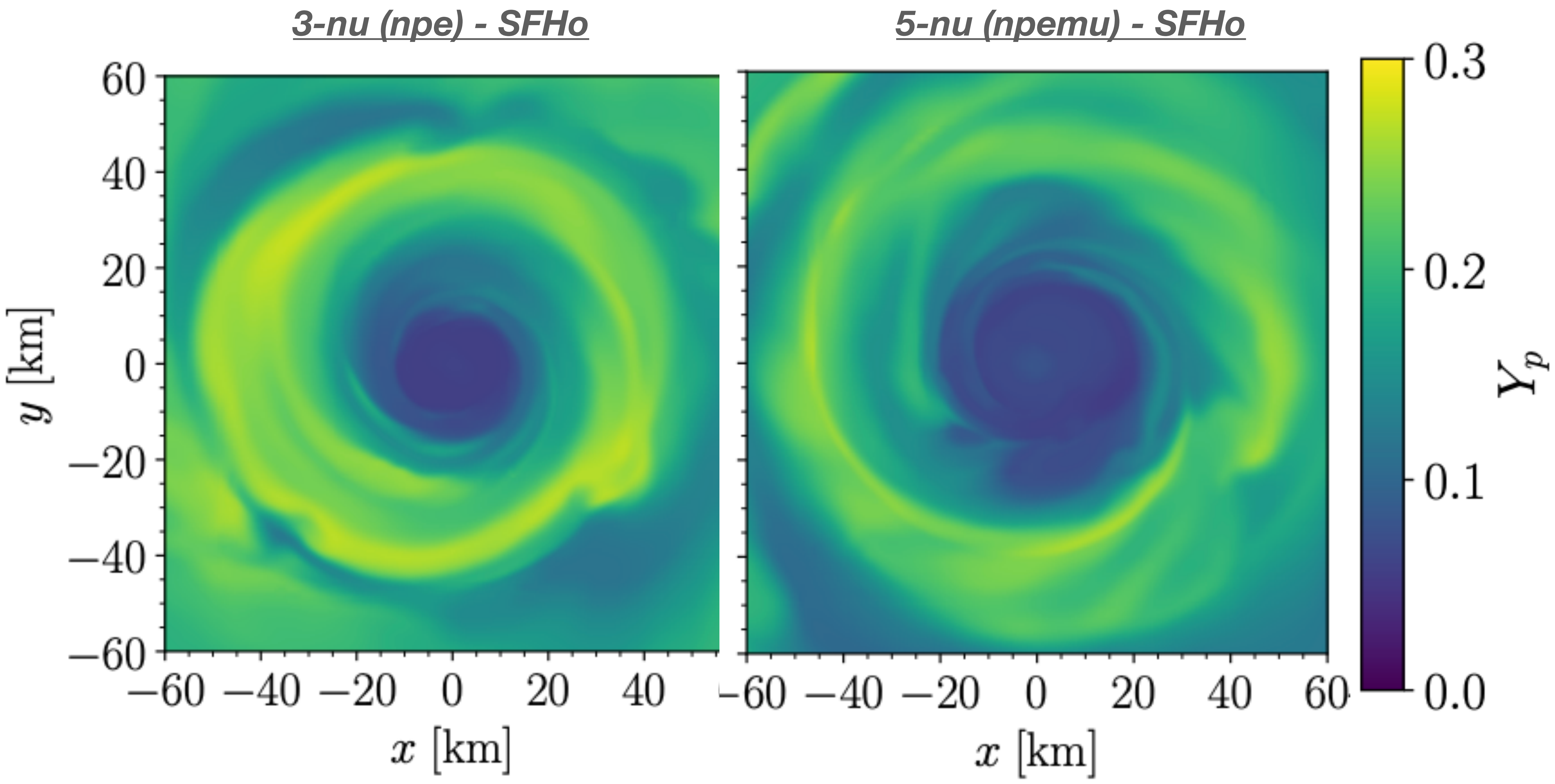
$$\mu_\Delta^{npl} := \mu_n - \mu_p - \mu_l$$
$$\mu_\Delta^{npl\nu} := \mu_n - \mu_p - \mu_l - \mu_{\bar{\nu}_l}$$

- **Trapped neutrino properties:**
  - **Bulk Viscosity contributing to GW signatures [Most+ ApJL 2024, Chabanov+ PRL 2025]: Attains trapped eqm => very weak bulk viscosity from dense regions due to β-processes**
  - **Hierarchy:** $\mu_{\bar{\nu}_e} > \mu_x > \mu_{\nu_e}$ **($3-\nu$ case)** **VS.** $\mu_{\bar{\nu}_\mu} > \mu_{\bar{\nu}_e} > \mu_{\nu_x} > \mu_{\nu_e} > \mu_{\nu_\mu}$ **($5-\nu$ case)**
- For $\mu_\Delta^{npe} > 0$ at **polar region and disk**, electrons are underproduced => Y_e slowly increases by reabsorption later (Slow electronization)， while $3-\nu$ mostly finished the electronization

Backup: Comparison to Leakage approach [Gieg+ PRD 2025]

Qualitative Similarities with muonic Leakage approach (Gieg+ 2025)

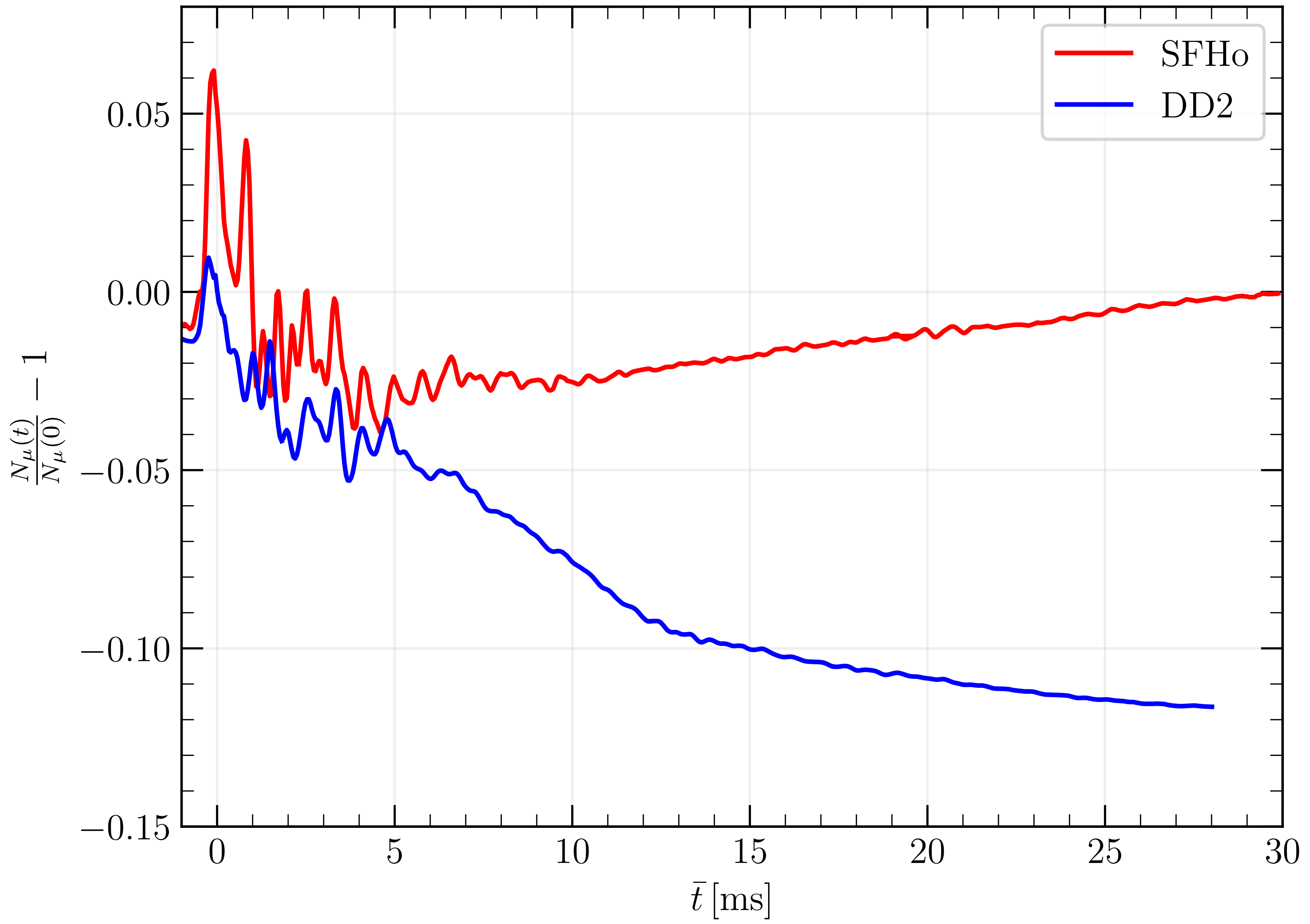
- Colder remnant
- Lower Y_p in the disk
- Lower total ejecta mass
- Rapid de-muonization in the disk
- Higher luminosity around merger time due to extra channels
- But quantitatively different

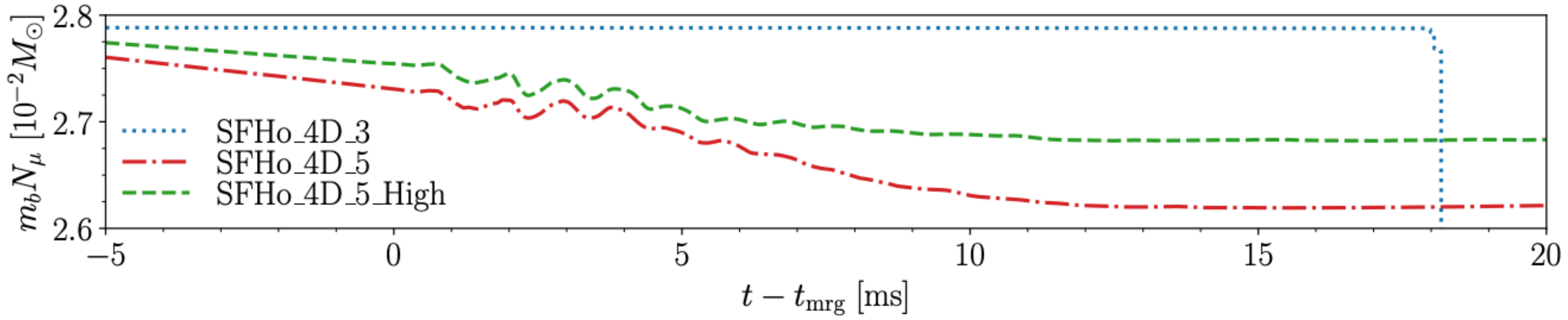


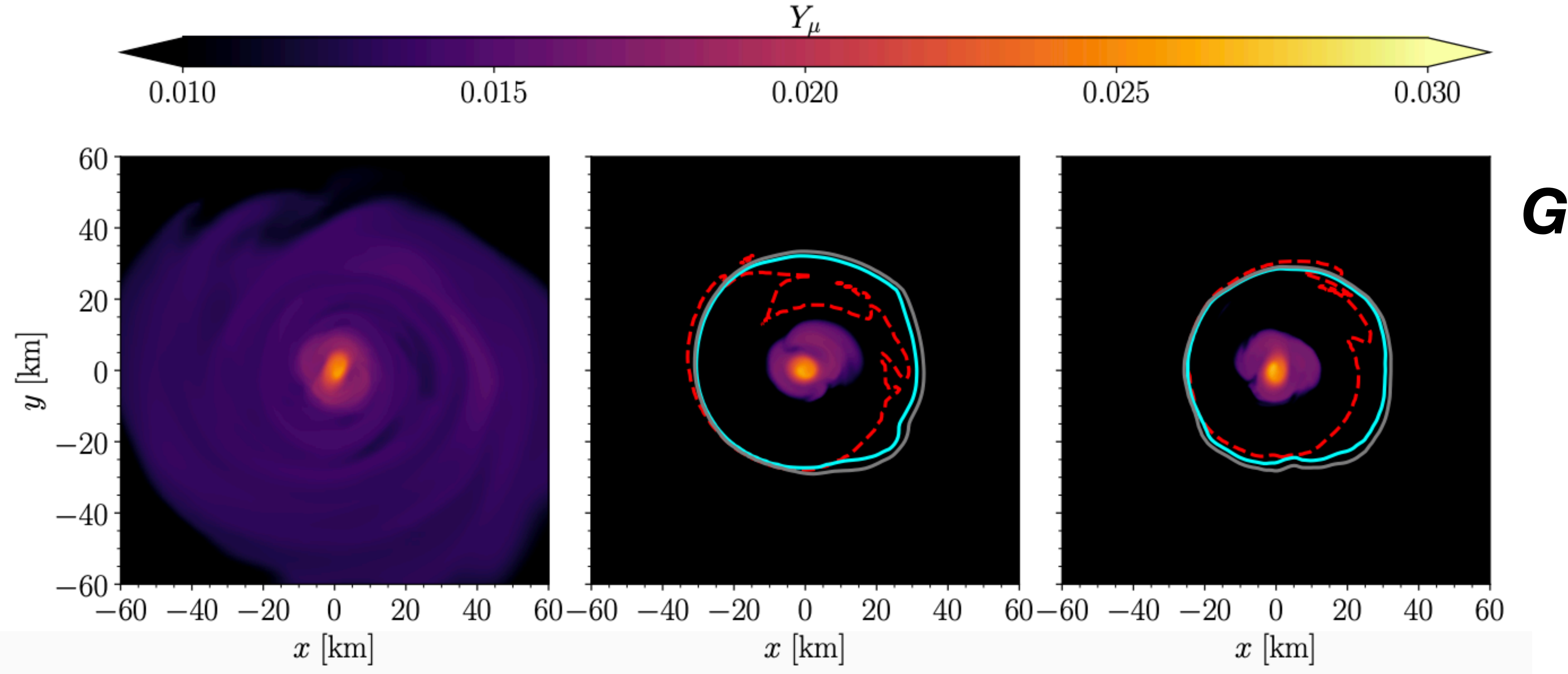
Both came out in similar period independently (~ 1 month difference) + different methods: a lot of similarities

=> Coincidence ? Robust effects ?

