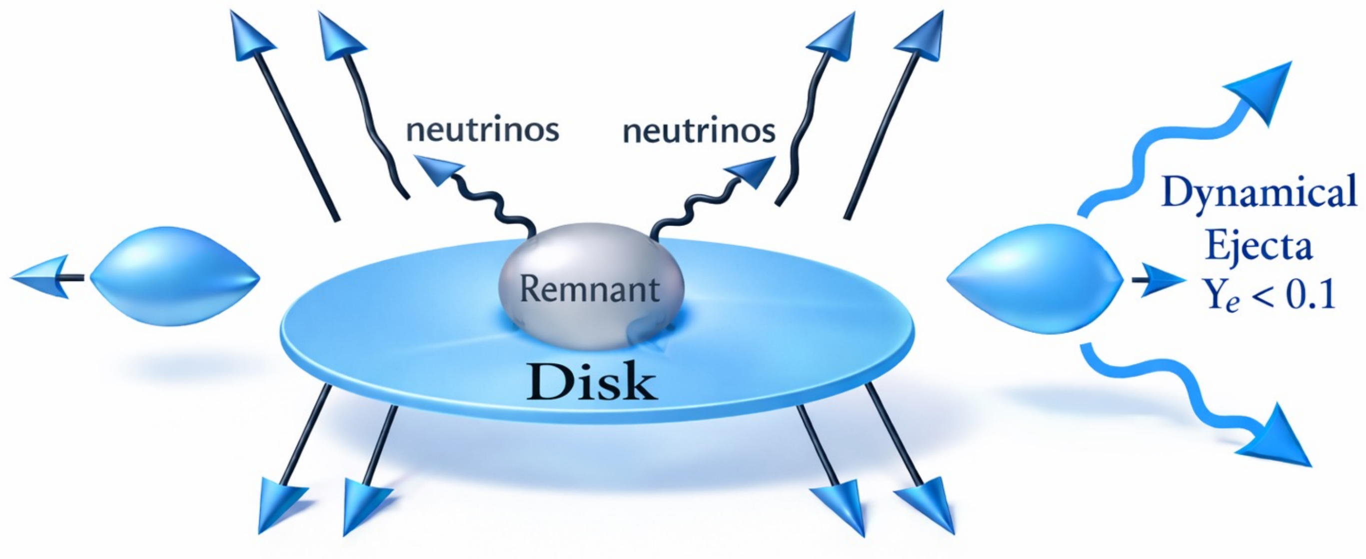
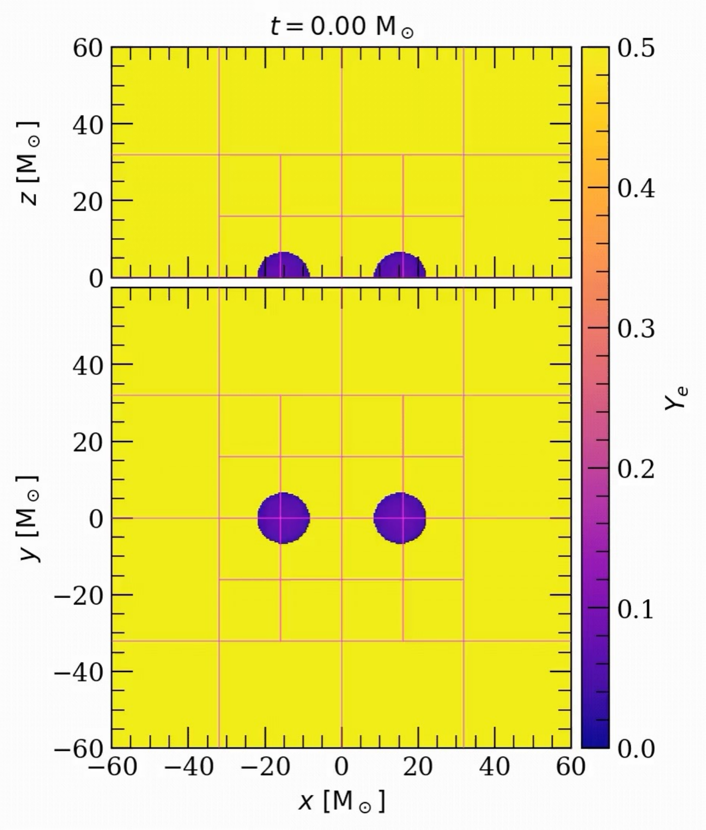
Subgrid Neutrino Oscillation in Neutron Star Merger Simulations

Collaborators: David Radice, Sherwood Richers, Albino Perego, Federico Maria Guercilena, Maitraya Bhattacharyya

Speaker: Yi Qiu

R-process and uncertainties





- BNS mergers are sites for rapid neutron capture (r-process) nucleosynthesis
- Ejecta electron fractions influenced by neutrinos through weak interations, e.g.,

$$p + e^- \leftrightarrow n + \nu_e, \qquad n + e^+ \leftrightarrow p + \bar{\nu}_e$$

- Large uncertainties remain: we missed out important ingradients in neutrino part

Numerical simulation of BNS in a nutshell

$$T_{\mu\nu} = T_{\mu\nu}^{\text{fluid}} + \boxed{T_{\mu\nu}^{\text{neutrino}}}$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

- Solving general relativistic (magneto-)hydrodynamics (GRMHD) equations
- Nuclear equation of state (EOS) describing the dense matter behavior

- Classical neutrino transport: moment based scheme or full Boltzmann or guided moments
- Quantum (neutrino flavor mixing) effects: missing!

Neutrino fast flavor conversions in BNS

Boltzmann equation of neutrinos

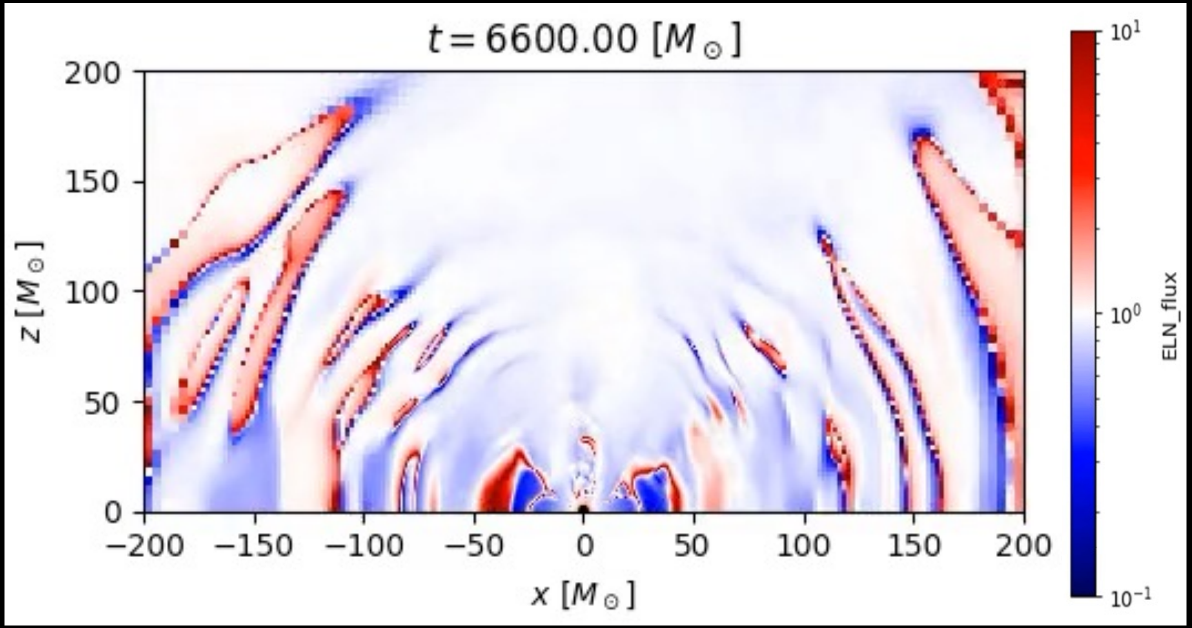
$$\frac{d\varrho}{dt} = Collisions + Oscillcations$$

Neutrino self-interaction leads to fast flavor instability (FFI), which emerges ubiquitously in the ejecta around remnant

Electron lepton number flux factor

$$f_{\mathrm{ELN}} = \frac{\frac{F_{\nu_e}}{E_{\nu_e}} N_{\nu_e} - \frac{F_{\bar{\nu}_e}}{E_{\bar{\nu}_e}} N_{\bar{\nu}_e}}{N_{\nu_e} - N_{\bar{\nu}_e}} > 1$$

See also Abbar+, Richers



BGK subgrid model

Challenge: neutrino fast flavor instability triggers ~ ***ns*** timescale flavor oscillations, but simulation time step **~*μ*s**

Our attempt: Bhatnagar-Gross-Krook (BGK) subgrid model

$$\textit{oscillation term} \rightarrow \frac{1}{\tau_a}(f - f^a)$$

See also Nagakura+, Wang+, Liu+

Toy models

- 4 neutrino species, i.e., $\nu_e, \nu_x, \bar{\nu}_e, \bar{\nu}_x$
- Relaxation time: fixed at 0.5 ns
- Flavor mixing conserves total lepton, heavy and electron lepton numbers
- Equilibrium states:
  - Maximal mixing (MX)

