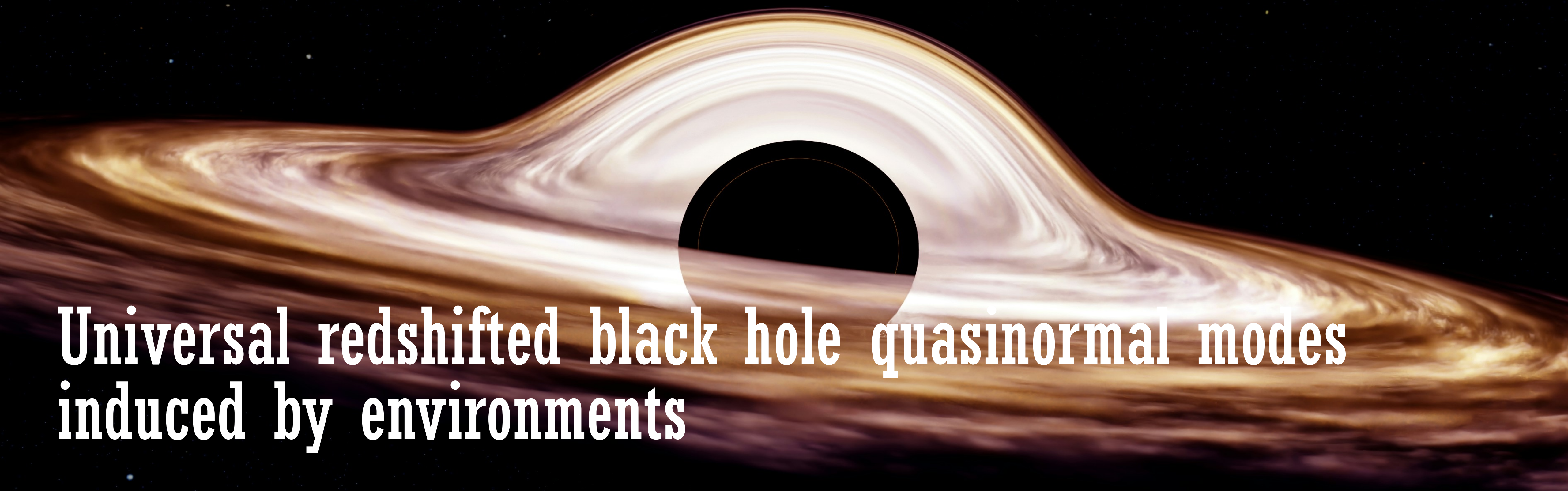

Universal redshifted black hole quasinormal modes induced by environments

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