Polarization of dileptons in the Z-boson and D-Y Productions

Jen-Chieh Peng

University of Illinois at Urbana-Champaign

and

National Central University

The 13th Circum-Pan Pacific Symposium on High Energy Spin Physics

Institute of Physics, Academia Sinica, Taiwan

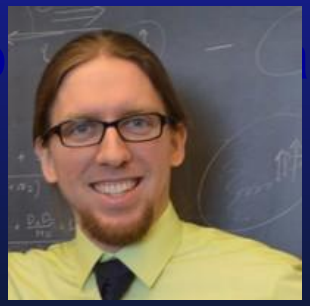
October 27 – 30, 2026

Insights on the lepton angular distributions in Drell-Yan and W/Z boson production in the "Geometric Model"

Work with Wen-Chen Chang, Evan McClellan, and Oleg Teryaev

P-3 Seminar, Los Ala... to ... to ... June ...





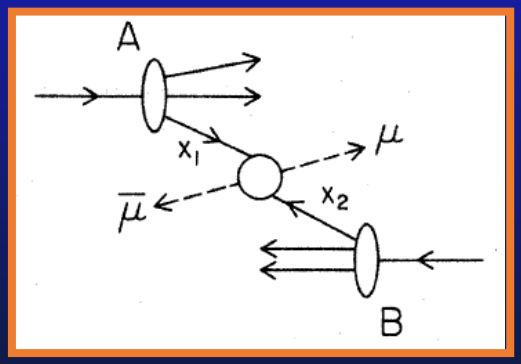


Phys. Lett. B758 (2016) 384;
Phys. Rev. D 96 (2017) 054020;
Phys. Lett. B789 (2019) 356;
Phys. Rev. D 99 (2019) 014032
Phys. Lett. B797 (2019) 134895
Phys. Rev. D 103 (2021) 034011
and recent results

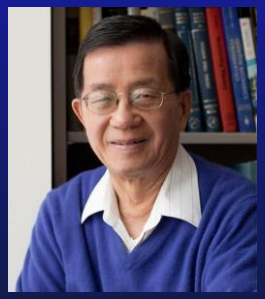
Angular Distribution in the "Naïve" Drell-Yan

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)





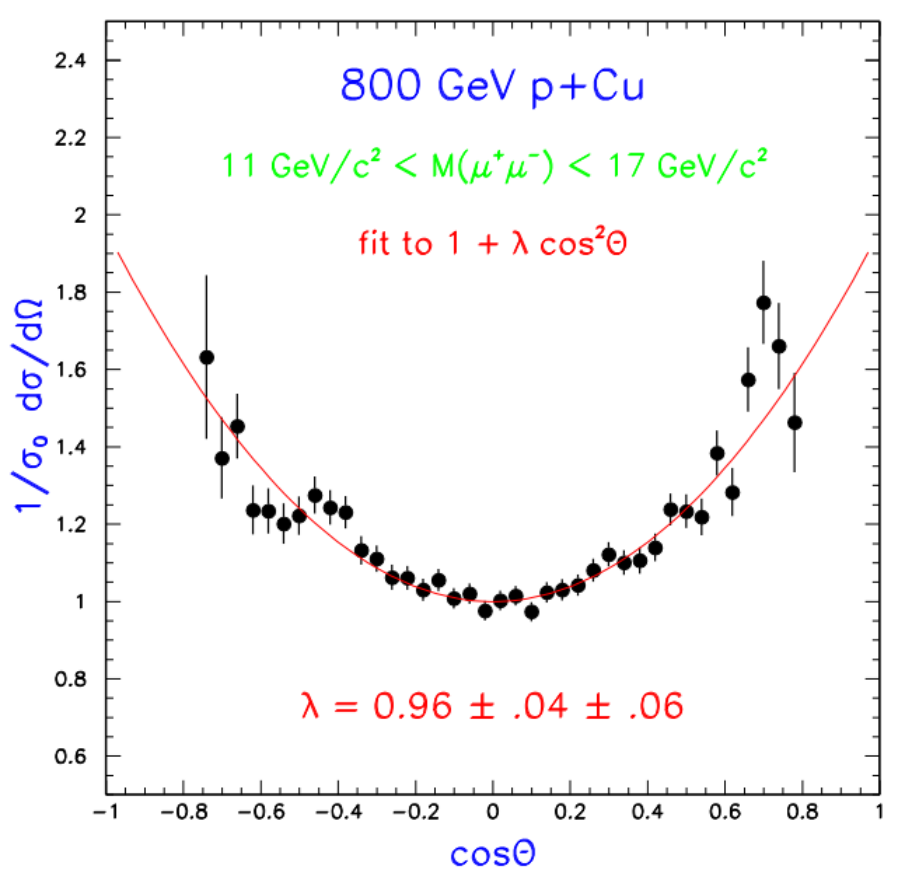


Lepton Angular Distribution in the "Naïve" Drell-Yan

(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin-$\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying as $(1+\cos^2\theta)$ rather than $\sin^2\theta$ as found in Sakurai's[10] vector-dominance model, where θ is the angle of the muon with respect to the time-like photon momentum.

Lepton Angular Distribution of "naïve" Drell-Yan:

$$\frac{d\sigma}{d\Omega} = \sigma_0(1+\lambda\cos^2\theta)$$



$\lambda = 1$:

transversely polarized

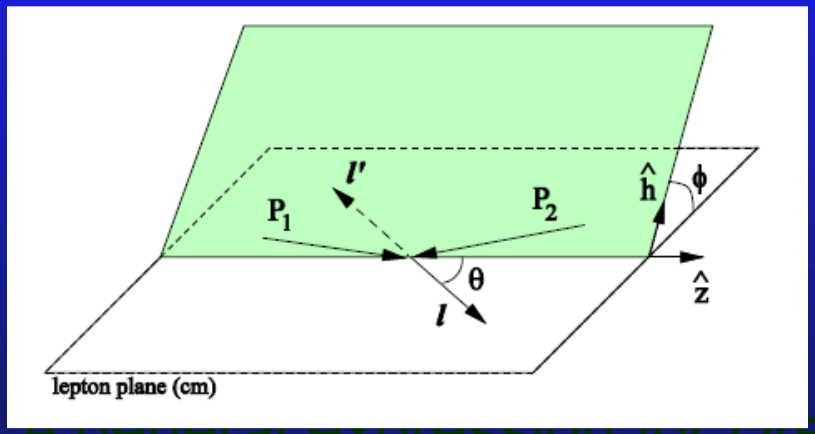
$\lambda = -1$:

longitudinally polarized

$\lambda = 0$:

unpolarized

Drell-Yan lepton angular distributions ($p_T > 0$)



Θ and Φ are the decay polar and azimuthal angles of the μ^- in the dilepton rest-frame

e.g., Collins-Soper frame

A general expression for Drell-Yan decay angular distributions:

