Transverse force densities in the nucleon from lattice QCD

Ross Young

Adelaide University · QCDSF Collaboration

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Institute of Physics, Academia Sinica



Collaborators



Joshua Crawford
Adelaide, PhD 2025



K. Utku Can
Adelaide



Alec Hannaford Gunn
Adelaide, PhD 2023



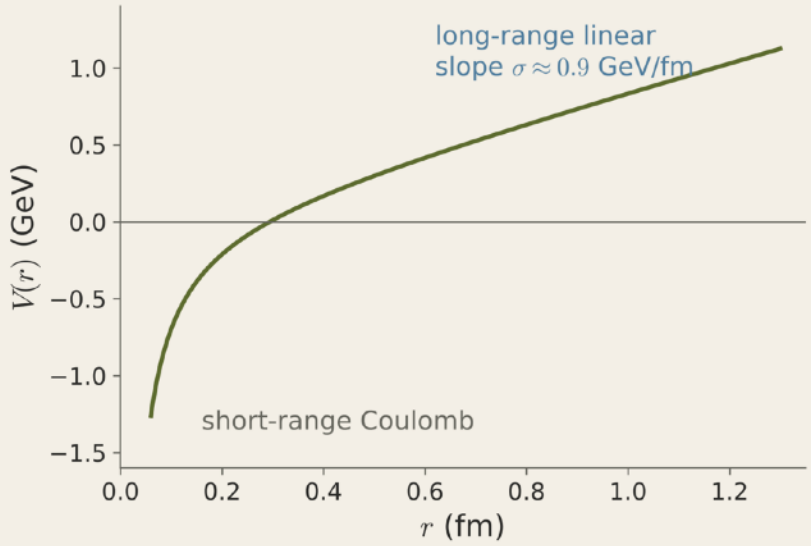
QCDSF at Lattice 2017, Granada: Stüben, Horsley, Rakow, RDY, Zanotti, Schierholz, Perlt, Nakamura

Confinement: Static quark potential

- Energy of a static quark–antiquark pair in pure Yang–Mills
- Coulomb at short range, linear at long range: the language of heavy-quark spectroscopy
- The slope is the string tension:

$$\sigma \approx 0.18\ \text{GeV}^2 \approx 0.9\ \text{GeV/fm}$$

Benchmark scale of colour forces; with light quarks the string breaks



What force acts on a *light* quarks in the *nucleon* at the instant we try to separate it?

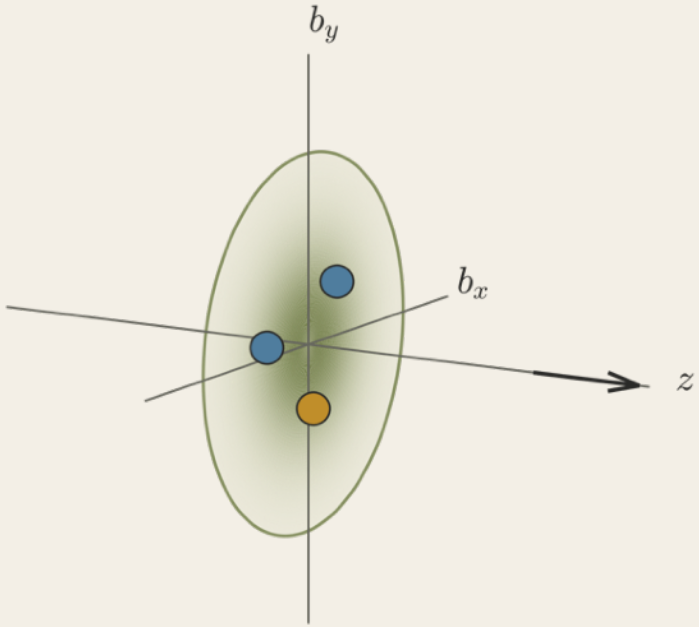
Colour forces in scattering

Colour forces in scattering · Forces on the lattice · Sum rules and higher twist

Imaging nucleon structure

- Generalised parton distributions: longitudinal momentum and transverse position of quarks
- Transverse-momentum distributions: their transverse motion

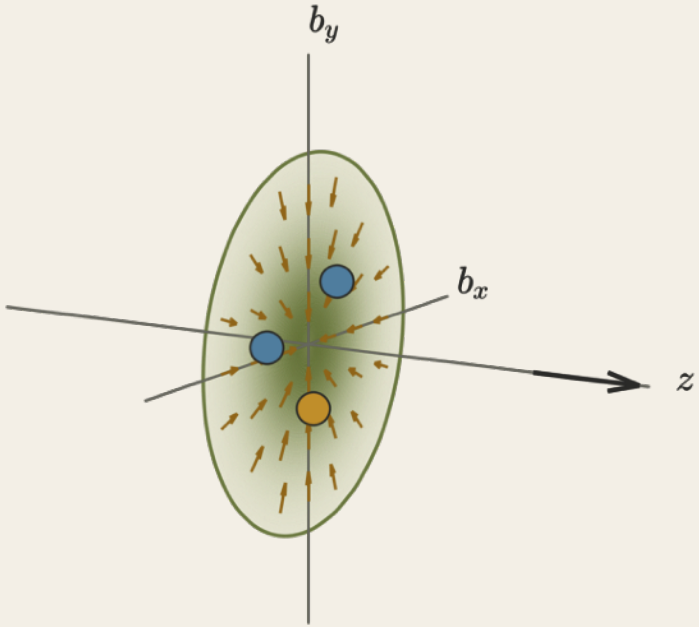
Can we go further, and describe the forces acting on these quarks?



Imaging nucleon structure

- Generalised parton distributions: longitudinal momentum and transverse position of quarks
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Can we go further, and describe the forces acting on these quarks?



Sivers (1990); Brodsky, Hwang & Schmidt; Collins (2002)

The Sivers asymmetry

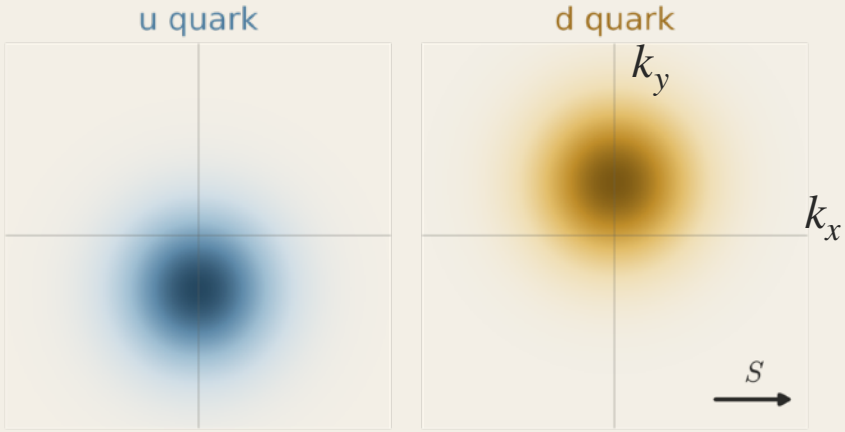
Measured

$$d\sigma \propto F_{UU}\left[1 + |S_T|\, A_{UT}^{\sin(\phi_h - \phi_S)} \sin(\phi_h - \phi_S) + \ \ldots\right]$$

- SIDIS off a transversely polarised target: hadrons emerge preferentially to one side
- Clearly non-zero asymmetry for π^+ production on transverse proton (HERMES, COMPASS)
- Naively time-reversal odd: zero without an interaction

