Global Analysis of Nucleon TMD PDFs

The 13th Circum-Pan-Pacific Symposium on High Energy Spin Physics

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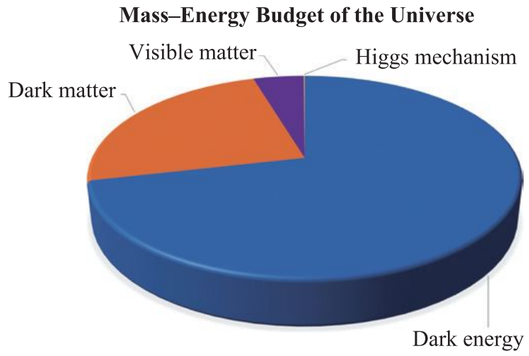
Transverse Nucleon Tomography Collaboration:
Hongxin Dong, Tianbo Liu, Bo-Qiang Ma, Peng Sun, Ke Yang, Chunhua Zeng, Yuxiang Zhao, Yiyu Zhou

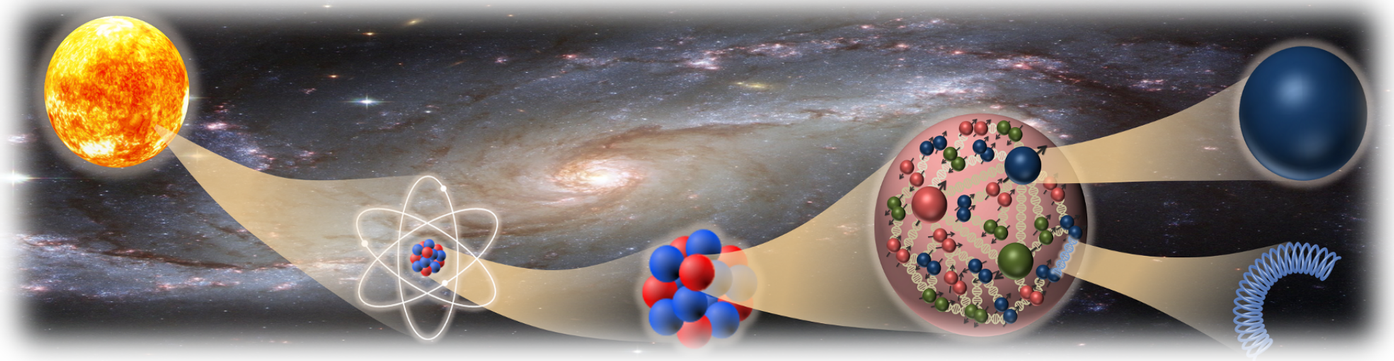
PacificSPIN

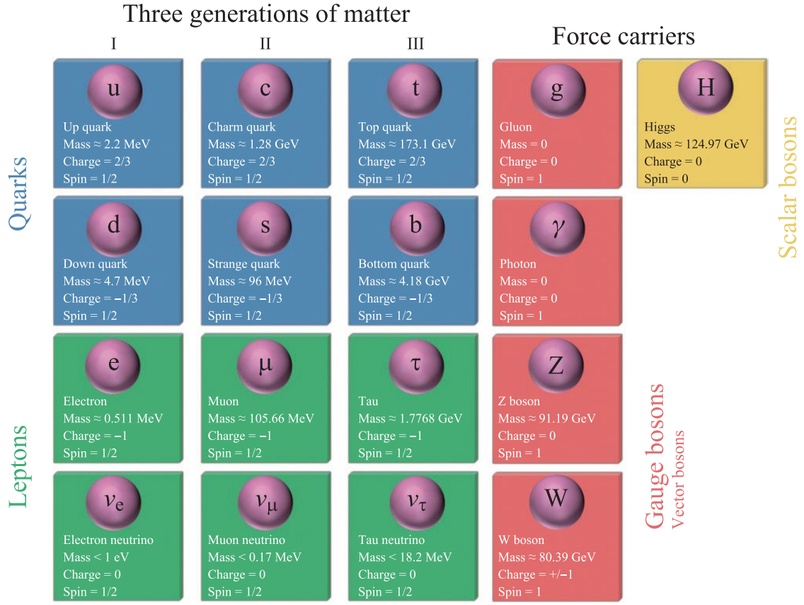
SCNT Southern Center for Nuclear-Science Theory

SHANDONG UNIVERSITY 山东大学

How much do we understand our world?



