The Collins-Soper kernel from a vacuum soft function

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with

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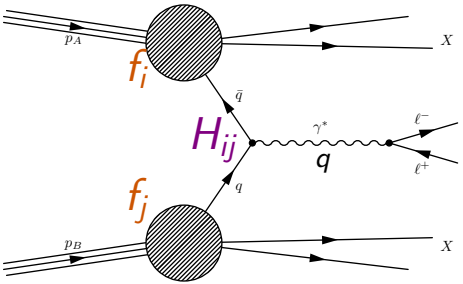


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TMD factorization in Drell-Yan scattering



- $\Lambda_{\rm QCD} \lesssim |\vec{q}_\perp| \ll Q = \sqrt{q^2}$
- Rapidity divergences in TMDPDFs need to be regulated
- Dependence on the Collins-Soper scale $\zeta_{a,b}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\mathrm{d}Y\mathrm{d}^2\vec{q}_\perp} = \sum_{i,j} H_{ij}\left(Q,\mu\right) \int \mathrm{d}^2\vec{b}_\perp e^{i\vec{b}_\perp\cdot\vec{q}_\perp} f_i\left(x_a,\vec{b}_\perp,\mu,\zeta_a\right) f_j\left(x_b,\vec{b}_\perp,\mu,\zeta_b\right)$$

$$\times\left[1+\mathcal{O}\left(\frac{q_\perp^2}{Q^2},\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)\right] \qquad \zeta_{a,b} = 2(x_{a,b}P^+e^{-y_n})^2$$

$$f_q\left(x_a, \vec{b}_\perp, \mu, \zeta_a\right) = \lim_{\substack{\epsilon\to 0 \\ y_B\to -\infty}} Z_{\rm UV}\left(\mu, \zeta_a, \epsilon\right) \frac{f_q^{\rm bare}\left(x_a, \vec{b}_\perp, \epsilon, y_B, x_a P^+\right)}{\sqrt{S_q\left(b_\perp, \epsilon, 2y_n - 2y_B\right)}}$$

UV scale evolution:

$$\gamma_\mu^q(\mu, \zeta) = \frac{\mathrm{d}\log f_q\left(x, \vec{b}_\perp, \mu, \zeta\right)}{\mathrm{d}\log \mu}$$

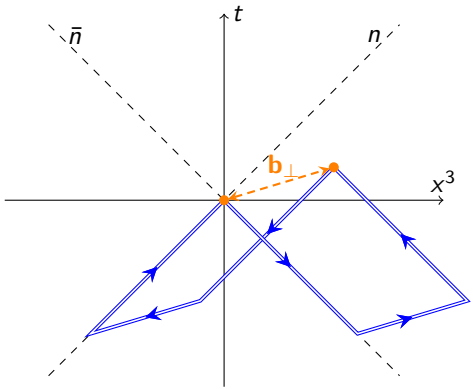
Rapidity evolution (CS kernel):

$$\gamma_q(b_\perp, \mu) = \frac{\mathrm{d}\log f_q\left(x, \vec{b}_\perp, \mu, \zeta\right)}{\mathrm{d}\log \sqrt{\zeta}}$$

The CS kernel controls the rapidity dependence of the soft function:

$$S_q\left(b_\perp, y_A, y_B, \mu\right) = S_I\left(b_\perp, \mu\right) e^{\gamma_q(b_\perp,\mu)(y_A - y_B)} \left(1 + \mathcal{O}\left(e^{-2(y_A - y_B)}\right)\right)$$

Regulating the soft function

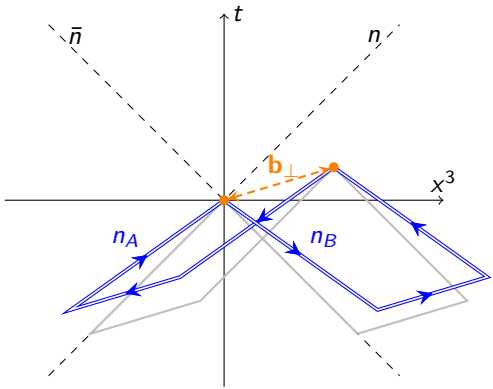


Soft function defined on the lightcone:

$$n^2 = \bar{n}^2 = 0$$

$$n = (1, 0, 0, 1),$$
$$\bar{n} = (1, 0, 0, -1)$$

Regulating the soft function



Soft function defined on the lightcone:

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$$n = (1, 0, 0, 1),$$
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Regulate rapidity divergence with spacelike Wilson lines:

$$n_A \equiv n - e^{-2y_A}\bar{n},$$
$$n_B \equiv \bar{n} - e^{+2y_B}n,$$
$$y_A, -y_B \to \infty$$

