

GPDs from Lattice QCD: Recent Progress with LaMET



HUEY-WEN LIN

PacificSPIN 2026

This work of HL is supported by the NSF under grant PHY 2209424 & 1653405, and DOE under DE-SC0024053 & DE-SC0024644

@LinQCD

NSF

U.S. DEPARTMENT OF ENERGY

EXCLAIM

The Ultimate QCD Microscope



Imaging of the proton

How are the sea quarks and gluons, and their spins, distributed in space and momentum inside the nucleon?

GPDs This talk

Next talk **TMDs**

EIC White Paper, 1212.1701



… (many other whitepapers)

The Present and Future of QCD (2303.02579)

Precision QCD with the EIC (2604.04765) → INT Program Report

What is Lattice QCD?

Image credit: Marvel Studios



What is Lattice QCD?

§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

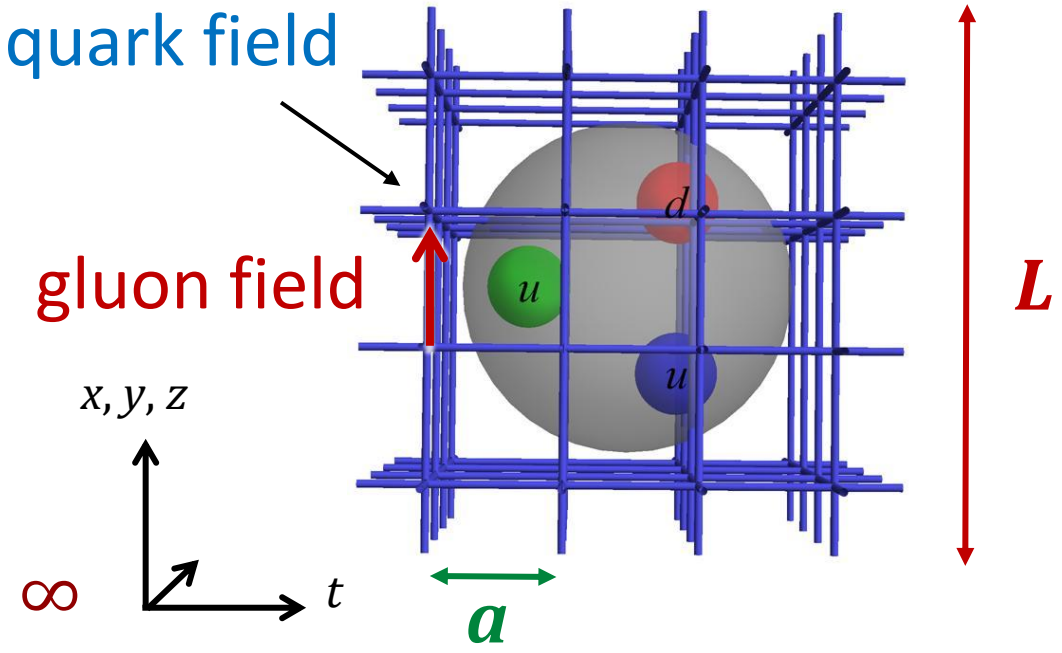
$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)} O\,(\bar{\psi},\psi,A)$$

in **Euclidean** space

- Quark mass parameter (described by m_π)
- Impose a UV cutoff
  discretize spacetime
- Impose an infrared cutoff
  finite volume

§ Recover physical limit

$m_\pi \to m_\pi^{\text{phys}}$, $\boldsymbol{a \to 0}$, $\boldsymbol{L \to \infty}$

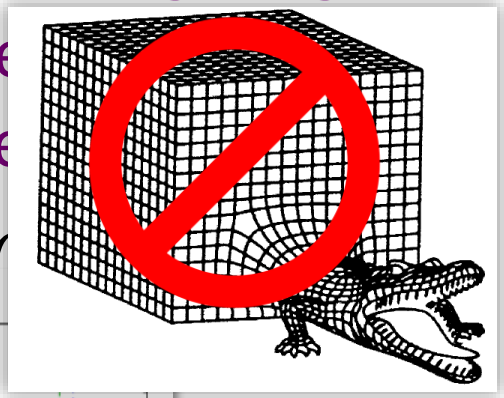


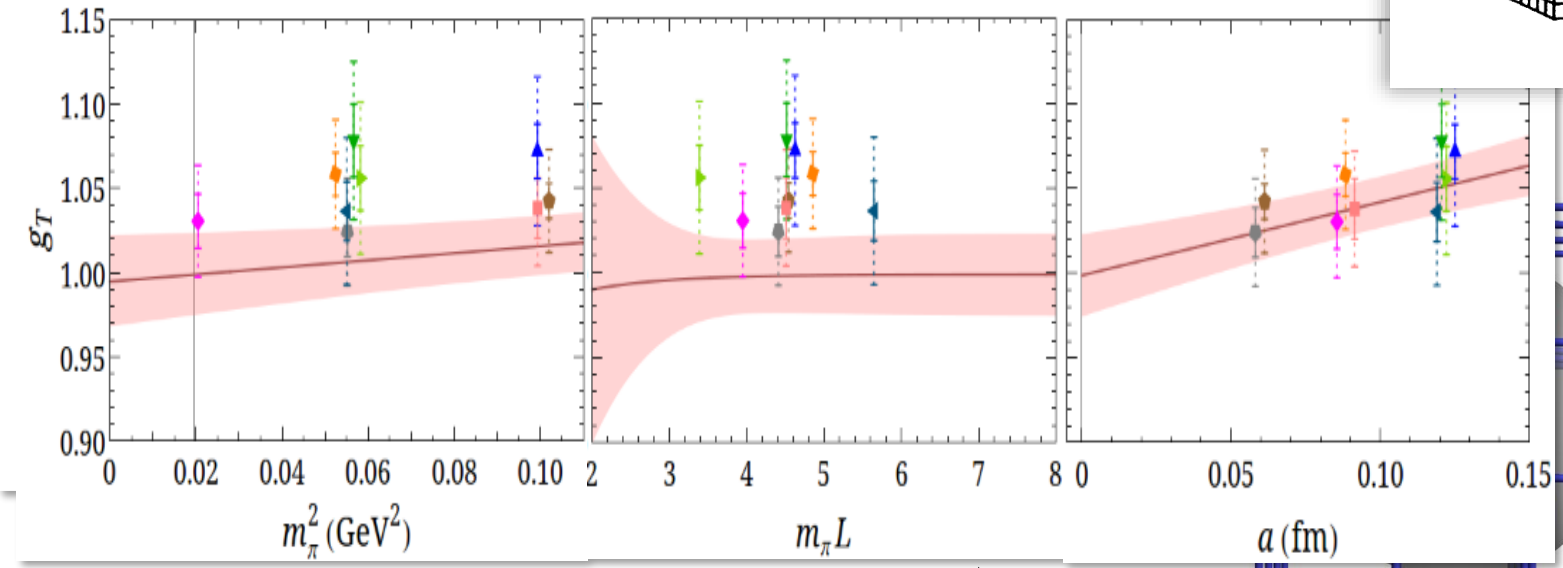
What is Lattice QCD?

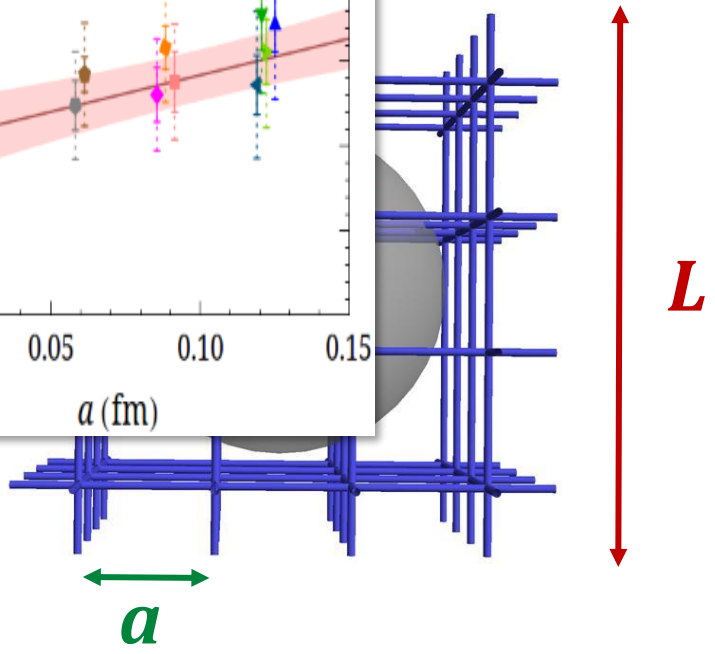
§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field the

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$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\, e^{iS(\bar{\psi},\psi,A)}($$

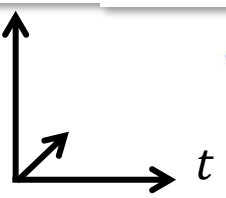






§ Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$



Moments of PDFs

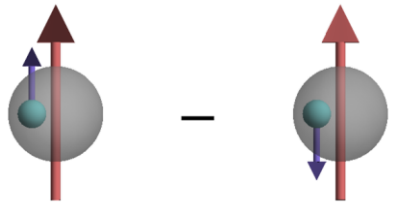
§ PDG-like rating system or average

§ LatticePDF Workshop

↝ Lattice representatives came together and devised a rating system

§ Lattice QCD/global fit status

$$\langle x^{n-1} \rangle_{\delta q} = \int_{-1}^{1} dx\, x^{n-1} \delta q(x)$$



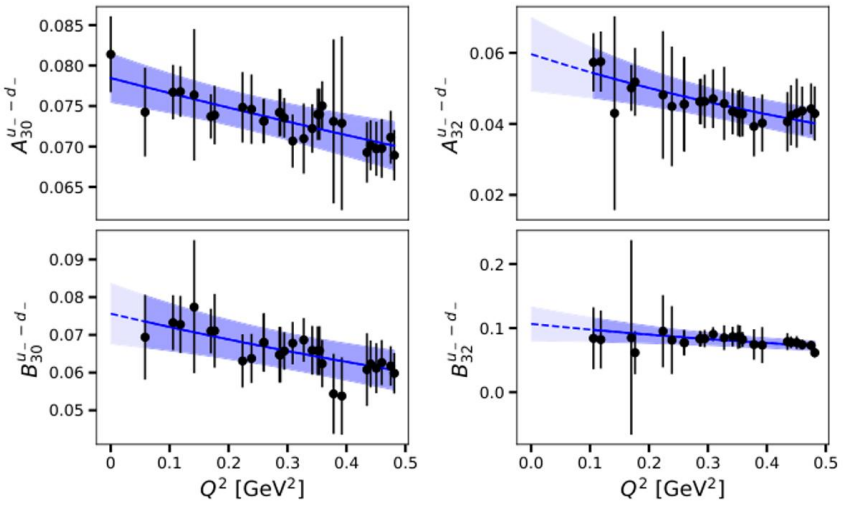
LatticePDF Report, 1711.07916, 2006.08636

Moment	Collaboration	Reference	N_f	DE	CE	FV	RE	ES		Value	Global Fit
g_T	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.926(32)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.989(32)(10)	
	χQCD 20	(Horkel *et al.*, 2020)	2+1	■	★	○	★	★	†	1.096(30)	
	LHPC 19	(Hasan *et al.*, 2019)	2+1	○	★	○	★	★	*	0.972(41)	
	Mainz 19	(Harris *et al.*, 2019)	2+1	★	○	★	★	★		$0.965(38)(^{+13}_{-41})$	0.10 — 1.1
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		1.08(3)(3)(9)	
	ETMC 19	(Alexandrou *et al.*, 2019b)	2	■	★	○	★	★	**	0.974(33)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		1.004(21)(02)(19)	
	RQCD 14	(Bali *et al.*, 2015)	2	○	★	★	★	■		1.005(17)(29)	
$\langle 1 \rangle_{\delta u-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	0.716(28)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	0.784(28)(10)	-0.14 — 0.91
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		0.85(3)(2)(7)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		0.782(16)(2)(13)	
$\langle 1 \rangle_{\delta d-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.210(11)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.204(11)(10)	-0.97 — 0.47
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.24(2)(0)(2)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.219(10)(2)(13)	
$\langle 1 \rangle_{\delta s-}$	ETMC 19	(Alexandrou *et al.*, 2019b)	2+1+1	■	★	○	★	★	**	-0.0027(58)	
	PNDME 18	(Gupta *et al.*, 2018)	2+1+1	★	★	★	★	★	*	-0.0027(16)	N/A
	JLQCD 18	(Yamanaka *et al.*, 2018)	2+1	■	○	○	★	★		-0.012(16)(8)	
	ETMC 17	(Alexandrou *et al.*, 2017d)	2	■	★	■	★	★		-0.00319(69)(2)(22)	

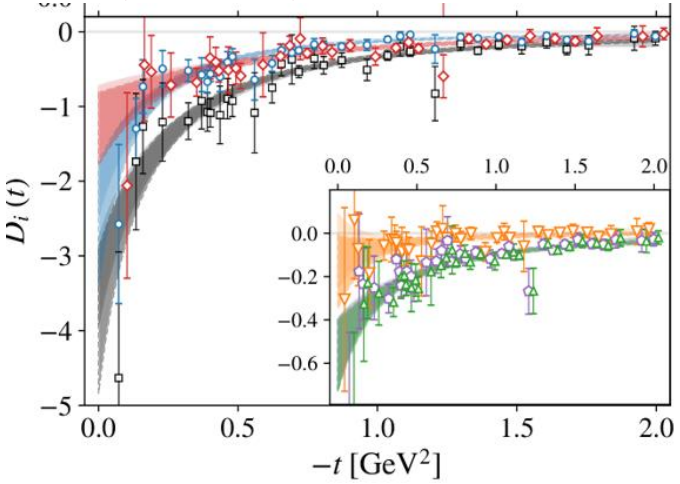
Progress on GPD Moments

§ Topics not covered in this talk

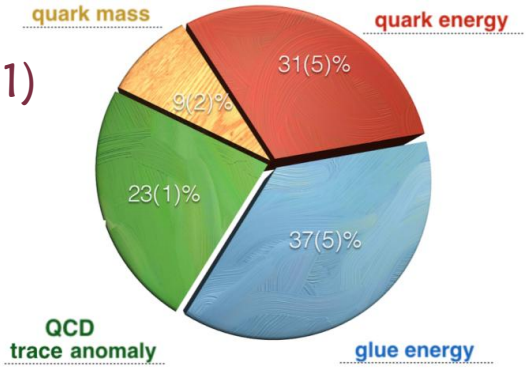
ETMC (2607.03458)



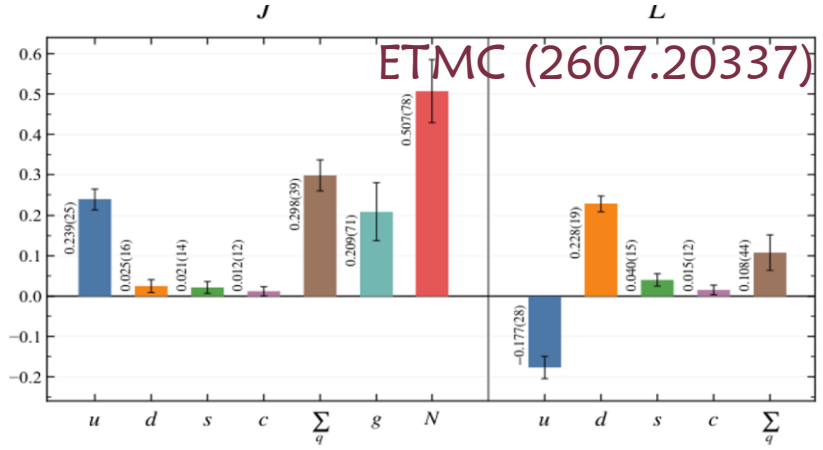
Hackett, Pefkou, Shanahan 2310.08484



XQCD (1710.09011)



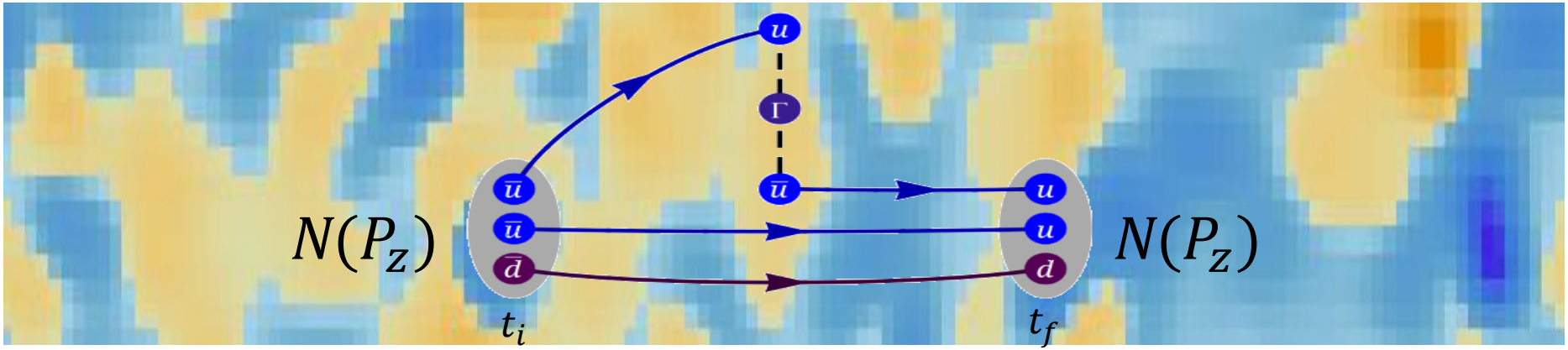
ETMC (2607.20337)



Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF

(X. Ji, 2013; See 2004.03543 for review); X. Ji's talk on Monday, this workshop



§ Compute quasi-distribution via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-izk_z} \left\langle P \left| \bar{\psi}(z)\Gamma \exp\left(-ig\int_0^z dz' A_z(z')\right)\psi(0) \right| P \right\rangle$$

§ Recover true distribution (take $P_z \to \infty$ limit)

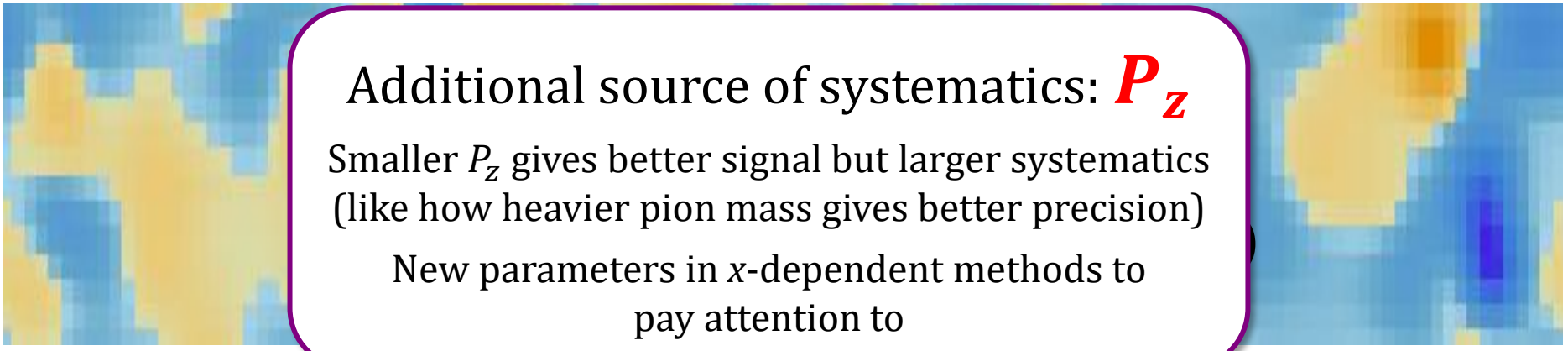
$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y},\frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(\frac{M_N^2}{\boldsymbol{P_z^2}}, \frac{\Lambda_{\rm QCD}^2}{(x\boldsymbol{P_z})^2}, \frac{\Lambda_{\rm QCD}^2}{\big((1-x)\boldsymbol{P_z}\big)^2}\right)$$

X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

Lattice Parton Method

§ Large-momentum effective theory (LaMET)/quasi-PDF

(X. Ji, 2013; See 2004.03543 for review)); X. Ji's talk on Monday, this worksho



Additional source of systematics: P_z

Smaller P_z gives better signal but larger systematics (like how heavier pion mass gives better precision)

New parameters in x-dependent methods to pay attention to

§ Compute quasi-distribution via

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X. Xiong e.a., 1310.7471; J.-W. Chen e.a., 1603.06664

Direct x-Dependent PDFs

§ Longstanding obstacle to lattice calculations!