All-the-way demuonization???





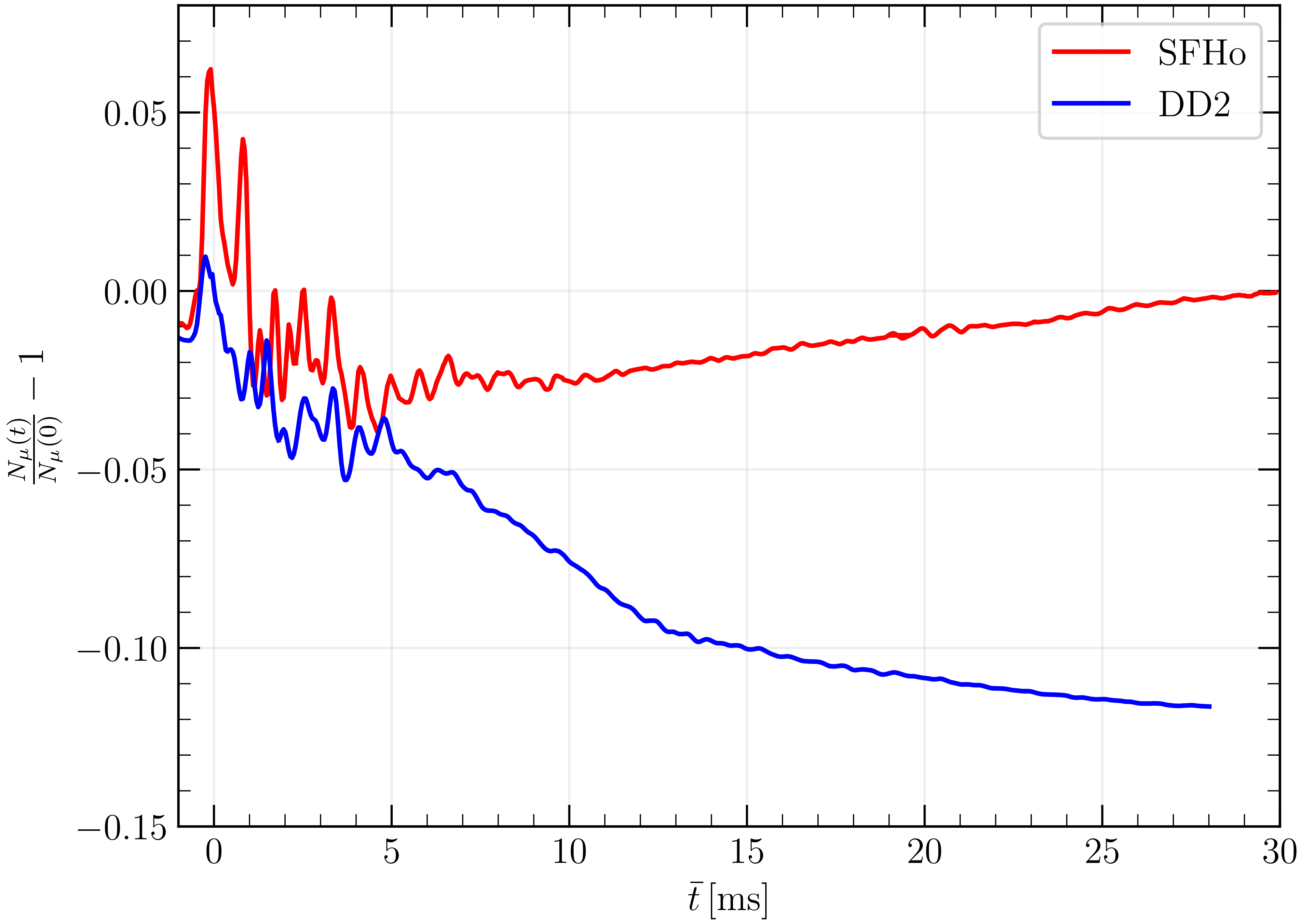


Gieg+ 2024

- Completely different evolution of N_μ compared to Leakage approach [Gieg+2024]
- Reasons?
  Not fixing degeneracy parameters η_ν and opacities in low-ρ-T regions, especially for grey schemes.
  [See backup slides]
- Need new atmospheric treatments!

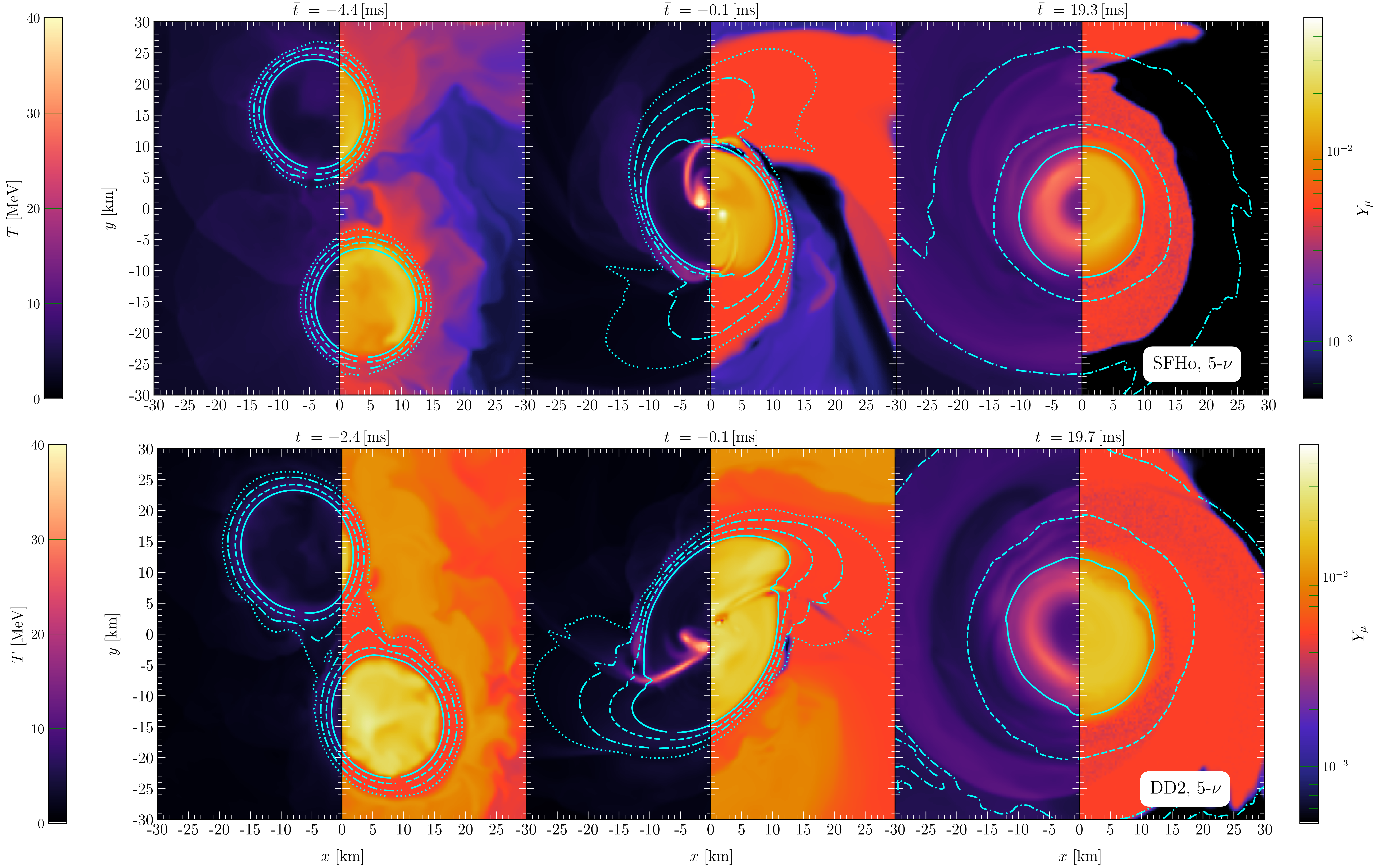
(De)muonization affecting Neutrino mean energy

- inferred from the neutrino mean energy in grey M1: $\left\langle \epsilon_\nu \right\rangle = W\left(\bar{E}_\nu - \bar{F}^i_\nu v_i\right)/N_\nu$, where N_ν is neutrino number density
- SFHo: $L_{\bar{\nu}_\mu} > L_{\nu_\mu}$, N_μ remains constant (or slightly increases) => $\left\langle \epsilon_{\bar{\nu}_\mu} \right\rangle >> \left\langle \epsilon_{\nu_\mu} \right\rangle$