Relevance to lattice extraction of TMDPDFs

In the LaMET framework:

$$\sqrt{S_I\left(b_\perp,\mu\right)}\,\tilde{f}_\Gamma\left(x,b_\perp,P^z,\mu\right) = f\left(x,b_\perp,\mu,\zeta\right)H_f\left(x,P^z,\mu\right)\exp\left[\frac{1}{2}\log\frac{(2xP^z)^2}{\zeta}\gamma_q\left(b_\perp,\mu\right)\right] + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{xP^z},\frac{1}{b_\perp(xP^z)}\right)$$

[Ebert, *et al.*, 2019], [Ji, Liu, Liu, 2019], [Ebert, *et al.*, 2022]

$\tilde{f}_\Gamma$: quasi-TMD beam function, H_f: perturbative hard kernel

S_I: intrinsic soft function [Ji, Liu, Liu, 2020]

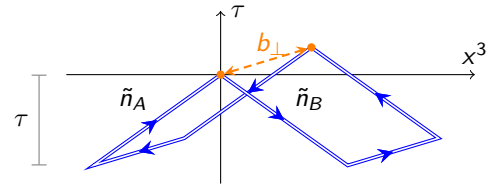
This talk: γ_q from a **vacuum** soft function, with no hadron states

Motivated by a proposal by Liu in [Ji, Liu, Liu, 2019]

Euclidean space directional vectors with purely imaginary time components

$$\tilde{n}_A = \left(in_A^0, 0, 0, n_A^3\right), \quad \tilde{n}_B = \left(in_B^0, 0, 0, n_B^3\right)$$

$$S_Q^{\text{lat}}\left(b_\perp, \tau; r_a, r_b; a\right) \sim$$



$$r_a = \frac{n_A^3}{n_A^0} = \frac{1 + e^{-2y_A}}{1 - e^{-2y_A}}, \quad r_b = \frac{n_B^3}{n_B^0} = \frac{1 + e^{2y_B}}{1 - e^{2y_B}}$$

Auxiliary field definition of the Wilson line

Write the Wilson line in terms of one-dimensional 'fermions' that live along the path:

$$P\exp\left\{-ig\int_{s_i}^{s_f}\mathrm{d}s n^\mu A_\mu(y(s))\right\}$$
$$= Z_\psi^{-1}\int \mathcal{D}\psi\mathcal{D}\bar{\psi}\,\psi\bar{\psi}\exp\left\{i\int_{s_i}^{s_f}\mathrm{d}s\bar{\psi}i\partial_s\psi - \bar{\psi}n\cdot A\psi\right\}$$

[Gervais, Neveu 1980], [Aref'eva 1980]

Auxiliary field propagator:

$$in\cdot DH_n(y) = \delta(y) \overset{\text{Euclidean space}}{\longrightarrow} -i\tilde{n}\cdot D_E H_{\tilde{n}}(y) = \delta(y), \qquad \tilde{n} = (in^0, \vec{n})$$

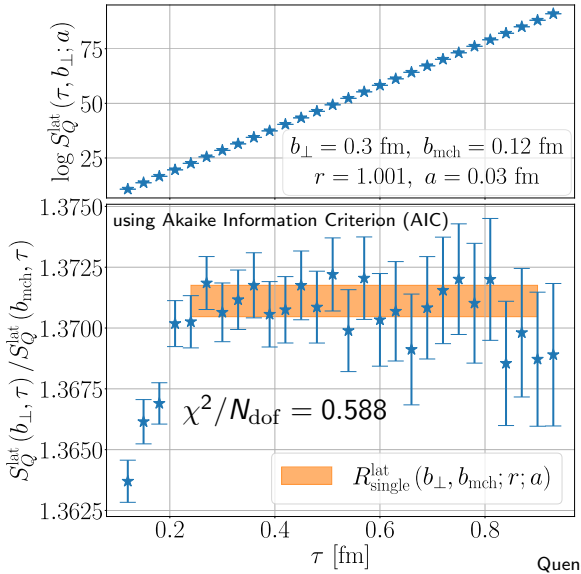
Meaningful solution only obtained with a UV cutoff, since the auxiliary field Hamiltonian is unbounded from below. [Aglietti, *et al.* 1992], [Aglietti, 1994]

$\Rightarrow$ Cutoff effect cancels in a ratio at fixed (τ, r, a) next slide

Perturbative computation of soft factor with auxiliary field propagators requires further investigation [Liu, 2022]