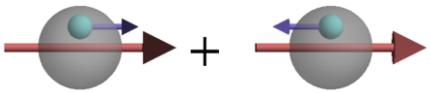


- quasi-PDFs/large-momentum effective theory (LaMET) (X. Ji, 2013; See 2004.03543 for review)
- **Pseudo-PDF** method: differs in FT (A. Radyushkin, 2017)
- Lattice cross-section method (**LCS**) (Y Ma and J. Qiu, 2014, 2017)
- Hadronic tensor currents (Liu et al., PRL72 (1994) 1790, … 1603.07352)
- Euclidean correlation functions (RQCD, 1709.04325)
- …

Lattice Example Results

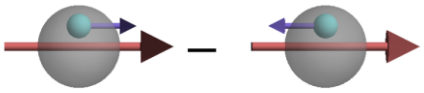
§ Summary of physical pion mass PDFs results

unpolarized



$u(x) - d(x)$

longitudinally polarized

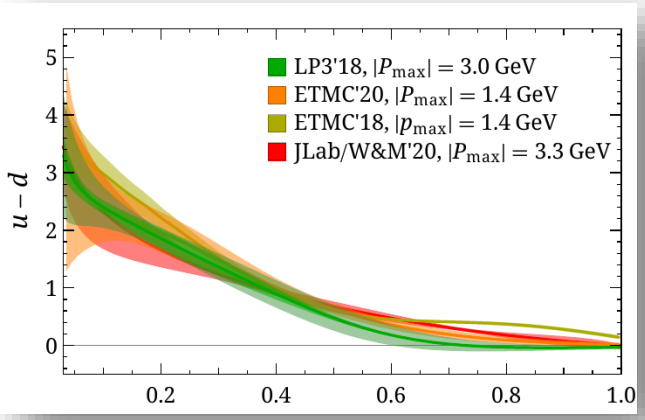


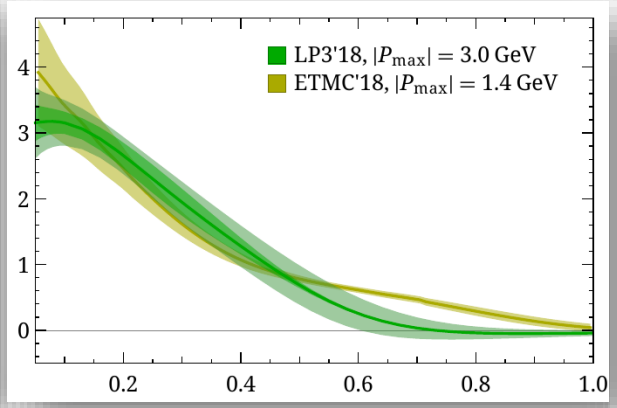
$\Delta u(x) - \Delta d(x)$

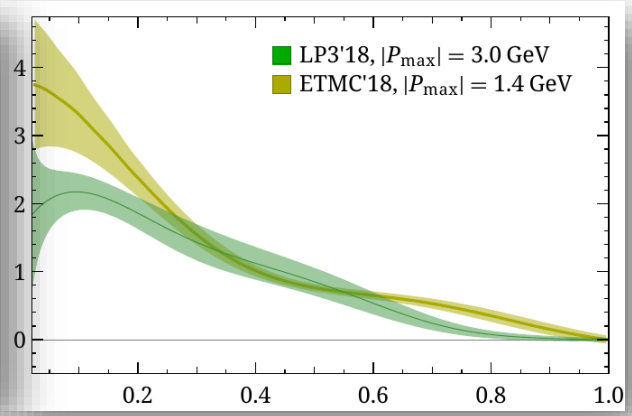
transversely polarized

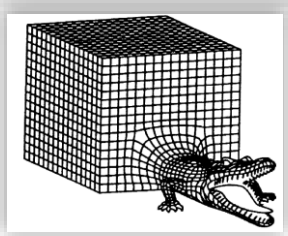


$\delta u(x) - \delta d(x)$









Finite volume,
Discretization,
...

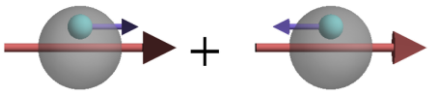


LatticePDF Report,
Prog.Part.Nucl.Phys.121 (2021) 103908

Lattice Example Results

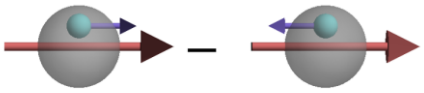
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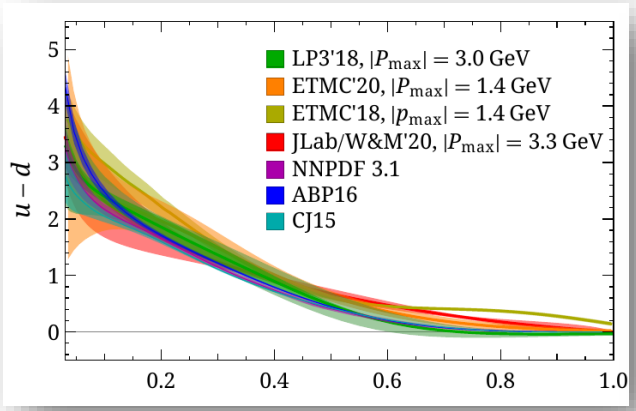


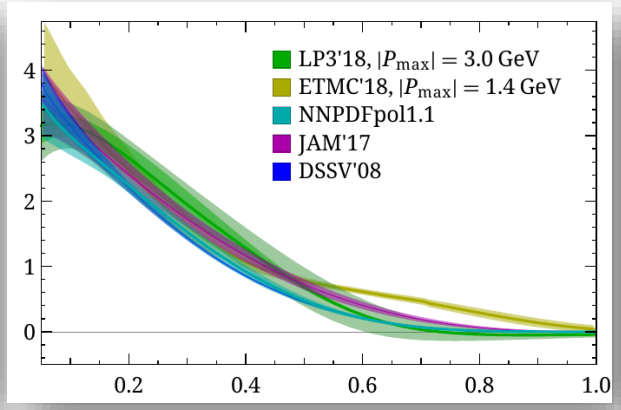
$\Delta u(x) - \Delta d(x)$

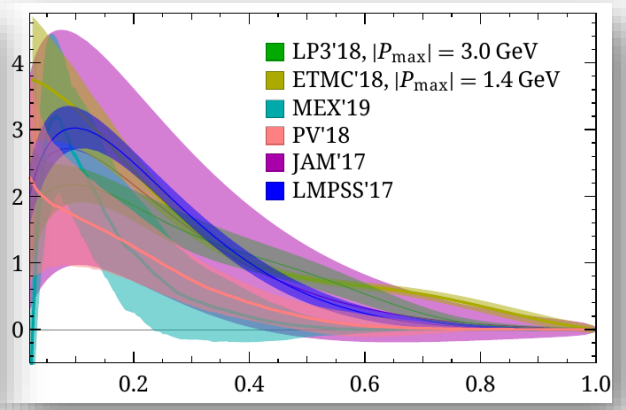
transversely polarized

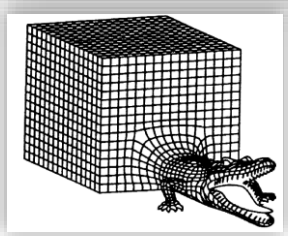


$\delta u(x) - \delta d(x)$

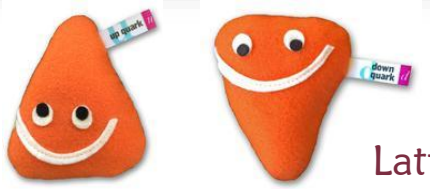








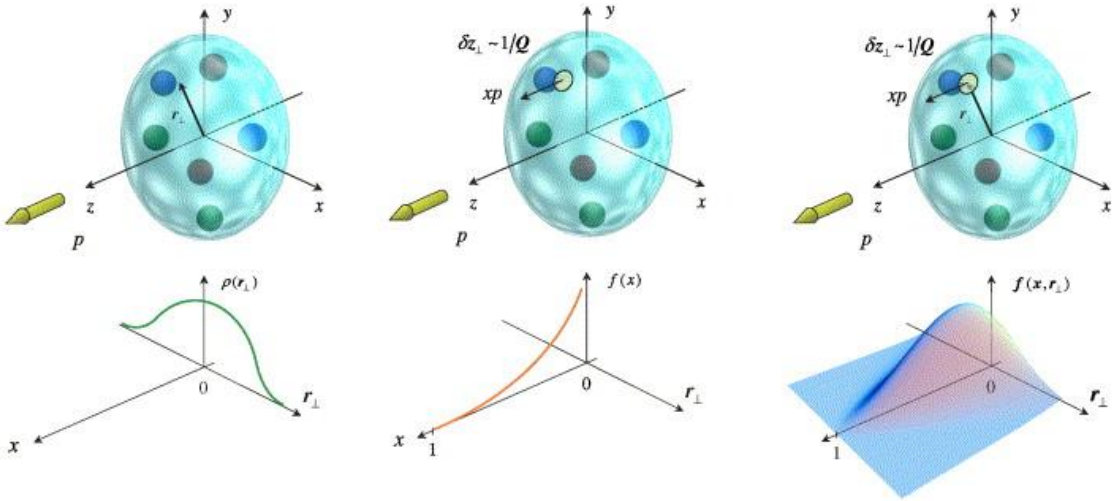
Finite volume,
Discretization,
...



LatticePDF Report, Prog.Part.Nucl.Phys. 121 (2021) 103908 [256 citations]

3D Scan of Proton

§ Generalized Parton Distributions (GPDs)



Picture from
Physics Report, 416 (2015)

§ Recent GPDs Review Papers

Three-dimensional imaging of hadrons with hard exclusive reactions: advances in experiment, theory, phenomenology, and lattice QCD

2512.15064

M. Boër[1,a], A. Camsonne[2,a], M. Constantinou[3,a], H.S. Jo[4,a,b], K. Joo[5,a], K. Semenov-Tian-Shansky[4,6,a,b], H.-D. Son[7,a], P. Sznajder[8,a], C. Van Hulse[9,a], E. Voutier[10,a], J. Wagner[8,a], A. Afanasev[11], J.S. Alvarado[10], S. Bhattacharya[12], D. Biswas[1], Xu Cao[13], H.-M. Choi[4], K. Cichy[14], N. Crnković[15], W. Hamdi[16], M. Hoballah[10], G.M. Huber[17], P. T. P. Hutauruk[18], A. Jentsch[19], C.-R. Ji[20], H.-Ch. Kim[7], B. Kriesten[21], Huey-Wen Lin[22], P.-J. Lin[23], V. Martínez-Fernández[24,25], M. Mazouz[16], Z.-E. Meziani[26], M. Nefedov[27], K. Passek-K.[15], B. Pire[28], P. Rossi[29,30], O. Teryaev[31], A. W. Thomas[32], N. Tomida[33], Z. W. Zhao[34]

Generalized parton distributions: Theory meets experiment

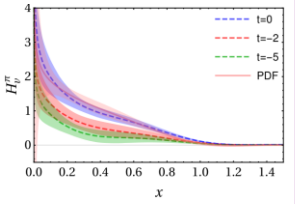
2609.19544

Yuxun Guo,[1,2,3,*] Xiangdong Ji,[4,†] Yao Ji,[5,‡] and Jialu Zhang[4,§]
[1] *Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA*
[2] *Physics Department, University of California, Berkeley, CA 94720, USA*
[3] *RIKEN BNL Research Center and Physics Department, Brookhaven National Laboratory, Upton, NY 11973, USA*
[4] *Tsung-Dao Lee Institute and School of Physics and Astronomy, Shanghai Jiao Tong Unviersity, Shanghai, China*
[5] *School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen 518172, China*

x-Dependent GPDs from Lattice QCD

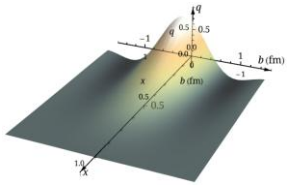
§ LaMET GPD milestones, placed by arXiv date

First pion GPD



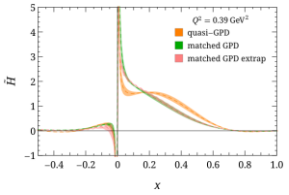
{310, 1.74}

First Nucleon tomography



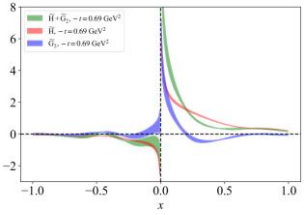
{135, 2.2}

Helicity GPD at physical m_π



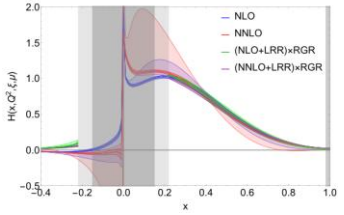
{135, 2.2}

Twist-3 GPDs



{260, 1.67}

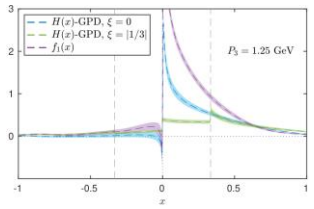
1st GPD w/ RGR + LRR



{135, 2.2}

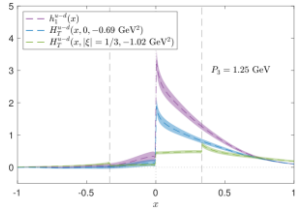
2019 2020 2021 2022 2023 2024 2025 2026

First nucleon GPDs



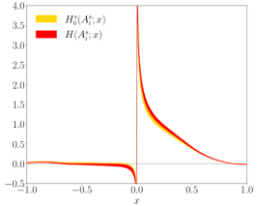
{260, 1.67}

Transversity GPDs



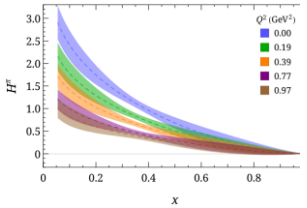
{260, 1.67}

Asymmetric frame



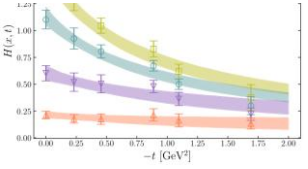
{260, 1.25}

Pion GPD at physical m_π



{135, 1.73}

1st Pion GPD w/ RGR + LRR



{300, 1.94}

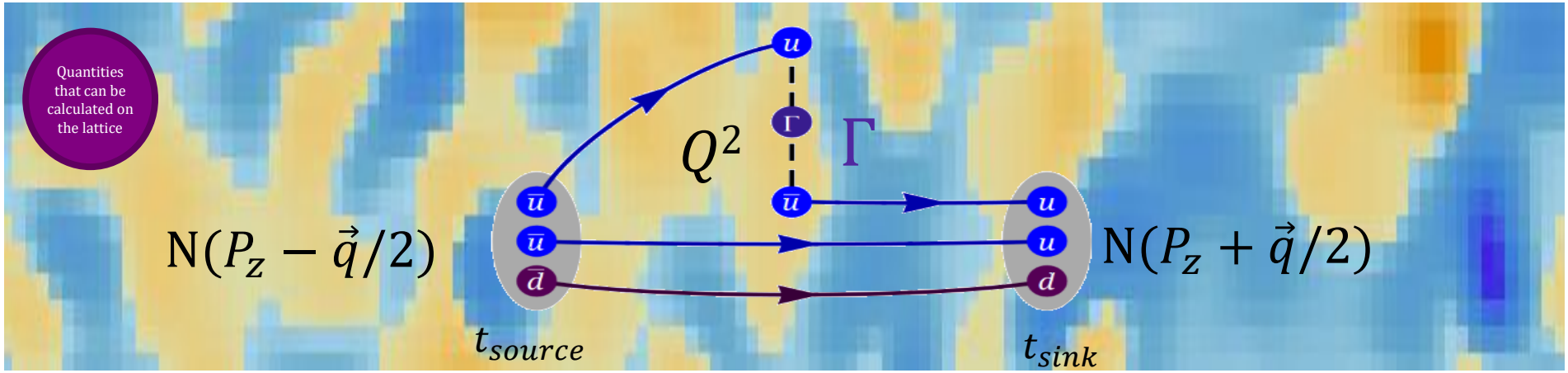
§ This talk covers selected results

Isovector Nucleon GPDs

§ Nucleon GPD using quasi-PDFs at **physical pion mass**

- MSULat: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon GPD results





$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{P}_z}{\bar{P}_0}\int \frac{dz}{4\pi} e^{ixz\bar{P}_z}\langle P'|\tilde{O}_{\gamma_0}(z)|P\rangle = \frac{\bar{u}(P')}{2\bar{P}^0}\left(H(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + E(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

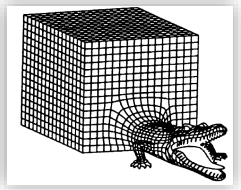
$$p^\mu = \frac{p''^\mu + p'^\mu}{2}, \qquad \Delta^\mu = p''^\mu - p'^\mu, \qquad t = \Delta^2, \qquad \xi = \frac{p''^+ - p'^+}{p''^+ + p'^+}$$

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Nucleon Tomography

§ Nucleon GPD using quasi-PDFs at physical pion mass

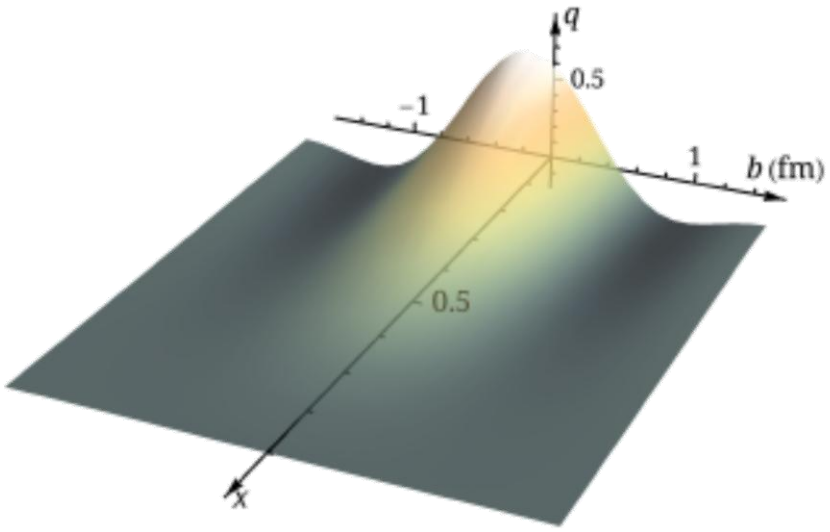
- Lattice details: clover/2+1+1 HISQ
  0.09 fm, 135-MeV pion mass, $P_z \approx 2$ GeV
- $\xi = 0$ isovector nucleon quasi-GPD results



finite-volume, discretization,



$$q(x,b) = \int \frac{d\vec{q}}{(2\pi)^2} H\left(x, \xi = 0, t = -\vec{q}^{\,2}\right) e^{i\vec{q}\cdot\vec{b}}$$



First Proton CT Scan from Lattice QCD!

HL, Phys.Rev.Lett. 127 (2021) 18, 182001

Tomography @ Physical Pion Mass

§ Lots of progress on tomography by many collaborations

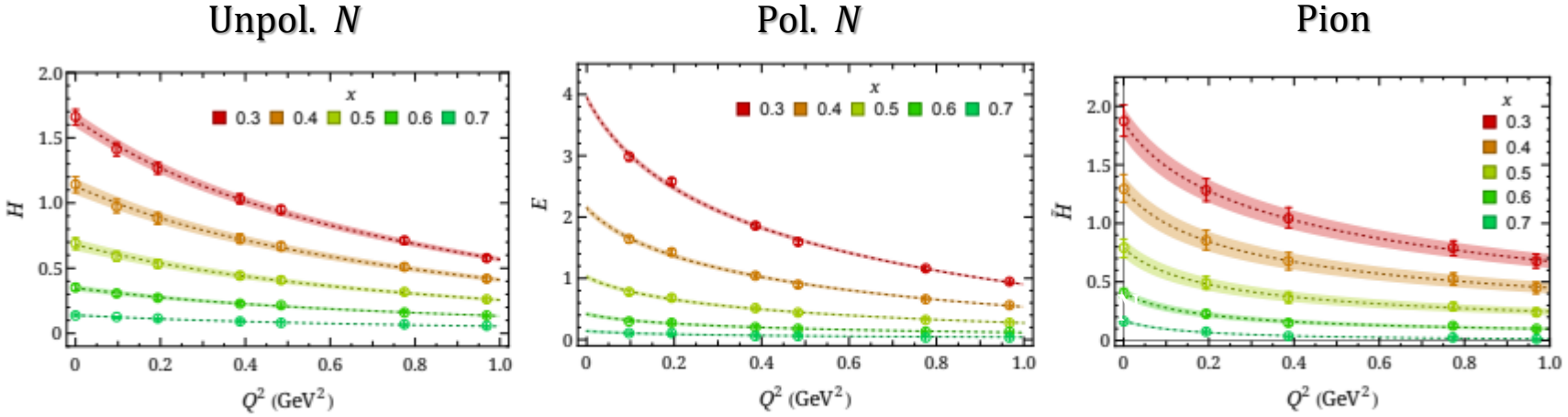


Figure 4.1: Nucleon isovector H (left), E (middle) and $\tilde{H}$ (right) GPDs at $\xi = 0$ with z-expansion to Q^2 at selected x values.