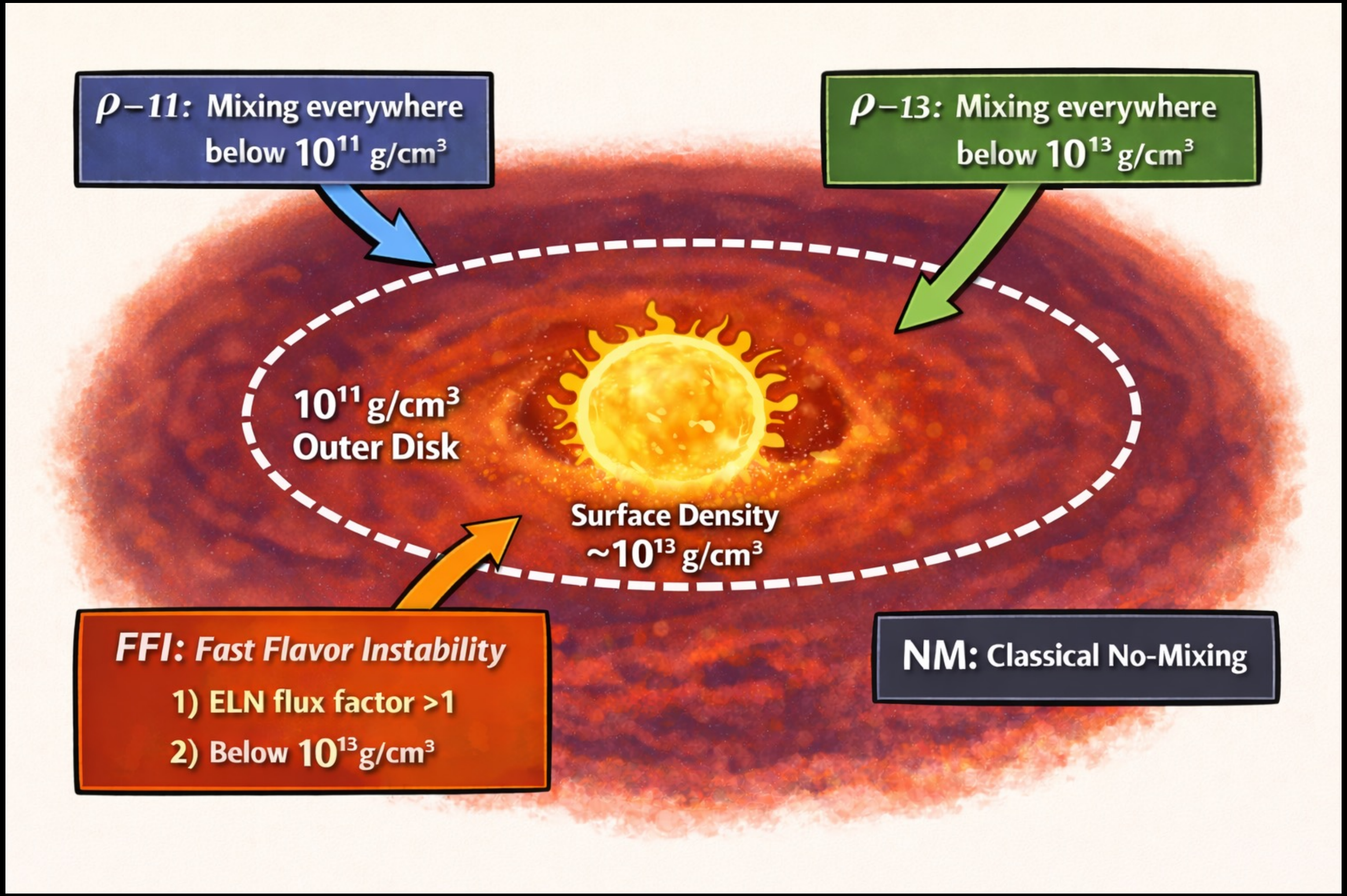
$$\begin{aligned} n_e^{\mathrm{eq}} &= N/6 + N_e/2 \\ \bar{n}_e^{\mathrm{eq}} &= N/6 - N_e/2 \\ n_x^{\mathrm{eq}} &= N/3 + N_x/2 \\ \bar{n}_x^{\mathrm{eq}} &= N/3 - N_x/2 \end{aligned}$$

  - Many-body (MB)

$$n_e^{\mathrm{eq}} \bar{n}_e^{\mathrm{eq}} = \frac{1}{4} n_x^{\mathrm{eq}} \bar{n}_x^{\mathrm{eq}}$$

Martin+ (2023)

Where to turn on mixing?

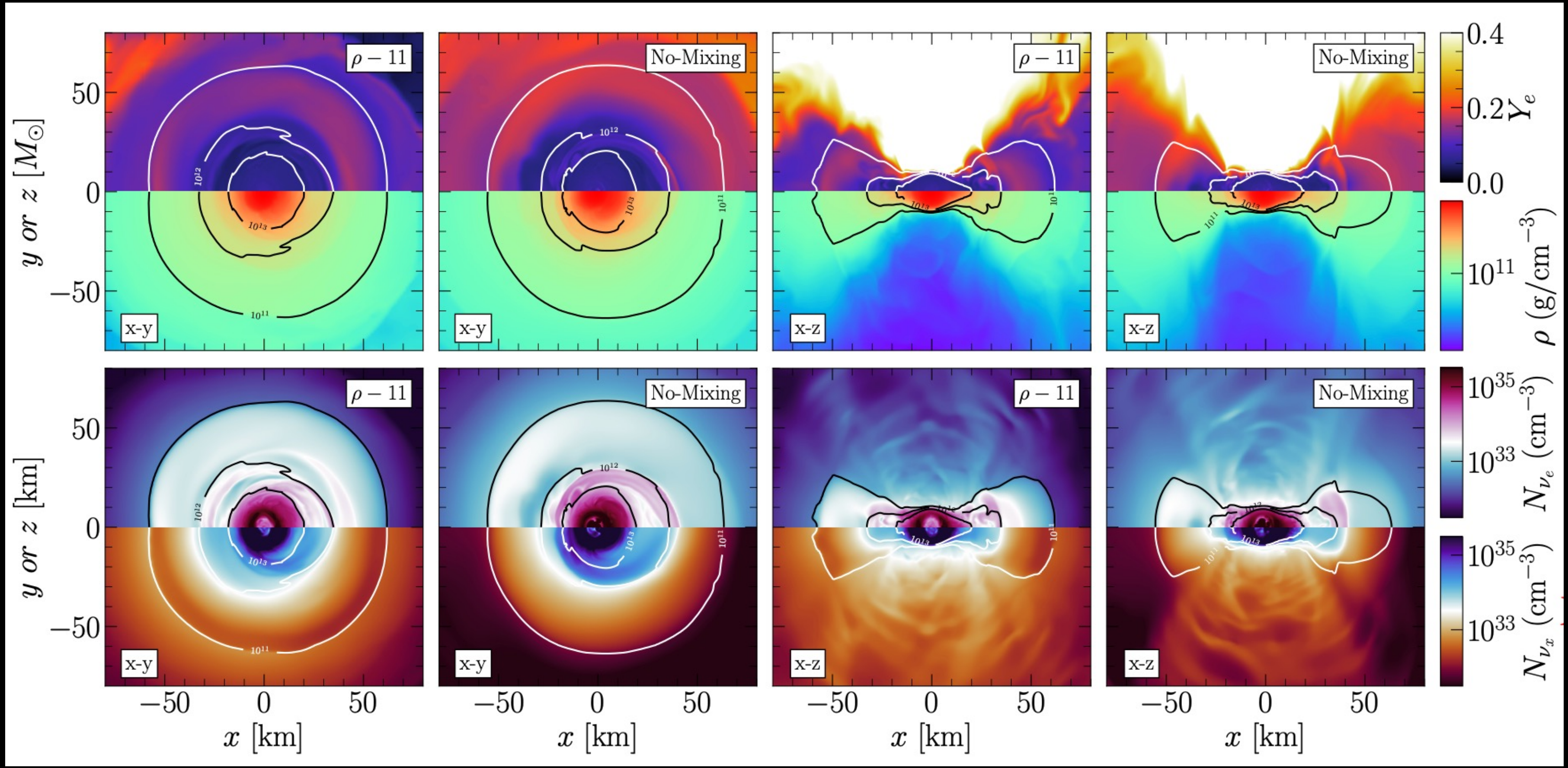


Similar as that in Ehring+(2023)

- WhiskyTHC_M1 with two equations of state: DD2 and SFHo
- Two (low/standard) resolutions (LR/SR), with spacing 246 m/184 m
- Prescriptions differ: FFI models use MX, Other mixing models use MB

