Quasi-steady states and miscidynamics in “once-in-a-lifetime encounter” neutrino models

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Collective Neutrino Oscillations in Supernovae and Neutron Star Mergers
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The mean-field approximation ignores entanglement

$$\underbrace{i(\partial_t + \vec{v} \cdot \nabla) f = [H_f, f]}_{\text{Mean-field approximation}} + C_f$$

$$f = \begin{pmatrix} f_{ee} & f_{ex} \\ f_{xe} & f_{xx} \end{pmatrix} (t, \vec{r}, \vec{p})$$

- C_f: loss of coherence/purity
  - Randomness (collisions)
  - Entanglement

Side note:

- Engineers: $3 \approx \pi$
- Physicists: $3 \approx 2$

Many-body models

$$|\psi\rangle_{N \text{ neutrinos}} \doteq \begin{pmatrix} c_{(1)} \\ c_{(2)} \\ c_{(3)} \\ \vdots \\ c_{(2^N)} \end{pmatrix}$$

$$\rho_a = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{pmatrix}_a = \mathrm{Tr}_{\text{not } a}(|\psi\rangle\langle\psi|)$$

- "Curse of dimensionality": $N \lesssim 20$

Side note:

- Engineers: $20.1 \approx 20$
- Physicists: $10^{58} \approx 20$

Is entanglement important?

- Millions–billions of moles of neutrinos per cubic centimeter
- We know they collectively oscillate
- Can neutrinos collectively entangle, on the scale of an entire star?

Two-neutrino forward scattering

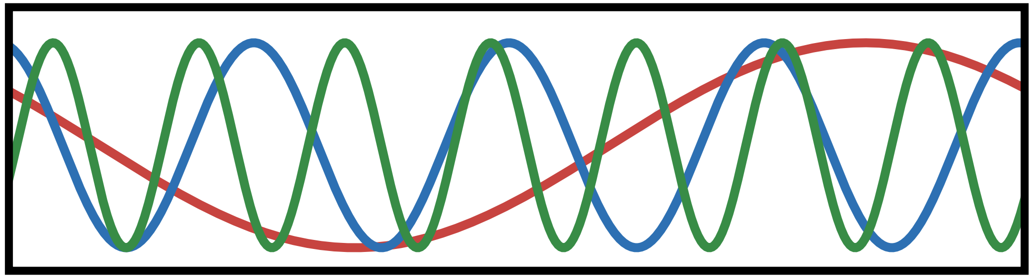
$$H_{12} = \mu(1 - \vec{v}_1 \cdot \vec{v}_2)\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

$$\begin{array}{c} \\ \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \propto \end{array} \begin{array}{c} \begin{matrix} ee & ex & xe & xx \end{matrix} \\ \begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ ee \\ ex \\ xe \\ xx \end{array}$$