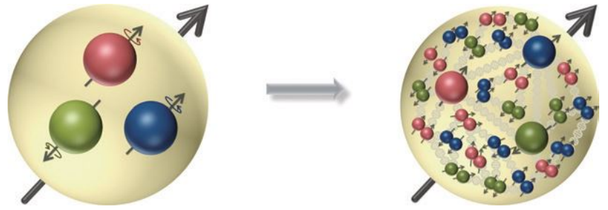




Mass



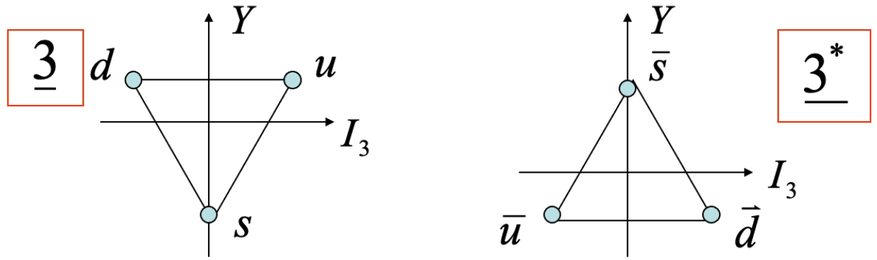
Spin



How do quarks and gluons make up a nucleon?
How can nucleon properties be explained at quarks and gluons degrees of freedom?

Proton Spin Structure in Naïve Quark Model

Quark model:



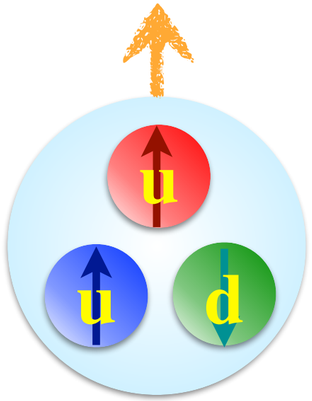
M. Gell-Mann, Phys. Lett. 8, 214 (1964);
G. Zweig, CERN Report No. TH-401 (1964).

ordinary baryons: $|qqq\rangle$, mesons: $|q\bar{q}\rangle$

Spin-flavor wave function of the proton:

$$\left|p_\uparrow\right\rangle = \frac{1}{\sqrt{18}}\Big[2\left|u_\uparrow d_\downarrow u_\uparrow\right\rangle + 2\left|u_\uparrow u_\uparrow d_\downarrow\right\rangle + 2\left|d_\downarrow u_\uparrow u_\uparrow\right\rangle - \left|u_\uparrow u_\downarrow d_\uparrow\right\rangle$$
$$- \left|u_\uparrow d_\uparrow u_\downarrow\right\rangle - \left|u_\downarrow d_\uparrow u_\uparrow\right\rangle - \left|d_\uparrow u_\downarrow u_\uparrow\right\rangle - \left|d_\uparrow u_\uparrow u_\downarrow\right\rangle - \left|u_\downarrow u_\uparrow d_\uparrow\right\rangle\Big].$$

$$\Delta u = u_\uparrow - u_\downarrow = \frac{4}{3} \qquad \Delta d = d_\uparrow - d_\downarrow = -\frac{1}{3}$$



Naïve quark model: the sum of quark spins gives the proton spin!

Nucleon Spin Structure

Proton spin puzzle

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s \sim 0.3$$

Quark spin only contributes a small fraction to the nucleon spin.