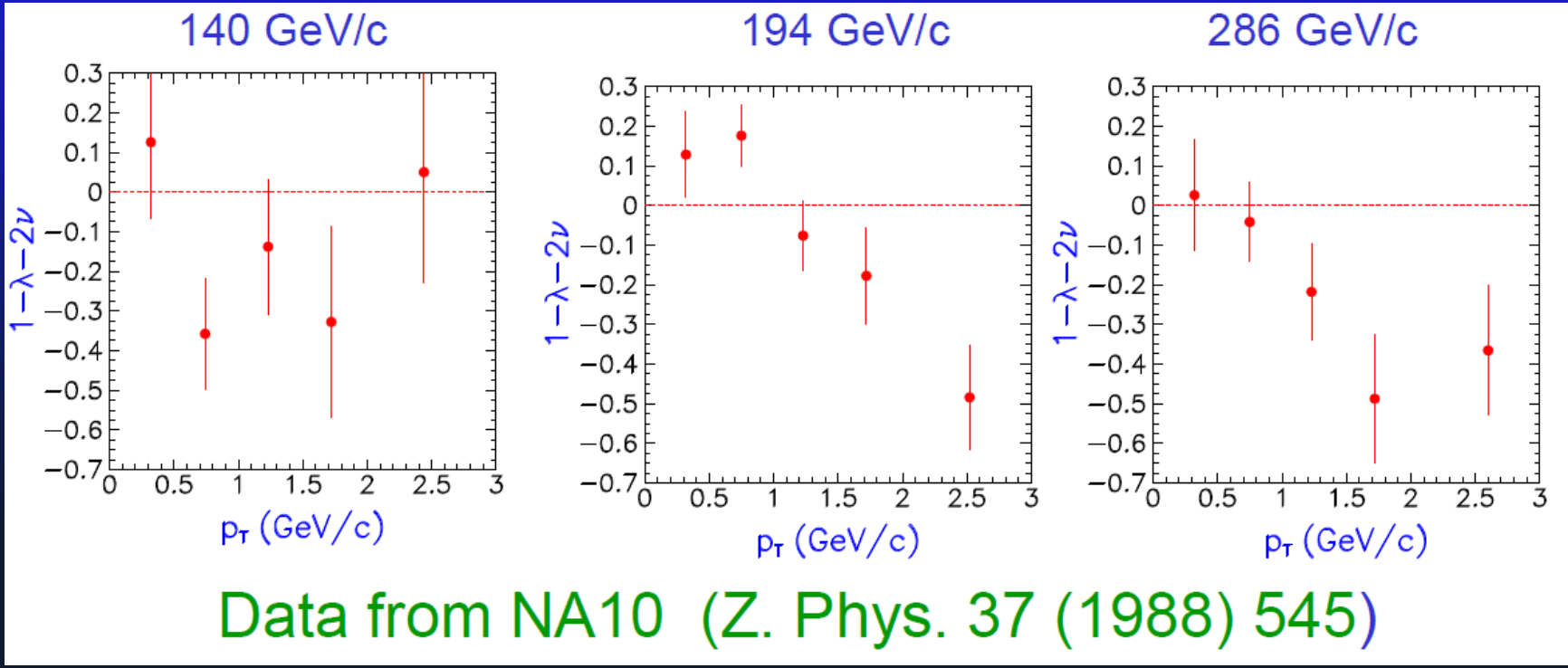
$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right) = \left[\frac{3}{4\pi}\right]\left[1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{\nu}{2}\sin^2\theta\cos 2\phi\right]$$

Lam-Tung relation: $1 - \lambda = 2\nu$

- Reflect the spin-1/2 nature of quarks
  (analog of the Callan-Gross relation in DIS)
- Insensitive to QCD - corrections

Lepton angular distributions in pion-induced Drell-Yan

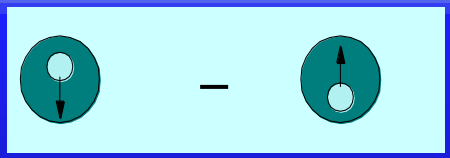
The Lam-Tung relation, $1-\lambda-2\nu=0$, is violated !



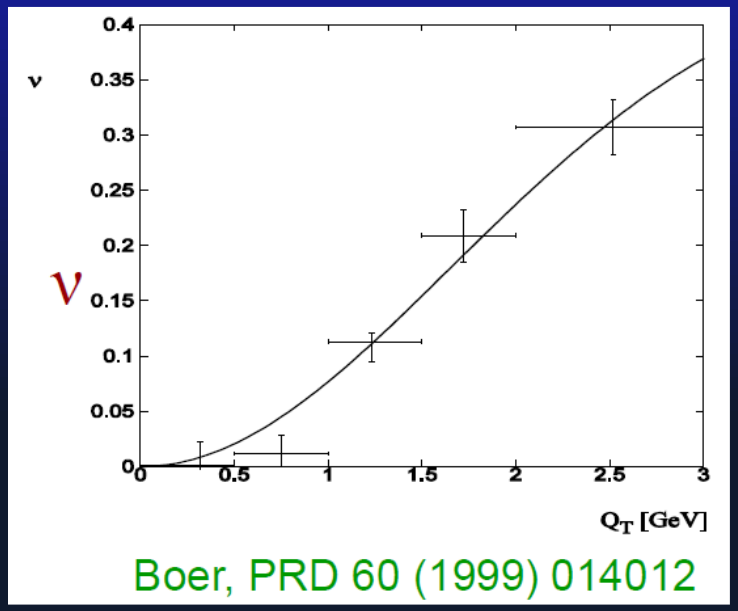
Data from NA10 (Z. Phys. 37 (1988) 545)

Violation of the Lam-Tung relation suggests interesting new origins (Brandenburg, Nachtmann, Mirkes, Brodsky, Khoze, Müller, Eskolar, Hoyer, Väntinnen, Vogt, etc.)

Boer-Mulders function $h_1^{\perp}$



- Boer pointed out that the cos2ϕ dependence can be caused by the presence of the Boer-Mulders function.
- $\langle \cos(2\phi) \rangle \sim \sum_{q,\bar{q}} [h_1^{\perp q}(x_1) h_1^{\perp \bar{q}}(x_2) + h_1^{\perp \bar{q}}(x_1) h_1^{\perp q}(x_2)]$