General dynamics

✅ ❌ ✅ ❌



Change

Lower Y_e

Similar

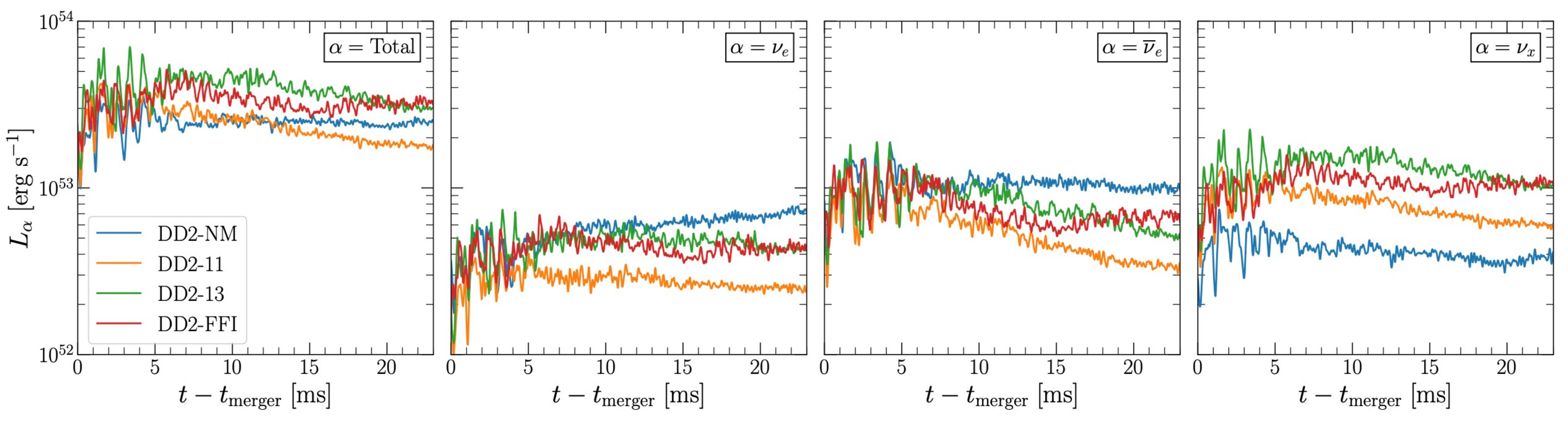
Lower N_{ν_e}

Higher N_{ν_x}

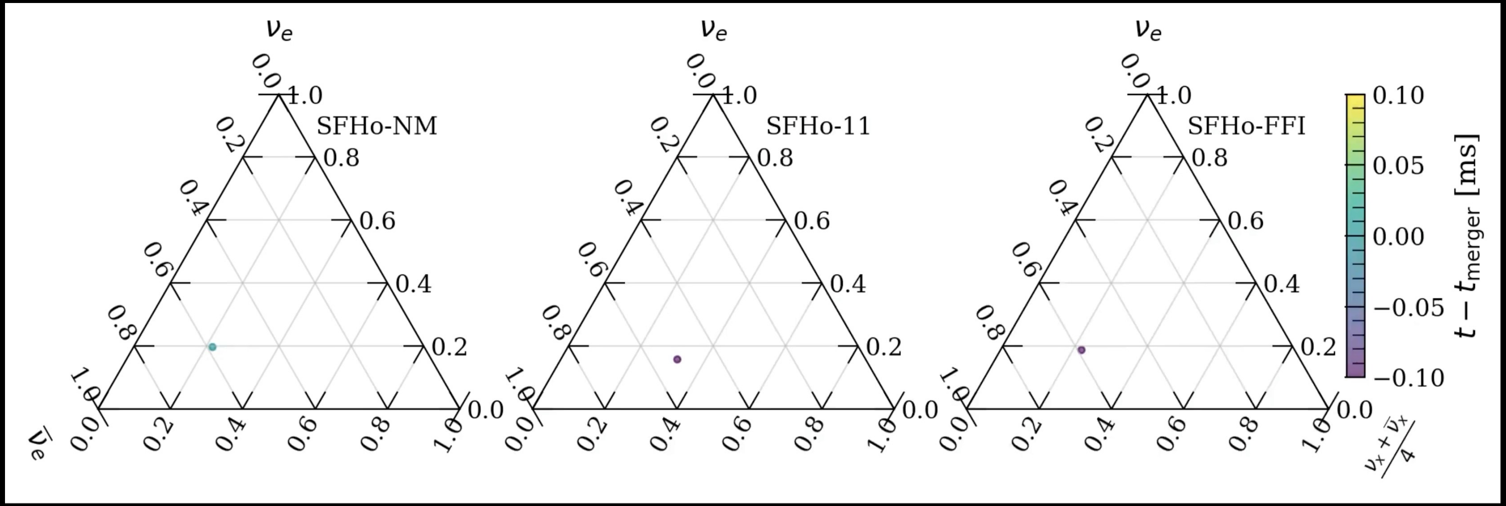
- Flavor conversions of $\nu_e, \bar{\nu}_e \rightarrow \nu_x, \bar{\nu}_x$
- What about the observables?

Sliced at ~10 ms after merger

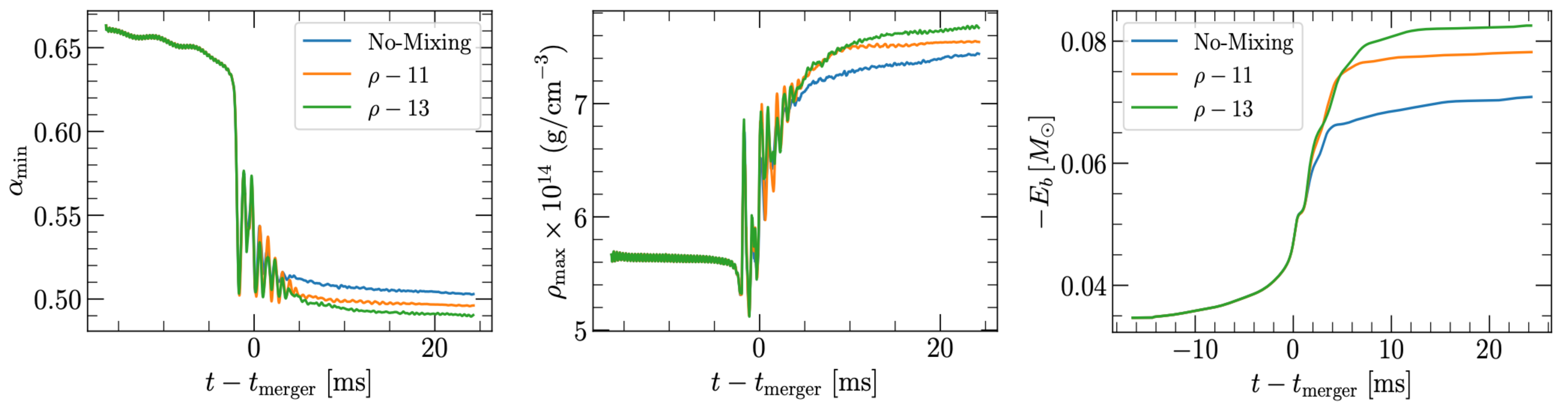
Neutrino luminosities



- Higher heavy lepton neutrino luminosities
- Lower electron (anti-)neutrino luminosities
- Final flavor distribution more "equipartition"

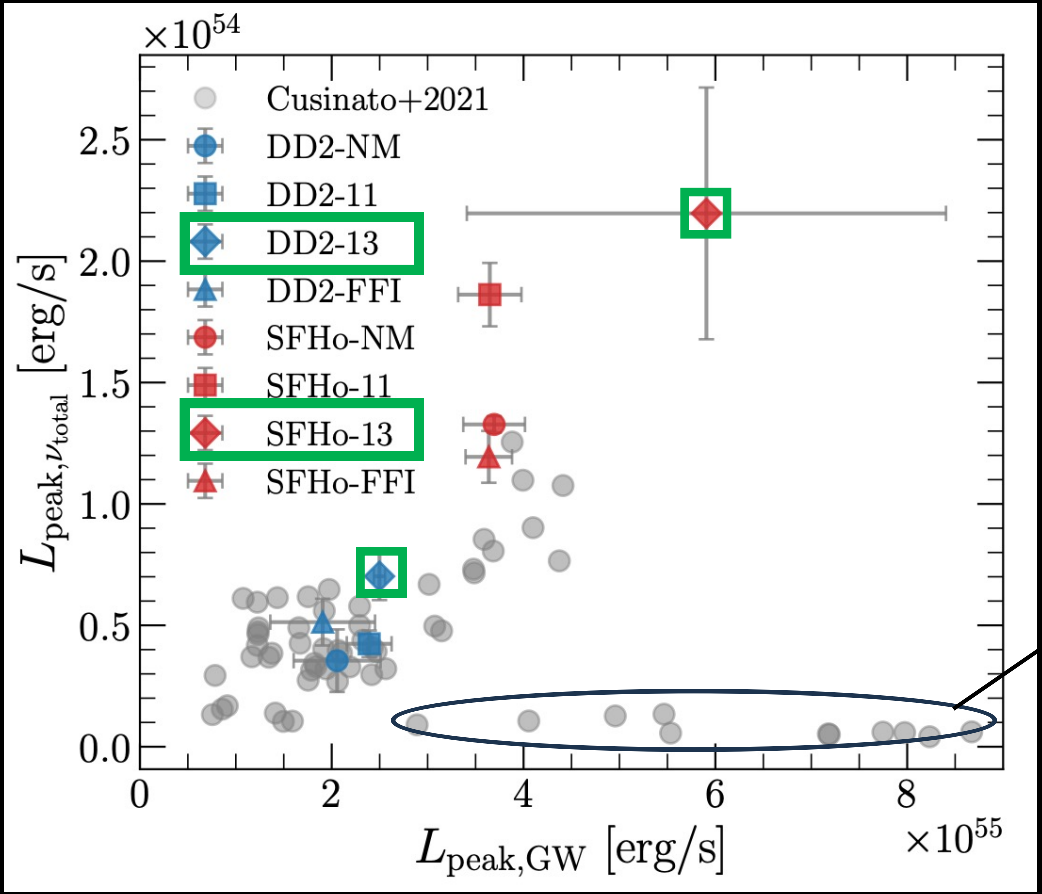


Impacts on the remnants



- More neutrino mixings → more compact remnants
- Flavor conversions alter electron type neutrinos number → affect convection → higher GW binding energy (also seen in Ehring et al. 2024)

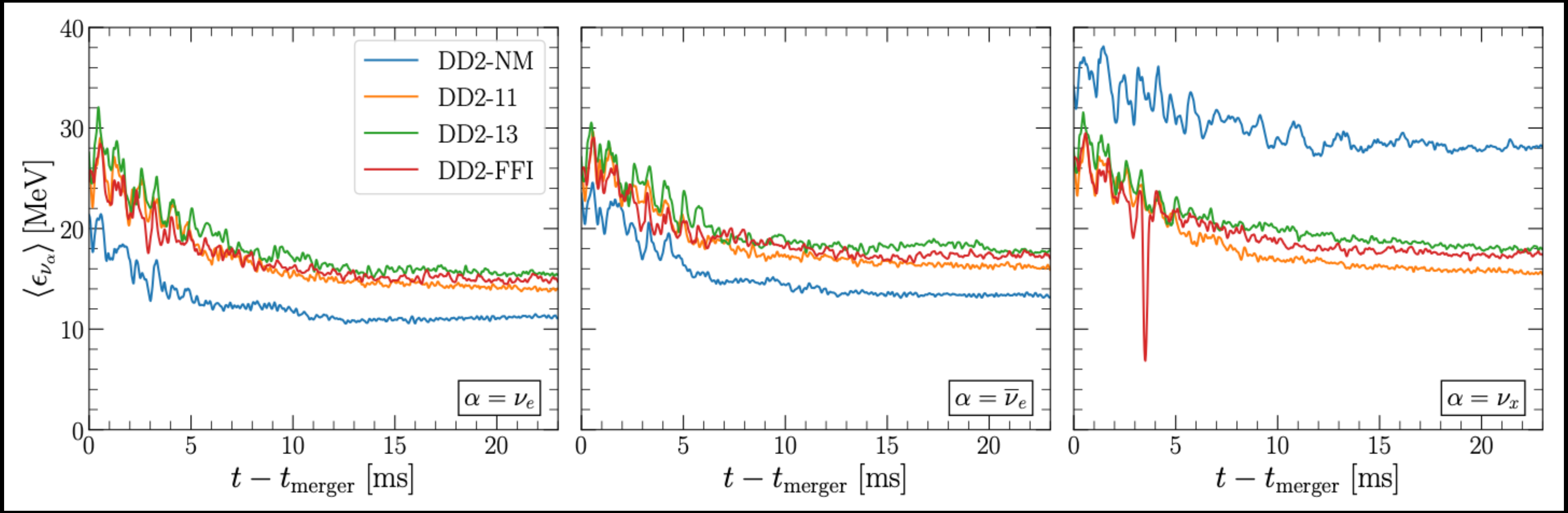
Multimessenger signatures



prompt collapse

- Correlation between neutrino and GW emissions
- Flavor conversions at high density regions increase peak luminosities

