GW-CMB 2025

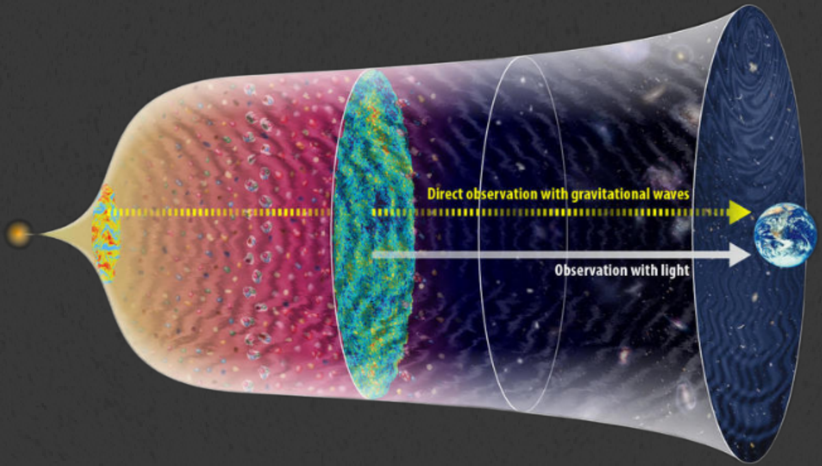
Robust constraints on tensor perturbations from cosmological data: A comparative analysis

Giacomo Galloni

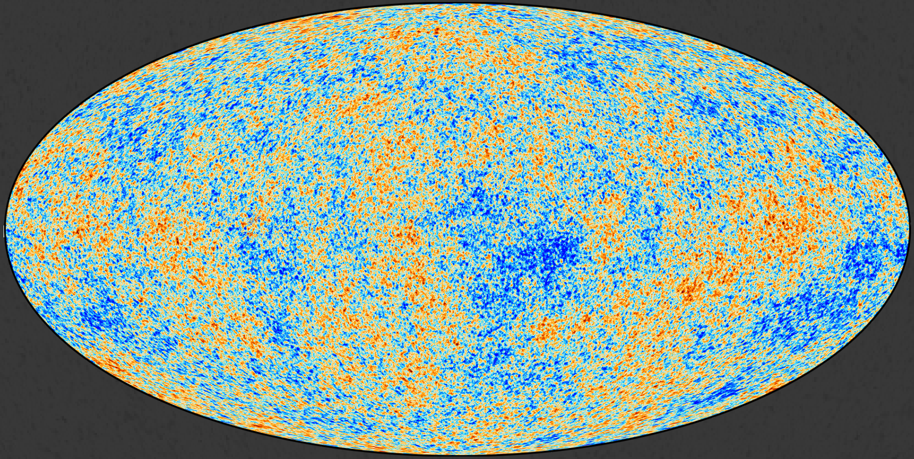
Department of Physics and Earth Science
University of Ferrara

