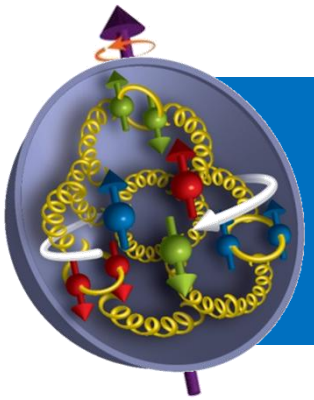
PacificSPIN 2026

The 13th Circum-Pan-Pacific Symposium on High Energy Spin Physics



Angular momentum decompositions & spin sum rules

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Oct 27-30, 2026

Some clarifications regarding terminology

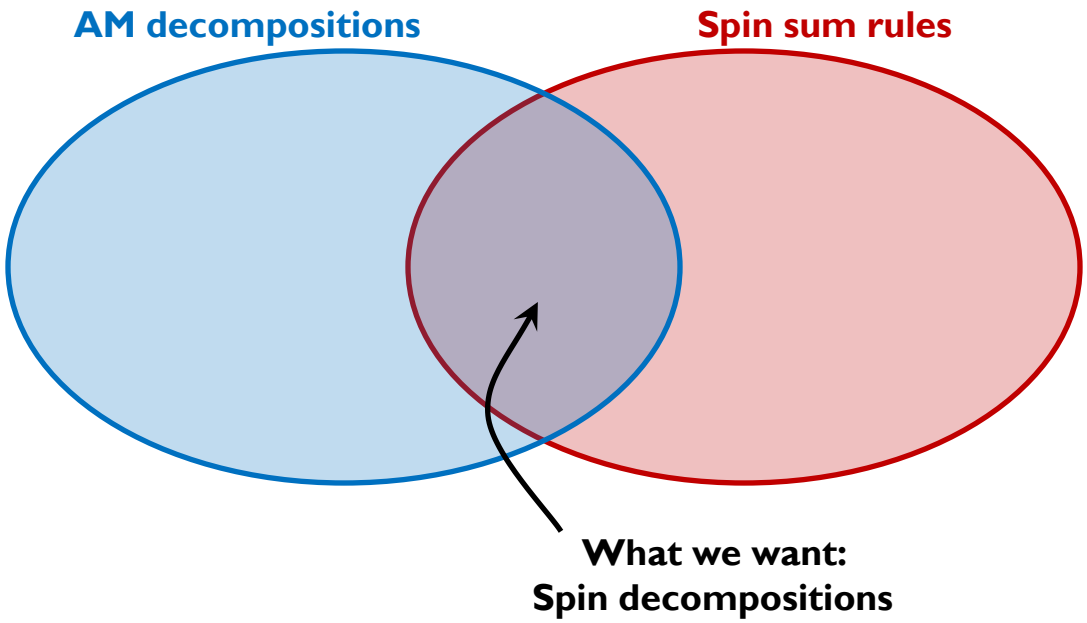
Spin ≡ ***internal*** **angular momentum (AM)**

i.e. AM about the « center » of the system

Sum rule ≡ **relation between *separately* « *measurable* » quantities**

Decomposition ≡ **sum rule based on a nontrivial relation at the *operator level***

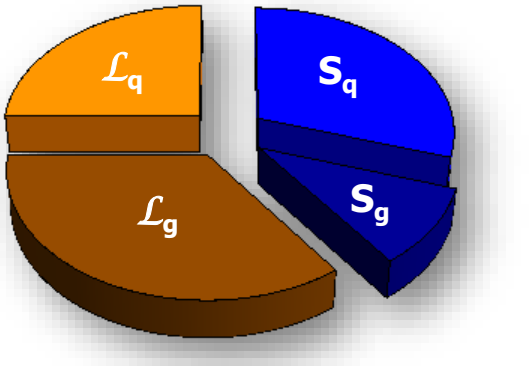
e.g. $A = (A-B)+B$ is trivial unless $(A-B)$ can be expressed in a different way



AM decompositions (1990-2008)

Canonical

$$\vec{p} = \frac{\partial L}{\partial \vec{v}}$$



$$\vec{S}_q = \int \mathrm{d}^3 r \, \psi^\dagger \tfrac{1}{2} \vec{\Sigma} \psi$$

$$\vec{\mathcal{L}}_q = \int \mathrm{d}^3 r \, \psi^\dagger \vec{r} \times (-i\vec{\nabla})\psi$$

$$\vec{S}_g = \int \mathrm{d}^3 r \, \vec{E}^a \times \vec{A}^a$$

$$\vec{\mathcal{L}}_g = \int \mathrm{d}^3 r \, E^{ai} (\vec{r} \times \vec{\nabla}) A^{ai}$$

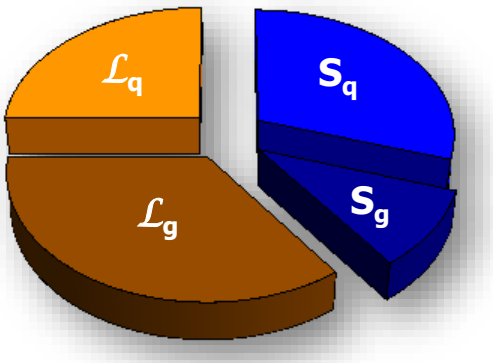
Gauge fixed ! $\boxed{A^+ = 0}$

[Jaffe, Manohar, NPB337 (1990) 509]