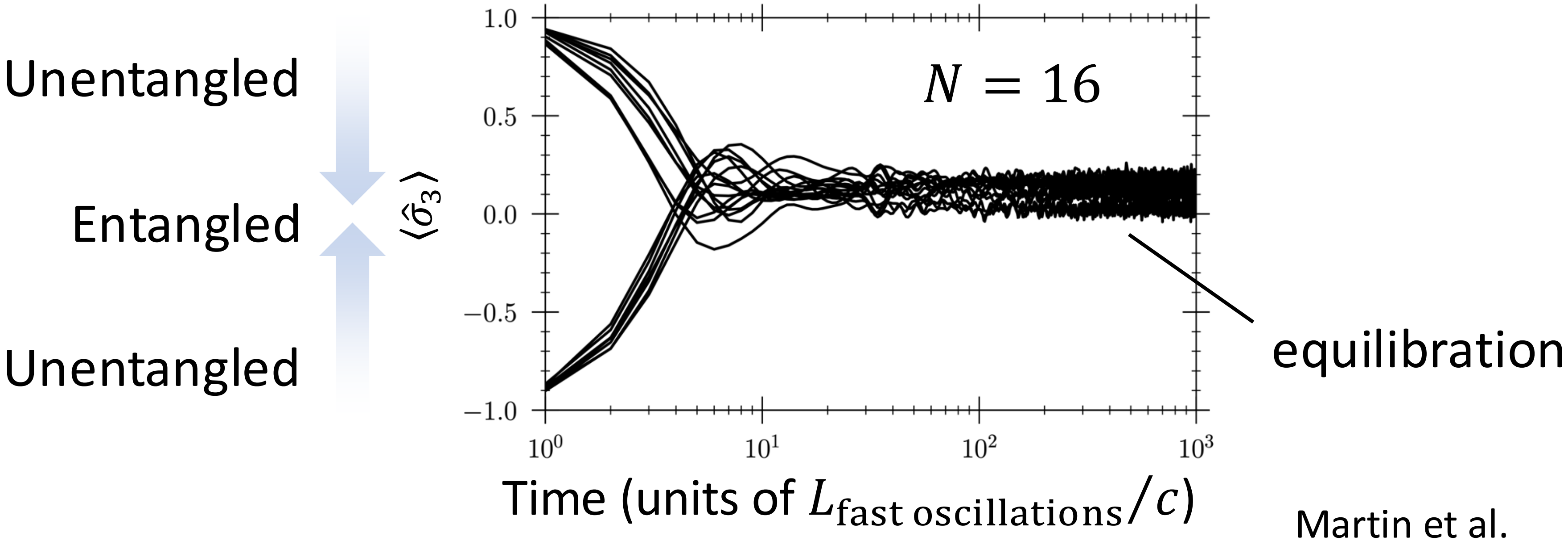
$|ex\rangle \rightarrow c_1|ex\rangle + c_2|xe\rangle$ (entanglement)

Plane wave models

$$H = \sum_{\text{pairs } i,j} H_{ij}$$



"Plane waves in a (periodic) box"

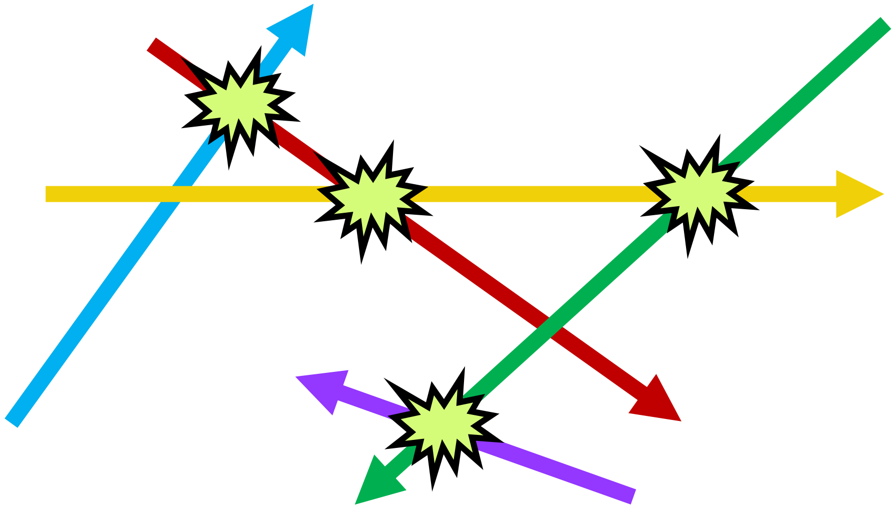


Martin et al.

