# Recent progress in the nucleon spin structure

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# Outline

- Spin sum rules
- 30 years of GPDs, DVCS, and EMT FFs
- Lattice QCD
- Longitudinal polarization and twist-3 GPDs
- Conclusion

# Spin sum rules

Frame dependence, gauge dependence, twist-2, 3, and 4

Longitudinal & transverse polarization

# Spin and Lorentz transformation

- Spin is simplest to describe in the rest frame of a particle

  non-relativistic spin wave functions

- However, in experiments or massless case, particles are often travelling with high velocity.
- The spin structure of a particle is generally frame dependent, following Lorentz transformation.

# Longitudinal and transverse spin

- When boost in the same direction as the spin, the projeciton is invariant under the boost.

$$J^z|p\lambda_z\rangle = \lambda_z|p\lambda_z\rangle$$

as $J^z$ commutes with boost operator $K^z$ : $[J^z, K^z] = 0$

- On the other hand, when boost in the transverse direction, the spin eigenstate in the rest frame will change

$$[J_\perp, K^z] \neq 0$$

transverse spin/polarization is NOT an eigenstate of transverse AM!

# Longitudinal spin

- Longitudinal spin (helicity) remains finite at large momentum.
  Therefore, it is intrinsically a twist-3 observable.
- The longitudinal spin sum rule a la Jaffe and manohar (1990)

$$\frac{1}{2} = \frac{1}{2}\ \Delta\Sigma + \Delta G + \ell_q^z + \ell_g^z$$

- However, both $\Delta\Sigma$ and $\Delta G$ can be associated with twist-2 observables and therefore has a partonic interpretation.

$$\Delta\Sigma = \sum_i \int dx\, \Delta q_i\,(x)\ ;\ \ \Delta G = \int dx\, \Delta g(x)$$

this is because $\Delta q$ can be defined from $\bar{\psi}\gamma^\mu\gamma_5\psi$ which is a twist-2

$\Delta G$ can be defined from $K^\mu$ (Chern-Simon's current), also a twist-2

# Orbital AM contribution to helicity

- When the nucleon is travelling in z –direction, the OAM of quarks & gluons

$$\vec{b}_\perp \times \vec{k}_\perp$$

remains finite. No simple partonic sum rule

**There is no simple partonic interpretation for the total nucleon helicity!**

- The helicity sum rule in terms of gauge invariant operators is frame independent, in particular, one has a similar sum rule in the rest frame (Ji, 1996)

# Transverse spin

- Transverse pol. state $|ps_\perp\rangle$ is not an eigenstate of transverse AM $J_\perp$
- However, the expectation value of

$$\langle ps_\perp | J_\perp | ps_\perp \rangle = \frac{\hbar}{2}\gamma$$

is well-defined! Ji & Yuan *Phys. Lett. B* 810 (2020) 135786

Transverse AM grows with particle's momentum, thus is a leading twist quantity!

$$k^z \times b_\perp \quad \text{(twist-2)}$$

thus has a simple partonic interpretation, but it is not related to the spin of partons!

# Transverse spin sum rule

- By introducing twist-2 GPDs, E and H, one can write (Ji, 1996)

$$\frac{1}{2} = \int dx \sum_i x(H_f(x) + E_f(x))$$

This is a partonic sum rule for the transverse spin.

Namly, the transverse AM carried by parton with momentum fraction x is

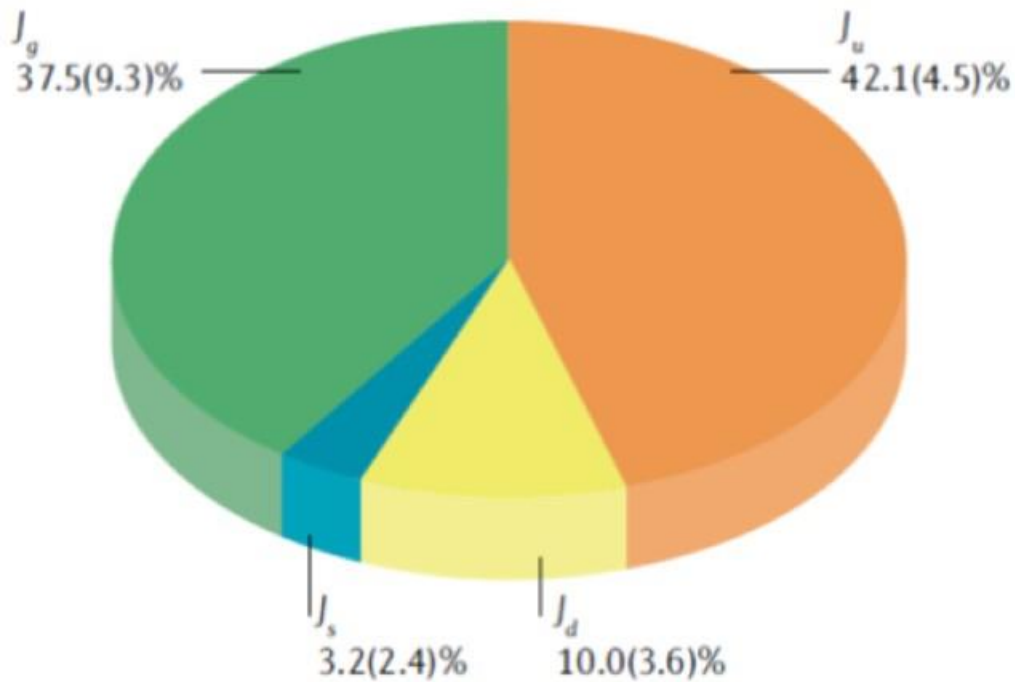
$$J_f(x) = x(H_f(x) + E_f(x))$$

The total is the sum from all partons!

- There is a twist-4 version of the transverse AM sum rule which will involve twist-3 parton spin.

# Gauge invariant spin decomposition

## ETMC & chi-QCD collaborations



# 30 years GPDs, DVCS and EMT form factors

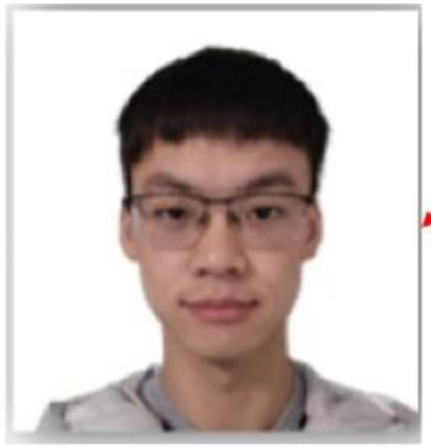
# Origin of GPDs

- In 1994, D. Mueller et al recognized GPDs as non-perturbative matrix elements in QCD factorization (D. Mueller et al, )
- In 1996, GPDs was independently introduced as nuclear observables allowing studying the spin structure for the nucleon (Ji, PRL1997)
  1. Form factors of quarks and gluon energy-momentum tensor (EMT)
  2. Spin sum rule
  3. Generalized parton distributions and its sum rules to measure FF of EMTs
  4. Deeply-virtual Compton scattering
  5. A class of new experiments (deep exclusive processes, including meson production)

# Existing Reviews (latest from 20 years ago)

- X. Ji, Off forward parton distributions *J. Phys. G* 24 (1998) 1181-1205
- K. Goeke, Maxim V. Polyakov, M. Vanderhaeghen, Hard exclusive reactions and the structure of hadrons, *Prog. Part. Nucl. Phys.* 47 (2001) 401-515
- M. Diehl, Generalized parton distribution, *Phys. Rept.* 388 (2003) 41-27
- X. Ji, Generalized parton distributions, *Ann. Rev. Nucl. Part. Sci.* 54 (2004) 413-450
- A.V. Belitsky ,A.V. Radyushkin, Unraveling hadron structure with generalized parton distributions, *Phys.Rept.* 418 (2005) 1-387

# Filling the 20-years gap: 2609.19544 [hep-ph]



## Generalized parton distributions: Theory meets experiment


Yuxun Guo,[1,2,3,*] Xiangdong Ji,[4,†] Yao Ji,[5,‡] and Jialu Zhang[4,§]

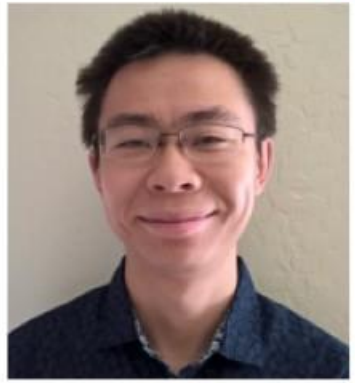
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] 17 Sep 2026



Over the past three decades, generalized parton distributions (GPDs) have emerged as one of the most active and important areas of research in nucleon structure and quantum chromodynamics (QCD). Since the last comprehensive review two decades ago, substantial progress has been made in experimental measurements of hard exclusive

# Form factors of quark-gluon EMT

- Complete form factors of quark & gluon EMTs

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{U}(P')\Big[A_{q,g}(\Delta^2)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}(\Delta^2)\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha/2M \\ +C_{q,g}(\Delta^2)(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2)/M + \bar{C}_{q,g}(\Delta^2)g^{\mu\nu}M\Big]U(P) \qquad (12)$$

  - A is related to mass/energy density and scalar field distribution
  - B related to quark and gluon angular momentum
  - C not discussed
  - C-bar is due to non-conservation effect
- It shall not be called gravitational form factors because gravity does distinguish between quarks and gluons.

# Generalized parton distributions

- **Introduce GPD in symmetric notation**

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu \psi(\lambda n/2)|P\rangle = H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu U(P) + E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\mu\nu}\Delta_\nu}{2M}U(P) + \ldots$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P'|\bar{\psi}(-\lambda n/2)\gamma^\mu \gamma_5 \psi(\lambda n/2)|P\rangle = \tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\gamma^\mu \gamma_5 U(P) + \tilde{E}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{\gamma_5\Delta^\mu}{2M}U(P) + \ldots \qquad (16)$$

- **Only H and E are related to the EMT form factors**

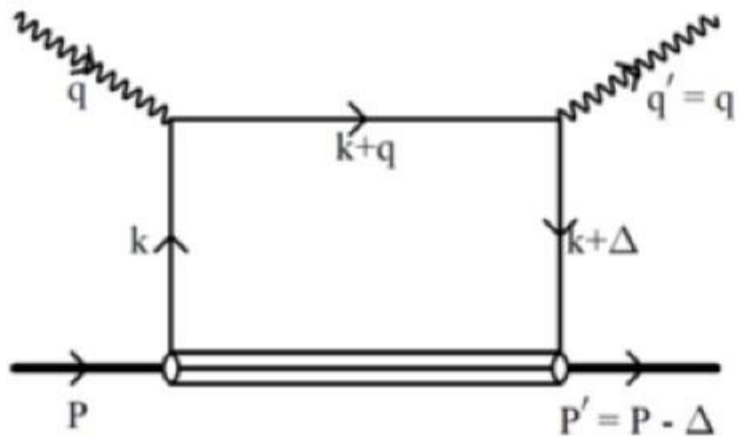
$$\int_{-1}^{1} dx\; xH(x,\xi,t) = A(t) + \xi^2\, C(t)\;,$$

$$\int_{-1}^{1} dx\; xE(x,\xi,t) = B(t) - \xi^2\, C(t)\;.$$

The off-forward parton distributions can be defined for quark helicity-flip (chiral-odd) correlations, for higher twists, and for gluons. DVCS provides one process to access to these