Nov 06, 2025

Inflation

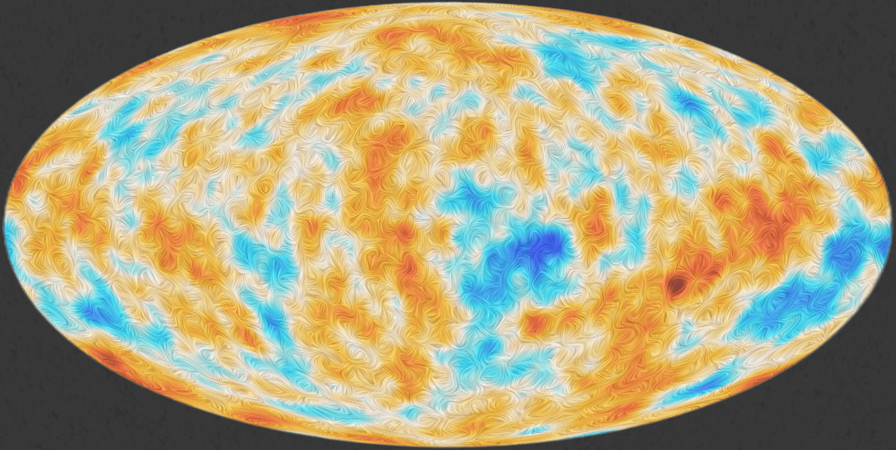

Direct observation with gravitational waves
Observation with light

Cosmic Microwave Background



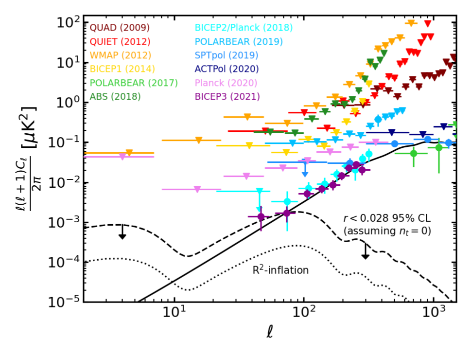
Credits: *Planck* collaboration

Cosmic Microwave Background Polarization



Credits: *Planck* collaboration

Observations



Scalar and tensor primordial spectra

Scalar

$$\Downarrow$$

$$P_s(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1}$$

$$\Downarrow$$

$$\{A_s, n_s\}$$

Tensor

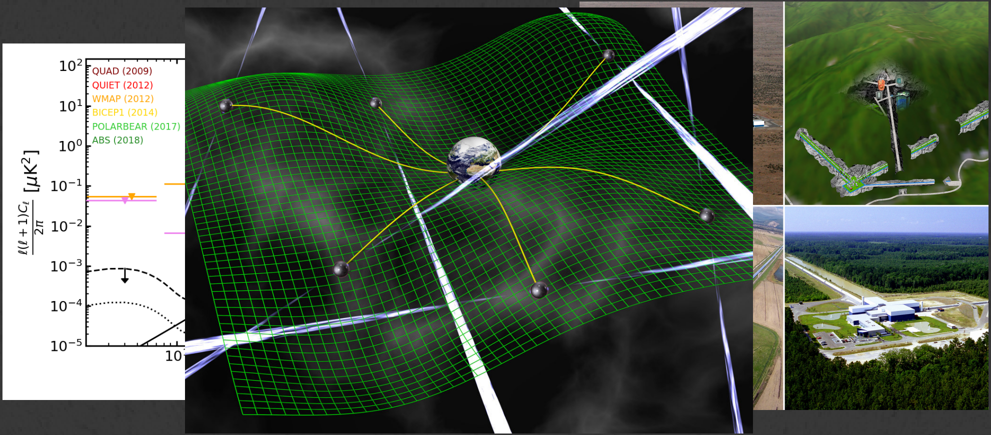
$$\Downarrow$$

$$P_t(k) = r A_s \left(\frac{k}{k_*}\right)^{n_t}$$

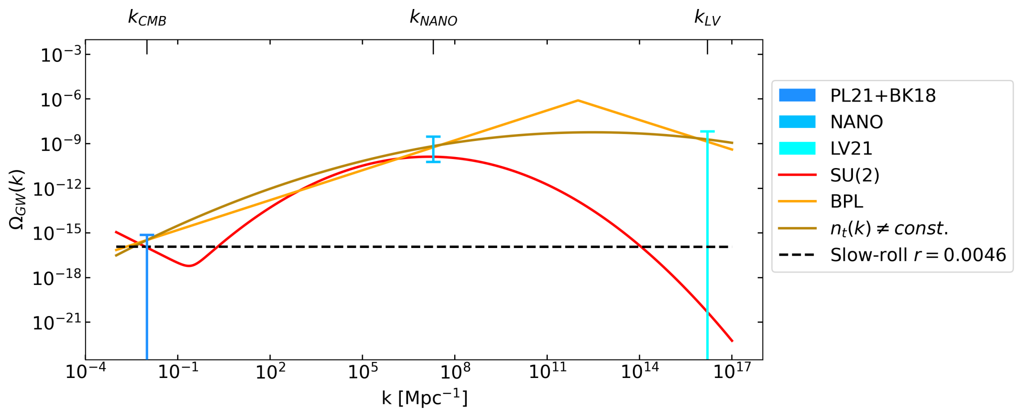
$$\Downarrow$$

$$\boxed{\{r, n_t\}}$$

Observations



Power law? Maybe not...



Likelihood: the key to extract information from data

$$\mathcal{L}(D|\vec{\theta})$$

$$\vec{\theta}$$

ΛCDM $+$ $r,\ n_t$

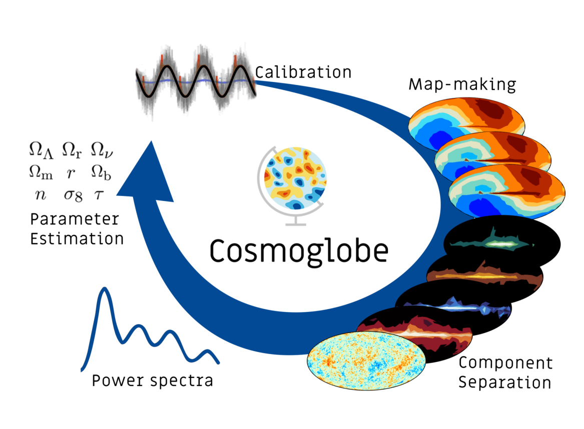
Likelihood: the key to extract information from data

$$\mathcal{L}(D|\vec{\theta})$$

$$\vec{\theta}$$

ΛCDM $\quad + \quad$ r, n_t

D = Time-domain data (?)



D. J. Watts et al. - arxiv:2303.08095

D = Time-domain data (?)

D = map: pixel-based likelihood

$$-2\log\mathcal{L} = m^{\mathrm{T}} COV^{-1} m + \log|COV|$$

$$m = \{T, Q, U\}$$

$$COV = S(C_\ell) + N$$

M. Gerbino et al. - arXiv:1909.09375

D $= C_\ell$: spectrum-based likelihood

$$a_{\ell m} \equiv \int Y^*_{\ell m}(\theta,\phi)\frac{\Delta T}{T_0}(\theta,\phi)\,\mathrm{d}(\cos\theta)\,\mathrm{d}\phi$$

$$\Downarrow$$

$$\langle a^*_{\ell m} a_{\ell' m'}\rangle = \delta_{\ell\ell'}\delta_{mm'}C_\ell$$

$$\Downarrow$$

$$-2\log\mathcal{L} = \sum_\ell (2\ell+1)\left[\frac{\hat{C}_\ell}{C_\ell} - \log\left(\frac{\hat{C}_\ell}{C_\ell}\right) - 1\right]$$

S. Hamimeche and A. Lewis - arXiv:0801.0554

D $= C_\ell$: spectrum-based likelihood

$$a_{\ell m} \equiv \int Y^*_{\ell m}(\theta,\phi)\frac{\Delta T}{T_0}(\theta,\phi)\,\mathrm{d}(\cos\theta)\,\mathrm{d}\phi$$

$$\Downarrow$$

$$\langle a^*_{\ell m} a_{\ell' m'}\rangle = \delta_{\ell\ell'}\delta_{mm'}C_\ell$$

$$\Downarrow$$

$$-2\log\mathcal{L} = \sum_\ell (2\ell+1)\left[\frac{\hat{C}_\ell}{C_\ell} - \log\left(\frac{\hat{C}_\ell}{C_\ell}\right) - 1\right]$$