J. Ashman *et al.*, PLB 206, 364 (1988); NP B328, 1 (1989).

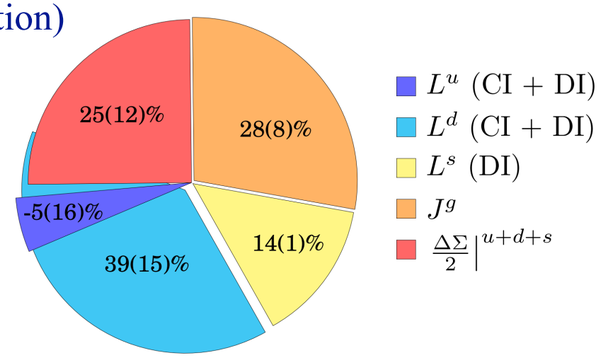
Spin decomposition

$$J = \frac{1}{2}\Delta\Sigma + \Delta G + L_q + L_g$$

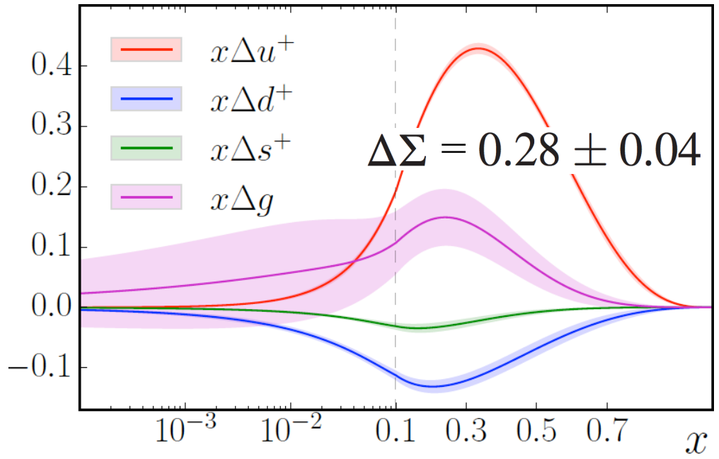
$$= \frac{1}{2}\Delta\Sigma + L_{q,\text{kin}} + J_g$$

R.L. Jaffe, A. Manohar, NPB 337, 509 (1990).
X. Ji, PRL 78, 610 (1997).

Lattice QCD
(kinetic decomposition)



χQCD Collaboration, PRD 91, 014505 (2015).



JAM Collaboration, PRD 93, 074005 (2016); PRL 119, 132001 (2017).

Gluon spin from LQCD:

$$S_g = 0.251(47)(16)$$

50% of total proton spin

Y.-B. Yang *et al.* (χQCD Collaboration), PRL 118, 102001 (2017).

Wigner Rotation Effect

Melosh-Wigner rotation

quark spin in a rest proton $\neq$ quark spin in a moving proton

If applying a kinetic boost, one may relate the spin states in *proton rest frame* to the spin states in *infinite momentum frame*

$$\chi_T^{\uparrow} = w\left[\left(k^+ + m\right)\chi_F^{\uparrow} - \left(k^1 + ik^2\right)\chi_F^{\downarrow}\right] \qquad k^+ = k^0 + k^3$$

$$\chi_T^{\downarrow} = w\left[\left(k^+ + m\right)\chi_F^{\downarrow} + \left(k^1 - ik^2\right)\chi_F^{\uparrow}\right] \qquad w = \left[2k^+\left(k^0 + m\right)\right]^{-1/2}$$

E.P. Wigner, Ann. Math 40 (1939) 149; H.J. Melosh, Phys. Rev. D 9 (1974) 1095.

The effect on quark polarization

$$\Delta q = \int \mathrm{d}^3\mathbf{k}\mathscr{M}\left[q^{\uparrow}(k) - q^{\downarrow}(k)\right] \qquad \mathscr{M} = \frac{(k^+ + m)^2 - k_T^2}{2k^+(k^0 + m)}$$

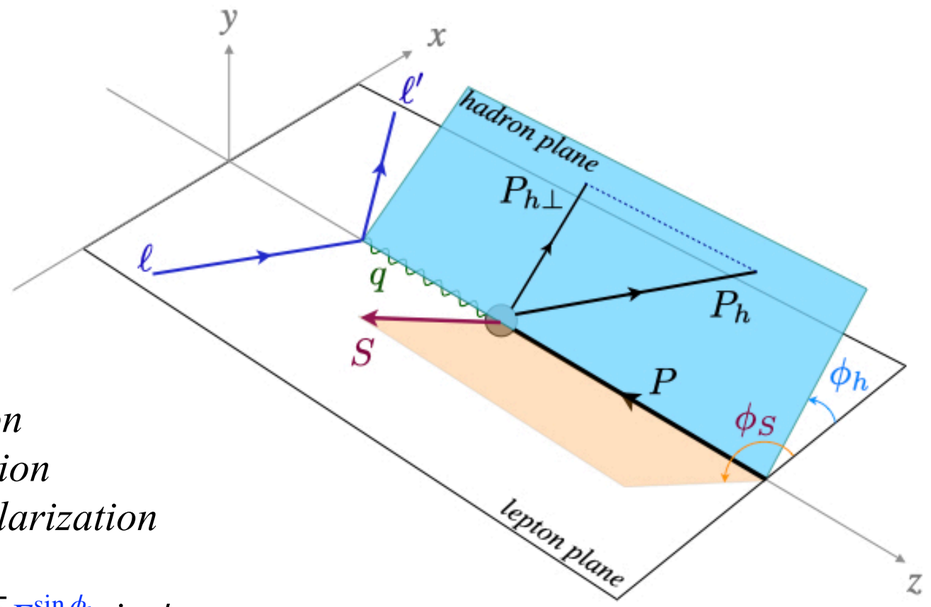
B.-Q. Ma, J. Phys. G 17 (1991) L53-L58; B.-Q. Ma, Q.-R. Zhang, Z. Phys. C 58 (1993) 479.