# Deeply virtual Compton scattering

- **Exclusive production of high-energy photon in deep-inelastic scattering kinematics (high-energy virtual photon in and high-energy real photon out): Diffraction**




$$T^{\mu\nu}(P,q,\Delta) = -\frac{1}{2}(p^\mu n^\nu + p^\nu n^\mu - g^{\mu\nu}) \int dx \left(\frac{1}{x-\xi/2+i\epsilon} + \frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times \left[H(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}U(P) + E(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\frac{i\sigma^{\alpha\beta}n_\alpha\Delta_\beta}{2M}U(P)\right]$$
$$-\frac{i}{2}\epsilon^{\mu\nu\alpha\beta}p_\alpha n_\beta \int dx \left(\frac{1}{x-\xi/2+i\epsilon} - \frac{1}{x+\xi/2+i\epsilon}\right)$$
$$\times \left[\tilde{H}(x,\Delta^2,\Delta\cdot n)\bar{U}(P')\not{n}\gamma_5 U(P) + \tilde{E}(x,\Delta^2,\Delta\cdot n)\frac{\Delta\cdot n}{2M}\bar{U}(P')\gamma_5 U(P)\right]$$

FIG. 1. Dominant scattering process in deeply-virtual Compton scattering

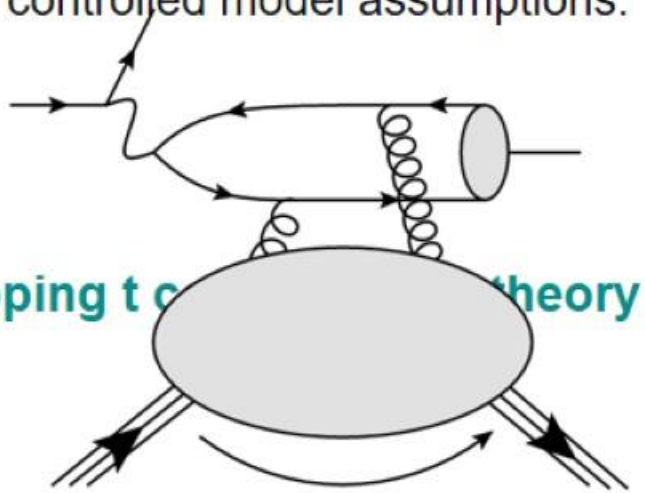
# Three routes to EMT information

## Exclusive scattering

Extracting EMT form factors as the Mellin moments via GPD sum rules, from e.g. dispersion relation and global analyses

## Threshold J/ψ

At large skewness, a leading spin-two approximation can expose EMT information with controlled model assumptions.



## Lattice QCD

Quark and gluon EMT matrix elements provide independent form-factor constraints and a cross-check.

**A joint extraction needs overlapping t c theory errors for each channel.**

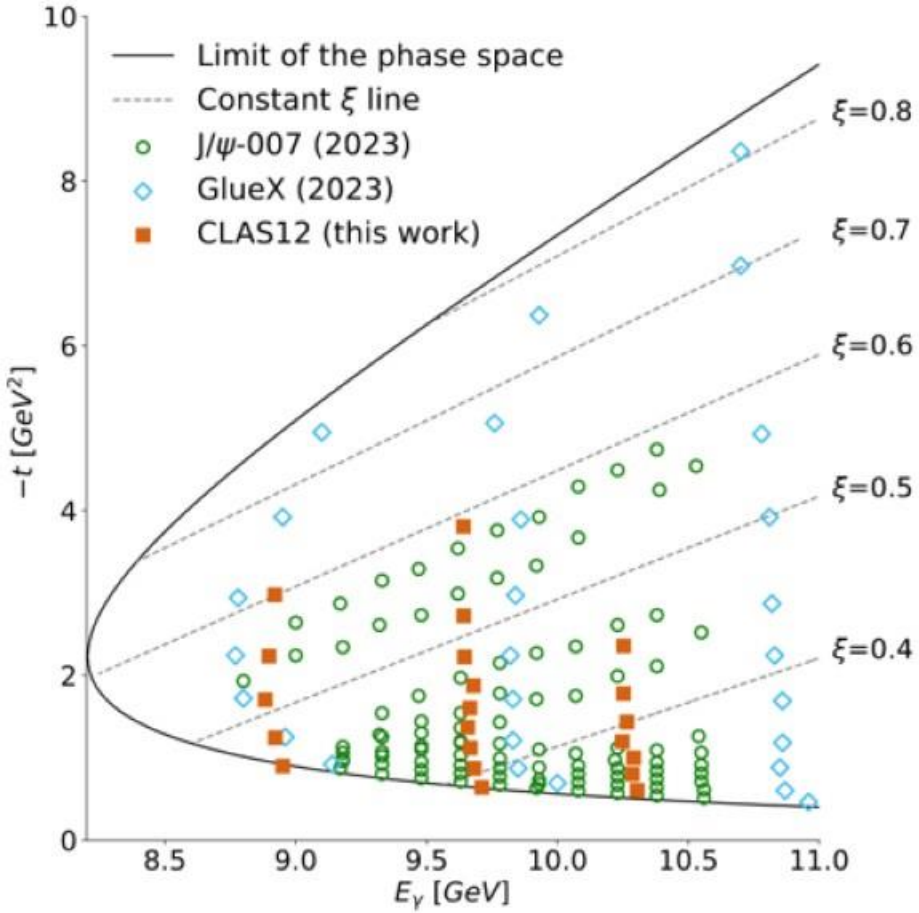
TABLE II Representative DVCS measurements, organized by kinematic regime, experiment, target, and principal observables. Qualitatively, HERA probes predominantly small $x_B$, HERMES and COMPASS intermediate $x_B$, and JLab the valence region.

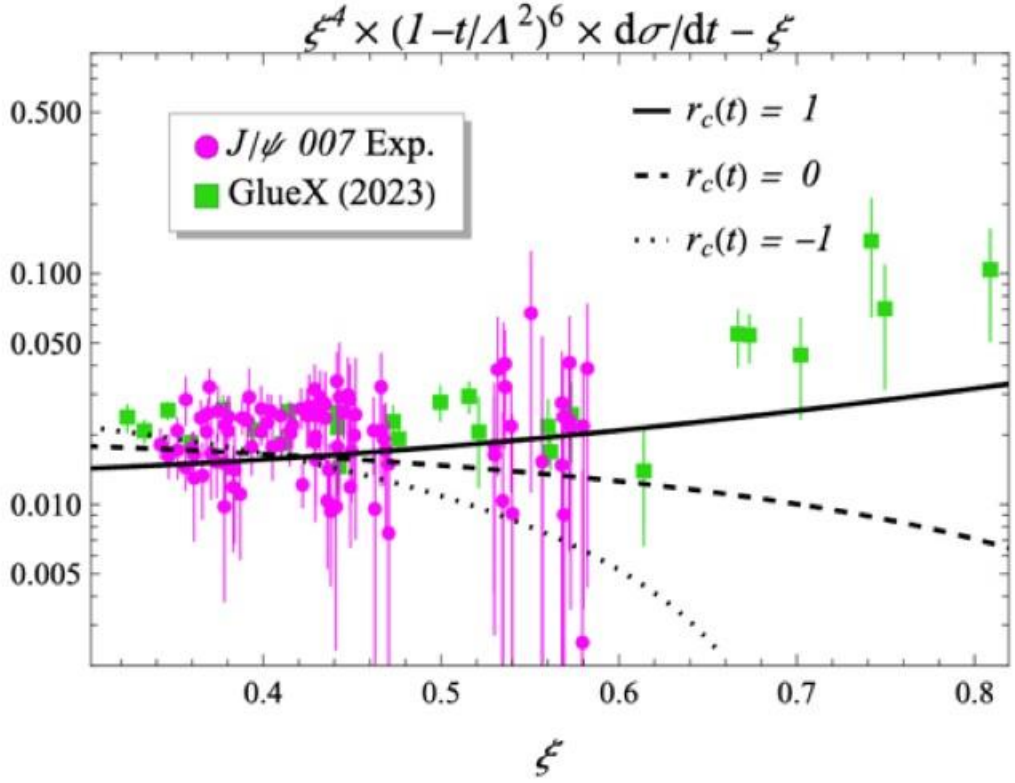
| **Kinematic regime** | **Experiment** | **Target** | **Principal measurements** | **References** |
|---|---|---|---|---|
| Small $x_B$ (HERA) | H1 | $p$ | Unpolarized differential cross sections with $Q^2$, $W$, and $t$ dependences | (Aaron *et al.*, 2008, 2009; Adloff *et al.*, 2001; Aktas *et al.*, 2005) |
| Small $x_B$ (HERA) | ZEUS | $p$ | Unpolarized differential cross sections with $Q^2$, $W$, and $t$ dependences | (Chekanov *et al.*, 2003, 2009a) |
| Intermediate $x_B$ | HERMES | $p, d$ | Beam-charge, beam-spin, and double-spin asymmetries; longitudinal- and transverse-target-spin asymmetries; | (Airapetian *et al.*, 2001, 2007, 2008b, 2009, 2010a, 2011, 2012a,b) |
| Intermediate $x_B$ | COMPASS | $p$ | Differential cross sections and their $t$ dependence | (Akhunzyanov *et al.*, 2019) |
| Valence | CLAS | $p$ | Beam- and target-spin and double-spin asymmetries; polarized and unpolarized $\phi$-dependent cross sections | (Chen *et al.*, 2006; Gavalian *et al.*, 2009; Hirlinger Saylor *et al.*, 2018; Jo *et al.*, 2015; Pisano *et al.*, 2015; Stepanyan *et al.*, 2001) |
| Valence | Hall A | $p$ | Unpolarized and beam-polarized $\phi$-dependent differential cross sections | (Defurne *et al.*, 2015, 2017; Muñoz Camacho *et al.*, 2006) |
| Valence | CLAS&Hall A | $p$ | Higher-$Q^2$ proton differential cross sections and asymmetries with extended multidimensional coverage | (Christiaens *et al.*, 2023; Georges *et al.*, 2022; Lee *et al.*, 2026) |
| Valence, neutron | CLAS&Hall A | $n$ | DVCS differential cross-section off the neutron; Beam-spin asymmetry with exclusive detection of the active neutron | (Benali *et al.*, 2020; Hobart *et al.*, 2024; Mazouz *et al.*, 2007) |
| Valence, nuclear | CLAS | $^4$He | Coherent DVCS on $^4$He and incoherent DVCS from bound protons | (Hattawy *et al.*, 2017, 2019) |

TABLE III Representative exclusive meson-production measurements, organized by final-state meson and kinematic regime.