Interpreted, in TMD factorisation

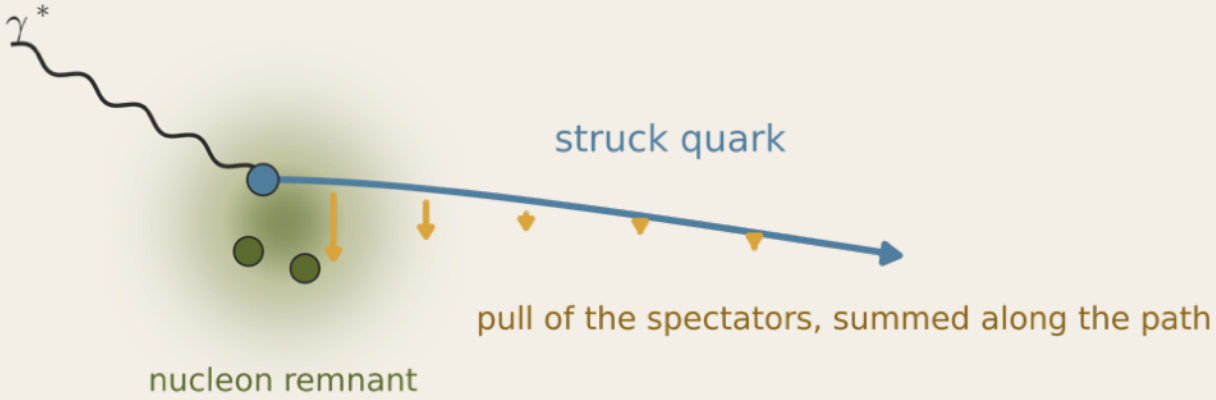
- A partonic correlation of nucleon spin with quark $k_\perp$: the Sivers function



Sivers function: partonic interpretation, schematic

Brodsky, Hwang & Schmidt; Collins (2002); Burkardt (2004)

Integrating the final-state interaction

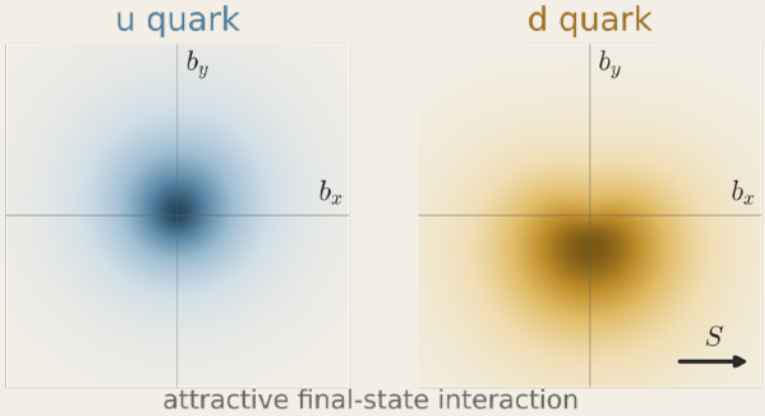


- The struck quark leaves through the colour field of the spectators
- The asymmetry needs the transverse pull accumulated along its whole path
- Before the collision instead (Drell–Yan), the sign reverses: process dependence
- On the lattice, the path is built into the operator (see Yong Zhao, ~1 hour ago)

Burkardt, PRD 66 (2002); NPA 735 (2004)

Chromodynamic lensing

- Transverse spin shifts u and d in opposite directions ($\kappa_u > 0, \kappa_d < 0$)
- An attractive final-state force turns the spatial shift into a momentum shift
- Predicts the signs of the u and d Sivers functions, consistent with the π^+ asymmetries



attractive final-state interaction

Sivers function ≈ spatial distortion × final-state force

Jaffe, Comments Nucl. Part. Phys. 19 (1990) 239
Jaffe & Ji, PRD 43 (1991) 724

g_2: clean access to twist three

$$W_{\mu\nu} = F_1\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) + \frac{F_2}{p\cdot q}\left(p_\mu - \frac{p\cdot q\, q_\mu}{q^2}\right)\left(p_\nu - \frac{p\cdot q\, q_\nu}{q^2}\right)$$
$$+ \frac{ig_1}{p\cdot q}\epsilon_{\mu\nu\lambda\sigma}q^\lambda s^\sigma + \frac{ig_2}{(p\cdot q)^2}\epsilon_{\mu\nu\lambda\sigma}q^\lambda(p\cdot q\, s^\sigma - s\cdot q\, p^\sigma)$$

- Deep inelastic: F_1, F_2 and g_1 map onto parton distributions; g_2 has no simple partonic interpretation
- Subtract the Wandzura–Wilczek part: the remainder $\bar{g}_2$ is genuine quark–gluon correlation

$$g_2 = g_2^{\mathrm{WW}}[g_1] + \bar{g}_2$$

Burkardt, PRD 88 (2013) 114502

Colour-Lorentz force

The x^2 moment of $\bar{g}_2$ is a local matrix element

$$\int dx\, x^2\, \bar{g}_2(x) = \frac{d_2}{3} \equiv \frac{1}{6} \sum_q e_q^2\, d_2^q$$

$$d_2^q = \frac{1}{2MP^+P^+S^x} \left\langle P,S \right| \bar{\psi}_q(0)\, \gamma^+\, gG^{+y}(0)\, \psi_q(0) \left| P,S \right\rangle$$

$$G^{+y} = \frac{1}{\sqrt{2}} (G^{0y} + G^{zy}) = -\frac{1}{\sqrt{2}} \left[\vec{E}_c + \vec{v} \times \vec{B}_c \right]^y$$

Quark current density coupled to the colour-Lorentz force

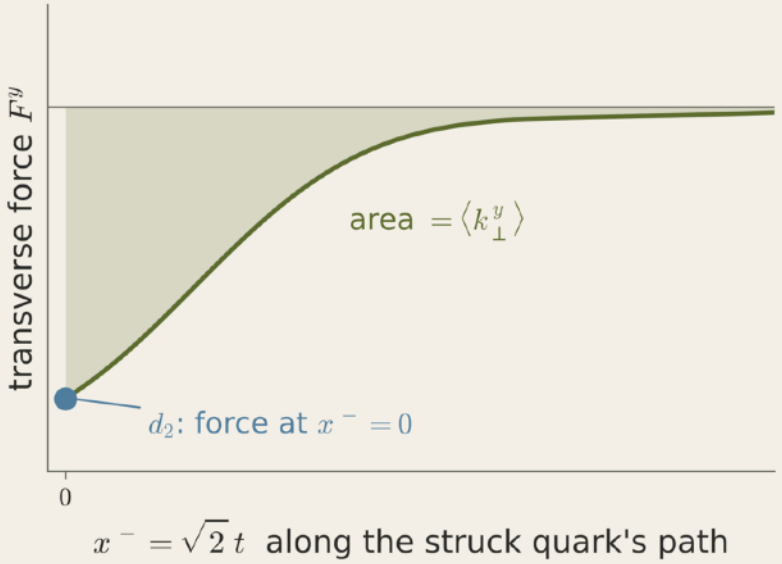
d_2: the force at the instant of the collision

$$\langle k_\perp^y \rangle = -\tfrac{1}{2P^+} \langle P,S| \bar\psi(0)\,\gamma^+ \int_0^\infty dx^-\, gG^{+y}(x^-)\,\psi(0)\,|P,S\rangle$$

- Partonic level: the $k_\perp$-weighted Sivers function is the force integrated along the path
- d_2 is the integrand at $x^- = 0$: the force just after the photon is absorbed

$$\langle k_\perp^y \rangle = \int_0^\infty dt\; F^y(t), \qquad x^- = \sqrt{2}\,t$$

$$F^y(0) = -M^2 d_2 \qquad \text{rest frame, S along x}$$

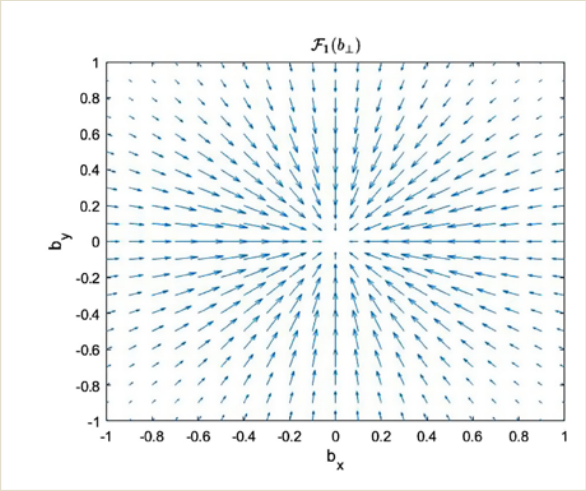


Aslan, Burkardt & Schlegel, PRD 100 (2019)