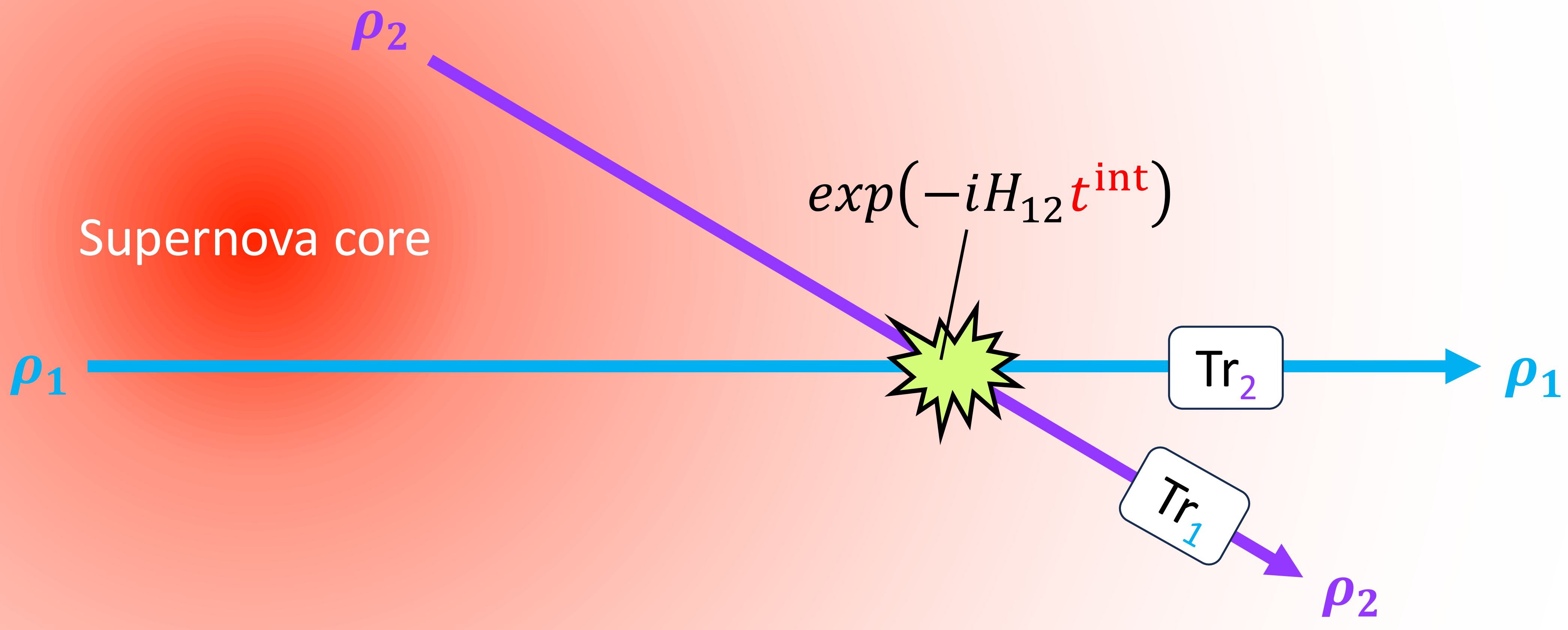
Dense but dilute gas

- Dense: Distance between neutrinos ~ 1 pm
- But also dilute: Wavepacket size ~ 0.1 pm (?)