| Kinematic regime | Experiment | Meson | Principal measurements | References |
|---|---|---|---|---|
| Small $x$ (HERA) | H1, ZEUS | $\rho^0$ | Exclusive electroproduction differential cross sections with $Q^2$, $W$, and $t$ dependences; polarization observables and spin-density matrix elements | (Aaron *et al.*, 2010b; Adloff *et al.*, 1997, 2000a; Aid *et al.*, 1996b; Breitweg *et al.*, 1999; Chekanov *et al.*, 2007) |
| Small $x$ (HERA) | H1, ZEUS | $\phi$ | Exclusive electroproduction differential cross sections; longitudinal–transverse separation and spin-density matrix elements | (Aaron *et al.*, 2010b; Adloff *et al.*, 1997; Chekanov *et al.*, 2005) |
| Small $x$ (HERA) | H1&ZEUS | $J/\psi$ | Exclusive electroproduction and photoproduction differential cross sections and their $Q^2$, $W$, and $t$ dependences | (Adloff *et al.*, 2000b; Aid *et al.*, 1996b; Aktas *et al.*, 2006; Alexa *et al.*, 2013; Breitweg *et al.*, 1999; Chekanov *et al.*, 2002, 2004) |
| Small $x$ (HERA) | H1, ZEUS | $\Upsilon$ | Exclusive photoproduction differential cross sections | (Adloff *et al.*, 2000b; Breitweg *et al.*, 1998; Chekanov *et al.*, 2009b) |
| Intermediate $x_B$ | HERMES | $\rho^0$ | Exclusive electroproduction differential cross sections at intermediate virtual-photon energies | (Airapetian *et al.*, 2000) |
| Intermediate $x_B$ | HERMES | $\pi^+$ | Differential cross sections and transverse-target-spin asymmetries | (Airapetian *et al.*, 2008a, 2010b) |
| Intermediate $x_B$ | COMPASS | $\pi^0$ | Differential cross sections | (Alexeev *et al.*, 2025, 2020) |
| Intermediate $x_B$ | COMPASS | $\omega$ | Transverse-target-spin asymmetries and spin-density matrix elements | (Adolph *et al.*, 2017; Alexeev *et al.*, 2021) |
| Intermediate $x_B$ | COMPASS | $\rho^0$ | Transverse-target-spin asymmetries and spin-density matrix elements | (Adolph *et al.*, 2014; Alexeev *et al.*, 2023) |
| Valence | CLAS | $\rho^0$ | Differential cross sections and polarization observables | (Morrow *et al.*, 2009) |
| Valence | CLAS | $\phi$ | Differential cross sections | (Lukashin *et al.*, 2001) |
| Valence | CLAS&Hall A | $\pi^0$ | Unseparated and Rosenbluth-separated differential cross sections, structure functions, and beam-spin asymmetries | (Bedlinskiy *et al.*, 2014; Defurne *et al.*, 2016; Dlamini *et al.*, 2021; Kim *et al.*, 2024; Mazouz *et al.*, 2017) |
| Valence | CLAS&Hall C | $\pi^+$ | Differential cross sections, longitudinal–transverse separations, and $Q^2$-scaling studies | (Blok *et al.*, 2008; Diehl *et al.*, 2023; Horn *et al.*, 2006, 2008; Huber *et al.*, 2008, 2014, 2015; Tadevosyan *et al.*, 2007; Volmer *et al.*, 2001) |
| Valence | CLAS | $\eta$ | Beam-spin asymmetries in exclusive electroproduction | (Zhao *et al.*, 2019) |

# J/psi production for Gluon EMT FF




*Guo, Ji, Yuan, 2023*

# GPDs parameterized in moments

GPDs can be formally expanded in the conformal moment space:

$$F(x,\xi,t) = \sum_{j=0}^{\infty} (-1)^j p_j(x,\xi) \mathcal{F}_j(\xi,t)$$

D. Mueller and A. Schafer 2005



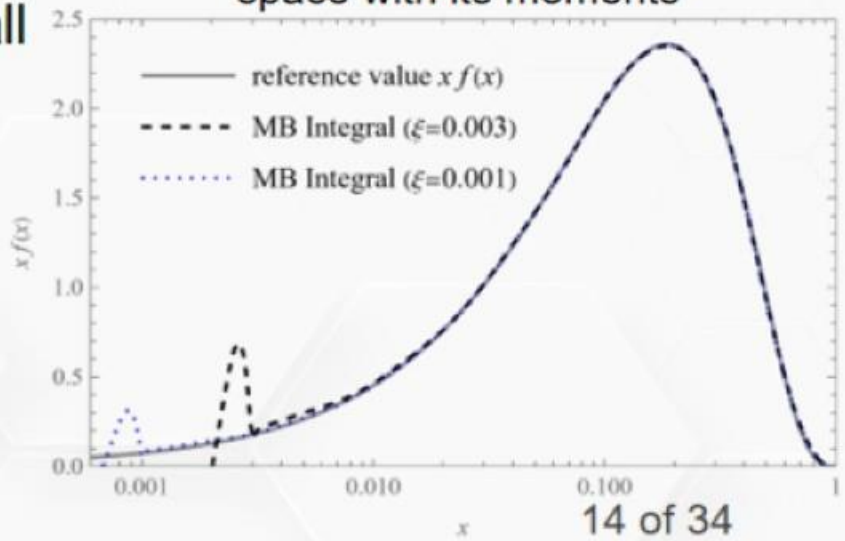

$p_j(x,\xi)$ : Orthogonal basis in terms of Gegenbauer polynomials

$\mathcal{F}_j(\xi,t)$ : Moments of GPDs to be parameterized

Whereas GPDs in x-space can be reconstructed by resumming all the moments through a complex integral in the moment space.

$$F(x,\xi,t) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \frac{p_j(x,\xi)}{\sin(\pi[j+1])} \mathcal{F}_j(\xi,t) \ ,$$

Example of reconstructed GPD in x-space with its moments



# GUMP parameterization

Moments of GPDs are polynomials of $\xi$, so they can be written as

$$\mathcal{F}_j(\xi,t) = \mathcal{F}_{j,0}(t) + \xi^2 \mathcal{F}_{j,2}(t) + \xi^4 \mathcal{F}_{j,4}(t) + \cdots$$

The first term describes GPDs at $\xi = 0$, and is parameterized as:

$$\mathcal{F}_{j,0}(t) = N \frac{B(j+1-\alpha'(t), 1+\beta)}{B(2-\alpha, 1+\beta)} R(t)$$

**Beta function** $NB(j+1-\alpha, 1+\beta)$

Correspond to the simple PDF ansatz in the forward limit

$$Nx^{-\alpha}(1-x)^{\beta}$$

**Regge trajectory** $\alpha(t) = \alpha + \alpha' t$

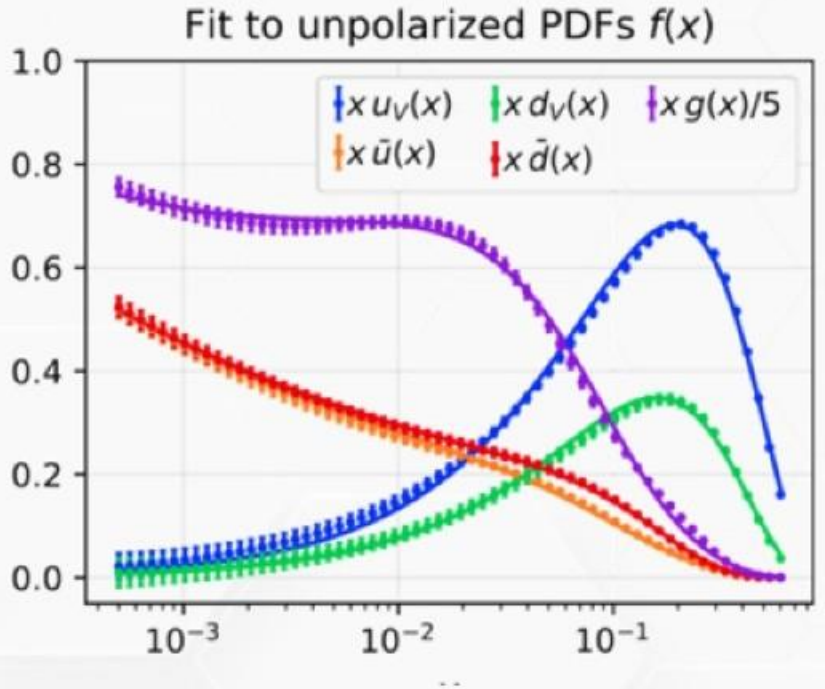
Modification from Regge theory in the form of

