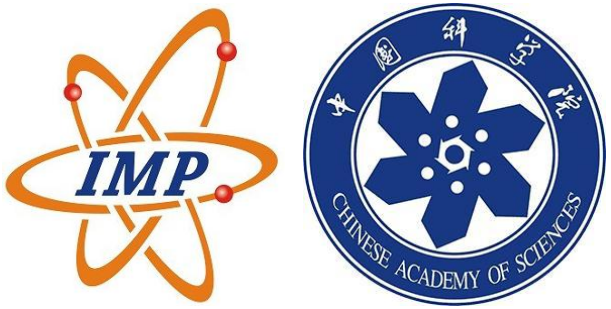
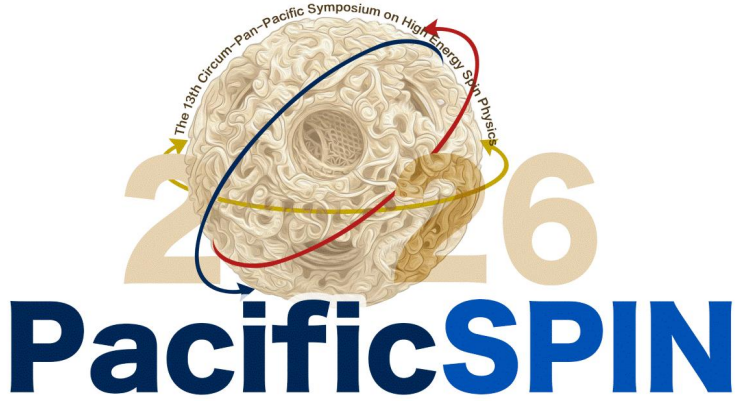
Higher-twist tensor polarization of the deuteron



S. Kumano, K. Kuroki, Phys. Lett. B 875 (2026) 140348
S. Kumano, K. Kuroki, Phys. Lett. B 880 (2026) 140741
S. Kumano, K. Kuroki, arXiv:2607.09237 (Phys. Rev. D, in press)

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Collaborator
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Taipei, Oct. 27-30, 2026

Introduction

- Spin Tensor
- Structure Functions
- Tensor-polarization Puzzle

Spin Tensor

Spin density matrix in the particle rest frame

- **Spin-1/2 (e.g., nucleon)**

$$\boldsymbol{\rho}_{1/2} = \frac{1}{2}\left(\mathbf{1} + s^i \boldsymbol{\sigma}^i\right)$$

$\boldsymbol{\sigma}$: Pauli matrices

$s^i = \mathrm{Tr}\left[\boldsymbol{\rho}_{1/2}\boldsymbol{\sigma}^i\right]$: **Spin vector**

- **Spin-1 (e.g., deuteron)**

$$\boldsymbol{\rho}_1 = \frac{1}{3}\left(\mathbf{1} + \frac{3}{2} S^i \boldsymbol{\Sigma}^i + 3 T^{ij} \boldsymbol{\Sigma}^{ij}\right)$$

$\boldsymbol{\Sigma}$: 3×3 spin matrices, $\boldsymbol{\Sigma}^{ij} = \frac{1}{2}\left(\boldsymbol{\Sigma}^i\boldsymbol{\Sigma}^j + \boldsymbol{\Sigma}^j\boldsymbol{\Sigma}^i\right) - \frac{2}{3}\mathbf{1}\delta^{ij}$

$S^i = \mathrm{Tr}\left[\boldsymbol{\rho}_1 \boldsymbol{\Sigma}^i\right]$, $T^{ij} = \mathrm{Tr}\left[\boldsymbol{\rho}_1 \boldsymbol{\Sigma}^{ij}\right]$: **Spin tensor**

Tensor polarization: Degrees of freedom unique to spin ≥ 1 particles

Covariant generalizations: $\boldsymbol{S} \to S^\mu$, $\boldsymbol{T} \to T^{\mu\nu}$ with $P_\mu S^\mu = 0$, $P_\mu T^{\mu\nu} = 0$

Structure Functions

Charged-lepton inclusive DIS P. Hoodbhoy, R. L. Jaffe, A. Manohar, NPB 312 (1989) 571

$$W_{\mu\nu} = -F_1\hat{g}_{\mu\nu} + F_2\frac{\hat{P}_\mu\hat{P}_\nu}{M\nu} + g_1\frac{i}{\nu}\varepsilon_{\mu\nu\alpha\beta}q^\alpha s^\beta + g_2\frac{i}{M\nu^2}\varepsilon_{\mu\nu\alpha\beta}q^\alpha\left(P\cdot q s^\beta - s\cdot q P^\beta\right)$$
$$-b_1 r_{\mu\nu} + b_2\frac{1}{6}\left(s_{\mu\nu}+t_{\mu\nu}+u_{\mu\nu}\right) + b_3\frac{1}{2}\left(s_{\mu\nu}-u_{\mu\nu}\right) + b_4\frac{1}{2}\left(s_{\mu\nu}-t_{\mu\nu}\right)$$

$$r_{\mu\nu} = \frac{1}{\nu^2}\left(q\cdot E^* q\cdot E - \frac{1}{3}\nu^2\kappa\right)\hat{g}_{\mu\nu}$$
$$s_{\mu\nu} = \frac{2}{\nu^2}\left(q\cdot E^* q\cdot E - \frac{1}{3}\nu^2\kappa\right)\frac{\hat{P}_\mu\hat{P}_\nu}{M\nu}$$
$$t_{\mu\nu} = \frac{1}{2\nu^2}\left(q\cdot E^*\hat{P}_{\{\mu}\hat{E}_{\nu\}} + q\cdot E\hat{P}_{\{\mu}\hat{E}^*_{\nu\}} - \frac{4\nu}{3M}\hat{P}_\mu\hat{P}_\nu\right)$$
$$u_{\mu\nu} = \frac{M}{\nu}\left(\hat{E}^*_{\{\mu}\hat{E}_{\nu\}} + \frac{2}{3}\hat{g}_{\mu\nu} - \frac{2}{3M^2}\hat{P}_\mu\hat{P}_\nu\right)$$

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu/q^2$$
$$\hat{a}_\mu = a_\mu - \left(a\cdot q/q^2\right)q_\mu$$
$$a^{\{\mu}b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$$

$$\nu = P\cdot q/M$$
$$\kappa = 1 + Q^2/\nu^2$$
$$P\cdot E = 0,\quad E^2 = -M^2$$
$$\sum_\lambda E^\mu(\lambda)E^{*\nu}(\lambda) = -g^{\mu\nu} + P^\mu P^\nu/M^2$$
$$s^\sigma = -\frac{i}{M}\varepsilon^{\sigma\alpha\beta\tau}E^*_\alpha E_\beta P_\tau$$

Tensor-polarized structure functions (SFs)

$$\boldsymbol{b_1, b_2, b_3, b_4}$$

- b_1, b_2: Leading-twist
  $2xb_1(x) = b_2(x)$ in the Bjorken limit
- b_3, b_4: Higher-twist

Symmetric under $\mu \leftrightarrow \nu$ and $E \leftrightarrow E^*$
→ Unpolarized lepton beams ✓

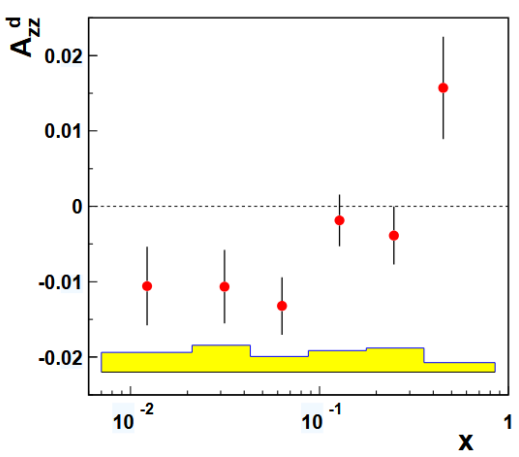
Tensor-polarization Puzzle

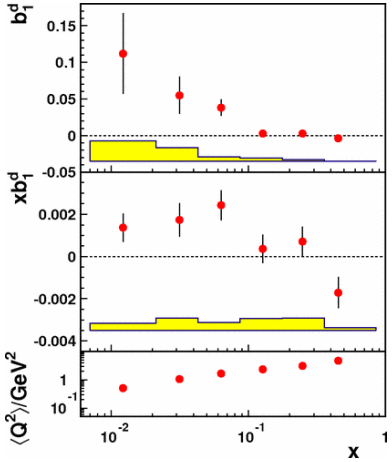
HERMES data

HERMES Collaboration, PRL 95 (2005) 242001

$$\frac{d^2\sigma}{dxdQ^2} = \frac{d^2\sigma^U}{dxdQ^2}\left(1 + \frac{1}{2}P_{ZZ}A_{ZZ}^D\right)$$