S. Hamimeche and A. Lewis - arXiv:0801.0554

Masking the sky

$$a_{\ell m} \equiv \int Y^*_{\ell m}(\theta,\phi)\frac{\Delta T}{T_0}(\theta,\phi)\,\mathrm{d}(\cos\theta)\,\mathrm{d}\phi$$

$$\Downarrow$$

$$\langle a^*_{\ell m} a_{\ell' m'}\rangle = \delta_{\ell\ell'}\delta_{mm'}C_\ell$$

$$\Downarrow$$

$$-2\log\mathcal{L} \neq \sum_\ell (2\ell+1)\left[\frac{\hat{C}_\ell}{C_\ell} - \log\left(\frac{\hat{C}_\ell}{C_\ell}\right) - 1\right]$$

S. Hamimeche and A. Lewis - arXiv:0801.0554

Fiducial Gaussian

$$-2\log\mathcal{L} = \left(\vec{C^{\rm obs}} - \vec{C^{\rm th}}\right) \times \mathsf{COV}_{\rm fid}^{-1} \times \left(\vec{C^{\rm obs}} - \vec{C^{\rm th}}\right)^{\rm T}$$

- Approximation in full-sky
- Accounts for multipole couplings
- Assumes Gaussianity of C_ℓ

Exact w/ $f_{\rm sky}$

$$-2\log\mathcal{L} = f_{\rm sky}\sum_\ell (2\ell+1)\left[\frac{C_\ell^{\rm obs}}{C_\ell^{\rm th}} - \log\left(\frac{C_\ell^{\rm obs}}{C_\ell^{\rm th}}\right) - 1\right]$$

- Exact in full-sky, simply scales as $f_{\rm sky}$ in cut-sky
- Disregards completely multipole couplings
- Captures the non-Gaussianity of C_ℓ
- The idea is to rescale the degrees of freedom

HL

$$-2\log\mathcal{L} = \left[g\left(\frac{\vec{C^{\rm obs}}}{C^{\rm th}}\right)C^{\rm fid}\right] \times \mathsf{COV}^{-1}_{\rm fid} \times \left[g\left(\frac{\vec{C^{\rm obs}}}{C^{\rm th}}\right)C^{\rm fid}\right]^{\rm T}$$

$$g(x) = sign(x-1)\sqrt{2(x-\log(x)-1)}$$

- Exact in full-sky
- Accounts for multipole couplings
- Captures non-Gaussianity of C_ℓ
- Negative values to account for

S. Hamimeche and A. Lewis - arXiv:0801.0554

Offset HL

$$-2\log\mathcal{L} = \left[g\left(\frac{C^{\mathrm{obs}}+O_\ell}{C^{\mathrm{th}}+O_\ell}\right)(C^{\mathrm{fid}}+O_\ell)\right]^{\rightarrow} \times \mathsf{COV}_{\mathrm{fid}}^{-1} \times \left[g\left(\frac{C^{\mathrm{obs}}+O_\ell}{C^{\mathrm{th}}+O_\ell}\right)(C^{\mathrm{fid}}+O_\ell)\right]^{\rightarrow\,\mathrm{T}}$$

$$g(x) = sign(x-1)\sqrt{2(x-\log(x)-1)}$$

- Exact in full-sky
- Accounts for multipole couplings
- Captures non-Gaussianity of C_ℓ
- Offsets everything avoiding negative values

A. Mangilli et al. - arXiv:1503.01347

LoLLiPoP

$$-2\log\mathcal{L} = \left[g\left(\frac{C^{\rm obs}+O_\ell}{C^{\rm th}+O_\ell}\right)\vec{}\,(C^{\rm fid}+O_\ell)\right] \times \mathsf{COV}^{-1}_{\rm fid} \times \left[g\left(\frac{C^{\rm obs}+O_\ell}{C^{\rm th}+O_\ell}\right)\vec{}\,(C^{\rm fid}+O_\ell)\right]^{\rm T}$$

$$g(x) = sign(x)sign(|x|-1)\sqrt{2(|x|-\log|x|-1)}$$

- Exact in full-sky
- Accounts for multipole couplings
- Captures non-Gaussianity of C_ℓ
- Offsets everything avoiding negative values

Planck collaboration - arXiv:1605.03507

D $= \hat{\Omega}_{\rm GW}$: likelihood on parameter

$$-2\log\mathcal{L}\left(\hat{\Omega}_{\rm GW}|\Omega_{\rm GW}\right) = \left(\frac{\hat{\Omega}_{\rm GW} - \Omega_{\rm GW}}{\sigma_{\Omega_{\rm GW}}}\right)^2$$

- Considering the results of an experiment
- Estimate of the parameter acts as data
- The width reflects the constraining power of the experiment
- Used to include the results of an experiment without its full complexities

Dataset

- Planck 2018 lowℓ — Commander samples / sims based
- Planck 2020 highℓ *plik* — Gaussian
- Planck 2020 highℓ HiLLiPoP / CamSpec — Gaussian
- Planck 2018/2020 lensing — Gaussian
- LoLLiPoP — modified oHL
- LIGO-Virgo-KAGRA interferometers — Gaussian on $\Omega_{\rm GW}$
- BICEP3/Keck — HL

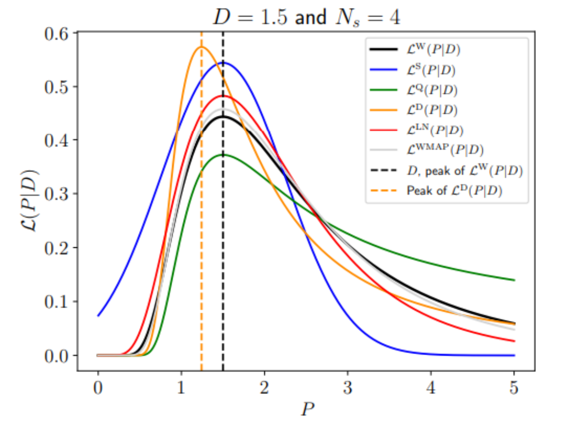
Planck collaboration - arXiv:1907.12875, 1807.06210