$$x^{-\alpha(t)}$$

**Residual terms** $R(t)$

Extra factorized t dependence for more flexibility

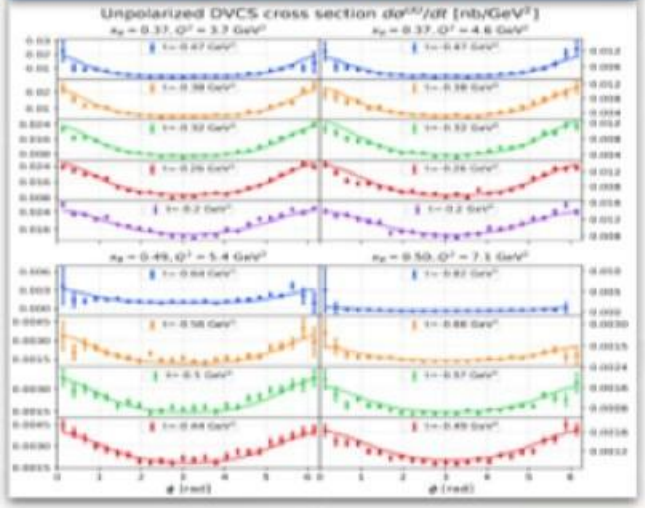
$$\exp(-bt)$$
$$(1 - t/M^2)^{-p}$$



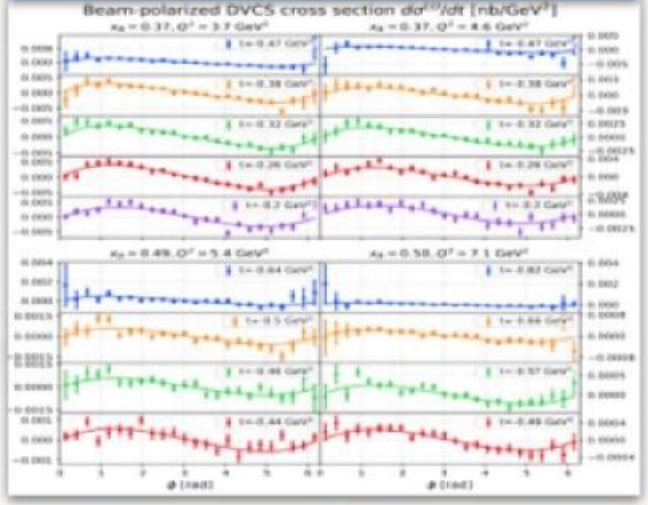
# Experimental inputs for GPDs

We include various kinds of exclusive measurements from JLab and HERA

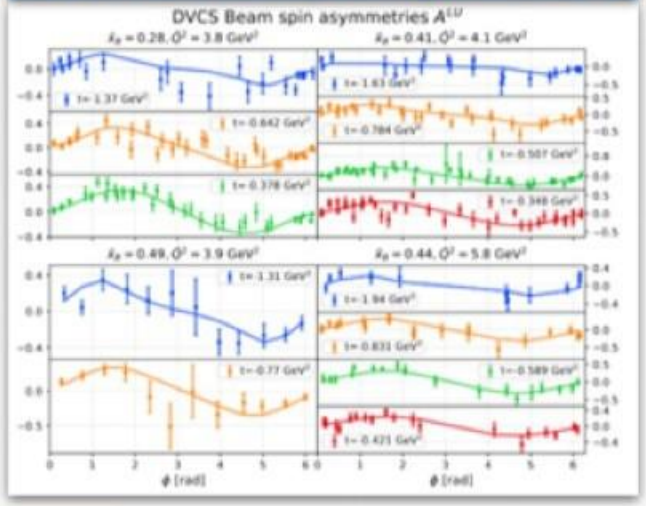
## Unpolarized DVCS



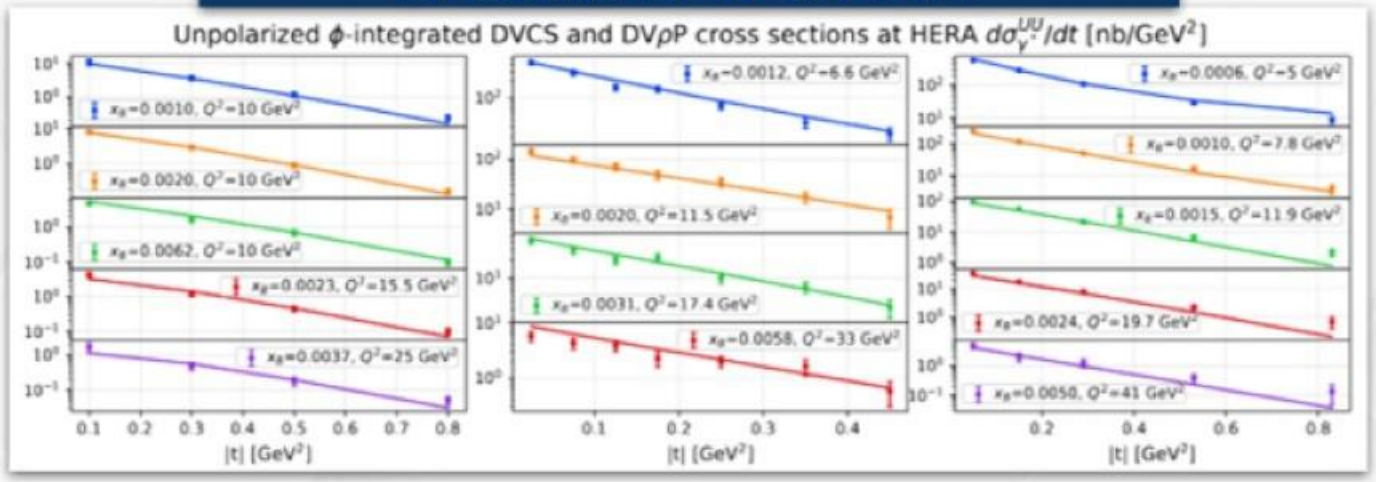

## Beam Polarized DVCS




## DVCS Beam-spin asymmetries



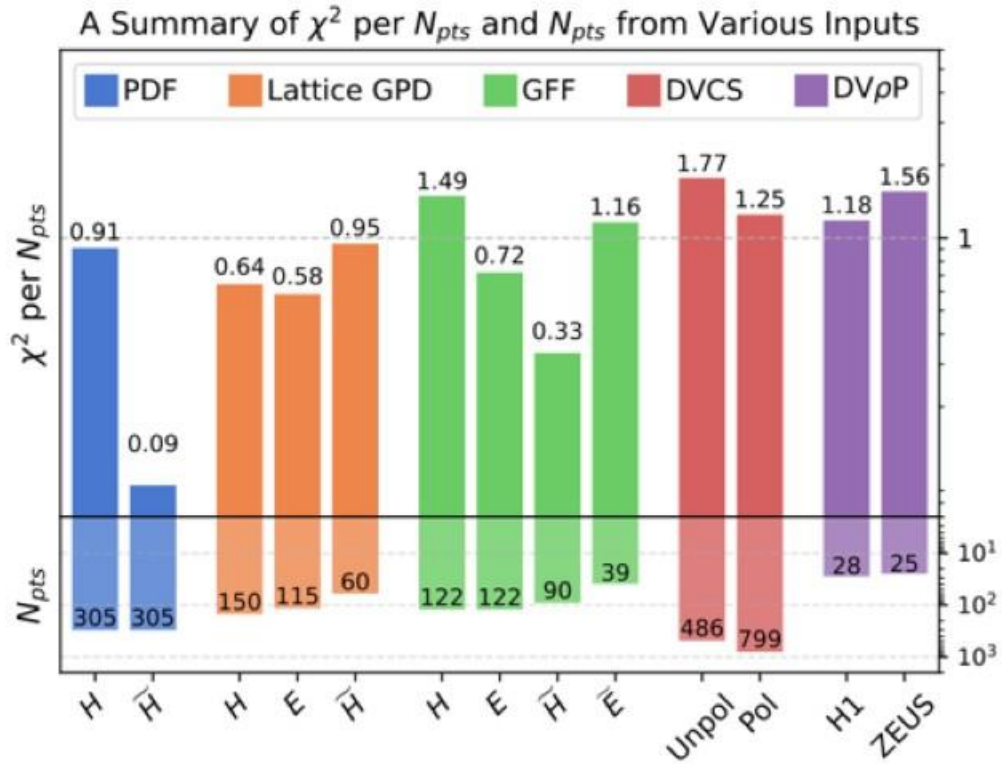

## DVCS and DVMP at HERA



# GPD extraction across exclusive processes

**High-precision data is an essential component for the extraction of GPDs from e.g. global analysis.**

The recent GUMP1.0 global extraction of GPDs using the high-precision JLab data of DVCS along with HERA DVCS and DVMP measurements and other inputs including lattice.



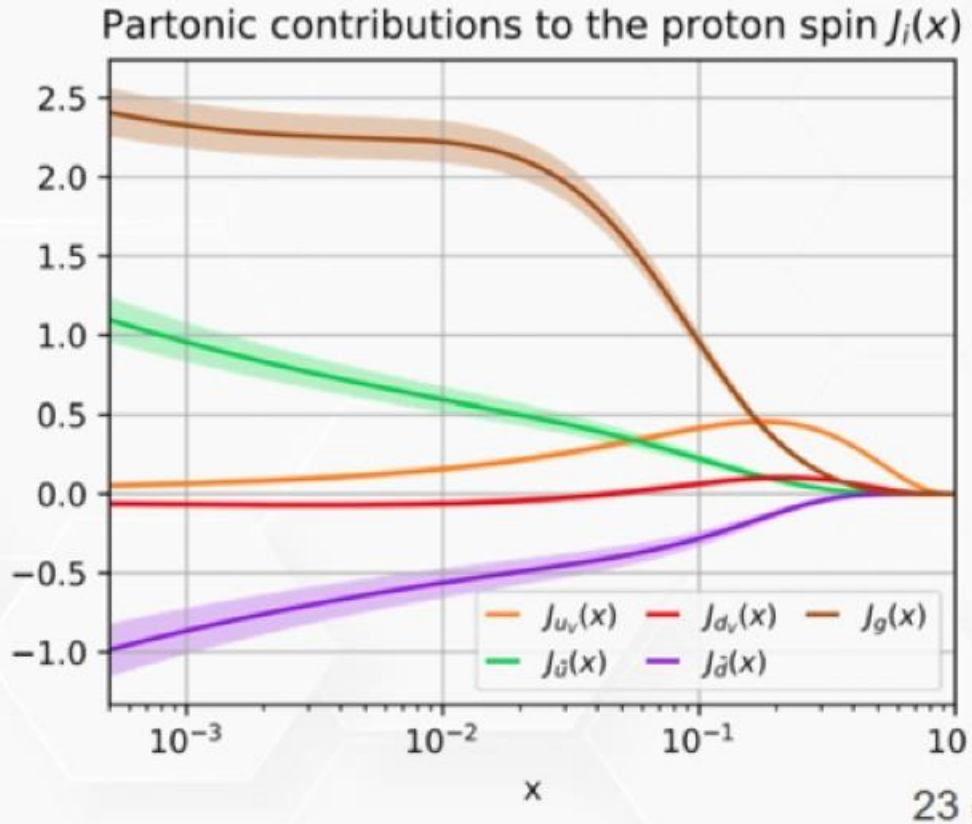

*Guo et al. (2025)*

# Partonic contributions to the proton spin

The partonic contributions to the proton spin can also be obtained from the extracted GUMP1.0 GPDs. Specifically, they are given by

$$J_{q/g}(x) = \frac{1}{2}\left(H_{q/g} + E_{q/g}\right)(x, \xi = 0, t = 0)$$

- Parametric biases/errors not considered
- Consistent with most existing constraints
- Some tension with the lattice GFFs (due to exp.)
- Proton spin sum rule satisfied!



# Nucleon tomography with GUMP1.0 GPDs

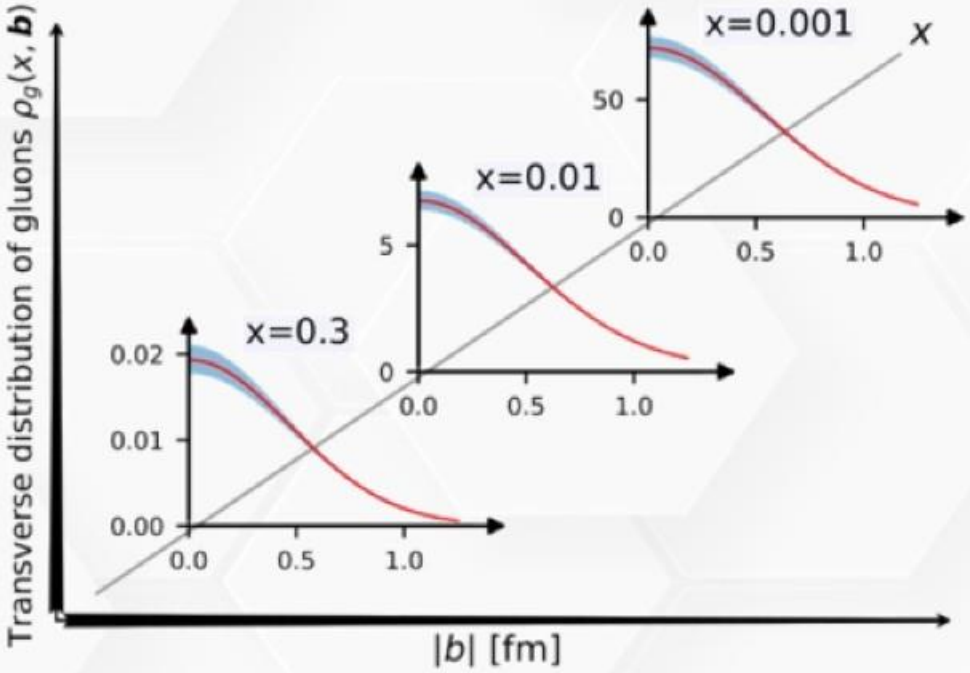
Of course we can also study the nucleon tomography via:

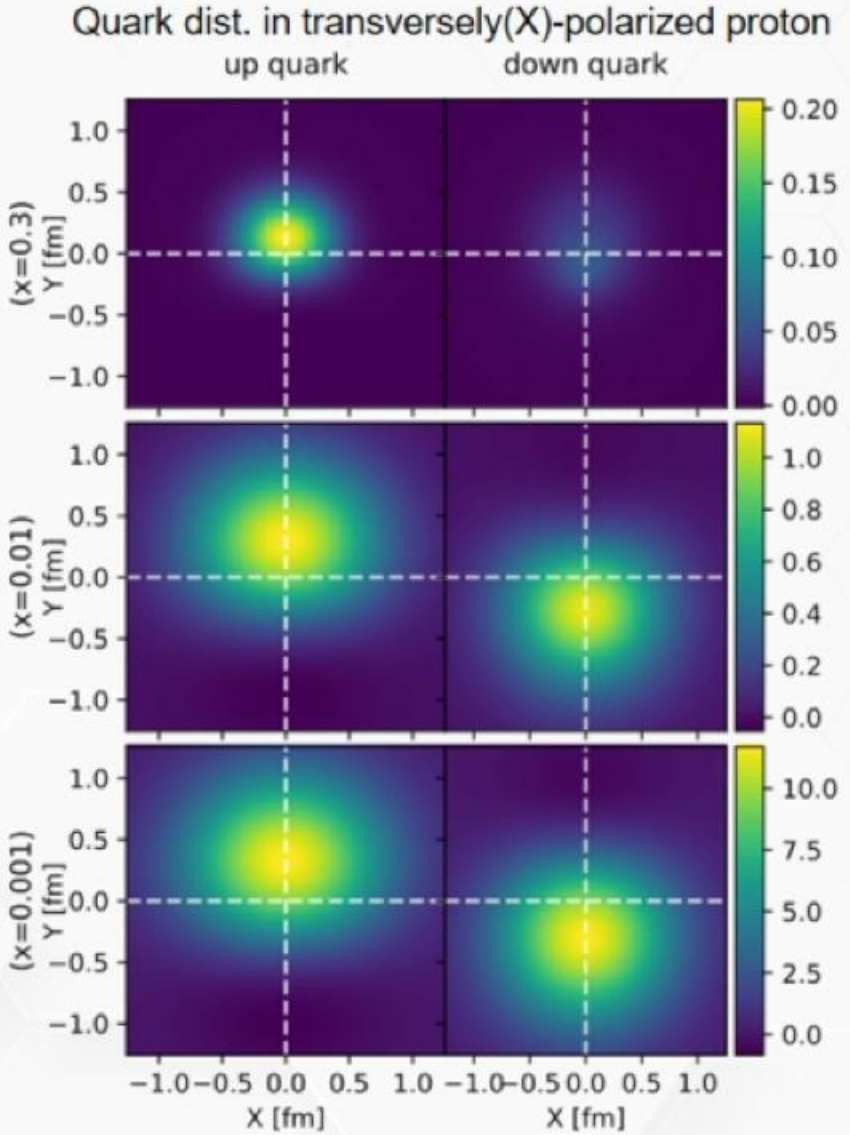
$$\rho_{q/g}(x, \boldsymbol{b}) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}} H_{q/g}(x, -\boldsymbol{\Delta}^2)$$