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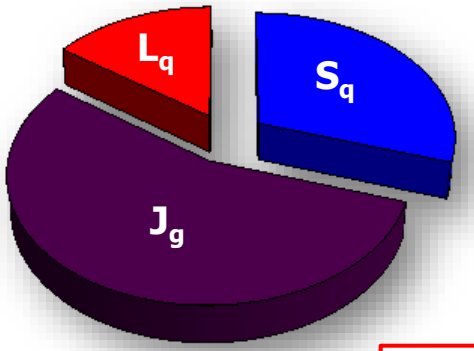
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Kinetic

$$\vec{\pi} = m\vec{v} = \vec{p} + g\vec{A}$$



$$\vec{D} = -\vec{\nabla} - ig\vec{A}$$

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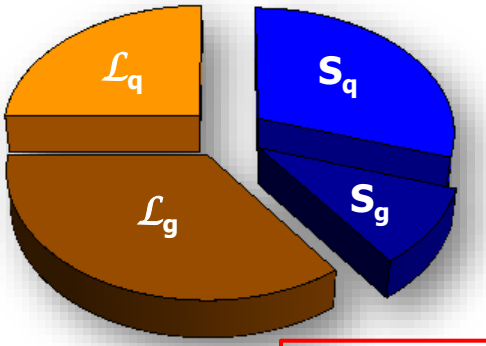
« Incomplete » ΔG **?**

[Ji, PRL78 (1997) 610]

AM decompositions (2008-now)

$$\vec{A} = \vec{A}_{\rm pure} + \vec{A}_{\rm phys}$$

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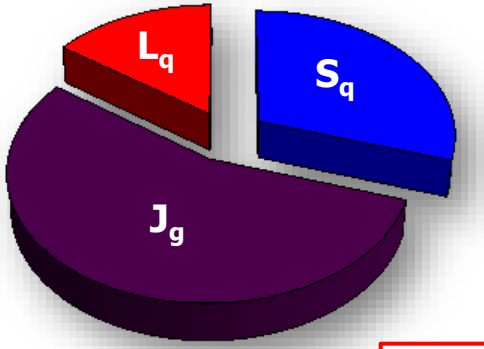
$$\vec{\mathcal{L}}_g = -\int \mathrm{d}^3 r\, E^{ai}(\vec{r} \times \vec{\mathcal{D}}^{ab}_{\rm pure}) A^{bi}_{\rm phys}$$

Gauge invariant !

[Chen, Lu, Sun, Wang, Goldstein, PRL100 (2008) 232002]

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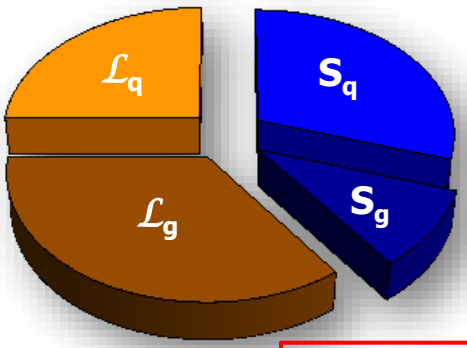
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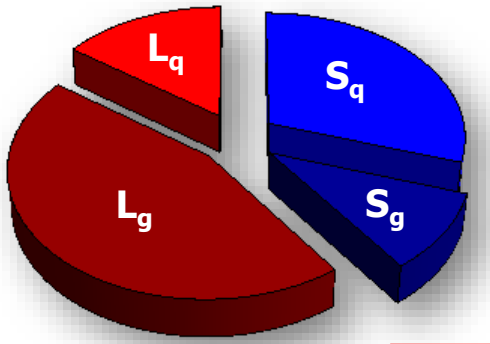
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$$\vec{L}_g = \vec{\mathcal{L}}_g - \int \mathrm{d}^3 r\, \rho^a \vec{r} \times \vec{A}^a_{\text{phys}}$$

« Complete »

« Potential » AM

[Wakamatsu, PRD81 (2010) 114010]

Ambiguity

$$A^\mu = A^\mu_{\text{pure}} + A^\mu_{\text{phys}}$$

Pure gauge field

$$A^\mu_{\text{pure}} \equiv U \tfrac{i}{g} \partial^\mu U^{-1} \quad \Leftrightarrow \quad F^{\mu\nu}_{\text{pure}} = 0$$

$$F^{\mu\nu} = \mathcal{D}^\mu_{\text{pure}} A^\nu_{\text{phys}} - \mathcal{D}^\nu_{\text{pure}} A^\mu_{\text{phys}} - ig[A^\mu_{\text{phys}}, A^\nu_{\text{phys}}]$$



The split is not unique and reflects the original gauge symmetry

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In general, $A^\mu_{\rm pure}$ and $A^\mu_{\rm phys}$ are non-local functionals of A^μ

[Hatta, PLB708 (2012) 186]
[C.L., PLB719 (2013) 185]
[C.L., PRD187 (2013) 034031]

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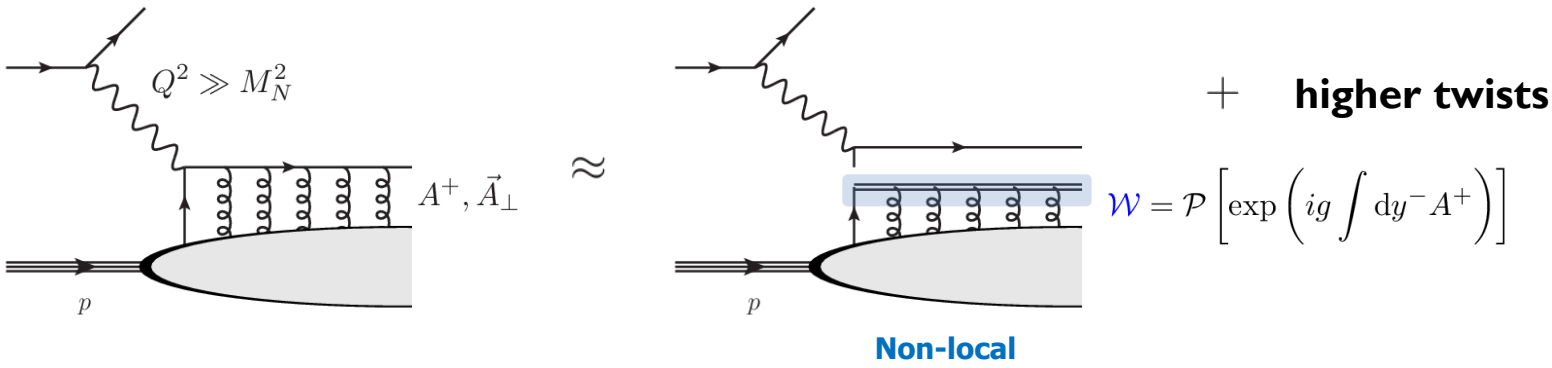
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Factorization in high-energy scattering