Tensor asymmetry: $A_{ZZ}^D \xrightarrow[\text{Bjorken limit}]{} A_{ZZ}^D = -\frac{2}{3}\frac{b_1^D}{F_1^D}$

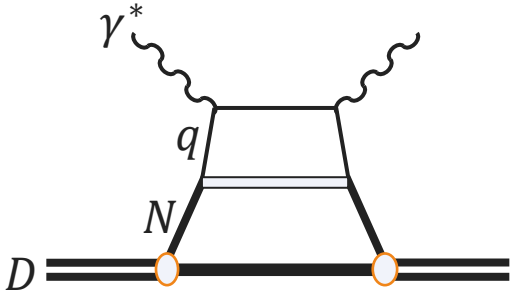


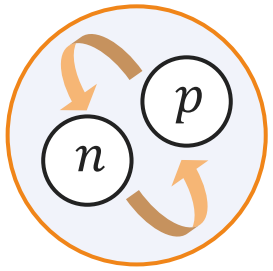


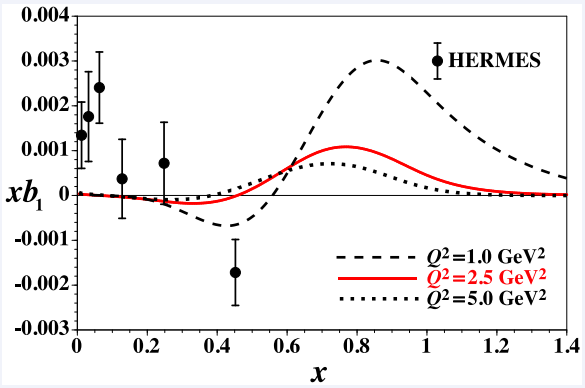
Convolution model

W. Cosyn, Y.-B. Dong, S. Kumano, M. Sargsian, PRD 95 (2017) 074036

Unpol. SF of the nucleon $\otimes$ Tensor-pol. nucleon dist. in the deuteron







indicate that these standard theoretical predications are much different from the HERMES measurements. Furthermore, significantly large distributions are predicted, at large x ($x > 0.8$) and even at extremely large x ($x > 1$). Since our results are very different from the HERMES measurement, new hadronic mechanisms could be needed for interpreting the data although there is still some room to improve the differences due to the higher-twist effects and the experimental extraction of b_1 from A_{zz}. The HERMES data have large uncertainties; however, upcoming JLab experimental measurements will improve on the size of the errors. In addition, there are

Upcoming JLab measurements and possible measurements at other facilities (e.g., EIC)

Importance of investigating higher-twist tensor polarization

Tensor-polarized PDFs and Twist-2′ Relations

- Tensor-polarized PDFs
- Wandzura-Wilczek-like Relation
- Burkhardt-Cottingham-like Sum Rule

Tensor-polarized PDFs

S. Kumano, Q.-T. Song, PRD 103 (2021) 014025

Collinear correlation function

$$\Phi_{ij}(x,P,T) = \int \frac{d\xi^-}{2\pi} e^{ixP^+\xi^-} \left\langle P,T \middle| \overline{\psi}_j(0)\, \psi_i(\xi) \middle| P,T \right\rangle_{\xi^+=0,\vec{\xi}_T=0}$$

$0 < x < 1$, $P^\mu = P^+\bar{n}^\mu + \frac{M^2}{2P^+} n^\mu$, the gauge link is not explicitly shown

$$n^\mu = \frac{1}{\sqrt{2}}(1,0,0,-1)$$
$$\bar{n}^\mu = \frac{1}{\sqrt{2}}(1,0,0,1)$$
$$g_T^{\mu\nu} = g^{\mu\nu} - \bar{n}^{\{\mu} n^{\nu\}}$$
$$a^{\{\mu} b^{\nu\}} = a^\mu b^\nu + a^\nu b^\mu$$

Spin tensor A. Bacchetta, P.J. Mulders, PRD 62 (2000) 114004

$$T^{\mu\nu} = \frac{1}{2}\left[\frac{4}{3} S_{LL} \frac{(P^+)^2}{M^2} \bar{n}^\mu \bar{n}^\nu - \frac{2}{3} S_{LL} \left(\bar{n}^{\{\mu} n^{\nu\}} - g_T^{\mu\nu} \right) + \frac{1}{3} S_{LL} \frac{M^2}{(P^+)^2} n^\mu n^\nu + \frac{P^+}{M} \bar{n}^{\{\mu} S_{LT}^{\nu\}} - \frac{M}{2P^+} n^{\{\mu} S_{LT}^{\nu\}} + S_{TT}^{\mu\nu} \right]$$

Tensor-polarization parameters: $S_{LL}, S_{LT}^\mu, S_{TT}^{\mu\nu}$

$$n \cdot S_{LT} = \bar{n} \cdot S_{LT} = 0,\ n_\mu S_{TT}^{\mu\nu} = \bar{n}_\mu S_{TT}^{\mu\nu} = 0,\ S_{TT}^{\mu\nu} = S_{TT}^{\nu\mu},\ g_{T\,\mu\nu} S_{TT}^{\mu\nu} = 0$$

Collinear parton distribution functions (PDFs)

$$\Phi(x,P,T) = \frac{1}{2}\left[S_{LL}(\bar{n}\cdot\gamma) f_{1LL}(x) + \frac{M}{P^+} S_{LL} 1 e_{LL}(x) + \frac{M}{P^+} (S_{LT}\cdot\gamma) f_{LT}(x) + \frac{M^2}{(P^+)^2} S_{LL}(n\cdot\gamma) f_{3LL}(x) \right]$$

Twist-2: f_{1LL} **Twist-3:** e_{LL}, f_{LT} **Twist-4:** f_{3LL}

e_{LL} is chiral-odd

Twist-2′ Relations

Twist-3 function = Twist-2′ part + Genuine twist-3′ part

e.g., Vector-polarized structure function

$$g_2 = g_2^{\mathrm{WW}} + g_2^{(\mathrm{HT})}$$

Wandzura-Wilczek (WW) relation

S. Wandzura, F. Wilczek, PLB 72 (1977) 195

$$g_2^{\mathrm{WW}}(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y)$$

WW approximation: $g_2 = g_2^{\mathrm{WW}} + g_2^{(\mathrm{HT})} \approx g_2^{\mathrm{WW}}$

Burkhardt-Cottingham (BC) sum rule

H. Burkhardt, W.N. Cottingham, Annals. Phys. 56 (1970) 453

$$\int_0^1 dx\, g_2(x) = 0$$

WW approximation ⇒ BC sum rule ✓

$$\int_0^1 dx\, g_2^{\mathrm{WW}}(x) = 0$$

Analogous relations for the tensor-polarized PDFs f_{1LL} and f_{LT}

Twist-2′ Relations

S. Kumano, KK, PLB 880 (2026) 140741

Moments of nonlocal matrix elements are related to moments of PDFs

$$\int_{-1}^{1} dx x^{n-1} \int \frac{d(P\cdot\xi)}{4\pi} e^{ixP\cdot\xi} \langle P,T|\bar{\psi}(0)\gamma^{\sigma}\psi(\xi)|P,T\rangle_{\substack{\xi^+=0\\ \vec{\xi}_T=0}} \cong S_{LL}\bar{n}^{\sigma}\int_{-1}^{1} dx x^{n-1} f_{1LL}(x) + \frac{M}{P\cdot n} S_{LT}^{\sigma}\int_{-1}^{1} dx x^{n-1} f_{LT}(x)$$