And the ones for transversely polarized proton

$$\rho^X_{q,\mathrm{In}}(x, \boldsymbol{b}) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}} \left[ H_q(x, -\boldsymbol{\Delta}^2) + \frac{i\Delta_y}{2M} \left(H_q + E_q\right)(x, -\boldsymbol{\Delta}^2) \right]$$

Y. Guo, X. Ji and K. Shells. Nucl. Phys. B 969 (2021)

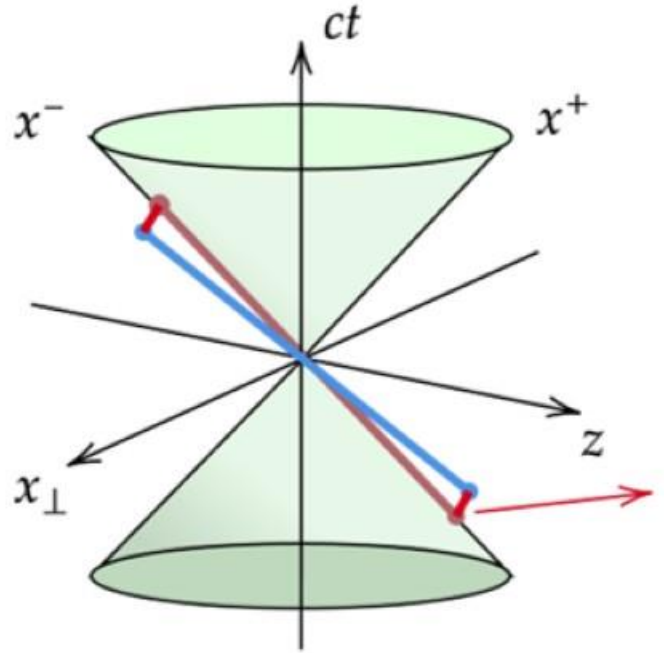


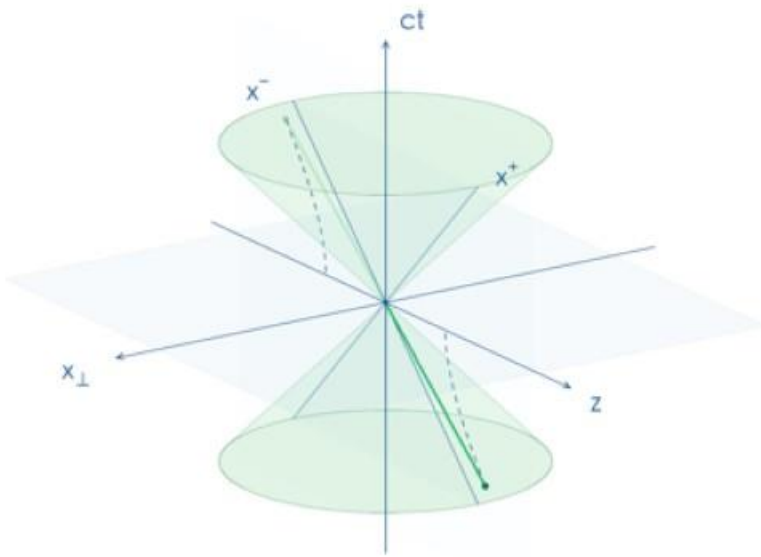



# Lattice calculations

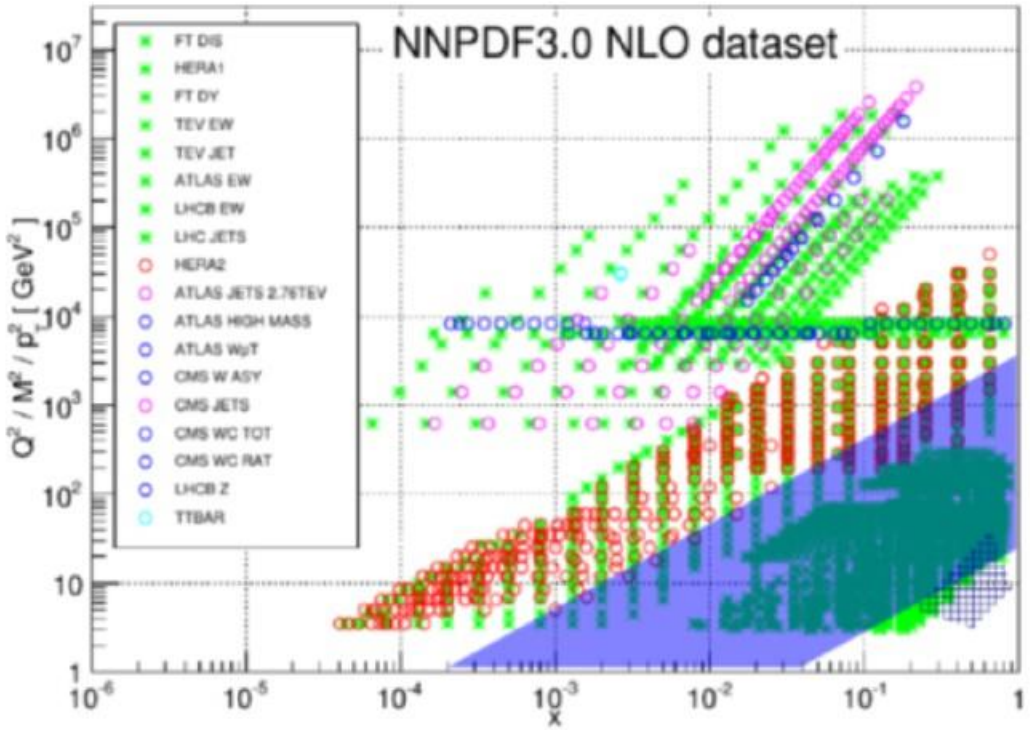
# Large momentum effective theory (LaMET)

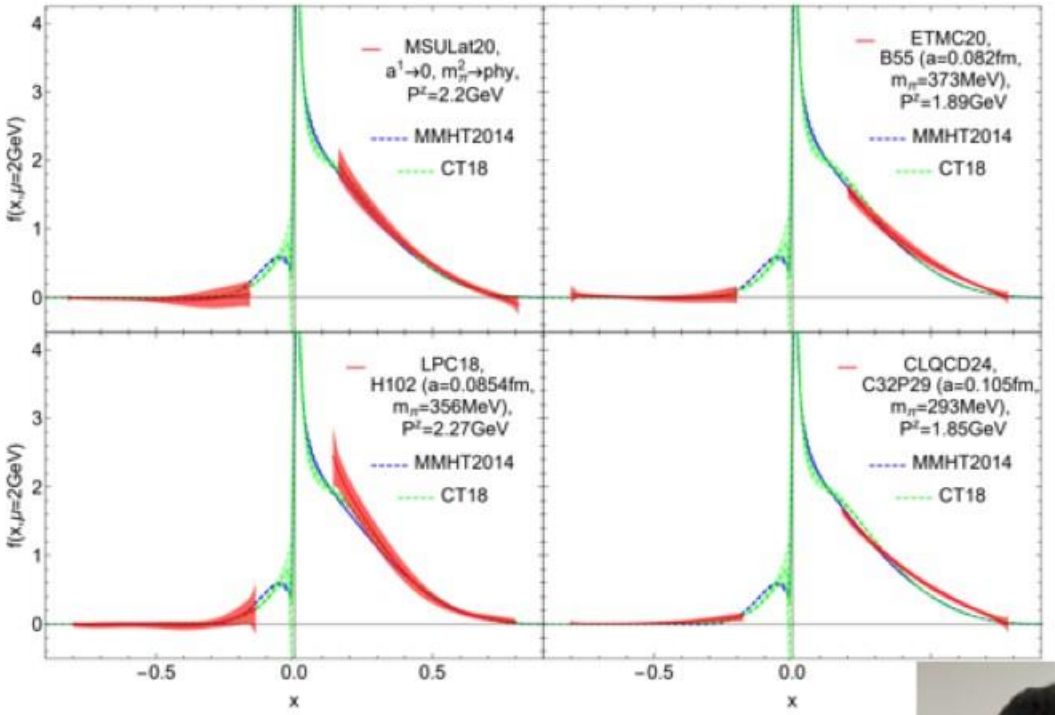
- A method to calculate x-dependent PDFs, GPDs, and TMDs (X. Ji, 2013)
- Light-cone physics is a critical phenomena