It predicts decreasing polarization with k_T, which should be tested by data.
This interpretation is based on a kinetic boost, but a complete boost including QCD dynamics is challenging.

Semi-inclusive Deep Inelastic Scattering

SIDIS process

$$l + N \to l' + h + X$$



Differential cross section
in terms of 18 structure functions

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}P_{hT}^2\,\mathrm{d}\phi_h\,\mathrm{d}\phi_S} = \frac{\alpha^2}{x_B y Q^2}\frac{y^2}{2(1-\epsilon)}\left(1+\frac{\gamma^2}{2x_B}\right)$$

$$\times\Big\{F_{UU,T}+\epsilon F_{UU,L}+\sqrt{2\epsilon(1+\epsilon)}F_{UU}^{\cos\phi_h}\cos\phi_h+\epsilon F_{UU}^{\cos 2\phi_h}\cos 2\phi_h+\lambda_e\sqrt{2\epsilon(1-\epsilon)}F_{LU}^{\sin\phi_h}\sin\phi_h$$

$$+S_L\left[\sqrt{2\epsilon(1+\epsilon)}F_{UL}^{\sin\phi_h}\sin\phi_h+\epsilon F_{UL}^{\sin 2\phi_h}\sin 2\phi_h\right]+\lambda_e S_L\left[\sqrt{1-\epsilon^2}F_{LL}+\sqrt{2\epsilon(1-\epsilon)}F_{LL}^{\cos\phi_h}\cos\phi_h\right]$$

$$+S_T\left[\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\epsilon F_{UT,L}^{\sin(\phi_h-\phi_S)}\right)\sin\left(\phi_h-\phi_S\right)+\epsilon F_{UT}^{\sin(\phi_h+\phi_S)}\sin\left(\phi_h+\phi_S\right)\right.$$

$$\left.+\epsilon F_{UT}^{\sin(3\phi_h-\phi_S)}\sin\left(3\phi_h-\phi_S\right)\quad+\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin\phi_S}\sin\phi_S+\sqrt{2\epsilon(1+\epsilon)}F_{UT}^{\sin(2\phi_h-\phi_S)}\sin\left(2\phi_h-\phi_S\right)\right]$$

$$+\lambda_e S_T\left[\sqrt{1-\epsilon^2}F_{LT}^{\cos(\phi_h-\phi_S)}\cos\left(\phi_h-\phi_S\right)\right.$$

$$\left.+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos\phi_S}\cos\phi_S+\sqrt{2\epsilon(1-\epsilon)}F_{LT}^{\cos(2\phi_h-\phi_S)}\cos\left(2\phi_h-\phi_S\right)\right]\Big\}$$

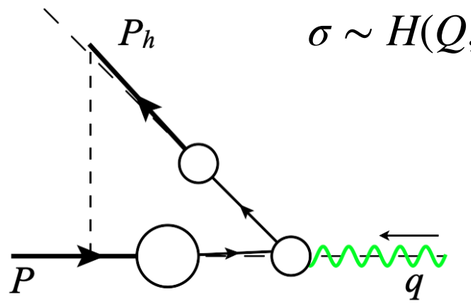
$$F_{AB,C}(x_B, z, P_{hT}^2, Q^2)$$

A: lepton polarization
B: nucleon polarization
C: virtual photon polarization

$$x_B = \frac{Q^2}{2P\cdot q}$$
$$y = \frac{P\cdot q}{P\cdot l}$$
$$z = \frac{P\cdot P_h}{P\cdot q}$$
$$\gamma = \frac{2x_B M}{Q}$$