Equivalent local-operator representation

$$\int_{-1}^{1} dx x^{n-1} \int \frac{d(P\cdot\xi)}{4\pi} e^{ixP\cdot\xi} \langle P,T|\bar{\psi}(0)\gamma^{\sigma}\psi(\xi)|P,T\rangle_{\substack{\xi^+=0\\ \vec{\xi}_T=0}} = \cdots = \frac{n_{\mu_1}\cdots n_{\mu_{n-1}}}{2(P\cdot n)^n}\langle P,T|R^{\sigma\{\mu_1\cdots\mu_{n-1}\}}|P,T\rangle$$

$$\psi(\xi) = \sum_{m=0}^{\infty}\frac{1}{m!}(\xi\cdot\partial)^m\psi(\xi)_{\xi=0}$$

$$\partial^{\mu_1}\cdots\partial^{\mu_{n-1}}\sum_{m=0}^{\infty}\frac{1}{m!}(\xi\cdot\partial)^m = \partial^{\{\mu_1}\cdots\partial^{\mu_{n-1}\}}\left(1+\sum_{m=1}^{\infty}\frac{1}{m!}(\xi\cdot\partial)^m\right)$$

$$a^{\{\mu_1}\cdots a^{\mu_{n-1}\}} = \frac{1}{n!}(a^{\mu_1}\cdots a^{\mu_{n-1}} + \text{permutations})$$

Local operator

$$R^{\sigma\{\mu_1\cdots\mu_{n-1}\}} = i^{n-1}\bar{\psi}(0)\gamma^{\sigma}D^{\{\mu_1}\cdots D^{\mu_{n-1}\}}\psi(0) - \text{traces}$$

$$= R^{\{\sigma\mu_1\cdots\mu_{n-1}\}} + R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}$$

$R^{\{\sigma\mu_1\cdots\mu_{n-1}\}}$	: Twist-2′
$R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}$	: Twist-3′
traces	: Twist $\geq 4'$

Twist-2′ Relations

S. Kumano, KK, PLB 880 (2026) 140741

Matrix-elements decomposition in terms of the momentum P^μ and spin tensor $T^{\mu\nu}$

$$\langle P,T|R^{\{\sigma\mu_1\cdots\mu_{n-1}\}}|P,T\rangle = \frac{2}{n}a_n M^2\left[\sum_{i=1}^{n-1}(T^{\sigma\mu_i}+T^{\mu_i\sigma})\prod_{j(\neq i)=1}^{n-1}P^{\mu_j} + \sum_{i=1}^{n-1}\sum_{j(\neq i)=1}^{n-1}T^{\mu_i\mu_j}P^\sigma\prod_{k(\neq i,j)=1}^{n-1}P^{\mu_k}\right]$$

$$\langle P,T|R^{[\sigma\{\mu_1]\cdots\mu_{n-1}\}}|P,T\rangle = \frac{2}{n}d_n M^2\left[\sum_{i=1}^{n-1}\sum_{j(\neq i)=1}^{n-1}(P^\sigma T^{\mu_i\mu_j}-P^{\mu_i}T^{\sigma\mu_j})\prod_{k(\neq i,j)=1}^{n-1}P^{\mu_k}\right]$$

a_n and d_n are overall coefficients associated with twist-2′ and twist-3′

As a result,

$$\frac{n_{\mu_1}\cdots n_{\mu_{n-1}}}{2(P\cdot n)^n}\langle P,T|R^{\sigma\{\mu_1\cdots\mu_{n-1}\}}|P,T\rangle \cong S_{LL}\bar{n}^\sigma\left[\frac{2(n-1)}{3}a_n\right] + \frac{M}{P\cdot n}S_{LT}^\sigma\left[\frac{n-1}{n}a_n - \frac{(n-1)(n-2)}{2n}d_n\right]$$

Moments of PDFs

$$\int_{-1}^{1}dx\,x^{n-1}f_{1LL}(x) = \int_0^1 dx\,x^{n-1}f_{1LL}^{\pm}(x) = \frac{2(n-1)}{3}a_n$$

$$\int_{-1}^{1}dx\,x^{n-1}f_{LT}(x) = \int_0^1 dx\,x^{n-1}f_{LT}^{\pm}(x) = \frac{n-1}{n}a_n - \frac{(n-1)(n-2)}{2n}d_n$$

$f^{\pm}(x) = f(x) \pm \bar{f}(x)$

$+\,(-)$ for $n =$ even (odd) integer

The twist-3 f_{LT} contains a twist-2′ contribution a_n

Twist-2′ Relations

S. Kumano, KK, PLB 880 (2026) 140741

$$f_{LT}(x) = f_{LT}^{\text{twist}-2'}(x) + f_{LT}^{(\text{HT})}(x)$$

Twist-2′ WW-like part **Genuine twist-3′ part**

WW-like relation

S. Kumano, Q.-T. Song, JHEP 09 (2021) 141

$$f_{LT}^{\text{twist}-2'}(x) = \frac{3}{2}\int_x^1 \frac{dy}{y} f_{1LL}(y)$$

Define $f_{2LT} \coloneqq \frac{2}{3} f_{LT} - f_{1LL}$,

$$f_{2LT}^{\text{twist}-2'}(x) = -f_{1LL}(x) + \int_x^1 \frac{dy}{y} f_{1LL}(y)$$

Same form as the original WW relation

BC-like sum rule

S. Kumano, Q.-T. Song, JHEP 09 (2021) 141

$$\int_0^1 dx\, f_{2LT}^{\text{twist}-2'}(x) = 0$$

$$\int_0^1 dx\, f_{2LT}(x) = \frac{2}{3}\int_0^1 dx\, f_{LT}^{(\text{HT})}(x)$$

Sum rule test

→ Genuine twist-3′ $f_{LT}^{(\text{HT})}$

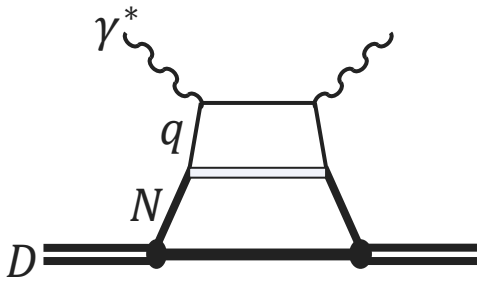
Numerical Application to the Deuteron

- Twist-2 PDF f_{1LL} by a Convolution Model
- Tensor-polarization Puzzle
- Twist-3 PDF f_{LT}
- Twist-3 Effects on Structure Functions

Convolution Model PDF f_{1LL}

W. Cosyn et al., PRD 95 (2017) 074036
S. Kumano, KK, arXiv:2607.09237

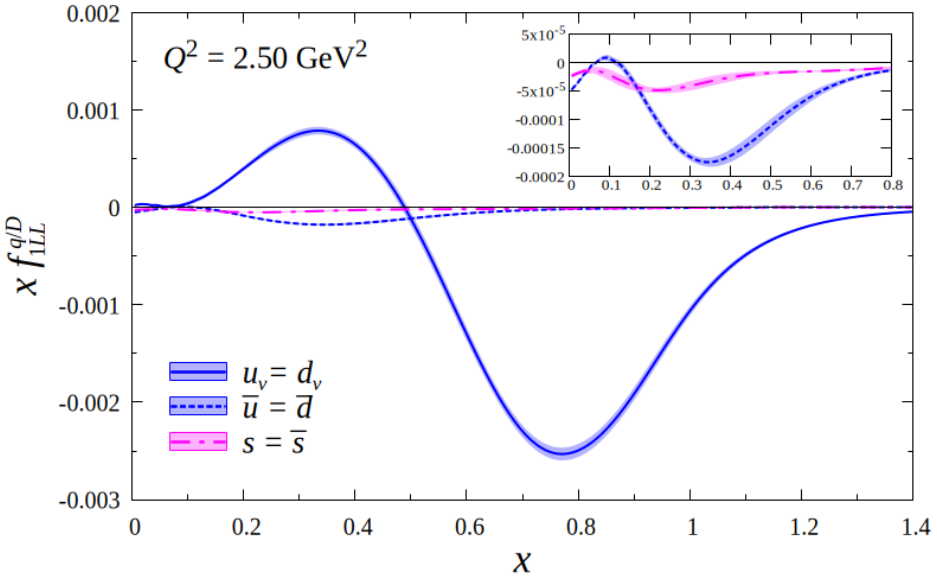
Tensor-pol. PDF in the deuteron = **Tensor-pol. nucleon dist. in the deuteron** ⊗ **Unpol. PDF in the nucleon**



$$\begin{pmatrix} f_{1LL}^{q/D}(x,Q^2) \\ f_{1LL}^{\bar{q}/D}(x,Q^2) \end{pmatrix} = -\frac{2}{3}\int_x^2 \frac{dy}{y}\,\delta_T f(y)\,\frac{1+4M_N^2(x/y)^2/Q^2}{1+R(x/y,Q^2)} \begin{pmatrix} q_i^N(x/y,Q^2) \\ \bar{q}_i^N(x/y,Q^2) \end{pmatrix}$$

$$\delta_T f(y) = \int d^3p\, y \left[-\frac{3}{4\sqrt{2}\pi}\phi_0(p)\phi_2(p) + \frac{3}{16\pi}|\phi_2(p)|^2 \right] (3\cos^2\theta - 1)\,\delta\left(y - \frac{p\cdot q}{M_N \nu}\right)$$

