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Extracting parity-violating gravitational waves from the large-scale structure of the universe

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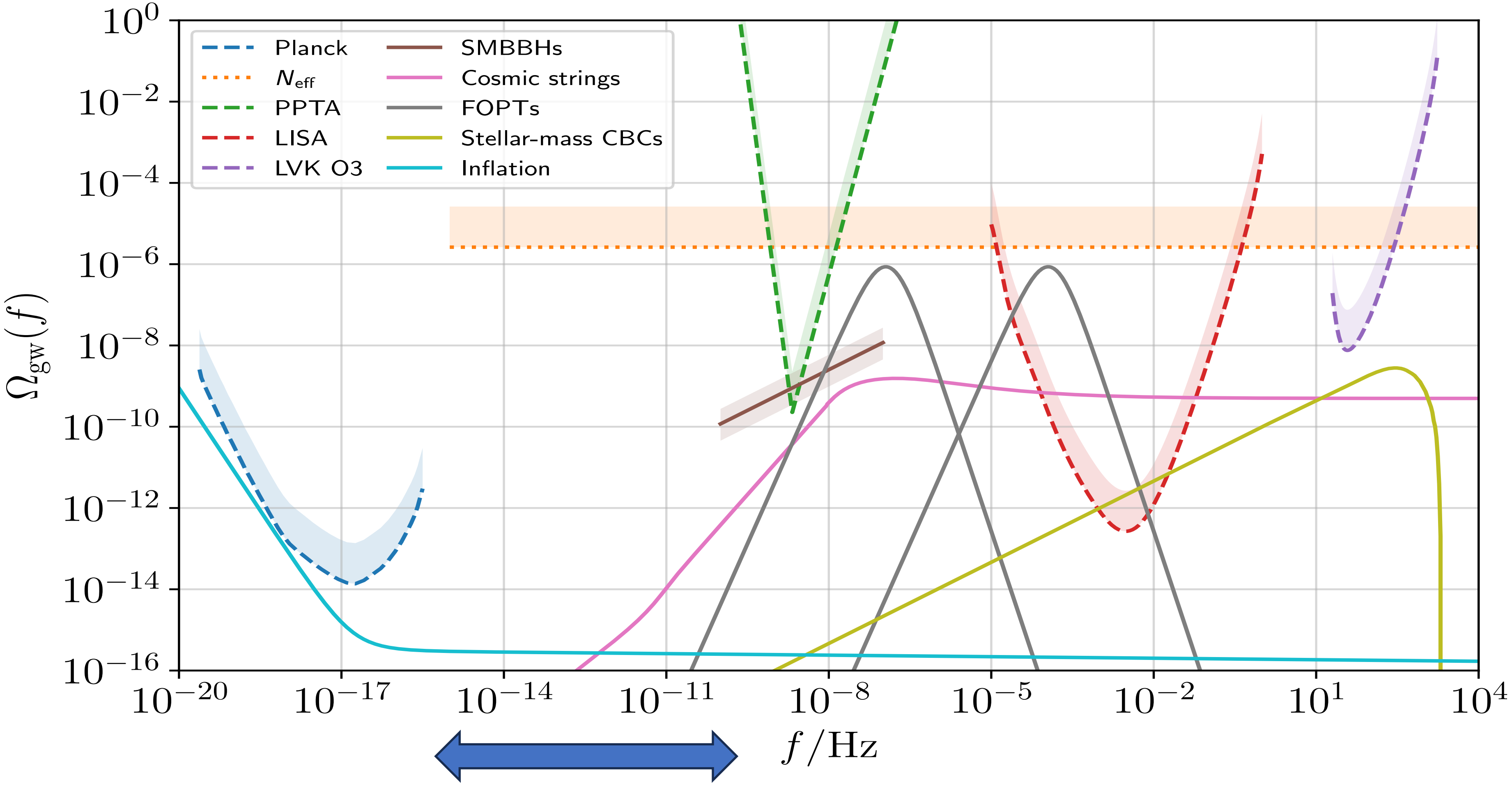
based on TO & M. Sasaki, JCAP 10 (2024) 060 [arXiv:2405.04210]

Multi-frequency Gravitational Wave Cosmology



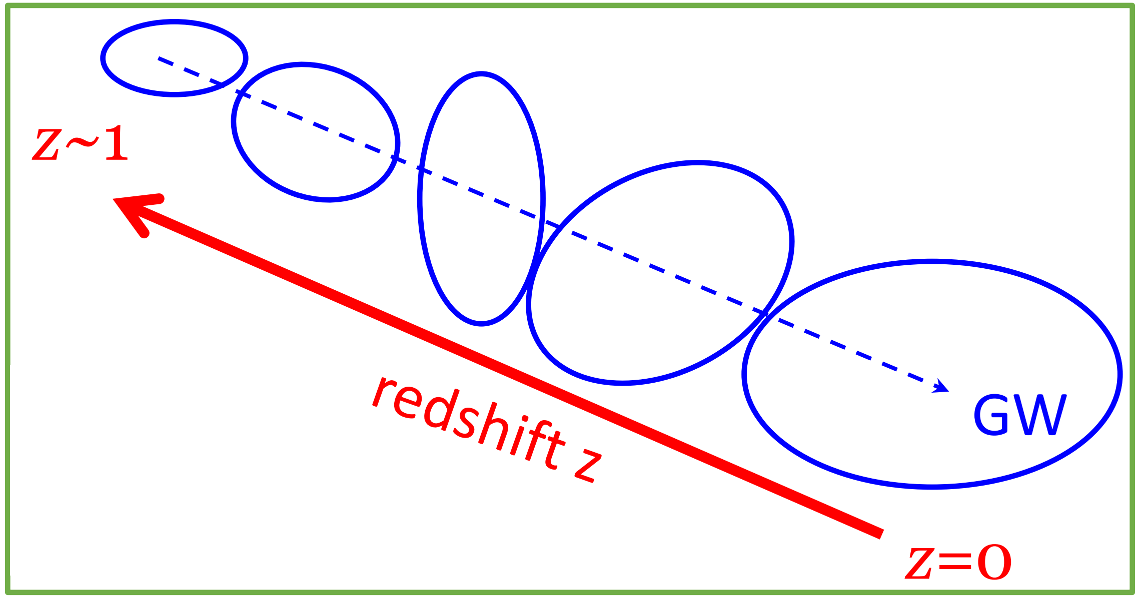
10^{-16} – 10^{-10} Hz: empty window

Renzini et al (2022)