$\Rightarrow$ Under investigation

Single ratio, time dependence



$$R_{\rm single}^{\rm lat}(b_\perp, b_{\rm mch}, r, a) = \lim_{\tau\to\infty} \frac{S_Q^{\rm lat}(b_\perp, \tau; r, r; a)}{S_Q^{\rm lat}(b_{\rm mch}, \tau; r, r; a)}$$

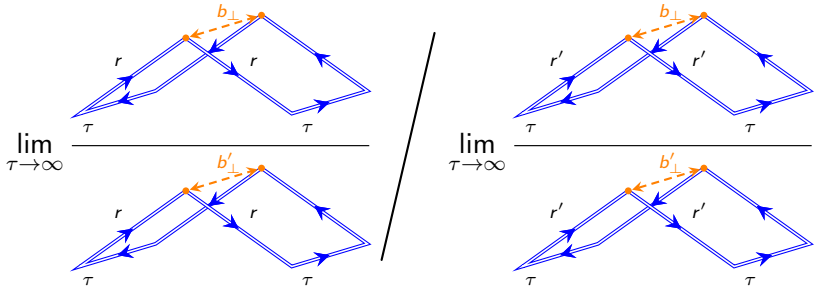
Same τ, r, a in numerator and denominator: cutoff-dependent growth cancels in the ratio

Quenched configurations from [Detmold, Endres, 2018]

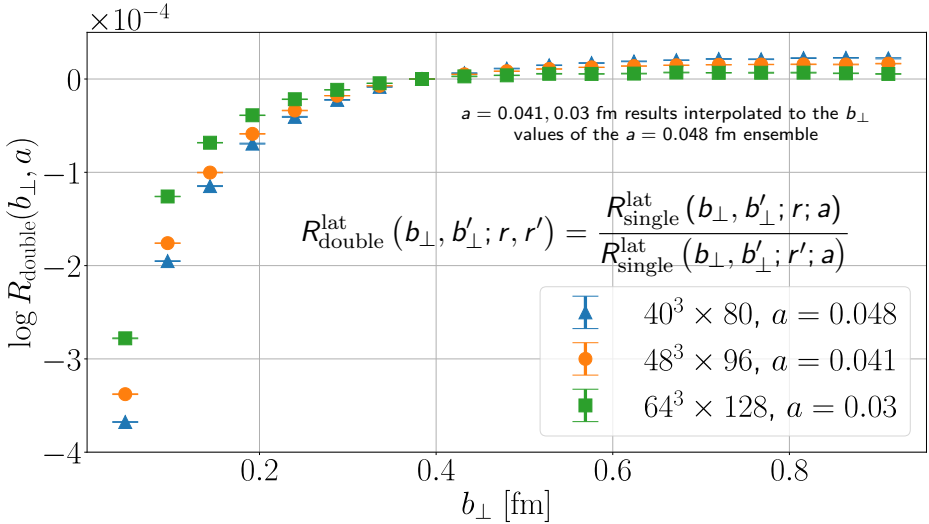
Treatment of divergences

$$R^{\text{lat}}_{\text{double}}\left(b_\perp, b'_\perp; r, r'\right) = \frac{R^{\text{lat}}_{\text{single}}\left(b_\perp, b'_\perp; r; a\right)}{R^{\text{lat}}_{\text{single}}\left(b_\perp, b'_\perp; r'; a\right)} = e^{\Delta\gamma_q(b_\perp, b'_\perp)\Delta Y(r,r')}$$

$$\Delta\gamma_q(b_\perp, b'_\perp) = \gamma_q\left(b_\perp, a\right) - \gamma_q\left(b'_\perp, a\right), \qquad \Delta Y(r, r') = \log\left(\frac{r+1}{r-1}\Big/\frac{r'+1}{r'-1}\right)$$



Interpolated double ratio, $b_\perp$ dependence



$b' = 0.384$ fm, $r = 1.002$, $r' = 1.001$

Matching procedure for double ratio

$$\gamma_q\left(b_{\perp,1},\mu\right)=\gamma_q\left(b_{\perp,2},\mu\right)+\frac{\log\left(R_{\text{double}}\left(b_{\perp,1},b_{\perp,2},\mu,r_1,r_2\right)\right)}{\log\left(\frac{r_1+1}{r_1-1}\Big/\frac{r_2+1}{r_2-1}\right)}$$

- Implies that we need at least one perturbative value of γ_q.
- Another perturbative value of γ_q is needed, because $r_{1,2}$ is renormalized due to $O(4)$ symmetry breaking. [Aglietti, *et al.* 1992], [Aglietti, 1994]