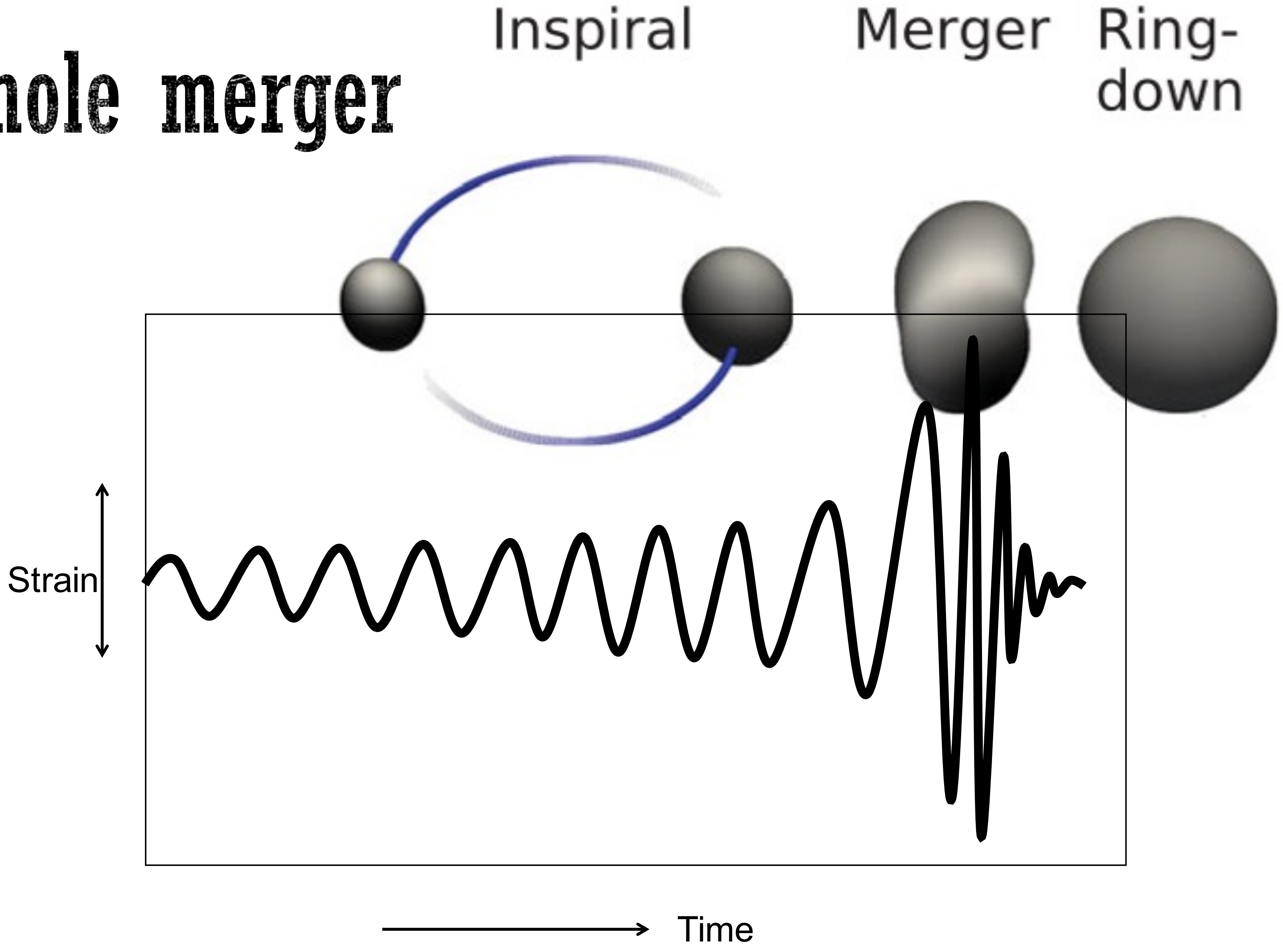
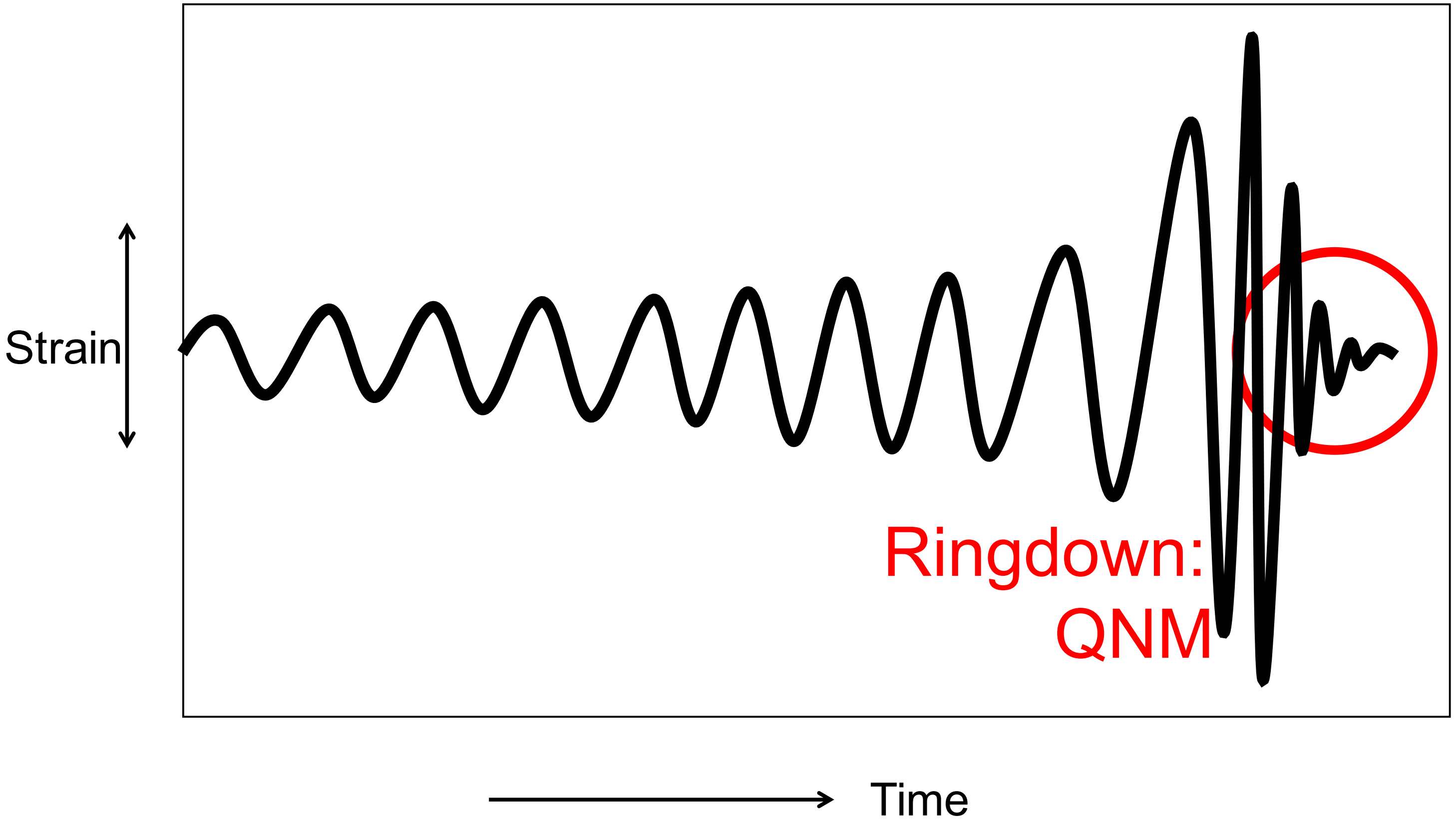
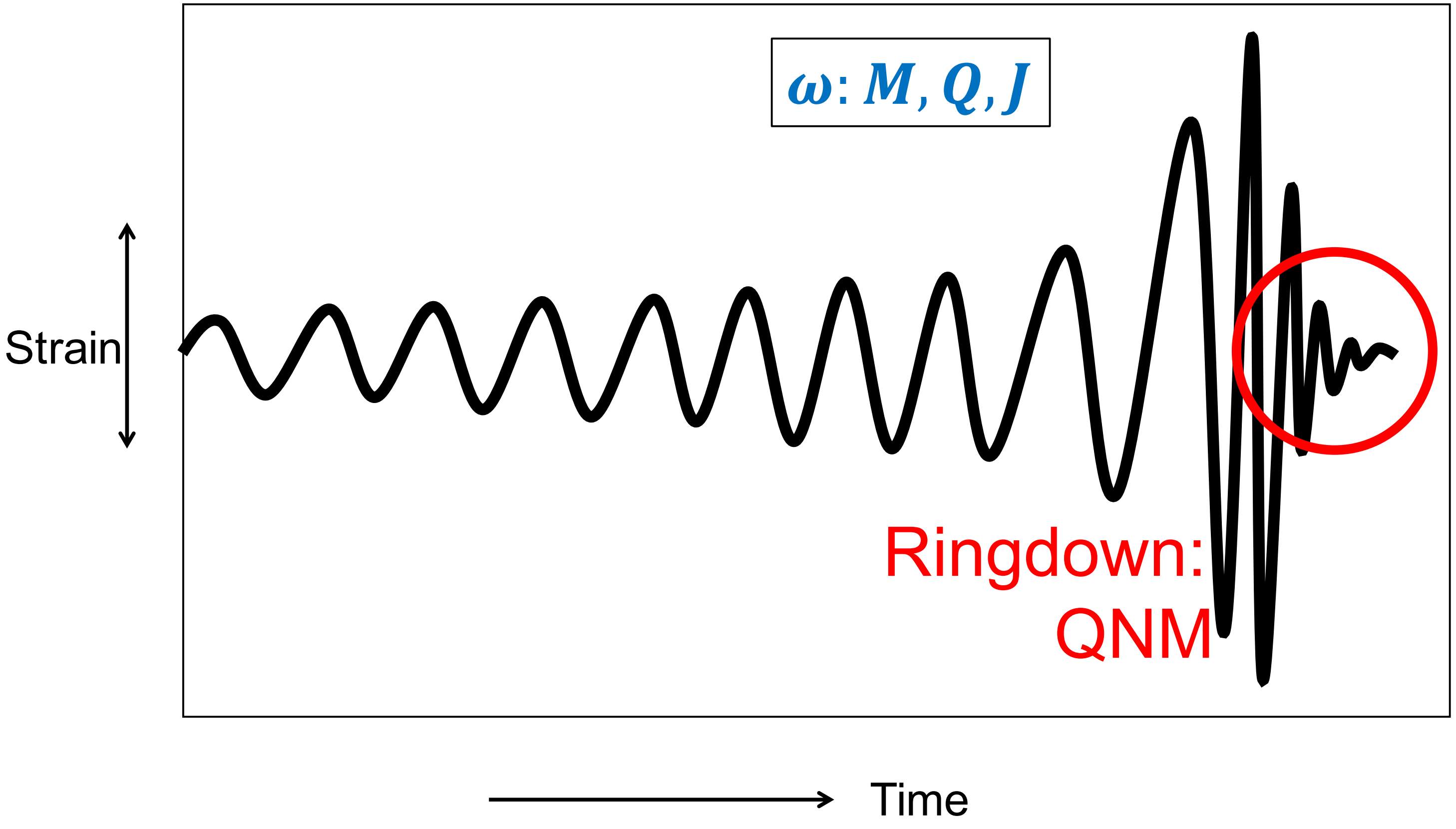
Black hole merger