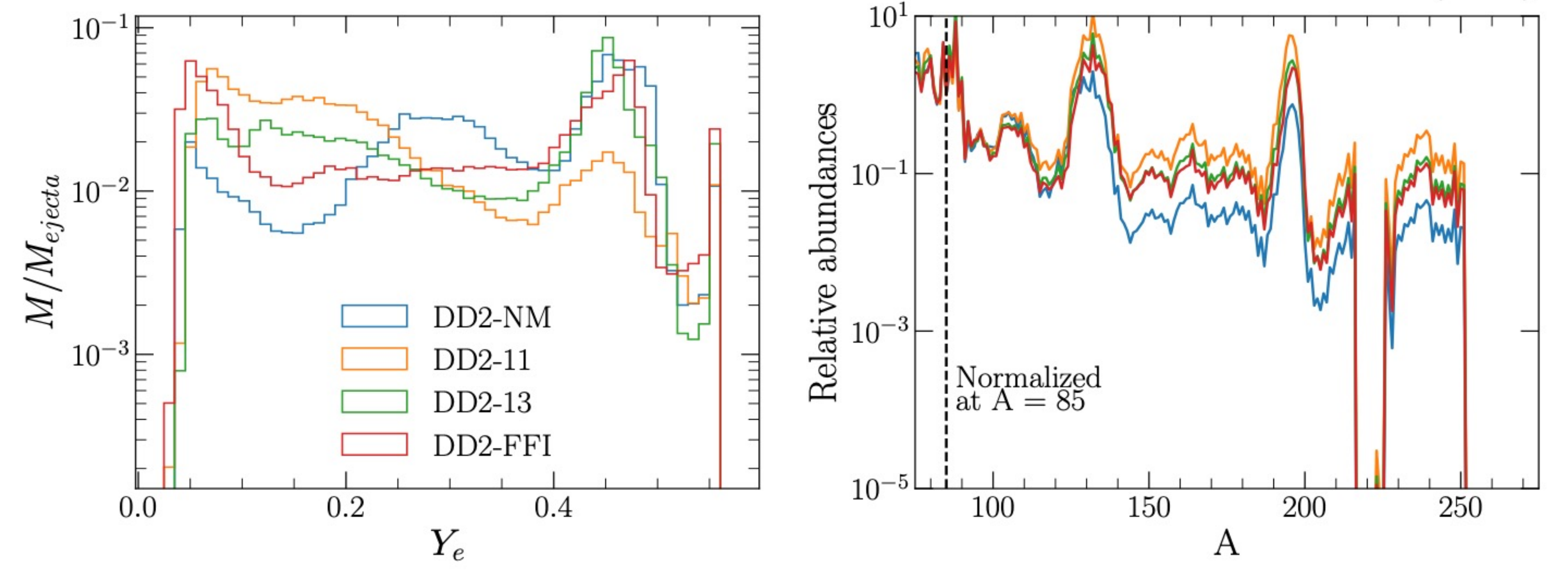
Neutrino mean energies



- Without flavor conversions, higher ν_x, lower $\nu_e, \bar{\nu}_e$
- With flavor conversions, all flavors comparable
- Flavor mixing effectively alter the decoupling surfaces of neutrinos

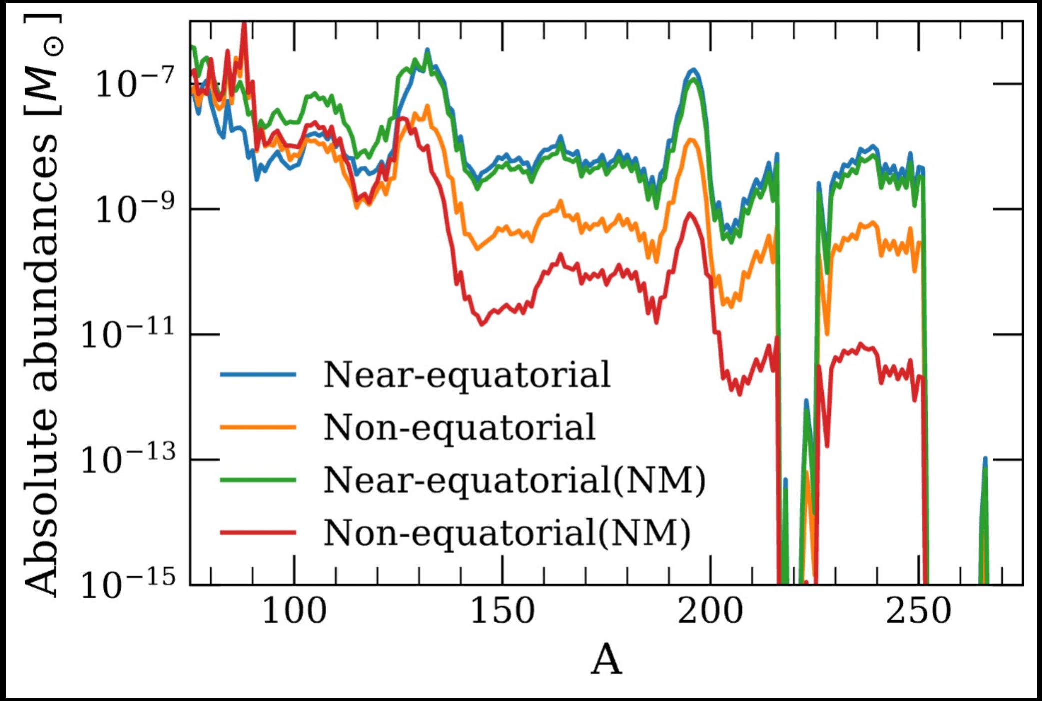
Impacts on the ejecta and nucleosynthesis

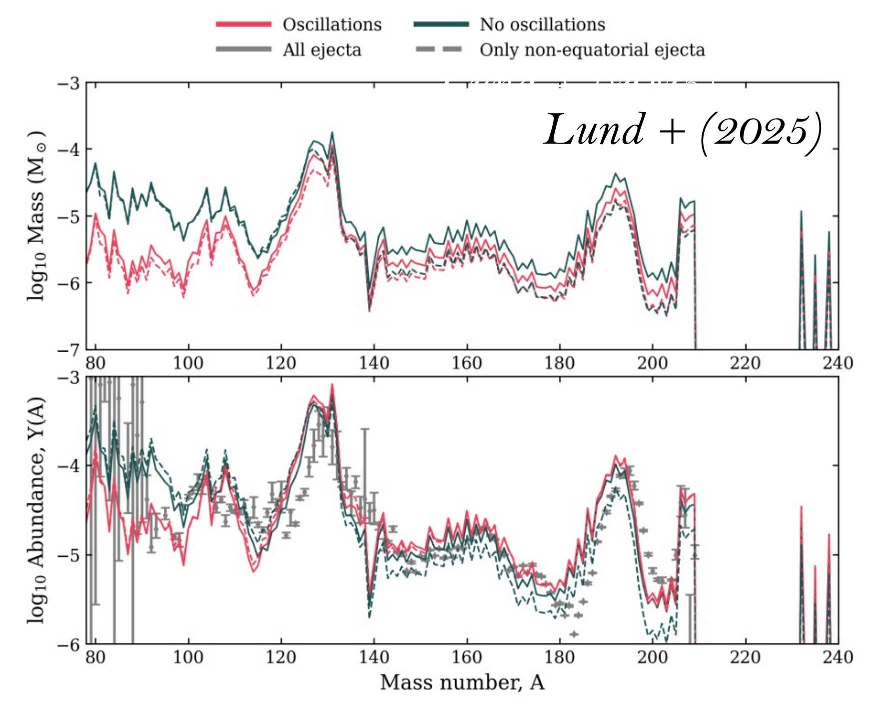
See also Li+, Just+, Fernandez+, Lund+



- Flavor conversions of $\nu_e, \bar{\nu}_e \rightarrow \nu_x, \bar{\nu}_x$ reduce neutrino reabsorption $\nu_e + n \rightarrow p + e^-$
- Flavor conversions → up to 5 times more very neutron-rich ($Y_e < 0.15$) material in ejecta, ~ 200% to 1000% more heavy element production
- Quantitative uncertainties remain for detailed mixing treatments

Angular dependence



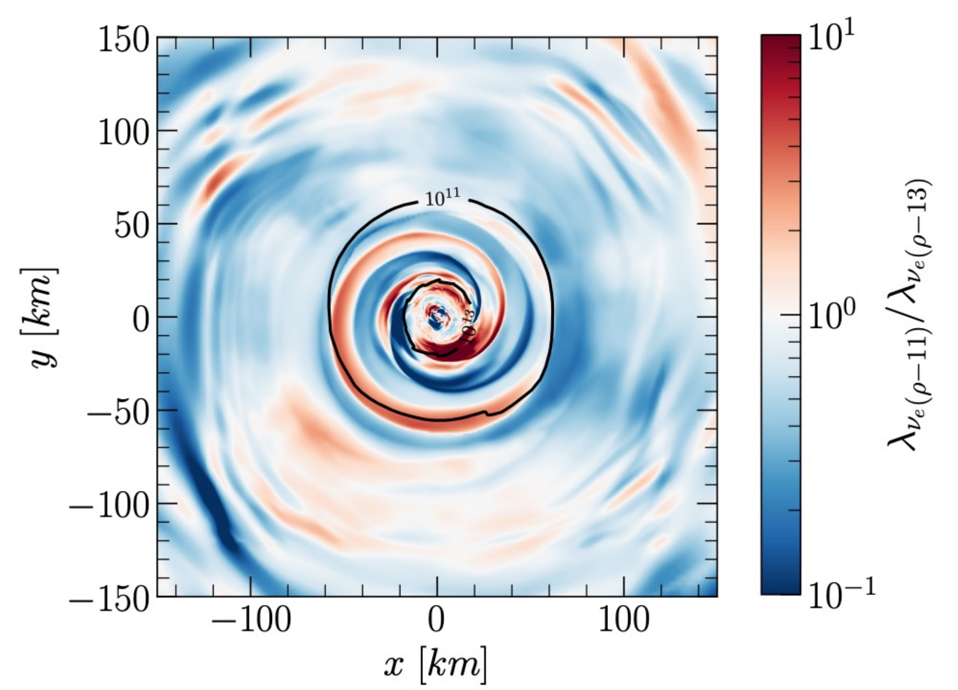


Lund + (2025)