Invited Review: HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

Tomography @ Physical Pion Mass

§ Lots of borations

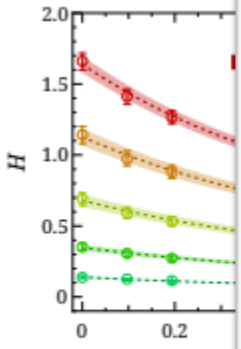
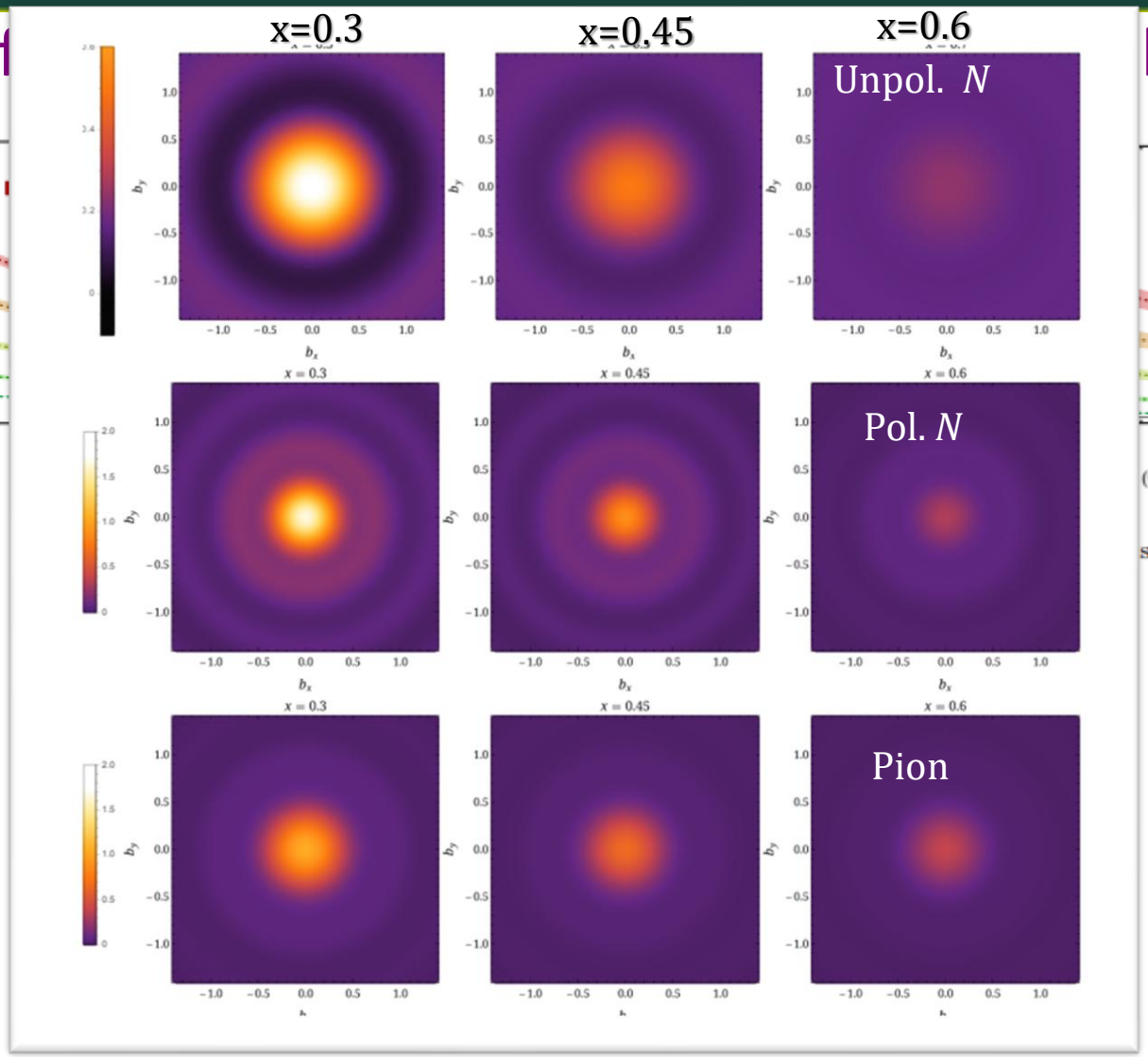
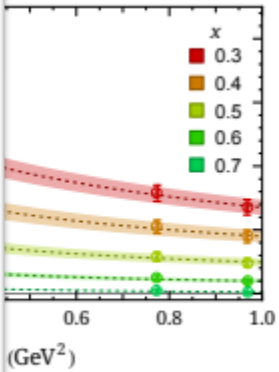


Figure 4.1:





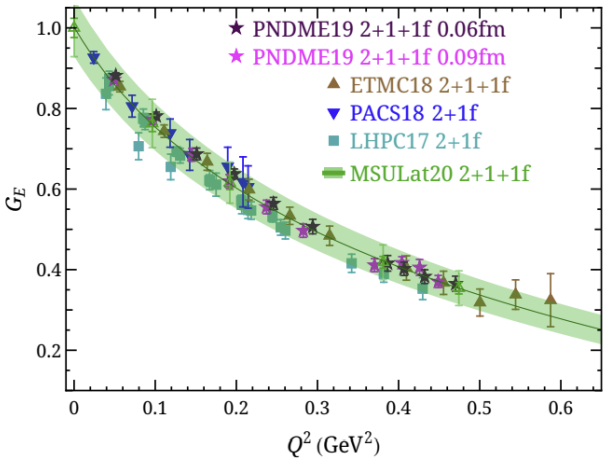
selected x values.

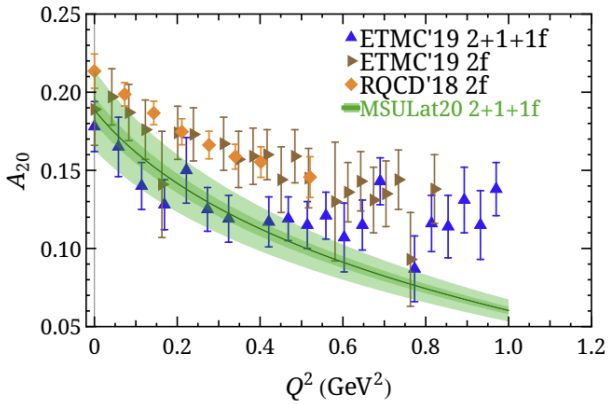
Invited Review: HL, Prog.Part.Nucl.Phys. 144 (2025) 104177

GPDs @ Physical Pion Mass

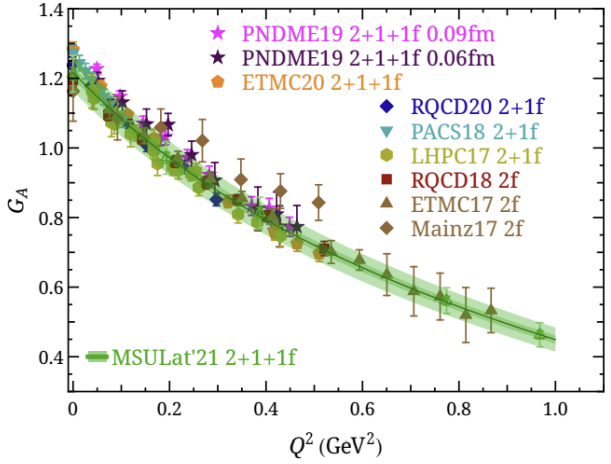
§ Integrate the GPDs over *x* to compare with LQCD form factors

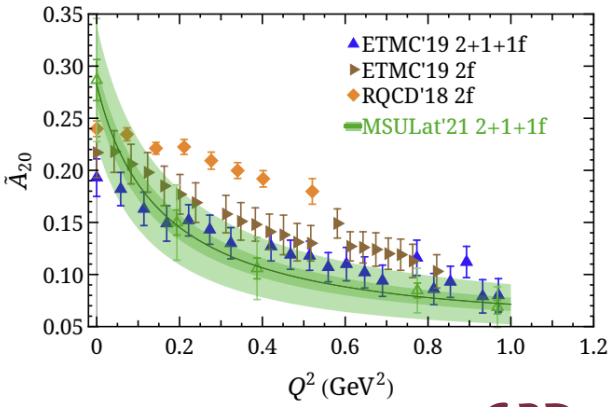
Unpol. *N*



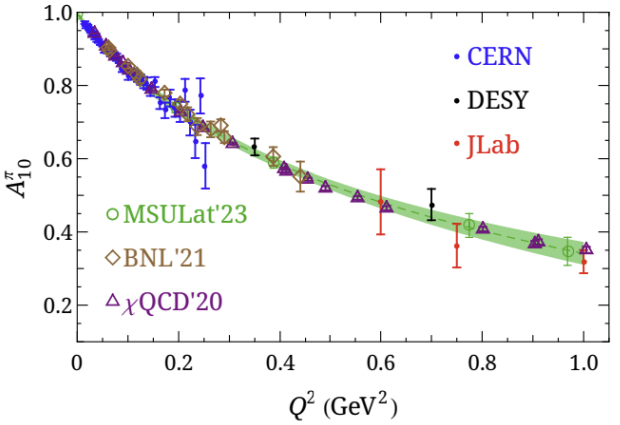


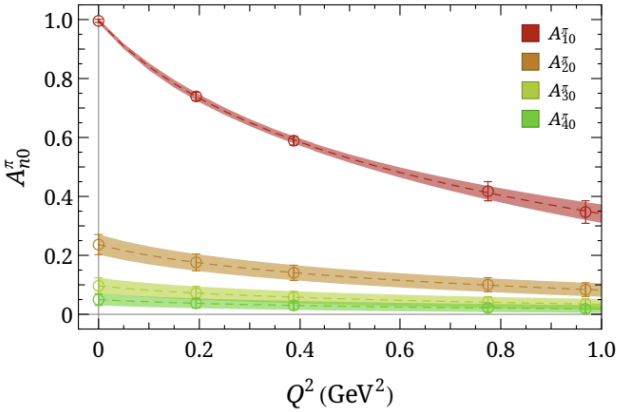
Pol. *N*





Pion





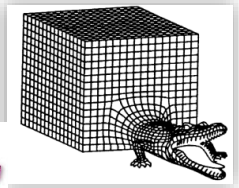
GPD review, 2512.15064

Asymmetric-Frame GPD

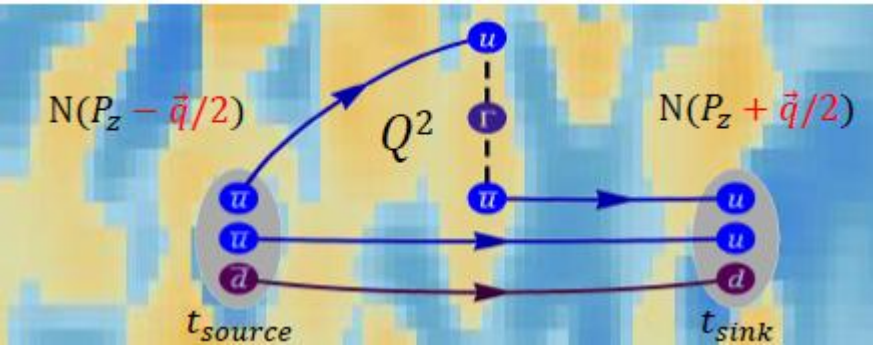
§ New calculations by ANL/BNL/ETMC using asymmetric frame

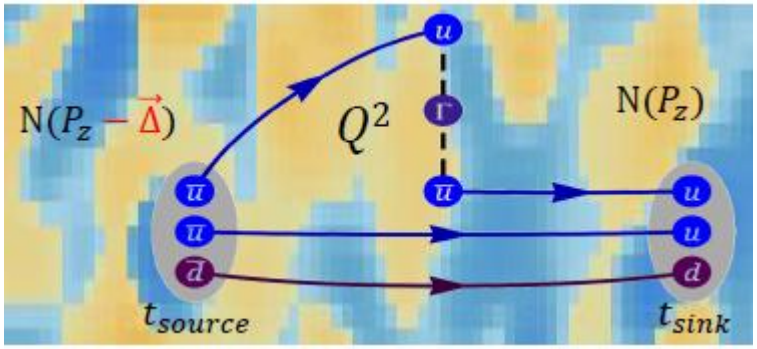
- 2+1+1 twisted-Wilson, 0.09 fm, $P_z \approx \mathbf{1.3}$ GeV
  260-MeV pion, one source-sink used
- $\xi = 0$ isovector nucleon GPD results





finite-volume, discretization, *heavy quark, excited-state, …*





$$\tilde{F}(x,\tilde{\xi},t,\bar{P}_z) = \frac{\bar{u}(P')}{2\bar{P}^0}\left(\boldsymbol{H}(x,\tilde{\xi},t,\bar{P}_z)\gamma^0 + \boldsymbol{E}(x,\tilde{\xi},t,\bar{P}_z)\frac{i\sigma^{0\mu}\Delta_\mu}{2M}\right)u(P'')$$

$$F^\mu(z,P,\Delta) = \bar{u}(p_f,\lambda')\left[\frac{P^\mu}{m}\boldsymbol{A_1} + mz^\mu\boldsymbol{A_2} + \frac{\Delta^\mu}{m}\boldsymbol{A_3} + im\sigma^{\mu z}\boldsymbol{A_4} + \frac{i\sigma^{\mu\Delta}}{m}\boldsymbol{A_5} + \frac{P^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_6} + mz^\mu i\sigma^{z\Delta}\boldsymbol{A_7} + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}\boldsymbol{A_8}\right]u(p_i,\lambda)$$

$$H(z\cdot P^{s/a}, z\cdot\Delta^{s/a}, (\Delta^{s/a})^2) = A_1 + \frac{\Delta^{s/a}\cdot z}{P^{s/a}\cdot z}A_3$$

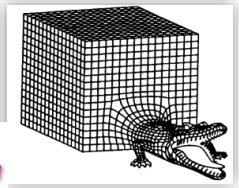
ANL/BNL/ETMC, 2209.05373 ,2310.13114

Asymmetric-Frame GPD

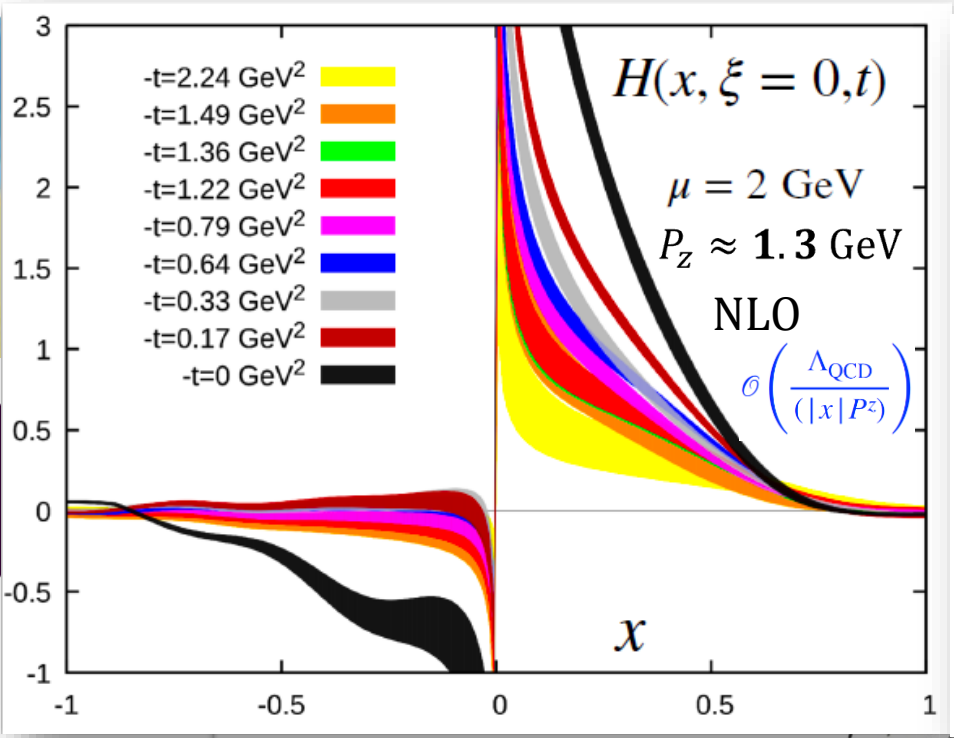
§ Results by ANL/BNL/ETMC using asymmetric framew/ quasi-PDFs

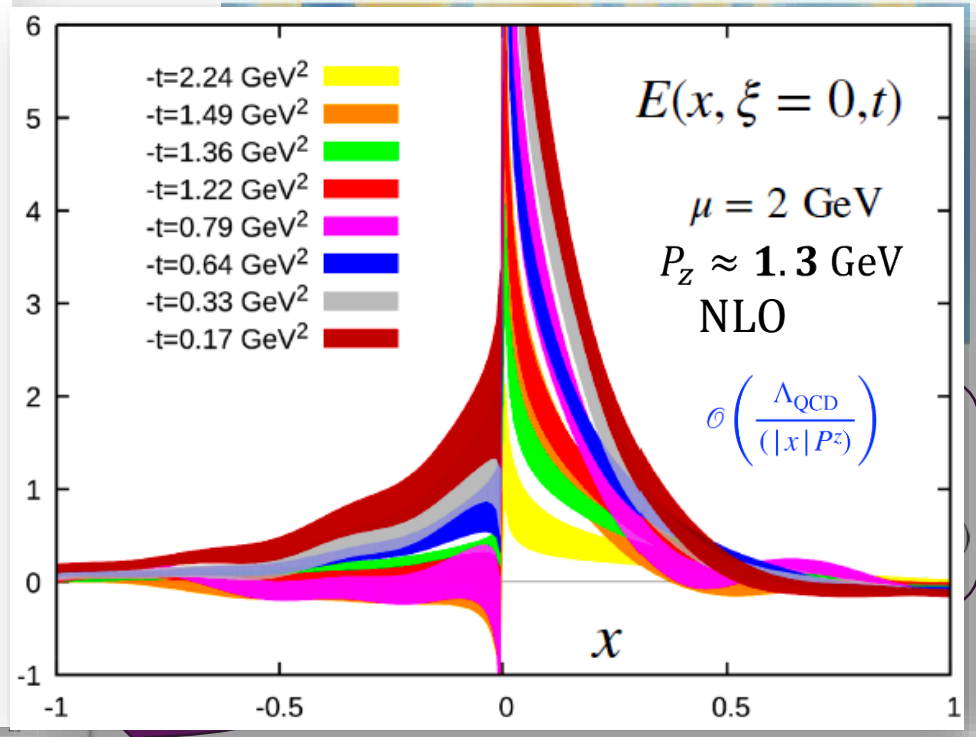
- 2+1+1 twisted-Wilson, 0.09 fm, $P_z \approx \mathbf{1.3}$ GeV
  260-MeV pion, one source-sink used
- $\xi = 0$ isovector nucleon GPD results





finite-volume, discretization, *heavy quark, excited-state, …*





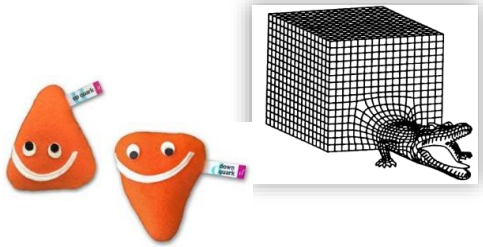
See Shohini Bhattacharya' talks@ Lattice 2024