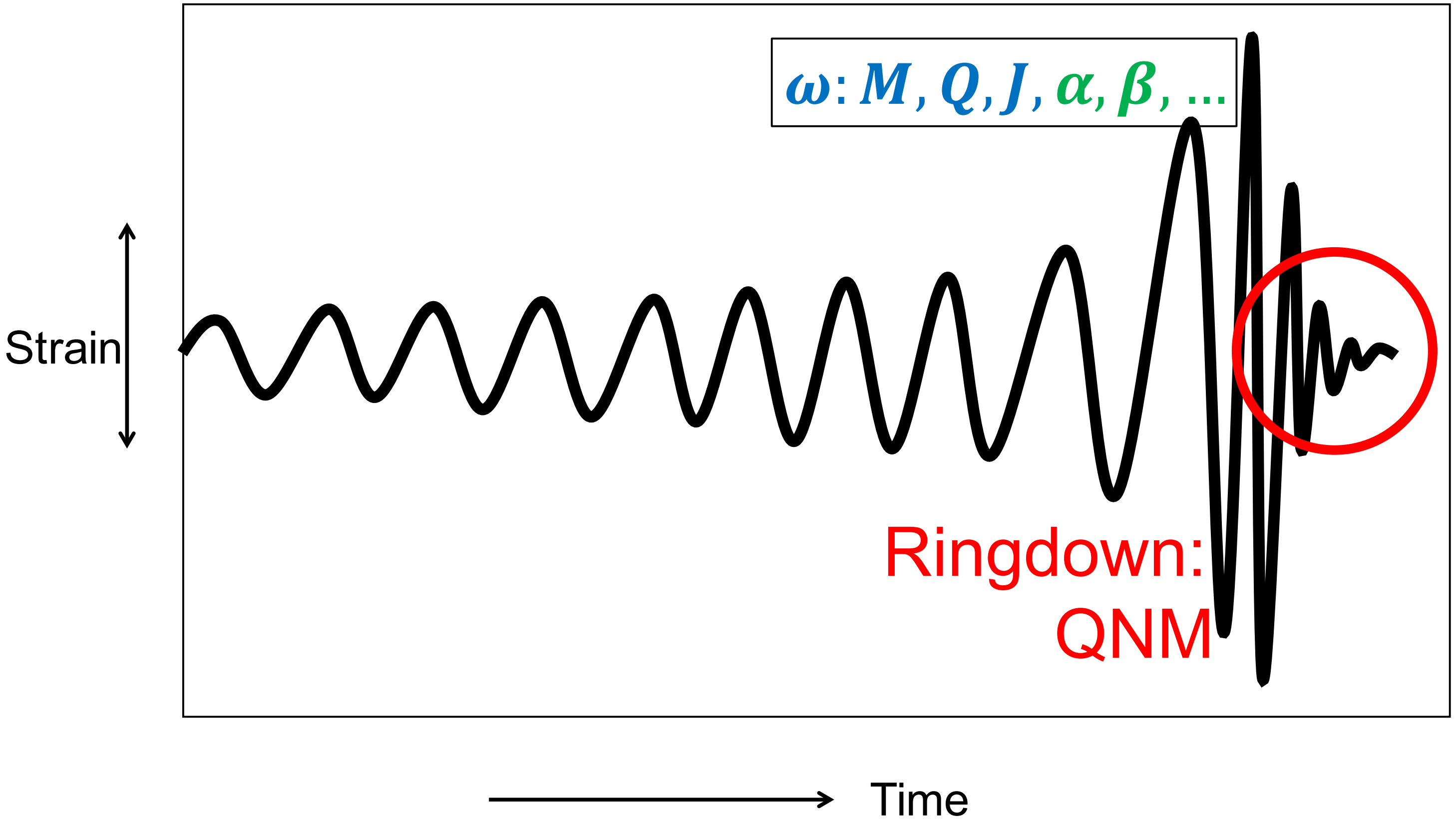
Quasinormal modes



Quasinormal modes



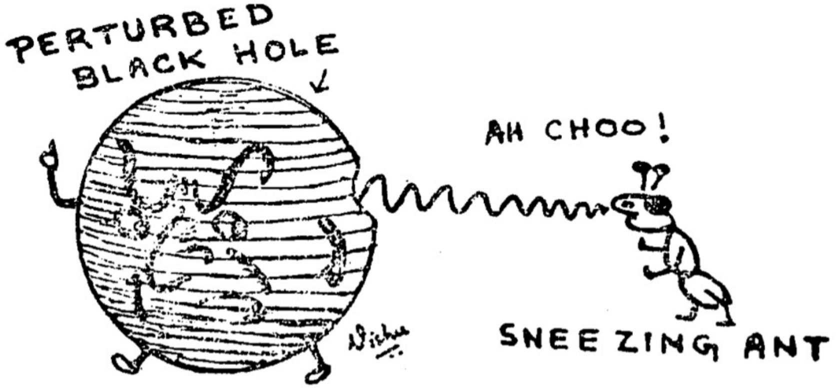
Black hole spectroscopy



Black hole QNMs: Master equations

- Gravitational perturbations

$$g_{\mu\nu} = g_{0\mu\nu} + h_{\mu\nu} \qquad |h| \ll 1$$

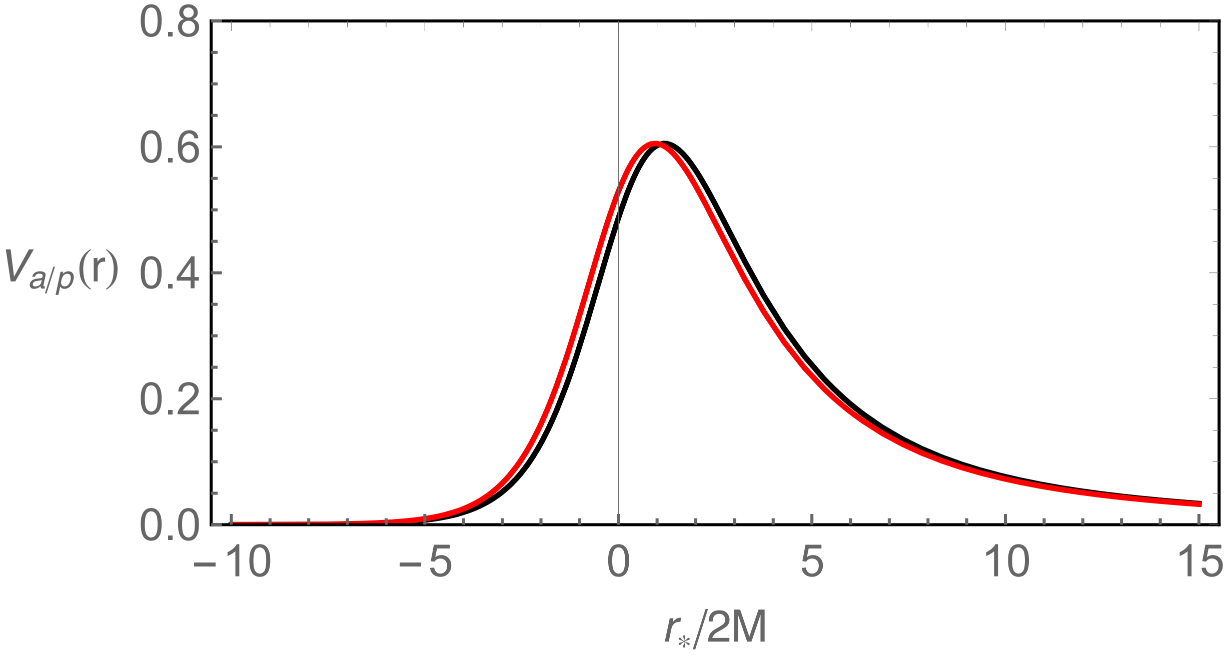