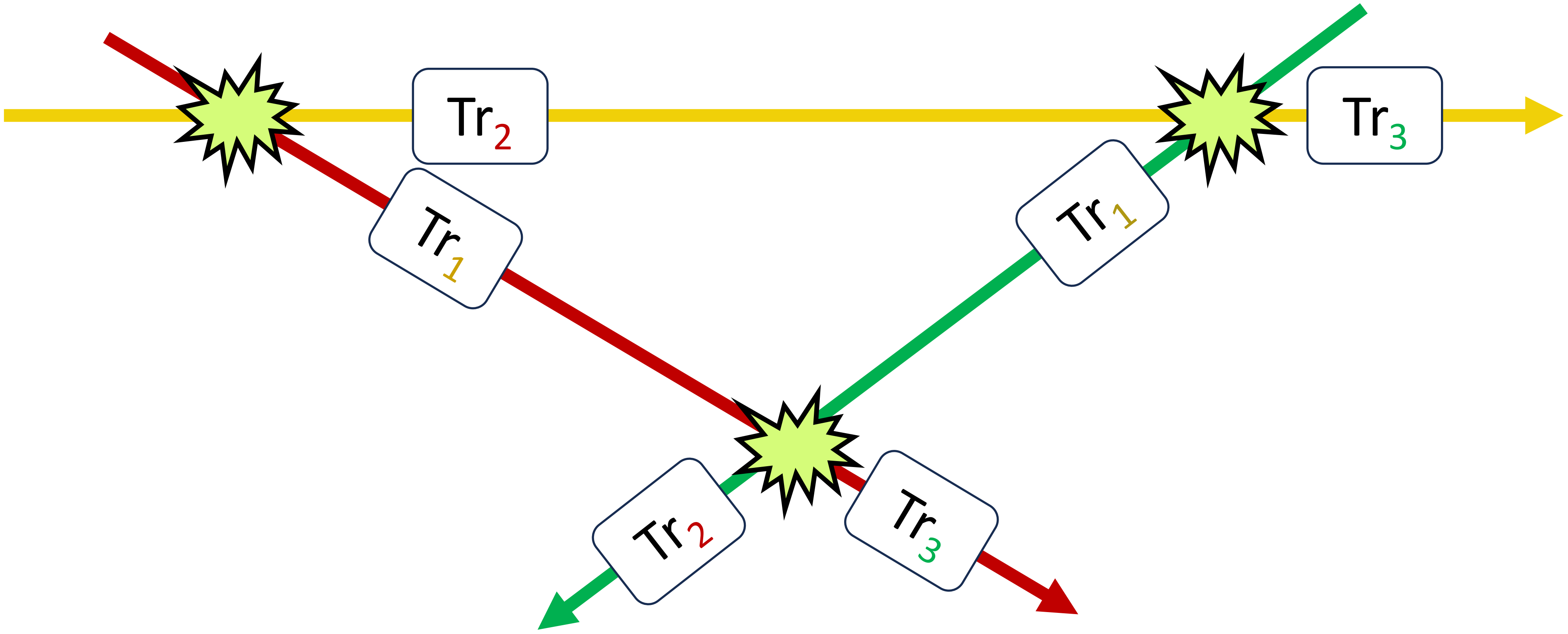
vs.



A once-in-a-lifetime encounter



Once-in-a-lifetime encounter (OILE) models



Discreteness parameter → cross section?

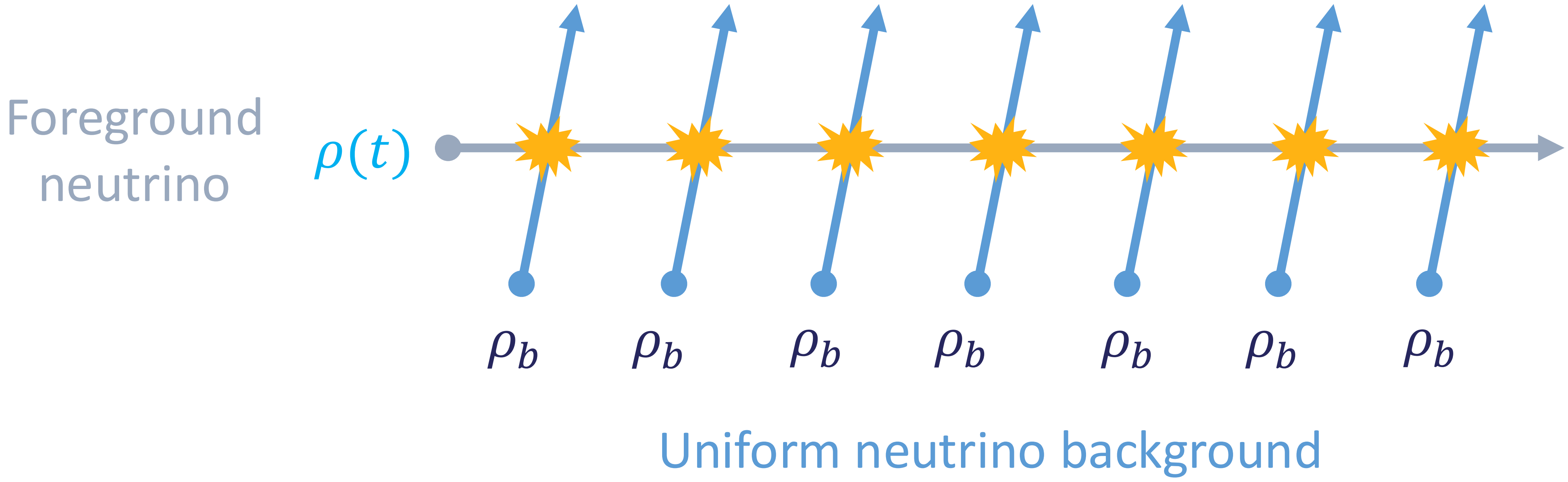
$$\gamma \equiv \mu t^{\text{int}} = \frac{\sqrt{2}G_F}{A \cdot L^{\text{wavepacket}}} t^{\text{int}} = \frac{\sqrt{2}G_F}{A}$$