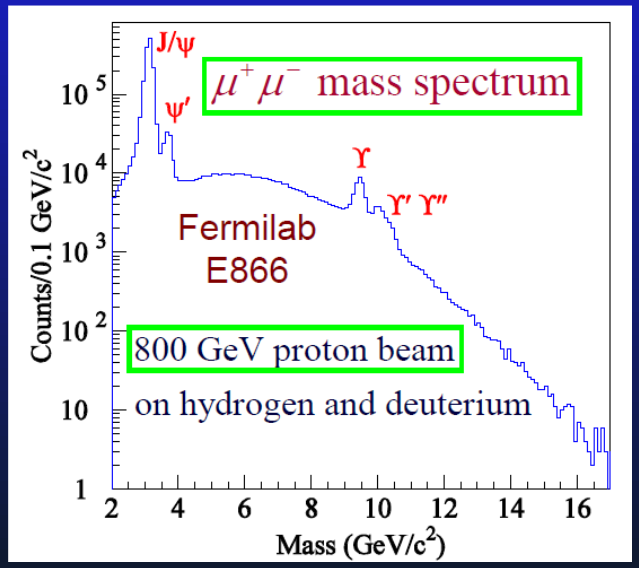
Boer, PRD 60 (1999) 014012

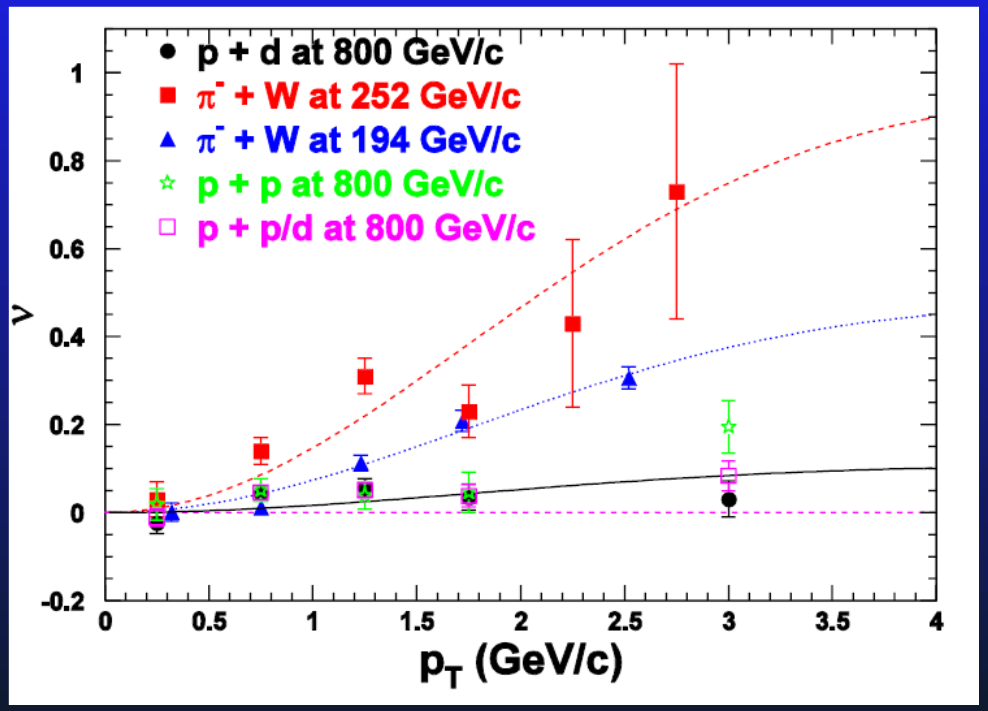
The violation of the Lam-Tung relation is due to the presence of the Boer-Mulders TMD function

The puzzle seems to be solved, and the first extraction of the novel Boer-Mulders function was obtained

$p+p$ and $p+d$ Drell-Yan from Fermilab E866

(L. Zhu, J. Peng et al. PRL 99; PRL. 102)





$$\nu(\pi^- + W \to \mu^+\mu^- X) \propto [\text{valence } h_1^\perp(\pi)] \times [\text{valence } h_1^\perp(p)]$$

$$\nu(p + p \to \mu^+\mu^- X) \propto [\text{valence } h_1^\perp(p)] \times [\text{sea } h_1^\perp(p)]$$

Sea-quark Boer-Mulders function is much smaller than the valence BM function, as expected

Angular coefficients of Z bosons produced in pp collisions at $\sqrt{s} = 8$ TeV and decaying to $\mu^+\mu^-$ as a function of transverse momentum and rapidity

CMS Collaboration *

Physics Letters B 758 (2016) 384–388

First measurement of Z-boson angular coefficients, A_0 up to A_4, at 8 TeV

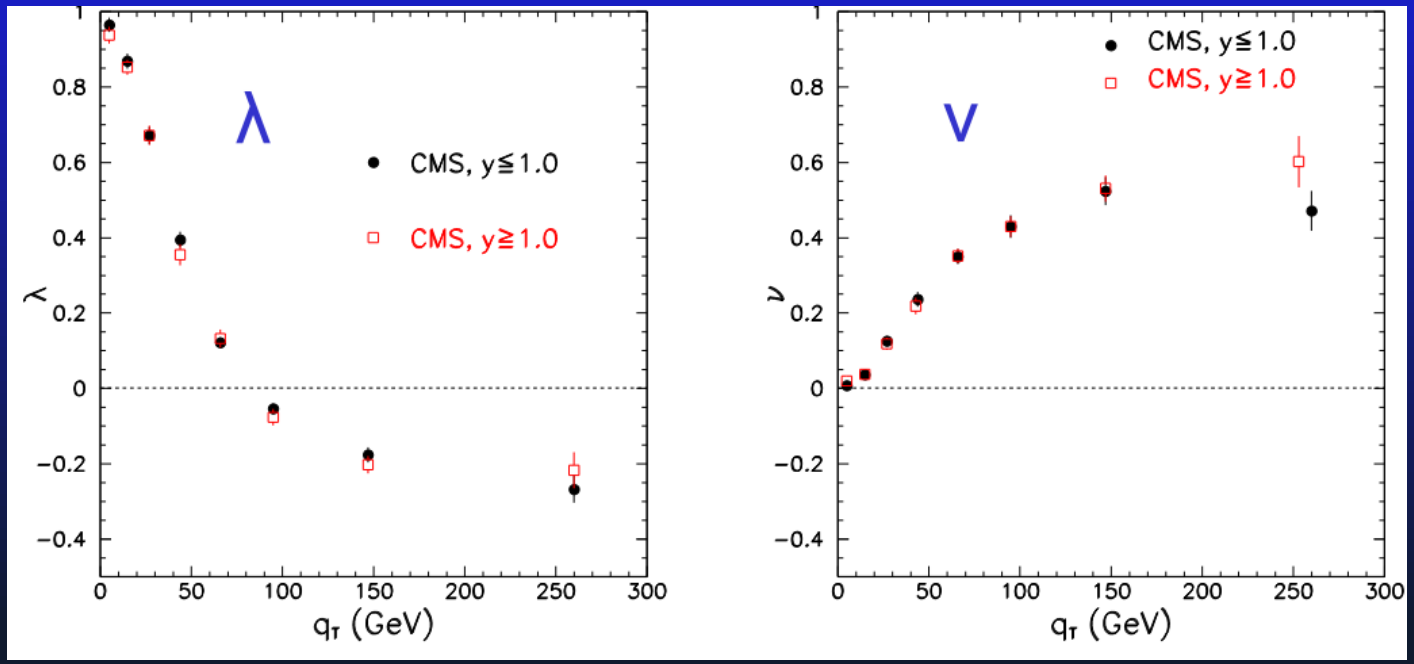
Measurement of the $Z \to \mu^+\mu^-$ angular coefficients in pp collisions at $\sqrt{s} = 13$ TeV as functions of transverse momentum and rapidity

The CMS Collaboration*

2604.25678

Measurement of Z-boson angular coefficients, A_0 up to A_7, at 13 TeV

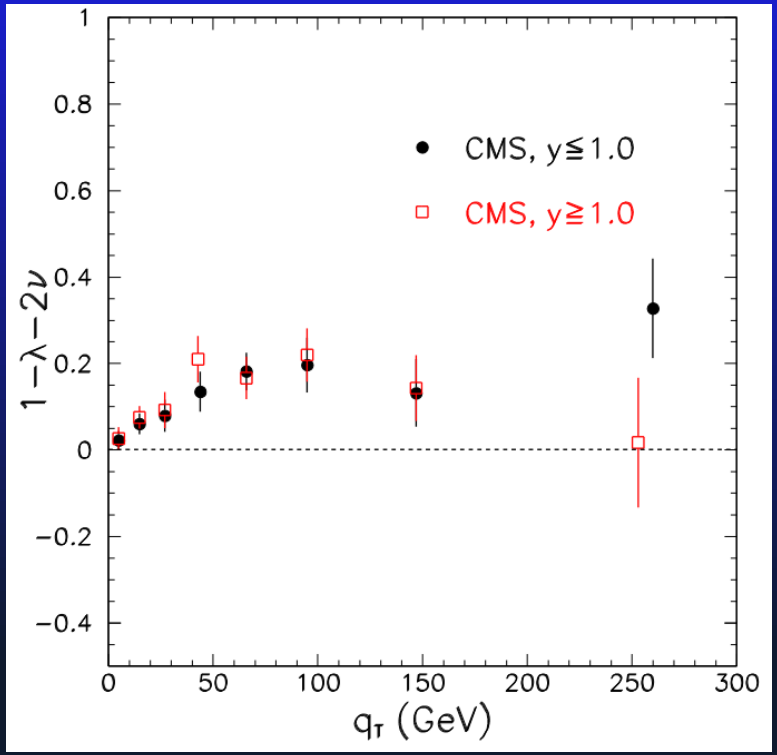
CMS (ATLAS) data for *Z*-boson production in p + p collision at 8 TeV



(arXiv:1504.03512, PL B 750 (2015) 154)

- Striking q_T (p_T) dependencies for λ and ν were observed at two rapidity regions
- Is Lam-Tung relation violated?