- If separable: $$\left(\frac{d^2}{dr_*^2} + \omega^2\right)\Psi = V_{a/p}\Psi$$

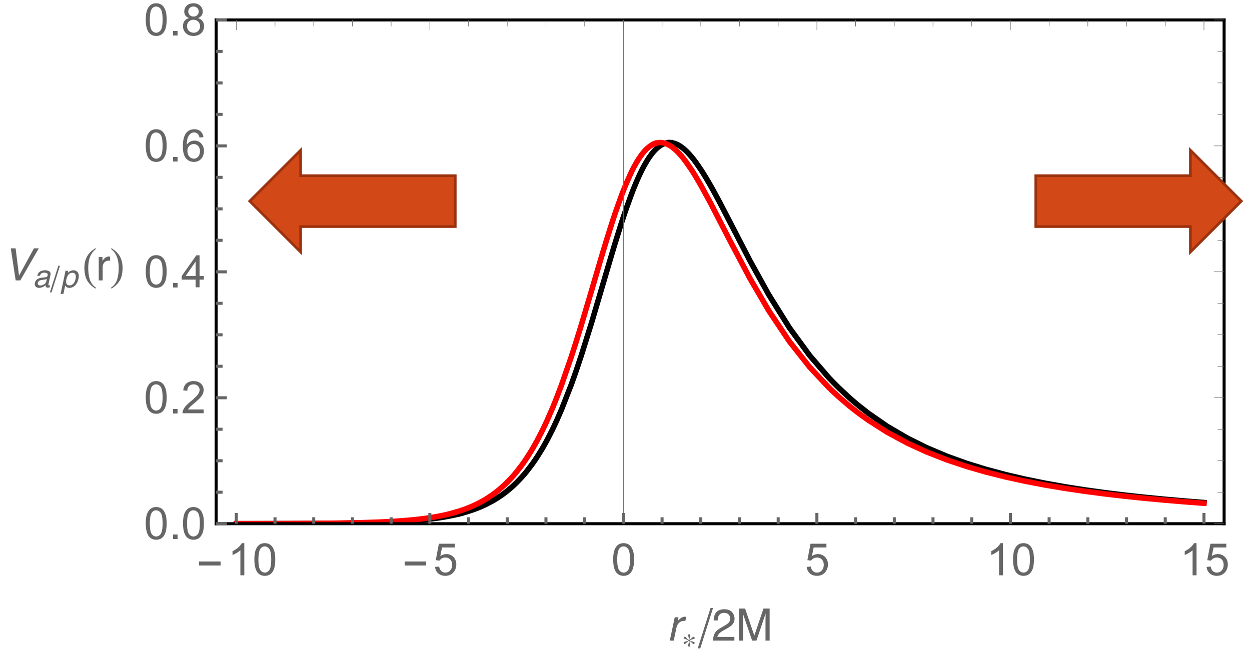
- Schwarzschild black hole:
  - Odd parity (axial): Regge-Wheeler equation
  - Even parity (polar): Zerilli equation

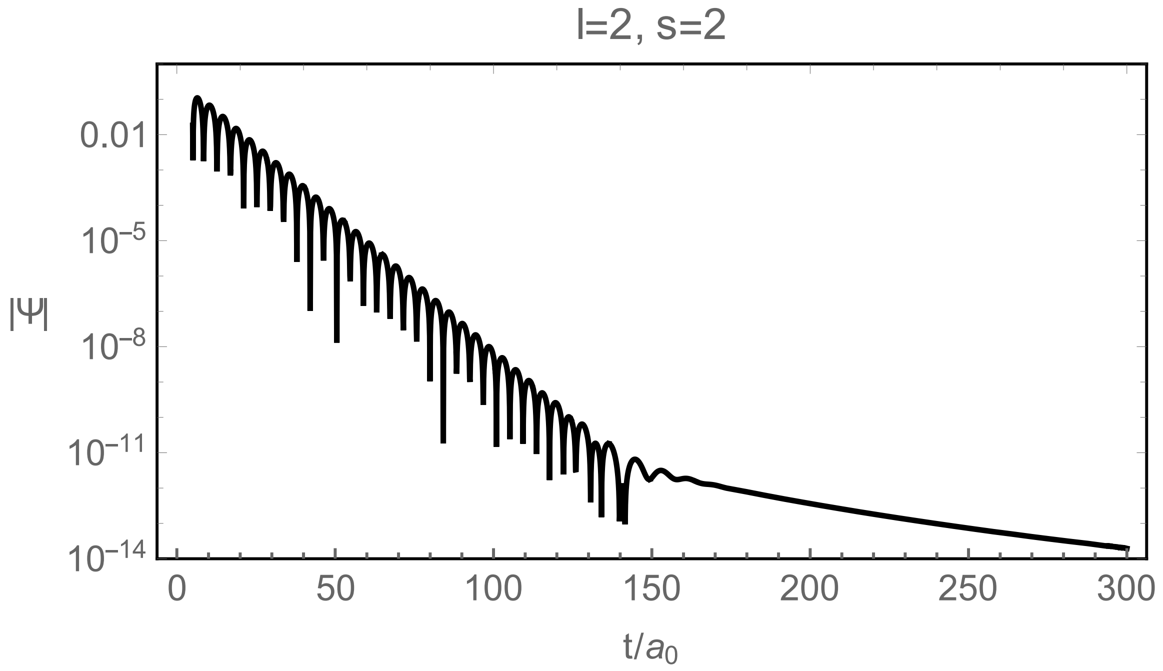
- In test field scenarios, the master equation can also be written in the Schrödinger-like form