Going off-forward: imaging the force

$$\langle p', s'|\bar{\psi}\gamma^+ igG^{+i}\psi|p, s\rangle = \bar{u}(p', s')\big[P^+\Delta^i\gamma^+\Phi_1(t) + MP^+ i\sigma^{+i}\Phi_2(t) + \tfrac{1}{M}P^+\Delta^i\, i\sigma^{+\Delta}\Phi_3(t)\big]u(p, s)$$

- Momentum transfer $\Delta_\perp$ is Fourier-conjugate to impact parameter $b_\perp$
- Partition by $\Delta_\perp$: three form factors, Φ_1, Φ_2, Φ_3
- Forward limit: $\Phi_2(0) = d_2$
- 2D Fourier transform: the force distribution in the transverse plane



Resolving forces from lattice QCD

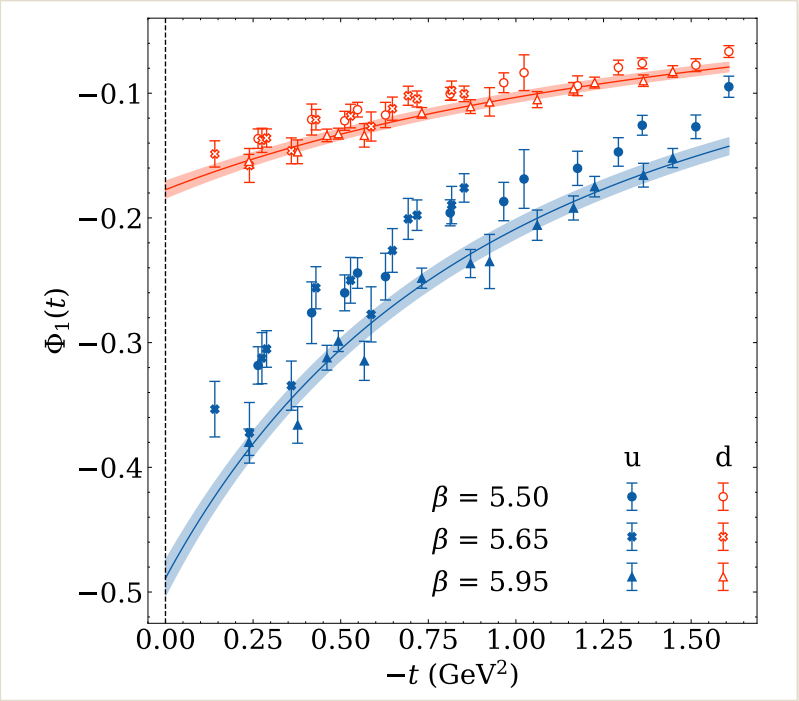
Colour forces in scattering · Forces on the lattice · Sum rules and higher twist

Crawford, RDY et al., PRL 134 (2025)

Φ_1: isotropic / spin-independent

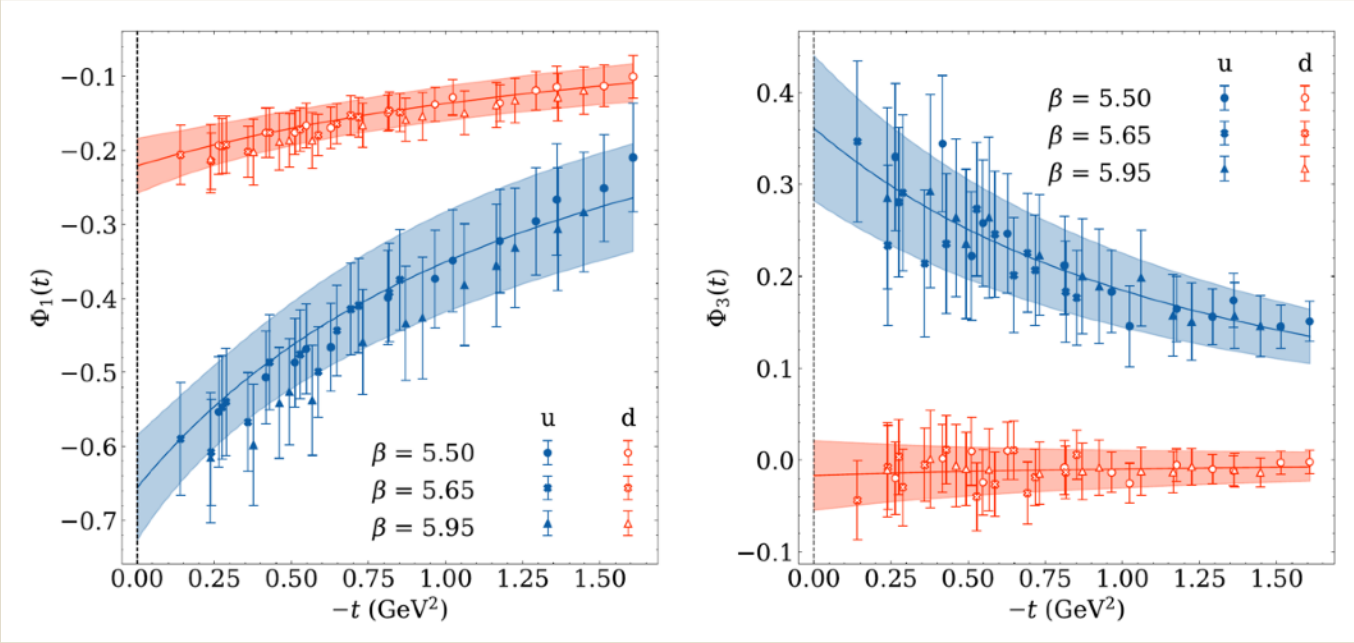
- Three-point functions, three source–sink separations
- $m_\pi \simeq 420\,\mathrm{MeV}$
- 3 lattice spacings: $\{0.052, 0.068, 0.074\}$ fm
- Dipole fit (finest lattice)

$\Phi_0 < 0$: attractive



Crawford, RDY et al., PRL 134 (2025)

Continuum extrapolation

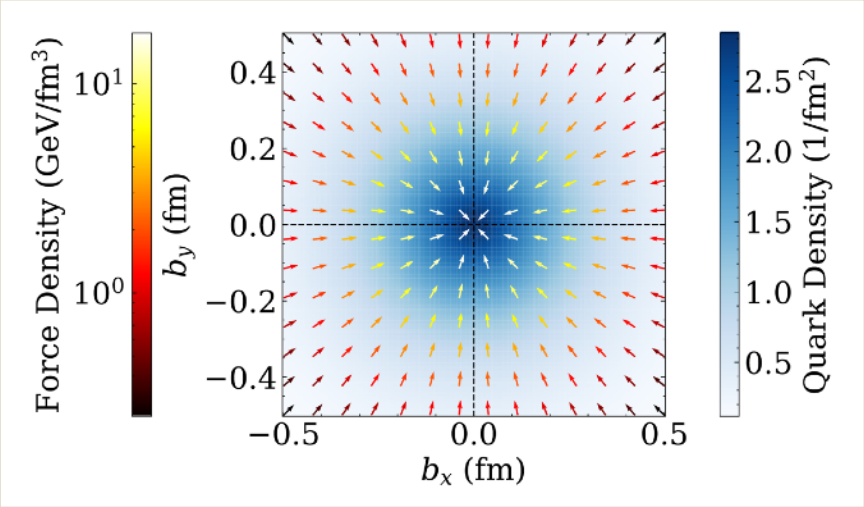


Three lattice spacings: Controlling discretisation broadens the bands: the price of a continuum result