- Materials near equatorial plane and intermediate latitudes show larger differences between no-mixing and mixing simulations
- Substantial differences in polar regions also, however it is not the main contribution to the boost in r-process

Comparisons of different mixing conditions

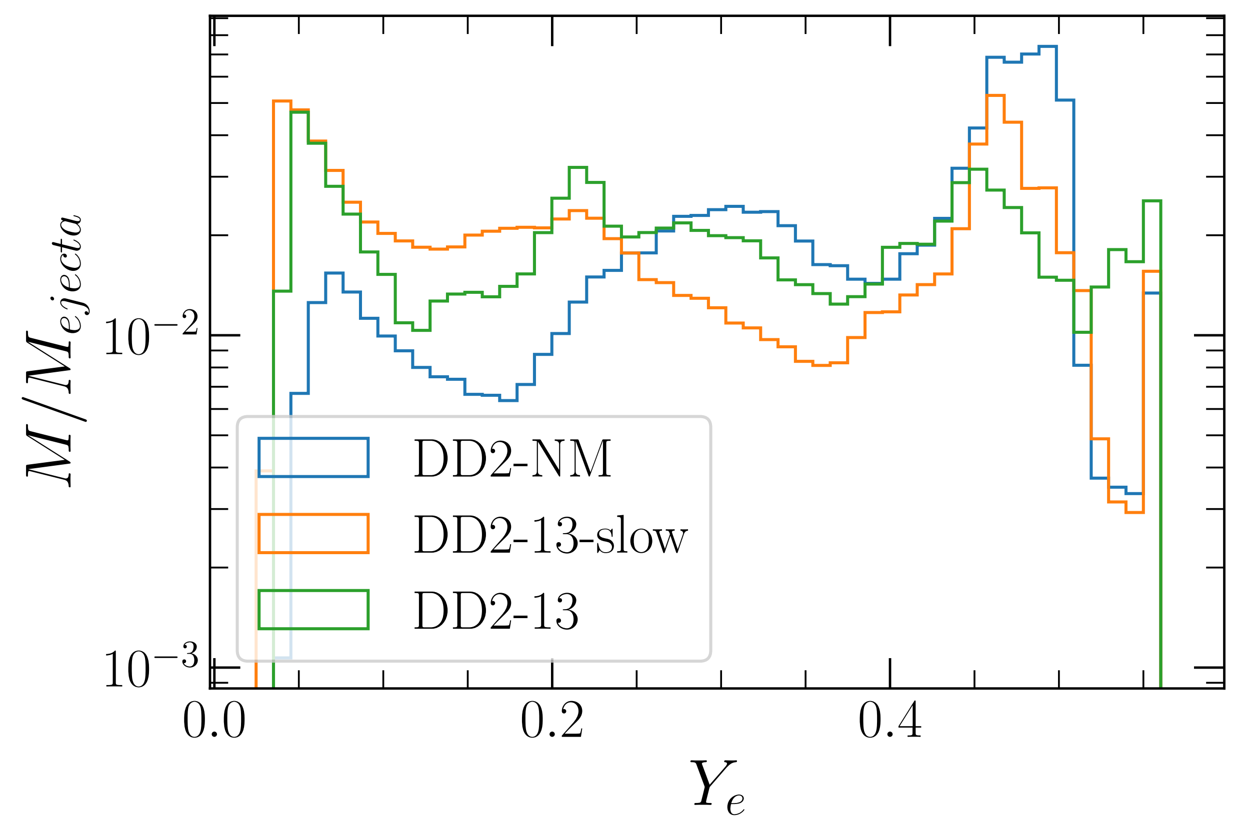
- Why $\rho - 11$ more neutron rich than $\rho - 13$?
- Inner disk (between 10^{11} and 10^{13} g/cm^3)
  - $\nu_e, \bar{\nu}_e \rightarrow \nu_x, \bar{\nu}_x$
  - $\nu_e, \bar{\nu}_e$ trapped, $\nu_x, \bar{\nu}_x$ optically thin
- Outer disk (below $\rho = 10^{11}$ g/cm^3)
  - Some $\nu_x, \bar{\nu}_x \rightarrow \nu_e, \bar{\nu}_e$
  - More $\nu_e + n \rightarrow p + e^-$
  - Less neutron rich ejecta



$$\frac{\mathrm{d}Y_e}{\mathrm{d}t} = \lambda_{\nu_\mathrm{e}}(1 - Y_\mathrm{e}) - \lambda_{\bar{\nu}_\mathrm{e}} Y_\mathrm{e} \approx \lambda_{\nu_\mathrm{e}}$$

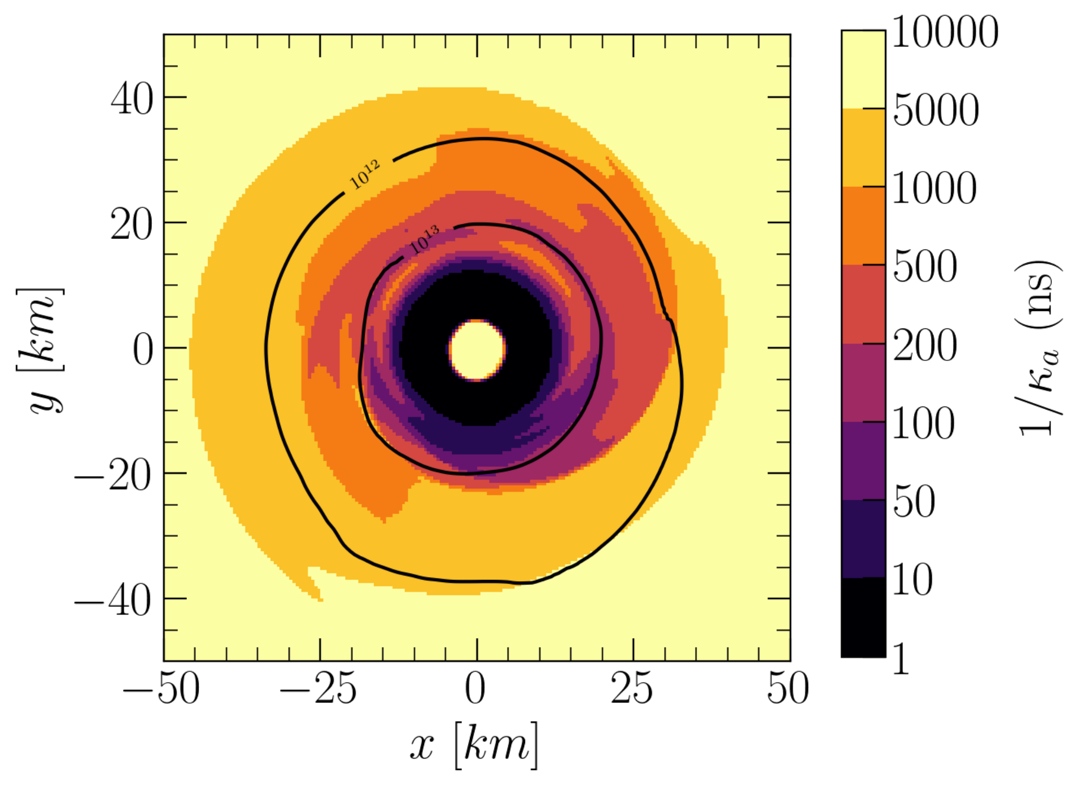
Y_e in $\rho - 13$ increase faster!

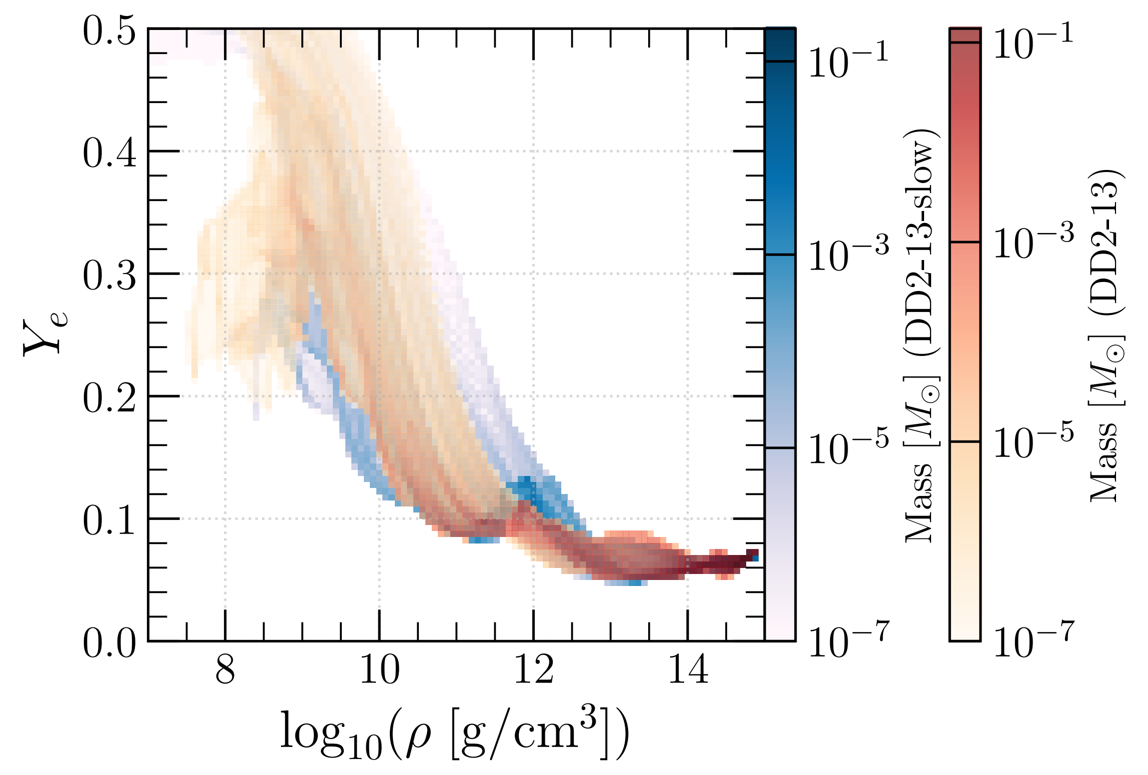
Comparisons of different relaxation times



- DD2-13-slow uses 50 ns flavor relaxation time, 100x than the default 0.5 ns used in DD2-13
- Slower flavor equilibration model's ejecta more neutron rich than that of DD2-13?