- Nucleon PDF q_i^N: MSTW2008 LO
  A.D. Martin et al., EPJ C 63 (2009) 189
- L/T ratio R: SLAC-R1998
  K. Abe et al., PLB 452 (1999) 194
- Deuteron WF ϕ_l: CD-Bonn
  R. Machleidt et al., PRC 63 (2001) 024001



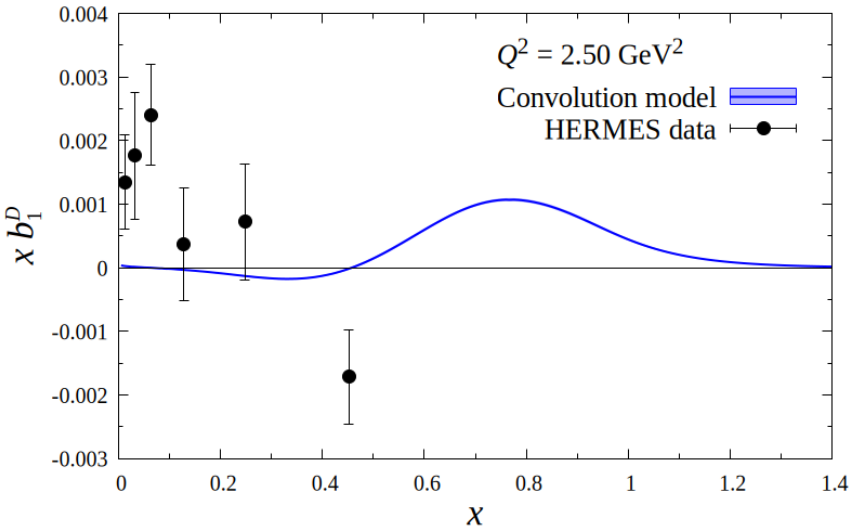
Tensor-polarization Puzzle

S. Kumano, KK, arXiv:2607.09237

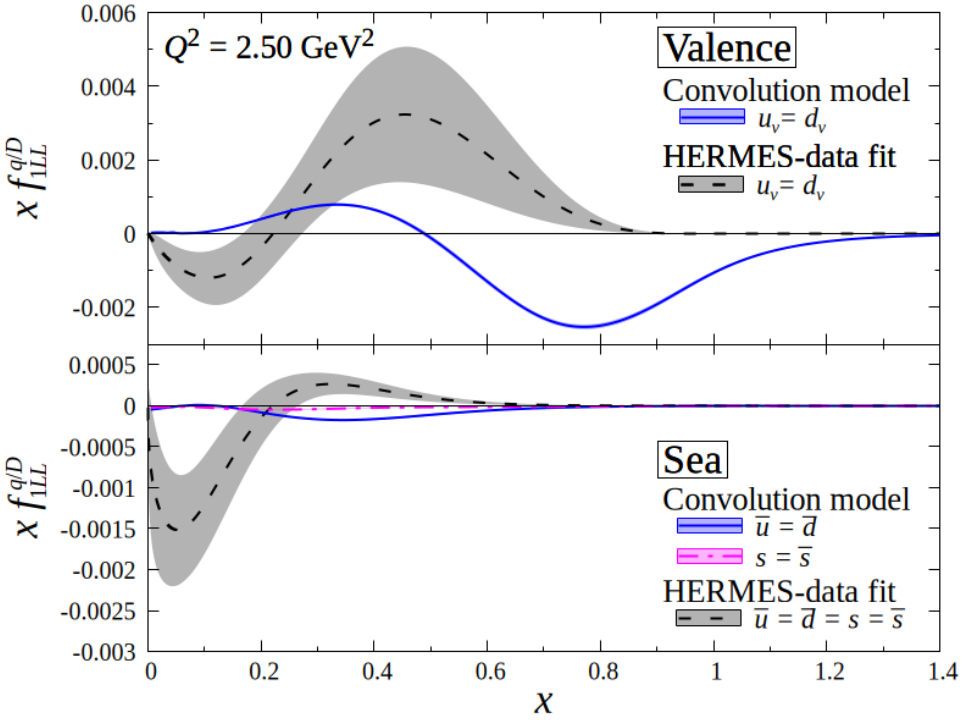
The convolution model fails to reproduce the HERMES measurement

■ Structure-function-level

$$b_1^D(x) = -\frac{3}{4}\sum_q e_q^2 \left[f_{1LL}^{q/D}(x) + f_{1LL}^{\bar{q}/D}(x) \right]$$



■ PDF-level (flavor-separated)



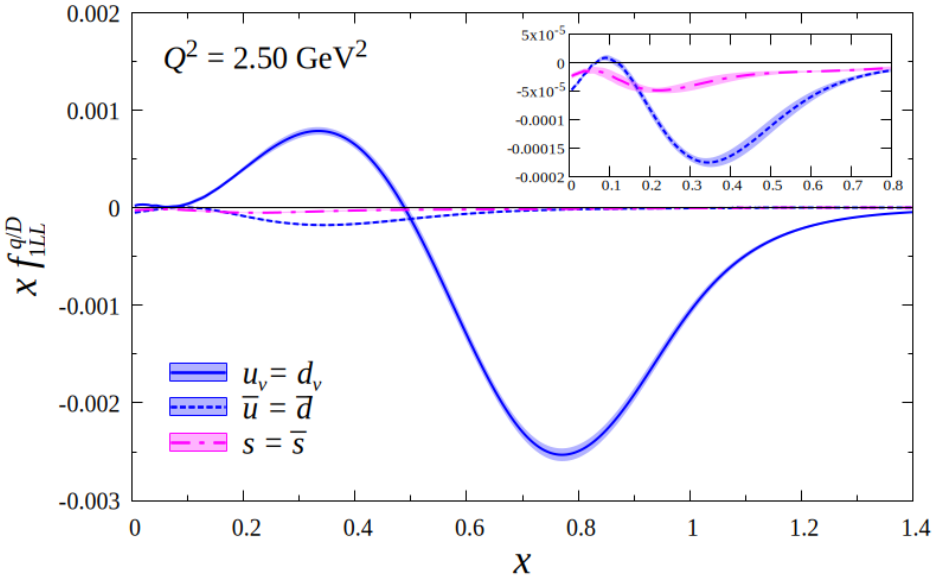
Valence: Significant at large x, opposite oscillatory behavior

Sea: Significant at small x in the HERMES-data fit, while very small in the convolution PDF

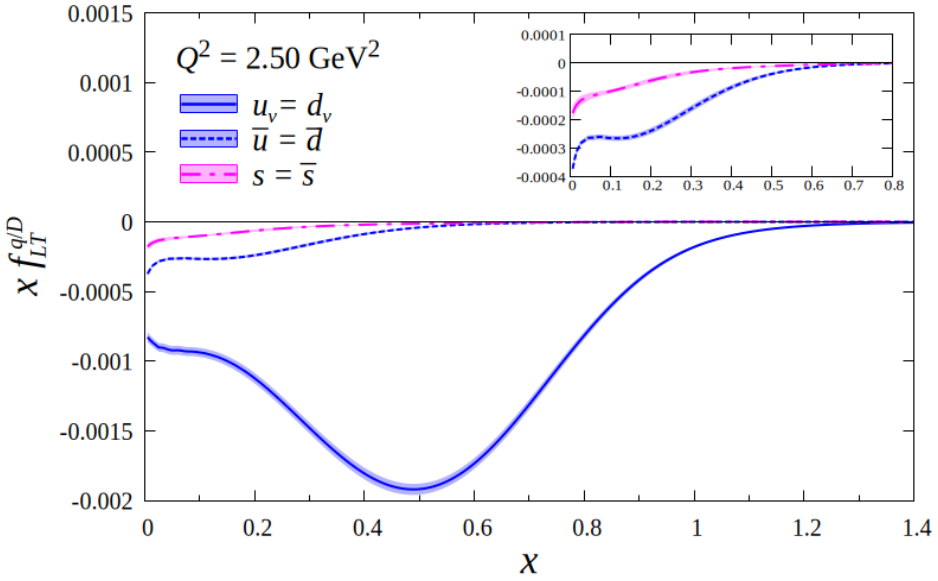
Twist-3 PDF f_{LT}

WW-like approximation: $f_{LT}(x) \cong f_{LT}^{\text{twist-2}'}(x) = \frac{3}{2}\int_x^2 \frac{dy}{y} f_{1LL}(y)$

■ Twist-2 PDF f_{1LL}



■ Twist-3 PDF f_{LT}



The twist-3 PDF f_{LT} is comparable in magnitude to the twist-2 PDF f_{1LL}

Potentially sizable twist-3 effects at $Q^2 \sim$ a few GeV^2 (e.g., at JLab)?