Data from CMS for Z-boson production in p+p collision at 8 TeV



- Yes, the Lam-Tung relation is violated ($1-\lambda > 2\nu$)!
- Can one understand the origin of the violation of the Lam-Tung relation (It cannot be due to the Boer-Mulders function)?

Interpretation of the CMS Z-production results

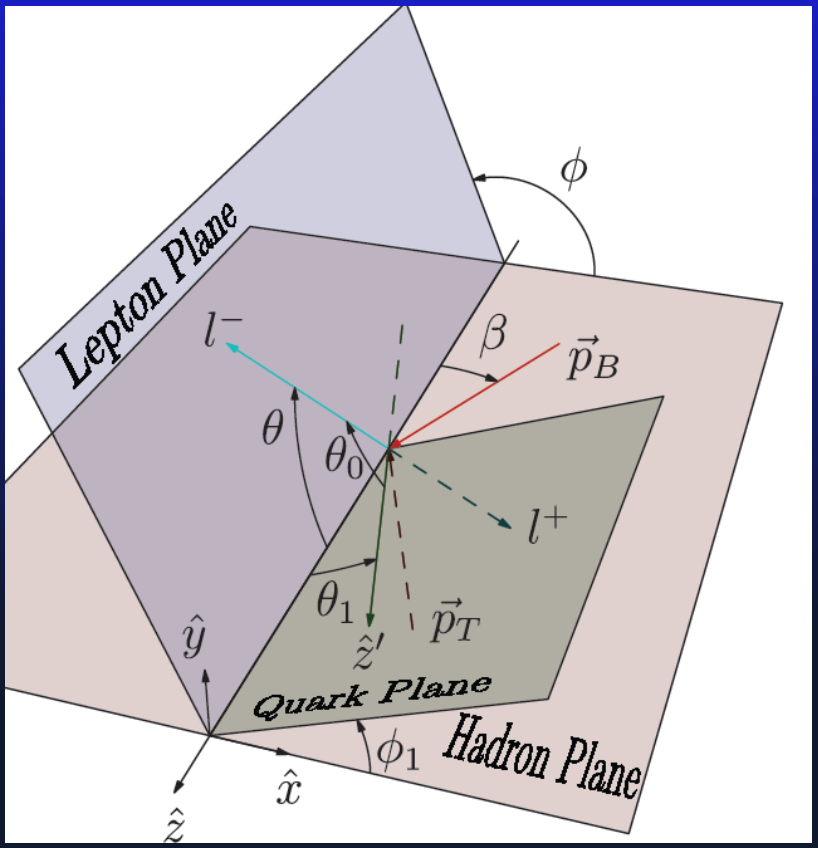
$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1 \sin 2\theta \cos\varphi$$
$$+ \frac{A_2}{2}\sin^2\theta \cos 2\varphi + A_3 \sin\theta \cos\varphi + A_4 \cos\theta$$
$$+ A_5 \sin^2\theta \sin 2\varphi + A_6 \sin 2\theta \sin\varphi + A_7 \sin\theta \sin\varphi$$

Questions:

- How is the above expression derived?
- Can one express $A_0 - A_7$ in terms of some quantities?
- Can one understand the q_T dependence of A_0, A_1, A_2, etc?
- Can one understand the origin of the violation of Lam-Tung relation?

$\lambda = \frac{2-3A_0}{2+A_0}$; $\nu = \frac{2A_2}{2+A_0}$; L-T relation, $1-\lambda = 2\nu$, becomes $A_0 = A_2$

How is the angular distribution expression derived?



- Define three planes in the Z-boson rest frame: hadron-plane, quark-plane, and lepton plane
- The z-axis of the quark-plane is along the quark-antiquark annihilation axis
- The $Z \to \mu^+ \mu^-$ decay angular distribution is azimuthally symmetric:

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

- Express this angular distribution in terms of the measured angles, θ and ϕ

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$

All eight angular distribution terms are obtained!

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta)$$
$$+ (\frac{1}{2}\sin 2\theta_1 \cos\phi_1)\sin 2\theta\cos\phi$$
$$+ (\frac{1}{2}\sin^2\theta_1 \cos 2\phi_1)\sin^2\theta\cos 2\phi$$
$$+ (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta$$
$$+ (\frac{1}{2}\sin^2\theta_1 \sin 2\phi_1)\sin^2\theta\sin 2\phi$$
$$+ (\frac{1}{2}\sin 2\theta_1 \sin\phi_1)\sin 2\theta\sin\phi$$
$$+ (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi.$$

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta)$$
$$+ A_1 \sin 2\theta\cos\varphi$$
$$+ \frac{A_2}{2}\sin^2\theta\cos 2\varphi$$
$$+ A_3 \sin\theta\cos\varphi + A_4\cos\theta$$
$$+ A_5 \sin^2\theta\sin 2\varphi$$
$$+ A_6 \sin 2\theta\sin\varphi$$
$$+ A_7 \sin\theta\sin\varphi$$

$A_0 - A_7$ are entirely described by θ_1, φ_1 and a

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

- $A_0 \geq A_2$ (or $1 - \lambda - 2\nu \geq 0$)
- Lam-Tung relation ($A_0 = A_2$) is satisfied when $\phi_1 = 0$
- Forward-backward asymmetry, a, is reduced by a factor of $\langle \cos\theta_1 \rangle$ for A_4
- Some inequality relations among $A_0 - A_7$ can be obatined

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

Some bounds on the coefficients can be obtained

$$0 < A_0 < 1$$

$$-1/2 < A_1 < 1/2$$

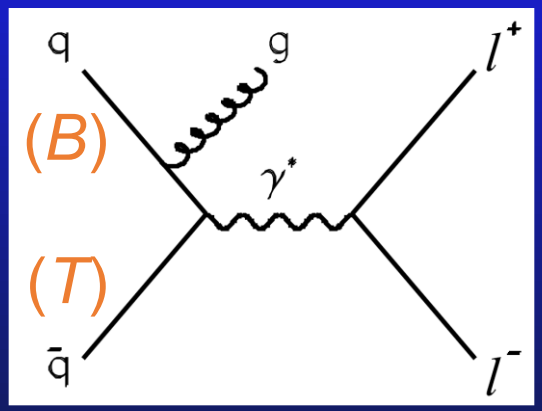
$$-1 < A_2 < 1$$

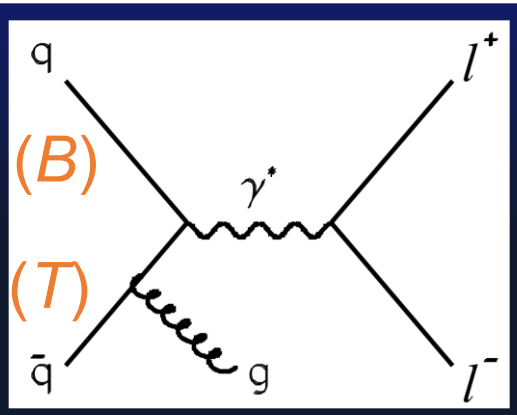
$$-a < A_3 < a$$

$$-a < A_4 < a$$

What are the values of θ_1 and ϕ_1 at order α_s?