We focus on λ = 1 – 100 Mpc (f = 10^{-14} – 10^{-16} Hz)

GWs = tidal force = tidal deformation

- GWs are locally identical to the scalar (Newtonian) tidal force
- The difference is its space-time dependence



- Different correlation functions
  - In linear theory, Newton potential Ψ(t,x) is almost constant in time:

$$\langle \Psi_k(t)\Psi_k(t')\rangle \sim P_\Psi(k)$$

(time-independent)

GWs propagate:

$$\langle h_k(t)h_k(t')\rangle \sim P_h(t,t';k)$$

(time-dependent)

Tidal force field

- metric perturbation

$$ds^2 = -a^2(\eta)\Big([1+2\Psi(\eta,\mathbf{x})]d\eta^2 + \{[1-2\Psi(\eta,\mathbf{x})]\delta_{ij} + h_{ij}^{\mathrm{TT}}(\eta,\mathbf{x})\}dx^i dx^j\Big)$$

- tidal force field

$$\delta R^k{}_{0j0}(\eta,\mathbf{x}) = \left[\Psi''(\eta,\mathbf{x}) + 2\mathcal{H}(\eta)\Psi'(\eta,\mathbf{x}) + \frac{1}{3}\nabla^2\Psi(\eta,\mathbf{x})\right]\delta_j^k + \delta^{ki}F_{ij}(\eta,\mathbf{x})$$

- dimensionless tidal force field

$$f_{ij}(\eta,\mathbf{x}) \equiv \frac{F_{ij}(\eta,\mathbf{x})}{4\pi G\bar{\rho}(\eta)a^2} = s_{ij}(\eta,\mathbf{x}) + t_{ij}(\eta,\mathbf{x})$$

$$s_{ij}(\eta,\mathbf{x}) \equiv \frac{1}{4\pi G\bar{\rho}a^2}\left(\partial_i\partial_j - \frac{1}{3}\delta_{ij}\nabla^2\right)\Psi(\eta,\mathbf{x}) \quad \cdots \text{ scalar}$$

$$t_{ij}(\eta,\mathbf{x}) \equiv -\frac{1}{8\pi G\bar{\rho}a^2}\left[h''_{ij}(\eta,\mathbf{x}) + \mathcal{H}(\eta)h'_{ij}(\eta,\mathbf{x})\right] \quad \cdots \text{ tensor}$$

- tensor Fourier component

$$\boxed{t_{ij}(\eta,\mathbf{k}) = \frac{k^2}{3\mathcal{H}^2}h_{ij}(\eta,\mathbf{k})}$$

for $k \gg \mathcal{H}$

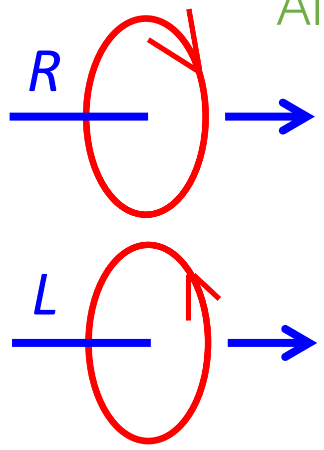
L-R helicity (polarization) decomposition

Creminelli+ (2014),
Alexander & Martin (2005)

- define * operation

$$X_{ij} \longrightarrow {}^{*}X_{ij}(\eta,\mathbf{x}) \equiv \frac{1}{2}\left(\epsilon_i{}^{mn}\partial_m X_{nj} + \epsilon_j{}^{mn}\partial_m X_{ni}\right)$$

($X = s$ for scalar, $X = h$ for tensor)



then,

$$h_{ij}(\eta,\mathbf{k}) = e_{ij}^{(R)}(\hat{\mathbf{k}})h_{(R)}(\eta,\mathbf{k}) + e_{ij}^{(L)}(\hat{\mathbf{k}})h_{(L)}(\eta,\mathbf{k})$$

$$\longrightarrow {}^{*}h_{ij}(\eta,\mathbf{k}) = \underline{k}\left[e_{ij}^{(R)}(\hat{\mathbf{k}})h_{(R)}(\eta,\mathbf{k}) - e_{ij}^{(L)}(\hat{\mathbf{k}})h_{(L)}(\eta,\mathbf{k})\right]$$