Potential and boundary conditions

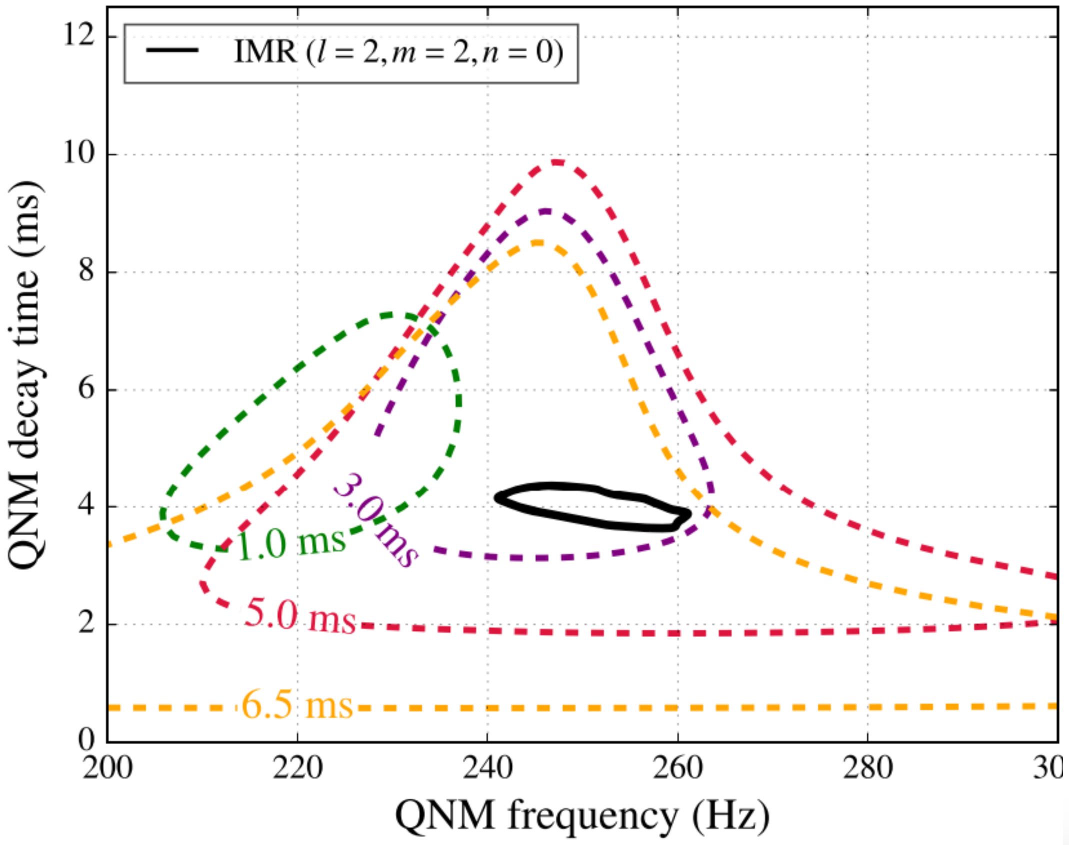


Potential and boundary conditions

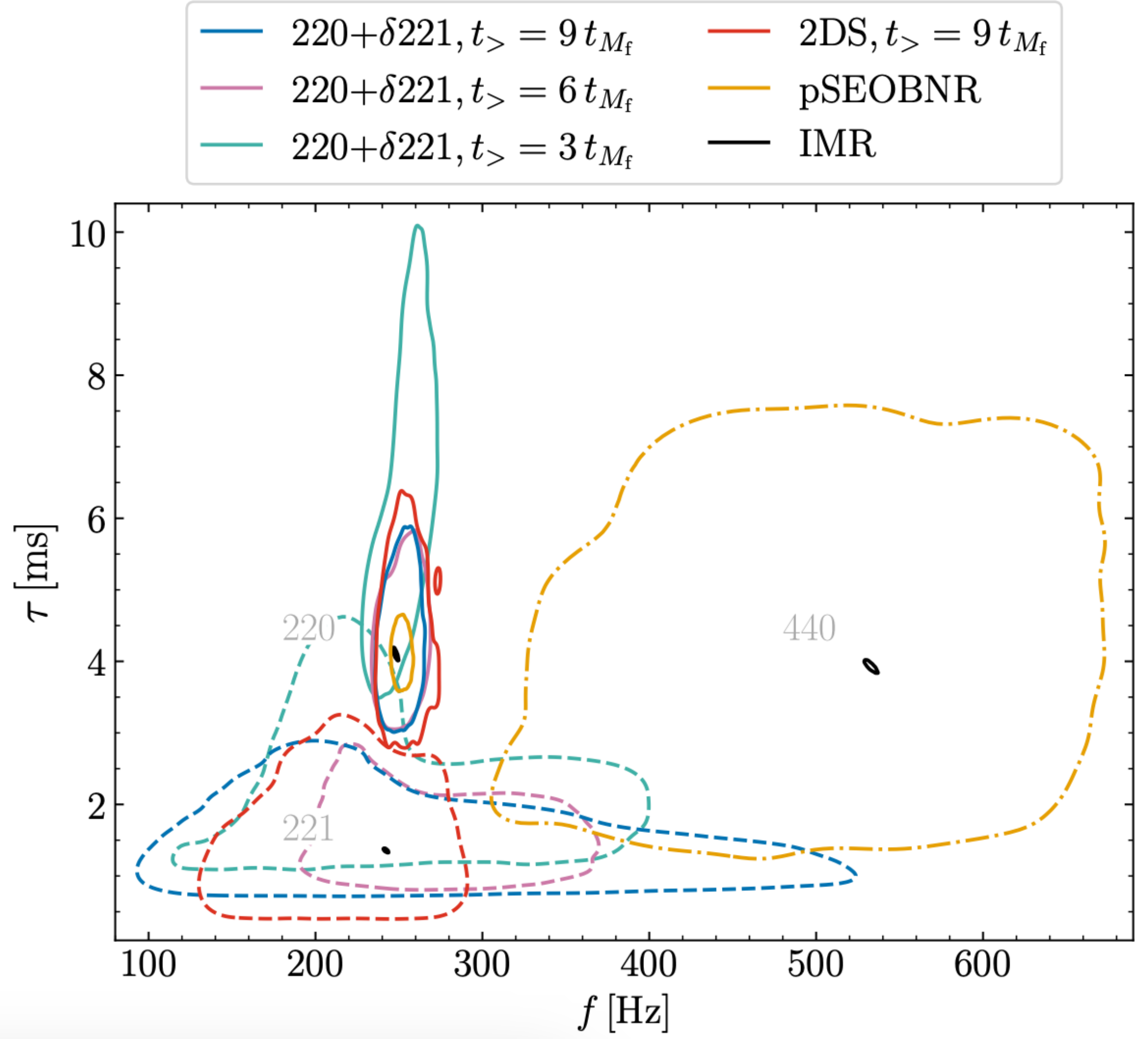




Black hole spectroscopy: GW150914 & GW250114

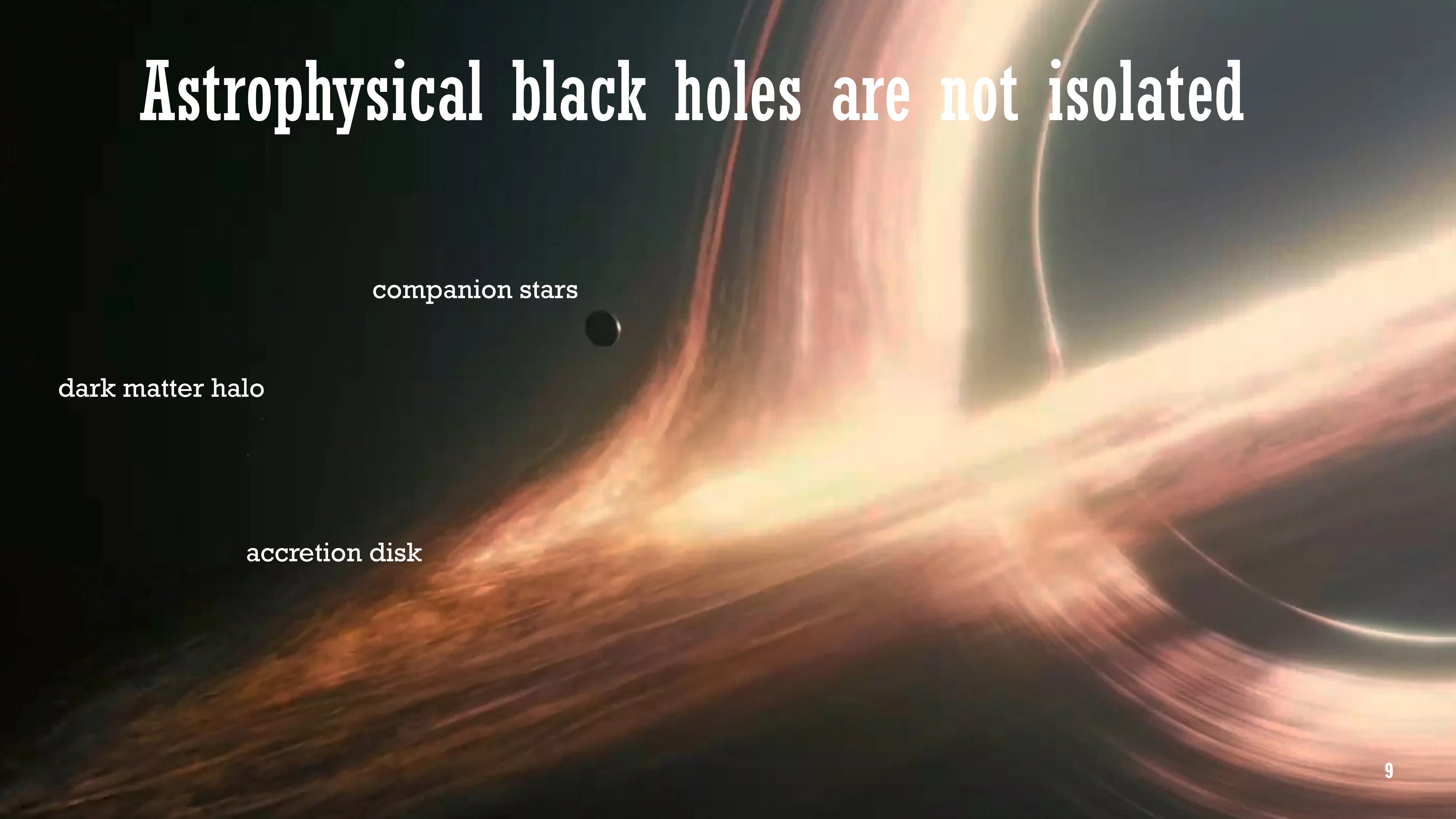


Abbott *et al.* (2016)