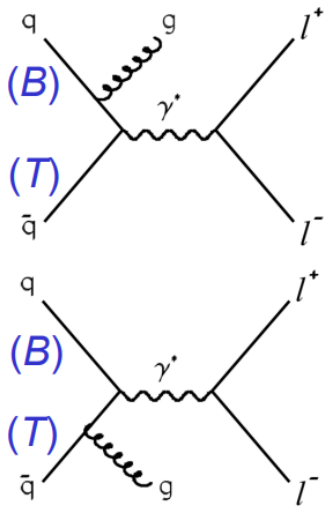
1) $q\bar{q} \to \gamma^*(Z^0)g$



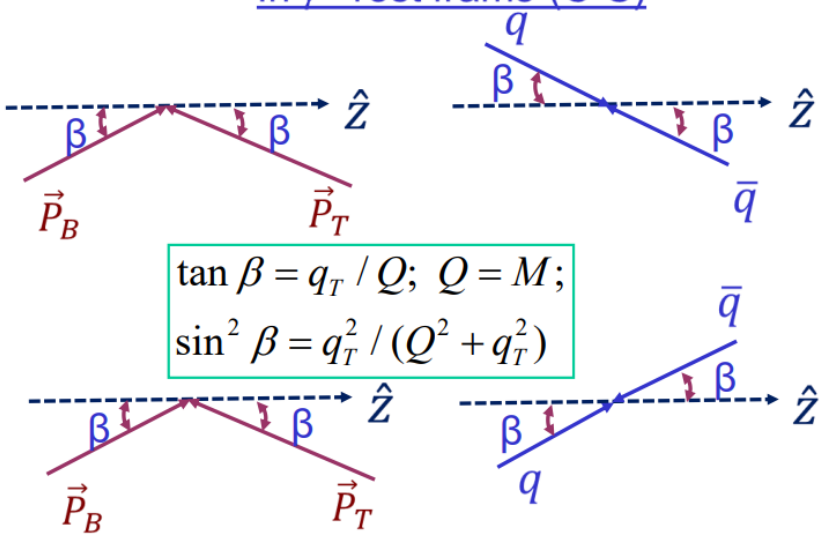


What are the values of θ_1 and ϕ_1 at order α_s?

1) $q\bar{q} \to \gamma^*(Z^0)g$



In γ^* rest frame (C-S)



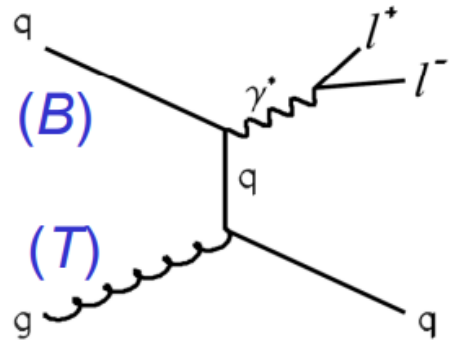
$\theta_1 = \beta$ and $\phi_1 = 0$; $A_0 = A_2 = \sin^2\beta$

$\theta_1 = \beta$ and $\phi_1 = 0$; $A_0 = A_2 = \sin^2\beta$

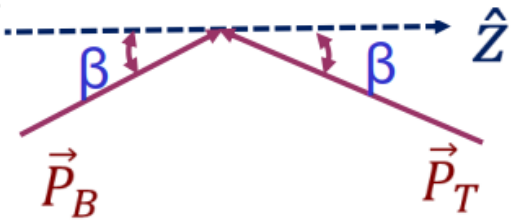
$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}; \qquad \nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

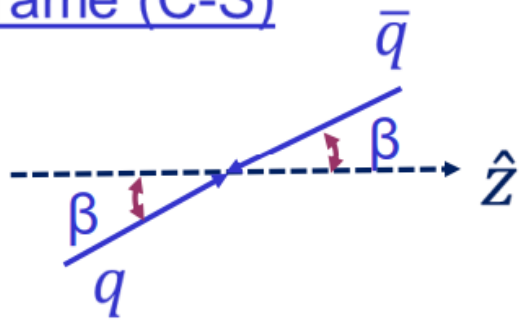
What are the values of θ_1 and ϕ_1 at order α_s?

2) $qg \to \gamma^*(Z^0)q$

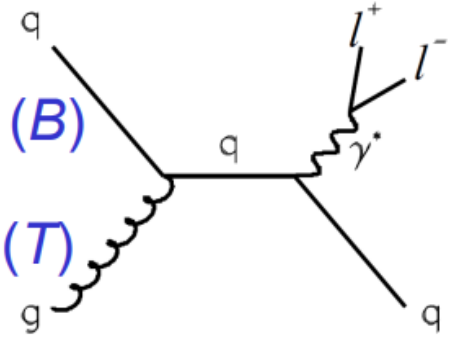


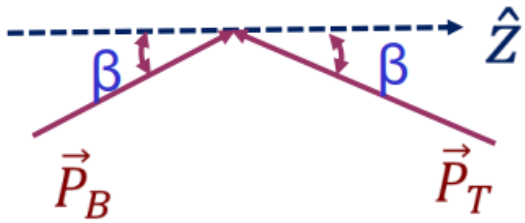
In γ^* rest frame (C-S)