Force density: unpolarised

- Φ_1 alone: isotropic
- Points to the centre: attractive
- Largest where the quarks are

$$\mathcal{F}^i(\mathbf{b}) = -2\sqrt{2}\, P^+\, b^i\, \tfrac{d}{db^2}\tilde{\Phi}_1(b^2)$$



up quark

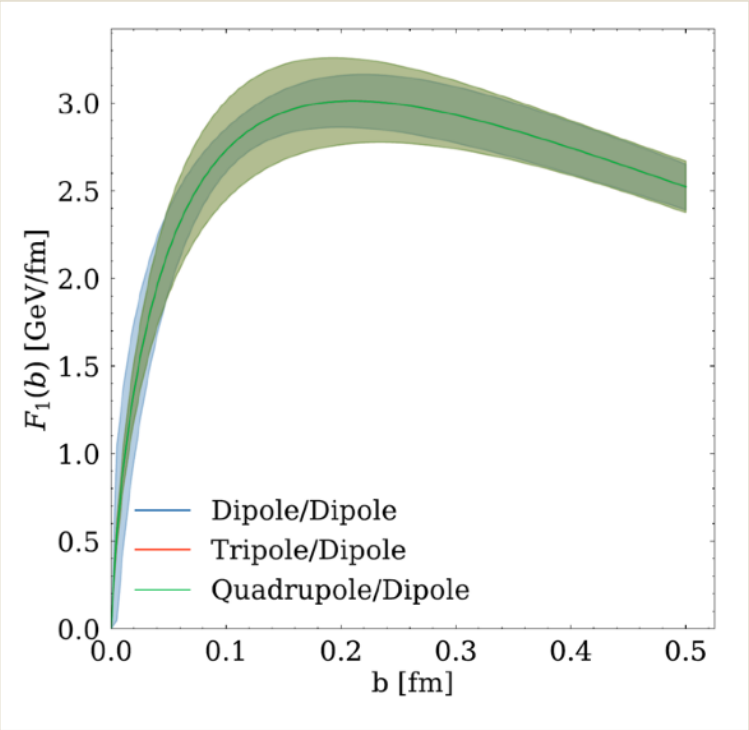
Local forces

- Divide out the quark density: the force per quark

$$F \sim \frac{\langle \rho F \rangle}{\langle \rho \rangle} = \frac{\langle \bar{\psi} \gamma^+ F^{+b} \psi \rangle}{\langle \bar{\psi} \gamma^+ \psi \rangle}$$

- Peak ≈ 3 GeV/fm at b ≈ 0.2 fm
- About three times the string tension

$3\,\mathrm{GeV/fm} \simeq 5 \times 10^5\,\mathrm{N}$: the weight of about ten elephants



In the press



Protons' Internal Forces Are As Strong As The Weight Of 5 Schoolbuses
Raw News Health, 26 Feb 2025
Protons sit in the nuclei of all atoms, but they are not fundamental particles: They are made of three quarks...

Protons' Internal Forces Are As Strong As The Weight Of 5 Schoolbuses
IFLScience, 26 Feb 2025
Space and Physicsphysics PUBLISHED31 minutes ago This is why you need very powerful particle accelerators to smash them! Dr.

Force As Strong As 10 Compressed Elephants Rests Inside A Proton, Suggests Study
Wonderful Engineering, 24 Feb 2025
Protons, the building blocks of all matter, hold some of the deepest mysteries in modern physics.

Illuminating the Inner Workings of the Proton
Space Daily, 24 Feb 2025
The international collaboration, which includes researchers from the University of Adelaide, is focused on uncovering the...

Mapping the forces inside protons
COSMOS magazine, 24 Feb 2025
You might be surprised to hear that there's anything 'inside' a proton. But scientists have mapped the forces between the...

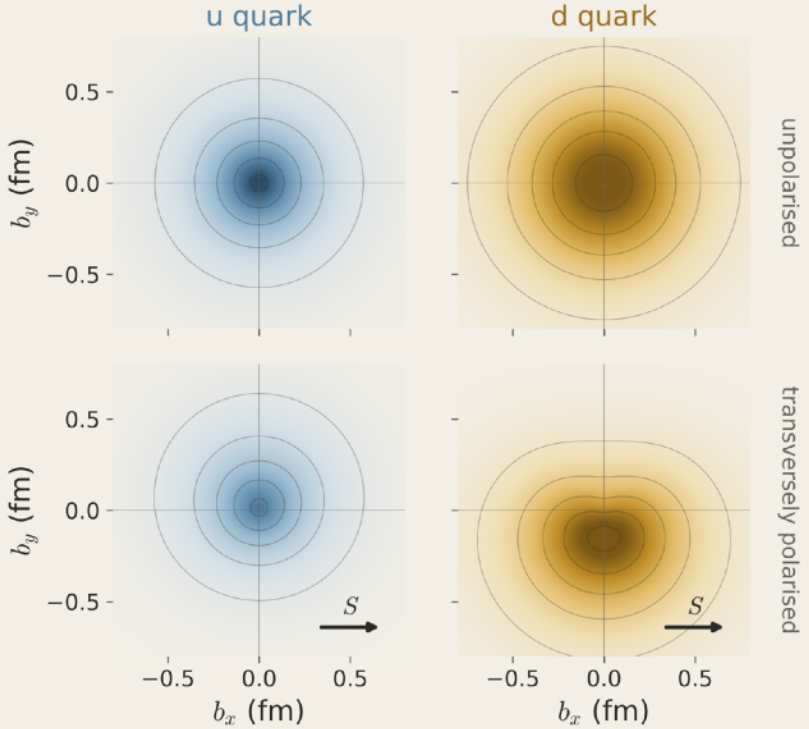
Illuminating the Inner Workings of the Proton
Sky Nightly , 24 Feb 2025
Illuminating the Inner Workings of the Proton by Simon Marsfield Sydney, Australia (SPX) Feb 21, 2025 A team of scientists has...

3 GeV/fm ≈ 5 × 10^5 N · PRL 134 (2025), Editors' Suggestion

Quark densities in the transverse plane

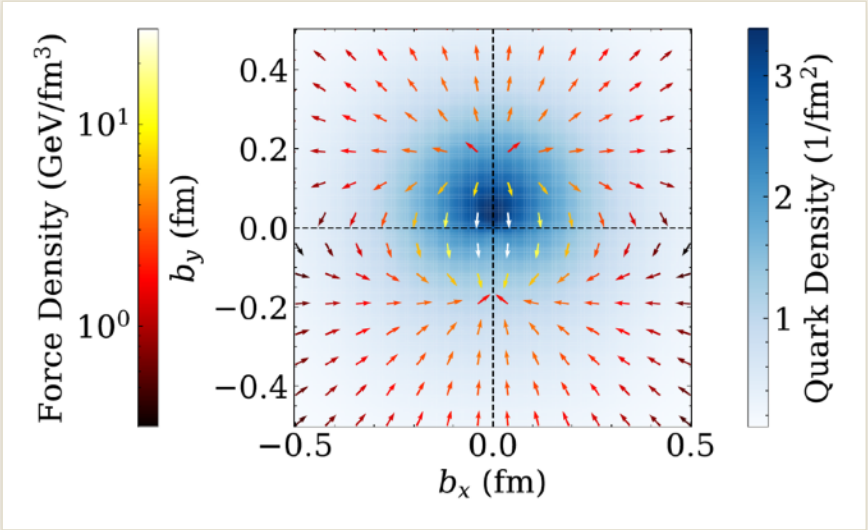
$$q(\mathbf{b}) = \tfrac{1}{2}\left[\tilde{F}_1(b^2) + \frac{b^j \epsilon^{ji} S^i}{m_N} \frac{d}{db^2}\tilde{F}_2(b^2)\right]$$

- Unpolarised: the Dirac form factor F_1
- Transverse spin: a shift set by the Pauli form factor F_2
- u up, d down: the distortion behind lensing