LVK (2025)

Astrophysical black holes are not isolated



Environmental effects

- How does astrophysics affect QNM spectra?
- Degeneracy with non-GR effects?
- QNMs of black hole + gravitating thin disk

Static and axisymmetric metric

$$ds^2 = -e^{2\nu}dt^2 + \rho^2 e^{-2\nu}d\phi^2 + e^{2\lambda-2\nu}(d\rho^2 + dz^2)$$

- Consider Weyl coordinates (t, ϕ, ρ, z)
- The metric functions $\nu = \nu(\rho, z), \ \ \lambda = \lambda(\rho, z)$

- Einstein equations in vacuum (outside the source):

$$\Delta\nu = 0$$

$$\lambda_{,\rho} = \rho\left(\nu_{,\rho}^2 - \nu_{,z}^2\right), \qquad \lambda_{,z} = 2\rho\nu_{,\rho}\nu_{,z}$$

CYC, Kotlařík (2023)

Static and axisymmetric metric

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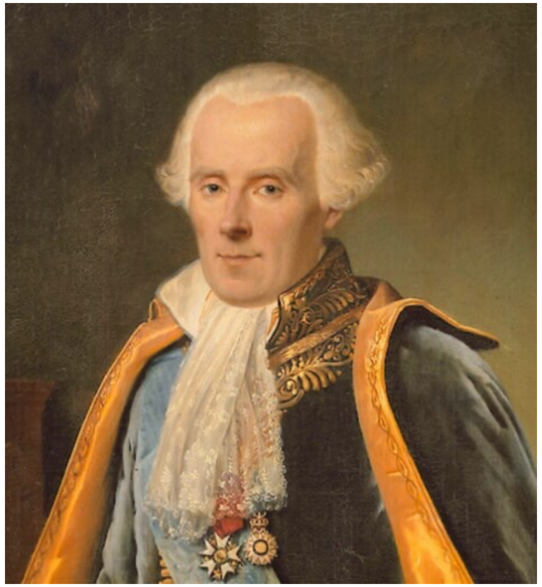


$$\Delta\nu = 0$$

Laplace eq. is linear

ν: Newtonian potential

$$\lambda_{,\rho} = \rho\left(\nu_{,\rho}^2 - \nu_{,z}^2\right), \qquad \lambda_{,z} = 2\rho\nu_{,\rho}\nu_{,z}$$

CYC, Kotlařík (2023)

Superposition of black hole and disk

- The Laplace equation is linear: simple sum to get the total gravitational potential

$$\nu = \nu_{\mathrm{Sch}} + \nu_{\mathrm{disk}}$$

- For λ, we have

$$\lambda = \lambda_{\mathrm{Sch}} + \lambda_{\mathrm{disk}} + \lambda_{\mathrm{int}}$$

where

$$\lambda_{\mathrm{int},\rho} = 2\rho(\nu_{\mathrm{Sch},\rho}\nu_{\mathrm{disk},\rho} - \nu_{\mathrm{Sch,z}}\nu_{\mathrm{disk,z}})$$
$$\lambda_{\mathrm{int},z} = 2\rho(\nu_{\mathrm{Sch},\rho}\nu_{\mathrm{disk},z} + \nu_{\mathrm{Sch,z}}\nu_{\mathrm{disk},\rho})$$

The Schwarzschild black hole

$$\nu_{\mathrm{Sch}} = \frac{1}{2}\ln\left(\frac{R_+ + R_- - 2M}{R_+ + R_- + 2M}\right), \qquad \lambda_{\mathrm{Sch}} = \frac{1}{2}\ln\left(\frac{(R_+ + R_-)^2 - 4M^2}{4R_+R_-}\right), \qquad R_\pm = \sqrt{\rho^2 + (|z| \mp M)^2}$$

Superposition of black hole and disk

- The Newtonian surface density profiles:

$$w^{(m,n)} = W^{(m,n)} \frac{b^{2m+1}\rho^{2n}}{2\pi(\rho^2 + b^2)^{m+n+3/2}}, \qquad m, n \in \mathbb{N}$$

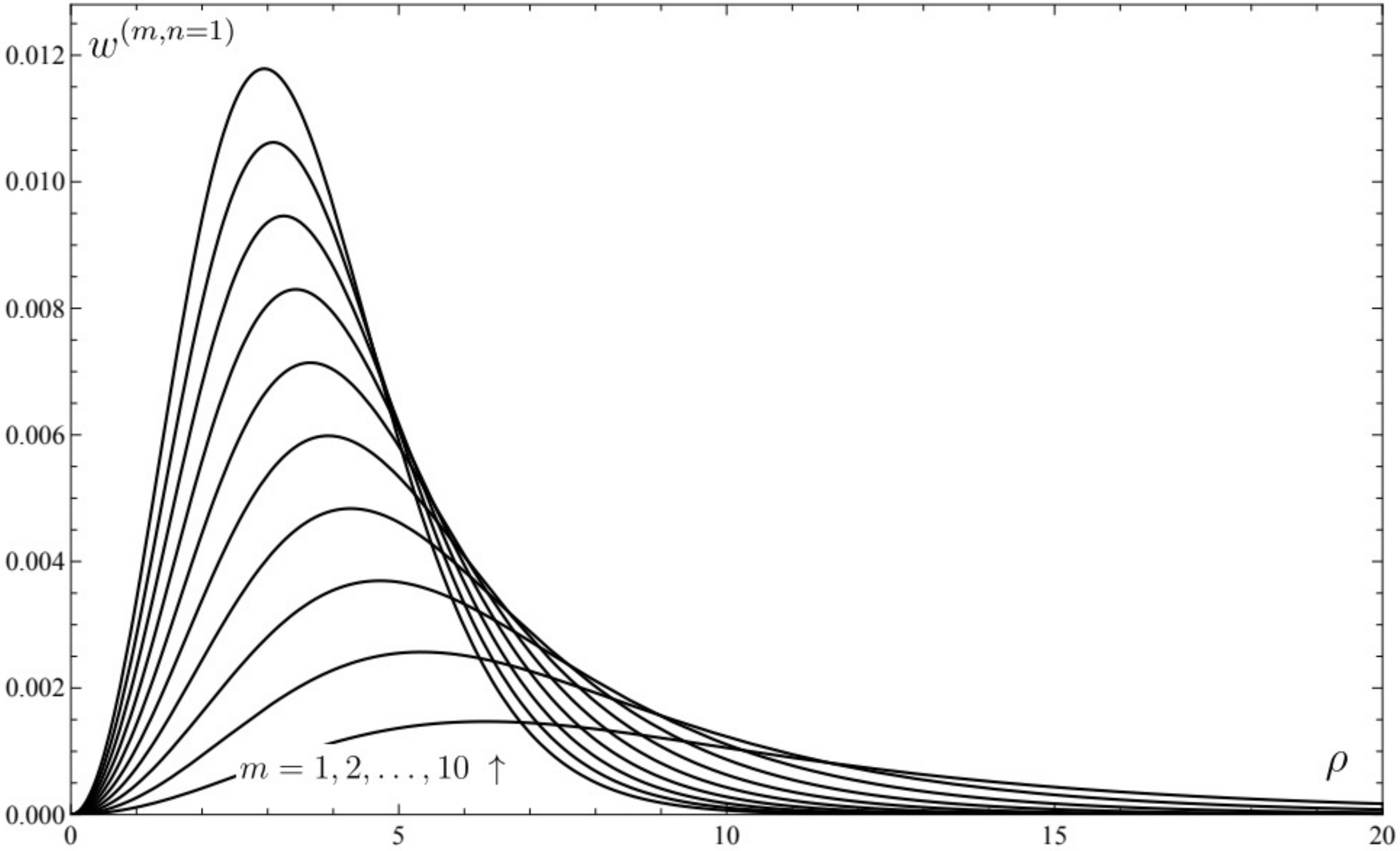
where

$$W^{(m,n)} = (2m+1)\binom{m+n+1/2}{n}\mathcal{M}$$

- Total mass of the disk is fixed: $2\pi \int_0^\infty w^{(m,n)}\rho d\rho = \mathcal{M}$
- The gravitational potential: $\Delta\nu = 4\pi w(\rho)\delta(z)$
- Exact and fully analytic ν and λ, but lengthy