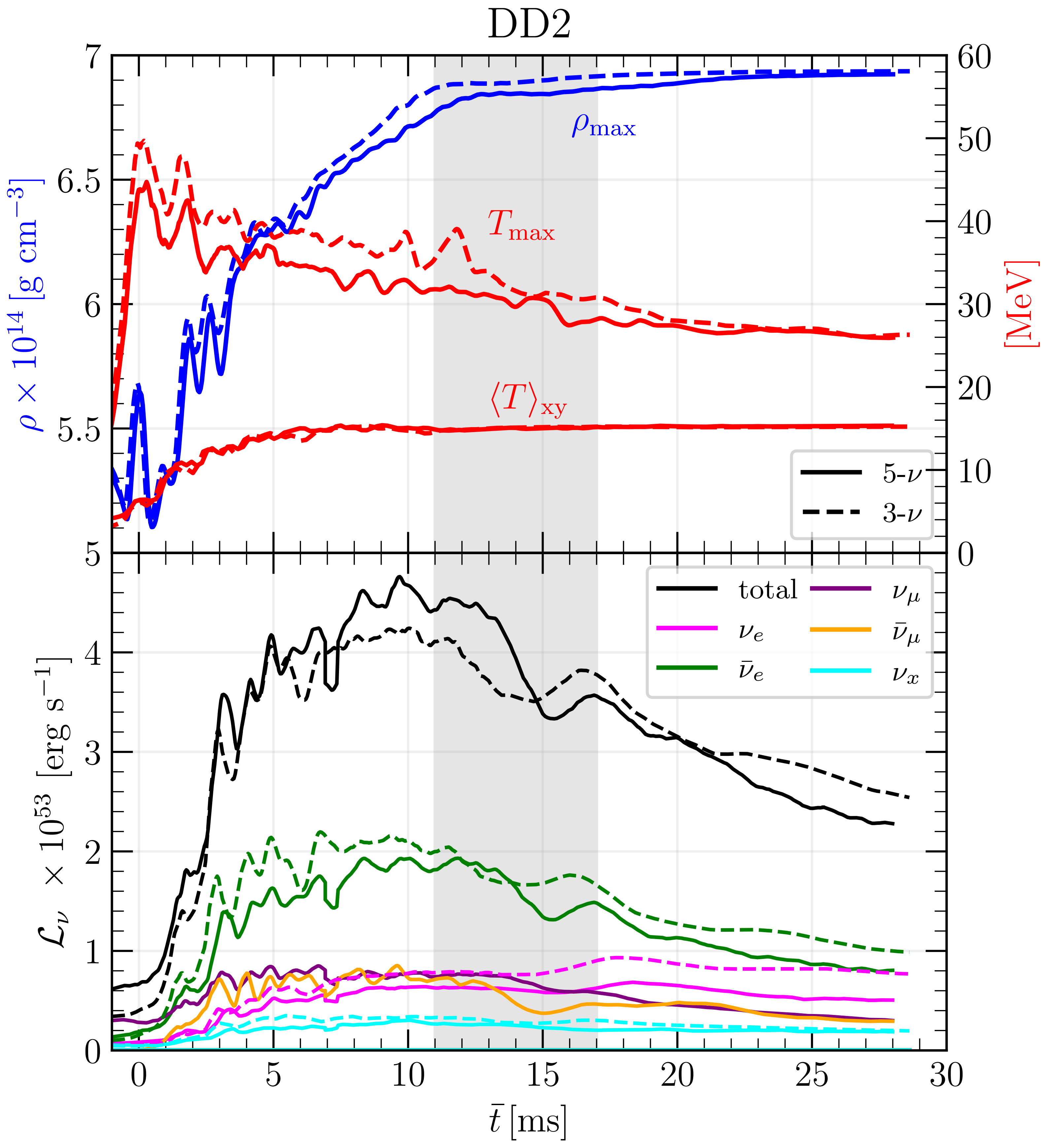


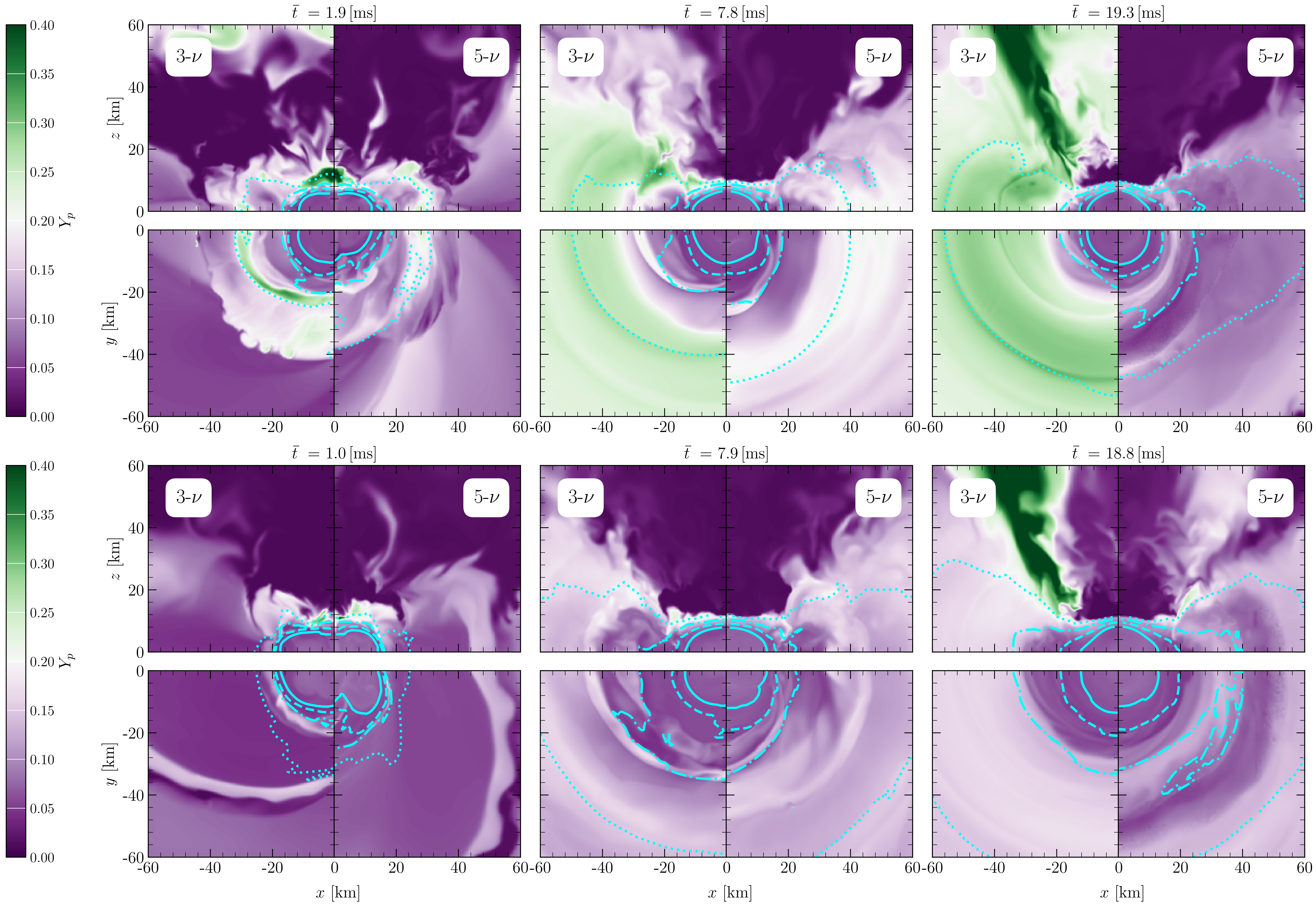
Backup: EOS dependence

Results: Temperature and muon fraction (Muonization or De-muonization?)