except for the factor k, * operation transforms R → R, L → – L

* operation can be used to detect parity violation (PV) in GWs

2-point functions

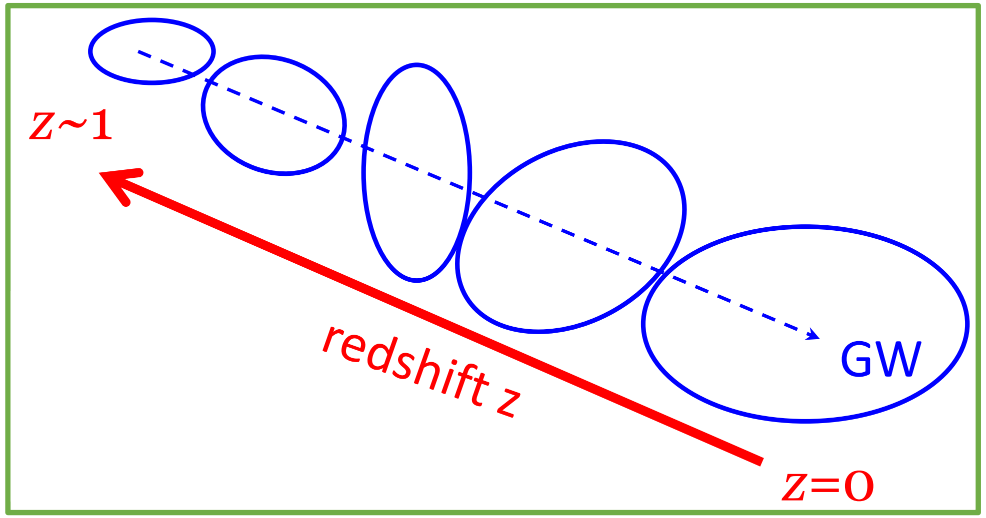
$$\langle h_{(\lambda)}(\eta,\mathbf{k})h_{(\lambda)}(\eta',\mathbf{k}')\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')P_{(\lambda)}(\eta,\eta',k) \qquad \lambda = R, L$$

$$\langle {}^*h(\eta,\mathbf{k})h(\eta',\mathbf{k}')\rangle = (2\pi)^3\delta(\mathbf{k}+\mathbf{k}')\, k\, \chi(\eta,\eta',k) P_{(R+L)}(\eta,\eta',k)$$

$$\chi(\eta,\eta',k) \equiv \frac{P_{(R)}(\eta,\eta',k) - P_{(L)}(\eta,\eta',k)}{P_{(R+L)}(\eta,\eta',k)}$$: degree of PV

- cosmological GW 2-pt functions exhibit unique time-dependence

$$P_{(\lambda)}(\eta,\eta',k) = \frac{a_0^2}{a(\eta)a(\eta')}\boxed{\cos[k(\eta-\eta')]}P_{(\lambda)}(k) \qquad \text{for } k\eta \gg 1$$



How can we probe cosmological GW background?

⇩

Intrinsic Alignment (IA) of galaxies

Effects of GWs on large-scale structure

- At 1st order, the density perturbation is not affected by GWs
- 2nd-order density field induced by the coupling between GWs and scalar tidal fields :

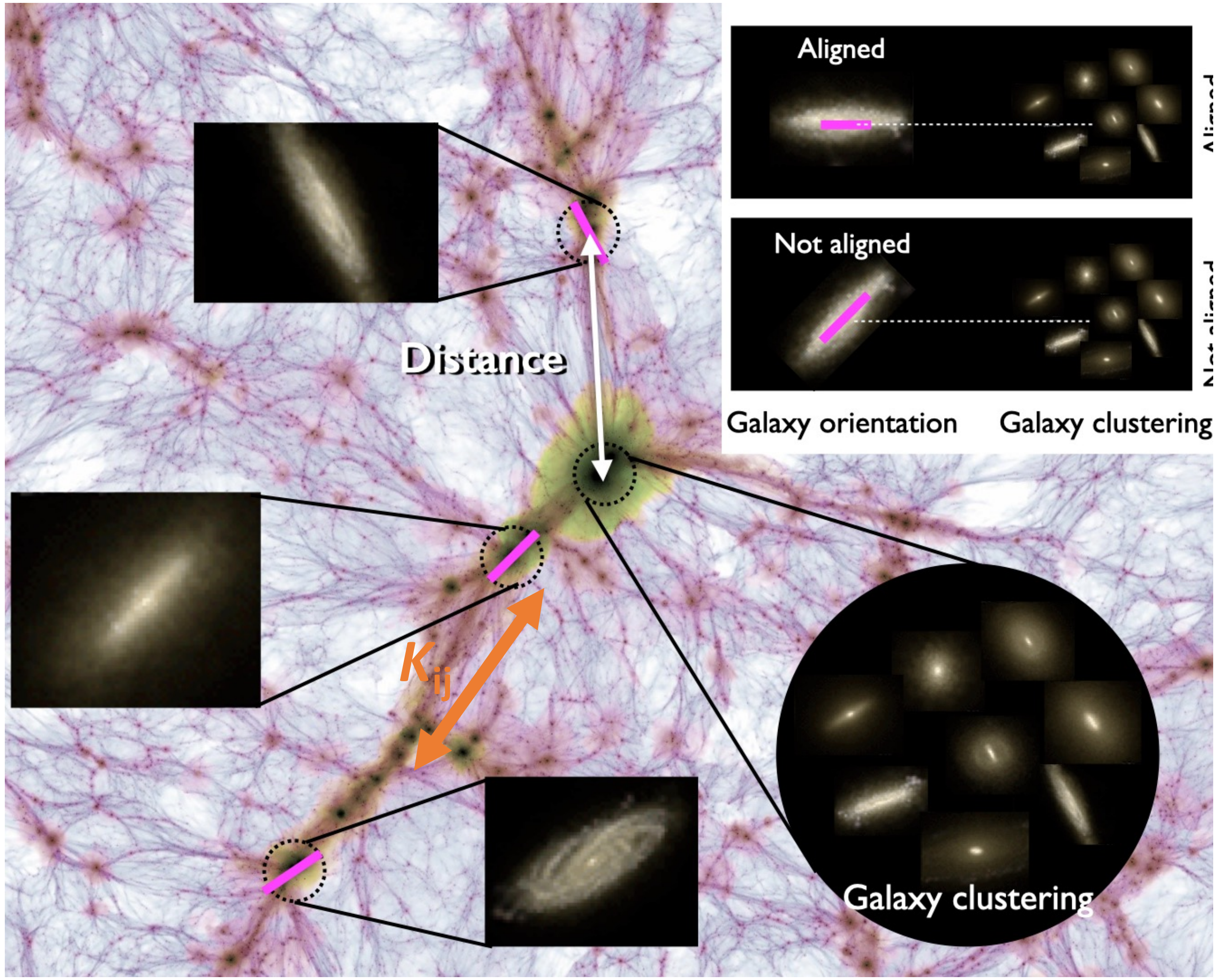
$$\delta^{(2)}(\eta) = \underbrace{\alpha(k_L,\eta)}_{\text{k-dependent}}\underbrace{h_{ij}^{\text{long}}(k_L,\eta_0)}_{\text{GWs at init time}}\underbrace{\frac{\partial_i\partial_j}{\partial^2}\delta_{\text{short}}^{(1)}(\eta)}_{\text{Scalar tidal field}}$$