E. Rosenberg et al. - arXiv:2205.10869 and M. Tristram et al. - arXiv:2309.10034

BICEP/*Keck* collaboration - arXiv:2110.00483

LIGO-Virgo-KAGRA collaboration - arXiv:2101.12130

GW likelihood



G. Franciolini et al - arXiv:2505.24695

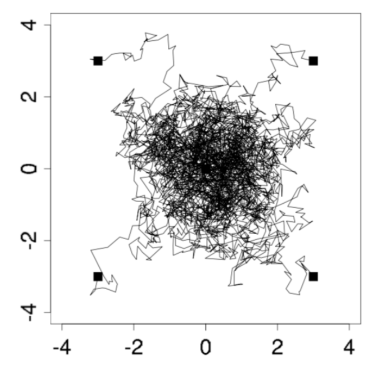
The Bayesian way

MCMC

$$p(\vec{\theta}|D) \propto \mathcal{L}(D|\vec{\theta})\pi(\vec{\theta})$$

$$\Downarrow$$

Posterior $\propto$ Likelihood $\times$ Prior



Bayesian credible intervals

$$\int_{\theta_{\text{low}}}^{\theta_{\text{up}}} p(\theta|D)d\theta = \alpha$$

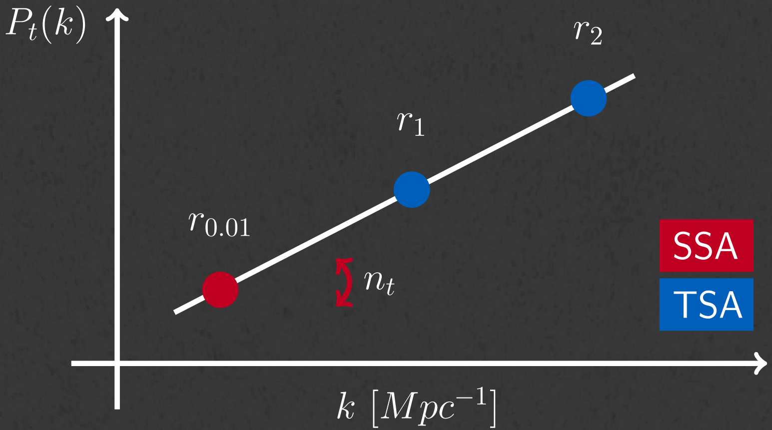
A physical boundary is encoded by the prior

$$\pi(\theta) = \begin{cases} 1 & \text{if} \quad \theta \geq 0 \\ 0 & \text{if} \quad \theta < 0 \end{cases}$$

Some details

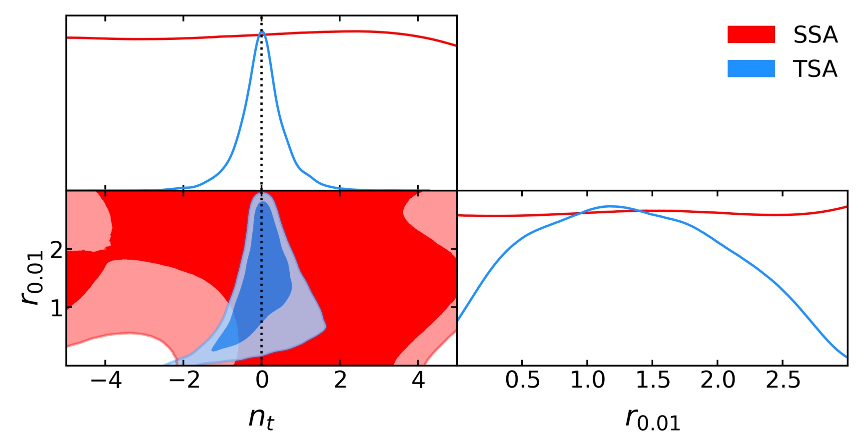
ΛCDM	$+$	r, n_t
$\Downarrow$		$\Downarrow$
6 $+\mathcal{O}(10)$		2 $+\mathcal{O}(10)$
$\Downarrow$		$\Downarrow$
Flat priors	VS	???

Single- and Two- Scale Approaches



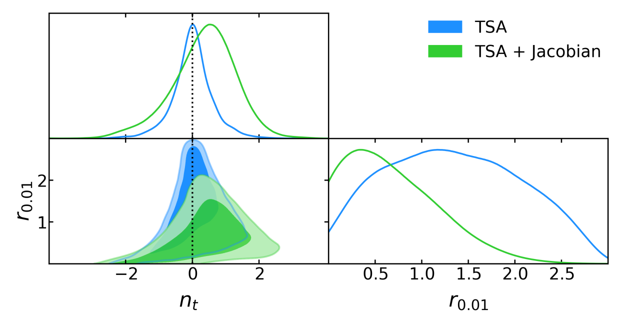
Cabass et al. - arXiv:1511.05146
Planck Collaboration - arXiv:1502.02114