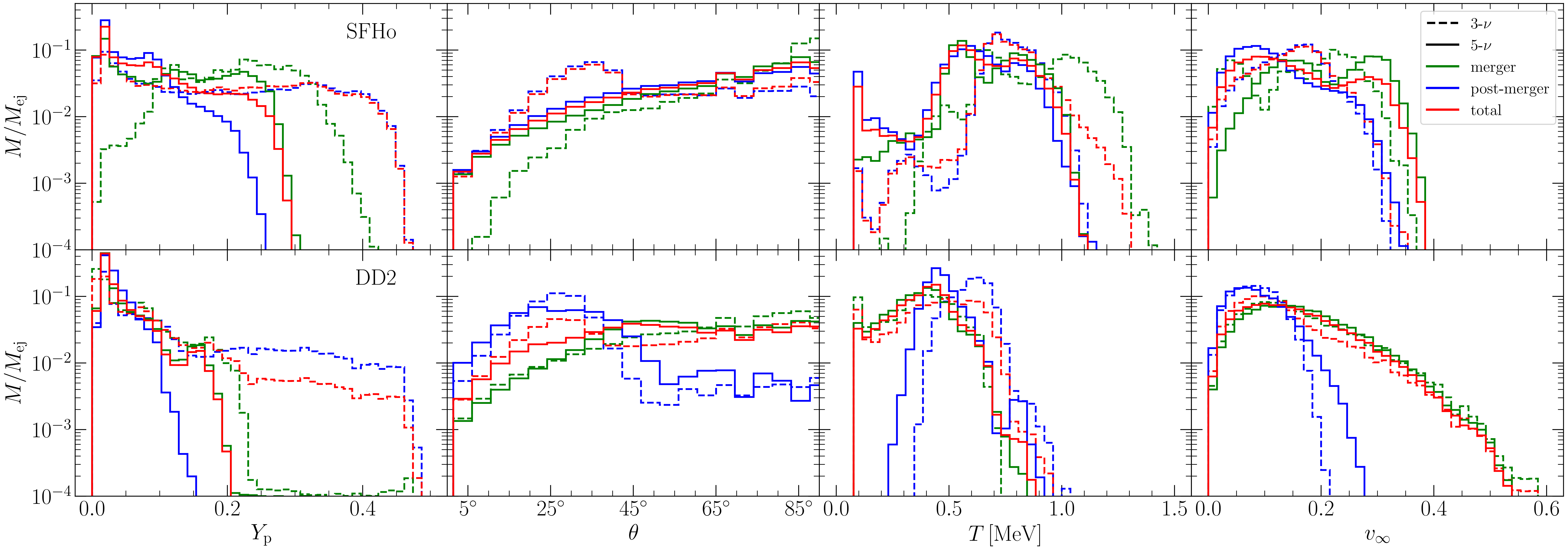


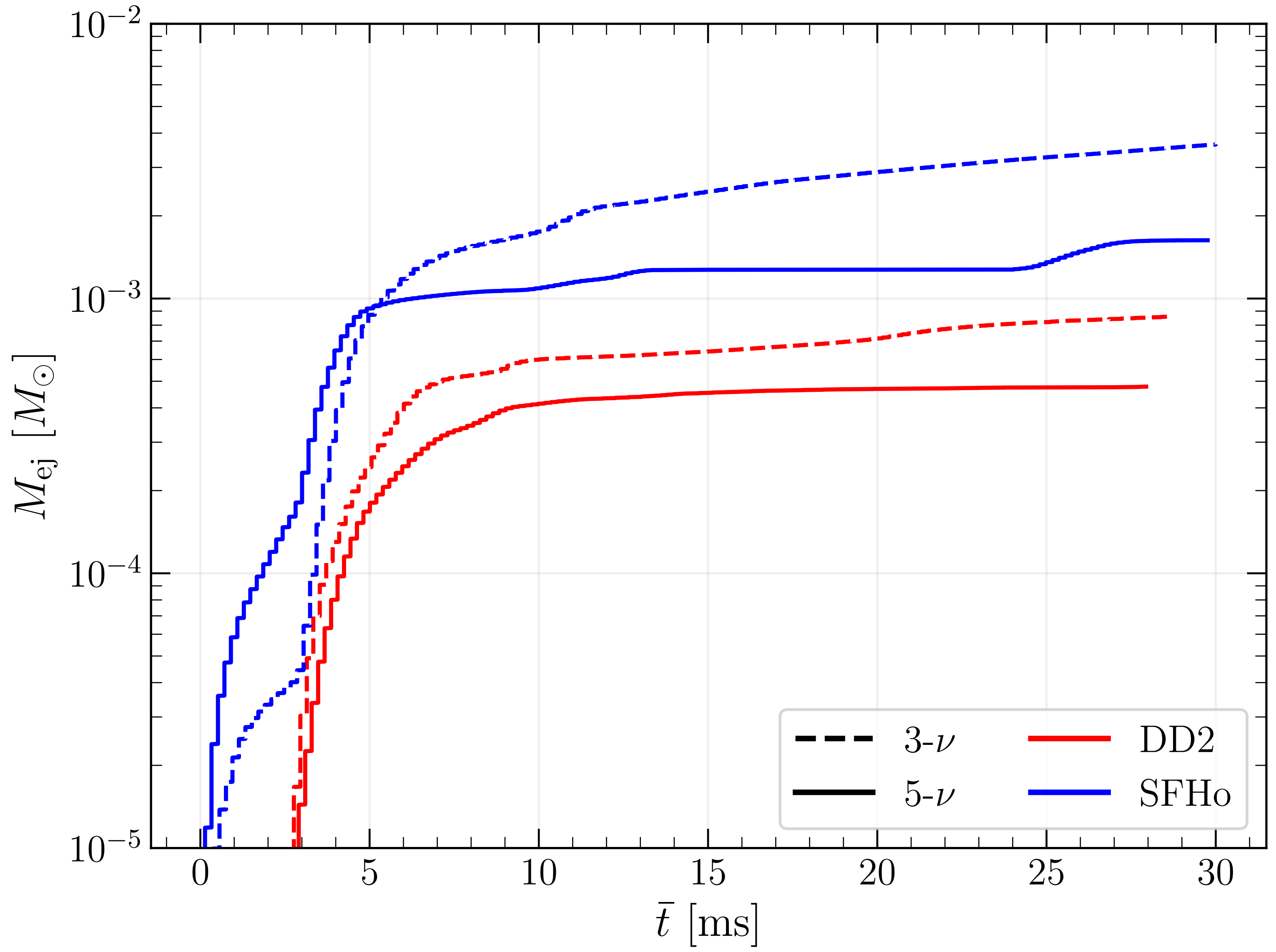
DD2 case: Composition





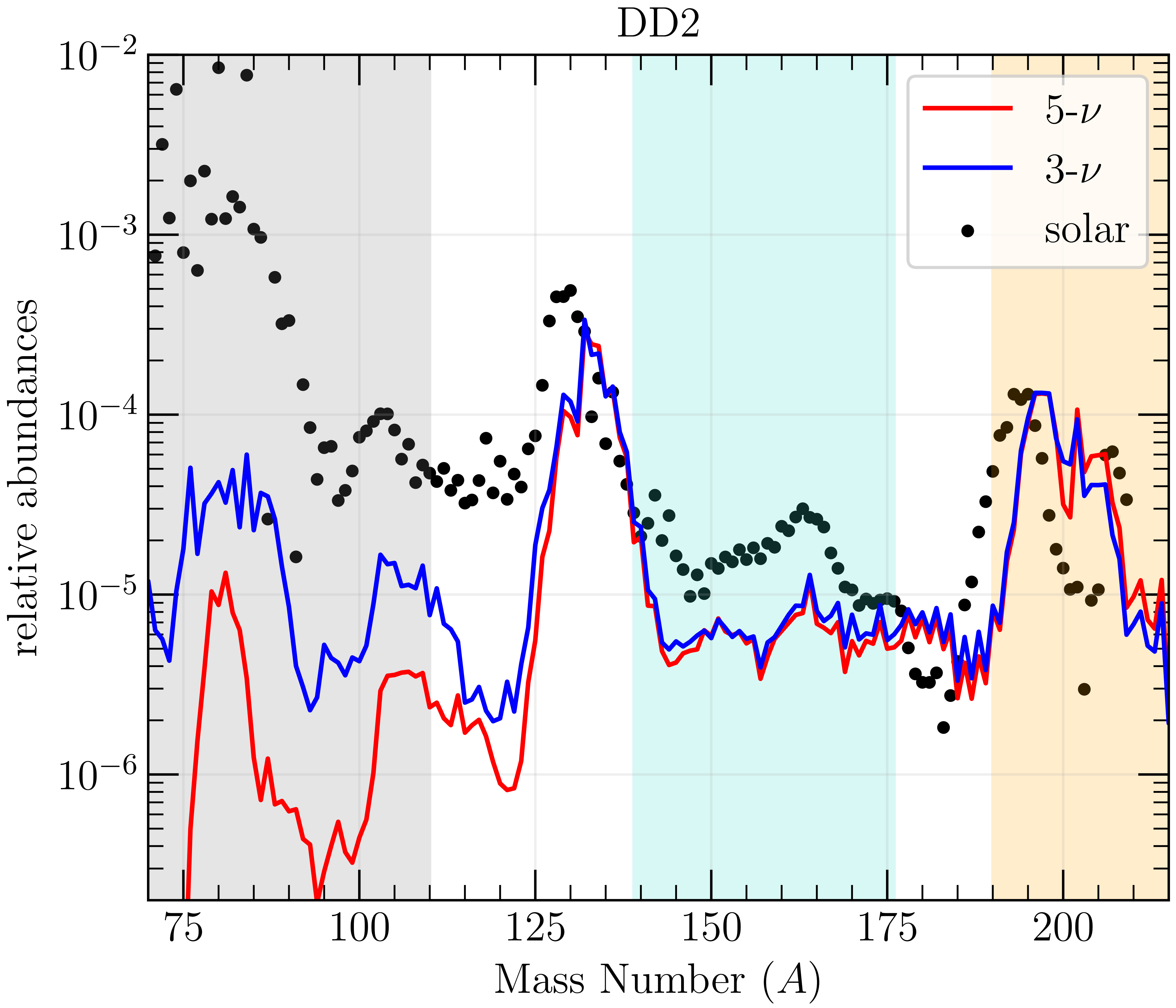
DD2 case: Comparison of Ejecta properties





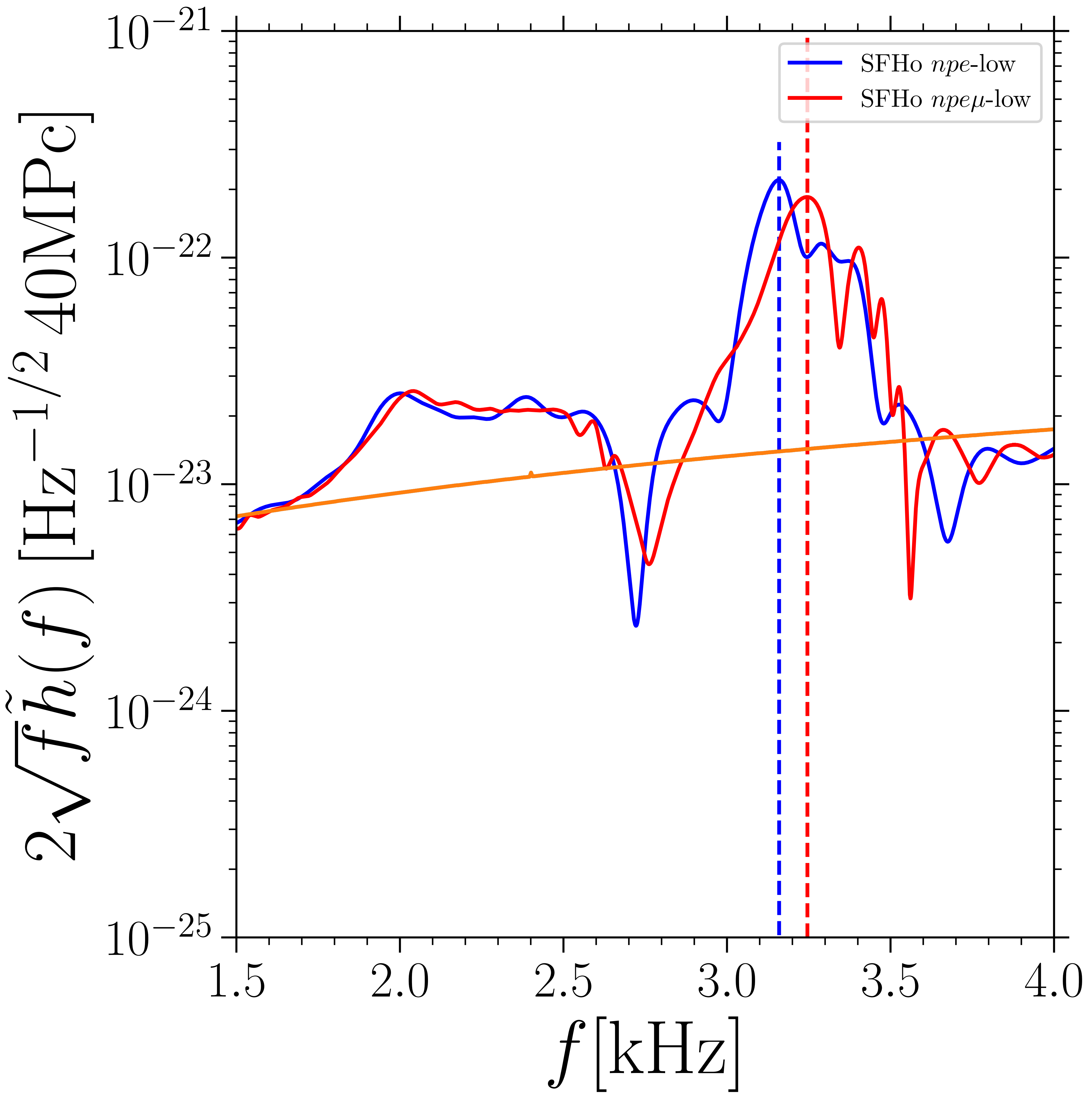
- Total Ejecta mass:
    - SFHo: $1.63 \times 10^{-3} M_\odot$ ($3.64 \times 10^{-3} M_\odot$) for $5-\nu$ ($3-\nu$)
    - DD2: $4.8 \times 10^{-4} M_\odot$ ($8.6 \times 10^{-4} M_\odot$) for $5-\nu$ ($3-\nu$)

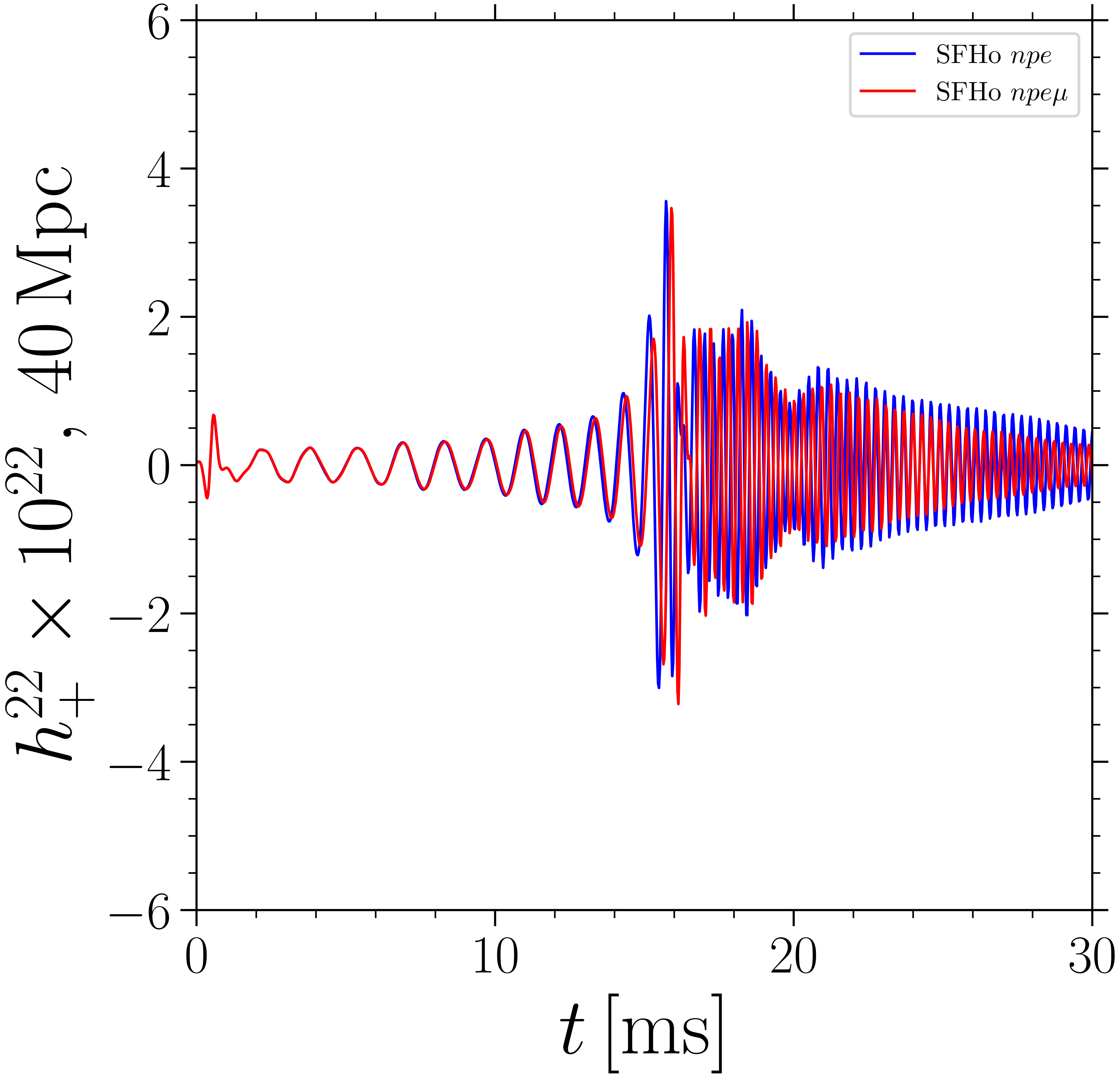
DD2 case: R-process nucleosysthesis



- Too less ejecta mass
- Too less muonization due to much less matter of DD2 EOS => pure de-muonization + temperature of remnant does not change so much