# u(x)-d(x) PDF (MSULat20, LPC18, ETMC20, CLQCD24)



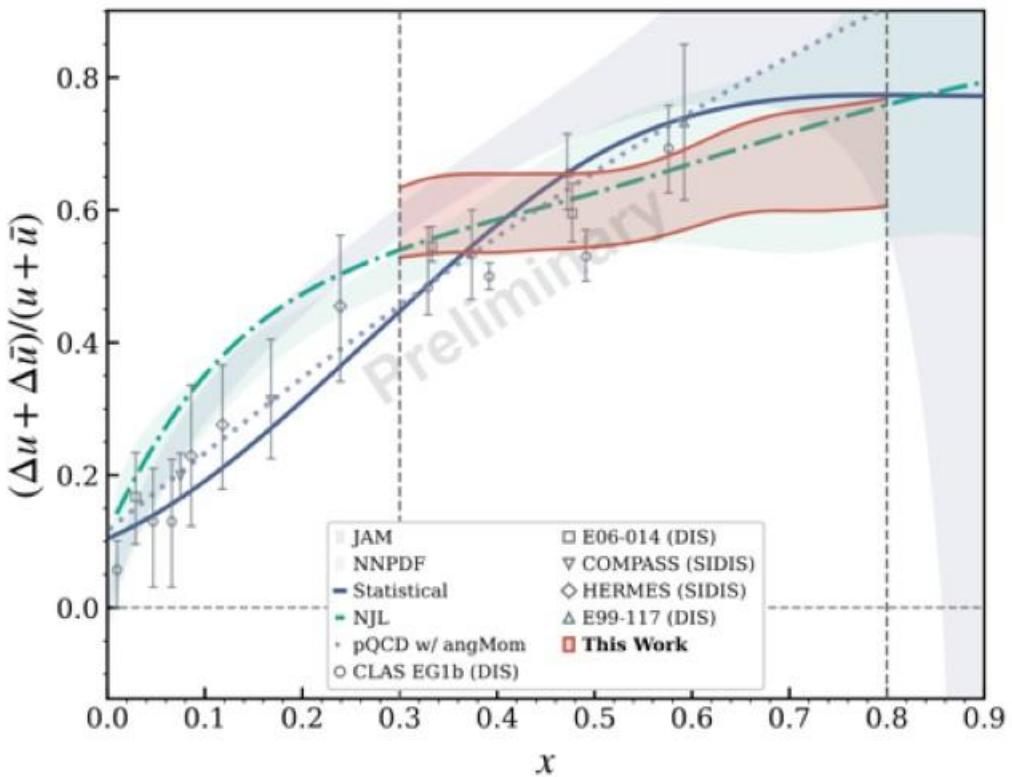



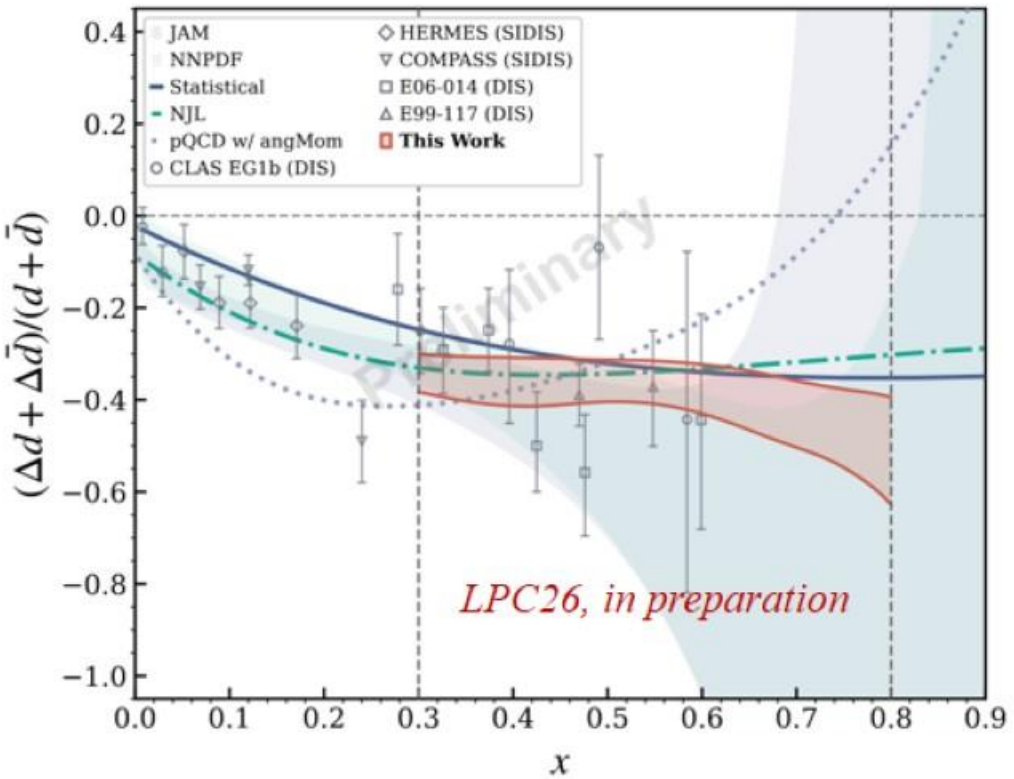

Ji & Su, 2016, e-Print: 2606.22224
to appear in PRL



# Ratios of Polarized PDFs from Lattice QCD (LPC)

- Polarization ratios $(\Delta q + \Delta\bar{q})/(q + \bar{q})$ at $\mu = 2$ GeV, from LPC 2026, to be published

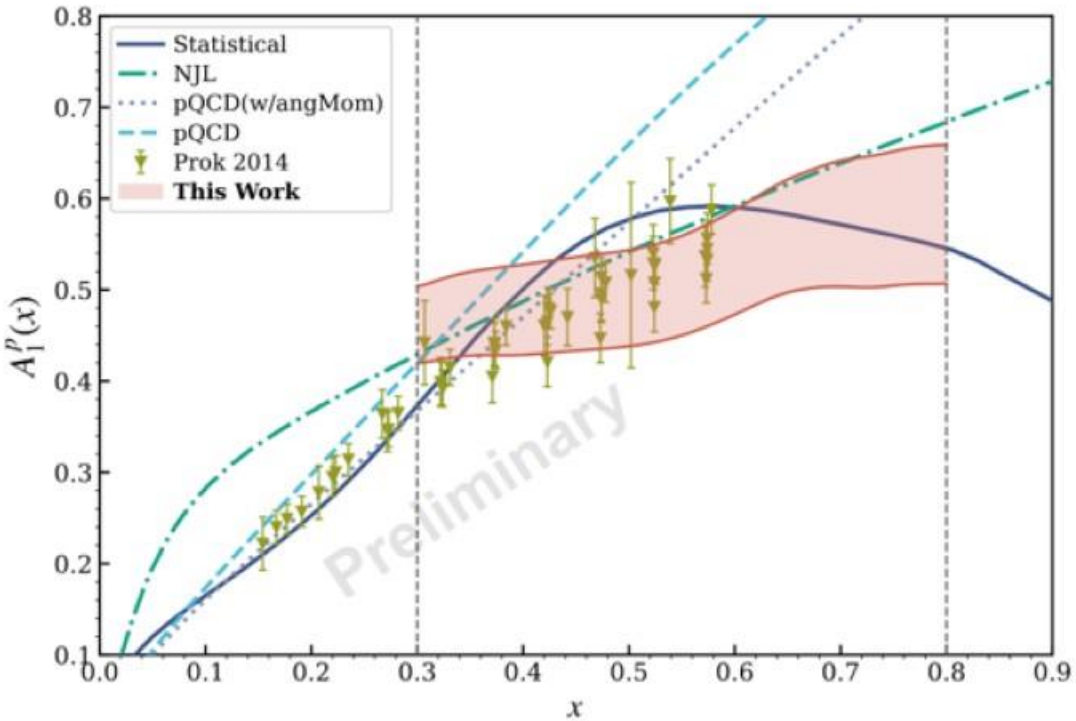


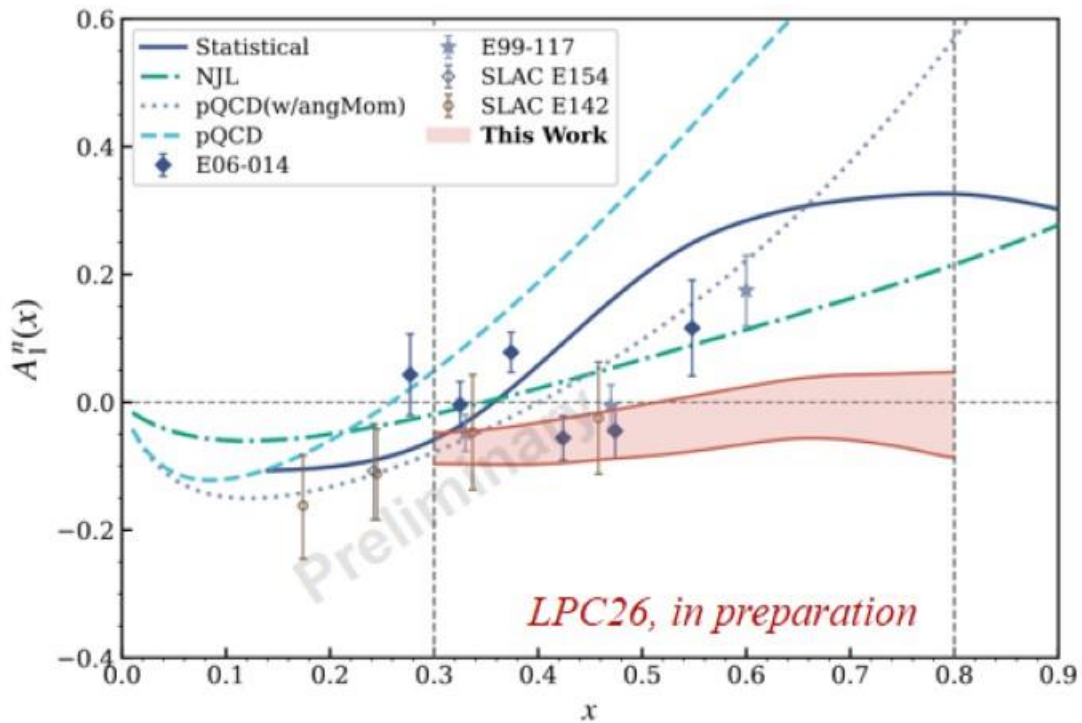



# Predictions for Nucleon Spin Asymmetries (LPC)

- Neglecting the strange quark, sea quark flavor separation and higher-twist corrections, the proton and neutron asymmetries:

$$A_1^p(x) \simeq \frac{4\Delta u(x) + \Delta d(x)}{4u(x) + d(x)}, \qquad A_1^n(x) \simeq \frac{\Delta u(x) + 4\Delta d(x)}{u(x) + 4d(x)}.$$



# Lattice GPD calculations

TABLE I Representative lattice calculations of $x$-dependent quark GPDs within LaMET. All nucleon results refer to the isovector $u-d$ combination. Blank entries within an ensemble block share the pion mass and lattice spacing specified in its first row.

| GPDs | Hadron | $\xi$ | $m_\pi$ | $a$ | Source |
|---|---|---|---|---|---|
| $H_v^\pi$ | $\pi$ | 0 | 310 MeV | 0.12 fm | (Chen *et al.*, 2020) |
| $H_v^\pi$ | $\pi$ | 0 | 135 MeV | 0.09 fm | (Lin, 2023) |
| $H_v^\pi$ | $\pi$ | 0 | 300 MeV | 0.04 fm | (Ding *et al.*, 2025) |
| $H, E, \tilde{H}, \tilde{E}$ | $N$ | 0, 1/3 | 260 MeV | 0.093 fm | (Alexandrou *et al.*, 2020c) |
| $H_T, E_T, \tilde{H}_T, \tilde{E}_T$ | $N$ | 0, 1/3 | 260 MeV | 0.093 fm | (Alexandrou *et al.*, 2022) |
| $H, E$ | $N$ | 0 | 260 MeV | 0.093 fm | (Bhattacharya *et al.*, 2022) |
| $\tilde{H}$ | $N$ | 0 | 260 MeV | 0.093 fm | (Bhattacharya *et al.*, 2024b) |
| $H_T, E_T, \tilde{H}_T$ | $N$ | 0 | 260 MeV | 0.093 fm | (Bhattacharya *et al.*, 2025b) |
| $H, E$ | $N$ | 1/7, $\pm$1/5, $\pm$1/2 | 260 MeV | 0.093 fm | (Chu *et al.*, 2025) |
| Twist-three GPDs | $N$ | 0 | 260 MeV | 0.093 fm | (Bhattacharya *et al.*, 2023a) |
| $H, E$ | $N$ | 0 | 135 MeV | 0.09 fm | (Lin, 2021) |
| $\tilde{H}$ | $N$ | 0 | 135 MeV | 0.09 fm | (Lin, 2022) |
| $H, E$ | $N$ | 0, 0.1 | 135 MeV | 0.09 fm | (Holligan and Lin, 2024) |

# $\xi = 0$ pion GPD result

- 在

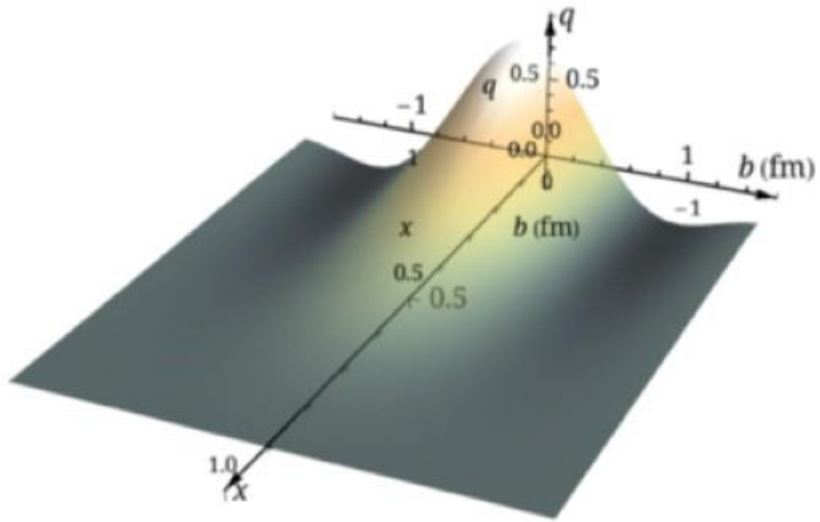



FIG. 13 The first lattice determination of the nucleon impact-parameter-space distribution as a function of $x$ and $b_\perp$, obtained from $H$ at the physical pion mass. From Lin (2021).