Twist-3 Effects on Structure Functions

SFs in terms of PDFs up to twist-3

J. Zhao et al., JHEP 12 (2025) 067

$$b_1 = -\frac{3}{4}\frac{1}{\gamma^2+1}\sum_q e_q^2 f_{1LL}^q \qquad \gamma = \frac{2Mx}{Q}$$

$$b_2 = -\frac{3}{2}\frac{x}{(\gamma^2+1)^2}\sum_q e_q^2 f_{1LL}^q + 8\frac{\gamma\, x}{(\gamma^2+1)^2}\frac{M}{Q}\sum_q e_q^2 f_{LT}^q$$

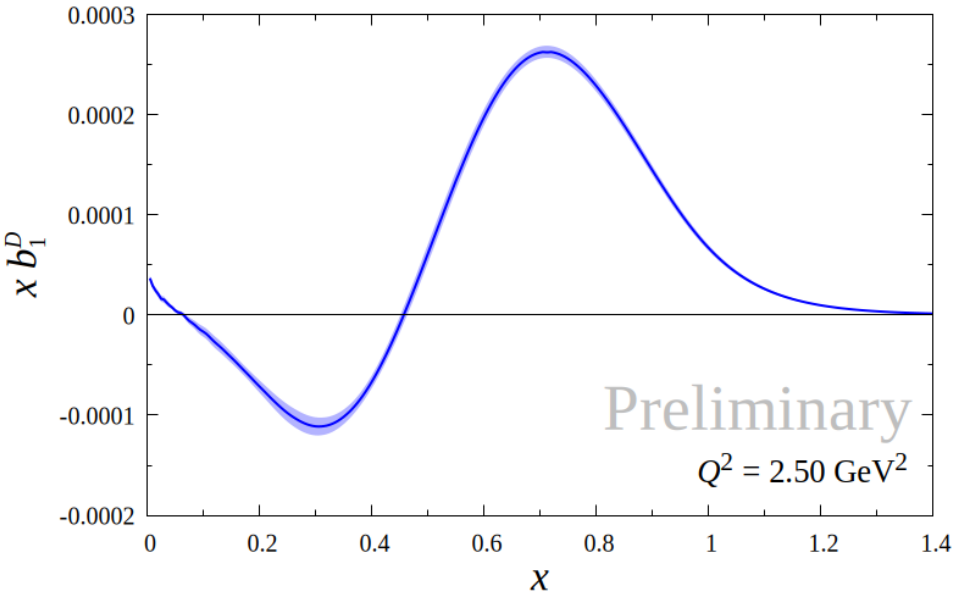
- **Callan-Gross-like relation** in the Bjorken limit

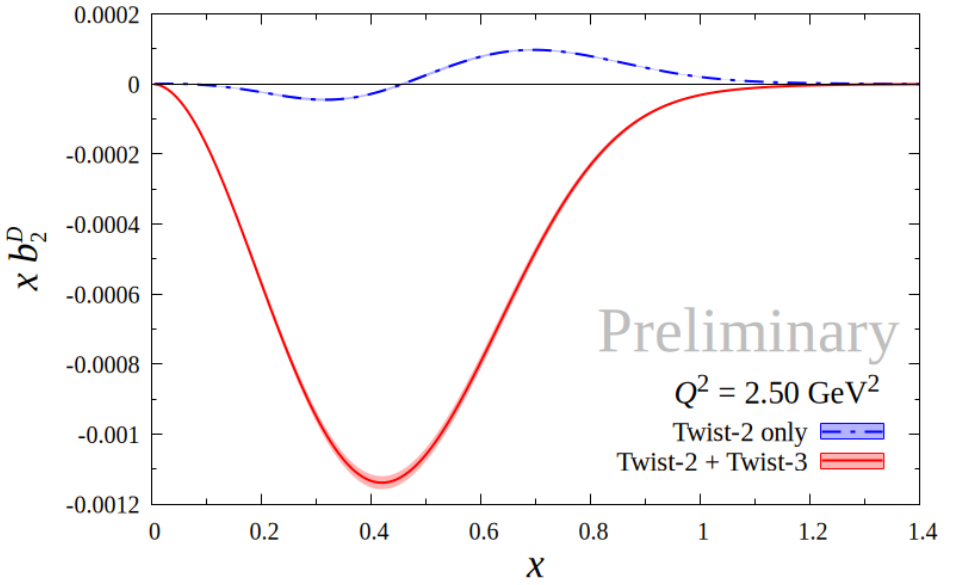
  $2xb_1(x) = b_2(x)$

  → b_2 becomes sizable (relative to b_1) at large x

- **Significant twist-3 contribution to b_2**

  Especially at large x, due to γ multiplying f_{LT}





Summary

Summary

Twist-2′ relations for the tensor-polarized twist-3 PDF and applications to the spin-1 deuteron

Twist-2′ relations

- **Wandzura-Wilczek-like relation:** $f_{LT}^{\text{twist}-2'}(x) = \frac{3}{2}\int_x^1 \frac{dy}{y} f_{1LL}(y)$

  The twist-3 PDF f_{LT} estimated from the twist-2 PDF f_{1LL}

- **Burkhardt-Cottingham-like sum rule:** $\int_0^1 dx\, f_{2LT}^{\text{twist}-2'}(x) = 0$

  A probe of genuine twist-3′ multiparton correlations

Numerical application to the deuteron at $Q^2 = 2.5\ \text{GeV}^2$

- Tensor-polarization puzzle at the PDF level
- Twist-3 PDF f_{LT} ≈ twist-2 PDF f_{1LL} → **Large effects on SF b_2**

Backup

Geometrical Interpretation

A. Bacchetta, P.J. Mulders, PRD 62 (2000) 114004

Spin operator along an arbitrary direction in the particle rest frame

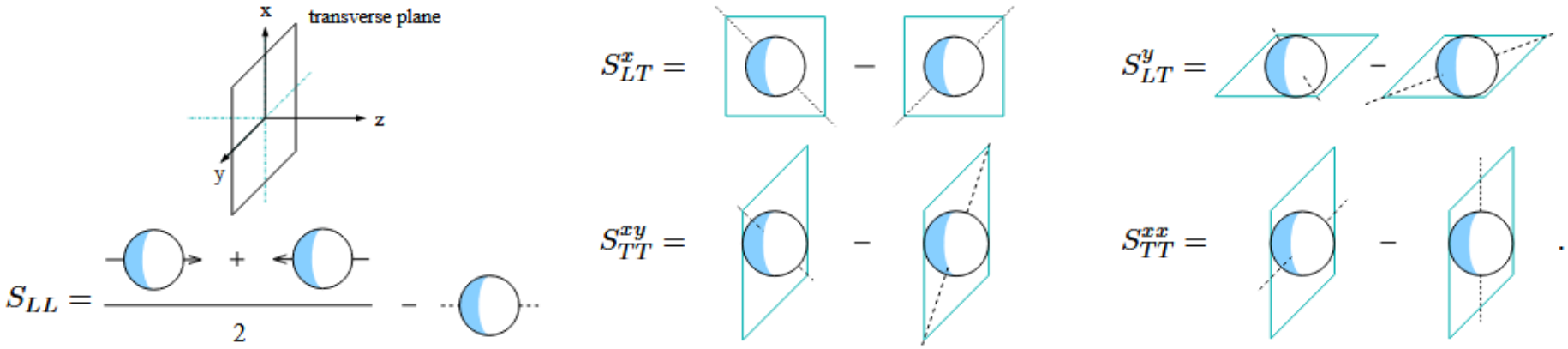
$$\boldsymbol{\Sigma}\cdot\hat{\boldsymbol{n}} = \Sigma_x \sin\theta\cos\phi + \Sigma_y \sin\theta\sin\phi + \Sigma_z\cos\theta, \quad \textbf{Eigenstate: } \left|m_{(\theta,\phi)}\right\rangle$$