Kotlařík, Kofroň (2022)

Superposition of black hole and disk



Surface density peaks at
$\rho_{max} = b\sqrt{2n/(3+2m)}$

Kotlařík, Kofroň (2022)

Superposition of black hole and disk

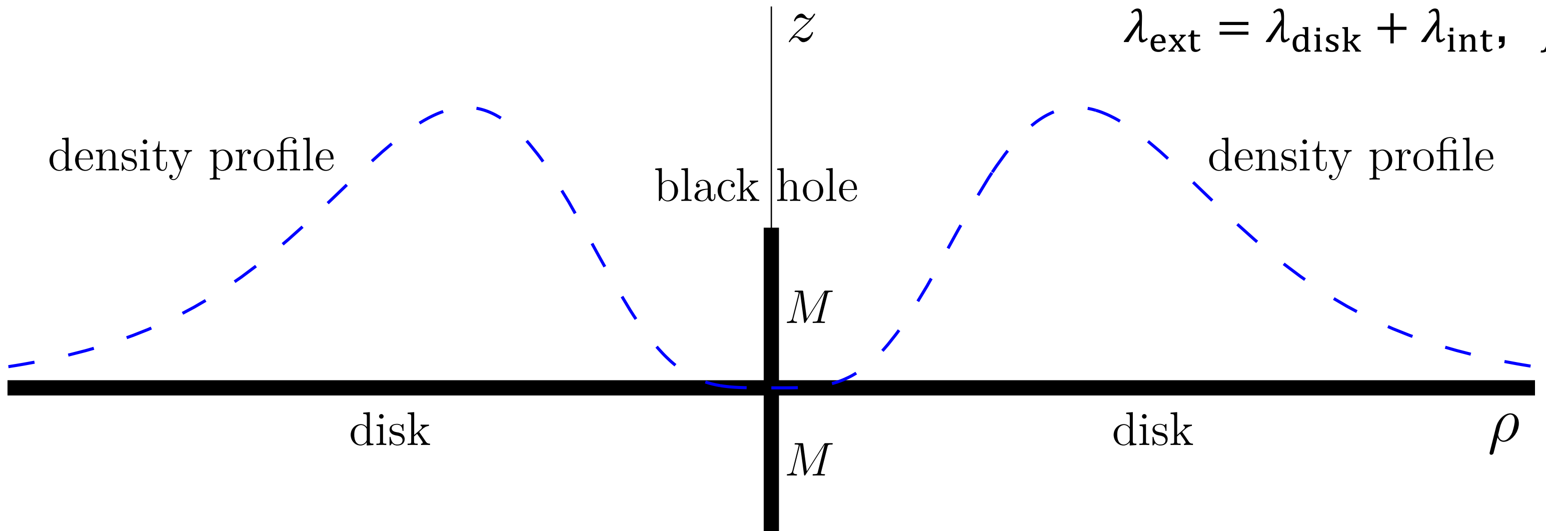
- Back to Schwarzschild coordinates

$$\rho = \sqrt{r(r-2M)}\sin\theta\,, \quad z = (r-M)\cos\theta$$

- The Superposed metric (SBH-disk) metric

$$ds^2 = -f(r)e^{2\nu_{\rm disk}}dt^2 + e^{2\lambda_{\rm ext}-2\nu_{\rm disk}}\frac{dr^2}{f(r)} + r^2e^{-2\nu_{\rm disk}}(e^{2\lambda_{\rm ext}}d\theta^2 + \sin^2\theta\, d\phi^2)$$

$\lambda_{\rm ext} = \lambda_{\rm disk} + \lambda_{\rm int}$, $f(r) = 1 - 2M/r$



Kotlařík, Kofroň (2022)

Scalar field QNMs

- Klein-Gordon equation
- Deformed black hole spacetime
- How to deal with the separability issue?
- Up to 1st order of $\epsilon \equiv \mathcal{M}/M$, one can use the projection method and obtain the ϵ-corrections on the zeroth order equation

Cano, Fransen, Hertog (2020) CYC, Chiang, Tsao (2022)

- A Schrödinger-like master equation is attainable (up to 1st order of ϵ)

$$\boxed{(\partial_{r_*}^2+\omega^2)\Psi_{lm} = V_{\text{eff}}(r,l,m)\Psi_{lm}}$$

$$\boxed{V_{\text{eff}}(r,l,m) = V_{\text{Sch}} + \delta V}$$

Scalar field QNMs

$$\boxed{(\partial_{r_*}^2 + \omega^2)\Psi_{lm} = V_{\text{eff}}(r,l,m)\Psi_{lm}} \qquad \boxed{V_{\text{eff}}(r,l,m) = V_{\text{Sch}} + \delta V}$$

- Series expansion w.r.t. $x \equiv \cos\theta$
- check convergence at a finite truncation j

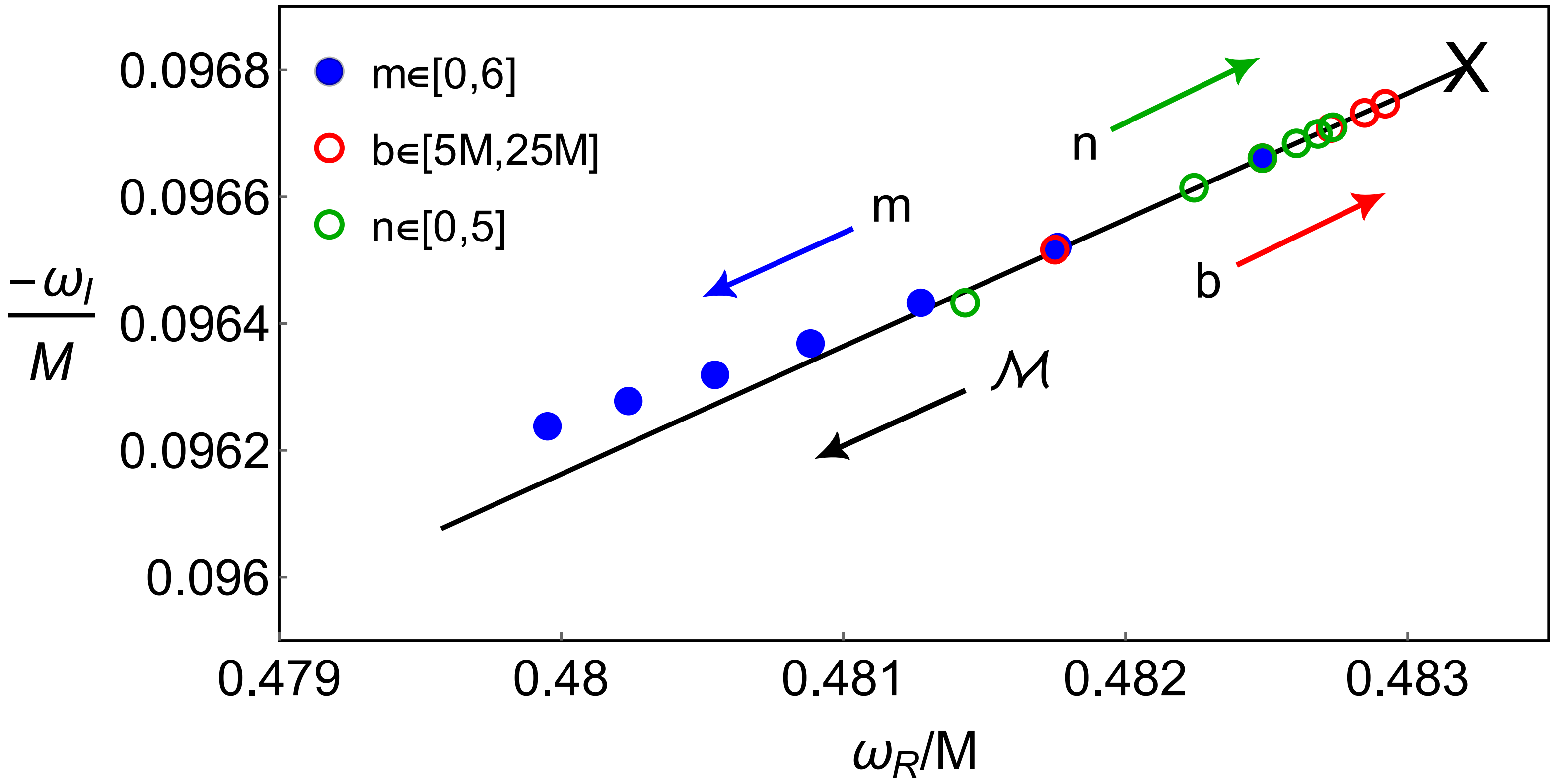
$$\nu_{\text{disk}} = \epsilon \mathcal{V}_j |x|^j, \qquad \lambda_{\text{int}} = \epsilon \mathcal{L}_j |x|^j$$