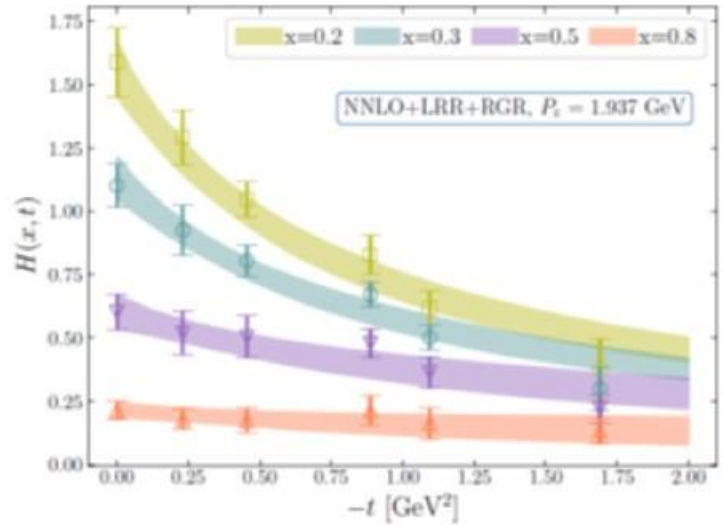



FIG. 12 Zero-skewness pion valence GPD as a function of $-t$ for several values of $x$. The shaded bands show monopole fits to the $t$ dependence. From Ding *et al.* (2025).

# GPDs when $x \neq \xi$

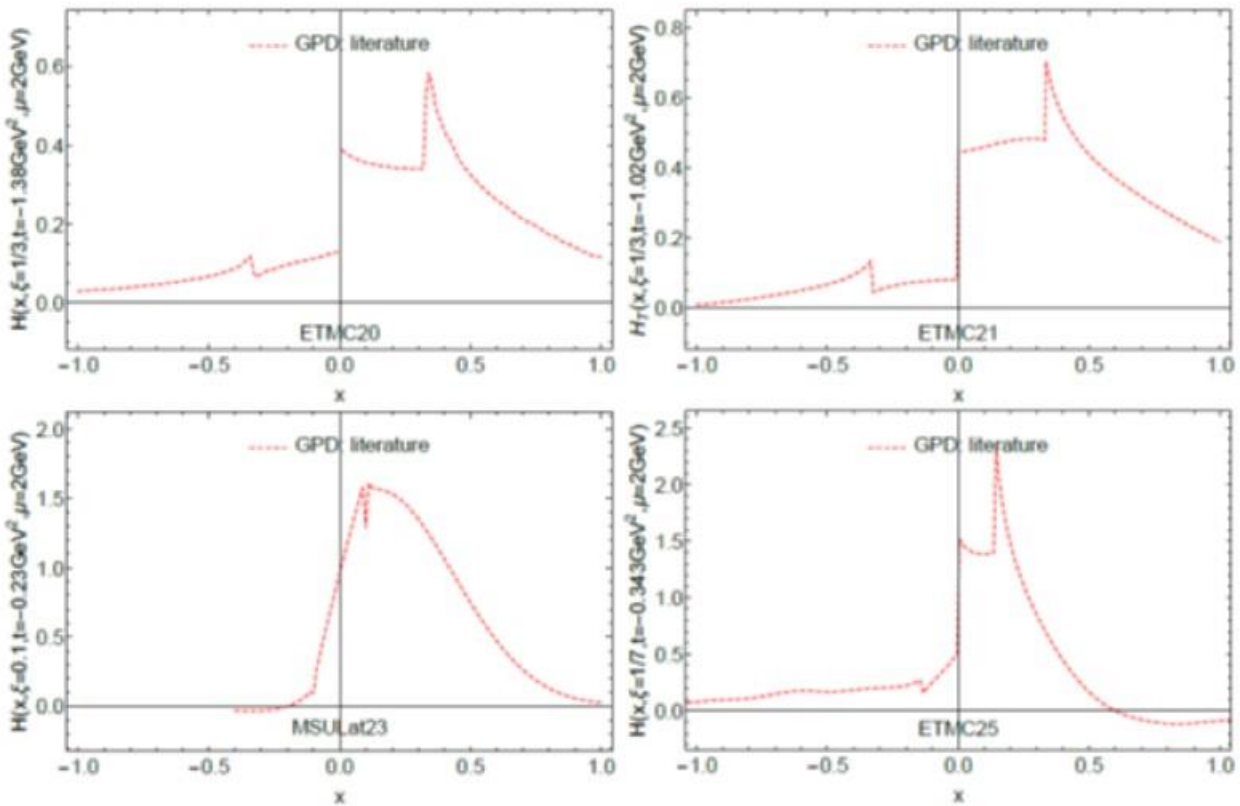
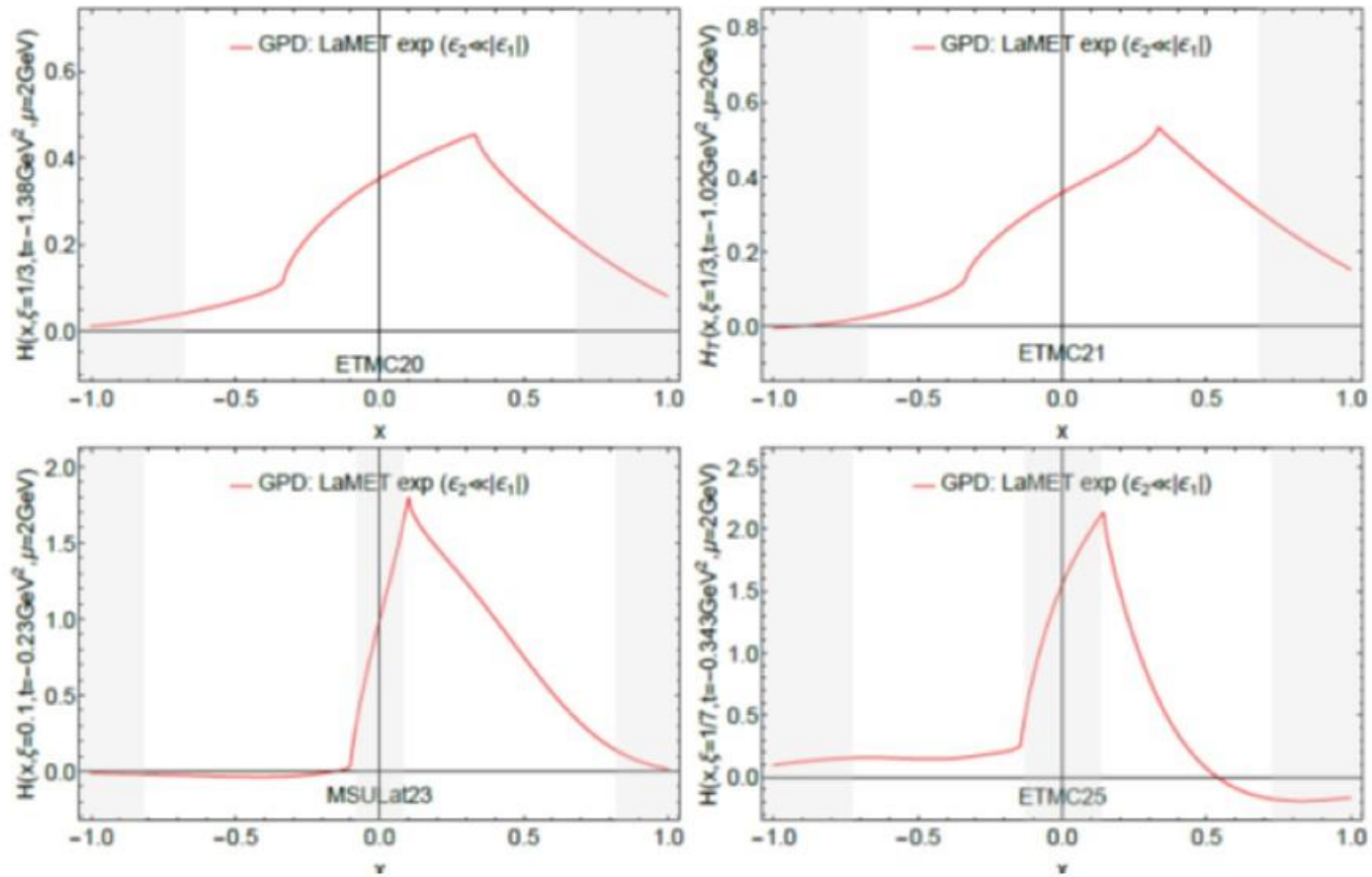



FIG. 2. Examples of non-zero skewness unpolarized $H$ or transversity $H_T$ GPDs from original works [26, 29, 40, 45]. The ETMC20 [26], ETMC21 [29], and MSULat23 [40] results from the relevant plots in the papers, and the ETMC25 [45] curve is obtained through private communication.

# Fixing the x=$\xi$ problem






**Y. Su, X. Ji, Y. Liu, R. Zhang, hep-latt/2609.20545**

# Progress in understanding $\Delta G$

- There is no gauge-invariant local axial vector corresponding to the gluon spin.
- There is a gauge-dependent candidate for $\Delta G$

$$\vec{S}_g = \overrightarrow{E_a} \times \vec{A}_a$$

- In the infinite-momentum limit, the gauge-dependent part of the above operator vanishes