ANL/BNL/ETMC, 2209.05373 ,2310.13114

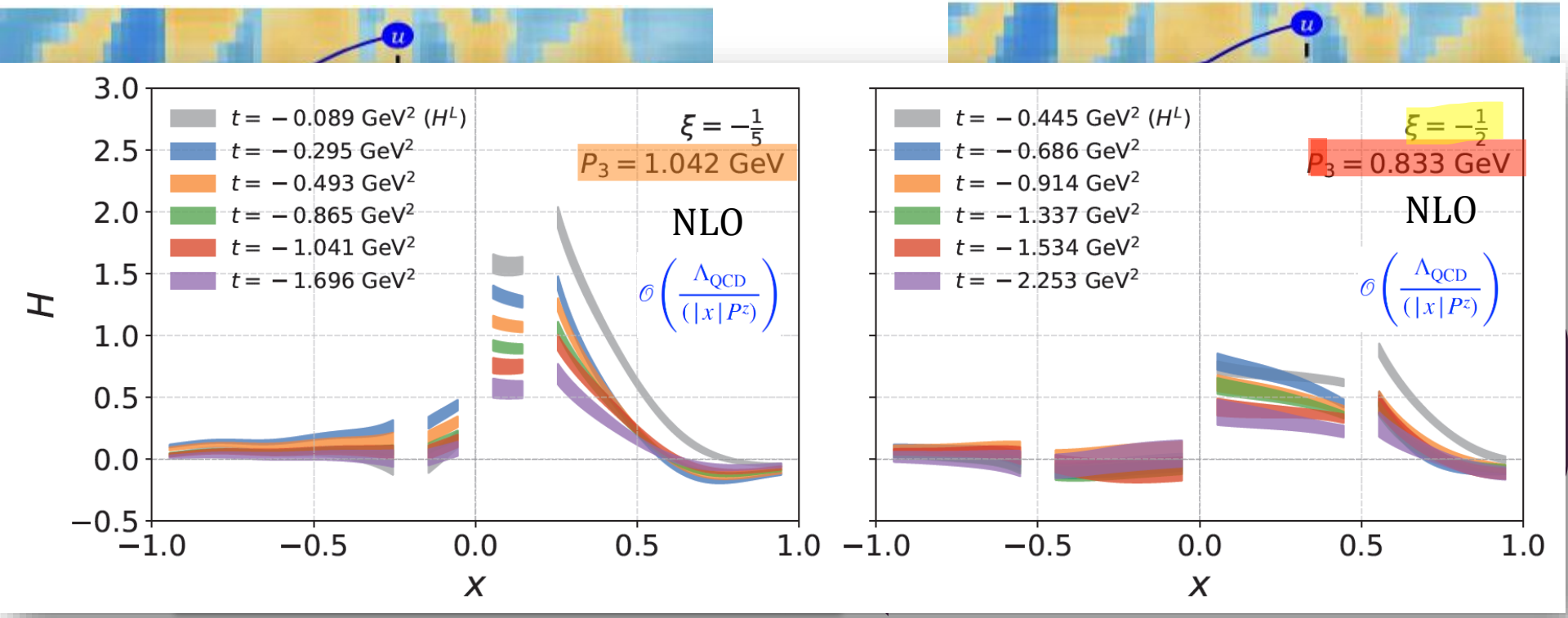
Asymmetric-Frame GPD

§ New calculations by ETMC using asymmetric frame w/ quasi-PDFs

- 2+1+1 twisted-Wilson, 0.09 fm, **260**-MeV pion, one source-sink used
- $\xi \neq 0$ isovector nucleon GPD results



finite-volume, discretization, *heavy quark, excited-state, …*

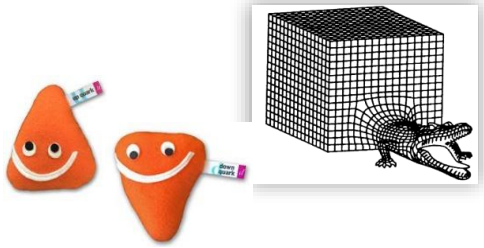


ETMC, 2508.17998

Asymmetric-Frame GPD

§ New calculations by HadStruc using asymmetric frame & pseudo-PDF

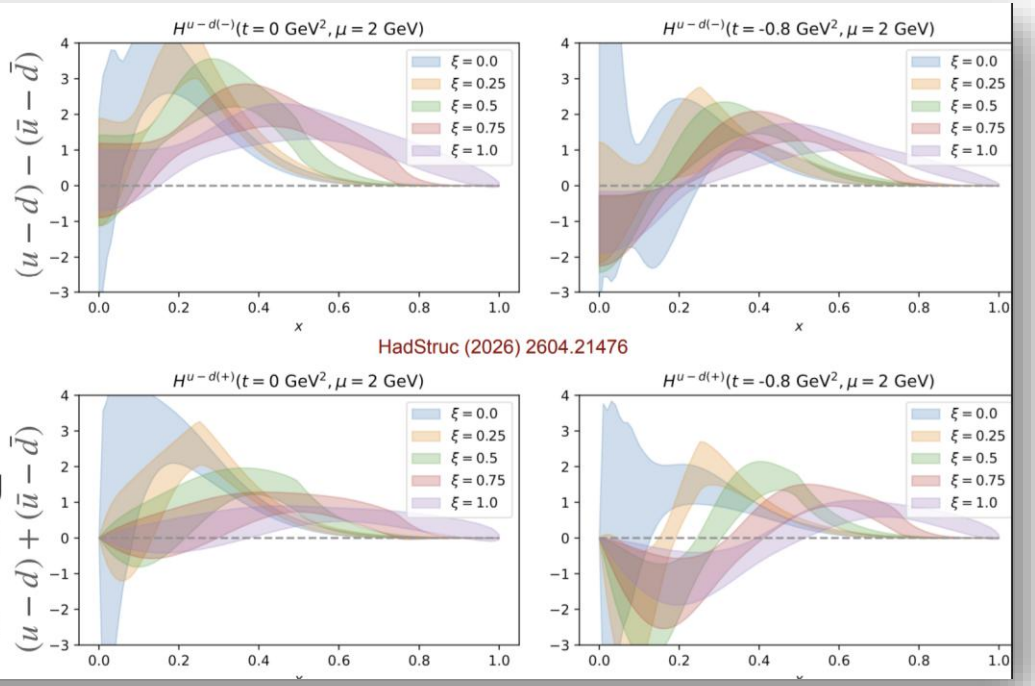
- 2+1 clover-Wilson, 0.094 fm, **358**-MeV pion
- $\xi \neq 0$ isovector nucleon GPD results
  - One-loop matching to H GPD



finite-volume, discretization, *heavy quark, excited-state, …*

…

- Matched at $\mu_0^2 = 1/z^2$ then RGE evolved to $\mu^2 = 4\ \text{GeV}^2$
- Uncertainty is statistical and excited state contamination
- Varying matching scale by $\sqrt{2}$ and changing hyperparameters were smaller



Slide from Joe Karpie @Lattice2026 Talk

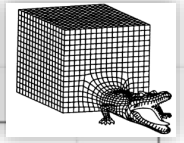
HadStruc, 2604.21476 [hep-lat]

Moments of Pion GPD

§ Skewness-dependent Moments of Pion GPD from Nonlocal Quark-Bilinear Correlators

X. Gao, et. al, 2511.01818





- N_f= 2+1+1 clover/HISQ, $a \approx 0.04$ fm, 300-MeV pion, $P_z \approx 2.4$ GeV

- The polynomial relation to Mellin moments:

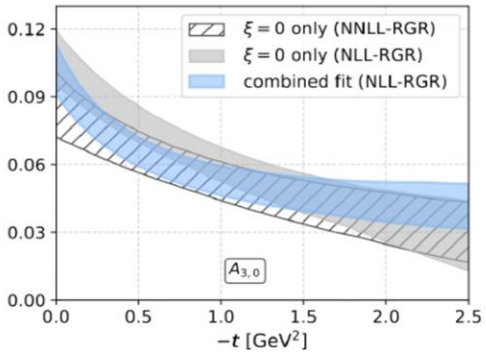
$$\langle x^2 \rangle = A_{3,0}(t) + (2\xi)^2 A_{3,2}(t),$$

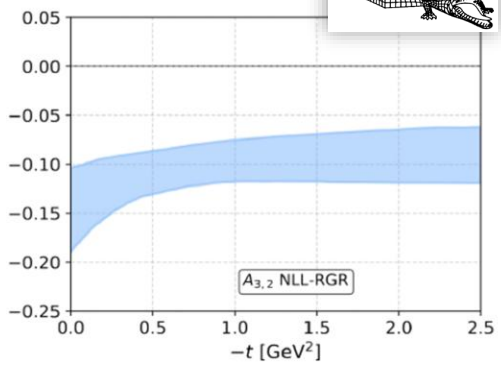
$$\langle x^4 \rangle = A_{5,0}(t) + (2\xi)^2 A_{5,2}(t) + (2\xi)^4 A_{5,4}(t)\,.$$

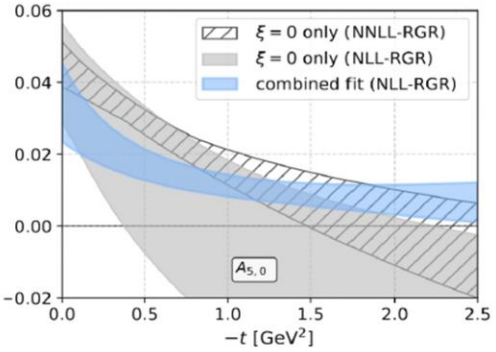
- Combine data at multiple values of ξ and $-t$ to extract GFFs $A_{3,i}$ and $A_{5,i}$.

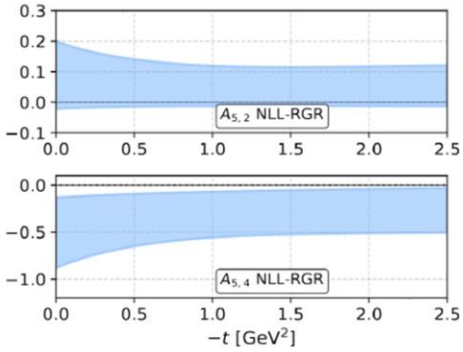
- Implement the NLL-resummation.

Slide provided by Fei Yao









GPD Systematics Update

§ Nucleon isovector $\xi = 0$ GPDs

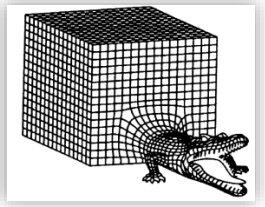
- **NNLO** matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

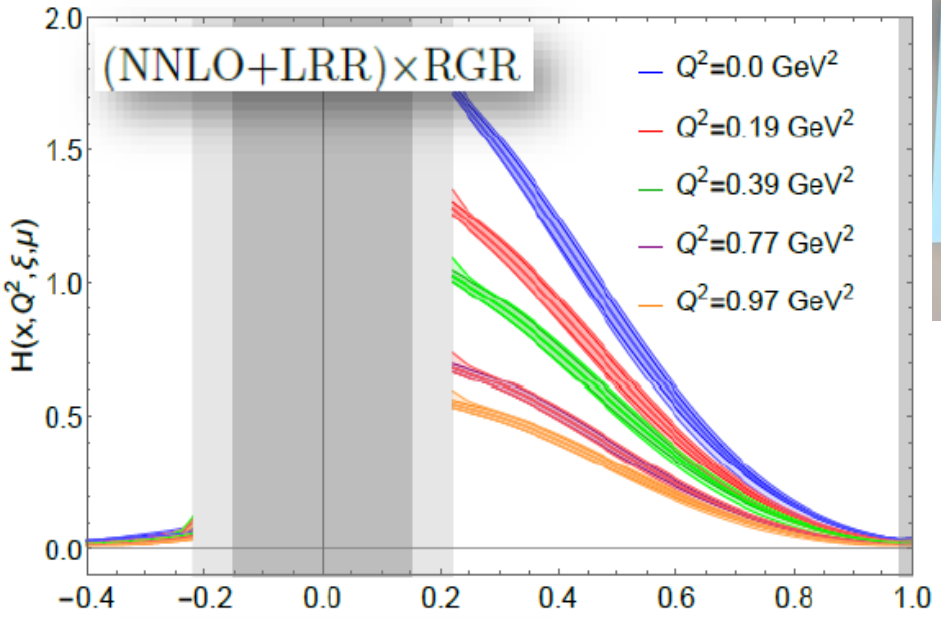
R. Zhang, et. al.
PLB 844, 138081 (2023)

- N_f= 2+1+1 clover/HISQ, $a \approx 0.09$ fm, 135-MeV pion, $P_z \approx 2$ GeV

J. Holligan, HL (MSULat), Phys.Rev.D 110 (2024) 3, 3

Wanted PDFs, GPDs, etc…







Talk by Jack @2023 Fall CTEQ Meeting

GPD Systematic Update

§ Pion valence-quark $\xi = 0$ GPDs

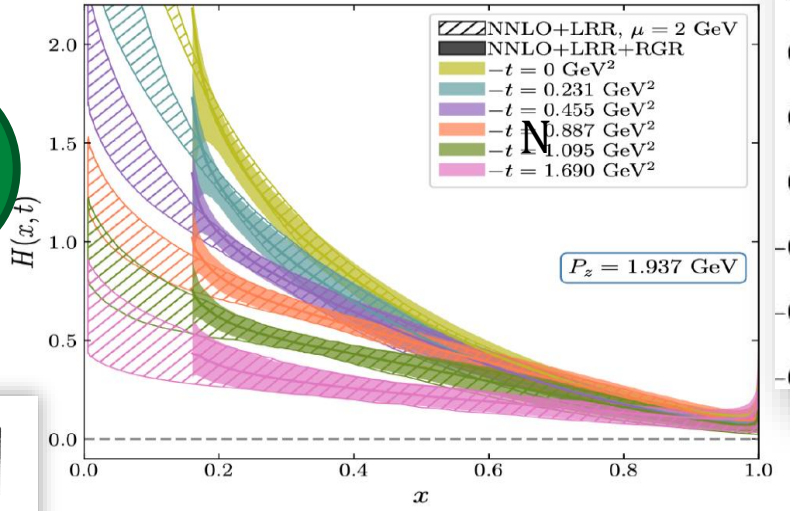
- NNLO matching & treat leading-renormalon effects
  - Leading-renormalon resummation (LRR)
  - Renormalization-group resummation (RGR)

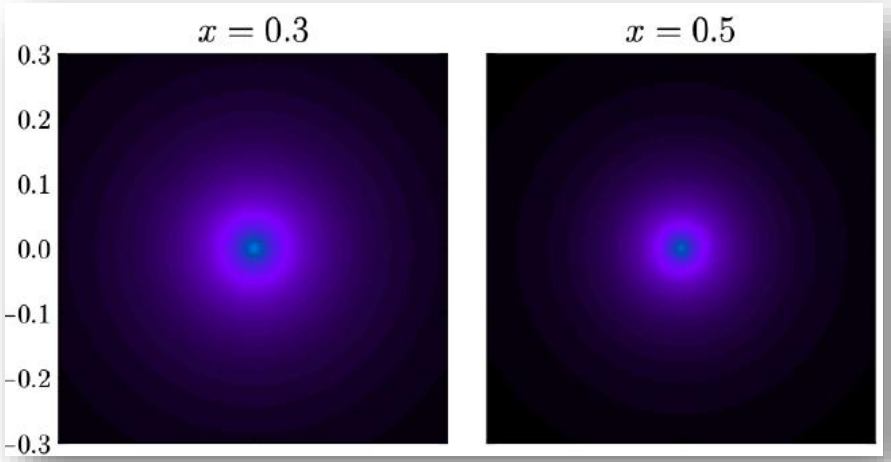
R. Zhang, et. al.
PLB 844, 138081 (2023)

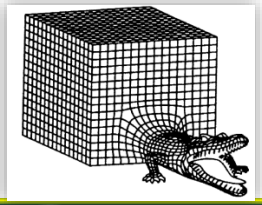
- N_f= 2+1 clover/HISQ, $a \approx 0.09$ fm, 300-MeV pion, $P_z \approx 2$ GeV

H. Ding et al (ANL/BNL/Wuhan),
2407.03516 [hep-lat]

Wanted PDFs, GPDs, etc…





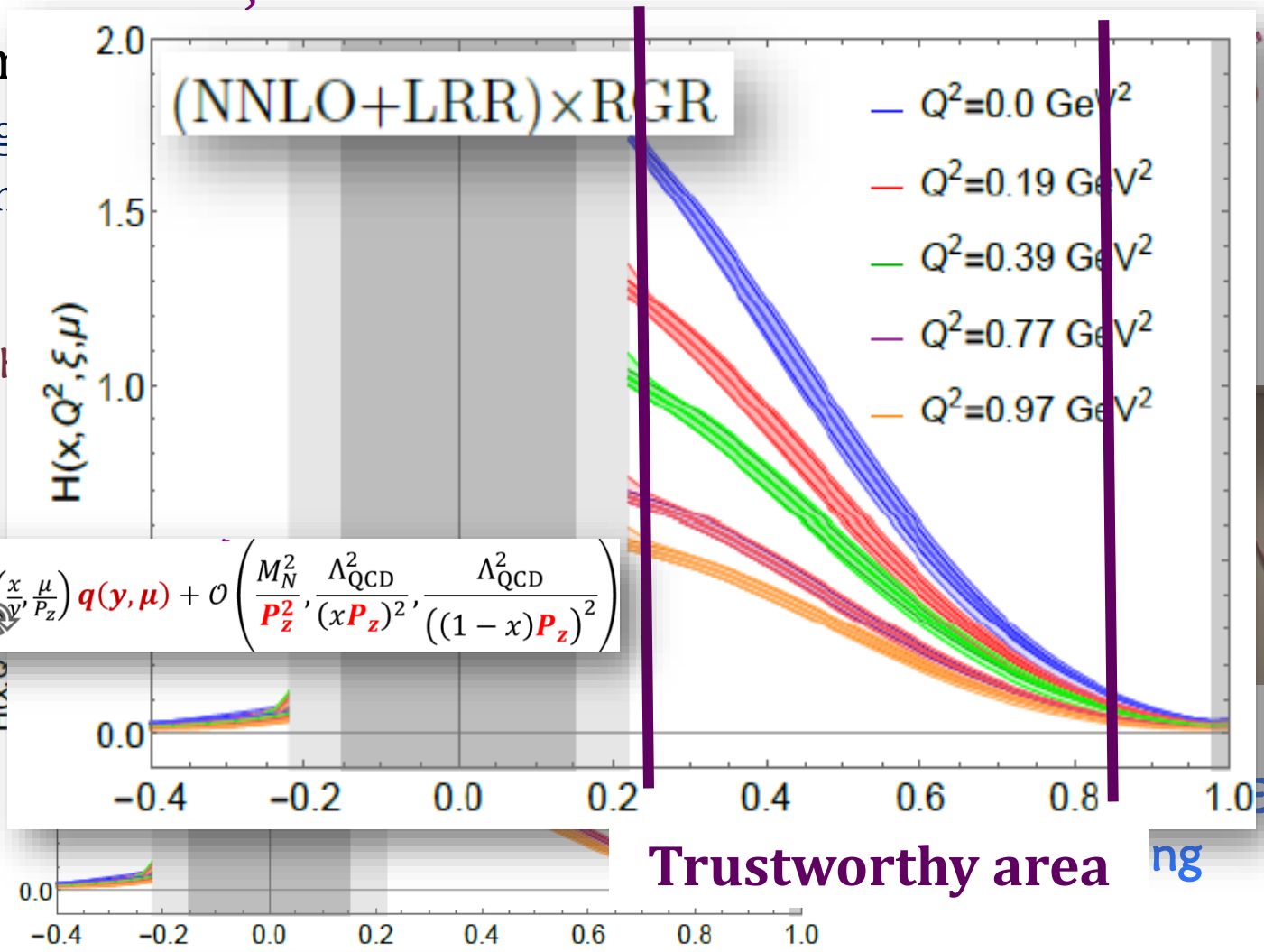


GPD Systematics Update

§ Nucleon isovector $\xi = 0$ GPDs

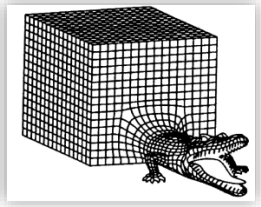
- **NNLO** n
  - Leading
  - Renorm
- N_f= 2+1+1

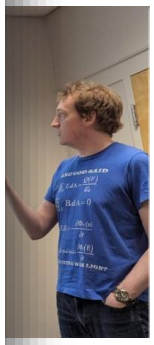
J. Holligan, H



Wanted

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{M_N^2}{P_z^2}, \frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P_z)^2}\right)$$





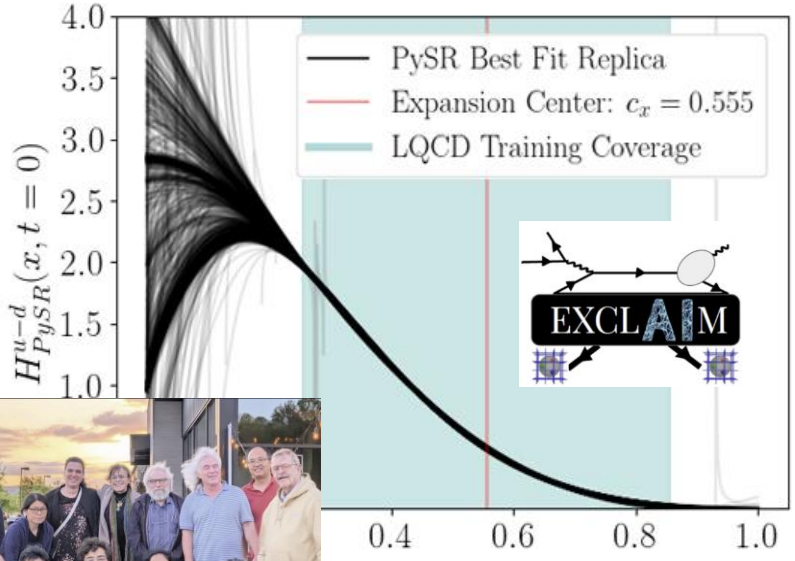
Overcoming LaMET Limitations

§ Symbolic Regression (SR) on **Quasi-PDF** (LaMET) method

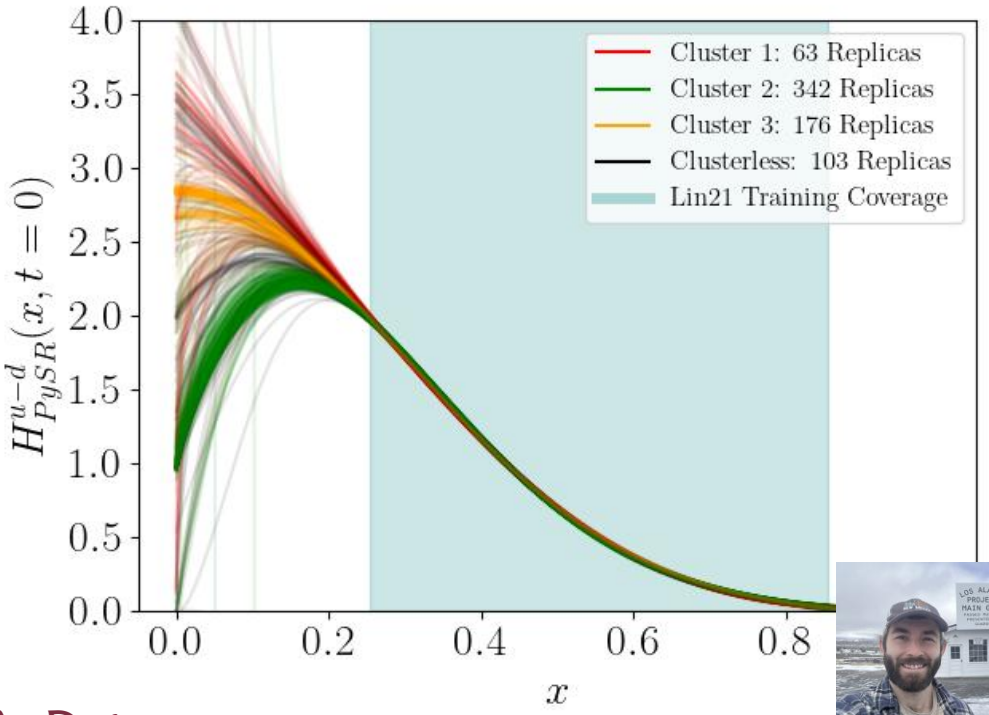
- Flexible function discovery with interpretable analytic forms
- Attractive for scientific modeling when appropriately constrained

