Circular Polarization and Doppler-Induced Dipole Anisotropies in the Stochastic Gravitational-Wave Background

Sevgi Karadağ[1], Chia-Hsuan Hsiung[2], Guo-Chin Liu[2], Kin-Wang Ng[1] and Henry Tsz King Wong.[1]

[1]*Institute of Physics, Academia Sinica, Taipei 11529, Taiwan*
[2] *Department of Physics, Tamkang University, New Taipei City 25137, Taiwan*

Outline

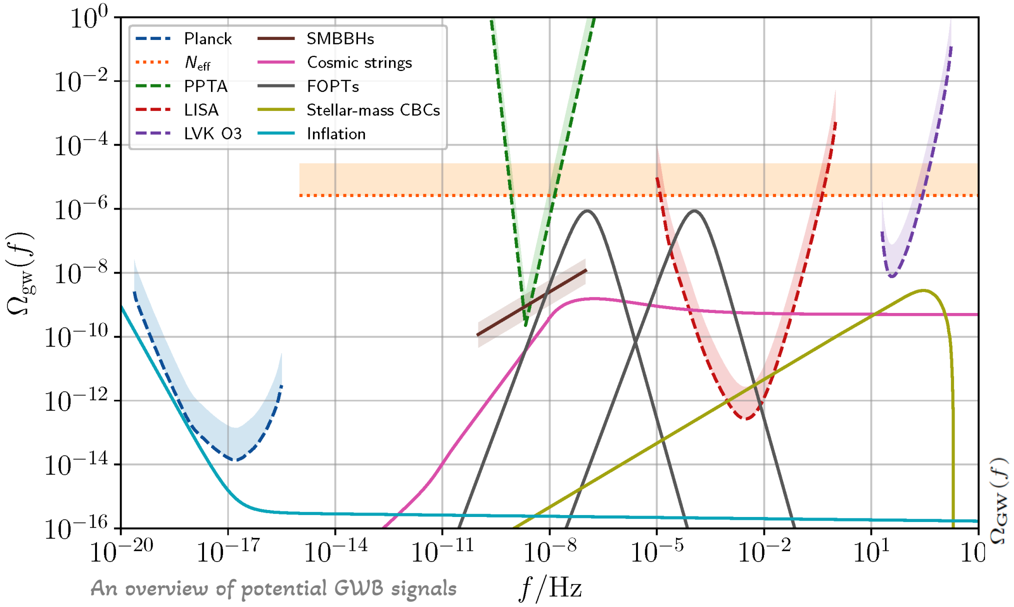
- Introduction
- Motivation & goals
- Theory & methodology
- Results
- Conclusions & …

Introduction

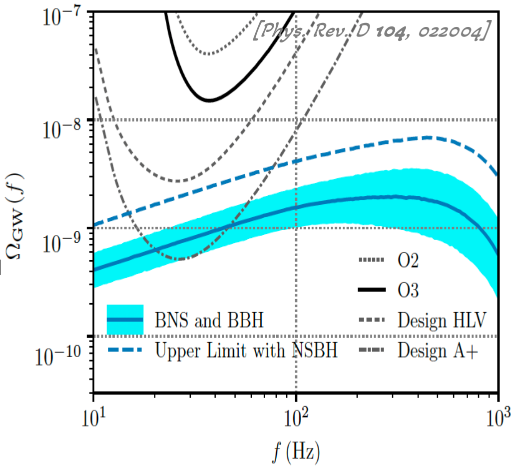
- **SGWB: superposition of many unresolved sources.**
- *Cosmological:* inflation, phase transitions, cosmic strings…
- *Astrophysical:* compact binaries, core-collapse supernovae, magnetars.
- **Statistical descriptions;**
  - "Vanilla": Gaussian-stationary, unpolarized, isotropic.
  - Beyond vanilla: anisotropy, non-Gaussianity, **circular polarization…**

Frequency ranges;

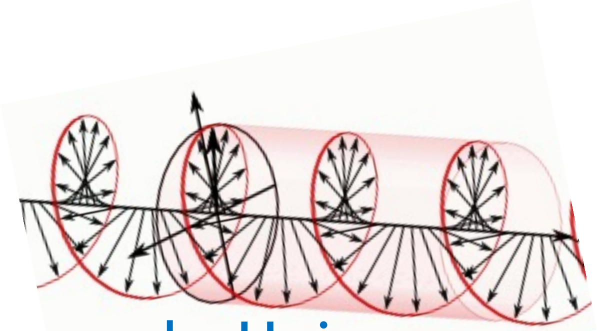
- Ground (**LIGO/Virgo/KAGRA**): ~*20 Hz - kHz.*
- Space **(LISA)**: $\sim 10^{-4} - 10^{-1}$ *Hz.*
- **PTAs**: $\sim 10^{-9} - 10^{-7}$ *Hz.*
- 3G**(CE/ET)**: ~*1-5 Hz to several kHz*