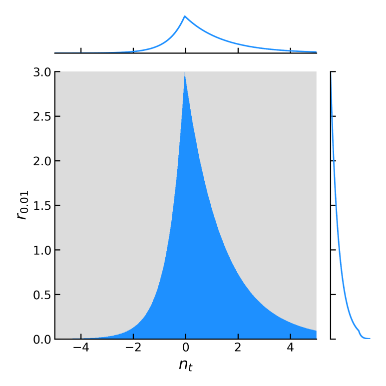
Robustness test: priors



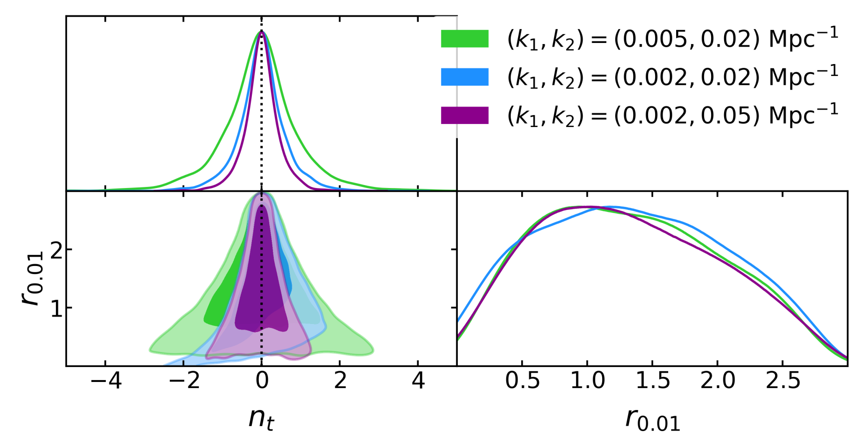
GG et al. - arXiv:2208.00188

Robustness test: Jacobian





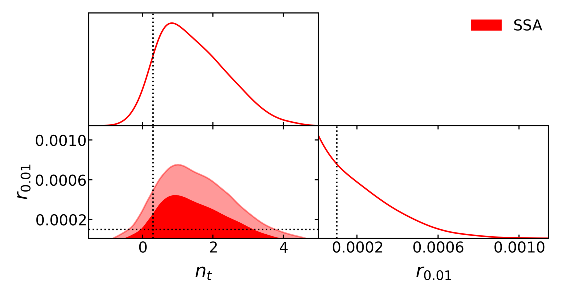
Robustness test: changing scales

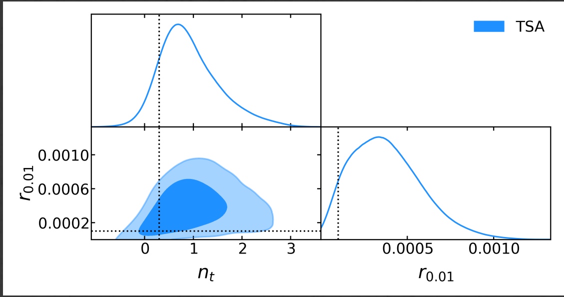


Larger leverage arm $k_2 - k_1 \Rightarrow$ More informative prior on $(r_{0.01}, n_t)$

Robustness test: mock dataset

$$-2\log\mathcal{L} = \sum_{\ell}(2\ell+1)\left[\frac{C_\ell^{\rm obs}}{C_\ell^{\rm th}} - \log\left(\frac{C_\ell^{\rm obs}}{C_\ell^{\rm th}}\right) - 1\right]$$



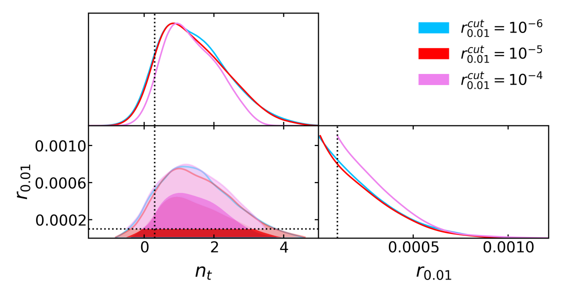


Squashed posterior towards $r = 0$

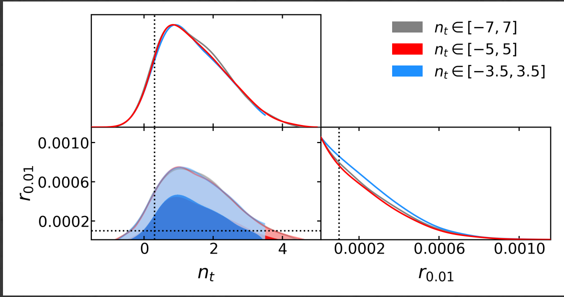
Fake detection of r, stricter bounds on n_t