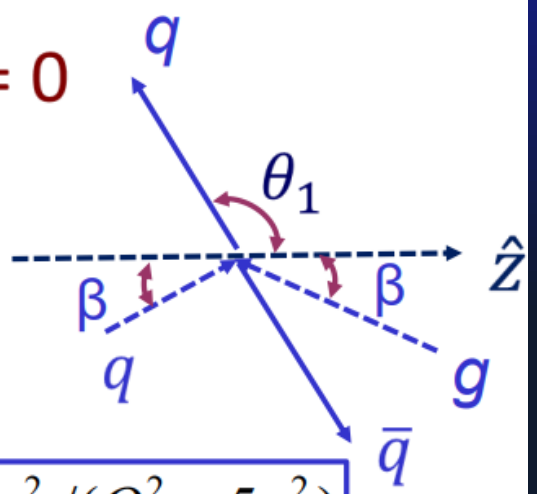




$\theta_1 = \beta$ and $\phi_1 = 0$





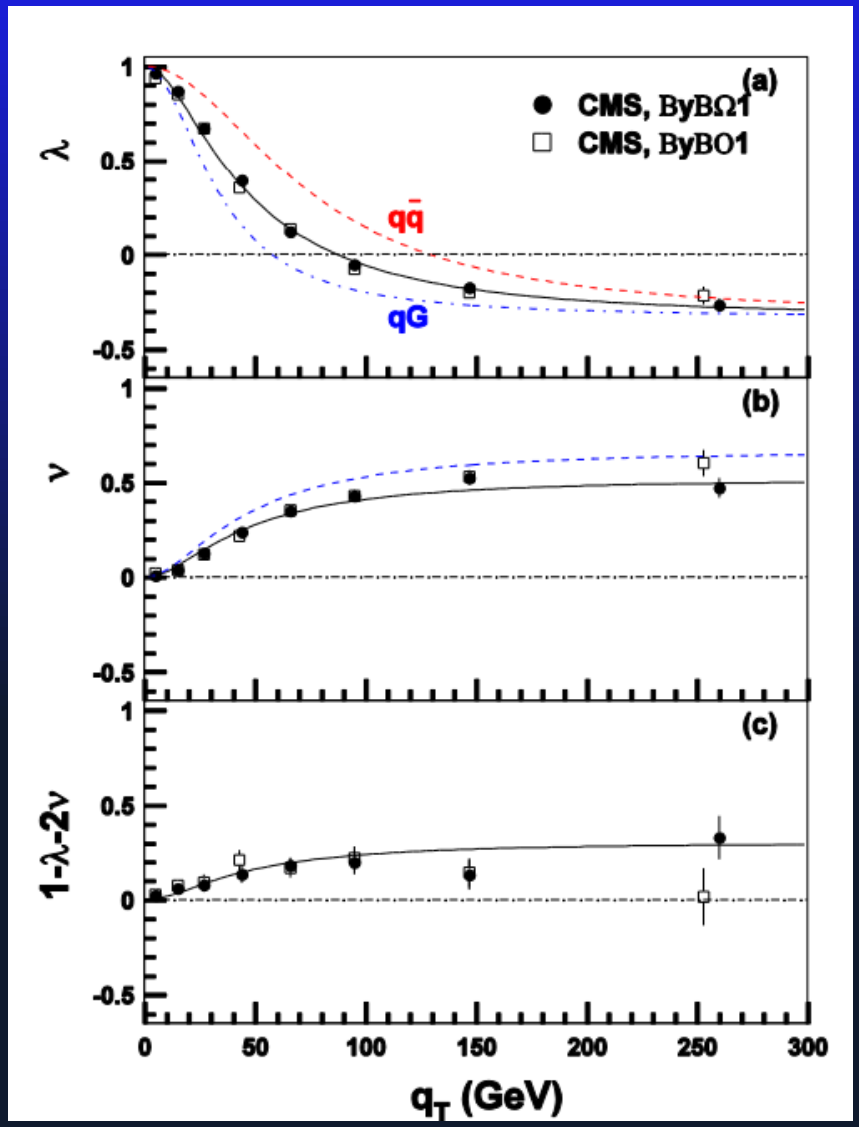


$\theta_1 > \beta$ and $\phi_1 = 0$; $A_0 = A_2 \approx 5q_T^2/(Q^2 + 5q_T^2)$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}; \qquad \nu = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

$A_0 = A_2 \approx 5q_T^2/(Q^2 + 5q_T^2)$

Compare with CMS data on Lam-Tung relation



Solid curves correspond to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes

Also,

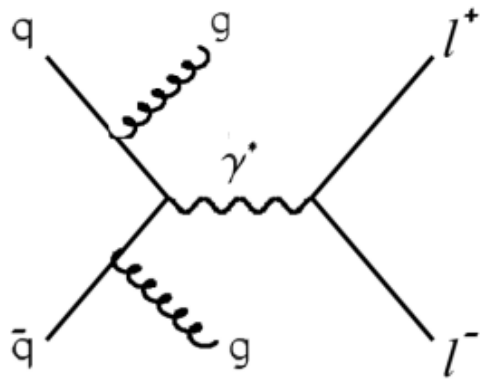
$$\left\langle \sin^2\theta_1 \cos 2\varphi_1 \right\rangle \Big/ \left\langle \sin^2\theta_1 \right\rangle = 0.77$$

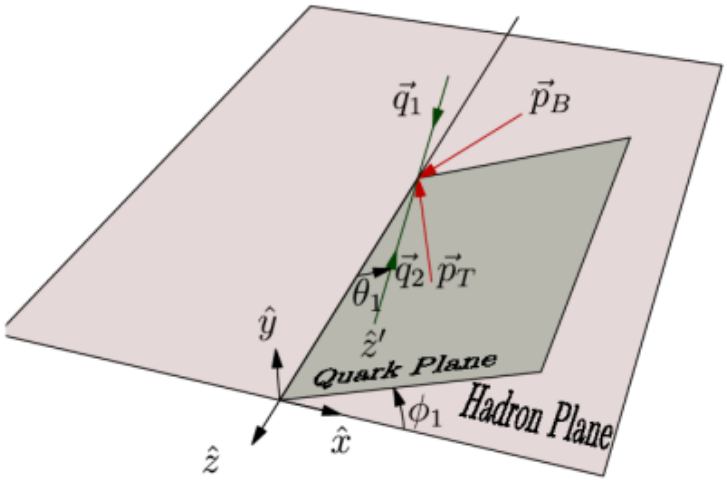
Violation of Lam-Tung relation is well described with a finite non-coplanarity angle

Phys. Lett. B758, 384

Origins of the non-coplanarity

1) Processes at order α_s^2 or higher





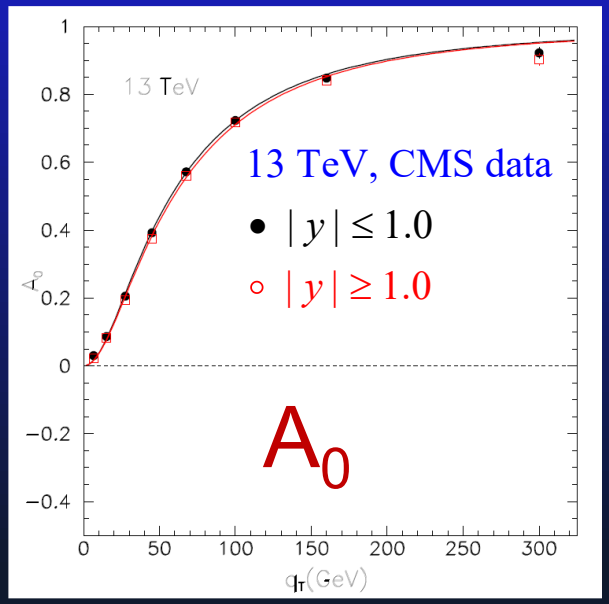
2) Intrinsic k_T from interacting partons

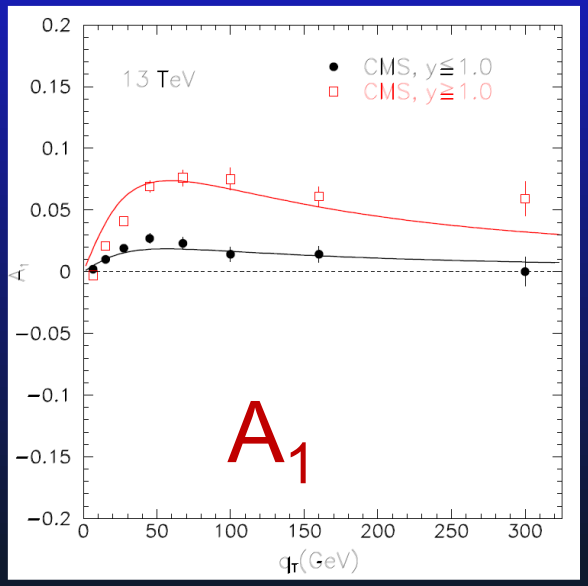
(Boer-Mulders functions in the beam and target hadrons)