An overview of potential GWB signals across the frequency spectrum [*Galaxies* 2022, *10*(1), 34]



Motivation & goals



- **Why circular polarization?**
  - Parity-violating processes in the early-Universe are expected to induce circular polarization in the SGWB ⟶ a feature predicted by certain inflationary models.
  - detecting such polarization, along with characterizing dipole anisotropies, would provide a great opportunity to enrich the scope of GW astronomy.

- ***Our focus,***
  - ✓ robust pipeline for circularly polarized SGWB ⟶ **simulations / LIGO O1-O3 .**
  - ✓ include Doppler-induced anisotropies in I and V.
  - ✓ parameter estimation (PE) on the **GW energy density parameter** and **polarization parameter.**
  - ✓ Forecasting for 3G (CE&ET) detectors.

Theoretical background (I)

- Power-law spectrum for SGWB,

$$\Omega_{\rm GW}(f) = \Omega_{\rm ref}\left(\frac{f}{f_{\rm ref}}\right)^{\alpha}$$

- **Cross-correlation?!**
  - Combining two (or more) detectors suppresses uncorrelated noise,

    while preserving any common SGWB signal.

$$\tilde{s}_i(f,t) = \tilde{h}_i(f,t) + \tilde{n}_i(f,t)$$
$$\tilde{s}_j(f,t) = \tilde{h}_j(f,t) + \tilde{n}_j(f,t),$$

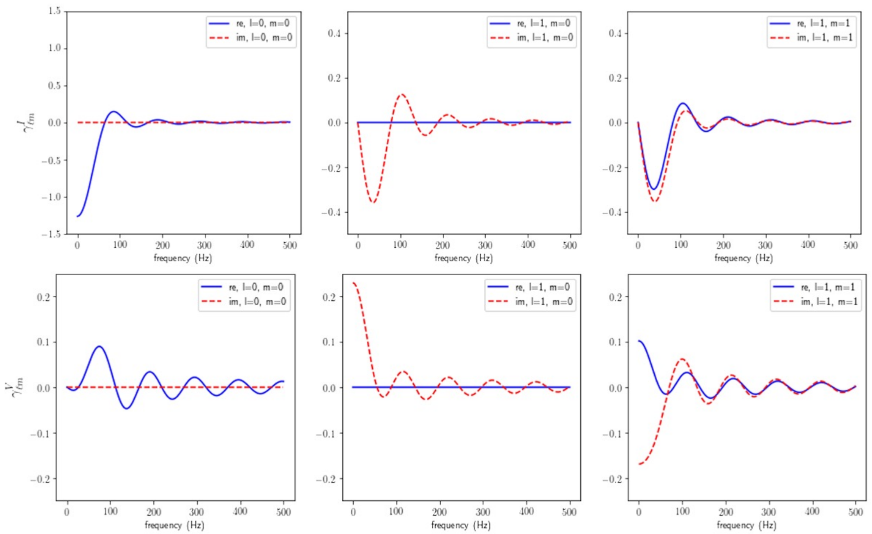
$$\langle \tilde{s}_i^*(f,t)\, \tilde{s}_j(f',t')\rangle = \langle \tilde{h}_i^*(f,t)\, \tilde{h}_j(f',t')\rangle$$

$$\langle C_{ij}(f,t)\rangle = \int d^2\hat{n}\; \mathcal{P}_{AA'}(f,\hat{n})\; \gamma_{ij}(f,t,\hat{n}),$$