Define $P\left(m_{(\theta,\phi)}\right) = \mathrm{Tr}\left\{\boldsymbol{\rho}\left|m_{(\theta,\phi)}\right\rangle\left\langle m_{(\theta,\phi)}\right|\right\}$,

$$S_{LL} = \frac{1}{2}P\left(1_{(0,0)}\right) + \frac{1}{2}P\left(-1_{(0,0)}\right) - P\left(0_{(0,0)}\right)$$

$$S_{LT}^{x} = P\left(0_{(-\pi/4,0)}\right) - P\left(0_{(\pi/4,0)}\right), \qquad S_{LT}^{y} = P\left(0_{(-\pi/4,\pi/2)}\right) - P\left(0_{(\pi/4,\pi/2)}\right)$$

$$S_{TT}^{xx} = P\left(0_{(\pi/2,-\pi/4)}\right) - P\left(0_{(\pi/2,\pi/4)}\right), \qquad S_{TT}^{xy} = P\left(0_{(\pi/2,\pi/2)}\right) - P\left(0_{(\pi/2,0)}\right)$$



$$-1 \le S_{LL} \le \frac{1}{2}, \quad -1 \le S_{LT}^{i} \le 1, \quad -1 \le S_{TT}^{ij} \le 1$$

HERMES-data Fit PDF f_{1LL}

S. Kumano, KK, PLB 875 (2026) 140348

Parametrization

S. Kumano, PRD 82 (2010) 017501

Assumptions: $q = u, d, s$, flavor-independent tensor polarization

$$f_{1LL}^{q/D}(x) = -\frac{2}{3}\delta_T w(x)\, q_v^D(x)$$

$$f_{1LL}^{\bar{q}/D}(x) = -\frac{2}{3}\alpha_{\bar{q}}\, \delta_T w(x)\, \bar{q}^D(x)$$

$$\delta_T w(x) = a x^b (1-x)^c (x_0 - x)$$

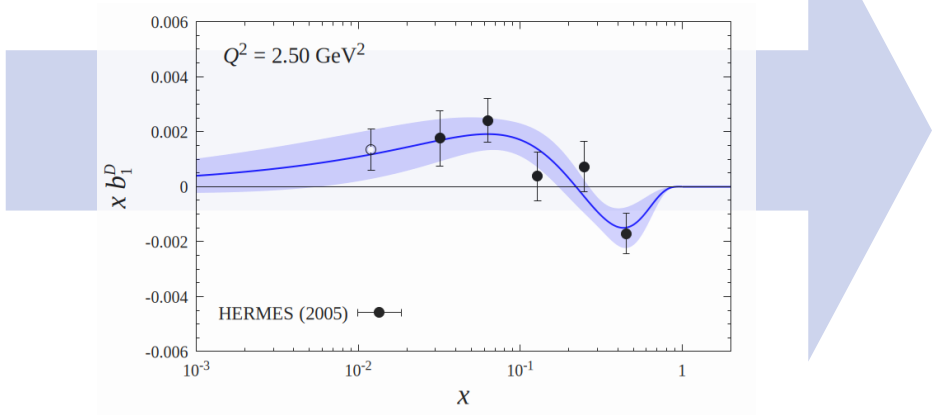
$q_v^D, \bar{q}^D$: Unpolarized PDF in nucleon

A.D. Martin et al., EPJ C 63 (2009) 189

Fit parameters

$a, b, c = 1\ (\text{fixed}), x_0, \alpha_{\bar{q}}$

$$b_1^D = -\frac{3}{4}\sum_q e_q^2 \left[f_{1LL}^{q/D} + f_{1LL}^{\bar{q}/D} \right]$$



$\chi^2/\text{d.o.f.} = 1.57$

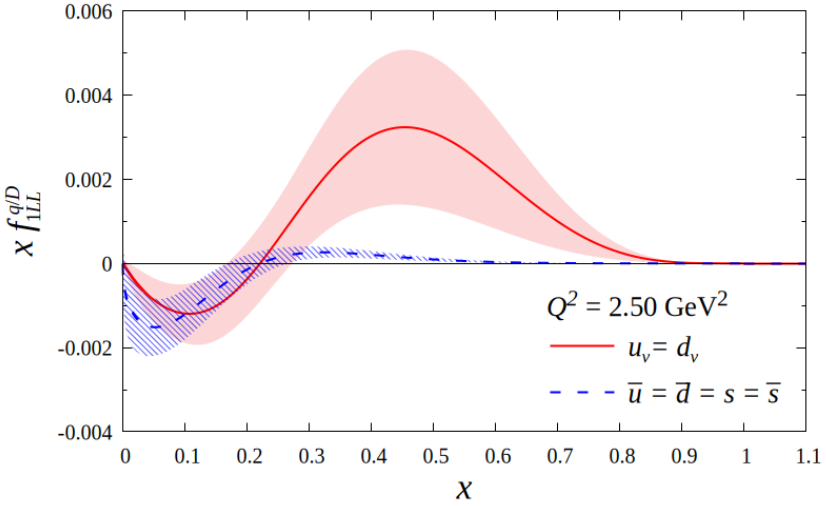
$a = 0.221 \pm 0.174$

$b = 0.648 \pm 0.342$

$c = 1.0$ (fixed)

$x_0 = 0.221$

$\alpha_{\bar{q}} = 3.20 \pm 2.75$

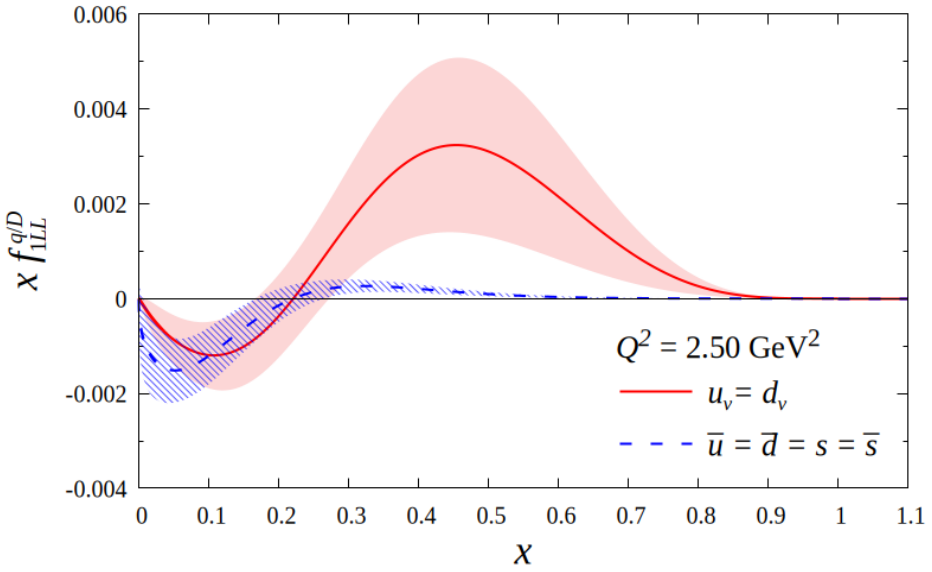


HERMES-data Fit PDF f_{LT}

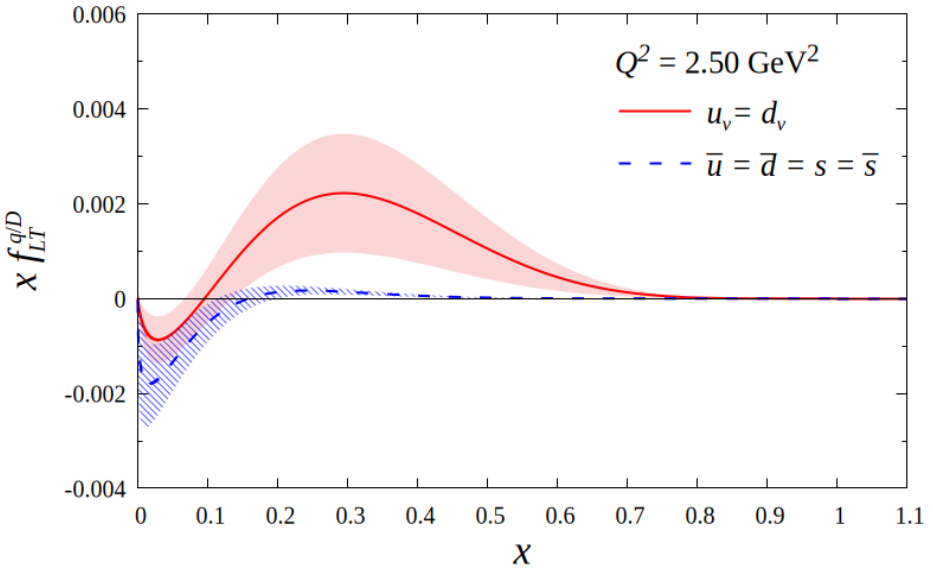
S. Kumano, KK, arXiv:2607.09237

WW-like approximation: $f_{LT}(x) \cong f_{LT}^{\text{twist-2}'}(x) = \frac{3}{2}\int_x^2 \frac{dy}{y} f_{1LL}(y)$