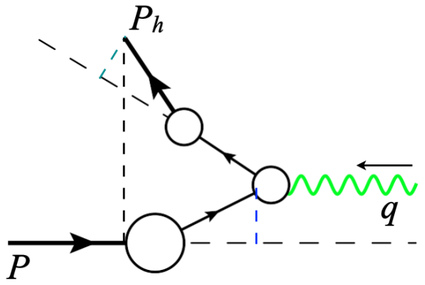
SIDIS Kinematic Regions

Sketch of kinematic regions of the produced hadron



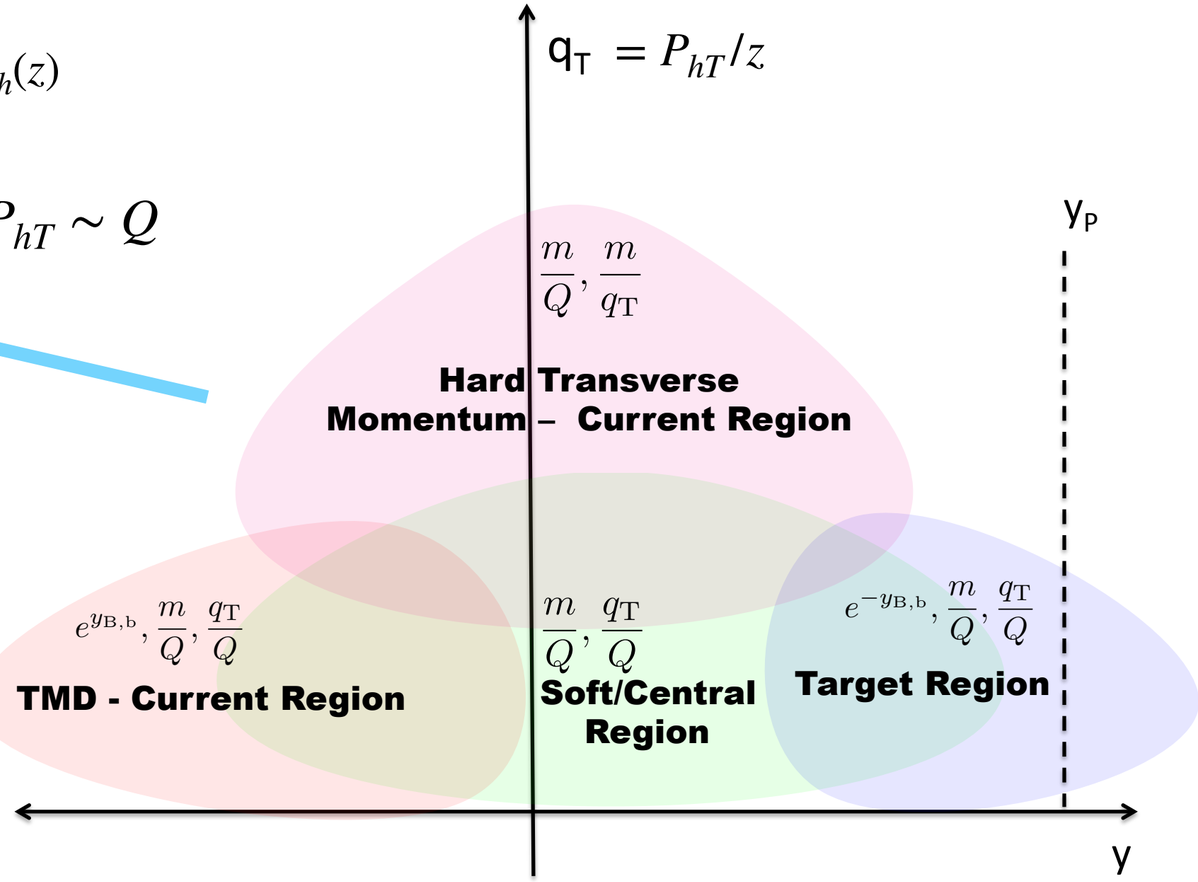
$$\sigma \sim H(Q, P_{hT}) \otimes f_{i/P}(x) \otimes D_{j\to h}(z)$$