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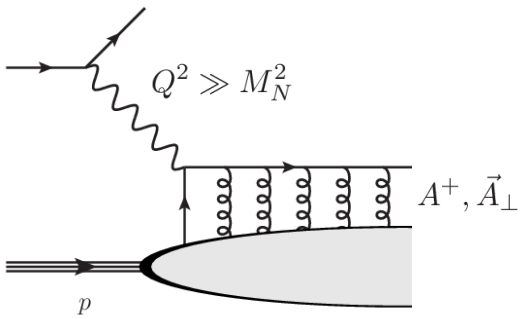
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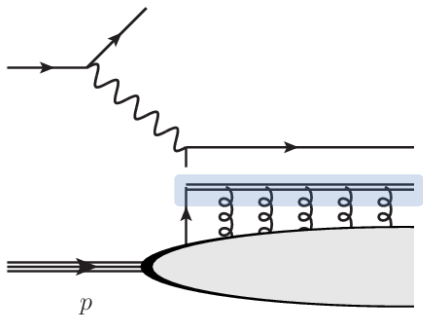
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Factorization in high-energy scattering



$\approx$



$+$ **higher twists**

$$\mathcal{W} = \mathcal{P}\left[\exp\left(ig\int \mathrm{d}y^- A^+\right)\right]$$

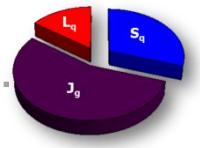
Non-local

$$A^{\rm pure}_\mu = \mathcal{W} \frac{i}{g} \partial_\mu \mathcal{W}^{-1}$$

$$A^+ = A^+_{\rm pure} \quad \Rightarrow \quad A^+_{\rm phys} = 0$$

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[C.L., PLB719 (2013) 185]
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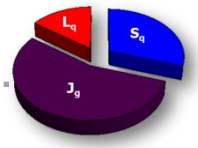
Generalized AM (*local* kinetic operators)



Poincaré symmetry implies that the following current is conserved

$$J^{\mu\alpha\beta} = \underbrace{x^{\alpha}T^{\mu\beta} - x^{\beta}T^{\mu\alpha}}_{\text{Orbital}} + \underbrace{S^{\mu\alpha\beta}}_{\text{Intrinsic}}$$

Generalized AM (*local* kinetic operators)



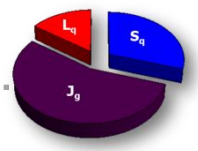
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Spin-1/2 target $\quad \langle p', s'|T_a^{\mu\nu}(0)|p, s\rangle = \overline{u}(p', s')\Gamma_a^{\mu\nu}(P, \Delta)u(p, s) \qquad a = q, g$

$$\Gamma_a^{\mu\nu}(P,\Delta) = \frac{P^\mu P^\nu}{M} A_a(t) + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} C_a(t) + Mg^{\mu\nu}\bar{C}_a(t) + \frac{P^{\{\mu}i\sigma^{\nu\}\lambda}\Delta_\lambda}{2M} J_a(t) - \frac{P^{[\mu}i\sigma^{\nu]\lambda}\Delta_\lambda}{2M} S_a(t)$$

$$\Rightarrow \boxed{\begin{aligned}S_q(t) &= \tfrac{1}{2} G_A^q(t)\\ S_g(t) &= 0\end{aligned}}$$

$F_a(t)$ **« Gravitational » form factors (GFFs)**

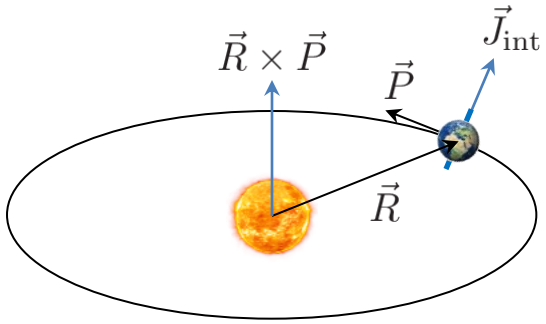
$$P = \frac{p' + p}{2}, \quad \Delta = p' - p, \quad t = \Delta^2$$

[Pagels, PR144 (1966) 4, 1250]
[Ji, PRL78 (1997) 610]
[Shore, White, NPB581 (2000) 409]
[Bakker, Leader, Trueman, PRD70 (2004) 114001]
[C.L., Leader, PR541 (2014) 163]

Total AM and spin

We can write in general

$$\vec{J} = \underbrace{\vec{R} \times \vec{P}}_{\text{External}} + \underbrace{\vec{J}_{\text{int}}}_{\text{Internal (spin)}}$$



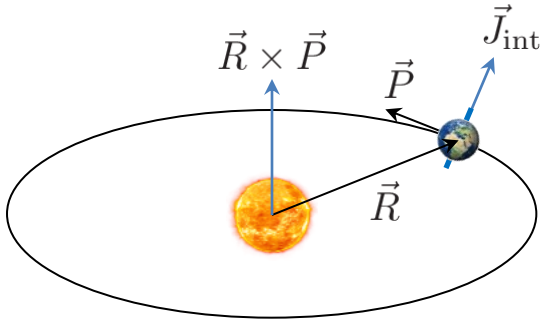
$\vec{R}$ **Position of the « center » of Earth**

$\vec{P}$ **Total momentum of Earth**

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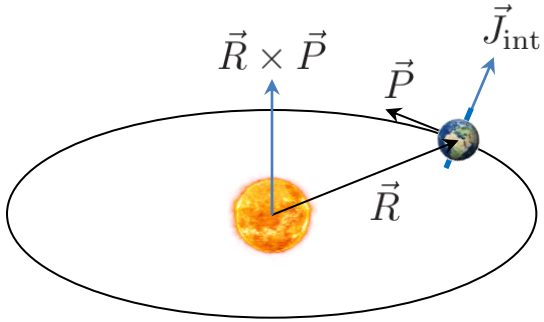
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Target at rest $\vec{P} = \vec{0}$ ➡ $\vec{J} = \vec{J}_{\text{int}}$