§ Work done with EXCLAIM

Before — Unconstrained SR



After — expansion coefficient clustering + physics filters

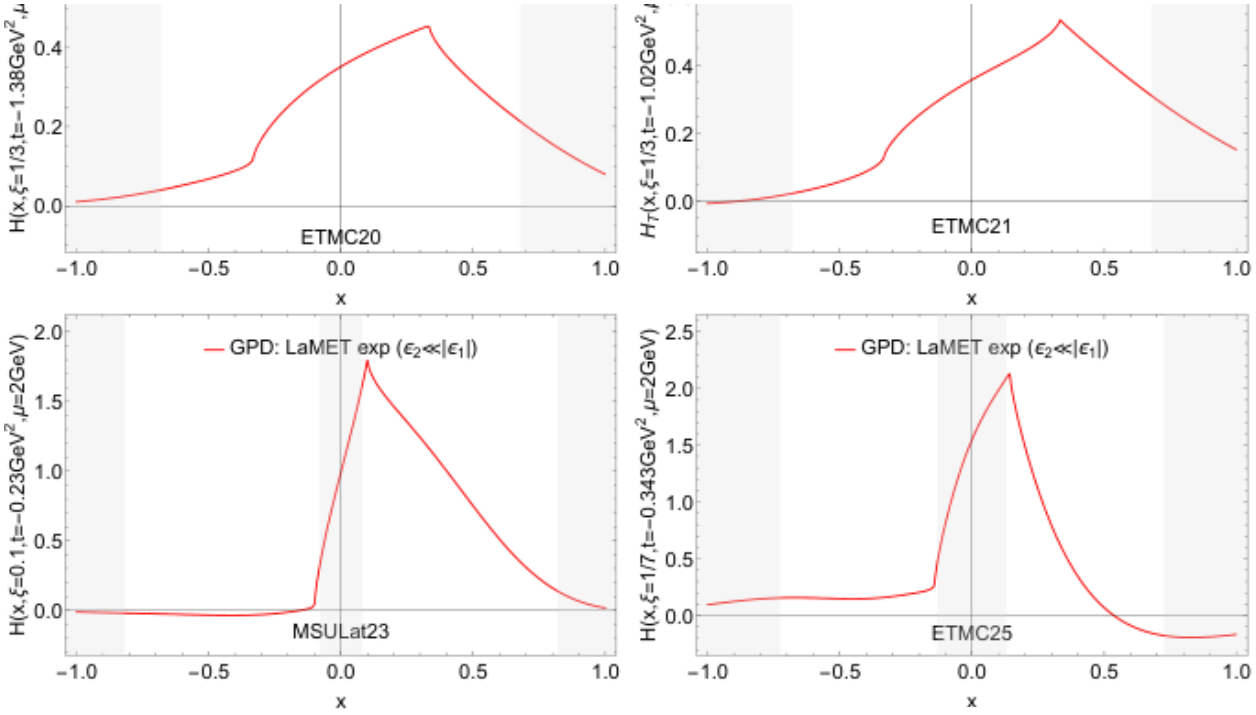




Plots by A. Dotson

$\xi \neq 0$ GPDs from LaMET

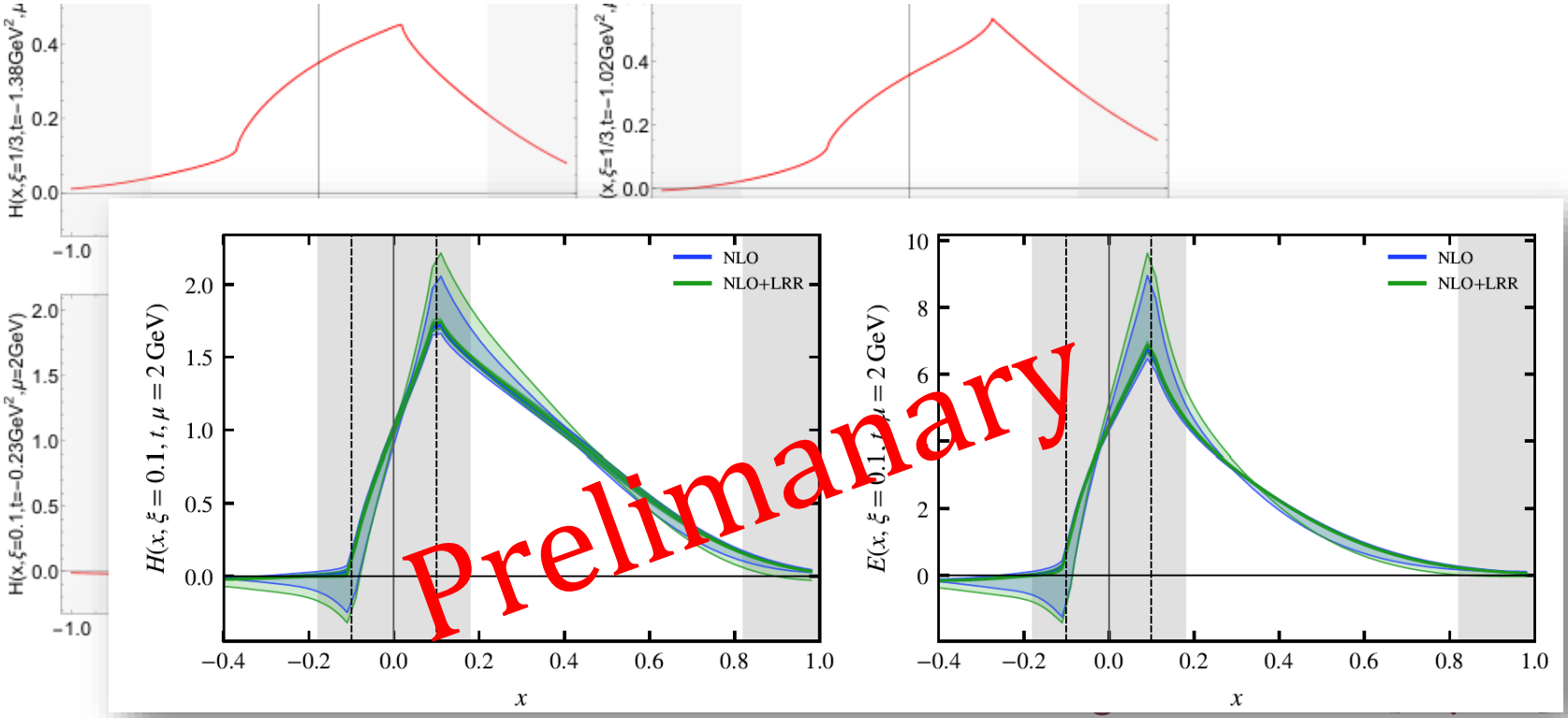
- LaMET gives the GPD at fixed x and ξ, not just its moments
  - Boost the hadron, compute an equal-time correlator, then match
- Several groups now report $H(x, \xi, t)$ directly
  - ETMC20, ETMC21, MSULat23 and ETMC25, at different ξ and t
  - Shaded bands mark where the expansion is not controlled



Y. Su, X. Ji, Y. Liu, R. Zhang 2609.20545 [hep-lat]

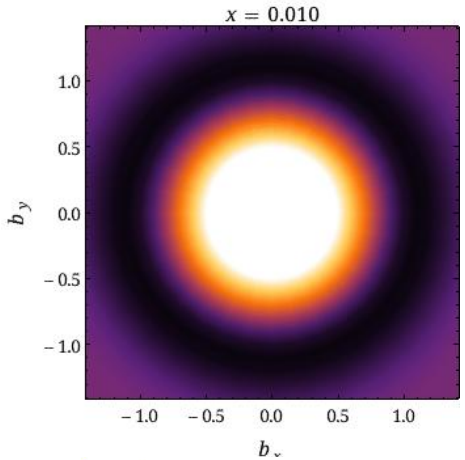
$\xi \neq 0$ GPDs from LaMET

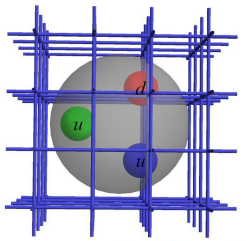
- LaMET gives the GPD at fixed x and ξ, not just its moments
  - Boost the hadron, compute an equal-time correlator, then match
- Several groups now report $H(x, \xi, t)$ directly
  - ETMC20, ETMC21, MSULat23 and ETMC25, at different ξ and t
  - Shaded bands mark where the expansion is not controlled



Summary and Outlook

- § Exciting era using LQCD to study the secret life of protons and others
  - Well-studied systematics → precision structures
- § Overcoming longstanding obstacles
  - Bjorken-x dependence of parton distributions are widely studied with LaMET and its variants
  - Continue AI/ML applications to further discovery
- § Precision and progress are limited on resources
  - Challenges = new opportunities quantities



Theory Input → **Global Analysis of PDFs/GPDs/TMDs** ← **Exp't Input** + 

EXCLAIM

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
The work of MSULat is supported by the NSF under grant PHY 2514533, 2209424 & 1653405, DOE under DE-SC0024053, DE-SC0024644 & DE-SC002405, and the Research Corporation for Science Advancement through the Cottrell Scholar Award

Summary and Outlook

§ LaMET summary of GPD calculations

GPDs	Hadron	ξ	m_π	a	N_f	P_z [GeV]	Matching	Renorm.	LRR	RGR	Source
H_v^π	π	0	310 MeV	0.12 fm	2+1+1	0.86, 1.32, **1.74**[a]	NLO	RI/MOM	No	No	LP^3/MSULat'19 [1904.12376]
H_v^π	π	0	135 MeV	0.09 fm	2+1+1	**1.73**	NNLO	Hybrid[b]	No	No	MSULat'23 [2310.10579]
H_v^π	π	0	300 MeV	0.04 fm	2+1	1.45, **1.94**[c]	NNLO	Hybrid[b]	Yes	Yes	ANL/BNL/Wuhan'24 [2407.03516]
H, E	N	0, 1/3	260 MeV	0.093 fm	2+1+1	0.83, 1.25, **1.67**[d]	NLO	RI′-MOM	No	No	ETMC'20c [2008.10573]
H, E	N	0	135 MeV	0.09 fm	2+1+1	**2.2**	NLO	RI/MOM	No	No	MSULat'20 [2008.12474]
H, E	N	0	260 MeV	0.093 fm	2+1+1	**1.25**	NLO	RI′-MOM	No	No	ANL/BNL/ETMC'22 [2209.05373]
H, E	N	0, 0.1	135 MeV	0.09 fm	2+1+1	**2.2**	NNLO/NLO[j]	Hybrid[b]	Yes	Yes[j]	MSULat'23 [2312.10829]
H, E	N	1/7, ±1/5, ±1/2	260 MeV	0.093 fm	2+1+1	0.42, 0.83, **1.25**[f]	NLO	RI′-MOM	No	No	ETMC'25 [2508.17998]
$\widetilde{H}, \widetilde{E}$	N	0, 1/3	260 MeV	0.093 fm	2+1+1	0.83, 1.25, **1.67**[d]	NLO	RI′-MOM	No	No	ETMC'20c [2008.10573]
$\widetilde{H}$	N	0	135 MeV	0.09 fm	2+1+1	**2.2**	NLO[i]	RI/MOM	No	No	MSULat'21 [2112.07519]
$\widetilde{H}$	N	0	260 MeV	0.093 fm	2+1+1	0.83, **1.25**, 1.67[e]	NLO	RI′-MOM	No	No	ANL/BNL/ETMC'23 [2310.13114]
$H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$	N	0, 1/3	260 MeV	0.093 fm	2+1+1	0.83, 1.25, **1.67**[d]	NLO	RI′-MOM	No	No	ETMC'21 [2108.10789]
$H_T, E_T, \widetilde{H}_T$	N	0	260 MeV	0.093 fm	2+1+1	0.83, **1.25**, 1.67[e]	NLO	RI′-MOM	No	No	BNL/ETMC'25b [2505.11288]
Twist-three GPDs	N	0	260 MeV	0.093 fm	2+1+1	0.83, **1.25**, 1.67[g]	NLO[h]	RI′-MOM	No	No	ETMC'23 [2306.05533]

[a] The paper does not say which boost the final GPD uses; 1.74 GeV ($4 \times 2\pi/L$) is inferred from its figures and from Lin (2023), which describes this work as having a largest boost of 1.7 GeV.

[b] Ratio scheme for $z \le z_s$ combined with Wilson-line mass subtraction at larger z; $z_s = 0.27$ fm in Lin (2023) and Holligan and Lin (2024), $z_s = 0.12$ fm in Ding *et al.* (2025).

[c] Asymmetric frame at $\xi = 0$, so $P_i^z = P_f^z$. Data at 0.97 GeV were also generated but not used in the GPD analysis.

[d] $\xi = 0$ at all three boosts; $\xi = 1/3$ only at $P_z = 1.25$ GeV.

[e] 0.83 and 1.67 GeV are symmetric-frame data at $-t = 0.69$ GeV^2 used to test the P_z dependence; the asymmetric-frame results over the full t range use $P_f^z = 1.25$ GeV.

[f] Final-state momentum P_f^z in the asymmetric frame. The matched GPDs at $\xi = -1/2, -1/5, 1/7$ use $P_f^z = 1.25$ GeV (average momenta 0.83–1.46 GeV).

[g] Symmetric frame. All three boosts at $-t = 0.69$ GeV^2; the larger momentum transfers only at 1.25 GeV.

[h] Simplified one-loop kernel of the twist-three PDF g_T, applied to all four GPDs; mixing with three-parton correlators is neglected and the perturbative zero-mode term is omitted.

[i] Order not stated in the paper; the cited matching kernels (Chen *et al.*, 2018; Lin *et al.*, 2018; Liu *et al.*, 2020) are one-loop.

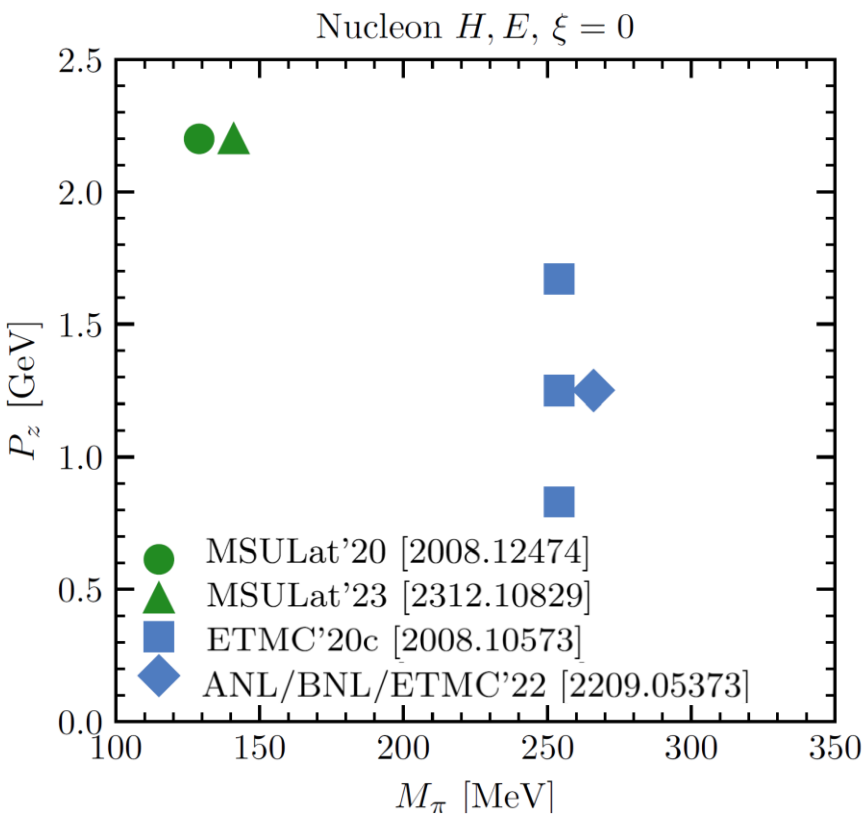
[j] NNLO matching at $\xi = 0$ and NLO at $\xi = 0.1$, where no NNLO kernel is available. RGR is applied in renormalization and matching at $\xi = 0$ only; at $\xi = 0.1$ the matching is NLO+LRR and RGR enters only the renormalization, since RGR matching was not available at nonzero skewness.

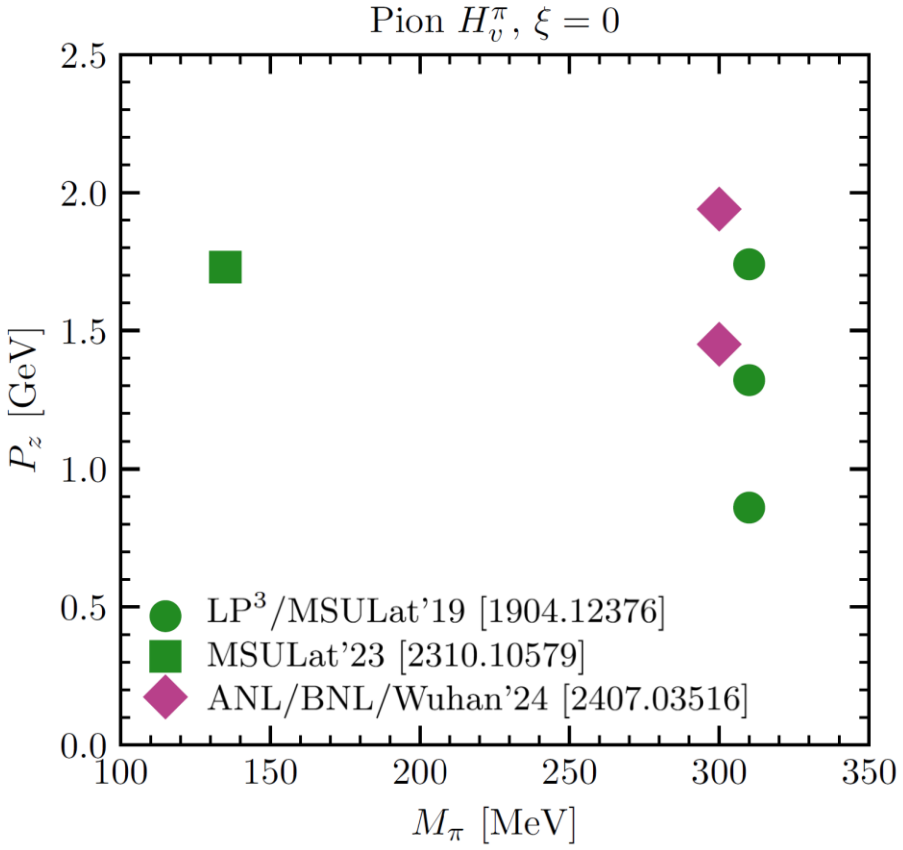
Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
The work of MSULat is supported by the NSF under grant PHY 2514533, 2209424 & 1653405, DOE under DE-SC0024053, DE-SC0024644 & DE-SC002405, and the Research Corporation for Science Advancement through the Cottrell Scholar Award

Summary and Outlook

§ LaMET summary of GPD calculations

- Rooms for more calculations





Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices & USQCD/NSF/DOE for computational resources
The work of MSULat is supported by the NSF under grant PHY 2514533, 2209424 & 1653405, DOE under DE-SC0024053, DE-SC0024644 & DE-SC002405, and the Research Corporation for Science Advancement through the Cottrell Scholar Award

Backup Slides



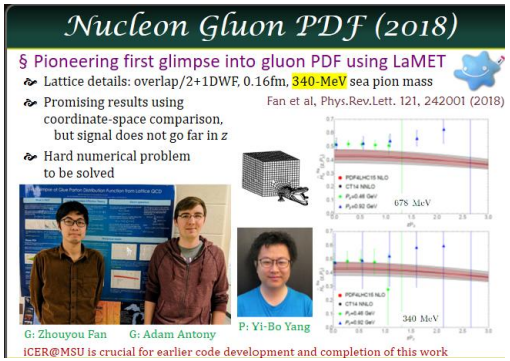
Gluon PDFs

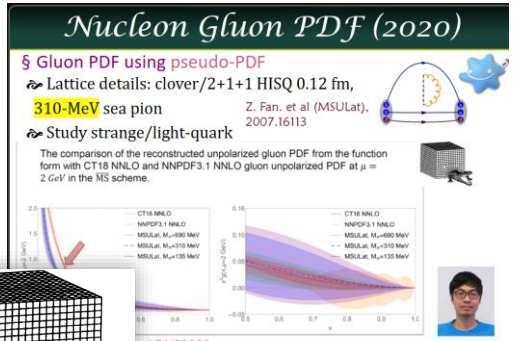
§ MSULat: Long history of pioneering gluon PDFs

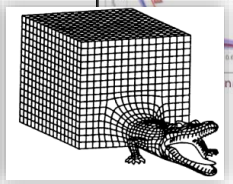
- Invited feature article under review



pseudo-PDF







MDPI

Article

Review of Pion and Nucleon Gluon Parton Distribution Functions from Lattice QCD

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[1] Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan 48824, USA
[2] Department of Computational Mathematics, Science and Engineering, Michigan State University, East Lansing, Michigan 48824, USA