Model for the double ratio

- Model the double ratio as a piecewise function:

$$\log R^{\text{lat}}_{\text{double}}\left(b_\perp, b_{\text{mch}}; r, r'\right) = \Delta Y_i^{\text{ren}}\left(\left\{\begin{array}{ll} \gamma_q^{\overline{\text{MS}}}(b_\perp,\mu) - \gamma_q^{\text{mch}} & , \; b_{\text{cut}} \le b_\perp \le b_{\text{th}} \\ \{d_\ell\} & , \quad b_\ell = b_\perp > b_{\text{th}} \end{array}\right\} + c_1\left(\frac{a^2}{b_\perp^2} - \frac{a^2}{b_{\text{mch}}^2}\right)\right),$$

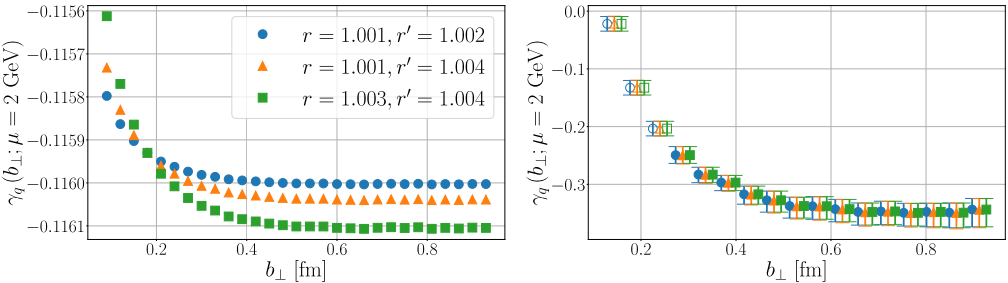
- Combined fit on three ensembles
- Model imposes a constraint on the large-$b_\perp$ region

$$b_{\text{cut}} = 0.144 \text{ fm}, \quad b_{\text{th}} = 0.24 \text{ fm}, \quad b_{\text{mch}} = 0.384 \text{ fm} > b_{\text{th}}$$

$$d_\ell = \gamma_q\left(b_\ell,\mu\right) - \gamma_q\left(b_{\text{mch}},\mu\right), \qquad \gamma_q^{\text{ext}}\left(b_\ell,\mu\right) = \gamma_q^{\text{mch}} + d_\ell$$

- Model does not adequately estimate the lattice spacing dependence $\Rightarrow$ assigned as a discretization uncertainty

Bare vs renormalized rapidity factor

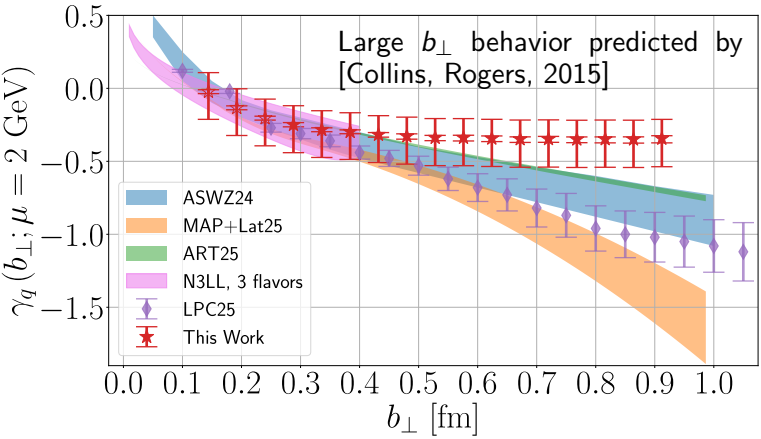


The CS kernel determined on the $a = 0.03$ fm ensemble with three sets of bare rapidities. Bare: γ_q varies by only $\sim 5 \times 10^{-4}$ and depends on (r, r'). Renormalized: the three sets agree.

- Left plot uses bare rapidity:

$$y - y' = \log\left(\frac{r+1}{r-1} \Big/ \frac{r'+1}{r'-1}\right)$$

- Right plot from model fit with renormalized rapidity
- Includes statistical, truncation and discretization errors.



- Our result is pure gauge. Comparison plots have $n_f = 3, 4$.
- Inner error bars include statistical, truncation and discretization errors.
- Outer error bars add the UV scale uncertainty (μ varied by a factor $\sqrt{2}$).

Conclusion and outlook

- Euclidean space soft function with complex directional vectors maps directly to Minkowski space result
- Lattice data at high statistical precision, low computational cost
- Double ratio method gives: $\gamma_q\left(b_\perp, \mu\right) - \gamma_q\left(b'_\perp, \mu\right)$
- Error primarily from UV scale uncertainty

Open questions and future direction:

- Ongoing exploration of perturbative results for auxiliary field computation
- Improve systematic uncertainties by simulating at finer lattices or with an alternative scheme for the rapidity renormalization