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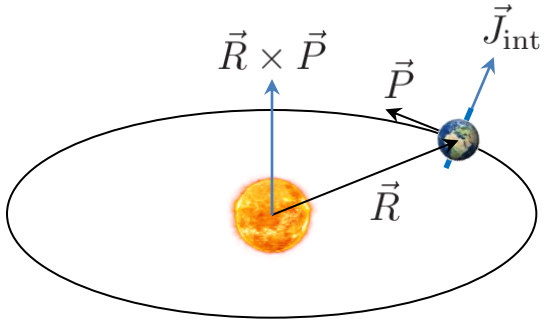
Moving target $\vec{P} = P_z \vec{e}_z$

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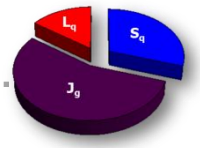
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Transversely polarized $\vec{J}_\perp = (\vec{R} \times \vec{P})_\perp + \vec{J}_{\text{int},\perp}$ $\qquad J_z = 0$

⚠ **Definition of** $\vec{R}_\perp$ **is essential !**

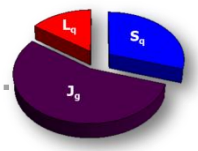
Longitudinal polarization (*local* kinetic operators)



Longitudinal spin decomposition $\langle J_z \rangle = \langle L_z^q \rangle + \langle S_z^q \rangle + \langle J_z^g \rangle$

[Ji, PRL78 (1997) 610]
[Bakker, Leader, Trueman, PRD70 (2004) 114001]

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For a wave-packet centered at the origin, one finds

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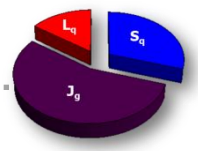
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Invariant under longitudinal boosts !

[Ji, PRL78 (1997) 610]
[Bakker, Leader, Trueman, PRD70 (2004) 114001]

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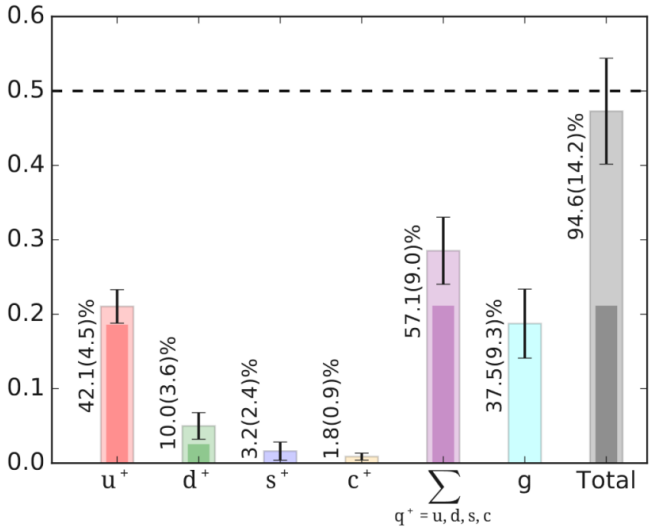
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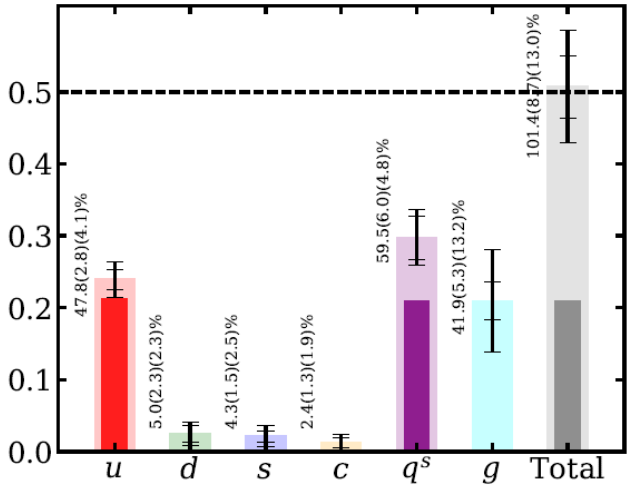
AM constraint $\displaystyle\sum_a J_a(0) = \frac{1}{2}$

[Ji, PRL78 (1997) 610]
[Bakker, Leader, Trueman, PRD70 (2004) 114001]