Abstract

Gluons play an essential role in the strong interaction that binds quarks into hadrons, as described by quantum chromodynamics (QCD). Understanding how the fundamental dynamics of QCD give rise to hadron structure remains one of the outstanding challenges in nuclear and particle physics. Parton distribution functions (PDFs) provide a successful framework for describing high-energy scattering processes involving hadrons. However, our knowledge of gluon PDFs remains comparatively limited because of the experimental challenges associated with directly constraining the gluon structure of hadrons. In recent years, a major breakthrough has enabled the calculation of the Bjorken-x dependence of PDFs directly from lattice QCD. Among the available approaches, the large-momentum effective theory (LaMET) and pseudo-PDF frameworks have emerged as the most widely used methods and have produced the majority of lattice determinations of parton distributions. These developments have sparked rapid progress in first-principles studies of the x-dependent structure of hadrons, including gluon PDFs of the nucleon and pseudoscalar mesons. In this review, we summarize recent advances in lattice-QCD calculations of gluon PDFs, discuss the current status and impacts of the field, and highlight the remaining challenges and future opportunities.

iCER@MSU is crucial for early code development and completion of our work

Gluon PDF in Nucleon

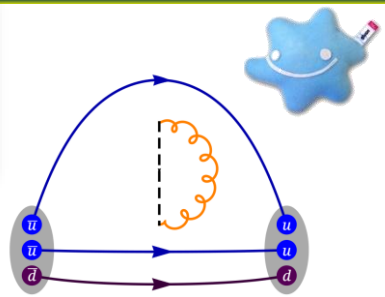
§ Continuum Gluon PDF w/ pseudo-PDF

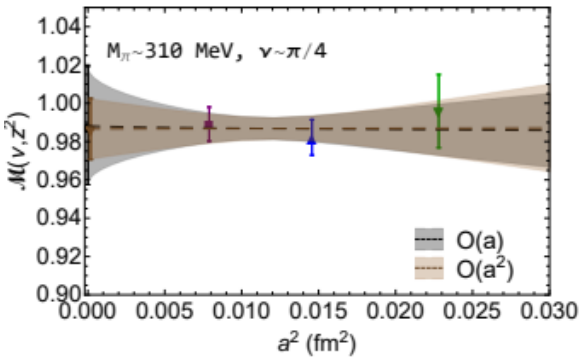
- 2+1+1 HISQ {0.09, 0.12, 0.15} fm

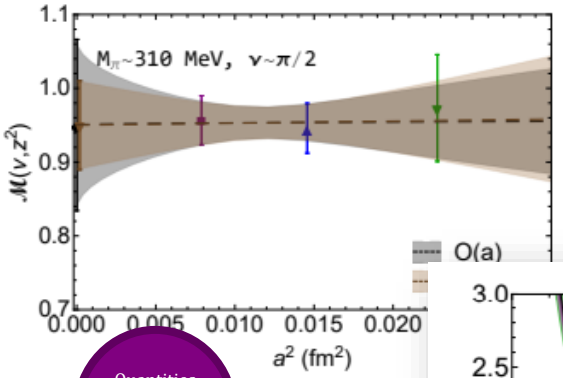
  [220, 310, 700]-MeV pion, 10^5–10^6 statistics

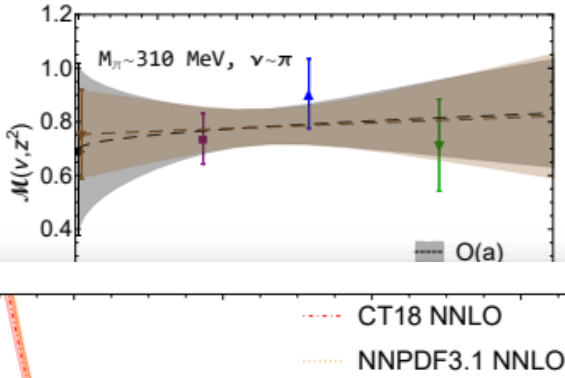
Z. Fan, W. Good, HL (MSULat), Phys.Rev.D 108 (2023) 1, 014508



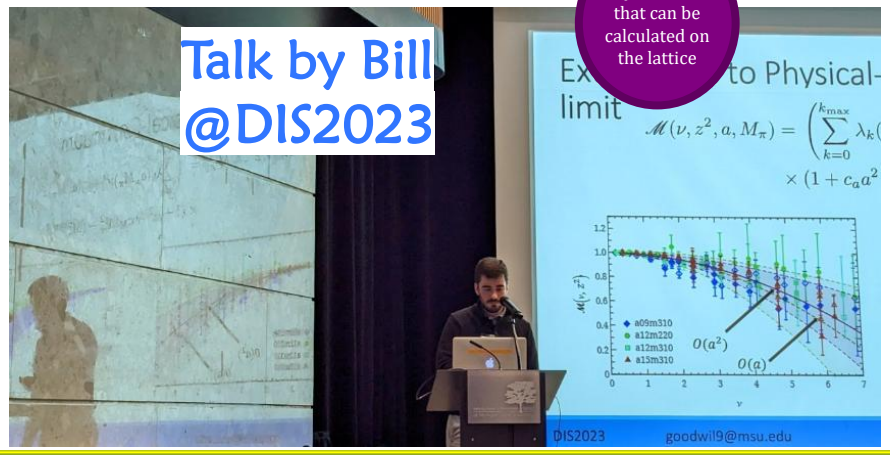


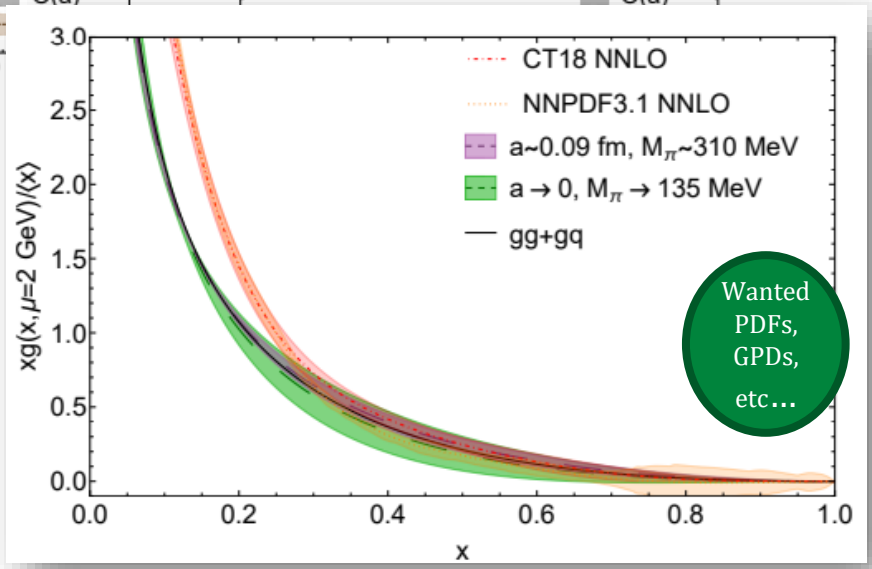






Quantities that can be calculated on the lattice





The Pseudo-PDF Method

§ We construct the reduced pseudo Ioffe-time distribution (RpITD), which removes kinematic factors and ultraviolet divergences,

$$\mathcal{M}(\nu = z \cdot P_z, z^2) = \frac{M(z \cdot P_z, z^2)/M(0 \cdot P_z, 0)}{M(z \cdot 0, z^2)/M(0 \cdot 0, 0)}$$

Radyushkin, Phys. Lett. B 781, (2018);
Balitsky et al., PRL 125, 135621 (2020)

§ The lightcone gluon PDF is obtained from the RpITD through the pseudo-PDF matching relationship,

$$\mathcal{M}(\nu, z^2) = \int dx\, C(x\nu, \mu^2 z^2) q(x, \mu^2) + O(\Lambda^2_{\rm QCD} z^2)$$

(μ is the renormalization scale in the $\overline{\rm MS}$ scheme)

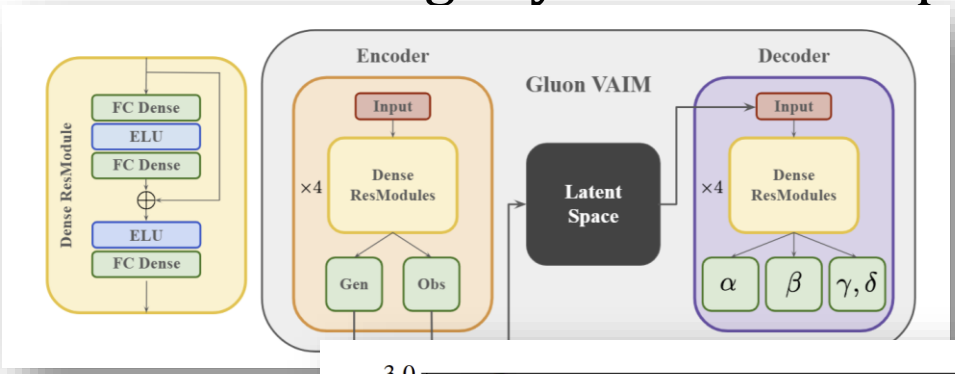
The parton-parton matching kernel; calculated using perturbation theory

Similar to the global fits, we use a functional form to describe the PDFs…

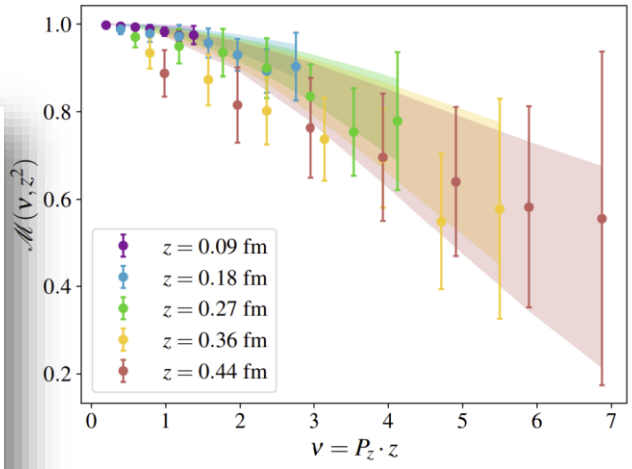
AI on Inverse Problems

§ Variational Autoencoder Inverse Mapper (VAIM)

- Injects observable data into the latent space during training
- Reduces ambiguity to solve complex inverse problems



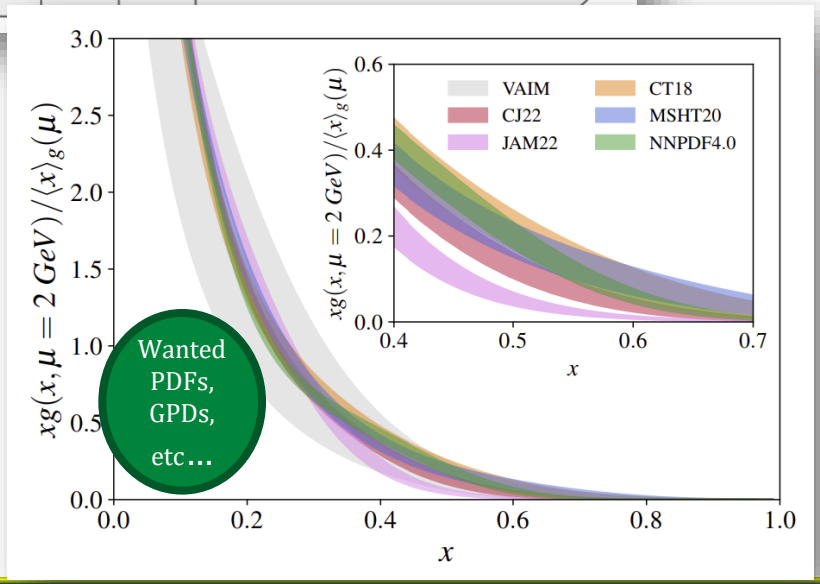
B. Kriesten, A. NieMiera et al,
Phys.Lett.B 874 (2026) 140132



Closure test



Plots by
Alex NieMiera

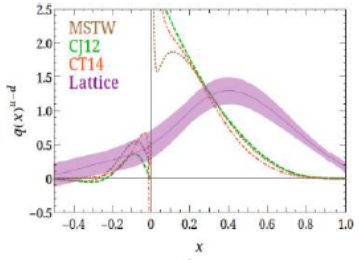


Lattice Parton Calculations

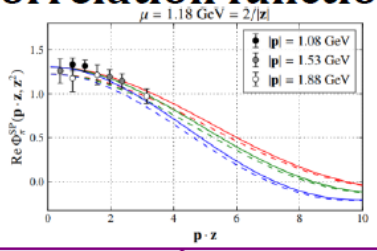
§ Rapid developments!

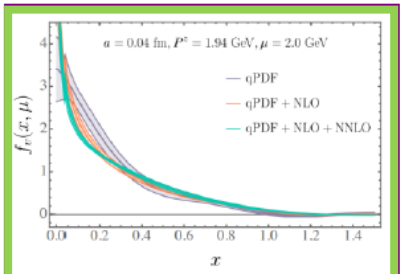
Invited Reivew: HL, Prog.Part.Nucl.Phys. 144 (2025)

LaMET/quasi-PDF lattice calculation



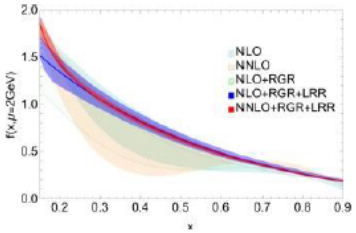
Euclidean correlation functions





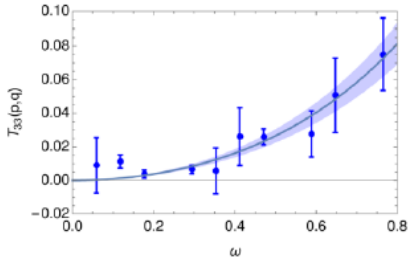
1st NNLO PDF

1stPDF w/ LRR+RGR

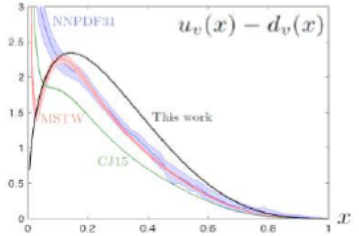


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024

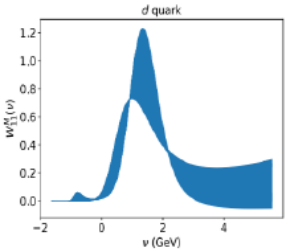
Compton amplitude



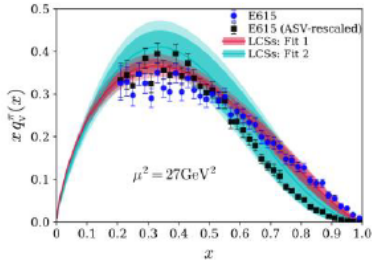
pseudo-PDFs



Hadronic tensor



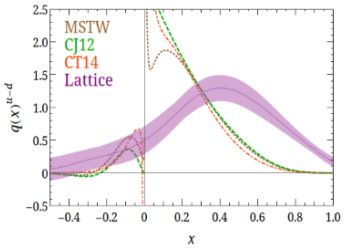
LCS



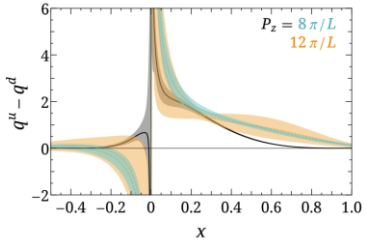
Lattice Parton Calculations

§ Physics-quantity milestones

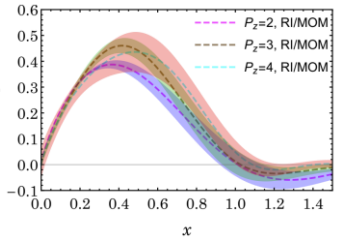
First unpol. lattice PDF



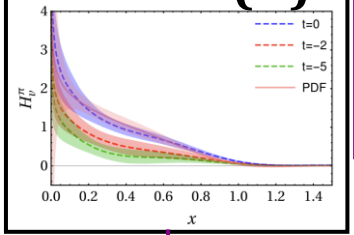
First PDFs at $M_\pi^{\rm phys}$



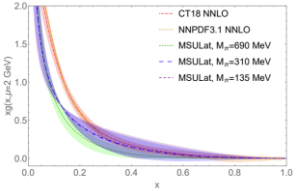
Pion v-PDF



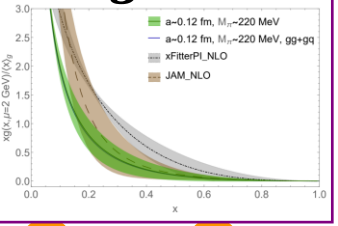
1st GPD (π)



N g-PDF

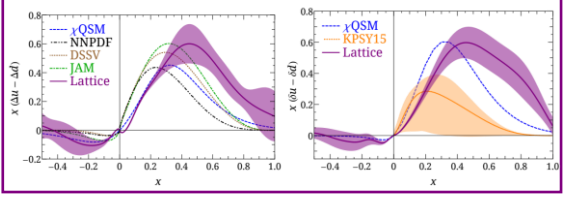


π g-PDF

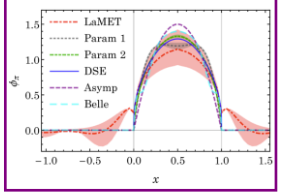


2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023

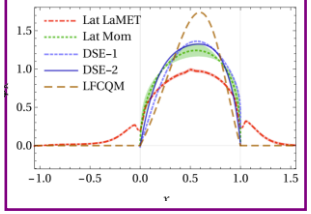
Pol. PDFs and mass corrections



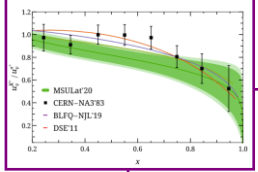
Pion DA



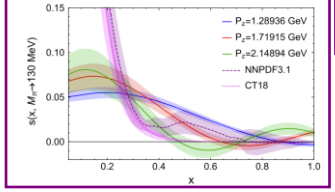
Kaon DA



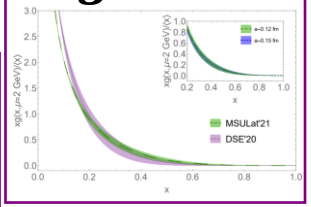
K PDF



s,c PDF



Kaon g-PDF

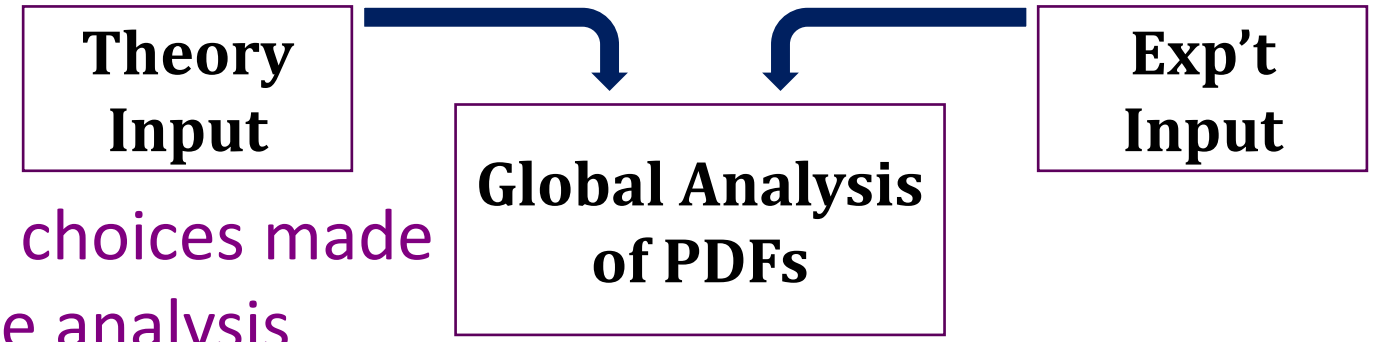


Invited Reivew: HL, Prog.Part.Nucl.Phys. 144 (2025)

1D: Parton Distribution Function

§ Experiments cover diverse kinematics of parton variables

- Global analysis takes advantage of all data sets



§ Some choices made for the analysis

- Choice of data sets and kinematic cuts
- Strong coupling constant $\alpha_s(M_Z)$
- How to parametrize the distribution

$$xf(x,\mu_0) = a_0 x^{a_1}(1-x)^{a_2} P(x)$$

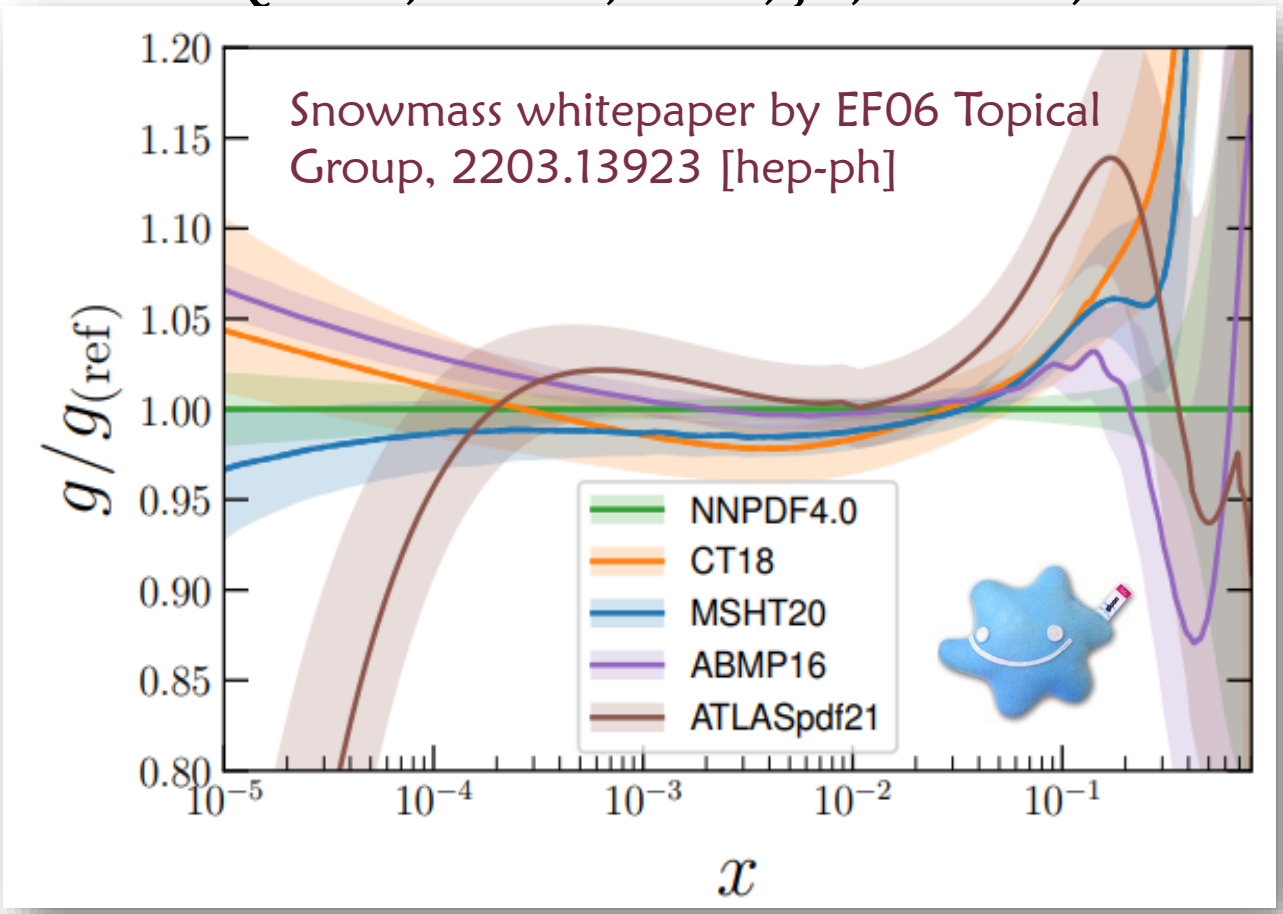
- Assumptions imposed

  SU(3) flavor symmetry, charge symmetry, strange and sea distributions

$$s = \bar{s} = \kappa(\bar{u} + \bar{d})$$

Why Lattice QCD?