Dai+ (2013), Schmidt+ (2014), Biagetti & Orlando (2020), Akitsu, Li & Okumura (2023)

  - cf. $$\delta^{(2)}(\eta) = \frac{4}{7}\frac{D(\eta)}{D(\eta_0)}\underbrace{\frac{\partial_i\partial_j}{\partial^2}\delta_{\text{long}}^{(1)}(\eta)}_{\text{long mode}}\underbrace{\frac{\partial_i\partial_j}{\partial^2}\delta_{\text{short}}^{(1)}(\eta)}_{\text{short mode}}$$

- Shapes of galaxies, γ_{ij}, are affected by GW at 1st order as an instantaneous response effect

Intrinsic Alignment of galaxies



- (scalar) tidal fields statistically align galaxies

$$\gamma_{ij}^{s} = b_K^{s} s_{ij}$$

$$s_{ij}(\eta, \mathbf{x}) = \left(\frac{\partial_i \partial_j}{\nabla^2} - \frac{1}{3}\delta_{ij} \right) \delta_m(\eta, \mathbf{x})$$

Catelan+ (2000),
Hirata & Seljak (2004),
Okumura & Taruya (2020)

- GWs also align with galaxies

$$\gamma_{ij}^{GW} = b_K^{GW} t_{ij}(\eta, \mathbf{x})$$

$$t_{ij}(\eta, \mathbf{k}) = \frac{k^2}{3\mathcal{H}^2} h_{ij}(\eta, \mathbf{k})$$

Schmidt, Pajer & Zaldarriaga (2014),
Akitsu, Li & Okumura (2023)

Projected tidal force field in 3D

- Observable = projected tidal field

$$\underbrace{e^{(1)} = \hat{x}\,,\; e^{(2)} = \hat{y}}_{\text{basis perpendicular to } n}\,,\; n = \hat{z}$$

basis perpendicular to n

line of sight

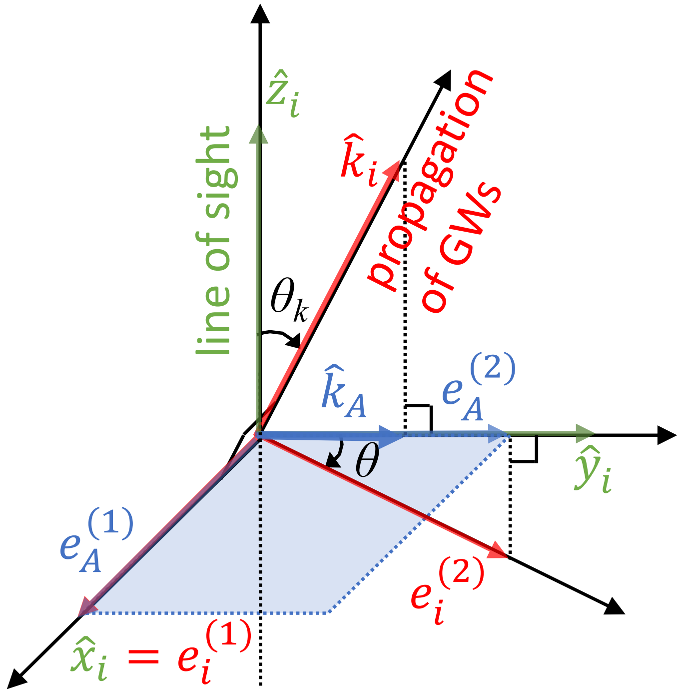
projection tensor: $P^i_m \equiv \delta^i_m - n^i n_m$

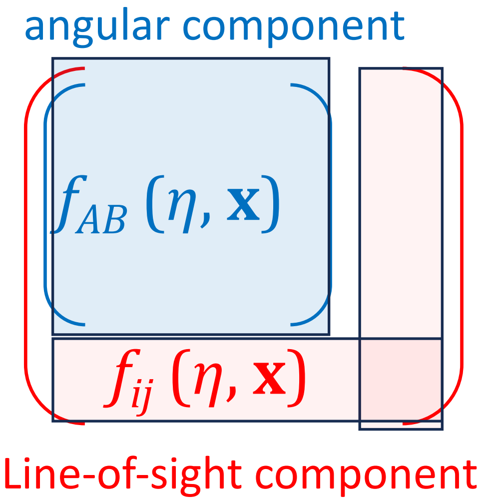
- Projected field in 3D

$$f_{AB} = P^i_A P^j_B f_{ij}$$

$$f^T_{AB} \equiv f_{AB} - \frac{1}{2}\delta_{AB} f^C_C$$

trace part is difficult to detect





- Projected (traceless) tidal force field

$$f^T_{AB} = s^T_{AB} + t^T_{AB}$$

scalar

tensor

How can we extract tensor part?