■ Twist-2 PDF f_{1LL}



■ Twist-3 PDF f_{LT}



The twist-3 PDF f_{LT} is comparable in magnitude to the twist-2 PDF f_{1LL}

Potentially sizable twist-3 effects at $Q^2 \sim$ a few GeV^2 (e.g., at JLab)?

Twist-3 Effects on SFs in HERMES-data Fit PDF

SFs in terms of PDFs up to twist-3

J. Zhao et al., JHEP 12 (2025) 067

$$b_1 = -\frac{3}{4}\,\frac{1}{\gamma^2+1}\,\sum_q e_q^2\, f_{1LL}^q \qquad \gamma = \frac{2Mx}{Q}$$

$$b_2 = -\frac{3}{2}\,\frac{x}{(\gamma^2+1)^2}\sum_q e_q^2\, f_{1LL}^q + 8\,\frac{\gamma x}{(\gamma^2+1)^2}\,\frac{M}{Q}\sum_q e_q^2\, f_{LT}^q$$

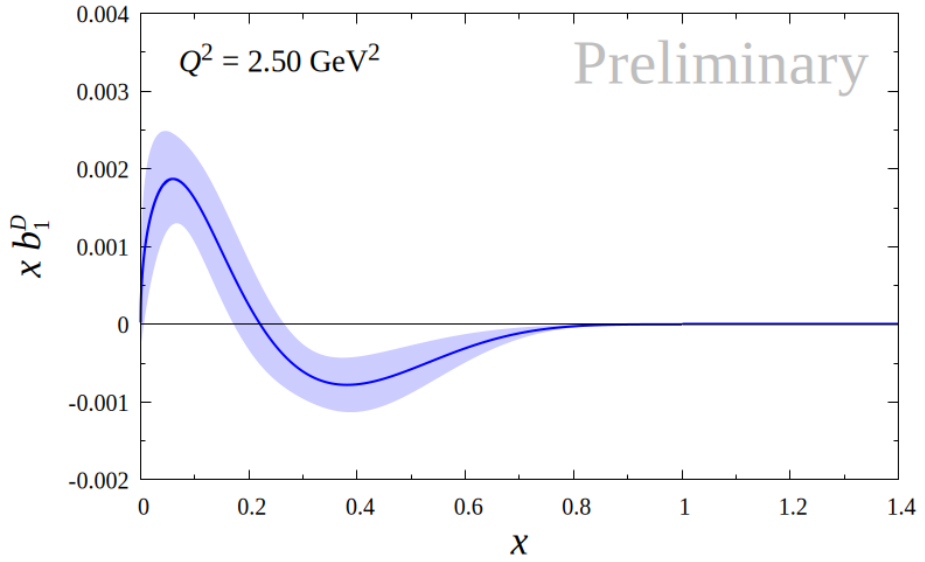
- **Callan-Gross-like relation** in the Bjorken limit

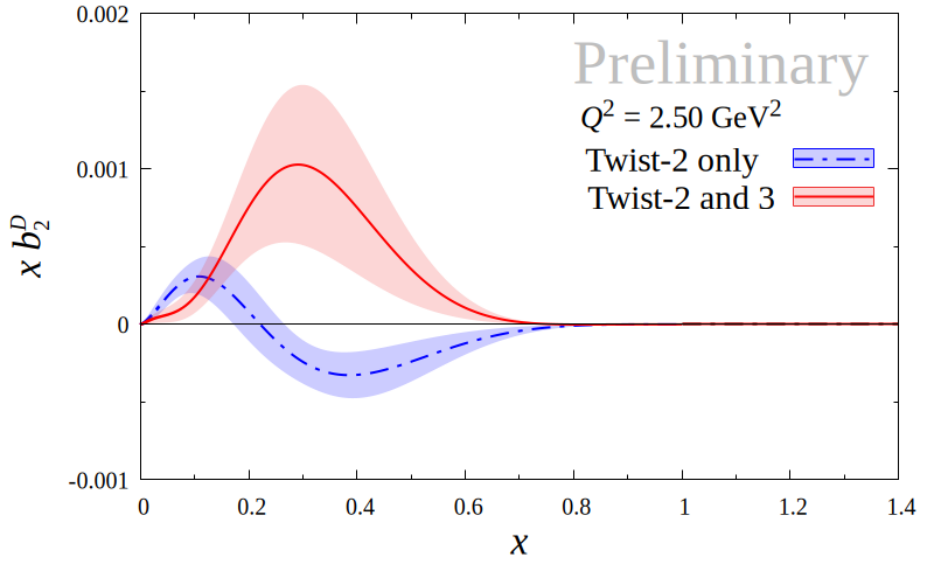
  $2xb_1(x) = b_2(x)$

  → b_2 becomes sizable (relative to b_1) at large x

- **Significant twist-3 contribution to b_2**

  Especially at large x, due to γ multiplying f_{LT}





Convolution: PDF dependence

S. Kumano, KK, arXiv:2607.09237

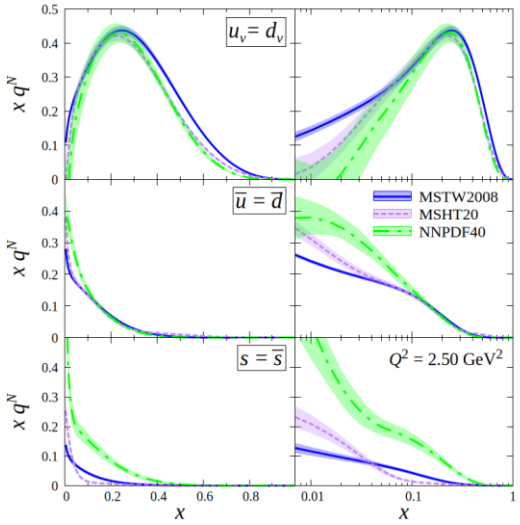
$$\begin{pmatrix} f_{1LL}^{q/D}(x,Q^2) \\ f_{1LL}^{\bar{q}/D}(x,Q^2) \end{pmatrix} = -\frac{2}{3}\int_x^2 \frac{dy}{y}\,\delta_T f(y)\,\frac{1+4M_N^2(x/y)^2/Q^2}{1+R(x/y,Q^2)} \begin{pmatrix} q_i^N(x/y,Q^2) \\ \bar{q}_i^N(x/y,Q^2) \end{pmatrix}$$

$$\delta_T f(y) = \int d^3p\, y\left[-\frac{3}{4\sqrt{2}\pi}\phi_0(p)\phi_2(p) + \frac{3}{16\pi}|\phi_2(p)|^2\right](3\cos^2\theta - 1)\delta\left(y - \frac{p\cdot q}{M_N\nu}\right)$$