Soften the EOS: Gravitational wave signatures



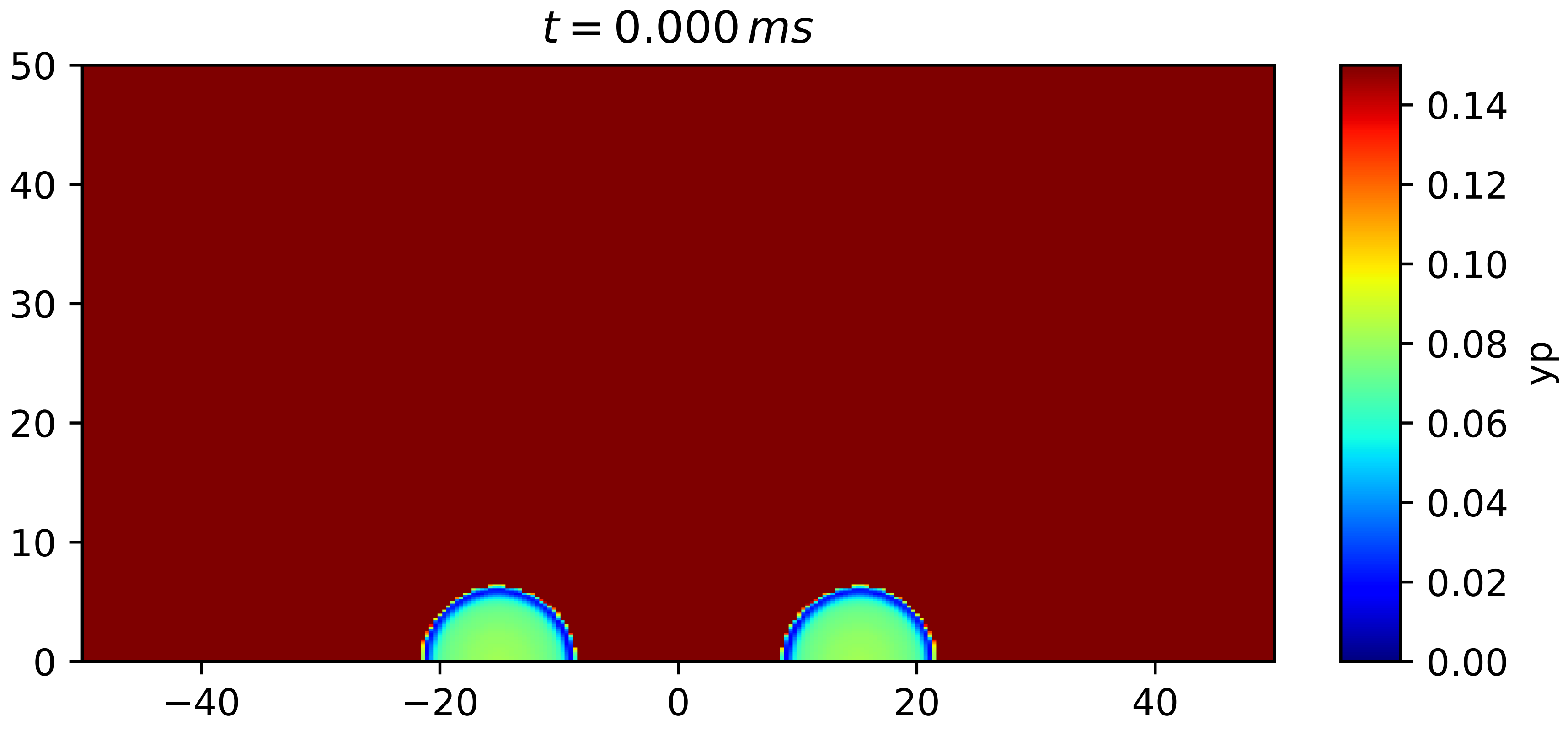


- f_2 peak: (3.25 vs 3.16 kHz) for (*npeμ*, *npe*)
- Soften the EOS (SFHo) and more stable remnant

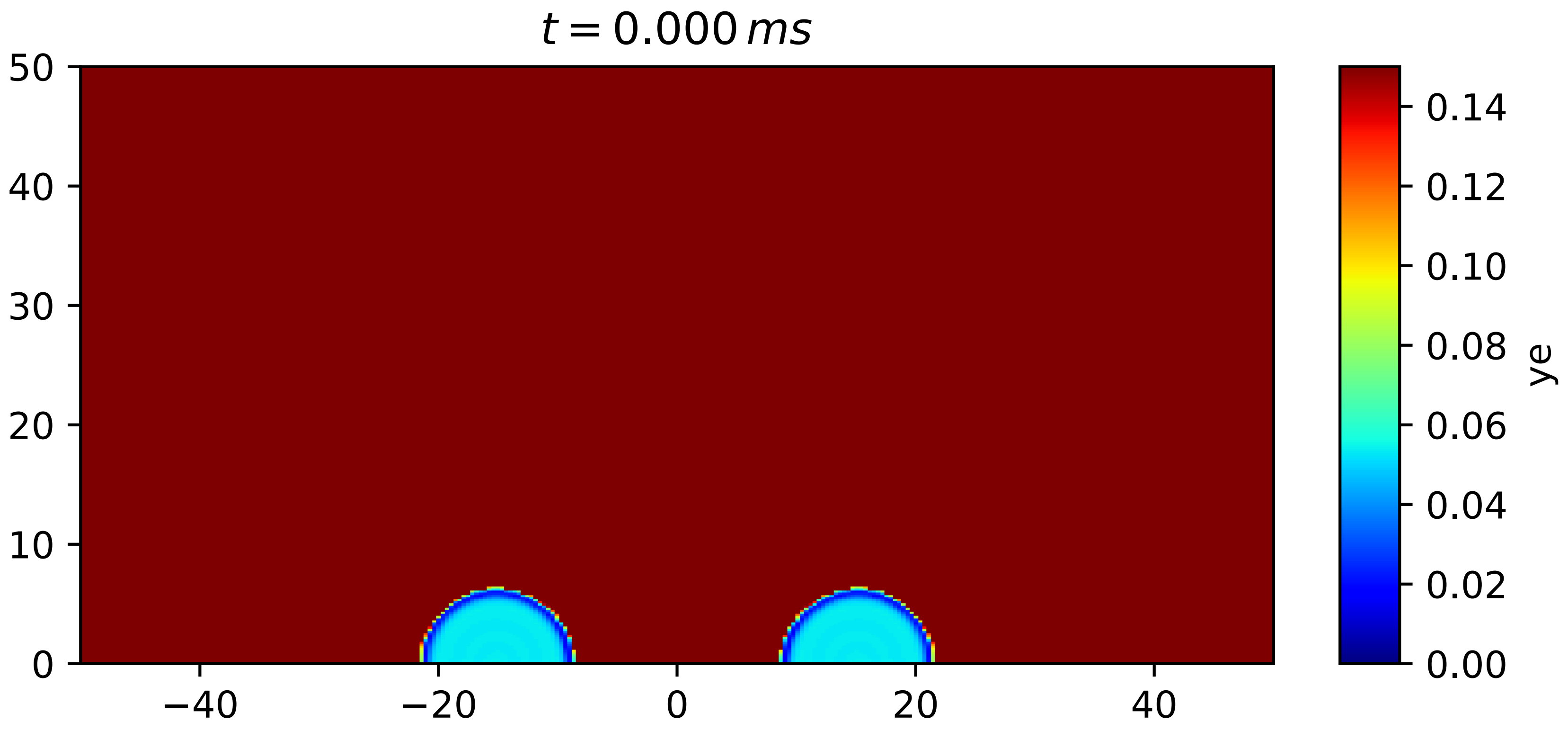
Not put-by-hand muon compositions of SFHo cold BNS