- **Overlap reduction functions (ORFs),**
  - ORFs encode geometry, separation, and polarization response of a detector pair:

$$\gamma_{IJ}(f) = \frac{5}{8\pi}\sum_{A}\int_{S^2} d\hat{\boldsymbol{n}} F_I^A(f,\hat{\boldsymbol{n}}) F_J^A(f,\hat{\boldsymbol{n}}) e^{-i2\pi f \hat{\boldsymbol{n}}\cdot(\boldsymbol{x}_I - \boldsymbol{x}_J)}$$

$$\gamma^{I,V}_{\ell m}(f,\mathbf{R}_{\mathbb{D}};\mathbf{R}_{ab}) = (4\pi)\sum_{m'} D^{\ell}_{m'm}(\mathbf{R}_{\mathbb{D}}^{-1})\sum_{\ell_e m_e}\mathbb{D}_0(\mathbf{R}_{ab})\cdot\mathbb{E}^{I,V}_{\ell_e m_e}\sum_{LM} i^L j_L\left(\frac{2\pi f r}{c}\right) Y_{LM}(\hat{r}_0)\left\langle\begin{matrix} L & \ell_e & \ell \\ M & 0m_e & 0m'\end{matrix}\right\rangle,$$



Physical Review D, 103(6), 063528

Theoretical background (II)

Boost transformations:

Under a boost velocity, $\vec{v} = \beta \hat{v}$
GW frequencies transform as,

$$f \to \mathcal{D}^{-1} f \qquad \mathcal{D} = \frac{\sqrt{1-\beta^2}}{1-\beta\, \hat{n}\cdot\hat{v}}$$

(where $\beta = v/c = v \; with \; c = 1$),

➢ **tilt parameters of the isotropic background,**

$$n_\Omega(f) = \frac{d \ln \Omega_{GW}(f)}{d \ln f}, \qquad \alpha_\Omega(f) = \frac{d n_\Omega(f)}{d \ln f}.$$

$$I(f) = \frac{\rho_c}{4\pi f^3}\, \Omega_{\rm GW}(f),$$

$$\rho_{\rm critical} = \frac{3c^2 H_0^2}{8\pi G} \approx 1.6 \times 10^{-8}\; h_{100}^2\; \frac{\rm ergs}{\rm cm^3}$$

$$M(f) = \frac{\beta^2}{6}\left(8 + n_\Omega\,(n_\Omega - 6) + \alpha_\Omega\right),$$
$$D(f) = \beta\,(4 - n_\Omega),$$

➢ Modified GW energy density spectrum acquires a **kinematic modulation** of its monopole and a **dipole term (“kinematic dipole”)** proportional to $D(f)$:

$$\Omega'_{GW}(f, \hat{\boldsymbol{n}}) = \sqrt{4\pi}\, \Omega_{GW}(f) \left[\left(1 + M(f)\right) Y_{00}(\theta,\varphi) + \frac{D(f)}{\sqrt{3}} \left\{ Y_{10}(\theta,\varphi) + \sqrt{2} Y_{11}(\theta,\varphi) + \sqrt{2} Y_{1-1}(\theta,\varphi) \right\} \right]$$

$$= \sqrt{4\pi}\, \Omega_{ref} H_{ref,\alpha}(f) \left[\left(1 + \frac{v^2}{6}\left\{8 + \alpha^2 - 6\alpha + \frac{\alpha^2}{f}\right\}\right) Y_{00}(\theta,\varphi) + \frac{v}{\sqrt{3}}(4-\alpha) \left\{ Y_{10}(\theta,\varphi) + \sqrt{2} Y_{11}(\theta,\varphi) + \sqrt{2} Y_{1-1}(\theta,\varphi) \right\} \right]$$

Methodology

- **correlation model (with monopole + dipole, I and V channels)** for a pair of detectors i and j,

$$C_{ij} = I'\gamma_{ij}^{I;\ell\leq 1} + V'\gamma_{ij}^{V;\ell\leq 1}$$
$$= I'\left[\gamma_{ij}^{I;\ell\leq 1} + \Pi\,\gamma_{ij}^{V;\ell\leq 1}\right]$$
$$\cong \Omega'_{GW}\left[\gamma_{ij}^{I;\ell\leq 1} + \Pi\,\gamma_{ij}^{V;\ell\leq 1}\right]$$

$$\Pi(f) = \frac{V}{I}$$

$$I \geq |V|$$

Boost-dipole coefficients;

$$(\alpha_{\rm sol}, \delta_{\rm sol}) = (167.942 \pm 0.007, -6.944 \pm 0.007)\ \text{deg}.$$

$$\begin{aligned}
\Omega_M &= \Omega_{00}\ \Omega_{ref},\\
&= \sqrt{4\pi}\ \Omega_{ref}\\
[\Omega_D^{10}]^{sol} &= \sqrt{\frac{4\pi}{3}}\cos(\pi/2 - \delta_{sol})\ \Omega_D\\
&= -0.24744\ \Omega_D,\\
[\Omega_D^{11}]^{sol} &= -\sqrt{\frac{2\pi}{3}}\sin(\pi/2 - \delta_{sol})e^{-i\alpha_{sol}}\ \Omega_D\\
&= (1.40489 + 0.30011i)\ \Omega_D,\\
[\Omega_D^{1-1}]^{sol} &= -[\Omega_D^{11}]^{*sol}
\end{aligned}$$