discreteness parameter

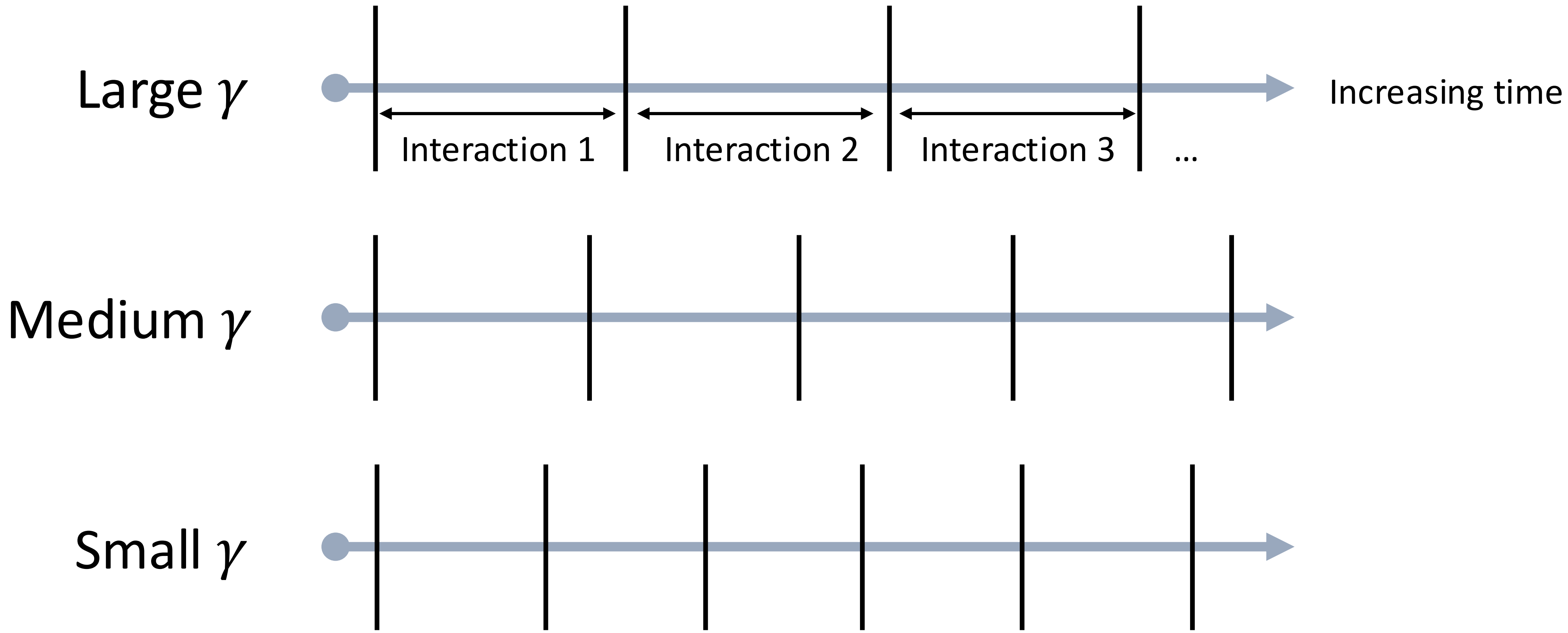
cross section

(using $L^{\text{wavepacket}} = t^{\text{int}}$)