Robustness test: mock dataset

$r_{0.01}$ cut-offs

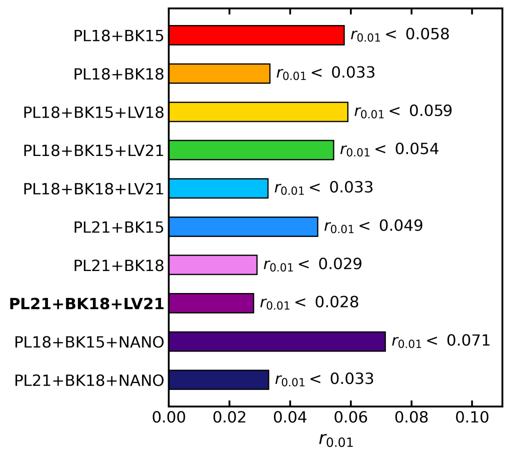


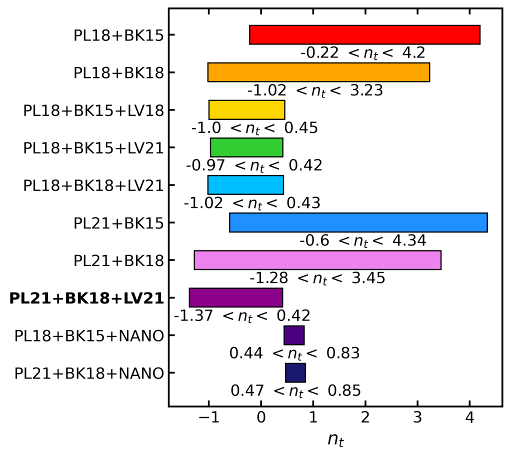
n_t prior



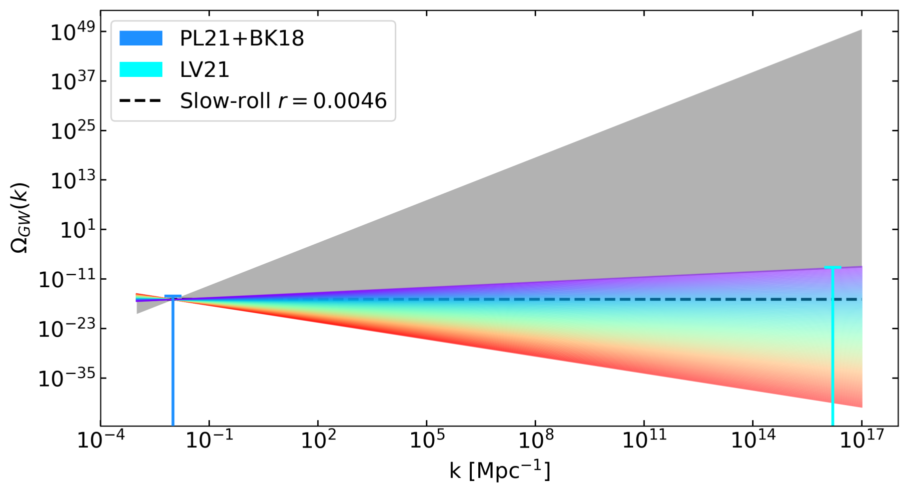
low $r_{0.01}$ cutoff + Large n_t prior $\Rightarrow$ Stable results

Results: r - n_t

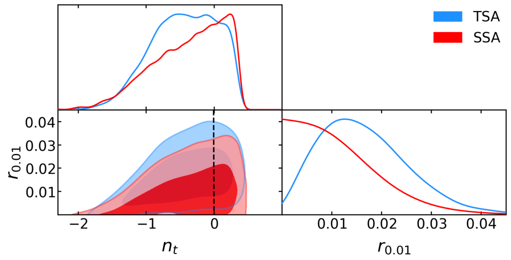

PL18+BK15
PL18+BK18
PL18+BK15+LV18
PL18+BK15+LV21
PL18+BK18+LV21
PL21+BK15
PL21+BK18
PL21+BK18+LV21
PL18+BK15+NANO
PL21+BK18+NANO
$r_{0.01} <$ 0.058
$r_{0.01} <$ 0.033
$r_{0.01} <$ 0.059
$r_{0.01} <$ 0.054
$r_{0.01} <$ 0.033
$r_{0.01} <$ 0.049
$r_{0.01} <$ 0.029
$r_{0.01} <$ 0.028
$r_{0.01} <$ 0.071
$r_{0.01} <$ 0.033
0.00
0.02
0.04
0.06
0.08
0.10
$r_{0.01}$

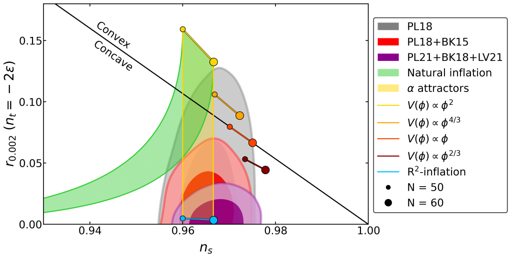

PL18+BK15
PL18+BK18
PL18+BK15+LV18
PL18+BK15+LV21
PL18+BK18+LV21
PL21+BK15
PL21+BK18
PL21+BK18+LV21
PL18+BK15+NANO
PL21+BK18+NANO
-0.22 $< n_t <$ 4.2
-1.02 $< n_t <$ 3.23
-1.0 $< n_t <$ 0.45
-0.97 $< n_t <$ 0.42
-1.02 $< n_t <$ 0.43
-0.6 $< n_t <$ 4.34
-1.28 $< n_t <$ 3.45
-1.37 $< n_t <$ 0.42
0.44 $< n_t <$ 0.83
0.47 $< n_t <$ 0.85
−1
0
1
2
3
4
n_t

GW energy density



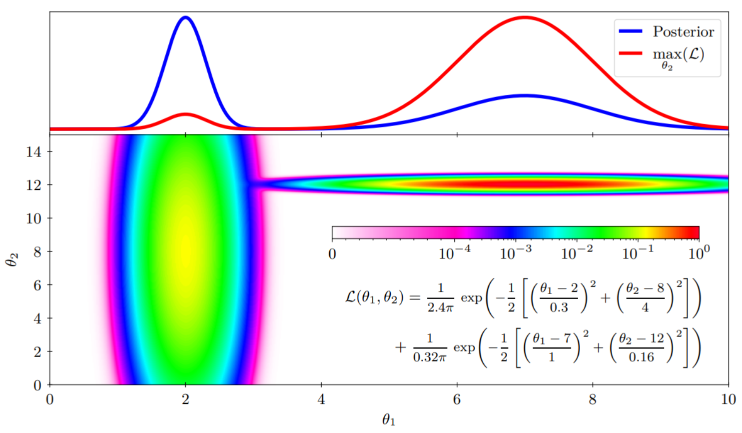
Results: r - n_t and r - n_s





The frequentist way

Volume effects



Credits: A. Nygaard et al. - arXiv:2308.06379

Results? Profile likelihood

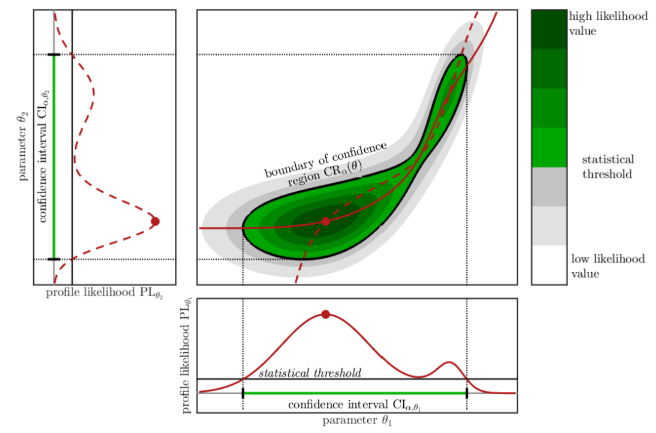
$$\mathcal{L}(D|\vec{\theta}) \to \mathcal{L}(D|\theta_1, \theta_2)$$