[Physical Review D, 106(8), 082005]

SIGNAL TEMPLATE:

$$C_{ij}(f,t) = \mathcal{H}_{ref}\left(\frac{f}{f_{ref}}\right)^{\beta}\sum_{l\leq 1}\left[\gamma_{ij}^{I}(f,t) + \Pi\gamma_{ij}^{V}(f,t)\right]_{lm}\Omega_{lm}(t)$$

$$\mathcal{H}_{ref} = \left(\frac{3H_0^2}{10\pi^2}f_{ref}^{-3}\right)\Omega_{ref}$$

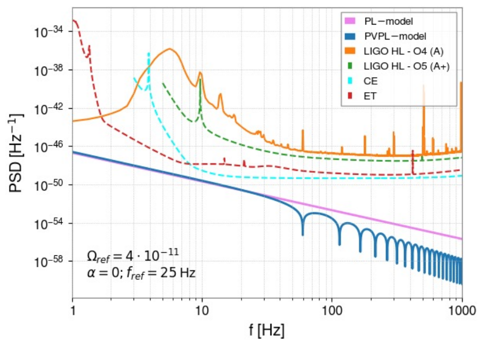
$$\beta = \alpha - 3$$

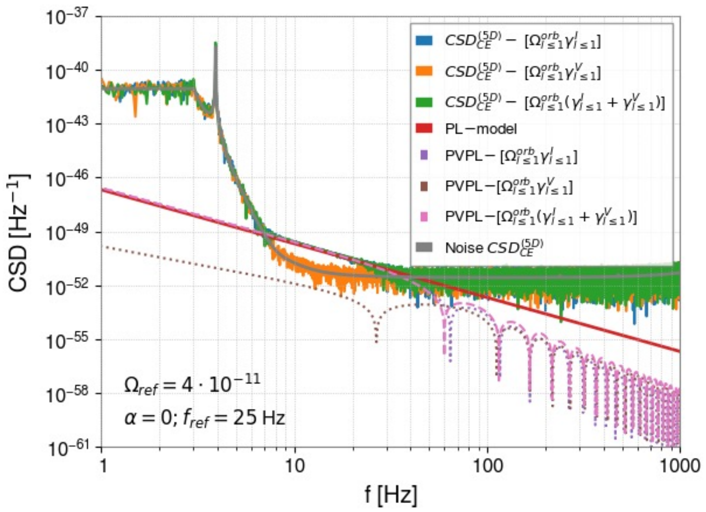
Preliminary results:

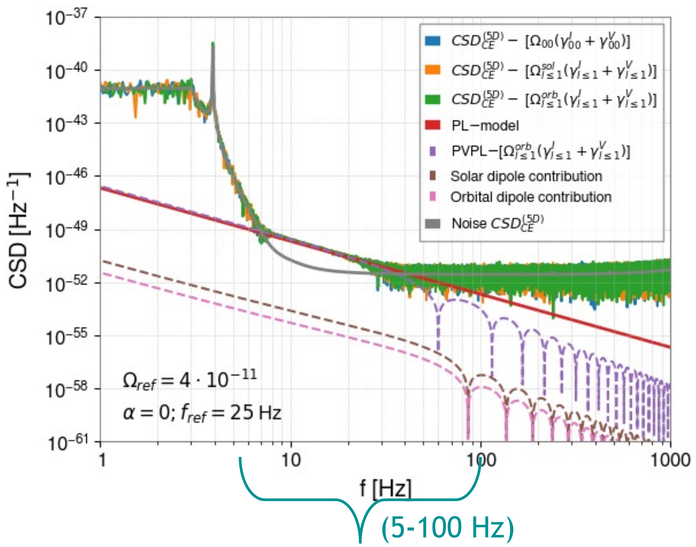
$$p\left(\hat{C}^{ij}_{GW,f}|\Theta\right) = exp\left[-\frac{1}{2}\sum_{ij}\sum_{f}\frac{\hat{C}^{ij}_{GW,f} - C^{ij}_{M}(f|\Theta)}{\sigma^{ij}_{GW,f}}\right]$$

TABLE I. Prior distribution for analysis.