$npe\mu$ **neutrinoless β-eqm**

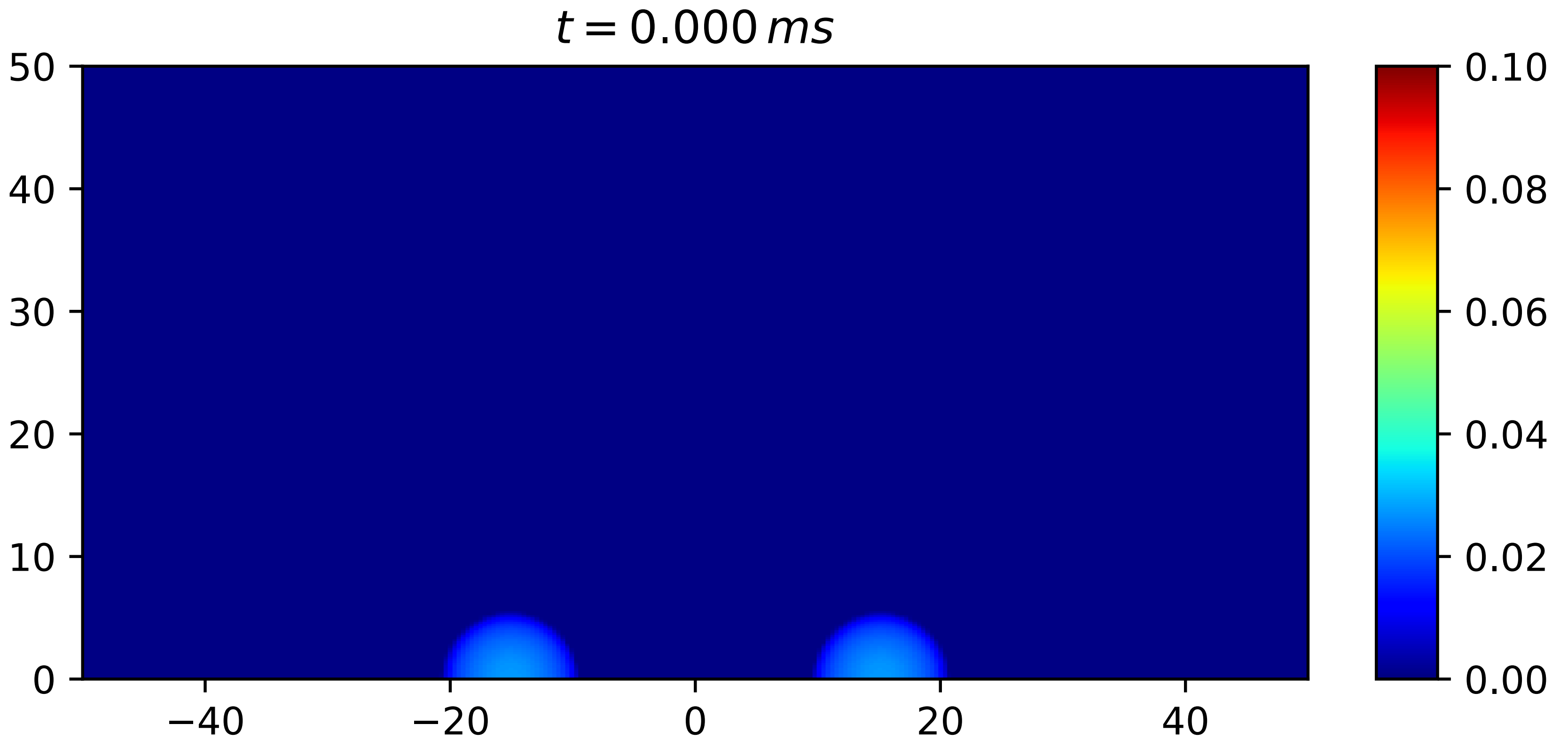
$Y_{\mu,\max} \approx 0.022$



$Y_p = Y_e + Y_\mu$

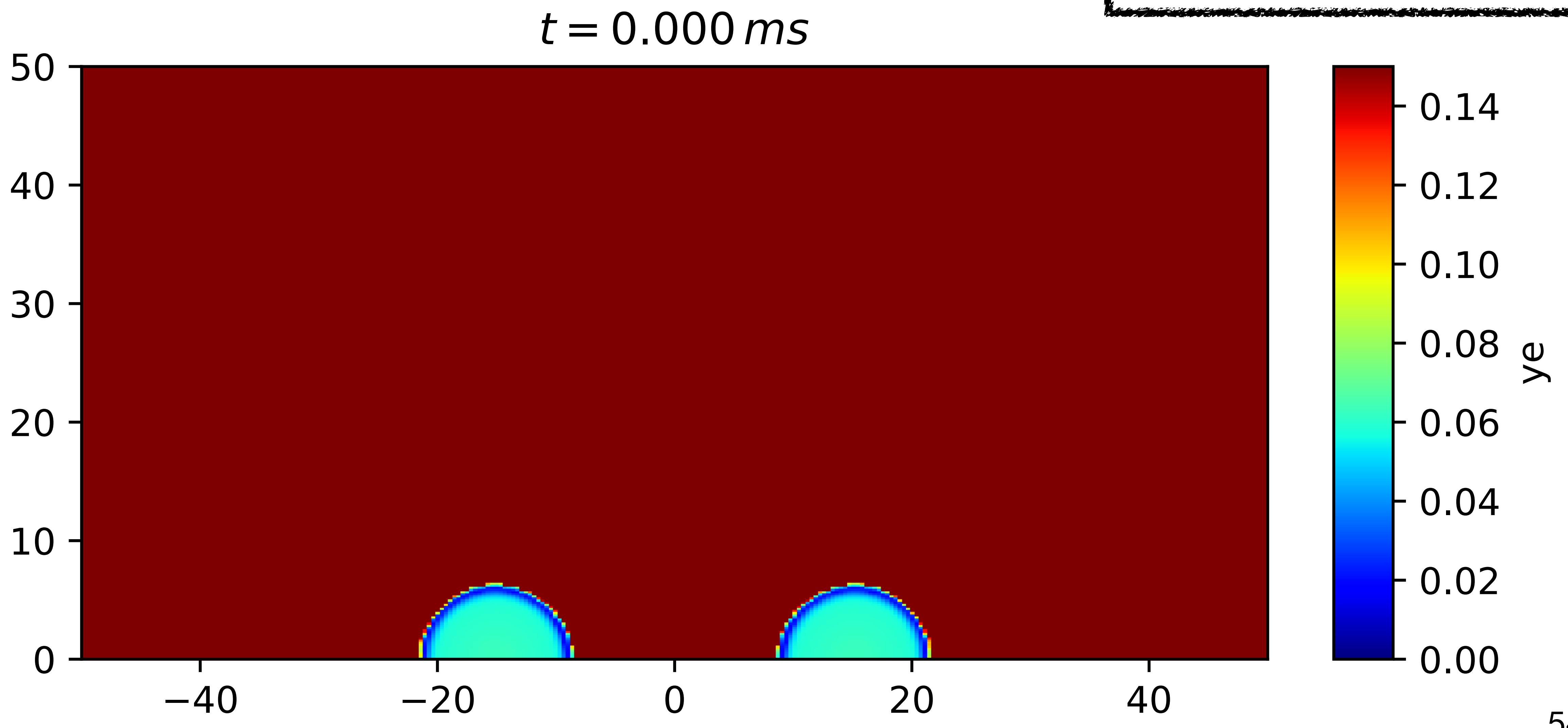


Y_e



Y_μ

npe **neutrinoless β-eqm**



$Y_p = Y_e$

Initial compositions of DD2 cold BNS

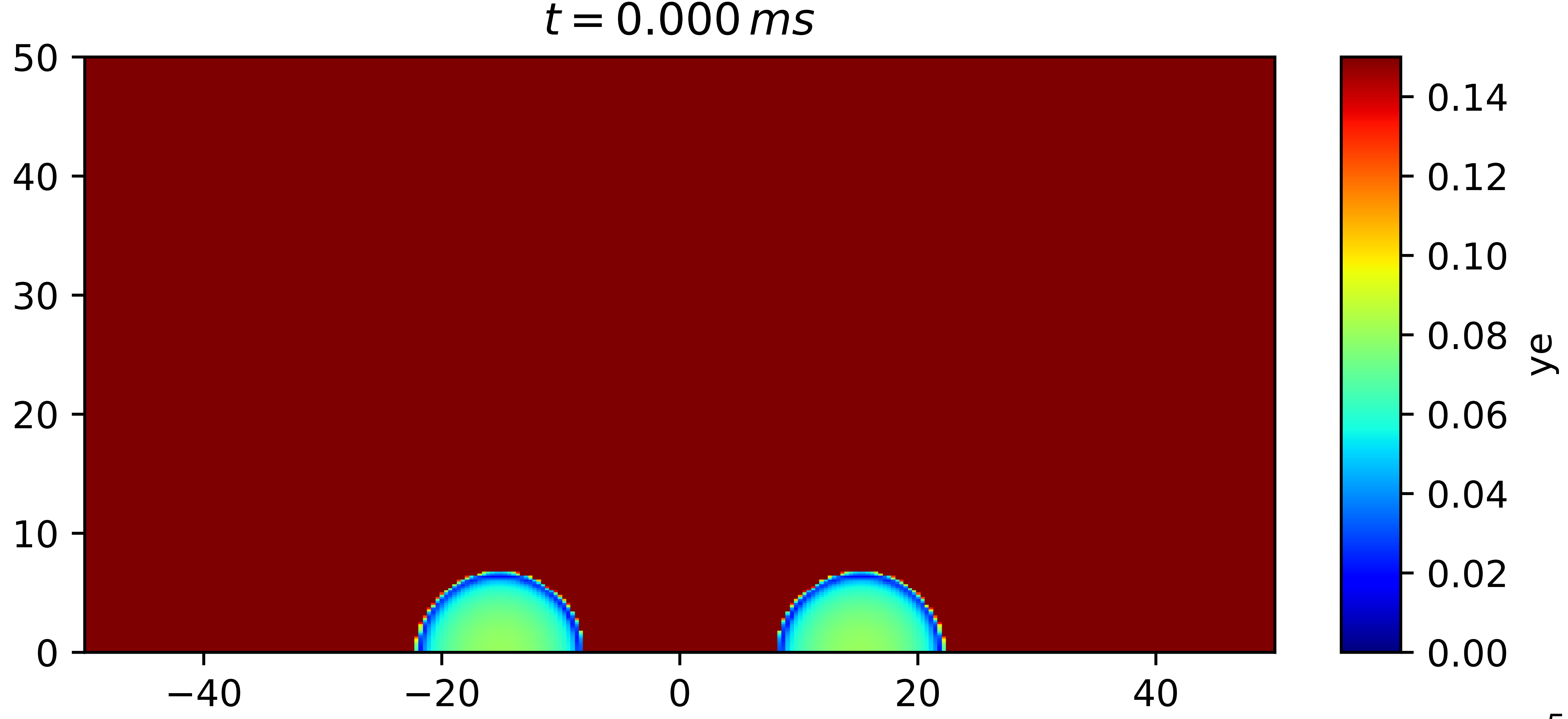
$npe\mu$ **neutrinoless** β**-eqm**

$Y_p = Y_e + Y_\mu$

Y_e

Y_μ

npe **neutrinoless** β**-eqm**



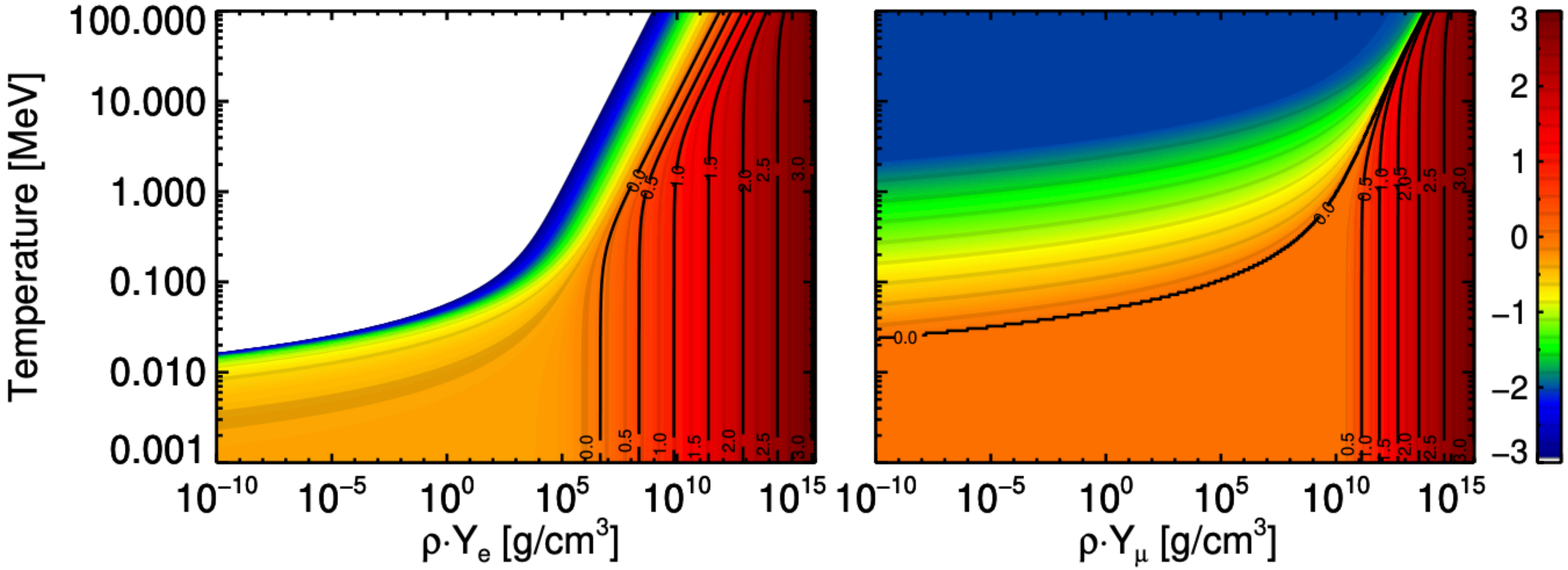
$Y_p = Y_e$

Neutron star is not as neutron rich as expected before!

Backup: Problems with muonic atmosphere

Treatment to prevent over-(de)muonization I

[Be careful, especially for grey scheme]



[Bolling PhD thesis 2018]

- $\mu_\mu^{\text{plotted}} = \text{sign}\left(\mu_\mu^{\text{NR}}\right)\log_{10}\left(\max\left(\left|\mu_\mu^{\text{NR}}\right|,1\right)\right)$
- $\mu_{\text{e}}^{\text{plotted}} = \log_{10}\left(\mu_{\text{e}}^{\text{R}}\right)$
- => Low-T low-rho: $\mu_e \sim 0$ MeV, $\mu_\mu^R \sim 100$ MeV, BUT infinitesimal Y_μ advected outward => Unphysical degeneracy parameter of muon neutrino in outward regions: $\eta_{\nu_\mu}^{eq} = \eta_\mu + \eta_p - \eta_n$ => **Over-(de)muonization / further and further away from trapped nu-eqm / Unphysical neutrino signatures / etc…**

Treatment to prevent over-(de)muonization II [Now, we have sabotaged η_ν]

- Extremely large and unphysical η_ν Huge problems:
  - Correction factor / Emissivities and opacities (Kirchhoff's laws), neutrino mean energy, etc…

$$\phi^{\text{in/out}}_{l,\text{ effective}} = \phi^{\text{in/out}}_{l,\text{ unmodified}} \frac{1}{1+\left(10^{10}\ \mathrm{g/cm^3}/\rho\right)^5} \frac{1}{1+(2.5\mathrm{MeV}/T)^6} .$$

[Eq 9.31 in Bolling PhD thesis 2018]

- **We must know, low-rho-T regimes should not have significant neutrino interactions!**
- Limiting opacities / interaction kernels: accurately evolving the vanishing muon number becomes numerically unfeasible anyway. => Changed to hard cutoffs of $10^{11}\ \mathrm{g\ cm^{-3}}$ to make them to be zero outside
- Limiting η_ν, e.g. optical depth with **Eikonal equation**
- $\tau_{\nu_\mu} > 1$, $\tau_{\bar{\nu}_\mu} > 1$
- Limiting pair processes, the blackbody factor: $\kappa_{\nu_x} = \eta_{\nu_x}/B_{\nu_x}$

Treatment/tricks (Some are particularly for grey scheme only)

- More challenging than CCSN and spectral-M1 with muons
- Highly dependent on $\eta_{\nu_\mu}, \eta_{\bar{\nu}_\mu}$
- Optical depth (Eikonal) => rescaled eta_nu [Useless in 3-species transport]
- $\tau_{\nu_\mu} > 1$, $\tau_{\bar{\nu}_\mu} > 1$
- $Y_\mu < 1\text{e-}4$ => set $\eta_{\nu_\mu}, \eta_{\bar{\nu}_\mu} = 0$ (~atmospheric treatment)
- Tnu^2/T^2
- Calculation of Pair processes [Same methods for nearly all grey scheme papers….]
- (Rescaled eta_nu)

Backup: Weakhub details

Neutrino emissivity and opacity: Weakhub (Ng+ ApJS 2024)

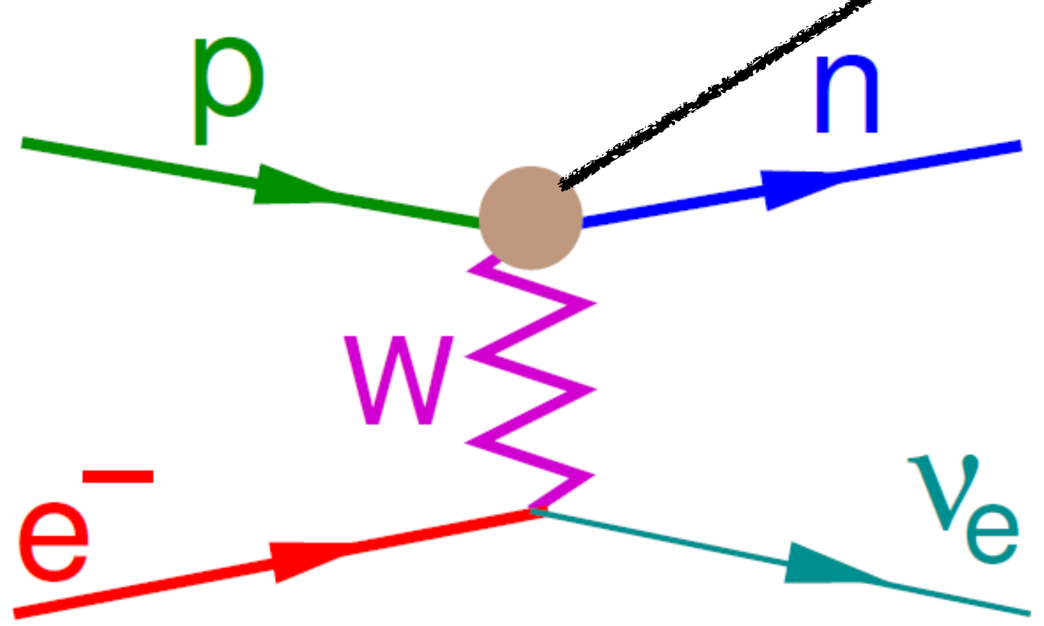
- Full kinematics calculation instead of elastic approx. for β-processes

$$\nu_e + n \leftrightarrow p + e^- \qquad \nu_\mu + n \leftrightarrow p + \mu^-$$
$$\bar{\nu}_e + p \leftrightarrow n + e^+ \qquad \bar{\nu}_\mu + p \leftrightarrow n + \mu^+$$