$$\theta_1 \in I = [0, 1, 2, 3, \ \ldots]$$

$$\mathsf{PL} = \mathsf{max}[\mathcal{L}(D|\theta_1 \in I, \theta_2)]$$

Particle Data Group - Phys.Rev.D 110 (2024) 3, 030001

Results? Profile likelihood

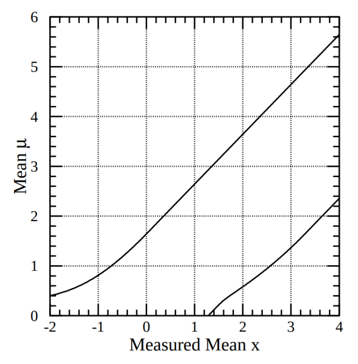


Credits: R. Boiger et al. - arXiv:1604.02894

Frequentist confidence Intervals

$$\int_{x_{\mathrm{low}}}^{x_{\mathrm{up}}} \mathcal{L}(D|\mu)dx = \alpha$$

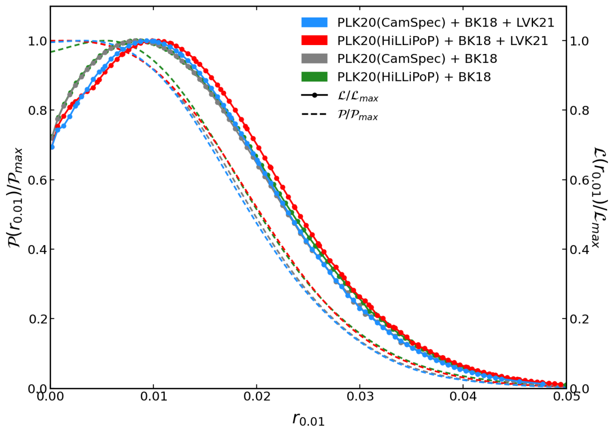
A physical boundary is encoded via the Feldman-Cousins prescription



$$\begin{cases} \lambda(x,\mu) = \dfrac{\mathcal{L}(x|\mu)}{\mathcal{L}(x|\mu_{\mathrm{best}})} \quad \text{with} \quad \mu_{\mathrm{best}} = \max(0,x) \\ \lambda(x_{\mathrm{low}},\mu) = \lambda(x_{\mathrm{up}},\mu) \end{cases}$$

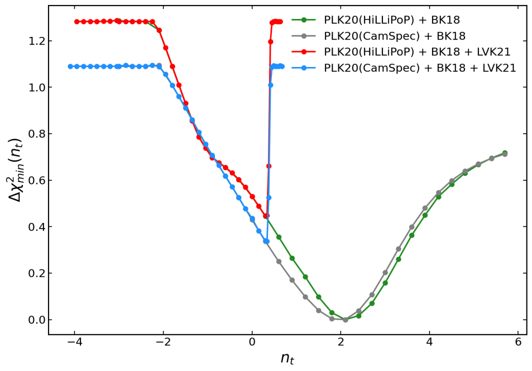
Gary J. Feldman and Robert D. Cousins - arXiv:physics/9711021