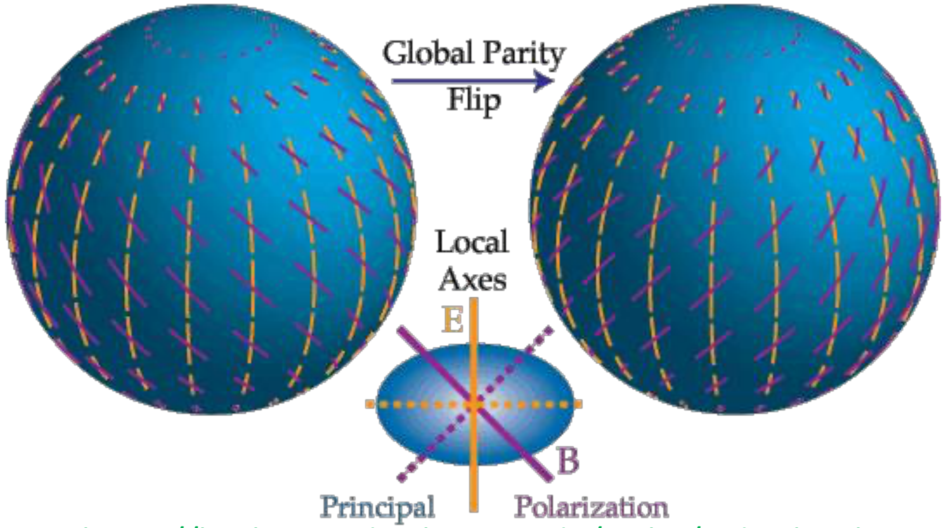
Extracting parity-violating GW components

- Conventional method: E/B-mode decomposition

  E-mode = scalar + tensor
  B-mode = tensor

  non-zero <B^2> = GW
  non-zero <E · B> = PVGW

  E/B-modes: defined globally



https://background.uchicago.edu/~whu/index.html

- New "local" method (complimentary to E/B method)

$$X_A^T(\eta,\mathbf{x}) \equiv \partial^B X_{BA}^T(\eta,\mathbf{x}),$$

$\nabla \cdot X$

$$\boxed{{}^*X_A^T(\eta,\mathbf{x}) \equiv \epsilon^{BC}\partial_B X_{CA}^T(\eta,\mathbf{x})}$$

$\nabla \times X$

$$\boxed{{}^*X^T(\eta,\mathbf{x}) \equiv \partial^A\,{}^*X_A^T(\eta,\mathbf{x})\epsilon^{BC}\partial^A\partial_B X_{CA}^T}$$

$\nabla \cdot \nabla \times X$

→ Pseudo-scalar, free from scalar components

2-point functions of projected tidal fields

- 2-point functions in terms of GW amplitude h

$$t_{ij}(\eta,\mathbf{k}) = \frac{k^2}{3\mathcal{H}^2} h_{ij}(\eta,\mathbf{k})$$

$$\Rightarrow \quad t_A = \frac{k^2}{3\mathcal{H}^2}\underbrace{h_A}_{\text{div } h}\,, \qquad {}^*t_A = \frac{k^2}{3\mathcal{H}^2}\underbrace{{}^*h_A}_{\text{rot } h}\,, \qquad \underbrace{\partial_A {}^*h^A = {}^*h^T}_{\text{div} \cdot \text{rot } h}$$

$$\langle {}^*h^T(\eta,\mathbf{x})\,{}^*h^T(\eta',\mathbf{x}')\rangle = \frac{a_0^2}{a(\eta)a(\eta')}\int \frac{k^6 dk}{2\pi^2} P_h(k)\Gamma_{{}^*h\,{}^*h}(kr, k\Delta\eta, \theta)$$

$$\langle {}^*h^{T,A}(\eta,\mathbf{x})\, h^T_A(\eta',\mathbf{x}')\rangle = \frac{a_0^2}{a(\eta)a(\eta')}\int \frac{k^4 dk}{2\pi^2} P_h(k)\underbrace{\chi(k)}_{\neq 0 \text{ if } \exists \text{PV}}\Gamma_{{}^*h\,h}(kr, k\Delta\eta, \theta)$$

$\Gamma_{XY}(kr, k\Delta\eta, \theta)$ $\cdots$ "Overlap Reduction Function" for $< XY >$

$$r = |\mathbf{x}-\mathbf{x}'|\,,\ \ \Delta\eta = \eta - \eta'\,,\ \ \cos\theta = \frac{\mathbf{x}\cdot\mathbf{x}'}{|\mathbf{x}||\mathbf{x}'|}$$