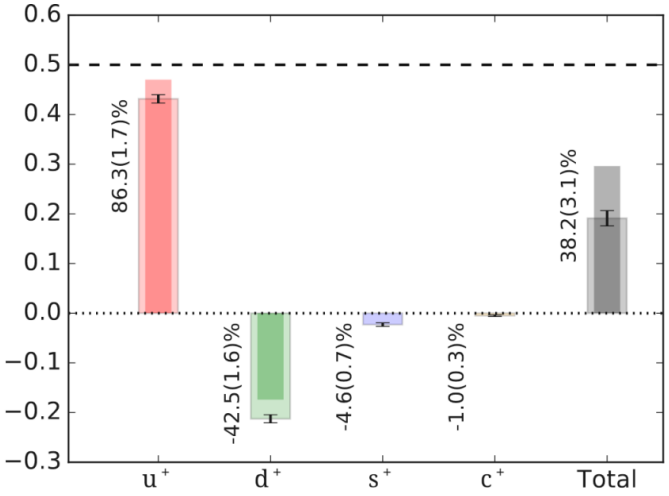
Recent Lattice QCD results

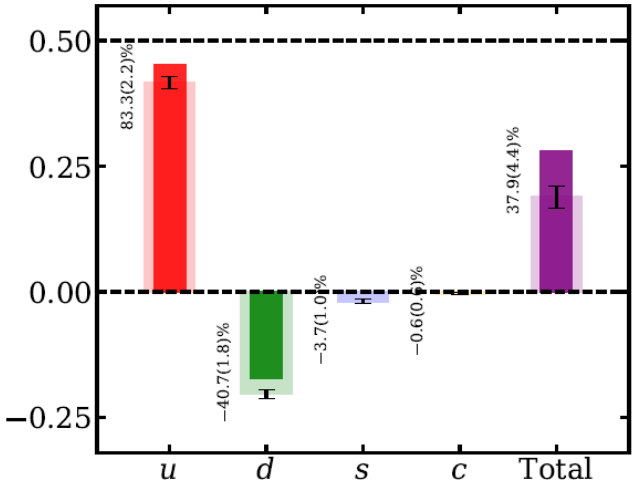
$J_a(0)$





$S_a(0)$

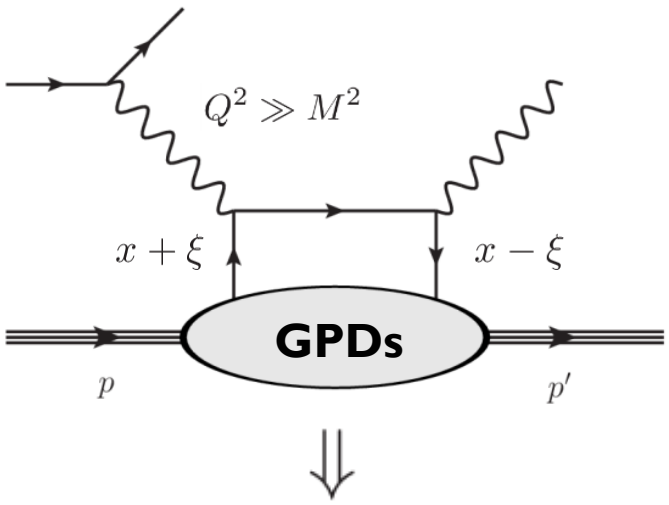




[Alexandrou *et al.*, PRD101 (2020) 094513]

[Alexandrou *et al.*, arXiv:2607.21230]

Generalized parton distributions (GPDs)



Deeply virtual Compton scattering (DVCS)

$$\frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \langle p', s'|\overline{\psi}(-\tfrac{z}{2})\gamma^+ \mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|p,s\rangle\big|_{z^+=|\vec{z}_\perp|=0}$$

$$= \frac{1}{2P^+}\,\overline{u}(p',s')\left[\gamma^+ H_q(x,\xi,t) + \frac{i\sigma^{+\lambda}\Delta_\lambda}{2M} E_q(x,\xi,t)\right] u(p,s)$$

[Mueller, Robaschik, Geyer, Dittes, Horejsi, FP42 (1994) 101]
[Radyushkin, PLB385 (1996) 333]
[Ji, PRL78 (1997) 610]
[Ji, PRD55 (1997) 7114]
[Radyushkin, PRD56 (1997) 5527]

Anomalous gravitomagnetic moment

Mellin moments $\xi = 0$

$$\int \mathrm{d}x\, H_q(x,0,t) = F_1^q(t)$$

$$\int \mathrm{d}x\, E_q(x,0,t) = F_2^q(t)$$

Anomalous magnetic moment

$$F_2^q(0) = \kappa_q$$

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[Ji, PRL78 (1997) 610]
[Teryaev, hep-ph/9904376]
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[Burkardt, IJMPA18 (2003) 2, 173]
[Lowdon, Chiu, Brodsky, PLB774 (2017) 1]
[C.L., EPJC78 (2018) 9, 785]
[Cotogno, C.L., Lowdon, PRD100 (2019) 045003]

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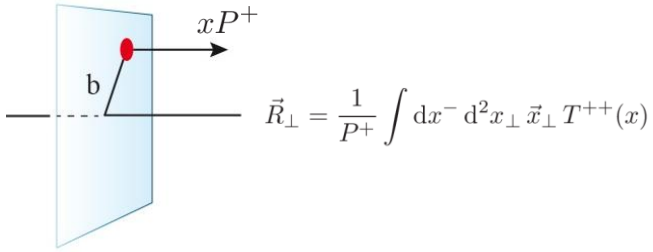
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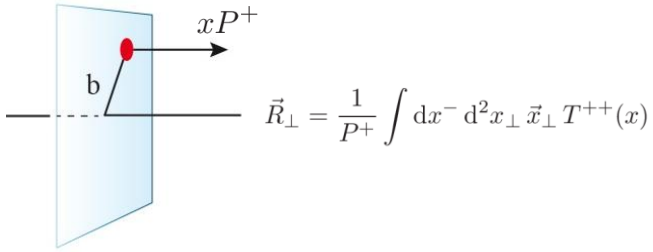
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Equivalence principle for spinning targets

➡ **No intrinsic energy dipole moment !**

See Oleg Teryaev's talk (Wed)

[Ji, PRL78 (1997) 610]
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[Brodsky, Hwang, Ma, Schmidt, NPB593 (2001) 311]
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Examples of « refined » spin sum rules

Ji's sum rule

$$\langle J_z^a \rangle = \int \mathrm{d}x \, \frac{x}{2} \left[H_a(x,0,0) + E_a(x,0,0) \right]$$

Examples of « refined » spin sum rules

Distribution in momentum space

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$$\langle J^a_{z} \rangle_{T} = \int \mathrm{d}x \, \underbrace{\frac{x}{2} \left[H_a(x,0,0) + E_a(x,0,0) \right]}_{\equiv J_a(x)}$$

[Hoodbhoy, Ji, Lu, PRD59 (1999) 014013]
[Burkardt, IJMPA18 (2003) 173]
[Hägler, Mukherjee, Schäfer, PLB582 (2004) 55]
[Burkardt, PRD72 (2005) 094020]
[Ji, Xiong, Yuan, PRL109 (2012) 152005]
[Ji, Xiong, Yuan, PLB717 (2012) 214]
[Hatta, Yoshida, JHEP02 (2012) 003]
[Ji, Xiong, Yuan, PRD88 (2013) 1, 014041]
[C.L., PLB719 (2013) 185]
[Leader, C.L., PR541 (2014) 3, 163]
[Liu, C.L., EPJA52 (2016) 6, 160]
[Ji, Yuan, PLB810 (2020) 135786]
[Ji, Yuan, Zhao, NRP3 (2021) 1, 27]
[Guo, Ji, Shiells, NPB969 (2021) 115440]