Force density: spin dependent

- Proton polarised along x: the spin-flip part, from Φ_2 and Φ_3
- Forward limit: $\Phi_2(0) = d_2$
- At the centre the force opposes the u-quark shift, as in lensing



up quark

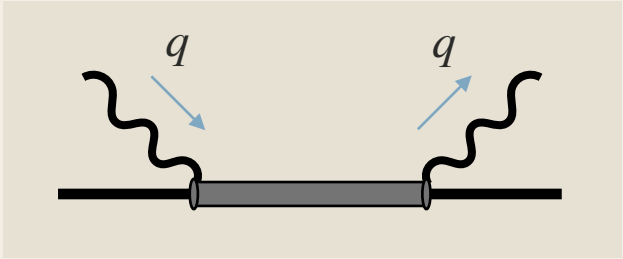
Suggestive of Sivers: off-forward, and only the $x^- = 0$ end of the final-state interaction

$$\mathcal{F}^i_{ss'}(\mathbf{b}) = -2\sqrt{2}P^+\frac{d}{db^2}\tilde{\Phi}_1(b^2) + \sqrt{2}m_N\epsilon^{ij}S^j\tilde{\Phi}_2(b^2) - \frac{2\sqrt{2}\epsilon^{jk}S^k}{m_N}\left[\delta^{ij}\frac{d}{db^2} + 2b^ib^j\frac{d^2}{(db^2)^2}\right]\tilde{\Phi}_3(b^2)$$

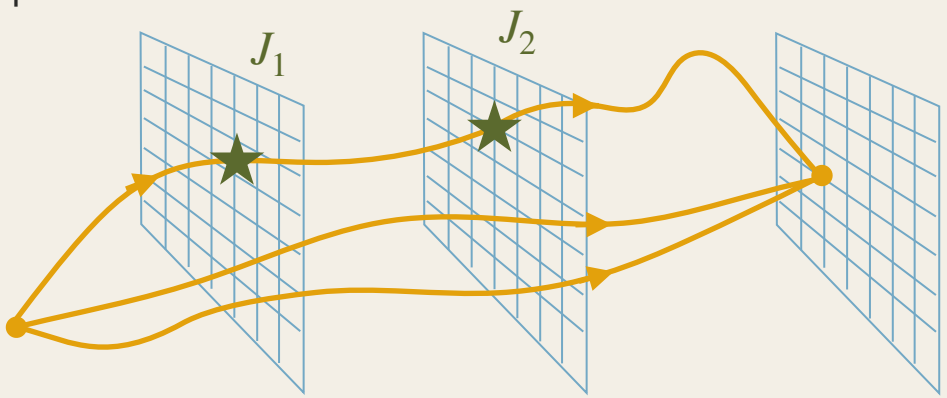
Sum rules and higher twist

Colour forces in scattering · Forces on the lattice · Sum rules and higher twist

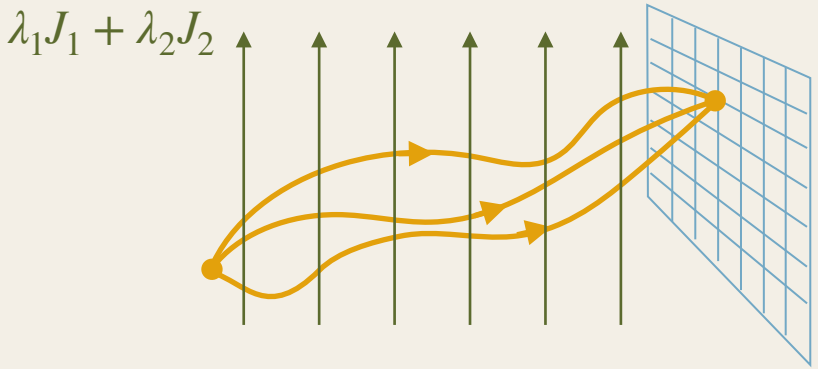
Compton amplitude on the lattice



4-pt function



Feynman-Hellmann: energy shift → matrix element



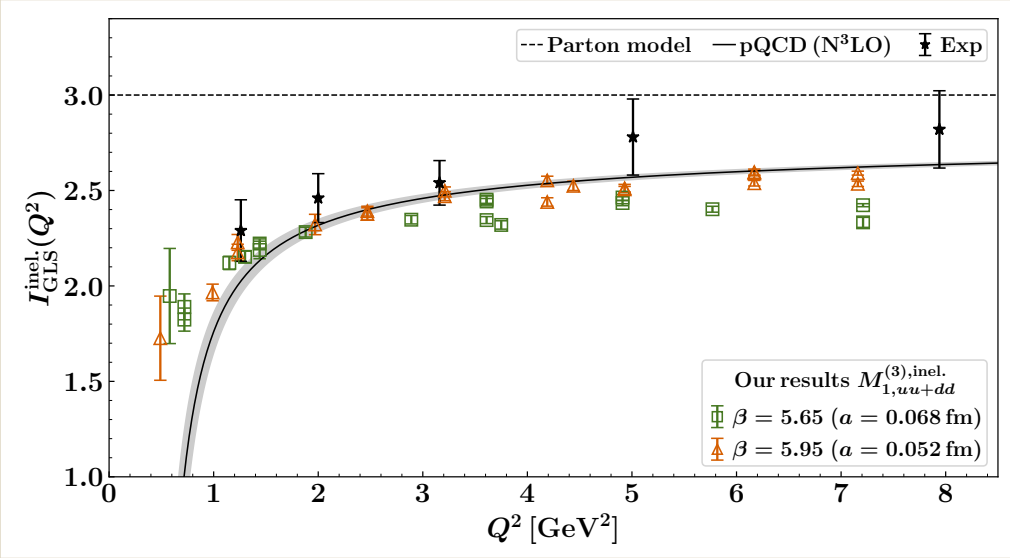
Can, RDY et al., PRD111 (2025)

GLS sum rule

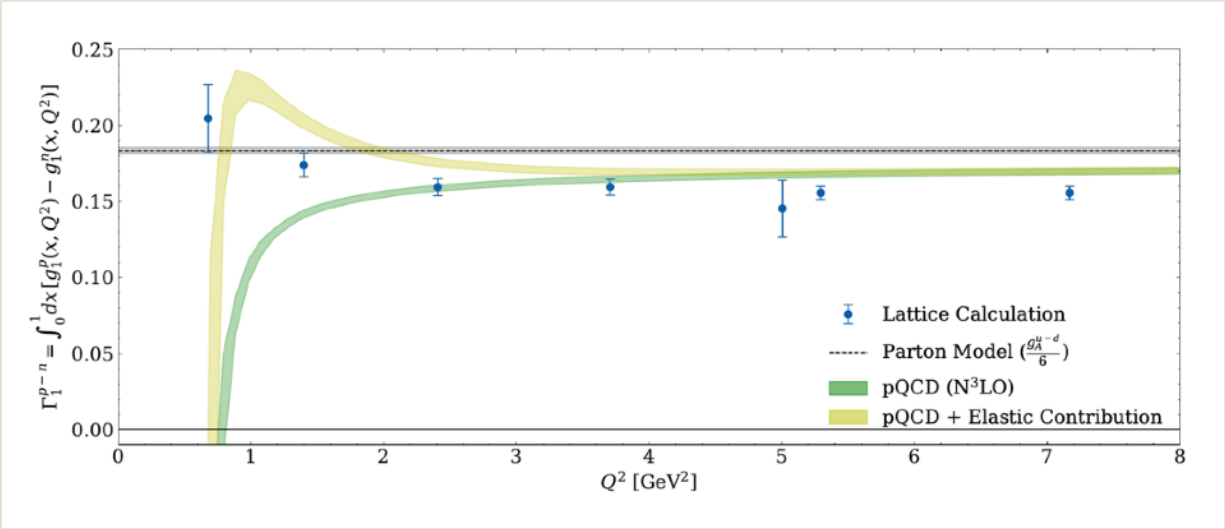
GLS sum rule: lowest moment of parity-violating structure function F_3