$\boxed{\text{The above 2-pt functions can be obtained by local operations}}$

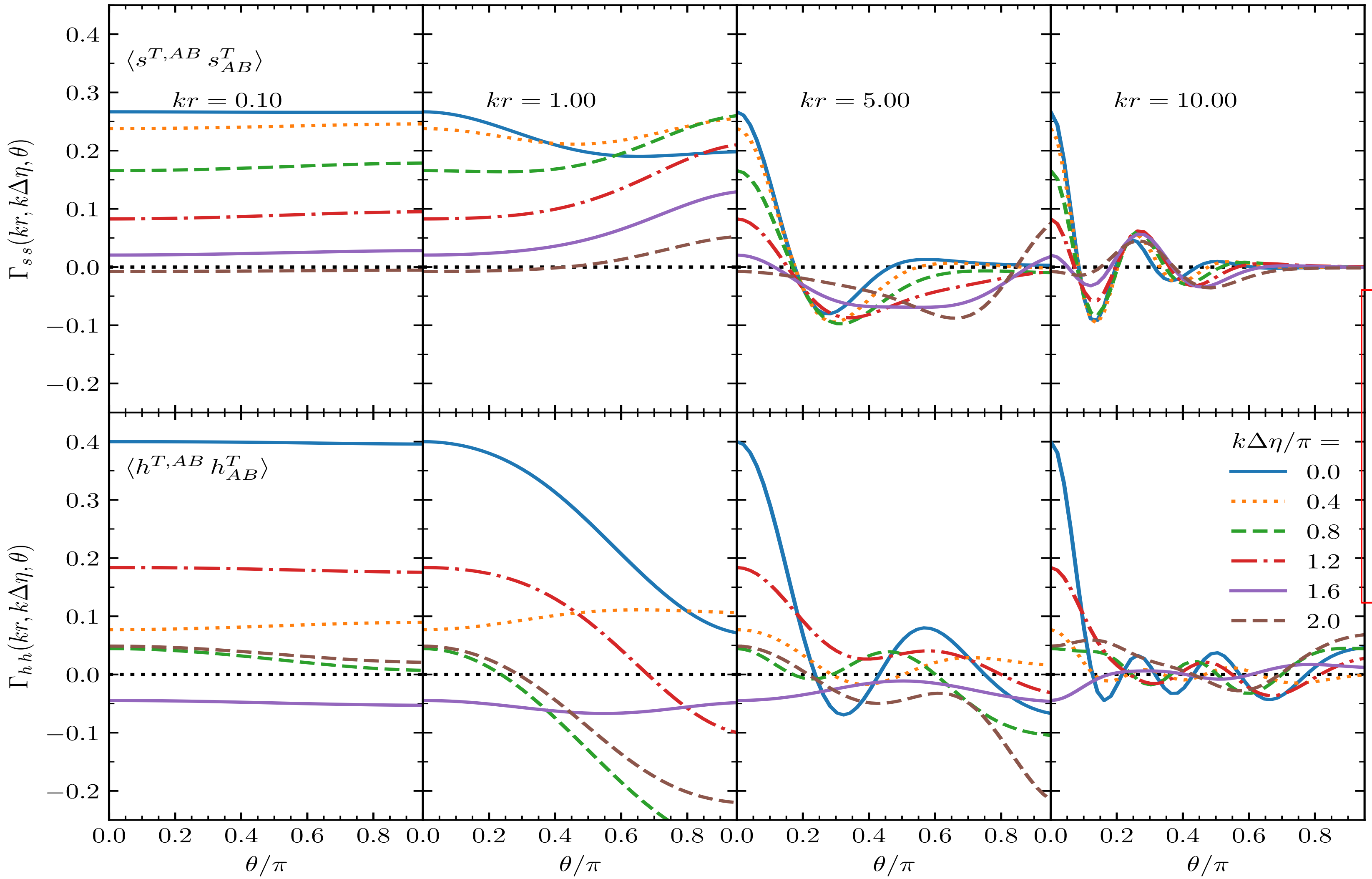
ORFs for scalar and tensor

$\langle s^{T,AB} s^{T}{}_{AB} \rangle$

scalar tidal field

$\langle h^{T,AB} h^{T}{}_{AB} \rangle$

tensor field



The phase shifts because GWs propagate through spacetime.

- In principle, it is possible to distinguish tensor from scalar contributions by template matching applied to the data.