Parameter	Prior
Ω_M	Log-Uniform $[10^{-14}, 10^{-6}]$
Ω_D	Log-Uniform $[10^{-16}, 10^{-8}]$
Π	Uniform $[-1, 1]$





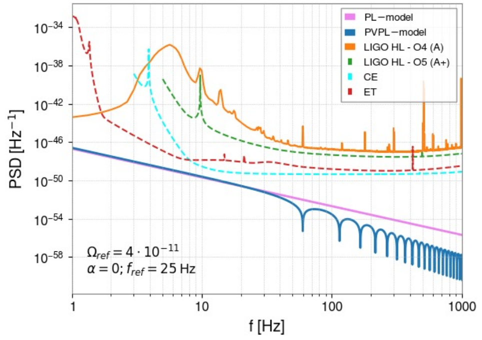


Preliminary results:

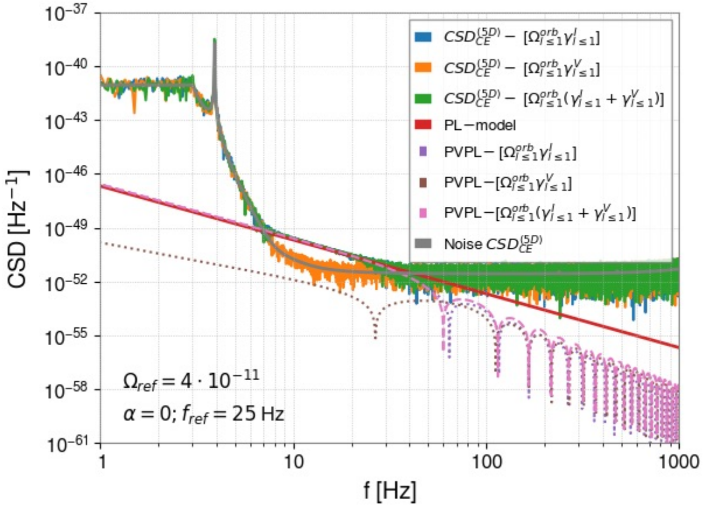
$$p\left(\hat{C}^{ij}_{GW,f}|\Theta\right) = exp\left[-\frac{1}{2}\sum_{ij}\sum_{f}\frac{\hat{C}^{ij}_{GW,f} - C^{ij}_{M}(f|\Theta)}{\sigma^{ij}_{GW,f}}\right]$$

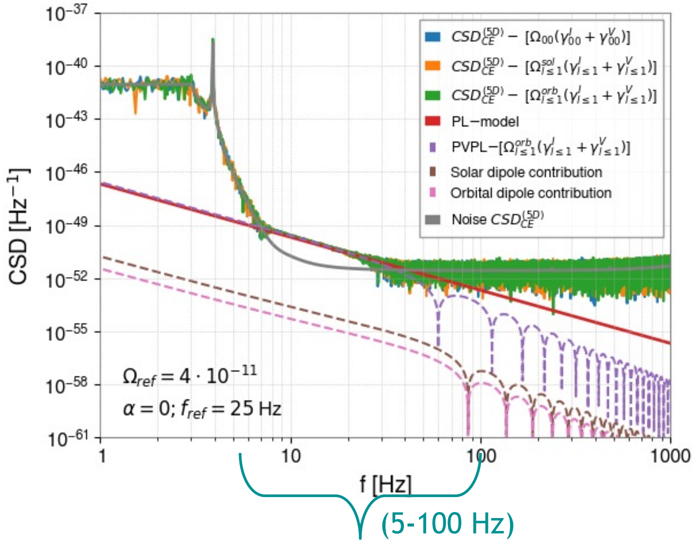
TABLE I. Prior distribution for analysis.