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$$\langle J^a_z \rangle_T = \int \mathrm{d}x \, \frac{x}{2} \, [H_a(x,0,0) + E_a(x,0,0)]$$

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[Burkardt, IJMPA18 (2003) 173]
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$[D^\mu, \mathcal{W}] \neq 0$  **Non-local version of covariant derivative is ambiguous and involves higher-twist distributions !**

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Distribution in position space

$$\langle J^a_z \rangle = J_a(0) = \int \mathrm{d}^3 r \underbrace{\int \frac{\mathrm{d}^3 \Delta}{(2\pi)^3} \, e^{-i\vec{\Delta}\cdot\vec{r}} \, J_a(t)}_{\equiv \mathcal{J}_a(\vec{r})}$$

[Polyakov, PLB555 (2003) 57]
[Goeke *et al.*, PRD75 (2007) 094021]
[Adhikari, Burkardt, NPBPS251 (2014) 105]
[Leader, C.L., PR541 (2014) 3, 163]
[Adhikari, Burkardt, PRD94 (2016) 11, 114021]
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[C.L., Mantovani, Pasquini, PLB776 (2018) 38]

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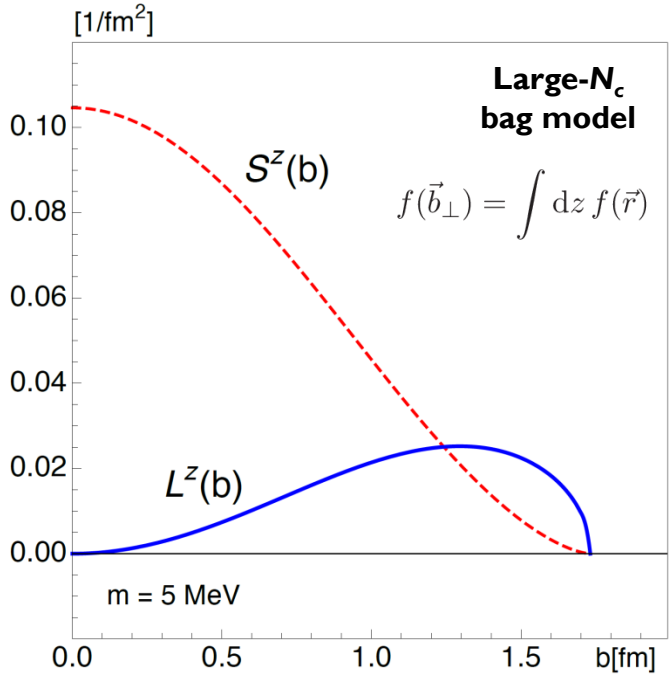
$\vec{\Delta}^2_\perp \, \frac{\mathrm{d}J_a(t)}{\mathrm{d}t}$ **does not contribute to the integral but affects the spatial distribution !**

AM spatial distributions

Orbital *vs* intrinsic

$$L^i(\vec{r}) = \epsilon^{ijk} r^j \langle T^{0k} \rangle_{\vec{0},\vec{0}}(\vec{r})$$

$$S^i(\vec{r}) = \tfrac{1}{2} \langle \overline{\psi} \gamma^i \gamma_5 \psi \rangle_{\vec{0},\vec{0}}(\vec{r})$$



Impact parameter $b = \sqrt{x^2 + y^2} = \sqrt{r^2 - z^2}$

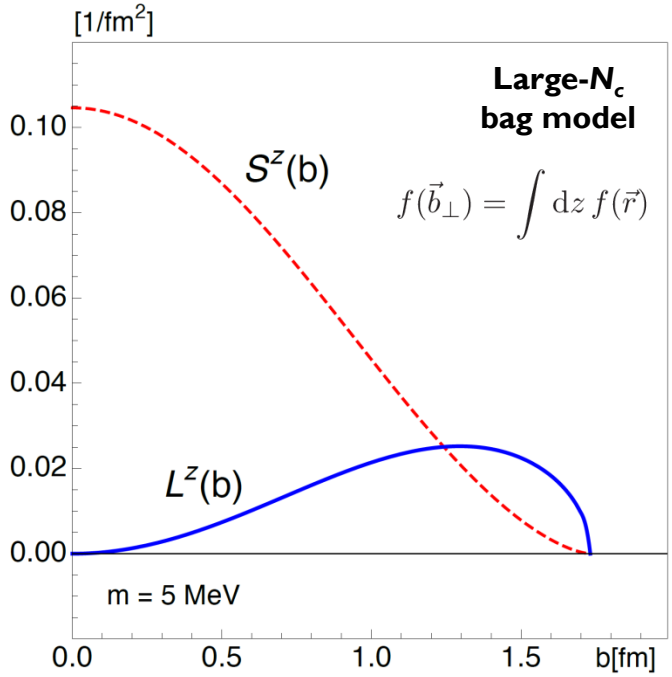
[C.L., Mantovani, Pasquini, PLB776 (2018) 38]
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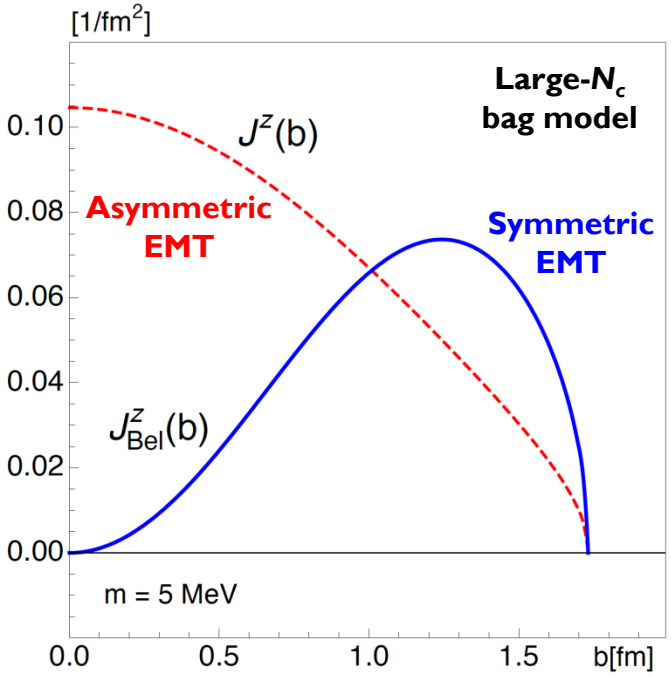