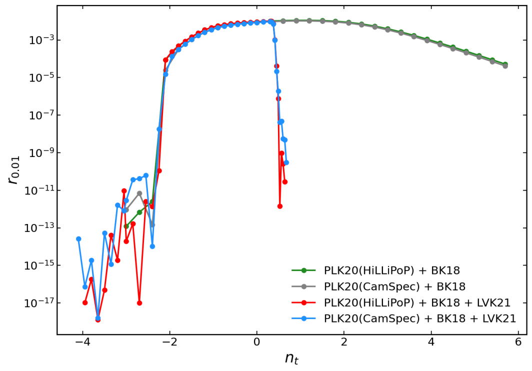
Results: PL on $r_{0.01}$



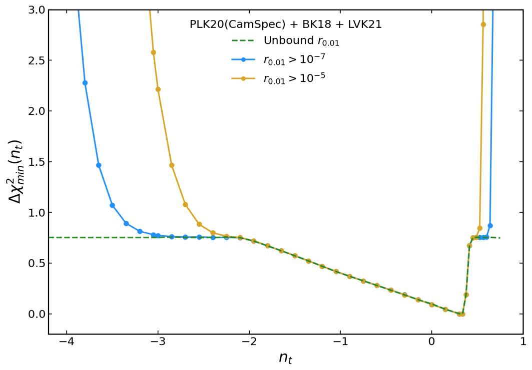
GG et al. - arXiv:2405.04455

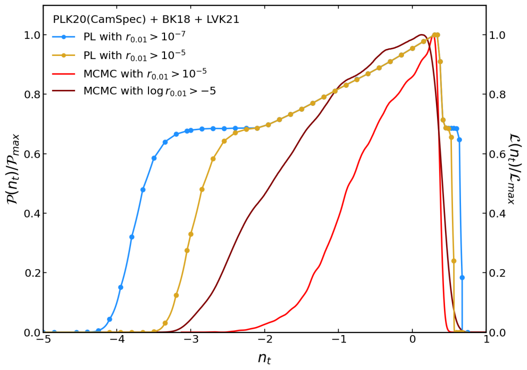
Results: PL on n_t



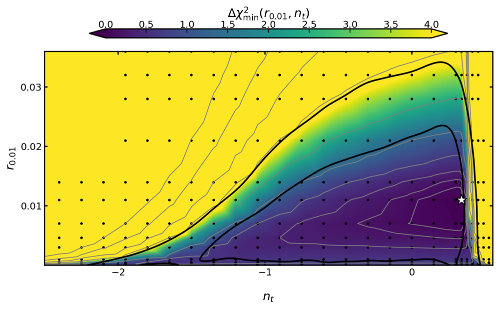


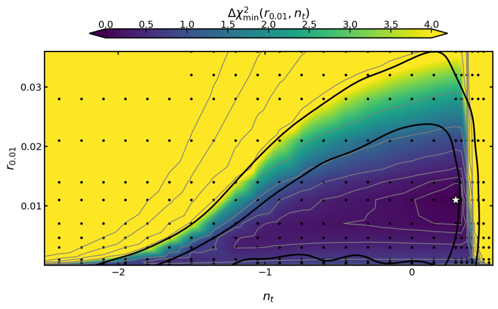
Results: PL on n_t





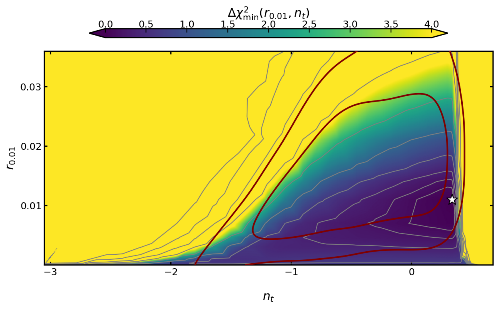
Results: 2D PL on $r_{0.01} - n_t$



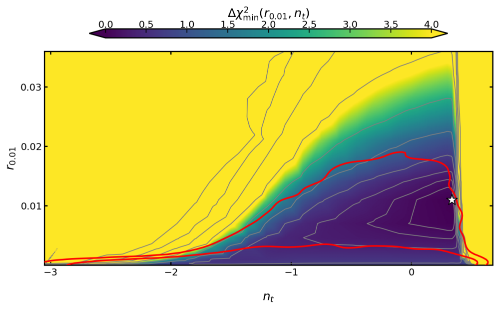


Results: alternative 2D PL on $r_{0.01} - n_t$

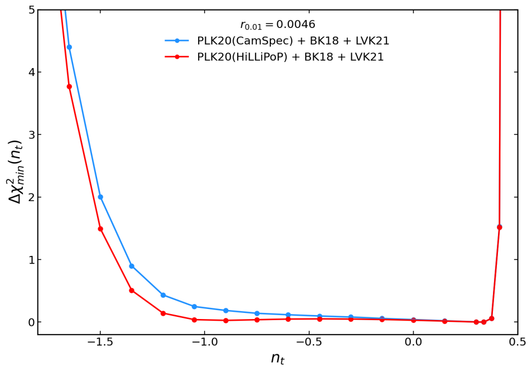
TSA

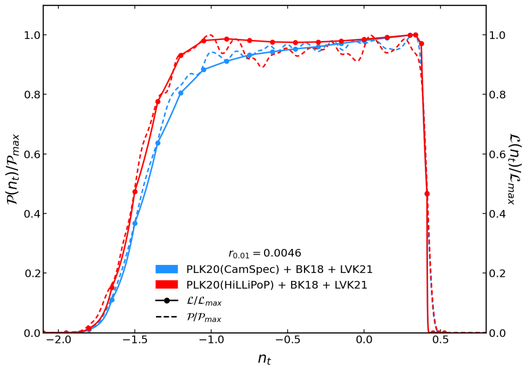


log-uniform

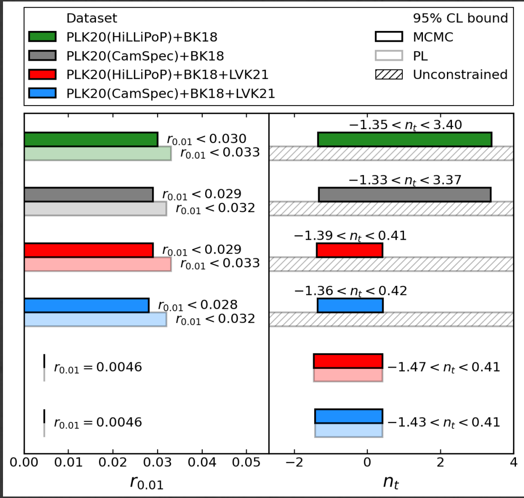


Results: PL on n_t for Starobinsky inflation





Results



Dataset
PLK20(HiLLiPoP)+BK18
PLK20(CamSpec)+BK18
PLK20(HiLLiPoP)+BK18+LVK21
PLK20(CamSpec)+BK18+LVK21
95% CL bound
MCMC
PL
Unconstrained
$r_{0.01} < 0.030$
$r_{0.01} < 0.033$
$r_{0.01} < 0.029$
$r_{0.01} < 0.032$
$r_{0.01} < 0.029$
$r_{0.01} < 0.033$
$r_{0.01} < 0.028$
$r_{0.01} < 0.032$
$r_{0.01} = 0.0046$
$r_{0.01} = 0.0046$
$-1.35 < n_t < 3.40$
$-1.33 < n_t < 3.37$
$-1.39 < n_t < 0.41$
$-1.36 < n_t < 0.42$
$-1.47 < n_t < 0.41$
$-1.43 < n_t < 0.41$
0.00 0.01 0.02 0.03 0.04 0.05
$r_{0.01}$
−2 0 2 4
n_t

Code is available!

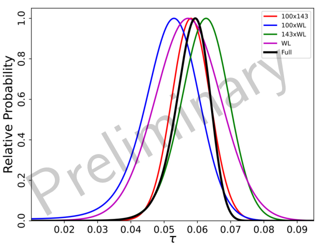
https://github.com/ggalloni/cobaya/tree/profile_sampler

https://github.com/ggalloni/profiler

Ads

Skip Ad ▶I

- New approximation of the likelihood at large scales
- E-mode polarization with WMAP + *Planck*
- Constraining the primordial tensor spectrum in presence of quantum decoherence effects
- Assessing CMB anomalies with 21cm cosmology
- Many others...



Credits: V. Genesini et al. - in prep.

Thank you for your attention!