  X. Ji, J. Zhang & Y. Zhao Phys.Rev.Lett. 111 (2013)

- This provides a recipe for lattice QCD calculation

# Total Gluon Helicity Contribution to the Proton Spin from Lattice QCD

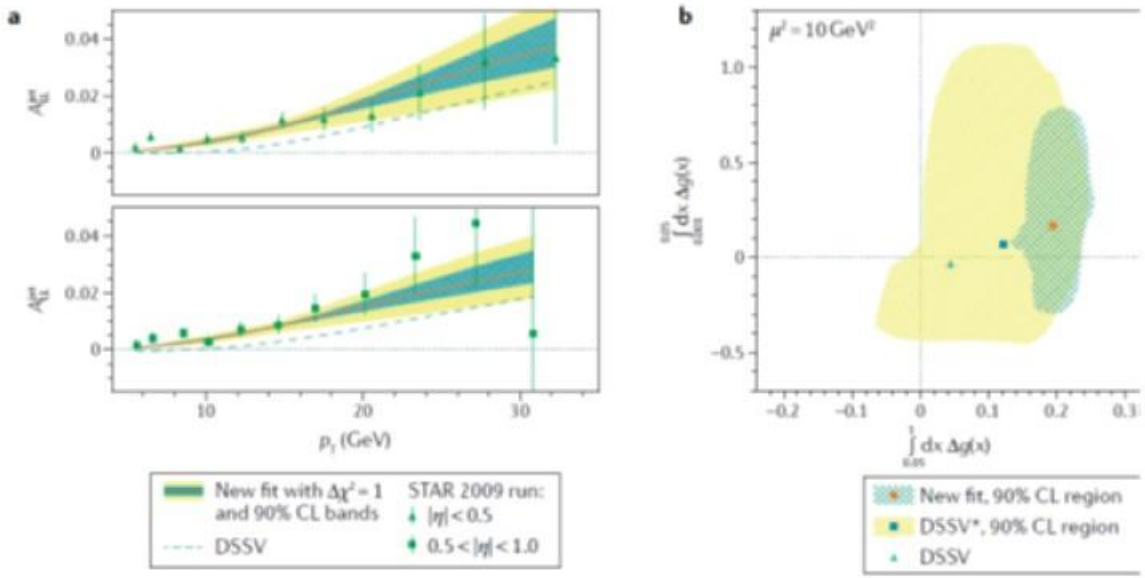
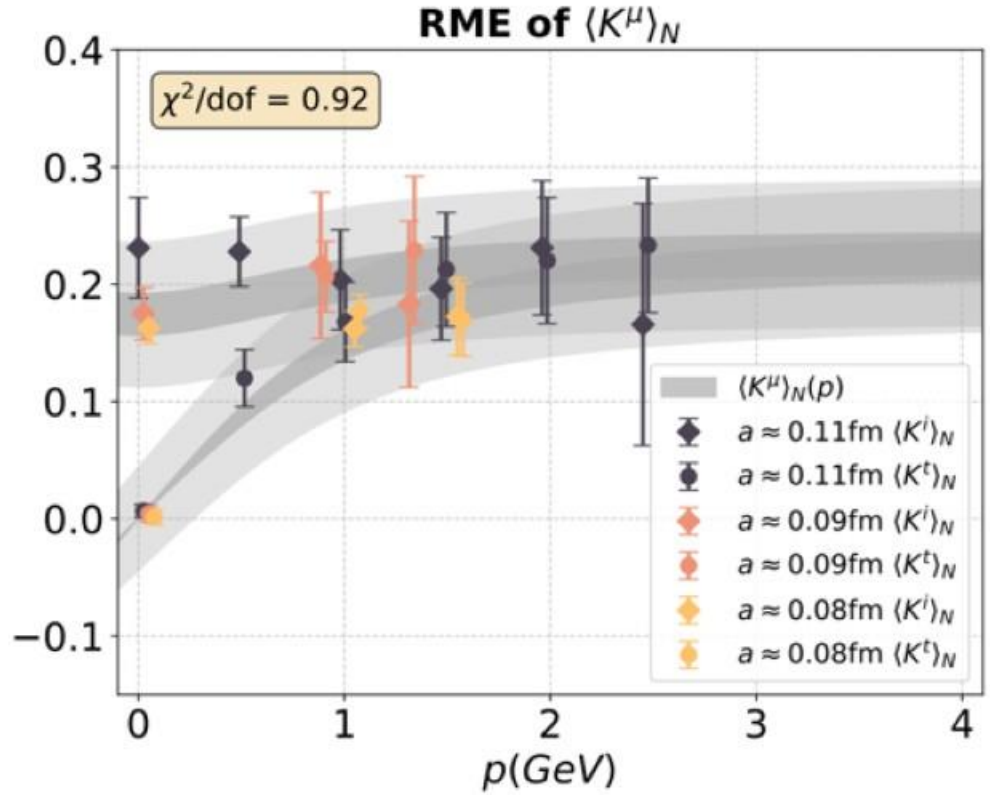



Fig. 2 | **The Relativistic Heavy Ion Collider at Brookhaven National Laboratory provides strong evidence for the gluon helicity contribution to the proton spin. a** | Double spin asymmetry $A_{LL}^{jet}$ in inclusive jet production measured



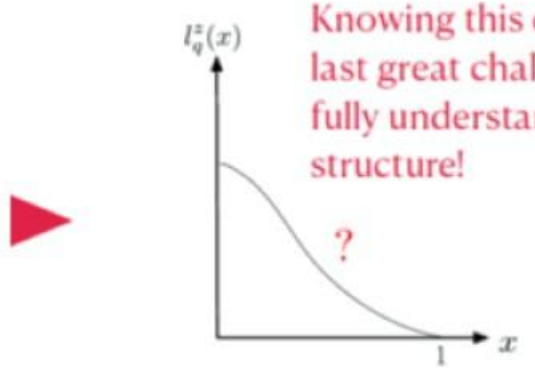

$$\Delta G\left(\mu^2 = 10\,\mathrm{GeV}^2\right) = 0.231(17)^{\mathrm{sta.}}(44)^{\mathrm{sym.}}$$

D.-J. Zhao et al., arXiv:2512.24315 (2025).

# Twist-3 GPDs & sum rules

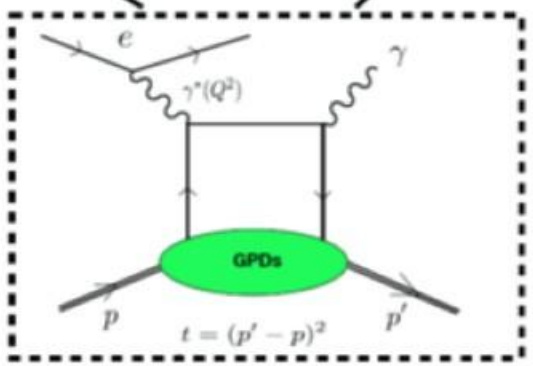
# Parton OAM & twist-3 GPD

$$l_q^z = \int dx l_q^z(x) \longrightarrow l_q^z(x) = \int dy \left[ G_{q,D,3}(x,y) + \mathcal{P}\frac{1}{y-x} G_{q,F,3}(x,y) \right]$$




OAM density

Knowing this curve will be the last great challenge in fully understanding proton spin structure!




Twist-3 GPDs extractable from DVCS & DVMP

Very very challenging! It will be a while before we learn how to measure twist-3 GPDs

# Transverse Polarization Sum Rules:

- Let's look again at transverse AM, but **split in terms of its 2 contributions**:

$$J^x = \underbrace{(\gamma - \frac{1}{2\gamma})J^{x(2)}}_{\xi^y T^{0z}(\xi)} + \underbrace{\frac{1}{2\gamma}J^{x(3)}}_{\xi^z T^{0y}(\xi)} = \gamma\frac{\hbar}{2}$$

Note: equal in rest frame by rotational symmetry

$J^{x(2)}$ & $J^{x(3)}$ ~ proton matrix elements of $q$ and $g$ fields

- Solution:

$$J^{x(2)} = J^{x(3)} = \frac{\hbar}{2}$$

Twist-2 Ji Sum Rule | Novel Twist-3 Sum Rule

Spin sum rules are $\gamma$-independent!

**2 Sum Rules:**

$$J_q^{x(2)} + J_g^{x(2)} = \frac{\hbar}{2} \quad \text{Twist-2}$$

$$J_q^{x(3)} + J_g^{x(3)} = \frac{\hbar}{2} \quad \text{Twist-3}$$

# Twist-3 transverse spin sum rule

- Rotated version of the Jaffe Manohar Longitudinal spin sum rule

X. Ji, Y. Guo & K. Shiells, Nuc. Phys. B 969, 115440 (2021)

Transverse Polarization: $\frac{1}{2}\Delta q_T + \Delta G_T + l_q^{x(3)} + l_g^{x(3)} = \frac{\hbar}{2}$

$\Delta q_T$ , $\Delta G_T$ Involve measurable PDFs in DIS and correspond to spin

$l_q^{x(3)}$ , $l_g^{x(3)}$ Involve twist-3 GPDs and correspond to canonical OAM

- $\Delta q_T$ is related to $g_2$ structure function
- $\Delta G_T$ is a twist-three, spin-dependent gluon distribution
- Twist-3 OAM are a challenge to measure

# $g_2$ structure function

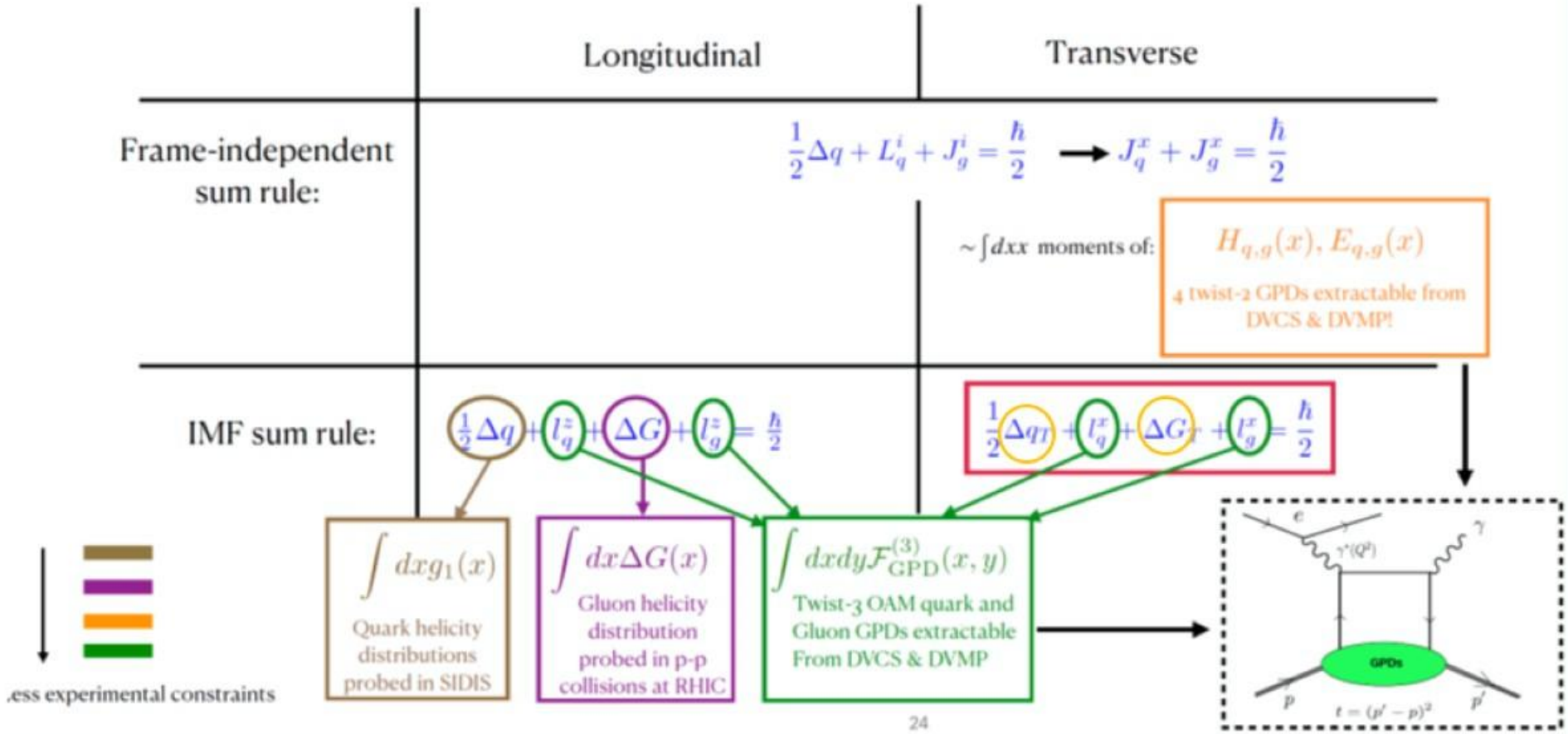
- Can be measured with a transversely polarized target.
- Jlab 6 GeV, 12 GeV
- Moments have been calculated on lattice in the past
- X-dependent distribution can be calculated using large momentum effective theory

# $\Delta G_T$: twist-3 gluon polarization

- When the target is transversely polarized, there is a twsit-3 gluon density.
- This mixes with g_2 in factorization formula.
- No experimental data.
- Can be calculated on lattice.

$$\langle PS | G^{+i}_{(0)} \tilde{G}^{\perp i}_{a,x} | P \rangle$$

# Experimental Roadmap for spin sums



# Conclusions

- The simplest spin sum rule is twist-2 transverse spin sum rule
- For long. pol., Jaffe-Manohar sum rule is a twist-3 one. and the frame independent sum rule does not have a simple parton interpretation,
- There is a frame-rotated version of transverse spin sum rule which involves $g_2$ and $\Delta G_T$
- All spin sum rules involve GPDs, but getting twist-2 GPDs will take a lot of efforts.