Comparisons of different relaxation times





- Flavor conversions at high density compete with thermodynamics

Summary

- Neutrino flavor conversions change the neutrino luminosities, mean energies and flavor hierarchy
- Neutrino flavor conversions give rise to a more neutron rich dynamical ejecta and boost the heavy element production
- Neutrino flavor conversion increase the remnant compactness, change the neutrino and GW peak luminosity and leave potential imprints on post-merger GW spectra
- *Where* and *how* neutrino flavor conversion happens change the results quantitatively -> we need better theoretical modeling!

Next steps

- Neutrino mixing
  - Better modeling? (see Richers+2024, Abbar+2024, Johns+2025, Liu+2025, Fiorillo+2025, Padilla-Gay+2025, Urquilla+2025)
- Microphysics
  - Muonic interactions? (see Gieg+2024, Pajkos+2024, Ng+2024)
  - Pair processes and inelastic scattering (see Cheong+2024, Chiesa+2024, Kawaguchi+2025)
- Magnetic field
  - Change neutrino opacities? (see Kumamoto+2025)
- Need longer runs!
- **AthenaK**: GPU-portable GRMHD+**M1** code is now ready! (see Fields+2025, Zhu+2025)

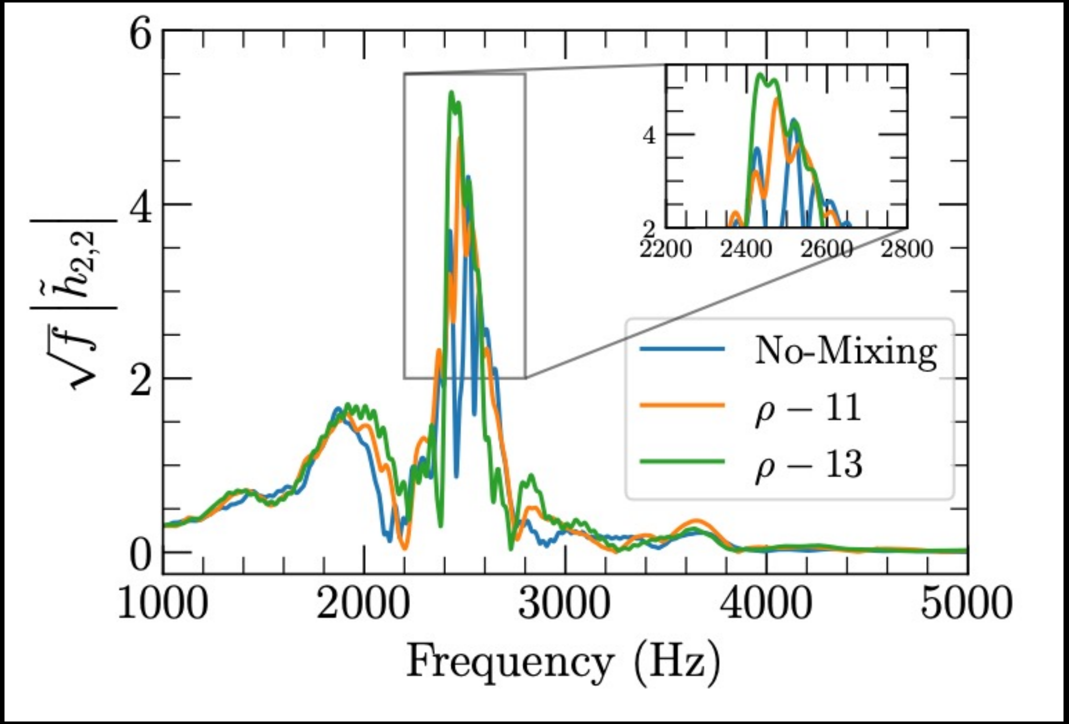
Thank you!

Phys. Rev. Lett. 135, 091401 (arXiv:2503.11758)
Phys. Rev. D 112 (2025) 12, 12 (arXiv:2510.15028)

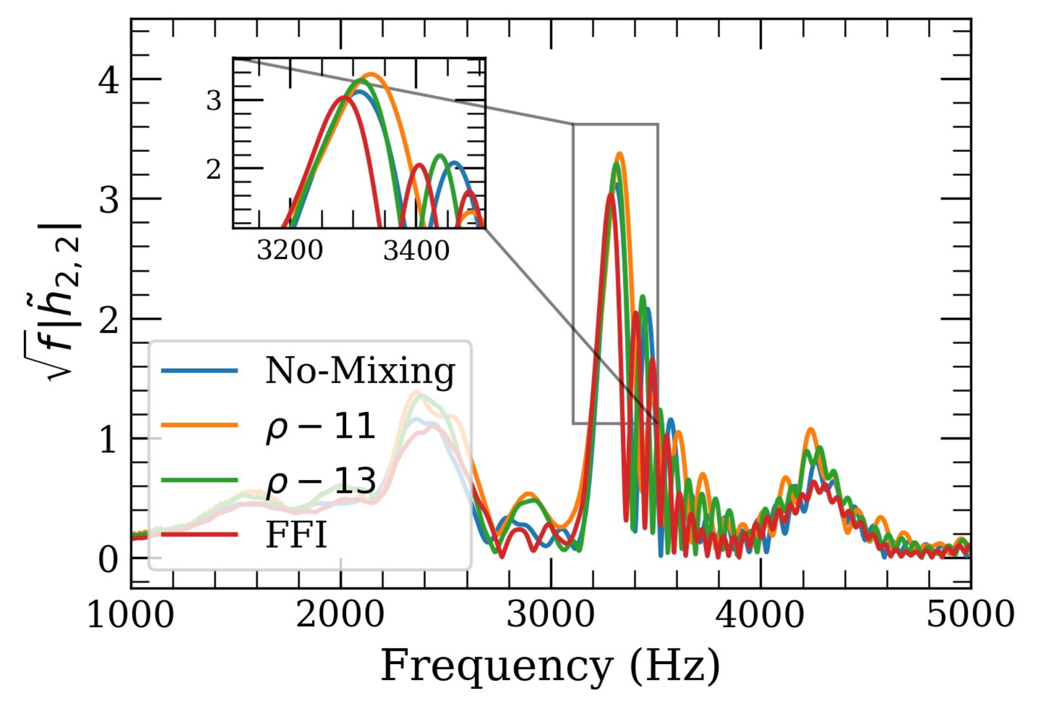
Please consider also cite the second paper where the FFI models and more analysis are actually included ☺