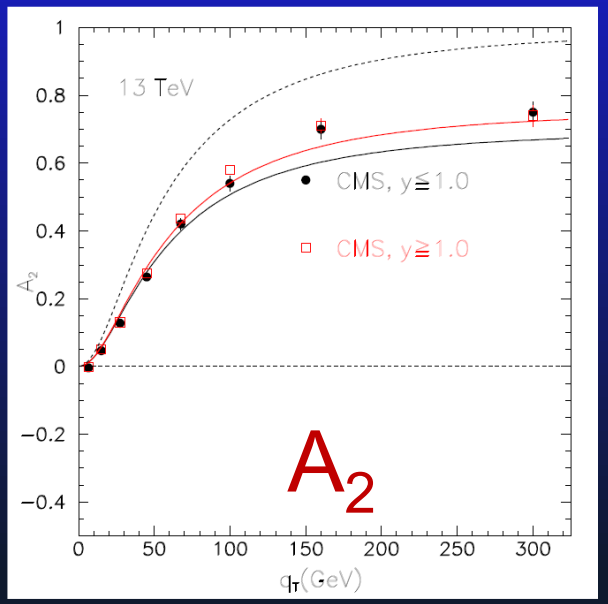
Measurement of the $Z \to \mu^+\mu^-$ angular coefficients in pp collisions at $\sqrt{s} = 13$ TeV as functions of transverse momentum and rapidity

The CMS Collaboration*

arXiv:2604.25678v1



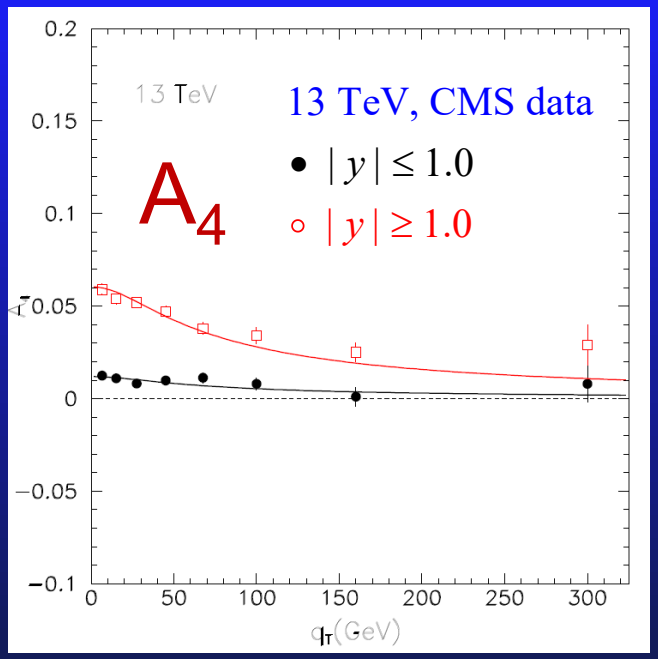


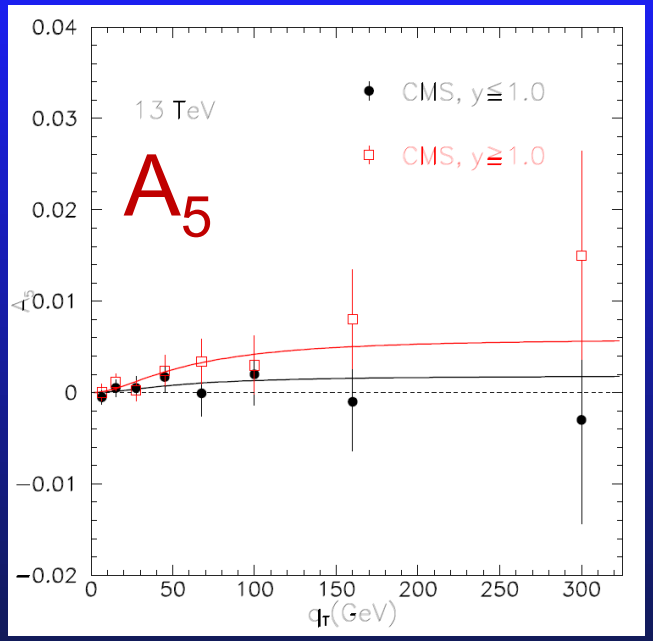


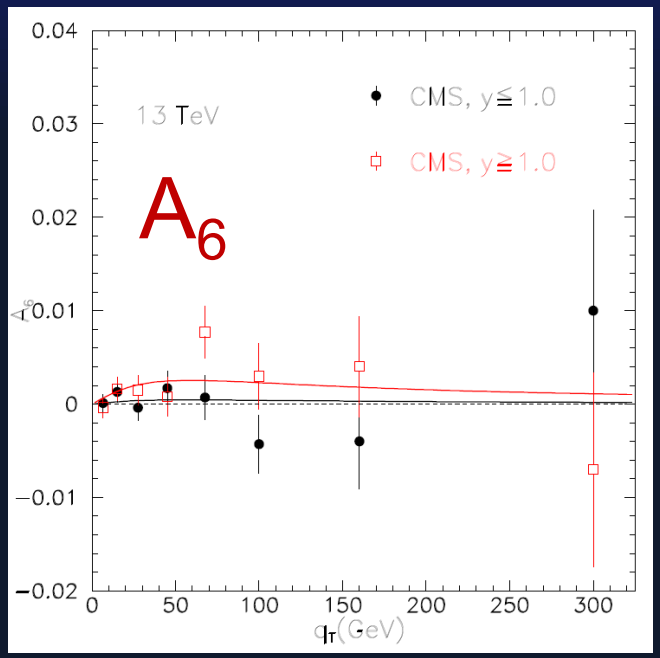
These new data at 13 TeV contain much higher statistics than the 8 TeV data

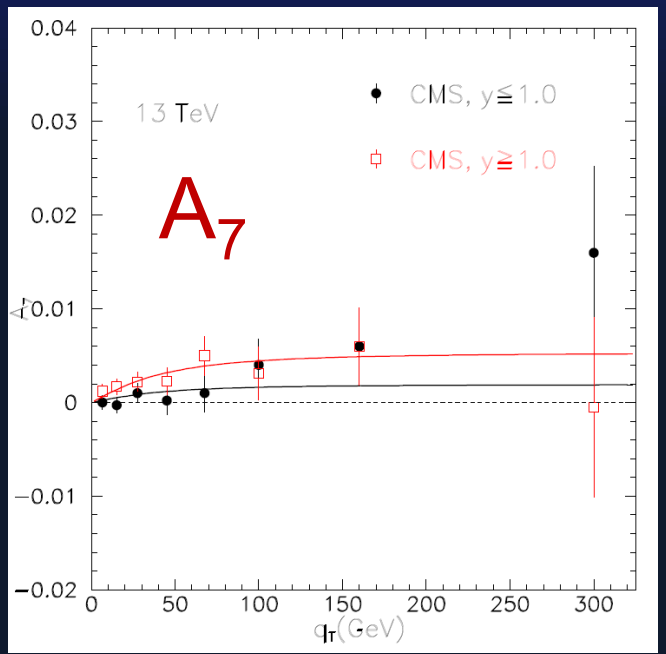
The q_T, rapidity, and beam-energy dependencies of the angular coefficients, and the violation of the Lam-Tung relation, can all be well described by the Geometric model

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This is the first observation of non-zero values of the T-odd coefficients: A_5, A_6, and A_7.