Excellent agreement with pQCD prediction over wide range of Q^2 (at finer lattice spacing)

Important building block toward electroweak radiative corrections



Bjorken sum rule down to the elastic region



- Full g_1 moment $(p-n)$ vs N³LO pQCD plus the elastic part, from form factors on the same ensemble ($a \simeq 0.068\,\mathrm{fm}$, $m_\pi \simeq 412\,\mathrm{MeV}$)
- High Q^2: the perturbative shape, slightly below
  low Q^2: a residual deviation shadows the elastic → *not much room for higher twist??*

Prospects: the leading higher twist

$$\Gamma_1^{p-n}(Q^2) = \frac{g_A}{6}\left[1 - \sum_{i=1}^{4} C_i\, a_s^i(Q^2)\right] + \frac{\Delta_{\mathrm{HT}}}{Q^2} + \ \ldots$$

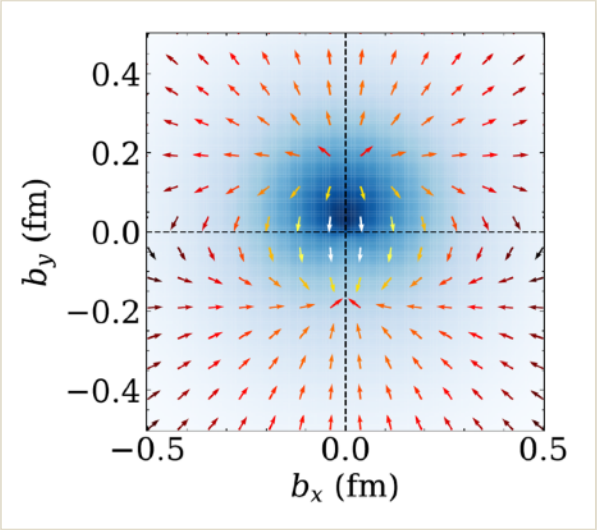
$$\Delta_{\mathrm{HT}} \ \propto \ M^2(a_2 + 4d_2 + 4f_2)$$

- Low-Q^2 deviation hints at higher twist, not yet quantified: needs continuum, quark-mass and finer Q^2 control
- a_2 and d_2 from three-point functions; the Q^2 dependence to isolate the twist-4 f_2

d_2 and f_2 together separate the chromo-electric and chromo-magnetic forces on the struck quark

Summary

- Sivers asymmetries come from final-state colour forces; d_2 is that force the instant the quark is struck
- First lattice maps of transverse force densities: local forces $\simeq 3\,\mathrm{GeV}$, three times the string tension
- Spin-dependent force densities supports the chromodynamic lensing perspective on Sivers
- Bjorken sum rule from the lattice down to the elastic region: a route to the leading higher twist, and to f_2



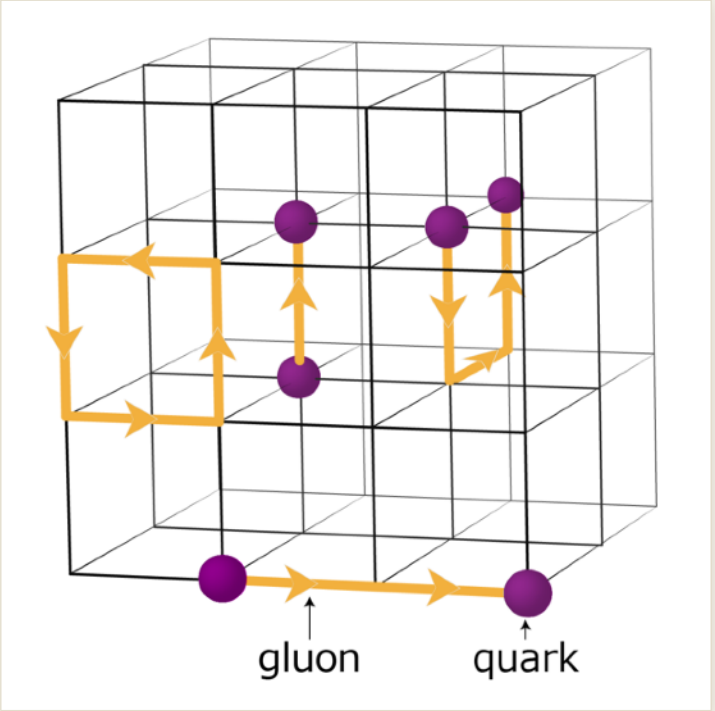
Next: lighter quark masses · control of discretisation · power corrections

Backup slides

Lattice QCD

- Numerical first-principles approach to non-perturbative QCD
- QCD discretised on a 4D space-time lattice; path integral by Monte Carlo
- Controlled systematics: lattice spacing, volume, quark masses

Here: $m_\pi \approx 420$ MeV, $L \approx 2.4$–3.4 fm, $a \approx 0.052$–0.074 fm



Lattice details

Nf	β	$L^3 \times T$	a (fm)	mπ, mK (MeV)	tsep / a	Nmeas
2+1	5.50	$32^3 \times 64$	0.074	465	11, 13, 15	3528
2+1	5.65	$48^3 \times 96$	0.068	412	11, 14, 17	1074
2+1	5.95	$48^3 \times 96$	0.052	418	14, 18, 22	1014

- CSSM/QCDSF/UKQCD ensembles; stout-smeared non-perturbatively O(a)-improved Wilson fermions (SLiNC)
- Tree-level Symanzik gluon action; all ensembles at the SU(3) symmetric point

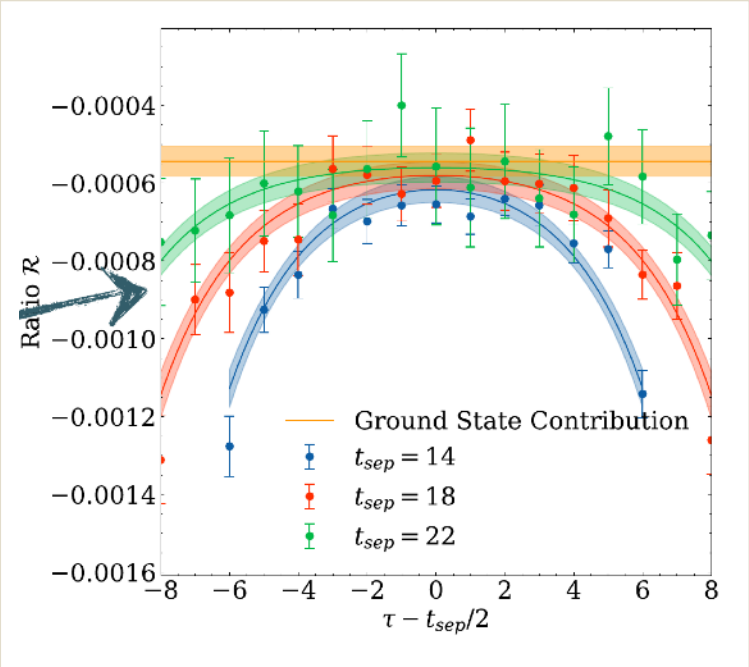
Haar, Nakamura & Stüben, EPJ Web Conf. (2018); Cundy et al., PRD (2009)

Three-point functions

$$\mathcal{O}^{[5]}_{[i\{j]4\}} = -\tfrac{g}{6}\,\bar{\psi}\big(\tilde{G}_{ij}\gamma_4 + \tilde{G}_{i4}\gamma_j\big)\psi - \text{traces}$$

$$\frac{\langle C_3(t,t')\rangle}{\langle C_2(t)\rangle\langle C_2(t')\rangle} \propto \langle N'|\mathcal{O}|N\rangle$$