Changes in gravitational waves

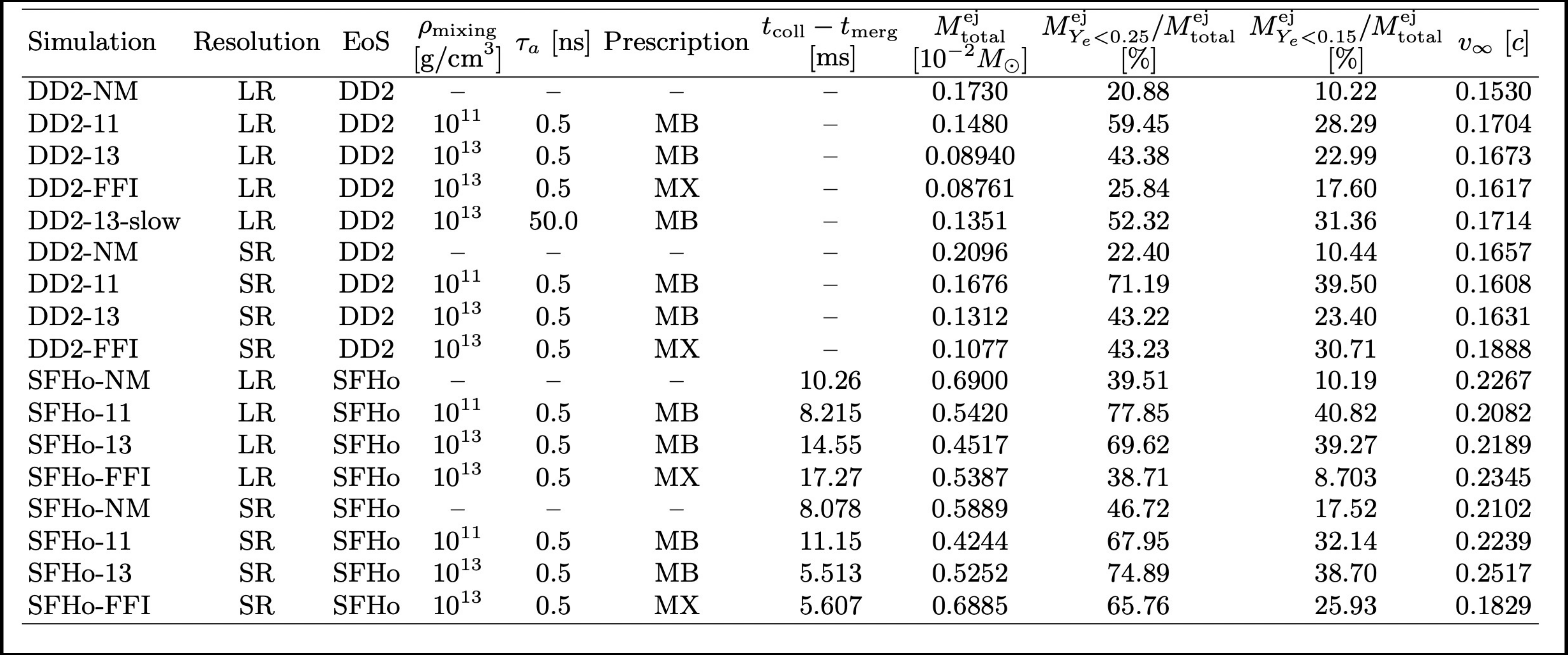
DD2



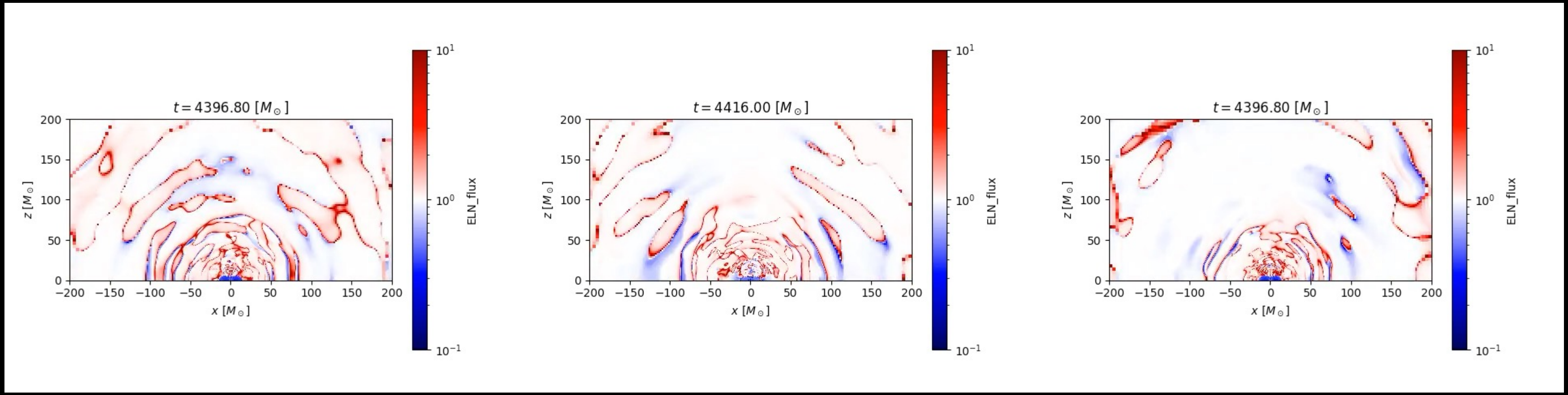
SFHo

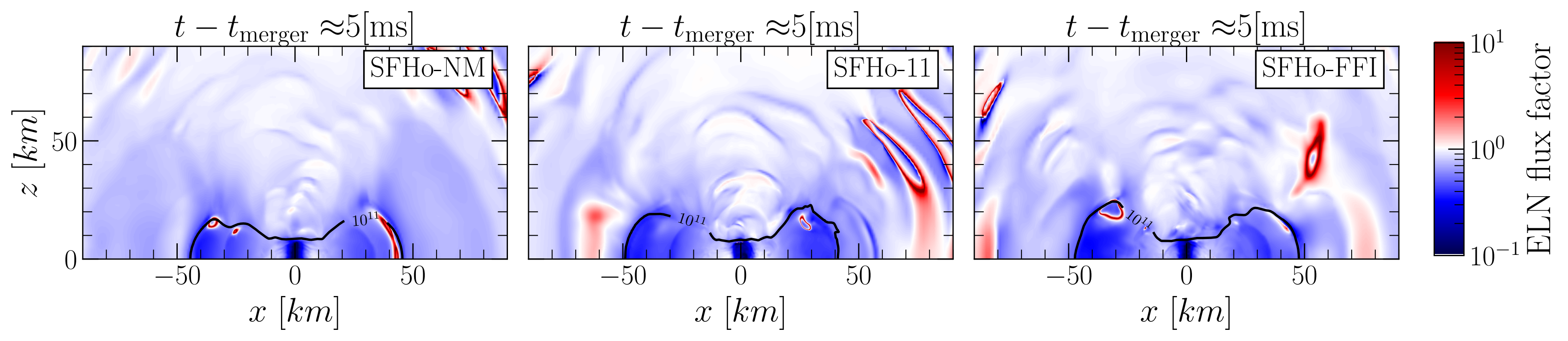


- Frequency discrepancy up to 100 Hz

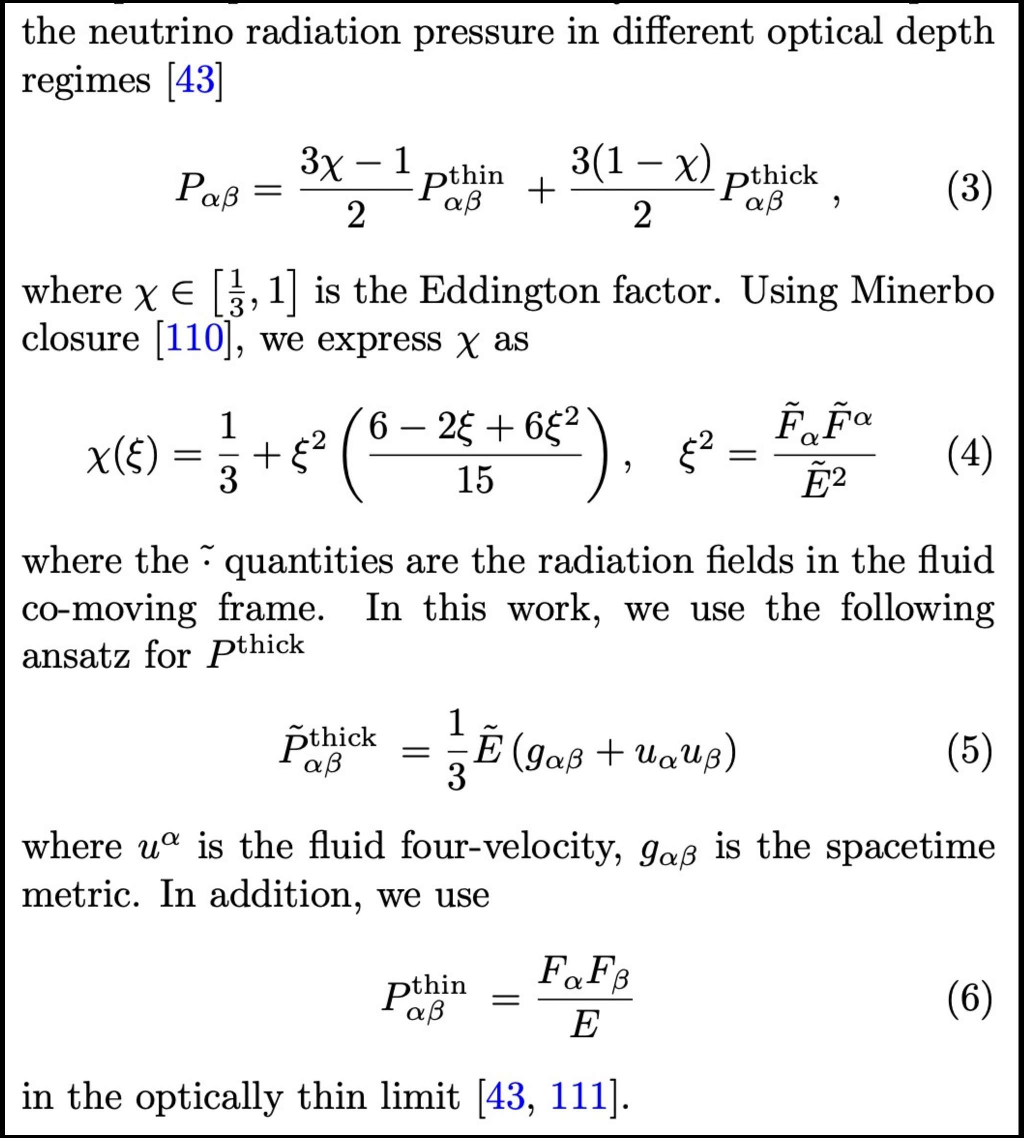


Simulation	Resolution	EoS	ρ_{mixing} [g/cm^3]	τ_a [ns]	Prescription	$t_{\mathrm{coll}} - t_{\mathrm{merg}}$ [ms]	$M^{\mathrm{ej}}_{\mathrm{total}}$ [$10^{-2} M_\odot$]	$M^{\mathrm{ej}}_{Y_e<0.25}/M^{\mathrm{ej}}_{\mathrm{total}}$ [%]	$M^{\mathrm{ej}}_{Y_e<0.15}/M^{\mathrm{ej}}_{\mathrm{total}}$ [%]	v_∞ [c]
DD2-NM	LR	DD2	–	–	–	–	0.1730	20.88	10.22	0.1530
DD2-11	LR	DD2	10^{11}	0.5	MB	–	0.1480	59.45	28.29	0.1704
DD2-13	LR	DD2	10^{13}	0.5	MB	–	0.08940	43.38	22.99	0.1673
DD2-FFI	LR	DD2	10^{13}	0.5	MX	–	0.08761	25.84	17.60	0.1617
DD2-13-slow	LR	DD2	10^{13}	50.0	MB	–	0.1351	52.32	31.36	0.1714
DD2-NM	SR	DD2	–	–	–	–	0.2096	22.40	10.44	0.1657
DD2-11	SR	DD2	10^{11}	0.5	MB	–	0.1676	71.19	39.50	0.1608
DD2-13	SR	DD2	10^{13}	0.5	MB	–	0.1312	43.22	23.40	0.1631
DD2-FFI	SR	DD2	10^{13}	0.5	MX	–	0.1077	43.23	30.71	0.1888
SFHo-NM	LR	SFHo	–	–	–	10.26	0.6900	39.51	10.19	0.2267
SFHo-11	LR	SFHo	10^{11}	0.5	MB	8.215	0.5420	77.85	40.82	0.2082
SFHo-13	LR	SFHo	10^{13}	0.5	MB	14.55	0.4517	69.62	39.27	0.2189
SFHo-FFI	LR	SFHo	10^{13}	0.5	MX	17.27	0.5387	38.71	8.703	0.2345
SFHo-NM	SR	SFHo	–	–	–	8.078	0.5889	46.72	17.52	0.2102
SFHo-11	SR	SFHo	10^{11}	0.5	MB	11.15	0.4244	67.95	32.14	0.2239
SFHo-13	SR	SFHo	10^{13}	0.5	MB	5.513	0.5252	74.89	38.70	0.2517
SFHo-FFI	SR	SFHo	10^{13}	0.5	MX	5.607	0.6885	65.76	25.93	0.1829





- ELN flux factor represents instabilities
- Fast flavor conversion instability targeted model still shows instabilities
- Dynamical effects like advections possibly regenerate the instabilities rapidly



the neutrino radiation pressure in different optical depth regimes [43]

$$P_{\alpha\beta} = \frac{3\chi - 1}{2} P_{\alpha\beta}^{\text{thin}} + \frac{3(1-\chi)}{2} P_{\alpha\beta}^{\text{thick}} , \qquad (3)$$

where $\chi \in \left[\frac{1}{3}, 1\right]$ is the Eddington factor. Using Minerbo closure [110], we express χ as

$$\chi(\xi) = \frac{1}{3} + \xi^2 \left(\frac{6 - 2\xi + 6\xi^2}{15} \right) , \quad \xi^2 = \frac{\tilde{F}_\alpha \tilde{F}^\alpha}{\tilde{E}^2} \qquad (4)$$