A simple once-in-a-lifetime encounter model



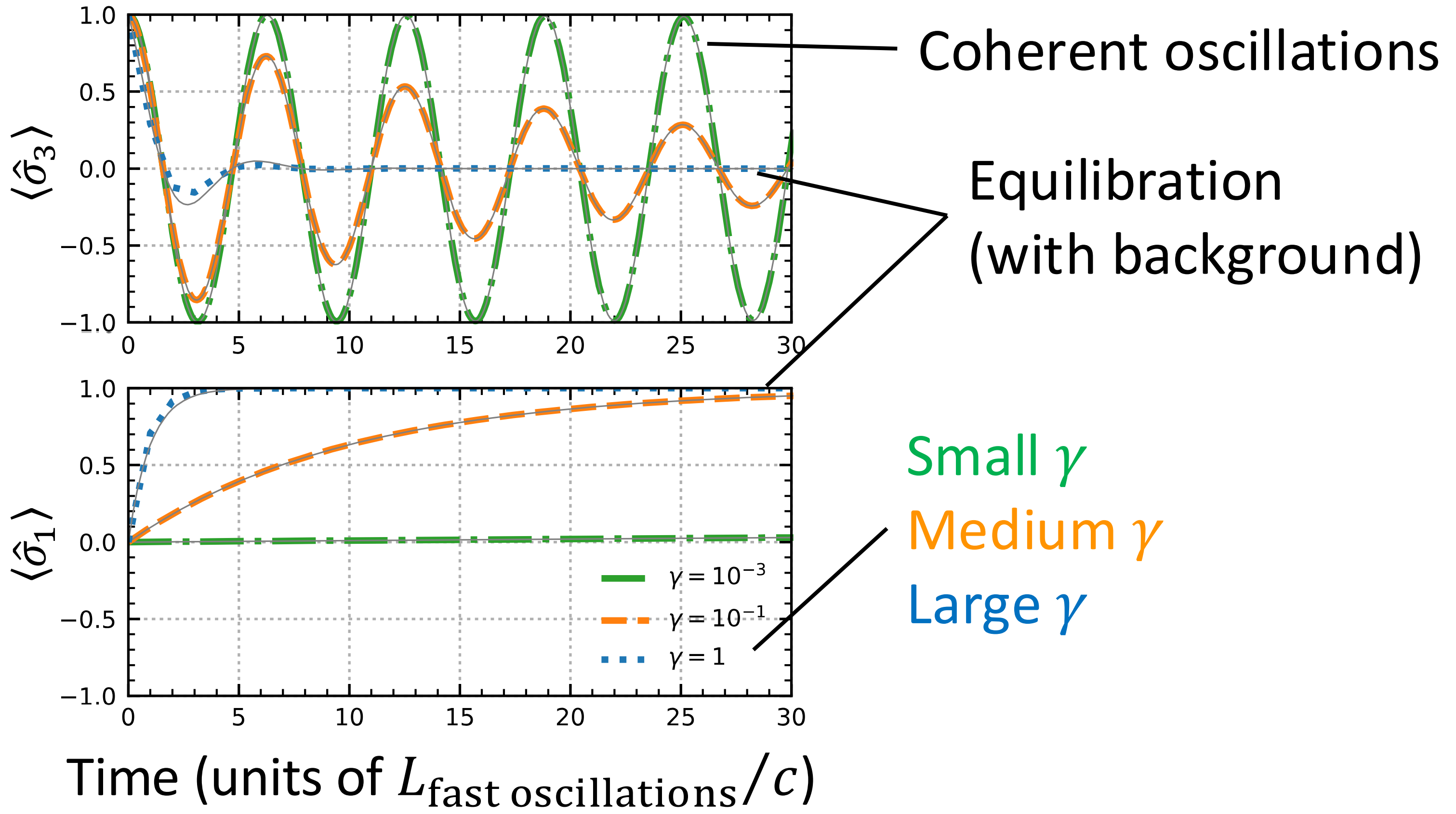
A (new) parameter γ



A numerical experiment with OILE (Result: Rate of entanglement ∝ γ)