$\epsilon \equiv \mathcal{M}/M$

$$V_{\text{eff}}(r) = l(l+1)\frac{f(r)}{r^2} + \frac{f(r)}{r}\frac{df}{dr}\left[1 + \epsilon b^j_{lm_z}\left(4\mathcal{V}_j(r) - 2\mathcal{L}_j(r)\right)\right]$$
$$+ \epsilon\left\{\frac{f}{r^2}\left[4a^j_{lm_z}\mathcal{V}_j(r) - c^j_{lm_z}\left(4\mathcal{V}_j(r) - 2\mathcal{L}_j(r)\right)\right] - \frac{b^j_{lm_z}}{2}\frac{d^2}{dr_*^2}\left[2\mathcal{V}_j(r) - \mathcal{L}_j(r)\right]\right\}$$

CYC, Kotlařík (2023)

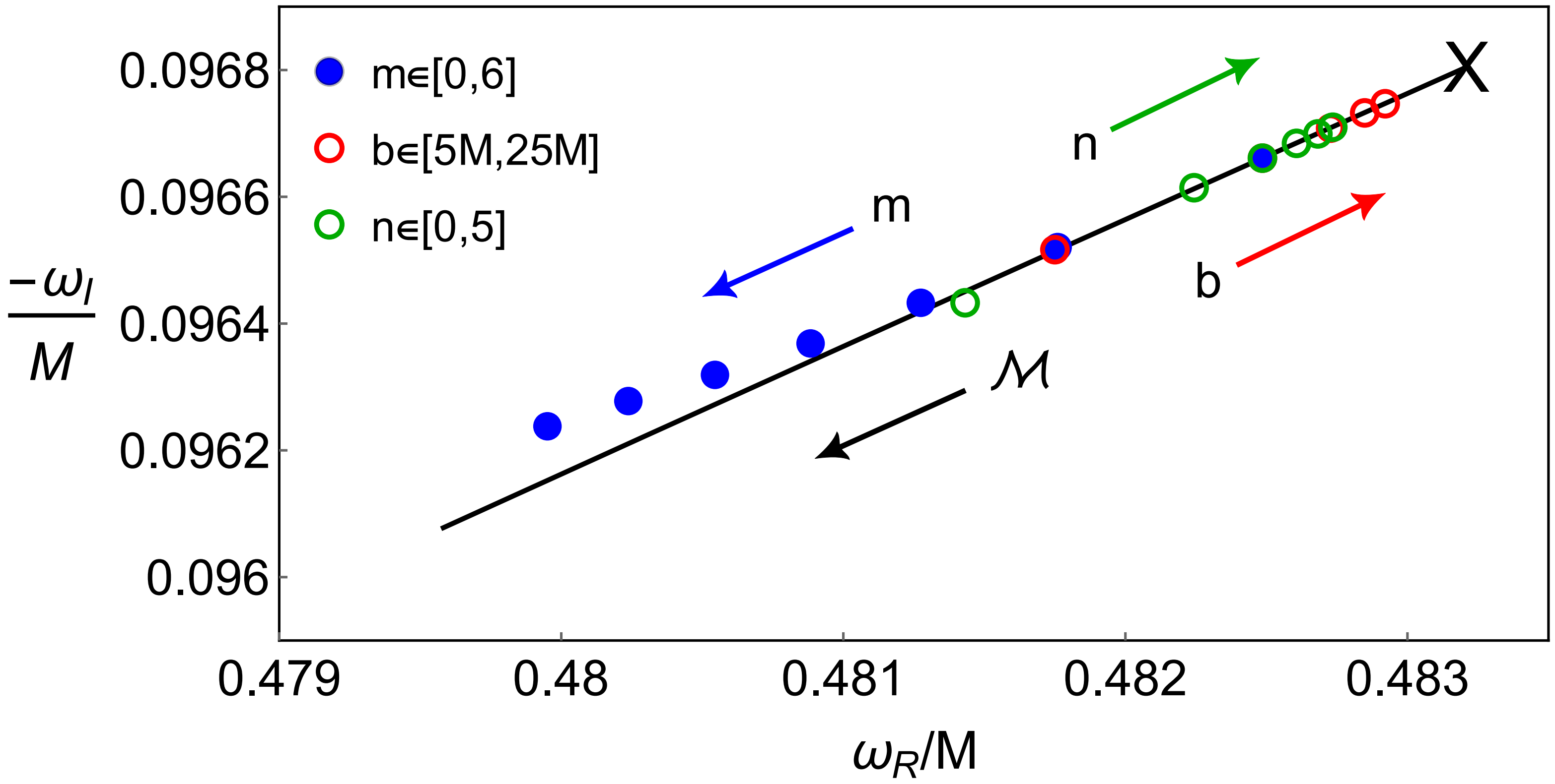
Universal relation?



$$\frac{\omega_I}{\omega_{Sch,I}} \approx \frac{\omega_R}{\omega_{Sch,R}} \approx 1 - O(\epsilon)$$

CYC, Kotlařík (2023)

Universal relation?



$$\frac{\omega_I}{\omega_{Sch,I}} \approx \frac{\omega_R}{\omega_{Sch,R}} \approx 1 - O(\epsilon)$$

CYC, Kotlařík (2023)

- Similar redshift behavior was also found for black hole embeded in spherically symmetric matter distrubition

Cardoso *et al*. (2022), Konoplya (2021), Pezzella (2025)

Conclusions

- Testing gravity through black hole spectroscopy
- We know little about environmental effects

- Superposition of black hole and gravitating thin disk
  - Universal redshift relation among disk parameters

  - Other disk models? linearly scale with disk mass? analytic proof?
  - Gravitational perturbations?
  - Accretion flow, rotation, geometrically thick disk