MSTW2008 LO A. D. Martin et al, EPJ C 63 (2009) 189

MSHT20 LO S. Bailey et al., EPJ C 81 (2021) 341

NNPDF4.0 LO NNPDF Collaboration, EPJ C 82 (2022) 428



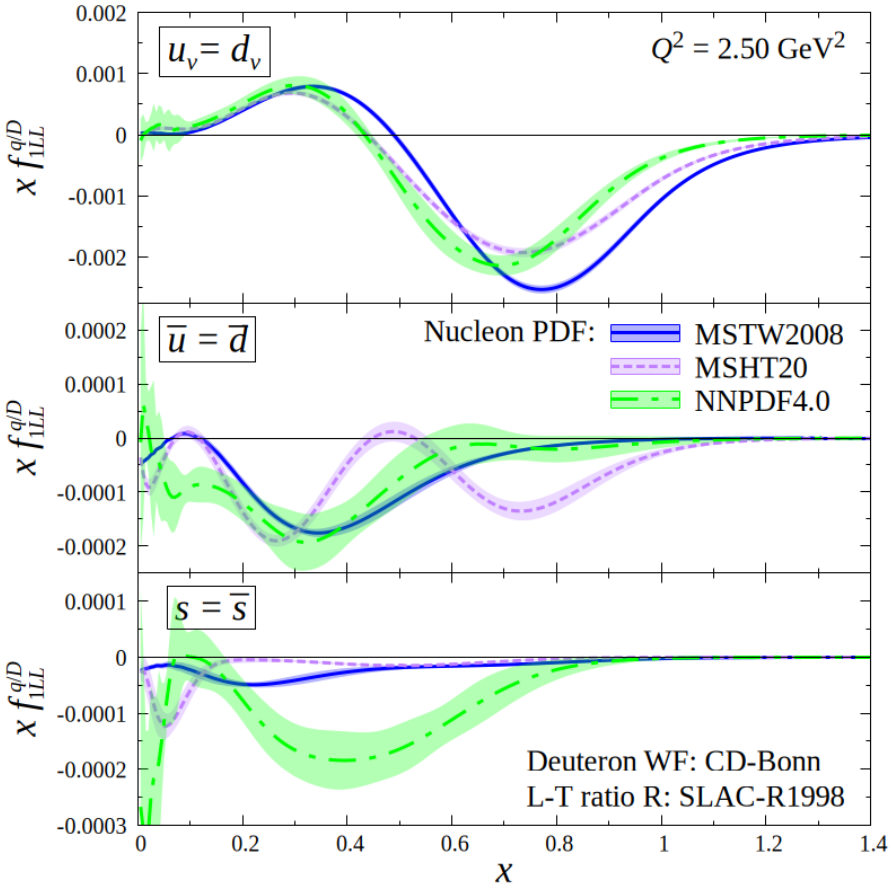
Valence

Almost the same results

Sea

Slight differences

Particularly for the strange with NNPDF



Convolution: WF dependence

S. Kumano, KK, arXiv:2607.09237

$$\begin{pmatrix} f_{1LL}^{q/D}(x,Q^2) \\ f_{1LL}^{\bar{q}/D}(x,Q^2) \end{pmatrix} = -\frac{2}{3}\int_x^2 \frac{dy}{y}\,\delta_T f(y)\,\frac{1+4M_N^2(x/y)^2/Q^2}{1+R(x/y,Q^2)} \begin{pmatrix} q_i^N(x/y,Q^2) \\ \bar{q}_i^N(x/y,Q^2) \end{pmatrix}$$

$$\delta_T f(y) = \int d^3p\, y\left[-\frac{3}{4\sqrt{2}\pi}\phi_0(p)\phi_2(p) + \frac{3}{16\pi}|\phi_2(p)|^2\right](3\cos^2\theta - 1)\delta\left(y - \frac{p\cdot q}{M_N\nu}\right)$$

CD-Bonn R. Machleidt, PRC 63 (2001) 024001

Relativistic, nonlocal, $\boldsymbol{P_D \sim 4.85\%}$

Nijm93 V.G.J. Stokes et al., PRC 49 (1994) 2950

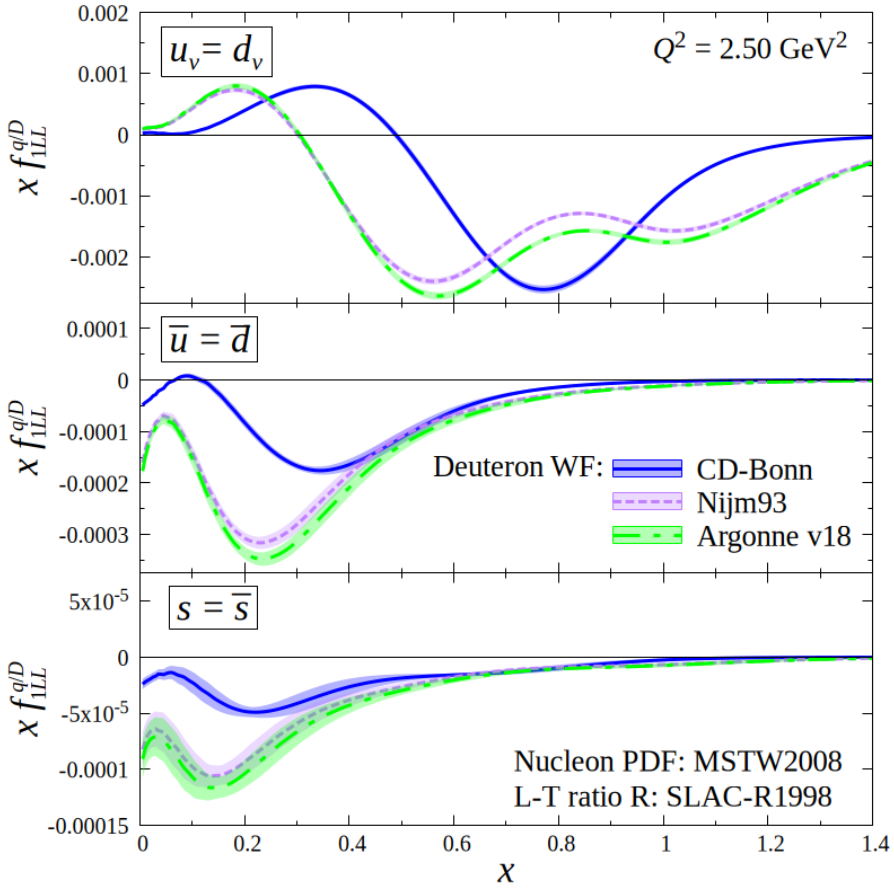
Non-relativistic, local, $\boldsymbol{P_D \sim 5.76\%}$

Argonne v18 R.B. Wiringa et al., PRC 51 (1995) 38

Non-relativistic, local, $\boldsymbol{P_D \sim 5.76\%}$

- Nijm93 and Argonne v18: **Nearly identical results**
- CD-Bonn: **Smaller than the other two**

  Due to the smaller D-wave probability P_D



Genuine Twist-3 PDF $f_{LT}^{(\mathrm{HT})}$

$$f_{LT}(x) = f_{LT}^{\mathrm{twist-2'}}(x) + f_{LT}^{(\mathrm{HT})}(x)$$

Twist-2′ WW-like part **Genuine twist-3′ part**

Multiparton distribution functions

S. Kumano, Q.-T. Song, JHEP 09 (2021) 141

G-type quark-gluon multiparton collinear correlation function

$$(\Phi_G^{\alpha})_{ij}(x_1,x_2) = \int \frac{d\xi_1^-}{2\pi}\frac{d\xi_2^-}{2\pi} e^{ix_1P^+\xi_1^-} e^{i(x_1-x_2)P^+\xi_2^-} \left\langle P,T\middle|\overline{\psi}_j(0)gG^{+\alpha}(\xi_2^-)\psi_i(\xi_1^-)\middle|P,T\right\rangle$$

$$\Phi_G^{\alpha}(x_1,x_2) = \frac{M}{2}\left[iS_{LT}^{\alpha}F_{G,LT}(x_1,x_2) - \epsilon_T^{\alpha\mu}S_{LT\,\mu}\gamma_5 G_{G,LT}(x_1,x_2) + iS_{LL}\gamma_T^{\alpha}H_{G,LL}^{\perp}(x_1,x_2) + iS_{TT}^{\alpha\mu}\gamma_{\mu}H_{G,TT}(x_1,x_2)\right]\bar{n}\cdot\gamma$$

$$f_{LT}^{(\mathrm{HT})}(x) = -\mathcal{P}\int_{-1}^{1} dy \frac{1}{x-y}\left[\frac{\partial}{\partial x}\left\{F_{G,LT}(x,y) + G_{G,LT}(x,y)\right\} + \frac{\partial}{\partial x}\left\{F_{G,LT}(y,x) + G_{G,LT}(y,x)\right\}\right]$$