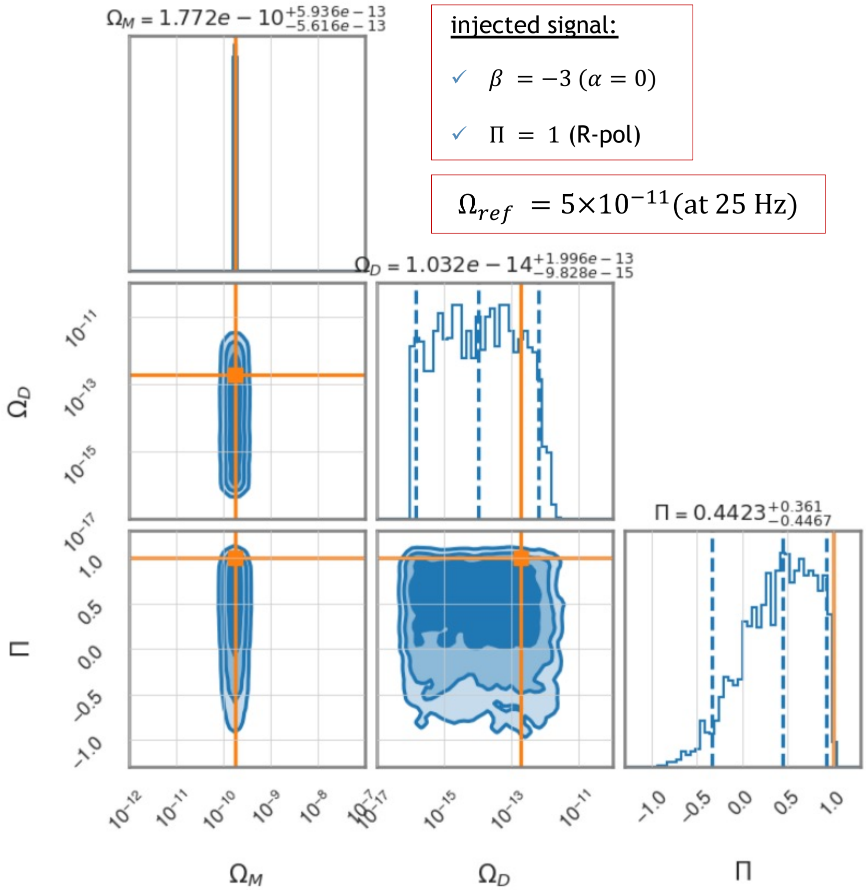
Parameter	Prior
Ω_M	Log-Uniform $[10^{-14}, 10^{-6}]$
Ω_D	Log-Uniform $[10^{-16}, 10^{-8}]$
Π	Uniform $[-1, 1]$



Simulated CE forecast







injected signal:

- ✓ $\beta = -3$ $(\alpha = 0)$
- ✓ $\Pi = 1$ (R-pol)

$\Omega_{ref} = 5\times10^{-11}$(at 25 Hz)

CE1–CE2 forecast for a 5-day run.
Posteriors recover injected the Ω_{ref} and Π.

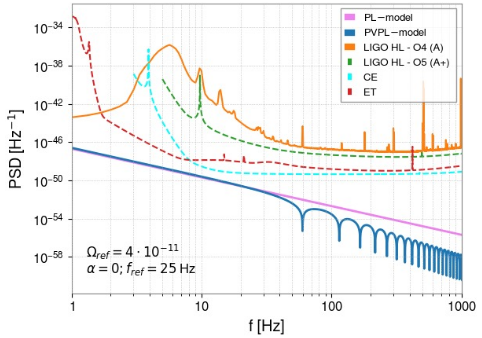
$\alpha = \mathbf{0}$:	Single-Baseline
Ω_M	1.782×10^{-10}
Ω_D	5.997×10^{-13}

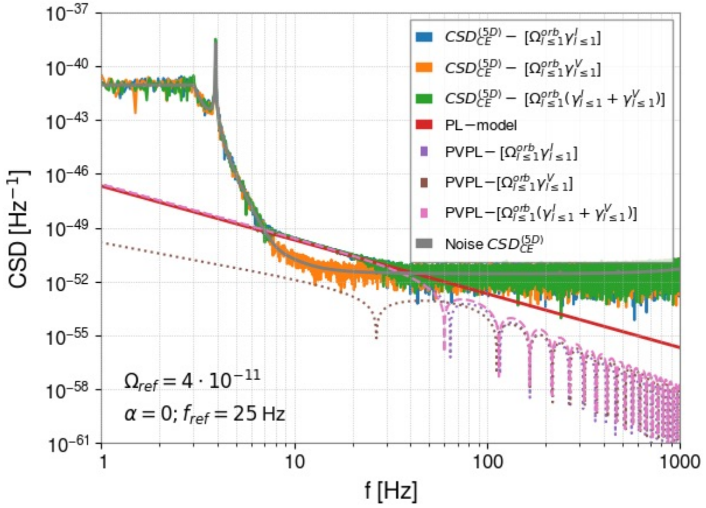
Preliminary results:

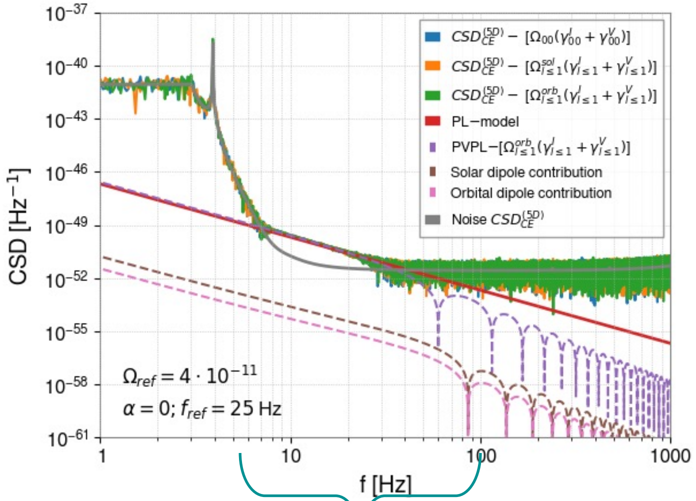
$$p\left(\hat{C}^{ij}_{GW,f}|\Theta\right) = exp\left[-\frac{1}{2}\sum_{ij}\sum_{f}\frac{\hat{C}^{ij}_{GW,f} - C^{ij}_{M}(f|\Theta)}{\sigma^{ij}_{GW,f}}\right]$$

TABLE I. Prior distribution for analysis.

Parameter	Prior
Ω_M	Log-Uniform $[10^{-14}, 10^{-6}]$
Ω_D	Log-Uniform $[10^{-16}, 10^{-8}]$
Π	Uniform $[-1, 1]$

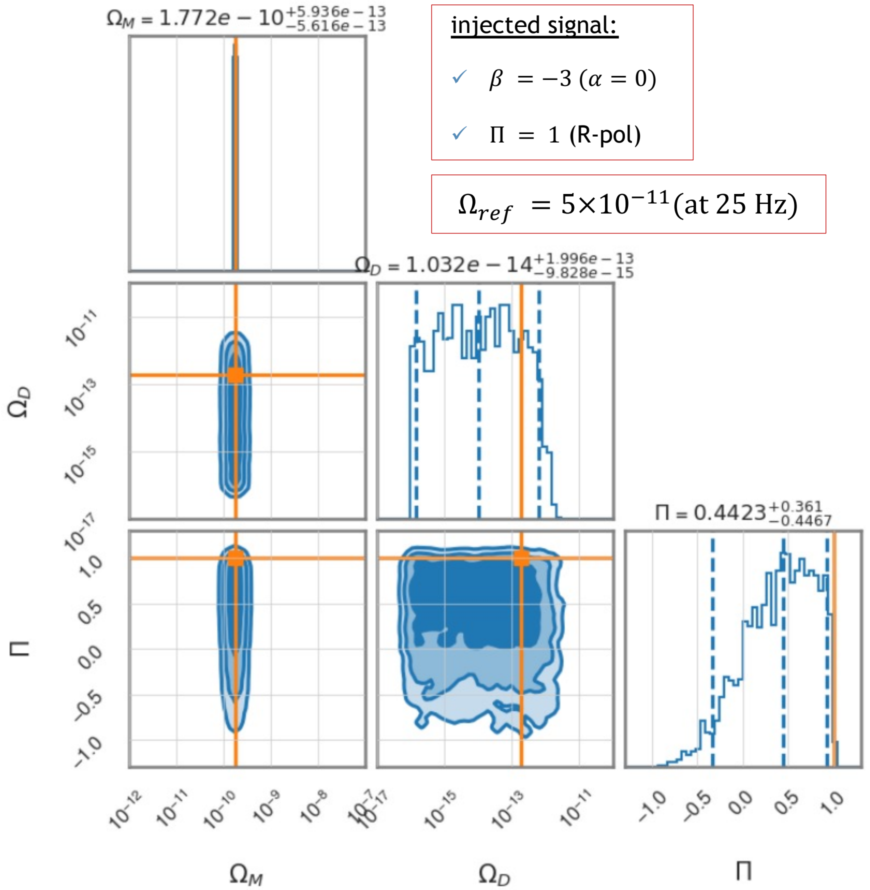






(5-100 Hz)