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Kinetic *vs* Belinfante

$$J^i(\vec{r}) = L^i(\vec{r}) + S^i(\vec{r})$$

$$J^i_{\mathrm{Bel}}(\vec{r}) = \epsilon^{ijk} r^j \langle \tfrac{1}{2}(T^{0k} + T^{k0}) \rangle_{\vec{0},\vec{0}}(\vec{r})$$



[C.L., Mantovani, Pasquini, PLB776 (2018) 38]
[C.L., Schweitzer, Tezgin, PRD106 (2022) 1, 014012]

Transverse polarization *(Belinfante form)*

Transverse spin decomposition $\langle J_{\mathrm{int},T} \rangle = \langle J^q_{\mathrm{int},T} \rangle + \langle J^g_{\mathrm{int},T} \rangle$

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[Leader, PRD85 (2012) 051501] $\langle J^a_{\mathrm{int},T}\rangle = \frac{1}{2}\left[A_a(0) + \frac{P^0}{M}B_a(0)\right]$

[Ji, Yuan, PLB810 (2020) 135786] $\langle J^a_{\mathrm{int},T}\rangle = \frac{P^0}{M}\frac{A_a(0)+B_a(0)}{2}$

[C.L., EPJC81 (2021) 5, 413] $\langle J^a_{\mathrm{int},T}\rangle = \frac{1}{2}\left[\left(\frac{P^0}{M}\right)^n A_a(0) + \frac{P^0}{M}B_a(0)\right]$

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Pauli-Lubanski pseudo-vector $W^\mu = \frac{1}{2}\epsilon^{\mu\alpha\beta\lambda}M_{\alpha\beta}P_\lambda$ $\gamma = P^0/M$

$\langle J_{\text{int},z}\rangle = \frac{1}{2}$

Lorentz-contracted four-vector	Antisymmetric rank-2 tensor	Generators of rotations $[J^i_{\text{int}}, J^j_{\text{int}}] = i\epsilon^{ijk}J^k_{\text{int}}$
$\vec{J}_{\text{int},E} \equiv \frac{\vec{W}}{P^0}$	$\vec{J}_{\text{int},M} \equiv \frac{P^0\vec{W} - \vec{P}W^0}{M^2}$	$\vec{J}_{\text{int},c} \equiv \frac{(P^0+M)\vec{W} - \vec{P}W^0}{M(P^0+M)}$
$\Rightarrow \langle J_{\text{int},T}\rangle = \frac{1}{2}\gamma^{-1}$	$\langle J_{\text{int},T}\rangle = \frac{1}{2}\gamma$	$\langle J_{\text{int},T}\rangle = \frac{1}{2}$

Pivot dependence

Spin definition $\vec{J}_{\mathrm{int},X} = \vec{J} - \vec{R}_X \times \vec{P}$

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Center of energy

$$\langle \vec{R}_E \rangle = \vec{\mathcal{R}} + \frac{\vec{P} \times \vec{s}}{2P^0(P^0+M)} \quad \Rightarrow \quad \langle \vec{J}_{\mathrm{int},E} \rangle = \frac{M}{2P^0}\left(\vec{s} + \frac{\vec{P}(\vec{P}\cdot\vec{s})}{M(P^0+M)}\right)$$

Center of mass

$$\langle \vec{R}_M \rangle = \vec{\mathcal{R}} - \frac{\vec{P} \times \vec{s}}{2M(P^0+M)} \quad \Rightarrow \quad \langle \vec{J}_{\mathrm{int},M} \rangle = \frac{P^0}{2M}\left(\vec{s} - \frac{\vec{P}(\vec{P}\cdot\vec{s})}{P^0(P^0+M)}\right)$$

Canonical center *aka* **center of spin**

$$\langle \vec{R}_c \rangle = \vec{\mathcal{R}} \quad \Rightarrow \quad \langle \vec{J}_{\mathrm{int},c} \rangle = \frac{\vec{s}}{2}$$

Newton-Wigner position

[C.L., JHEP78 (2018) 9, 785]
[C.L., EPJC81 (2021) 5, 413]
[Chen, C.L., PRD107 (2023) 9, 096003]

Pivot dependence

Spin definition $\vec{J}_{\mathrm{int},X} = \vec{J} - \vec{R}_X \times \vec{P}$

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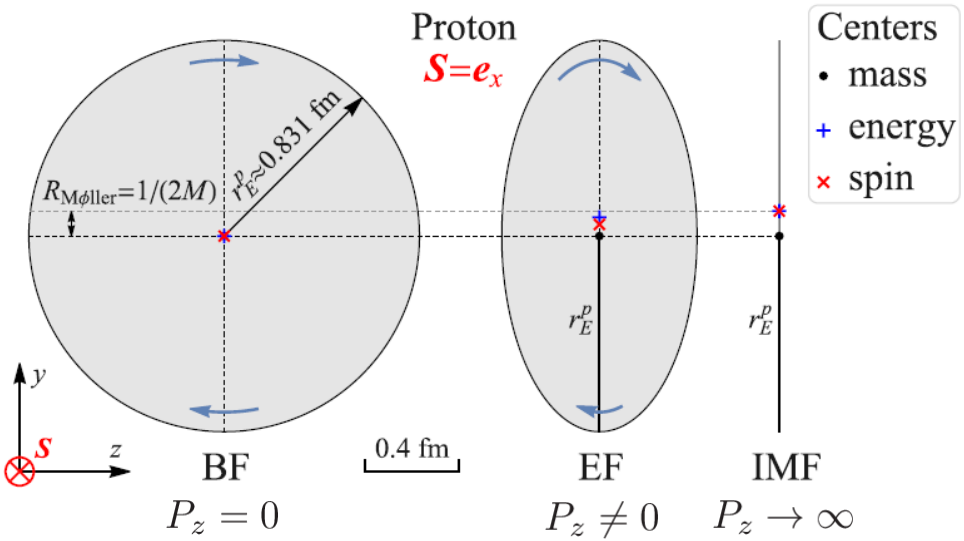
Center of mass