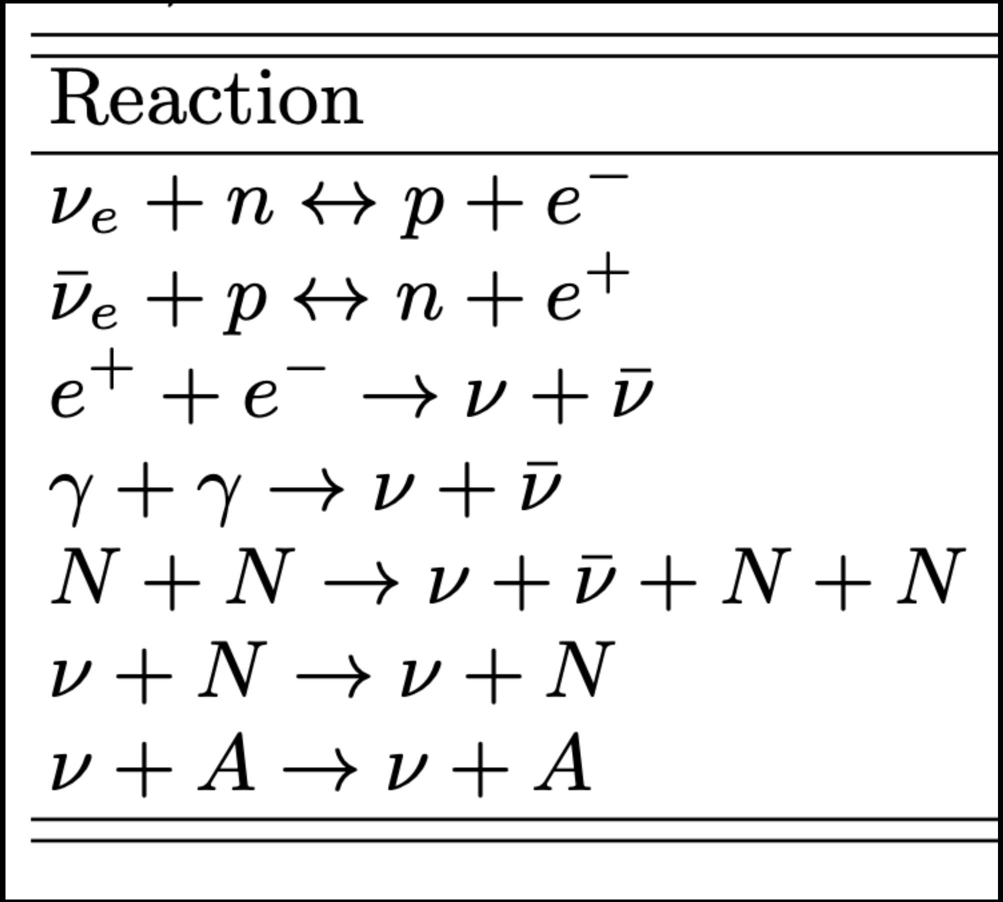
where the $\tilde{}$ quantities are the radiation fields in the fluid co-moving frame. In this work, we use the following ansatz for P^{thick}

$$\tilde{P}_{\alpha\beta}^{\text{thick}} = \frac{1}{3} \tilde{E} \left(g_{\alpha\beta} + u_\alpha u_\beta \right) \qquad (5)$$

where u^α is the fluid four-velocity, $g_{\alpha\beta}$ is the spacetime metric. In addition, we use

$$P_{\alpha\beta}^{\text{thin}} = \frac{F_\alpha F_\beta}{E} \qquad (6)$$

in the optically thin limit [43, 111].



Reaction
$\nu_e + n \leftrightarrow p + e^-$
$\bar{\nu}_e + p \leftrightarrow n + e^+$
$e^+ + e^- \rightarrow \nu + \bar{\nu}$
$\gamma + \gamma \rightarrow \nu + \bar{\nu}$
$N + N \rightarrow \nu + \bar{\nu} + N + N$
$\nu + N \rightarrow \nu + N$
$\nu + A \rightarrow \nu + A$

M1 neutrino transport

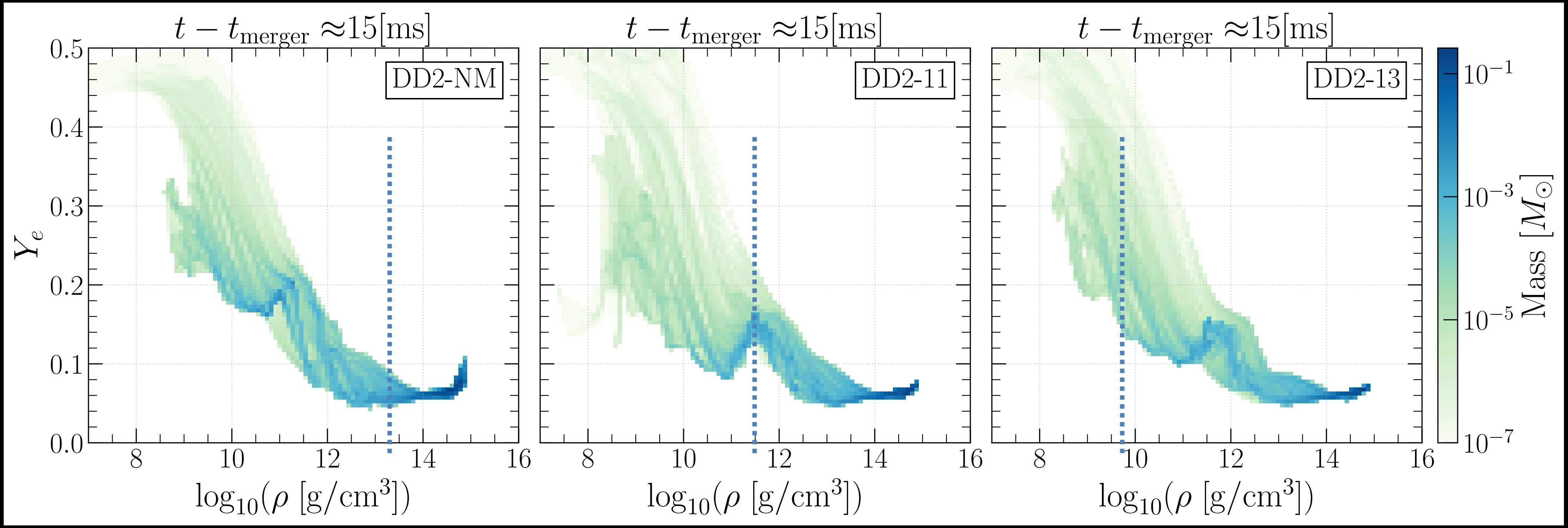
Radiation stress energy tensor in the lab frame

$$T^{\alpha\beta} = En^{\alpha}n^{\beta} + F^{\alpha}n^{\beta} + n^{\alpha}F^{\beta} + P^{\alpha\beta},$$

E, F^{α} and $P^{\alpha\beta}$ correspond to truncated moments. The system is reduced to 3+1 dimensions. Now evolving E and F^{α} with analytical closure $P^{\alpha\beta} = f(E, F^{\alpha})$

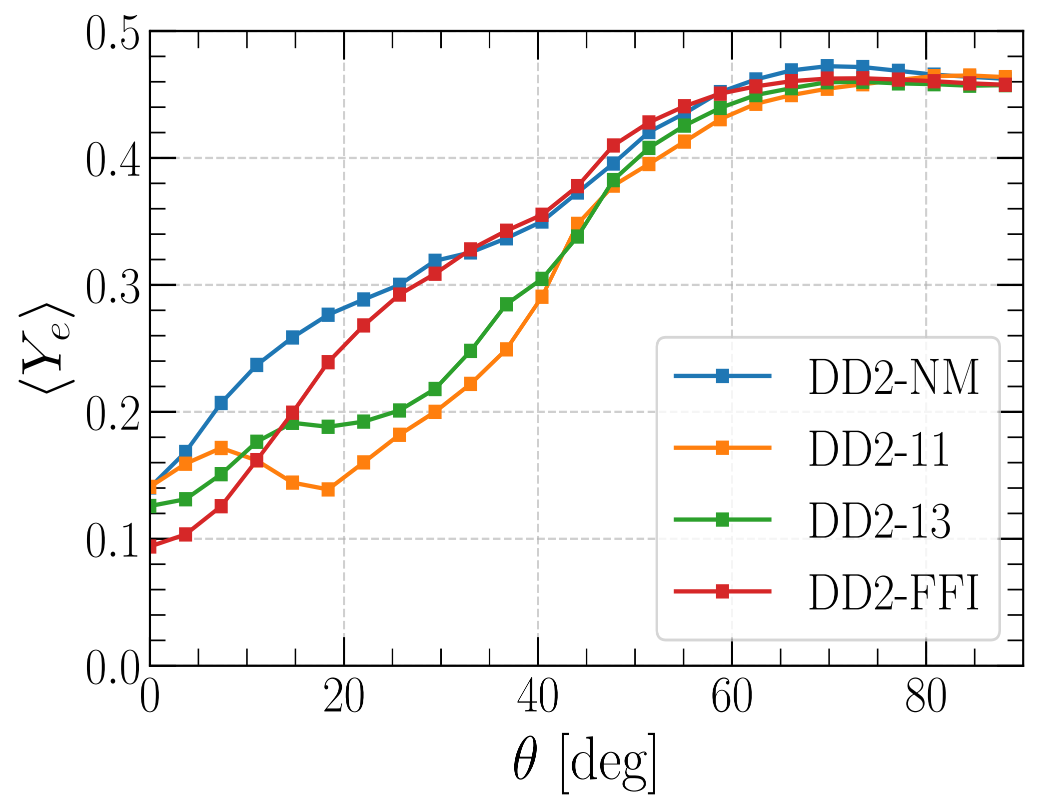
See also Radice+, Foucarzt+

Density dependence



- Difference between no-mixing and mixing models mainly appear at low density regions (below 10^{11} g/cm^3)

Angular dependence



- Materials near equatorial plane and intermediate latitudes show larger differences between no-mixing and mixing simulations