Three source–sink separations establish ground-state dominance

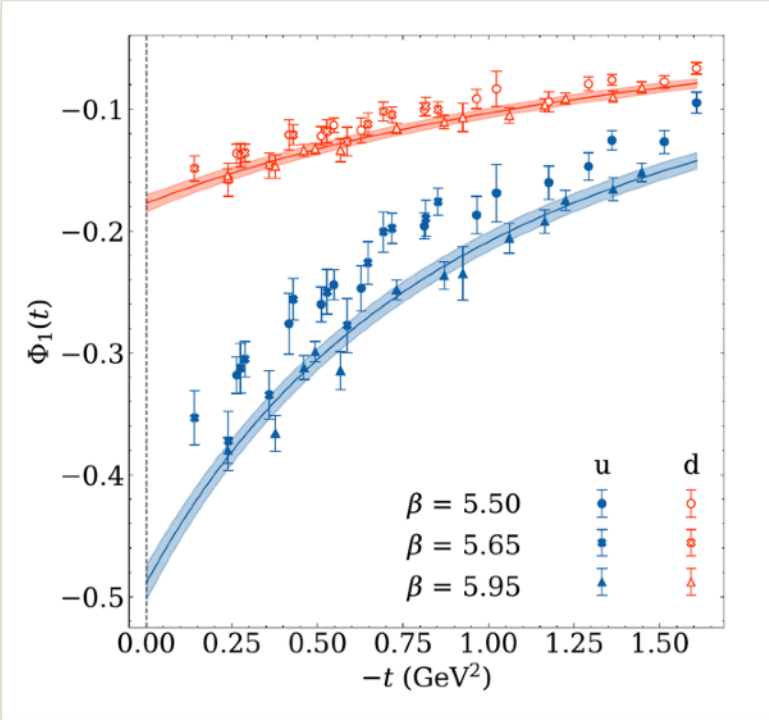


Discretisation artefacts

- Form factors at three lattice spacings, a ≈ 0.074, 0.068, 0.052 fm
- Model the a dependence in magnitude and slope

$$\Phi_i(t,a) = \frac{\Phi_i(0) + b_i a}{\left(1 - t\left(\Lambda_i^{-2} + c_i a\right)\right)^2}$$

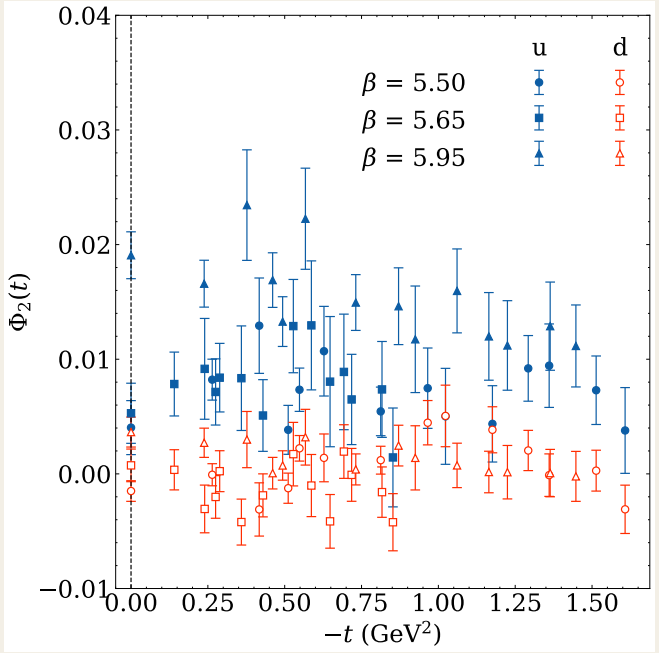
Some tension between lattices, mostly in overall normalisation



Φ_2 is hard

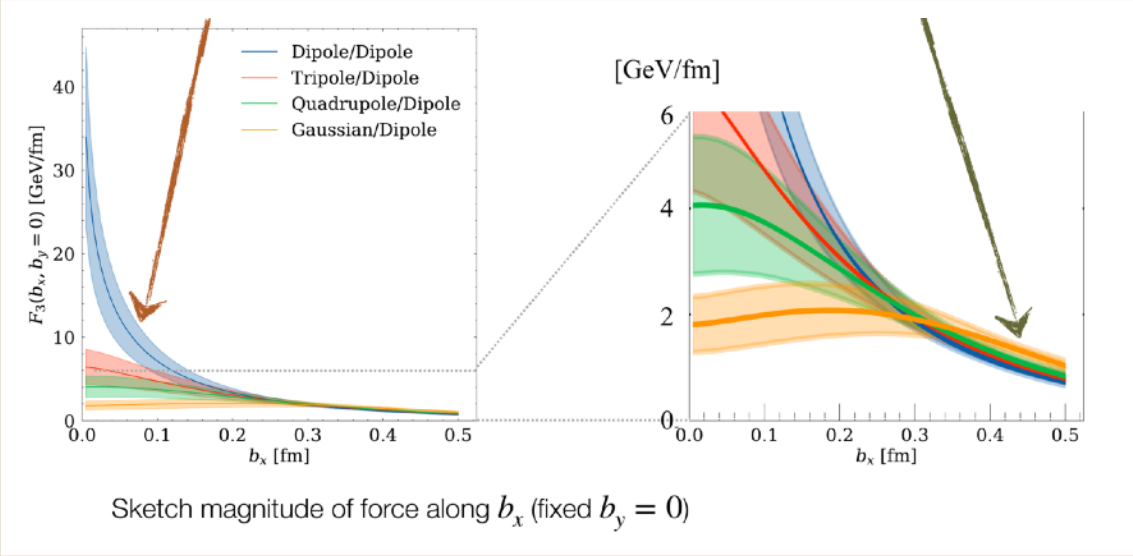
$\Phi_2(0) = d_2$

This form factor is small – systematic uncertainties as big as signal



Crawford, RDY et al., PRL 134 (2025)

Local forces: spin dependent (Φ_3)



Spin-dependent forces ≈ 2 GeV/fm at b ≈ 0.3 fm, where the models agree

Short distances: model dependent (need larger Q^2). Long distances: need smaller Q^2

Transverse force densities

Quark (charge) density

$$q(\mathbf{b}) = \frac{1}{2}\left[\tilde{F}_1(b^2) + \frac{b^j \epsilon^{ji} S^i}{m_N} \frac{d}{db^2}\tilde{F}_2(b^2)\right]$$

Force density

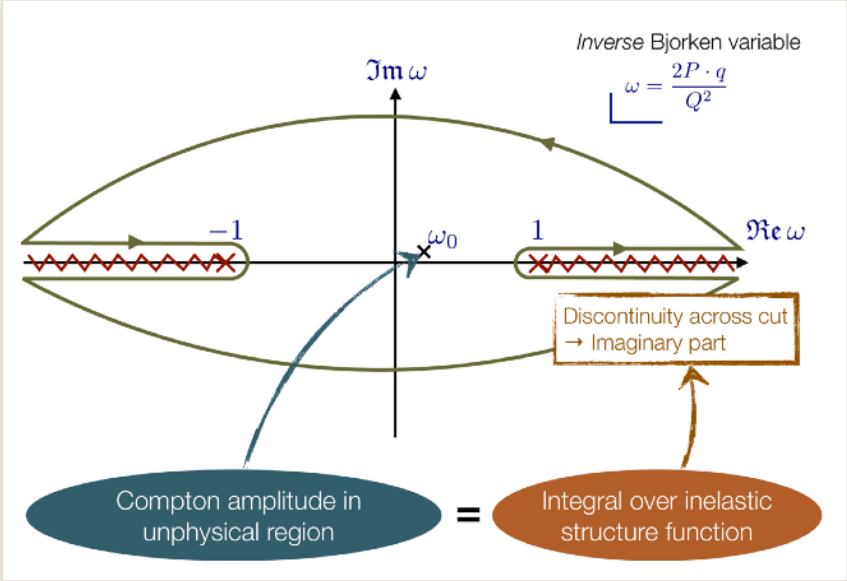
$$\mathcal{F}^i(\mathbf{b}) = -2\sqrt{2}\,P^+ b^i \frac{d}{db^2}\tilde{\Phi}_1 + \sqrt{2}\,m_N \epsilon^{ij} S^j \tilde{\Phi}_2 - \frac{2\sqrt{2}\epsilon^{jk} S^k}{m_N}\left[\delta^{ij}\frac{d}{db^2} + 2b^i b^j \frac{d^2}{(db^2)^2}\right]\tilde{\Phi}_3$$