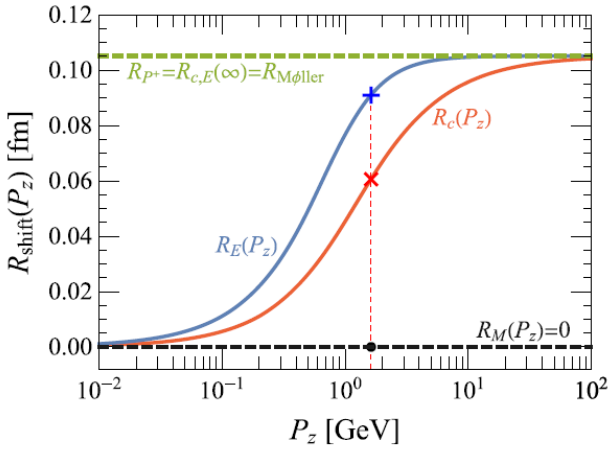
$$\langle \vec{R}_M \rangle = \vec{\mathcal{R}} - \frac{\vec{P} \times \vec{s}}{2M(P^0+M)} \quad \Rightarrow \quad \langle \vec{J}_{\mathrm{int},M} \rangle = \frac{P^0}{2M}\left(\vec{s} - \frac{\vec{P}(\vec{P}\cdot\vec{s})}{P^0(P^0+M)}\right)$$

Canonical center *aka* **center of spin**

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Newton-Wigner position

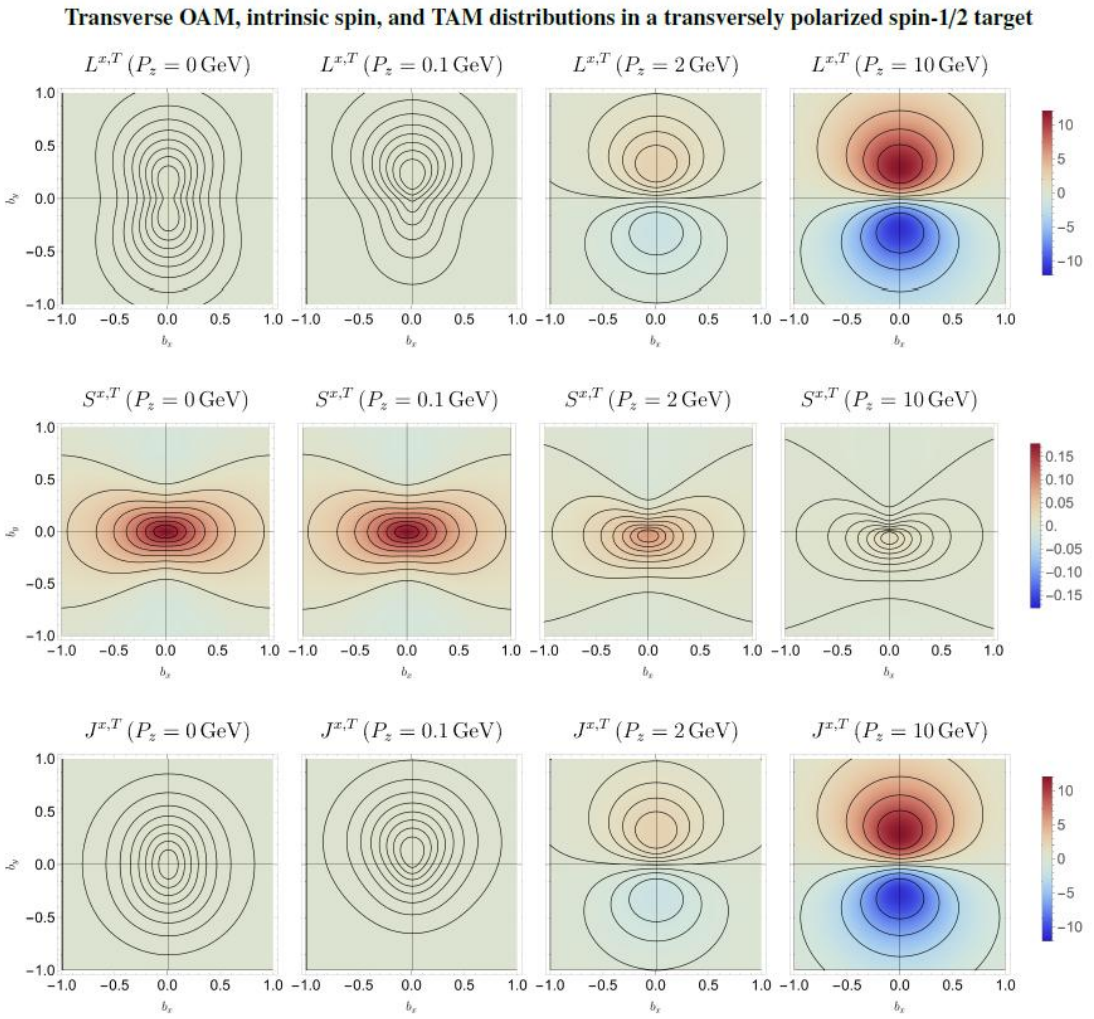




[C.L., JHEP78 (2018) 9, 785]
[C.L., EPJC81 (2021) 5, 413]
[Chen, C.L., PRD107 (2023) 9, 096003]

Recent refinements

- **L+S decomposition**
- **Spatial distribution**
- **Frame dependence**
- **Pivot contribution**



See Ravi Singh's talk (Fri)

Thank you!

[C.L., Mukherjee, Singh, Won, PLB868 (2025) 139792]
[C.L., Mukherjee, Singh, Won, arXiv:2609.08902]