Increasing entanglement

Increasing entanglement



$$\dot{P} = \underbrace{\mu \boldsymbol{P}_{bg} \times}_{\text{mean-field}} \underbrace{P + \mu\gamma(\boldsymbol{P}_{bg}}_{\text{equilibrate to } \boldsymbol{P}_{bg}} - P)$$

Kost, Johns, Duan (2024)

Two limits of once-in-a-lifetime encounter models

Mean-field approach

Many-body models

Increasing γ

Miscidynamics (a flash talk)

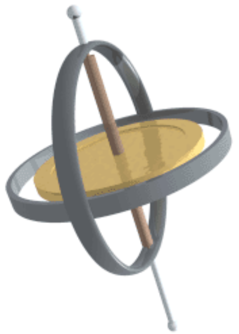
Another perturbative expansion

- Separation of timescales:
  - Possible in OILE models, but not plane wave models

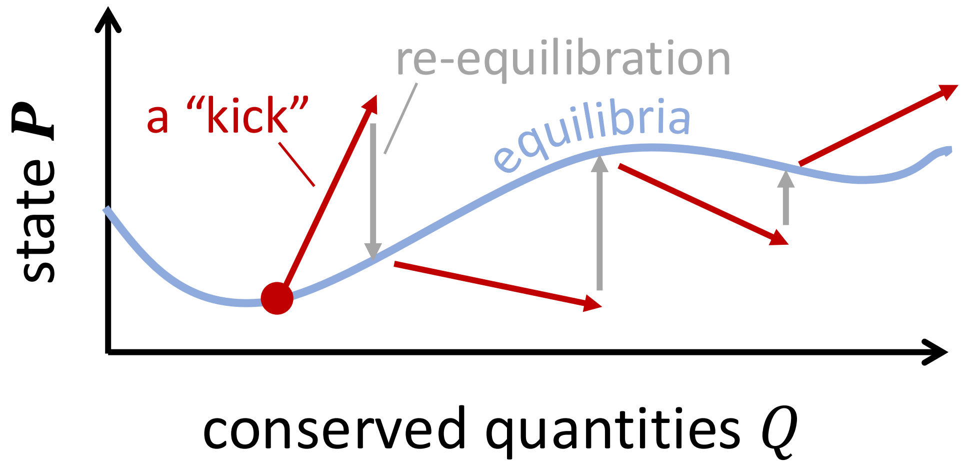
$$\mu\boldsymbol{\gamma} \text{ (entanglement rate)} \ll \mu \text{ (mean-field rate)}$$

- First order in $\boldsymbol{\gamma}$: The adiabatic approximation
  - Like in thermodynamics or quantum mechanics
  - Evolution along a **path of** different mean-field **equilibria**

What path is taken?



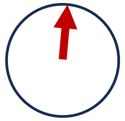
- Mean-field limit:
  equilibria parameterized by conserved quantities Q
- A series of small "kicks" away from equilibrium
  - Each "kick" changes Q
  - System quickly re-equilibrates with the new Q

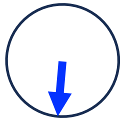


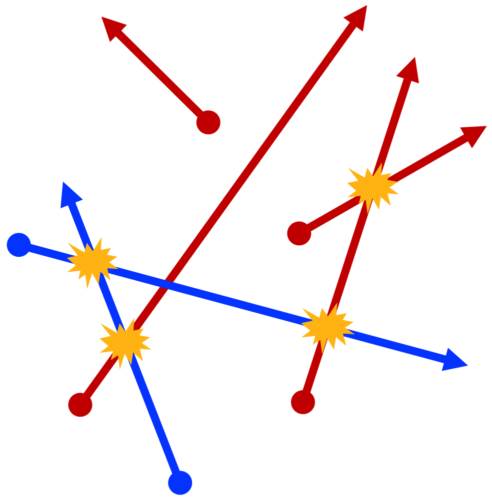
$$\rightarrow \quad \frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{P}_{\mathrm{eq}}(Q)}{\mathrm{d}Q} \cdot \frac{\mathrm{d}Q}{\mathrm{d}t}$$