Simulated CE forecast



injected signal:

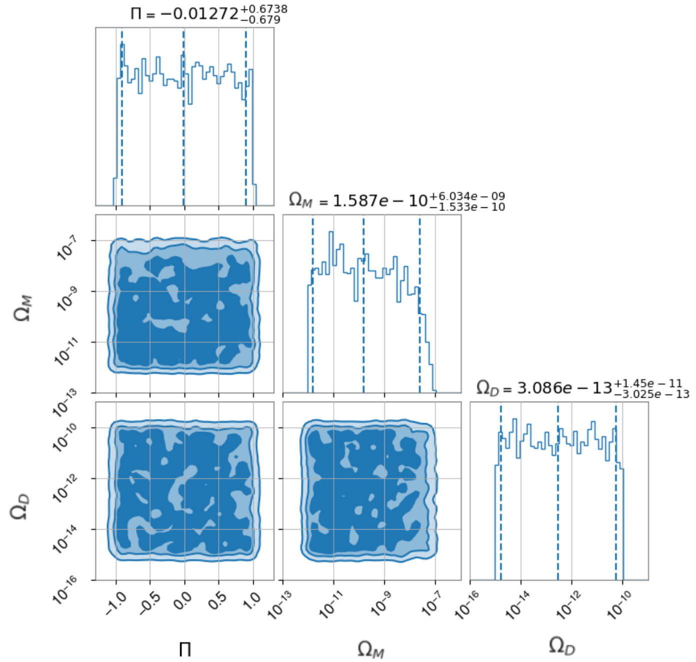
- ✓ $\beta = -3$ ($\alpha = 0$)
- ✓ $\Pi = 1$ (R-pol)

$\Omega_{ref} = 5\times10^{-11}$(at 25 Hz)

CE1–CE2 forecast for a 5-day run.
Posteriors recover injected the Ω_{ref} and Π.

$\alpha = 0$:	Single-Baseline
Ω_M	1.782×10^{-10}
Ω_D	5.997×10^{-13}

LIGO O1-O3 (20-1726 Hz)



Posteriors for constant Π with $\alpha = 0$.
UL @95% CL ➝ $\Omega_M = 2.4\times10^{-8}, \Omega_D = 5.2\times10^{-11}, \Pi = 0.89$ $(cnst.)$

Kinematic expectation
$r_D \sim \frac{\Omega_D}{\Omega_M} \approx 5\times10^{-3}$

Conclusions

- Circularly polarized SGWB is a probe of parity-violating early-Universe physics.
- **What we did?**
  - Built PE on Ω_M, Ω_D and Π & incorporate Doppler-induced dipole anisotropies.
  - No detection in O1-O3 data/set UL .
  - Circularly-polarized SGWB remains unconstrained by O1–O3, but our limits approach the **kinematic dipole regime** ➛ provides a meaningful benchmark for future searches.
  - **Simulations indicate 3G (CE)** with realistic observing time can **constrain or detect** circular polarization.
- **Next!**
  - analysis strategies has to be improved to enhance **sensitivity to the kinematic dipole anisotropies**.
  - **frequency-dependent circular-polarization fraction** $\Pi \rightarrow \Pi(f)$.
  - Multi baseline combination + longer observing time

Questions, comments, or suggestions are welcome…

References

[1] Romano, J. D., & Cornish, N. J. (2017). Detection methods for stochastic gravitational-wave backgrounds: a unified treatment. *Living reviews in relativity*, *20*, 1-223.

[2] Cusin, G., & Tasinato, G. (2022). Doppler boosting the stochastic gravitational wave background. *Journal of Cosmology and Astroparticle Physics*, *2022*(08), 036.

[3] Chung, A. K. W., Jenkins, A. C., Romano, J. D., & Sakellariadou, M. (2022). Targeted search for the kinematic dipole of the gravitational-wave background. *Physical Review D*, *106*(8), 082005.

[4] Chu, Y. K., Liu, G. C., & Ng, K. W. (2021). Spherical harmonic analysis of anisotropies in polarized stochastic gravitational-wave background with interferometry experiments. *Physical Review D*, *103*(6), 063528.

[5] Crowder, S. G., Namba, R., Mandic, V., Mukohyama, S., & Peloso, M. (2013). Measurement of parity violation in the early universe using gravitational-wave detectors. *Physics Letters B*, *726*(1-3), 66-71.