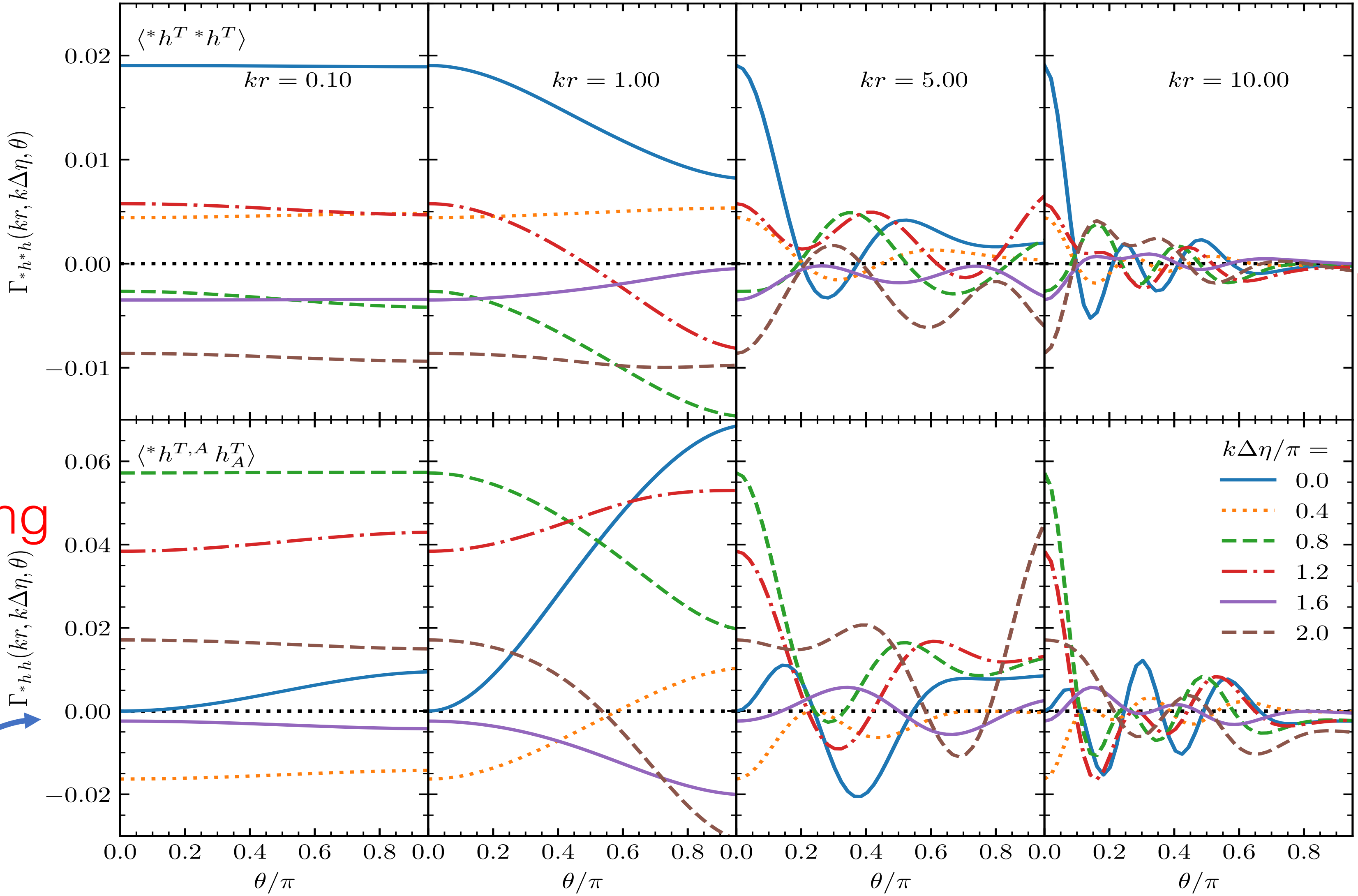
ORFs for <*h*h> and <*hh>

$\langle {}^*h^T \, {}^*h^T \rangle$

GW signal

$\left\langle {}^*h^{T,A} \, h^T{}_A \right\rangle$

parity-violating GW signal



The phase shifts because GWs propagate through spacetime.

PV signal vanishes in the limit $(\theta, k\Delta\eta) \rightarrow (0, 0)$

$$r = |\mathbf{x} - \mathbf{x}'| \,, \ \Delta\eta = \eta - \eta' \,, \ \cos\theta = \frac{\mathbf{x} \cdot \mathbf{x}'}{|\mathbf{x}||\mathbf{x}'|}$$

Polarization degrees of freedom of GWs

Mikura, Okumura & Sasaki (2025) [arXiv:2507.23302]

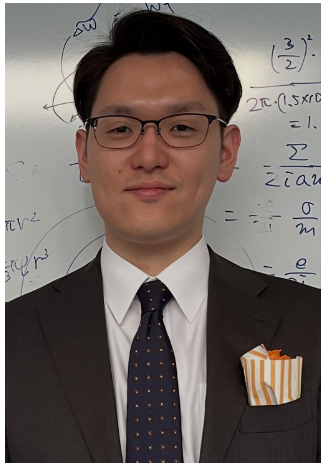
$$h_{ij}(\eta, \boldsymbol{k}) = \sum_{\lambda} e^{(\lambda)}{}_{ij} h_{(\lambda)}(\eta, \boldsymbol{k})$$

$$e^{(\pm 2)}{}_{ij} := \frac{1}{2}\left[\left(e^{(1)}{}_i \otimes e^{(1)}{}_j - e^{(2)}{}_i \otimes e^{(2)}{}_j\right) \mp i\left(e^{(1)}{}_i \otimes e^{(2)}{}_j + e^{(2)}{}_i \otimes e^{(1)}{}_j\right)\right]$$

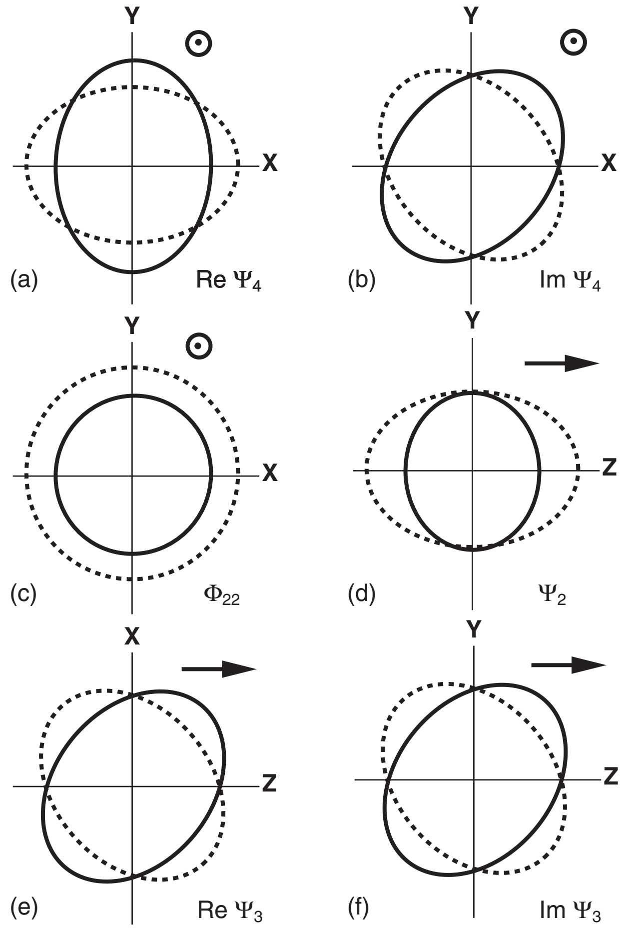
$$e^{(\pm 1)}{}_{ij} := \frac{1}{2}\left[\left(e^{(1)}{}_i \otimes \hat{k}_j + \hat{k}_i \otimes e^{(1)}{}_j\right) \mp i\left(e^{(2)}{}_i \otimes \hat{k}_j + \hat{k}_i \otimes e^{(2)}{}_j\right)\right] ,$$

$$e^{(b)}{}_{ij} := \frac{1}{\sqrt{2}}\left(e^{(1)}{}_i \otimes e^{(1)}{}_j + e^{(2)}{}_i \otimes e^{(2)}{}_j\right) ,$$

$$e^{(\ell)}{}_{ij} := \hat{k}_i \otimes \hat{k}_j .$$



Yusuke Mikura
(ASIAA)



Eardley et al (1973a,b)

We can extract the extra GW polarizations if one can resolve the three-dimensional shapes of galaxies!
--> a direct test of the modified theory of gravity

Summary

- Taking the divergence of the curl of the projected tidal tensor field X : $\nabla \cdot \nabla \times X$, allows us to extract GW signals locally, free from scalar (gravitational potential) tidal force field.