- § Discrepancies appear when data is scarce
- § Many groups have tackled the analysis
  - CTEQ-TEA, MSTW, ABM, JR, NNPDF, etc.



P5

Decipher the Quantum Realm

Elucidate the Mysteries of Neutrinos

Reveal the Secrets of the Higgs Boson

Explore New Paradigms in Physics

Search for Direct Evidence of New Particles

Pursue Quantum Imprints of New Phenomena

NP Long Range Plan

3 QUARKS AND GLUONS: UNDERSTANDING THE STRONG NUCLEAR FORCE

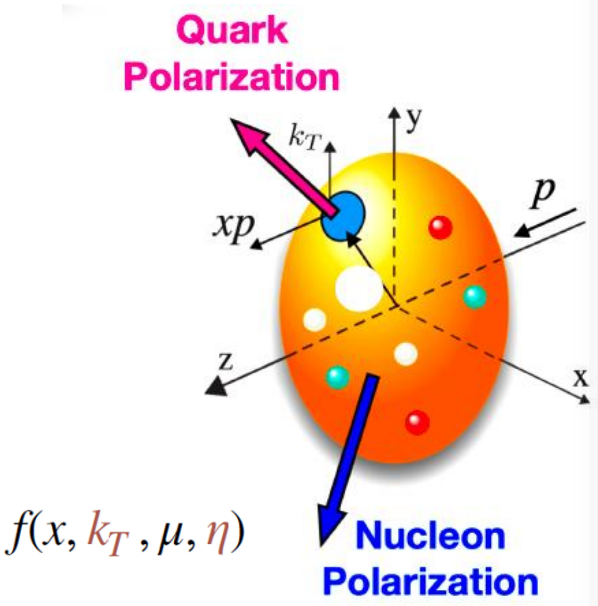
3.1 OVERVIEW

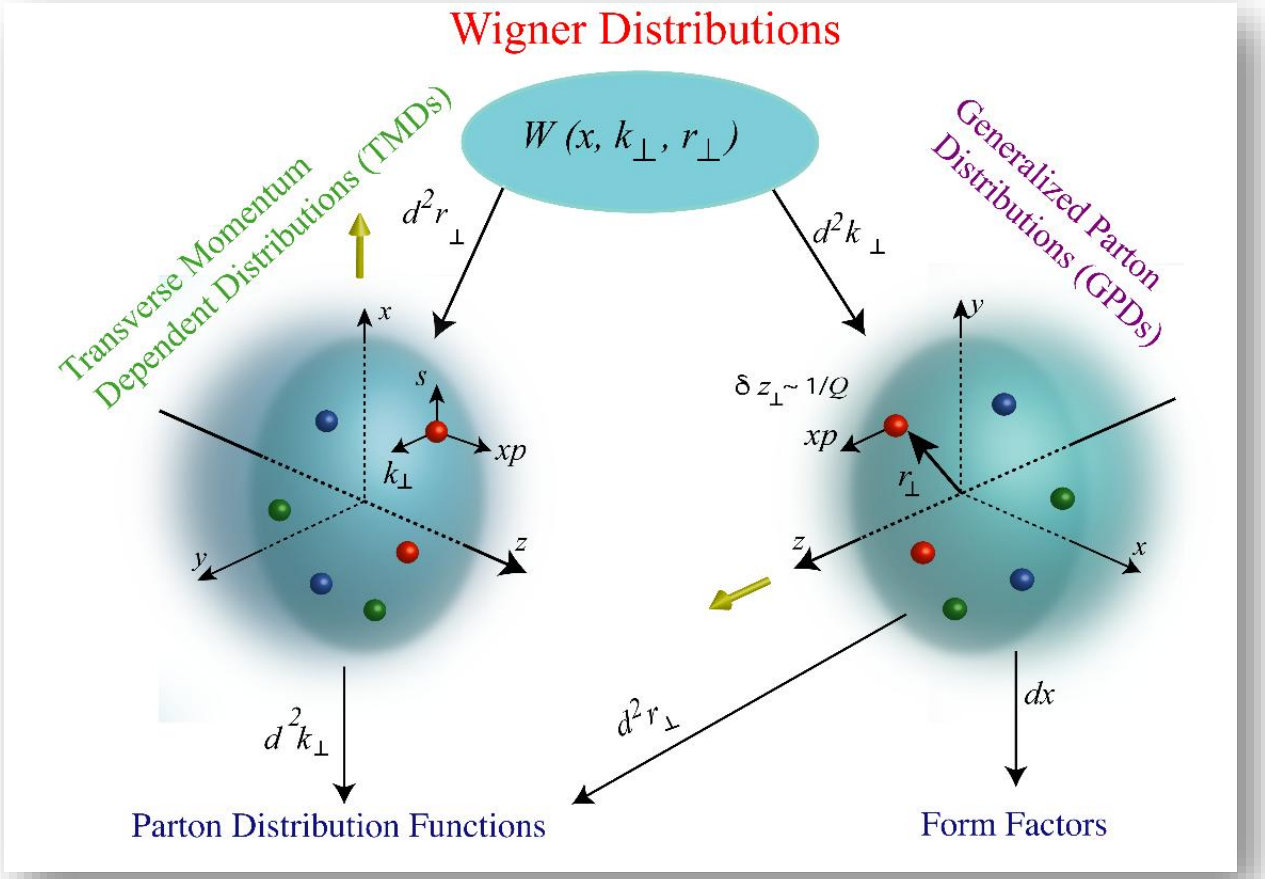
The quest to understand the nature of matter and the nucleus of the atom begins with quarks and gluons. These elementary particles are the building blocks of protons and neutrons, which in turn form nuclei, from hydrogen to the heavy elements (such as uranium or lead). The four basic forces of nature—gravitational, electromagnetic, strong, and weak—govern how objects or particles interact and how particles decay.

Transverse Momentum Distributions

§ TMDs inform on confined motion of partons in hadron

- Complementary to structure provided by PDFs and GPDs





Picture from INT Program INT–17–3

Transverse Momentum Distributions

§ TMDs inform on confined motion of partons in hadron

- Complementary to structure provided by PDFs and GPDs

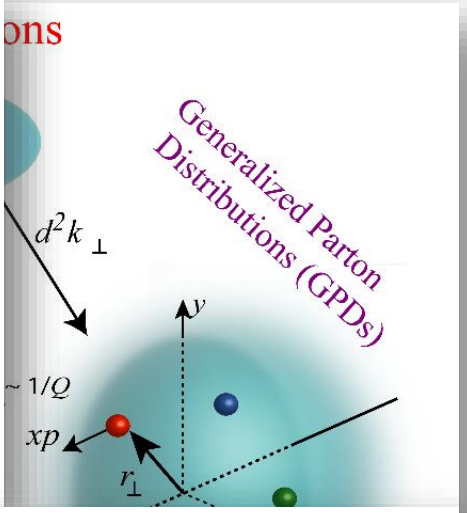
2304.03302

TMD Collaboration

Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

Renaud Boussarie[1], Matthias Burkardt[2], Martha Constantinou[3], William Detmold[4], Markus Ebert[4,5], Michael Engelhardt[2], Sean Fleming[6], Leonard Gamberg[7], Xiangdong Ji[8], Zhong-Bo Kang[9], Christopher Lee[10], Keh-Fei Liu[11], Simonetta Liuti[12], Thomas Mehen[13], Andreas Metz[3], John Negele[4], Daniel Pitonyak[14], Alexei Prokudin[7,16], Jian-Wei Qiu[16,17], Abha Rajan[12,18], Marc Schlegel[2,19], Phiala Shanahan[4], Peter Schweitzer[20], Iain W. Stewart[4], Andrey Tarasov[21,22], Raju Venugopalan[18], Ivan Vitev[10], Feng Yuan[23], Yong Zhao[24,4,18]



$f(x, k_T, \mu, \eta)$

Recent Advances in Transverse-Momentum-Dependent Distributions in Lattice QCD

Huey-Wen Lin[a,*], Wei Wang[b], Yong Zhao[c]

[a] *Department of Physics and Astronomy, Michigan State University, East Lansing, MI 48824*
[b] *State Key Laboratory of Dark Matter Physics, Shanghai Key Laboratory for Particle Physics and Cosmology, Key Laboratory for Particle Astrophysics and Cosmology (MOE), Physics-AI Institute School of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China*
Southern Center for Nuclear-Science Theory (SCNT), Institute of Modern Physics, Chinese Academy of Sciences, Huizhou 516000, Guangdong Province, China
[c] *Physics Division, Argonne National Laboratory, Lemont, IL 60439, USA*

HL, W. Wang, Y. Zhao, Invited Review by Prog. Part. Nucl. Phys. (2026)

What a TMD adds to a parton distribution

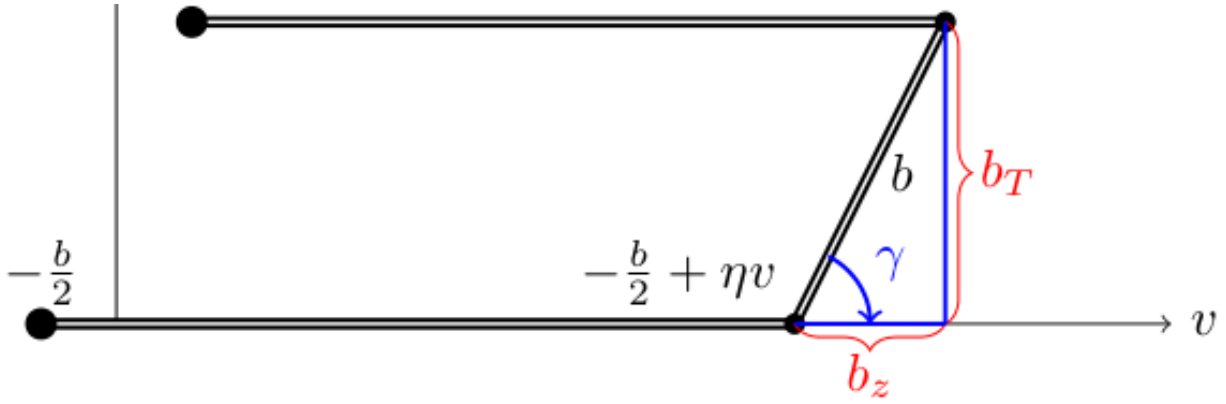
§ TMDPDF $f(x, k_\perp; \mu, \zeta)$

- x — parton longitudinal momentum fraction
- $k_\perp$ — intrinsic transverse momentum; the lattice works in the conjugate $b_\perp$
- μ — renormalization scale; ζ — Collins-Soper (rapidity) scale

§ Collinear PDFs integrate $k_\perp$ away, losing all transverse structure

§ The operator needs staple-shaped gauge links

- Staple direction encodes final/initial-state interactions
- Predicts the Sivers sign flip between SIDIS and Drell-Yan



Ebert, Schindler, Stewart & Zhao, JHEP 04 (2022) 178

What a TMD adds to a parton distribution

§ TMDPDF $f(x, k_\perp; \mu, \zeta)$

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$$f^{\mathrm{TMD}}(x, b_\perp, \mu, \zeta) = H\left(\frac{\zeta_z}{\mu^2}\right) e^{-\ln(\zeta_z/\zeta)K(b_\perp,\mu)} \tilde{f}(x, b_\perp, \mu, \zeta_z) S_r^{1/2}(b_\perp, \mu) + \cdots$$

The physical **lightcone TMDPDF**

Perturbative matching kernel; it resums the large logarithms of P^z and is known to NNLO.

Collins-Soper evolution from the lattice rapidity scale ζ_z to the physical scale ζ, governed by K.

The **quasi-TMDPDF** obtained on the lattice.

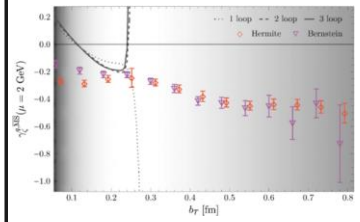
Square root of the **reduced soft function**.

Ebert, Schindler, Stewart & Zhao, JHEP 04 (2022) 178

Lattice TMD Calculations

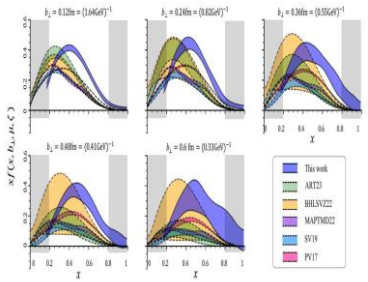
§ Physics-quantity milestones

Collins-Soper Kernel



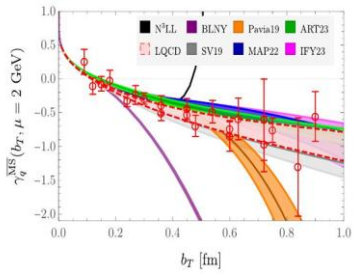
MIT'20 (March 2020)

1st Nucleon TMDPDF



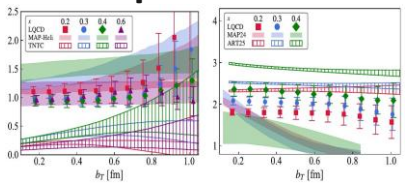
LPC'22 (Nov 2022)

1st CL CS Kernel



MIT/ANL'24 (Feb 2024)

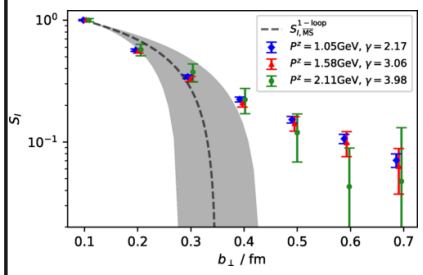
1st Helicity & Flavor-Dep TMDPDF



BNL/ANL'25 (May 2025)

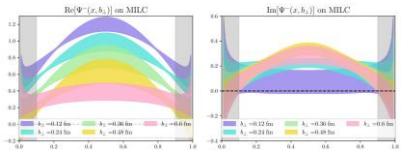
2020 2021 2022 2023 2024 2025 2026

Intrinsic Soft Function



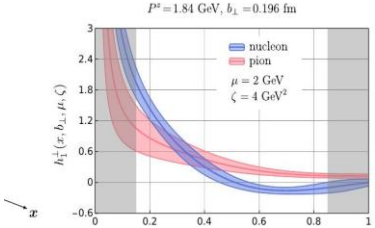
LPC'20 (May 2020)

π TMD Wave Function



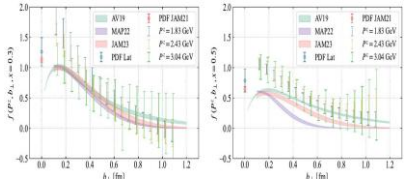
LPC'23 (Feb 2023)

First x-dep. Boer-Mulders TMD



LPC'24 (Dec 2024)

π TMDPDF (Coulomb-gauge)



BNL/ANL'25 (Apr 2025)

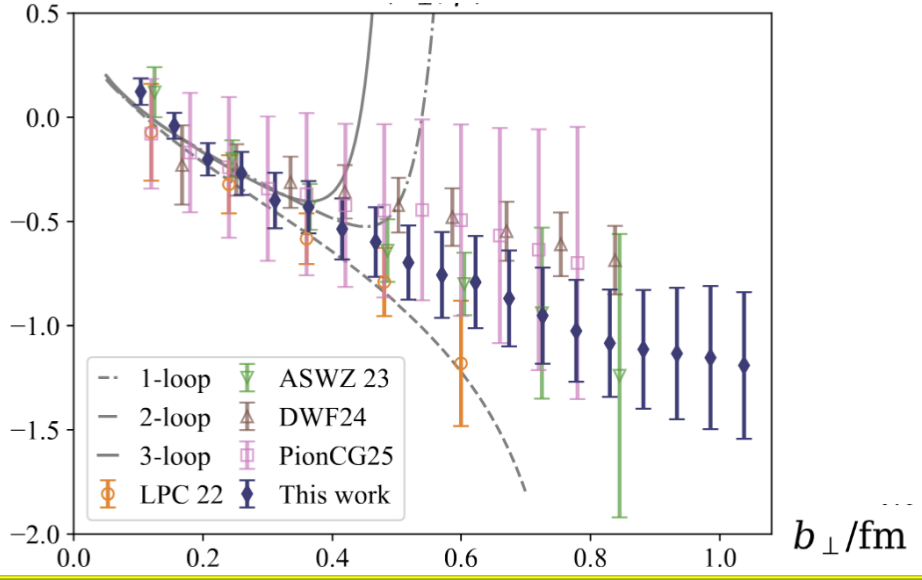
The Collins-Soper kernel: the evolution clock

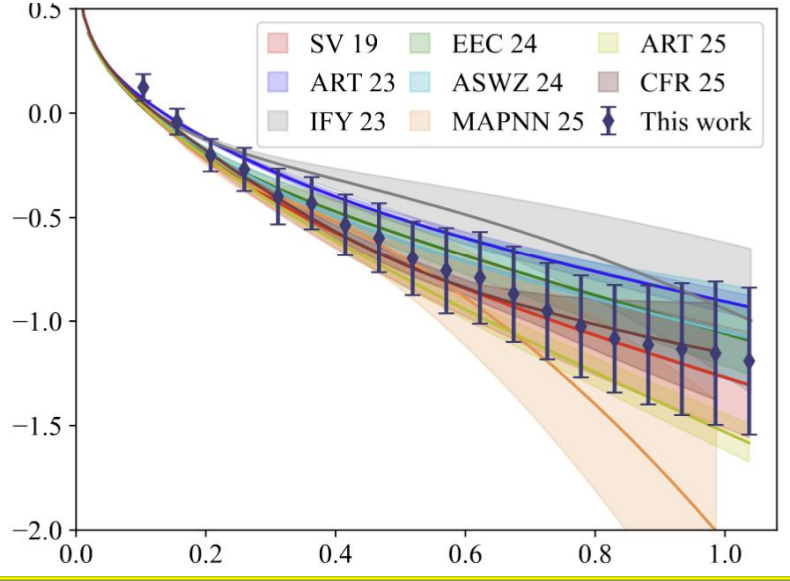
§ $K(b_\perp, \mu)$ governs how a TMD evolves in rapidity

- Universal: one K for every quark TMD, hadron and process
- It has no direct experimental observable
  - Global fits must model it at large $b_\perp$ — a leading uncertainty

§ State-of-the-art Lattce $K(b_\perp, \mu)$ Tan et al. (LPC'25), PRD113 (2026) 054505

- 2+1 clover, [0.05,0.10] fm, [**136**, 320]-MeV pion
- Agrees with three-loop perturbation theory at small $b_\perp$
- Lattice uncertainties are now competitive with global fits