Φ_1: isotropic Φ_2, Φ_3: spin dependent

Compton amplitude and structure functions

- Optical theorem: structure functions are the imaginary part of the forward Compton amplitude
- Dispersion relation: the amplitude at $|\omega| < 1$ is an integral over the inelastic structure function
- Spin-dependent amplitudes ↔

  moments of g_1 and g_2, at fixed Q^2

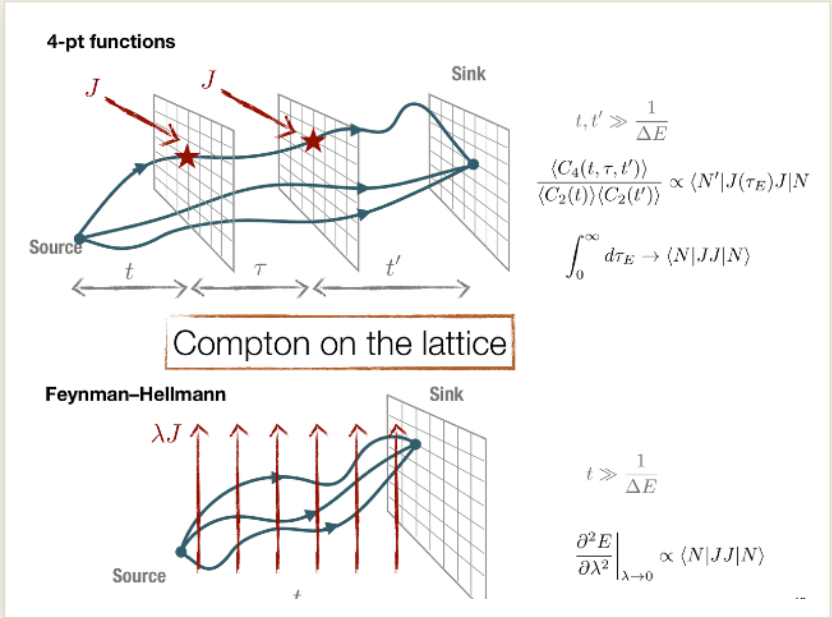


Chambers et al., PRL (2017); Can et al., PRD (2020)

Compton on the lattice: Feynman–Hellmann

- Couple a weak background current λJ to the quark fields
- The second-order energy shift is the forward Compton amplitude: no four-point functions
- Euclidean kinematics with |ω| < 1: intermediate states can't go on shell

$$\frac{\partial^2 E_{\mathbf{p}}}{\partial \lambda^2} = -\frac{1}{2E_{\mathbf{p}}} \int d^4\xi \, (e^{iq\cdot\xi} + e^{-iq\cdot\xi}) \langle \mathbf{p}|\mathrm{T}\, J(\xi) J(0)|\mathbf{p}\rangle$$



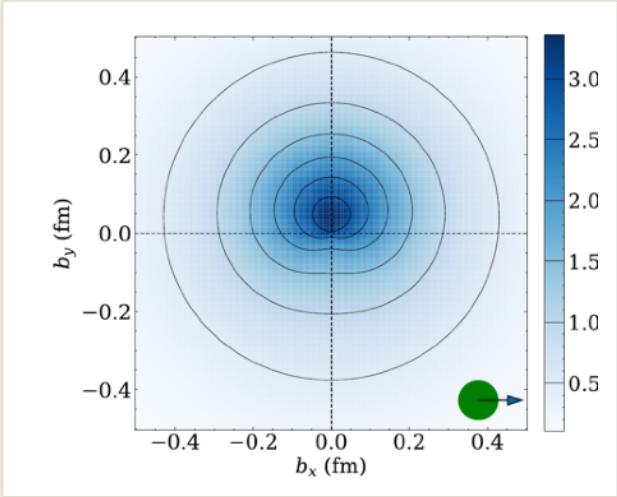
Transverse densities

Electromagnetic current

$$\langle p', s' | \bar{\psi} \gamma^\mu \psi | p, s \rangle = \bar{u}(p', s') \left[\gamma^\mu F_1(t) + \frac{i \sigma^{\mu\nu} q_\nu}{2 m_N} F_2(t) \right] u(p, s)$$

2D Fourier transform → quark density in the transverse plane

$$q(\mathbf{b}) = \frac{1}{2} \left[\tilde{F}_1(b^2) + \frac{b^j \epsilon^{ji} S^i}{m_N} \frac{d}{db^2} \tilde{F}_2(b^2) \right]$$



Proton spin along x: orbital motion shifts the quark density

Operator mixing and renormalisation

- The operator mixes with a lower-dimensional operator; mixing included in RI′-MOM

$$\mathcal{O}_R^{[5]}(\mu) = Z^{[5]}(a\mu)\left(\mathcal{O}^{[5]}(a) + \tfrac{1}{a}\frac{Z^{\sigma}(a\mu)}{Z^{[5]}(a\mu)}\,\mathcal{O}^{\sigma}(a)\right)$$

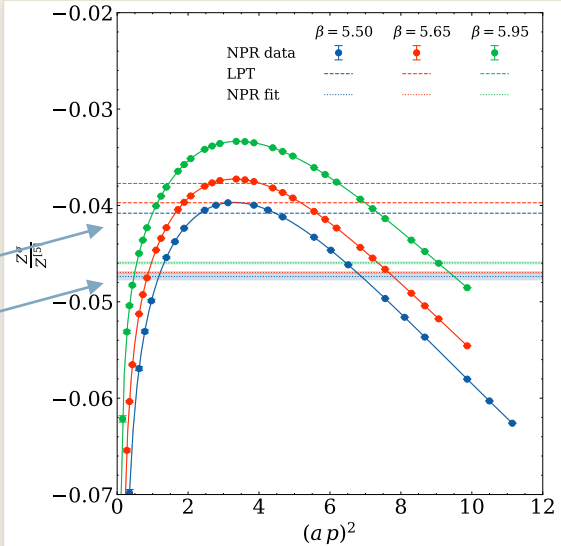
- Mixing coefficient determined in lattice perturbation theory and non-perturbatively
- $Z^{(5)}(a\mu)$ following RQCD; evolved to MS-bar at $\mu = 2$ GeV

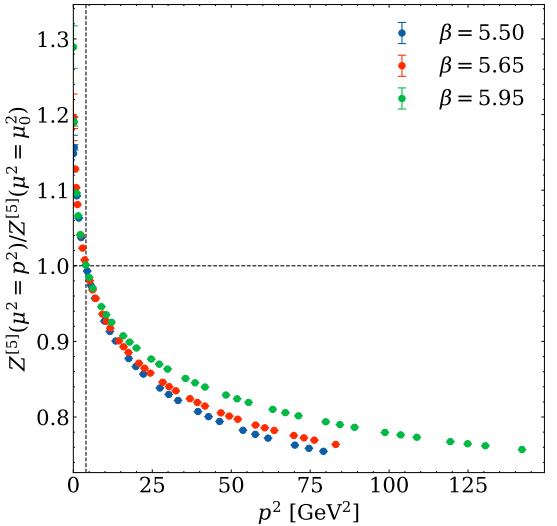
Bürger et al., PRD (2022)

Mixing coefficient

Lattice pert

Non-pert Renorm





- Amputated 3-pt Green's function matched to the continuum tree-level result
- Fit, and keep the constant B:

$$\frac{Z^\sigma}{Z^{[5]}} = \frac{A}{(ap)^2} + B + C(ap)^2 + D(ap)^4$$

Crawford, RDY et al., PRL 134 (2025)