The geometric model can describe these T-odd coefficients

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Other implications

On the Rotational Invariance and Non-Invariance of Lepton Angular Distributions in Drell-Yan and Quarkonium Production

Jen-Chieh Peng[a], Daniël Boer[b], Wen-Chen Chang[c], Randall Evan McClellan[a,d], Oleg Teryaev[e]

(Phys Lett B789, 352)

Three quantities are shown to be invariant for various reference frames (Faccioli et al.)

$$\mathcal{F} = \frac{1+\lambda+\nu}{3+\lambda}$$

$$\tilde{\lambda} = \frac{2\lambda+3\nu}{2-\nu}$$

$$\tilde{\lambda}' = \frac{(\lambda-\nu/2)^2+4\mu^2}{(3+\lambda)^2}$$

$$\mathcal{F} = \frac{1+\lambda_0-2\lambda_0\sin^2\theta_1\sin^2\phi_1}{3+\lambda_0} = \frac{1+\lambda_0-2\lambda_0 y_1^2}{3+\lambda_0}$$

$$\tilde{\lambda}' = \frac{\lambda_0^2(z_1^2+x_1^2)^2}{(3+\lambda_0)^2} = \frac{\lambda_0^2(1-y_1^2)^2}{(3+\lambda_0)^2}$$

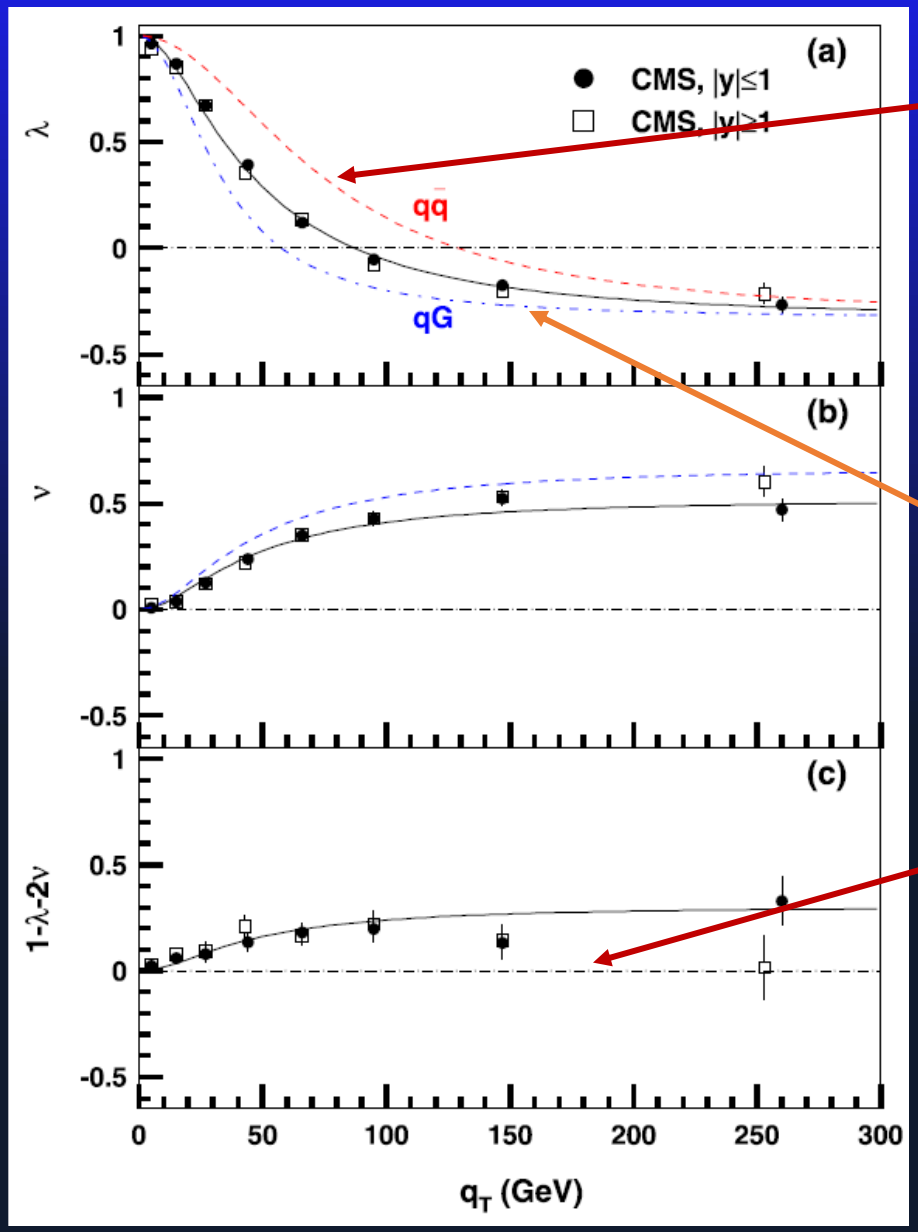
Using the geometric model, we obtain a natural explanation for such invariance. In fact, we show that there are infinite numbers of such invariant quantities, not just three!

Other implications

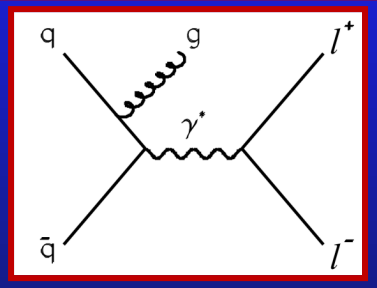
Extend this study to Z plus jets data at LHC

- The angular distribution coefficients are expected to be different, in general, for Z plus single jet and Z plus multi-jets events
- Lam-Tung relation is expected to be satisfied by Z plus single jet events. It is also expected to be badly violated by Z plus two or more jets.
- The q_T dependence of A_0 would be different for Z plus a single quark jet events and Z plus a single gluon jet events (can lead to the validation of various algorithms for quark/gluon jets separation)
- Would be great to analyze these data from LHC!

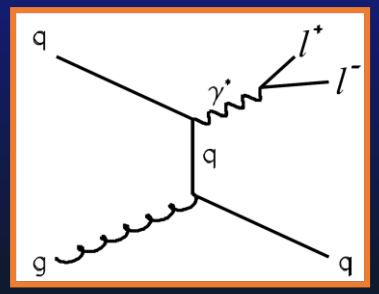
Expected Z plus jets results



Z plus single gluon jet



Z plus single quark jet



Very different results for Z plus single jet, depending on whether it is a quark jet or a gluon jet

Z plus single jet (no L-T violation)

Phys. Lett. B797, 134895

Summary

- The lepton angular distribution coefficients $A_0 - A_7$ can be described in terms of the polar and azimuthal angles of the $q - \bar{q}$ axis in an intuitive geometric approach
- Violation of the Lam-Tung relation is due to the acoplanarity between the quark plane and the hadron plane. This can come from order α_s^2 or higher processes, or from intrinsic k_T (TMD)
- It would be very interesting to obtain the angular distribution coefficients for Z-plus-single jet and Z-plus-multi-jets events
- This approach can be extended to many other hard-processes, including DIS or SIDIS at the EIC