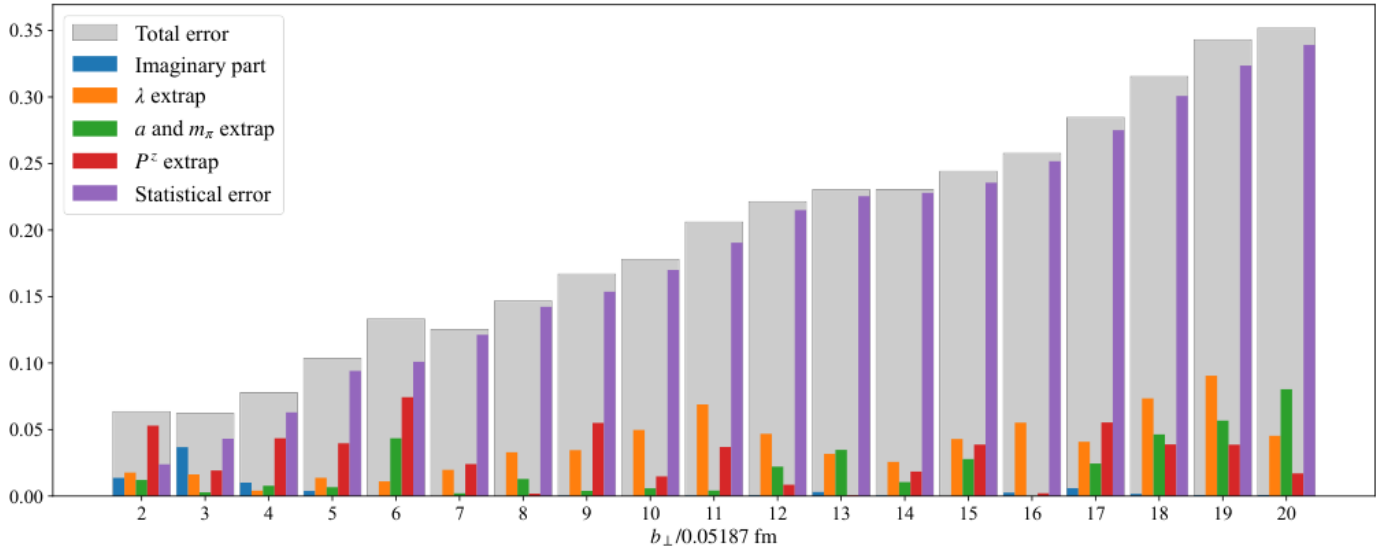
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- 2+1 clover, [0.05,0.10] fm, [**136**, 320]-MeV pion
- Reliable out to $b_\perp \approx 1$ fm — deep in the nonperturbative region
- Error budget is dominated by statistics, not by method



First lattice TMDPDFs of the nucleon

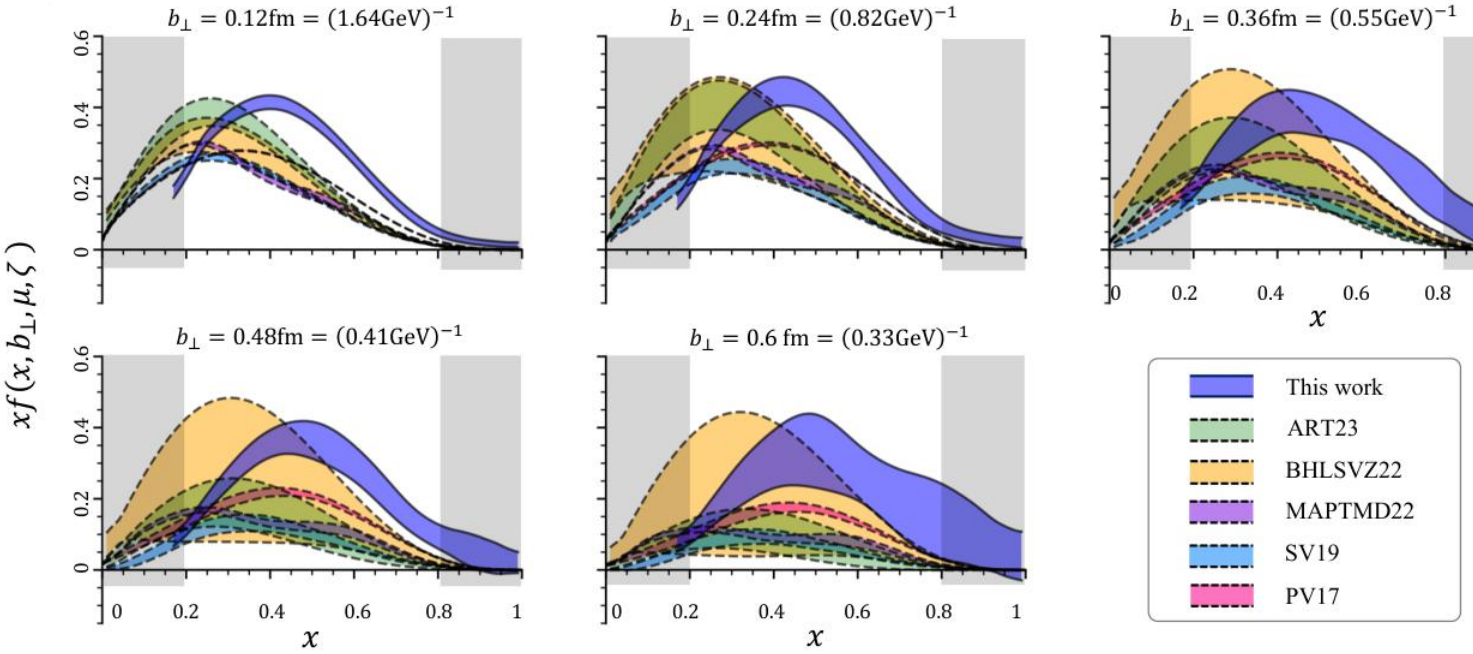
§ LPC'22: isovector unpolarized TMDPDF $x\,f(x, b_\perp, \mu, \zeta)$

- $N_f = 2+1+1$ (a=0.12 fm) MILC HISQ with {220,310}-MeV pion mass
- Extrapolated to $m_\pi = 135$ MeV and to infinite momentum
- NNLO matching with RG resummation

He et al. (LPC'22), PRD 109 (2024) 114513

§ Qualitative agreement with phenomenological fits

- Grey bands mark $x < 0.2$ and $x > 0.8$, where LaMET is unreliable



Pion TMD wave functions

§ TMDWF $\psi(x, b_\perp)$ — the exclusive-process counterpart of a TMDPDF

- Amplitude to find the $q\bar{q}$ pair at momentum fraction x, separation $b_\perp$

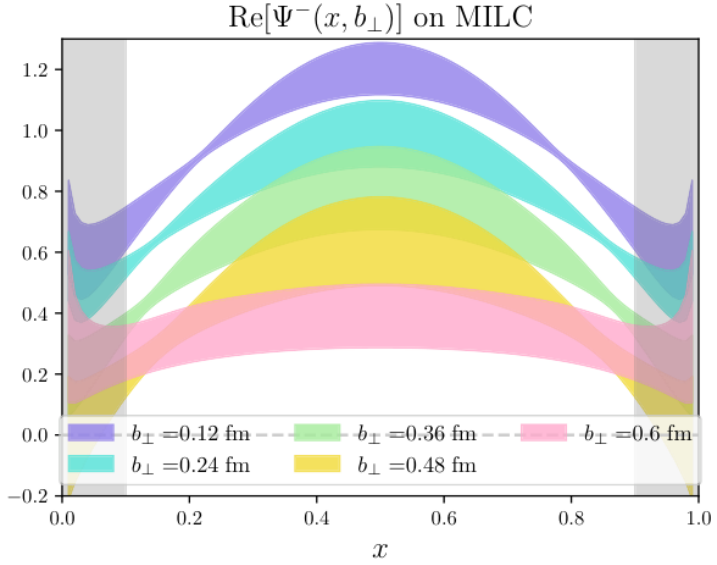
§ LPC'23: first lattice determination, on two independent ensembles

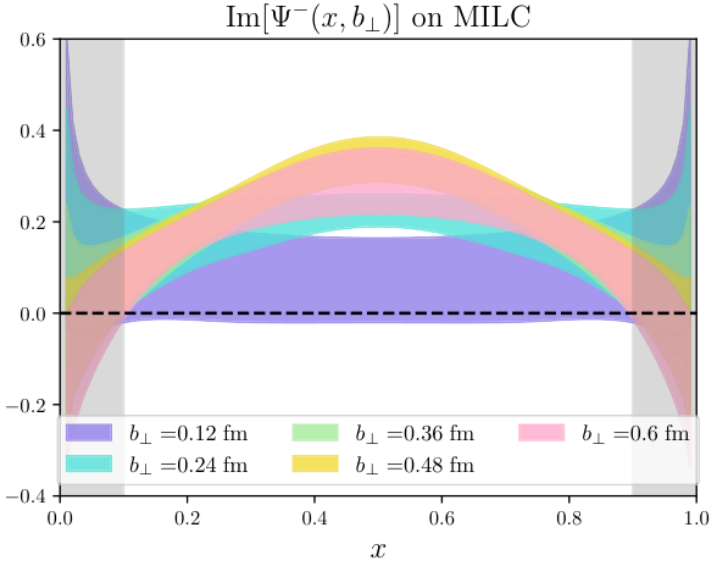
- MILC 2+1+1 HISQ (0.12fm) and CLS 2+1 clover (0.09fm) — a nontrivial cross-check

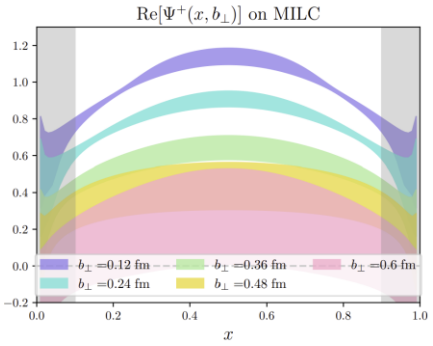
§ Reveals a non-negligible imaginary part

- Routinely neglected in phenomenological models
- ψ^+ and ψ^- differ, from the direction of the Wilson lines

Chu et al. (LPC'23), PRD 109 (2024) L091503







TMDPDFs at physical quark mass

§ BNL/ANL'25: proton helicity and flavour-separated TMDPDFs

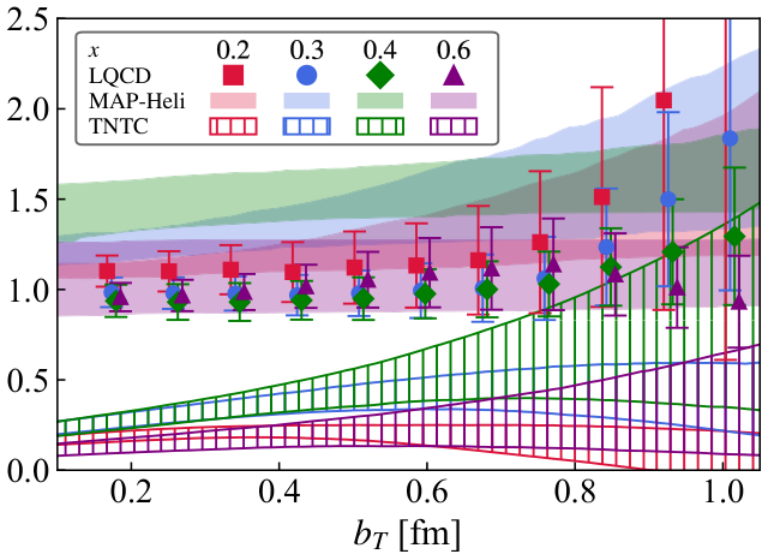
- Domain-wall fermions at the physical point, 2+1 flavours
- Coulomb-gauge correlators: no Wilson line, so the signal survives

§ Reaches $b_\perp \approx 1$ fm (i.e. $k_\perp \approx 200$ MeV)

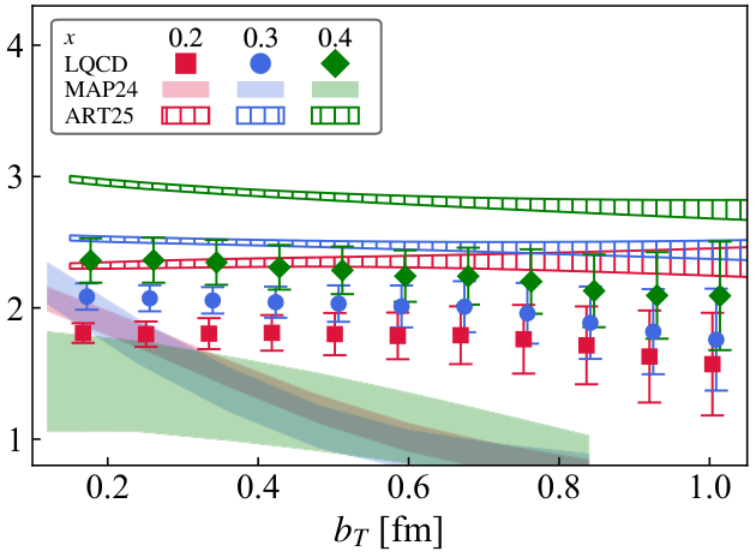
Bollweg et al., PRL 135 (2025) 201901

- Helicity and unpolarized transverse structure nearly identical
- Only mild flavour dependence between up and down quarks

$$\frac{g_{1L}^{\Delta u_+ - \Delta d_+}(x, b_T, \zeta, \mu)}{f_1^{u_v - d_v}(x, b_T, \zeta, \mu)}$$



$$\frac{f_1^{u_v}(x, b_T, \zeta, \mu)}{f_1^{d_v}(x, b_T, \zeta, \mu)}$$



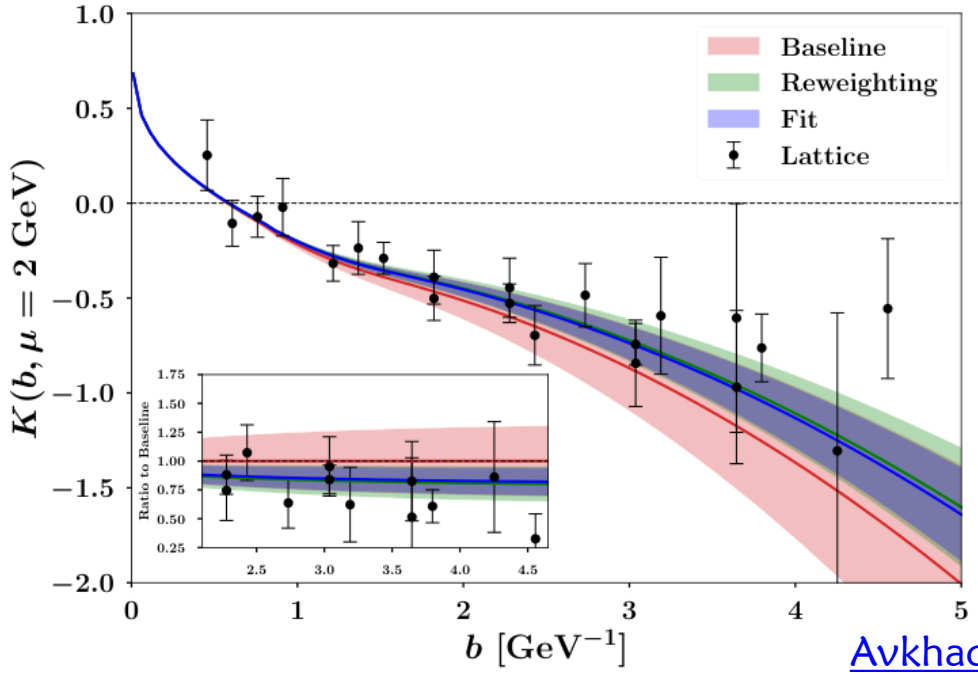
Lattice input to the global fits

§ First joint fit of Drell-Yan data together with lattice $K(b_\perp, \mu)$

- Use multiple LQCD data $a \in \{0.15, 0.12, 0.09\}$ extrapolation to physical pion
- Neural-network parametrization; reweighting and simultaneous fitting

§ Lattice helps reduce the fits uncertainty by 40-50%

- With no tension in the description of the Drell-Yan data
- Addresses a longstanding problem: K has no direct observable



Avkhadiev et al., PRL 136 (2026) 171902

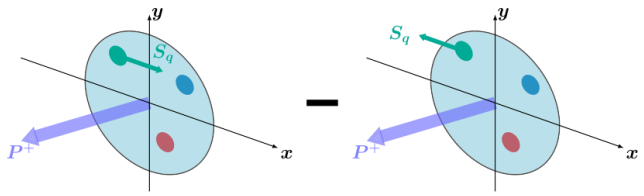
The Boer-Mulders function

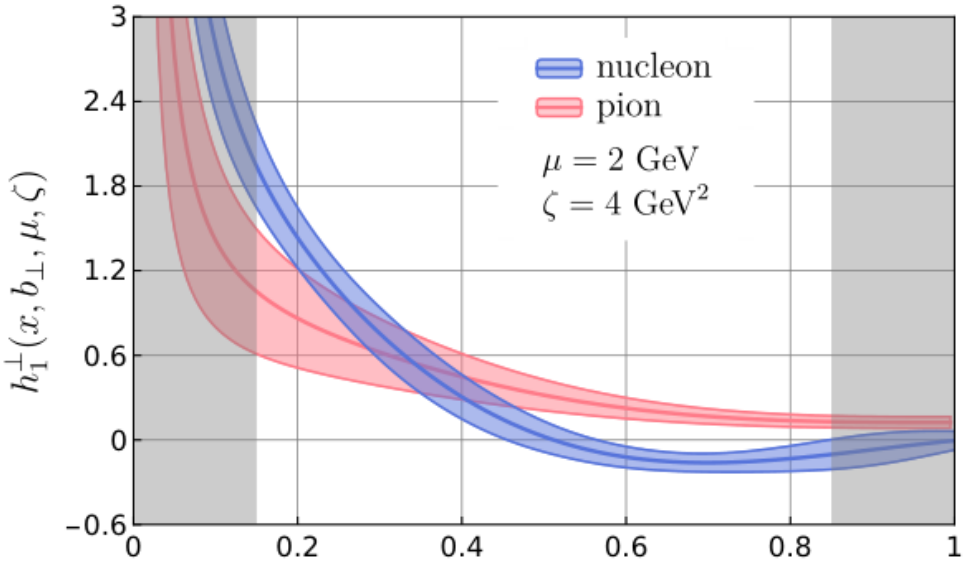
§ Boer-Mulders $h_1^\perp(\mathbf{x}, b_\perp)$: a time-reversal-odd TMD

- Correlates quark transverse polarization with transverse momentum
- In an unpolarized hadron — it needs the staple link to be non-zero

§ Computed for both the pion and the nucleon

- 2+1f clover fermion, a=0.098 fm, 338-MeV pion mass
- NLO matching with RGR; boosted momenta up to 2.11 GeV





LPC, 2502.11807

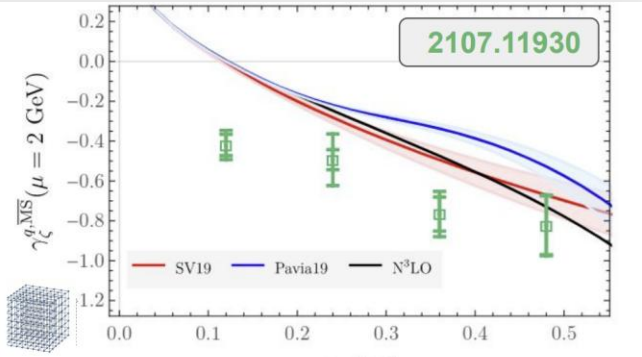
Gluon Collins-Soper Kernel

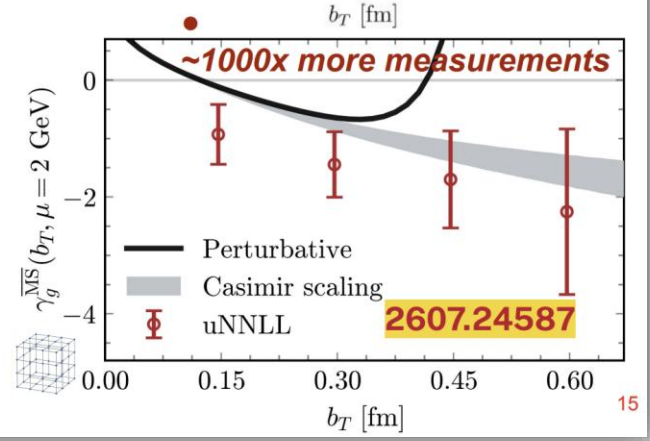
§ First Lattice-QCD calculations of gluon CS Kernel

- Clover-Wilson on 2+1+1 MILC HISQ physical pion ensemble
- Calculation done with 178-MeV pion mass w/ a~0.15 fm

Outlook

- Similar to quark CS kernel ~5 years ago
  - Need better perturbative matching
  - Need bigger boxes to control FV effects, Fourier transform and discretization artifacts
- But:
  - Statistics more challenging for gluon
  - CG method promising for statistics and systematic control
  - No pheno. results yet
- **Ongoing pheno. impact of quark results provides motivation for gluon case!**





Artur Avkhadiev, ANL

15

Slide from Artur Avkhadiev @Lattice2026 Talk