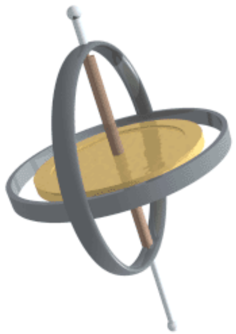
given by terms involving $\boldsymbol{\gamma}$

OILE and miscidynamics agree

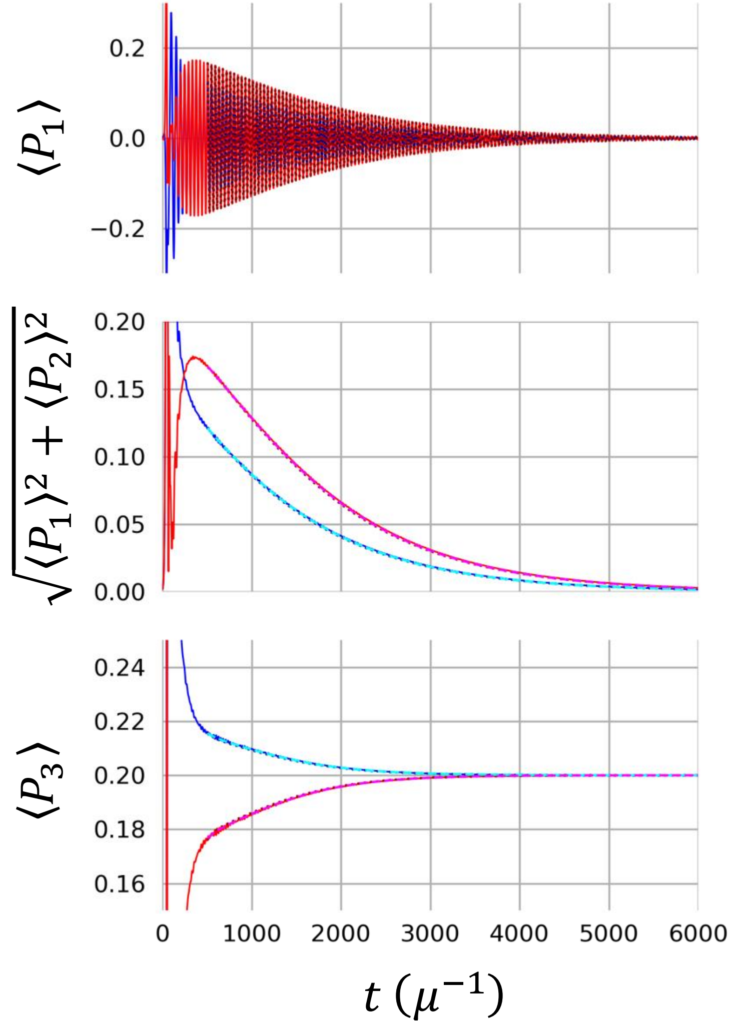
Group 1
60 neutrinos, initially 

Group 2
40 neutrinos, initially 





Group 1 & 2, $\boldsymbol{P}$ vs. t
Miscidynamics: dashed curves, 1 & 2



Adiabatic flavor evolution

Phenomenon	Adiabatic evolution of neutrinos driven by:
The MSW effect	Changing matter density
Spectral splits	Changing neutrino density
OILE/Miscidynamics	“Decoherence” through entanglement

Conclusions

- The standard approach to collective neutrino oscillations, the mean-field approach, ignores entanglement
- Once-in-a-lifetime encounter models show that entanglement can depend on the finite size of neutrino wavepackets
  - Small wavepacket size: mean-field approximation might be okay!
- Miscidynamics “factors out” small-scale dynamics by tracking large-scale changes in equilibria
  - Similar to the flavomon approach
  - Preliminary success with once-in-a-lifetime encounter models as a testing ground