- Taking the cross-correlation between the divergence and the curl of X : $< \nabla f \cdot \nabla \times X >$, we can extract parity-violating GW signals.

- Our method is complementary to the E/B-mode decomposition, as it can extract GWs locally, in principle.

- Calculating the expected signal-to-noise ratio is an ongoing work.

- Placing an upper limit on $\Omega_{\rm GW}$ from the current data of galaxy surveys will be the next step.

Okumura & Sasaki, JCAP 10 (2024) 060 [arXiv:2405.04210]
See also, Mikura, Okumura & Sasaki (2025) [arXiv:2507.23302]

Appendix

Relation to E/B-modes

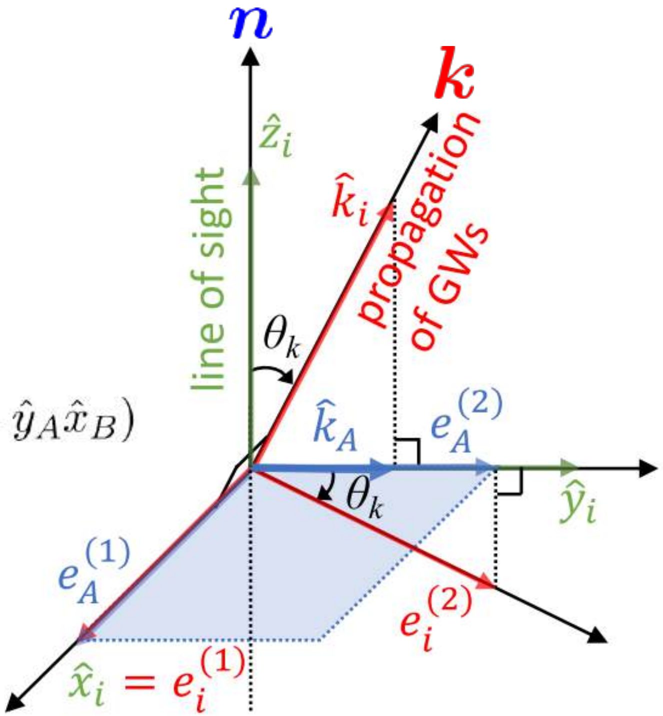
- Fourier modes

$$h_{(E)} = h_{AB}e^{AB}_{(E)} = \frac{1}{\sqrt{2}}\frac{1+\mu_k^2}{2}(h_{(R)} + h_{(L)}),$$

$$h_{(B)} = h_{AB}e^{AB}_{(B)} = \frac{i}{\sqrt{2}}\mu_k(h_{(R)} - h_{(L)}).$$

$$e^{(E)}_{AB} = \frac{1}{\sqrt{2}}\left(\hat{x}_A\hat{x}_B - \hat{y}_A\hat{y}_B\right), \quad e^{(B)}_{AB} = \frac{1}{\sqrt{2}}\left(\hat{x}_A\hat{y}_B + \hat{y}_A\hat{x}_B\right)$$

$$\mu_k \equiv \frac{\boldsymbol{k}\cdot\boldsymbol{n}}{k}$$



- Power spectra

$$P_{EE}(\eta,\eta',\mathbf{k}) = \frac{1}{8}(1+\mu_k^2)^2(P_{(R)} + P_{(L)}) = \frac{1}{8}(1+\mu_k^2)^2 P_h(\eta,\eta',k),$$

$$P_{BB}(\eta,\eta',\mathbf{k}) = \frac{1}{4}\mu_k^2(P_{(R)} + P_{(L)}) = \frac{1}{4}\mu_k^2 P_h(\eta,\eta',k),$$

$$P_{EB}(\eta,\eta',\mathbf{k}) = \frac{i}{4}\mu_k(1+\mu_k^2)(P_{(R)} - P_{(L)}) = \frac{i}{4}\mu_k(1+\mu_k^2)\chi(k)P_h(\eta,\eta',k)$$

Parity-violating GW encoded in the galaxy shape power spectrum

Akitsu, Li & Okumura (2023)

- E/B-mode decomposition

$$E(\mathbf{k},\hat{n}) \pm iB(\mathbf{k},\hat{n}) \equiv {}_{\pm2}\gamma(\mathbf{k},\hat{n})e^{\mp 2i\phi_k}$$

$${}_{\pm2}\gamma(\mathbf{k},\hat{n}) \equiv m^i_{\mp}(\hat{n})m^j_{\mp}(\hat{n})\gamma_{ij}(\mathbf{k})$$

- GW contributions

$$E(\mathbf{k}_L,\hat{n}) = b_K^{\rm GW}(k_L)\frac{1}{8}(1+\mu_L^2)\sum_\lambda h_{(\lambda)}(\mathbf{k}_L)$$

$$B(\mathbf{k}_L,\hat{n}) = -b_K^{\rm GW}(k_L)\frac{i}{2}\mu_L\sum_\lambda \frac{\lambda}{2}h_{(\lambda)}(\mathbf{k}_L)$$

- Power spectra

$$\langle X(\mathbf{k})Y^*(\mathbf{k}')\rangle \equiv (2\pi)^3\delta_{\rm D}^{(3)}(\mathbf{k}-\mathbf{k}')P_{XY}(\mathbf{k})$$

$$P_{EB}(k_L,\mu;z) = \frac{i}{16}\mu_L(1+\mu_L^2)(b_K^{\rm GW}(z))^2\chi(k_L)P_h(k_L)$$
$$\simeq i\frac{49}{256}\mu_L(1+\mu_L^2)\alpha^2(k_L;z)(b_K^{\rm scalar}(z))^2$$
$$\times\chi(k_L)P_h(k_L),$$