  - Strong interaction correction
  - High rest-mass of muons **=>** 4-D phase space integral [Fischer+2020, Guo+ 2021]
  - Weak magnetism, pseudoscalar, Form factor, etc
  - **Emissivity :**

$$Q\left(E_{\nu_e}\right) = 2\int \frac{d^3\boldsymbol{p}_p^*}{(2\pi)^3}\int \frac{d^3\boldsymbol{p}_e}{(2\pi)^3}\int \frac{d^3\boldsymbol{p}_n^*}{(2\pi)^3}\frac{\left\langle |\mathscr{M}|^2 \right\rangle}{16E_eE_p^*E_{\nu_e}E_n^*}$$
$$\times (2\pi)^4\delta^{(4)}\left(p_e + p_p - p_{\nu_e} - p_n\right)$$
$$\times f_p\left(1 - f_{\nu_e}\right)\left(1 - f_n\right),$$

Where
$$\begin{aligned}\langle|\mathcal{M}|^2\rangle = {} & \langle|\mathcal{M}|^2\rangle_{VV} \pm \langle|\mathcal{M}|^2\rangle_{VA} + \langle|\mathcal{M}|^2\rangle_{AA} \\ & + \langle|\mathcal{M}|^2\rangle_{VF} \pm \langle|\mathcal{M}|^2\rangle_{AF} + \langle|\mathcal{M}|^2\rangle_{FF} \\ & + \langle|\mathcal{M}|^2\rangle_{AP} + \langle|\mathcal{M}|^2\rangle_{PP},\end{aligned}$$



(*) Strong interaction corrections at dense matter for nucleon propagator

Detailed balance

$$Q(E_{\nu_e}) <=> \kappa(E_{\nu_e})$$

Neutrino opacity: Elastic approx. Vs Full kinematics calc.

Simplest Example: $\nu_e + n \leftrightarrow p + e^-$

- Golden rules with Pauli blocking → opacity

$$\kappa_a\left(E_1\right) = 2\int\frac{d^3\boldsymbol{p}_2}{(2\pi)^3}\int\frac{d^3\boldsymbol{p}_3}{(2\pi)^3}\int\frac{d^3\boldsymbol{p}_4}{(2\pi)^3}\frac{\left\langle|\mathscr{M}|^2\right\rangle}{16E_1E_2^*E_3E_4^*}$$
$$\times(2\pi)^4\delta^{(4)}\left(p_1+p_2-p_3-p_4\right)$$
$$\times f_2\left(1-f_3\right)\left(1-f_4\right),$$

Simple Elastic Approx.
(no momentum transferred to nucleons)
+ Drop out most of corrections

$$\kappa_a^*(\varepsilon) = \sigma_0 V_{ud}^2\eta_{12}\left(\frac{g_V^2+3g_A^2}{4}\right)\frac{1-f_{\rm FD}\left(E_l,\mu_l\right)}{1-f_{\rm FD}\left(\varepsilon,\mu_\nu^{eq}\right)}$$
$$\times\left(\frac{E_l}{m_e c^2}\right)^2\left[1-\left(\frac{m_l c^2}{E_l}\right)^2\right]^{1/2} W_{\rm M,\nu}^{\rm CC}W_{\rm R,\nu}^{\rm CC}\Theta\left(E_l-m_l c^2\right)$$

Full kinematics approach

$$\kappa_a\left(E_1\right) = \frac{G^2}{4\pi^3}\frac{1}{E_1^2}\int_{E_{3-}}^{E_{3+}}dE_3\int_{E_{2-}}^{E_{2+}}dE_2 f_2\left(1-f_3\right)\left(1-f_4\right)$$
$$\times\left[\mathscr{A}I_{\mathscr{A}}+\mathscr{B}I_{\mathscr{B}}+\cdots+\mathscr{E}I_{\mathscr{E}}+\mathscr{H}I_{\mathscr{H}}\right.$$
$$+\left(\mathscr{F}I_{\mathscr{F}}+\mathscr{F}^{PP}I_{\mathscr{F}}^{PP}\right)+\left(\mathscr{J}I_{\mathscr{J}}+\mathscr{J}^{AP}I_{J}^{AP}\right)$$
$$+\left(\mathscr{K}I_{\mathscr{K}}+\mathscr{K}^{AP}I_{\mathscr{K}}^{AP}+\mathscr{K}^{PP}I_{\mathscr{K}}^{PP}\right)$$
$$\left.+\left(\mathscr{L}I_{\mathscr{L}}+\mathscr{L}^{AP}I_{\mathscr{L}}^{AP}+\mathscr{L}^{PP}I_{\mathscr{L}}^{PP}\right)\right]$$

1. Matrix element + Weak/Strong corrections
2. All Phase Space dependences
3. 4D Monte Carlo integral

BNS pin point analysis (ν_e and $\bar{\nu}_e$) with DD2 EOS

Point	ρ	T	Y_e	Y_μ	μ_e	μ_μ	μ_n	μ_p	X_n	X_p	X_H	$X_{\rm light}$	A	Z	$U_n - U_p$	m_n^*	m_p^*
III	5.47×10^{14}	28.5	0.066	0.0001	154	4.57	1100	898	0.934	0.0663	0.00	0.00	0.00	0.00	28.6	344	343

- Inverse β decay activated $\because$ in-medium corrections of E_N and m_N^*, to satisfy the threshold: $k_{F,n} < k_{F,p} + k_{F,e}$ (Threshold of dUrca process)

$\boxed{\rho \sim 2n_s}$

BNS pin-point Opacity (ν_μ and $\bar{\nu}_\mu$)

DD2 EOS

Point	ρ	T	Y_e	Y_μ	μ_e	μ_μ	μ_n	μ_p	X_n	X_p	X_H	$X_{\rm light}$	A	Z	$U_n - U_p$	m_n^*	m_p^*
III	5.47 $\times 10^{14}$	28.5	0.066	0.0001	154	4.57	1100	898	0.934	0.0663	0.00	0.00	0.00	0.00	28.6	344	343

$\boxed{\rho \sim 2n_s}$

Two treatments with order-of-magnitude differences!

Weak interactions in `Weakhub`

β-processes

$\nu_e + n \leftrightarrow p + e^-$

$\bar{\nu}_e + p \leftrightarrow n + e^+$

$\bar{\nu}_e + e^- + p \leftrightarrow n$

$\nu_e + A'(N+1, Z-1) \leftrightarrow A(N,Z) + e^-$

1. Medium modifications at mean field level
2. Full kinematics approach (Fischer+ 2020, Guo+ 2020)
3. Weak magnetism, pseudoscalar, Form factor

Pair processes (Kernels/Emiss)

$e^- + e^+ \leftrightarrow \nu + \bar{\nu}$

$N + N \leftrightarrow N + N + \nu + \bar{\nu}$

$\gamma^*_{\mathrm{T/A/M/L}} \leftrightarrow \nu + \bar{\nu}$

$A^* \leftrightarrow A + \nu + \bar{\nu}$

$\nu_e + \bar{\nu}_e \leftrightarrow \nu_{\mu/\tau} + \bar{\nu}_{\mu/\tau}$

Elastic scattering

$\nu + N \leftrightarrow \nu + N$ ← 1. Viral expansion+RPA, 2. strange quark coupling, 3. WM, 4. recoil, 5. interpolated nucleon-nucleon blocking

$\nu + A \leftrightarrow \nu + A$ ← 1. Electron-polarization correction, 2. Heavy nuclei form factor)

$\nu + A_{\mathrm{light}} \leftrightarrow \nu + A_{\mathrm{light}}$

Inelastic scattering

Electronic	**Muonic**
$\nu + e^- \leftrightarrow \nu + e^-$	$\nu + \mu^- \rightleftarrows \nu + \mu^-$
$\nu + e^+ \leftrightarrow \nu + e^+$	$\nu + \mu^+ \rightleftarrows \nu + \mu^+$

Conventional Weak interactions, Mostly used CCSNe (`NuLib`)

$\nu_e + n \leftrightarrow p + e^-$

$\bar{\nu}_e + p \leftrightarrow n + e^+$

} Elastic approx. + WM + Recoil

$\nu_e + A'(N+1, Z-1) \leftrightarrow A(N,Z) + e^-$

$e^- + e^+ \leftrightarrow \nu + \bar{\nu}$ ← Emissivity/kernel approach

$N + N \leftrightarrow N + N + \nu + \bar{\nu}$ ← Emissivity approach

$\nu + N \leftrightarrow \nu + N$ ← 1. Viral expansion+RPA, 2. strange quark coupling, 3. WM, 4. recoil

$\nu + A \leftrightarrow \nu + A$

$\nu + e^- \leftrightarrow \nu + e^-$

Muonic

Lepton flavour exchange

$\nu_\mu + e^- \rightleftarrows \nu_e + \mu^-$

$\bar{\nu}_\mu + e^+ \rightleftarrows \bar{\nu}_e + \mu^+$

Lepton flavour conversion

$\bar{\nu}_e + e^- \rightleftarrows \bar{\nu}_\mu + \mu^-$

$\nu_e + e^+ \rightleftarrows \nu_\mu + \mu^+$

Inverse lepton decay

$\bar{\nu}_e + e^- + \nu_\mu \leftrightarrow \mu^-$

$\nu_e + e^+ + \bar{\nu}_\mu \leftrightarrow \mu^+$

β-processes

$\nu_\mu + n \leftrightarrow p + \mu^-$

$\bar{\nu}_\mu + p \leftrightarrow n + \mu^+$

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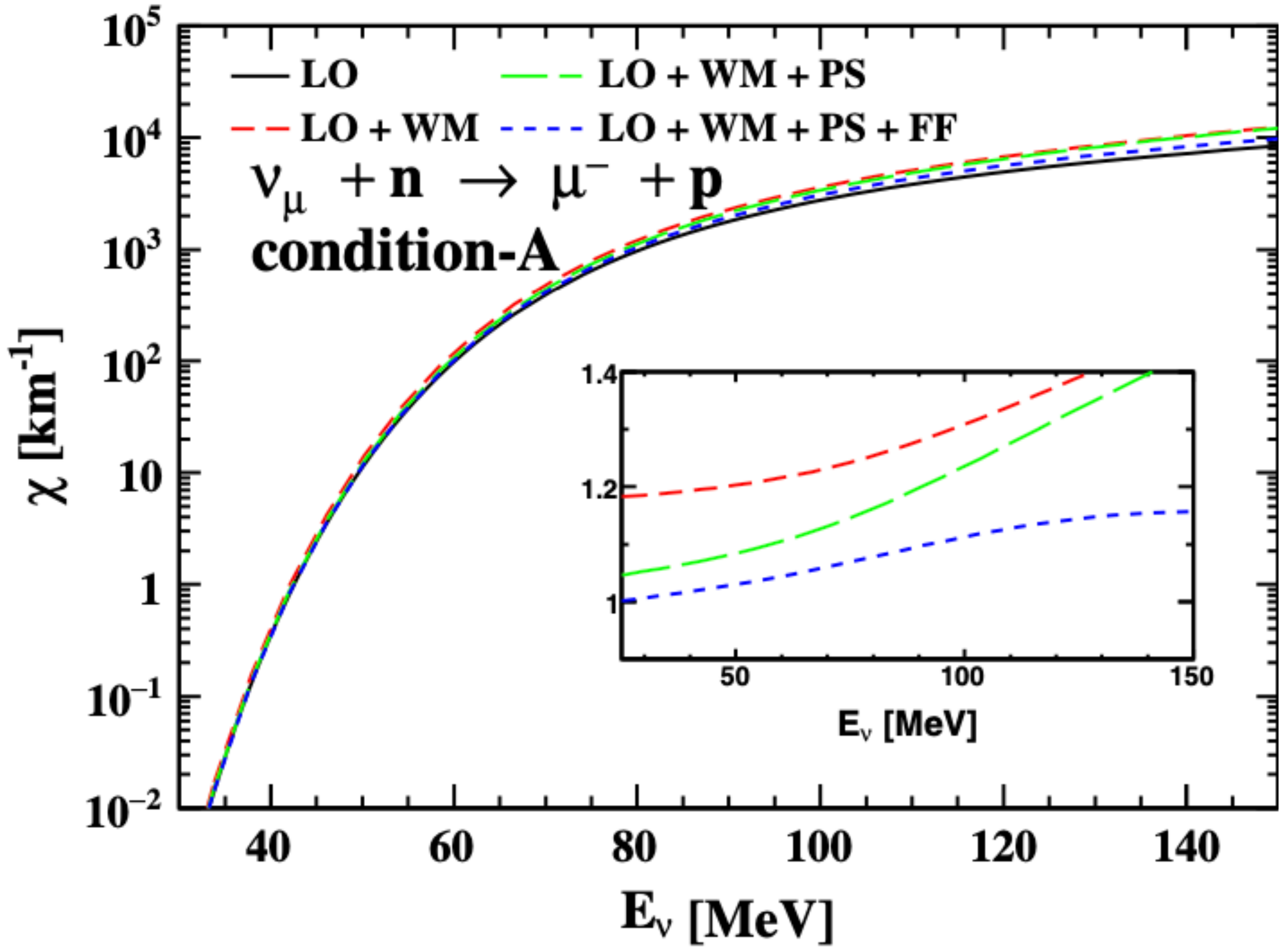
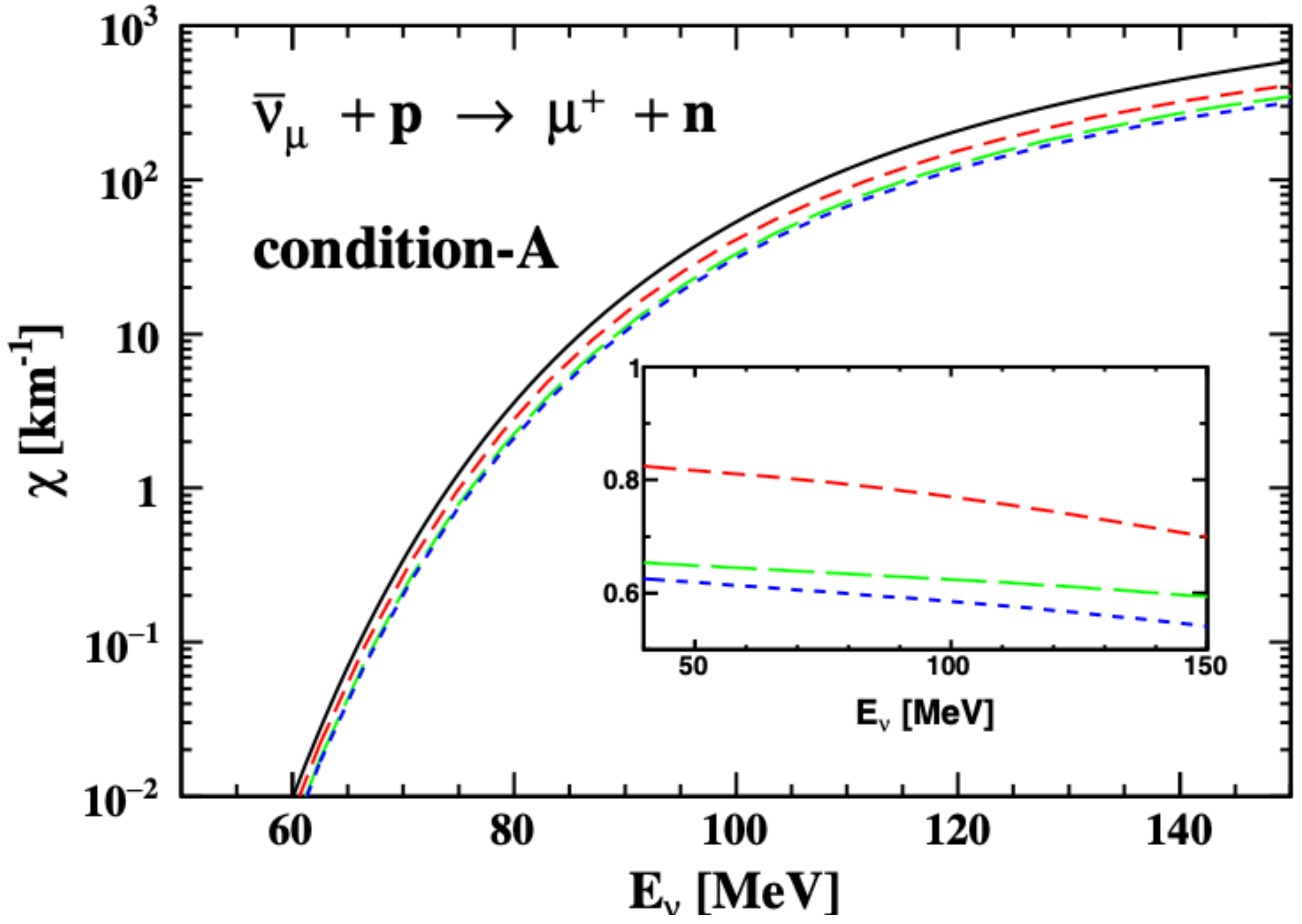
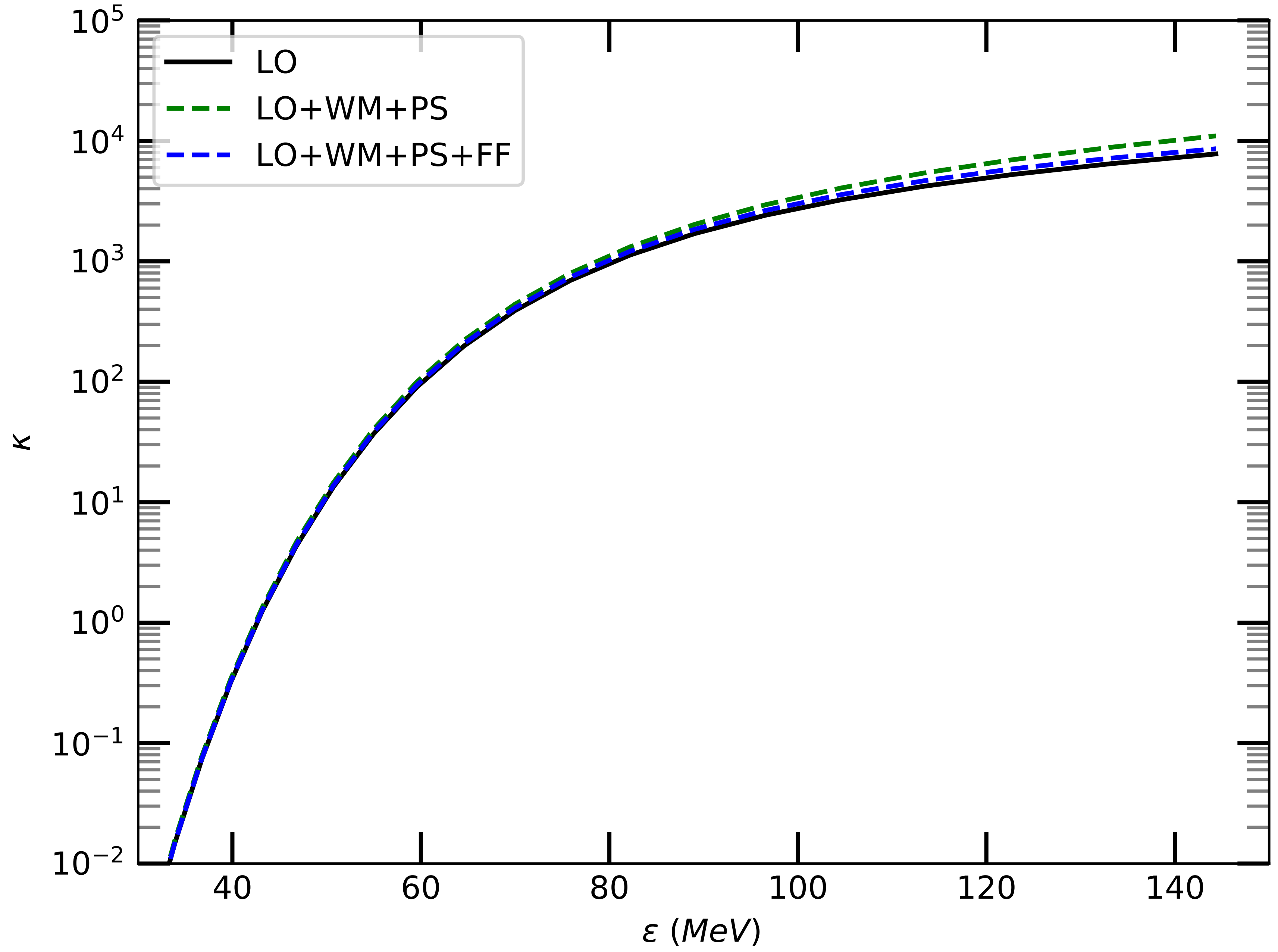
Tests of Full kinematics opacities with Guo+ 2020

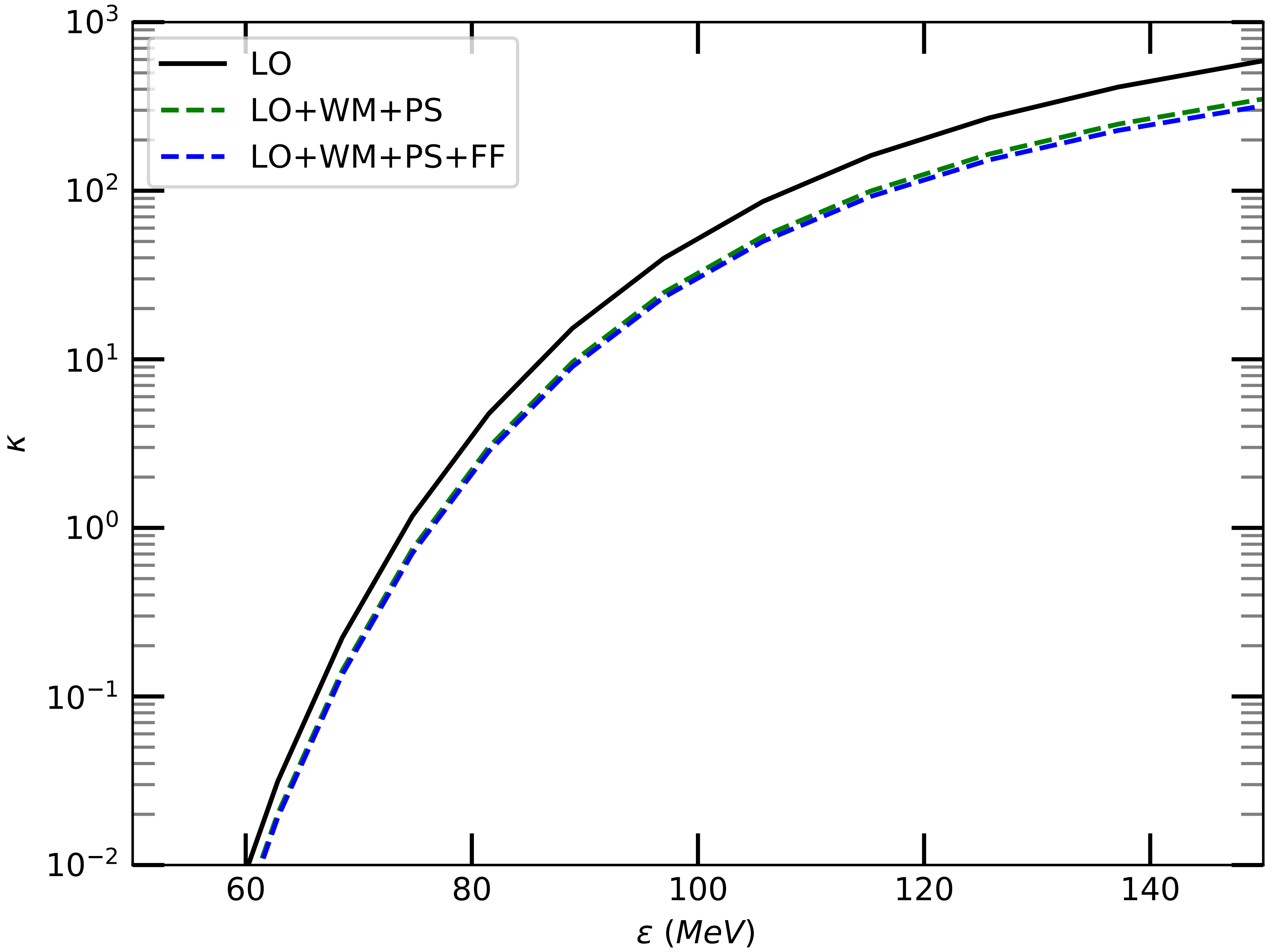
Guo+2020





Weakhub





Backup: Weakhub tabulation tricks

- Memory is sufficient even
  [Boltzmann solver 2D + muonic Weakhub + leptonic EOS + Baryonic EOS]
- Different resolution for e-flavor arrays and mu-flavor arrays for opacity table.