$$P_{hT} \sim Q$$



$$P_{hT} \ll Q$$

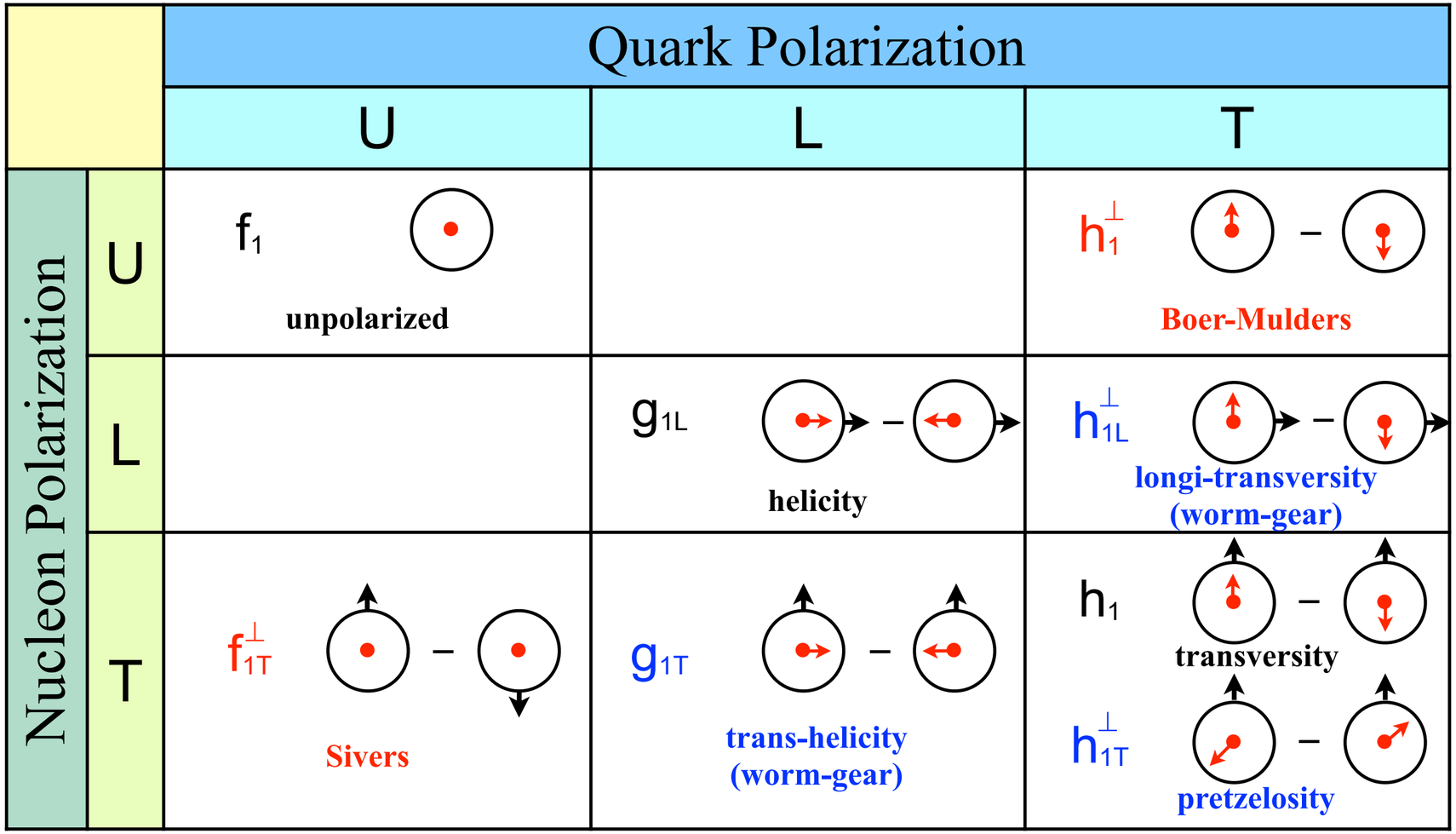
$$\sigma \sim H(Q) \otimes f_{i/P}(x, k_T) \otimes D_{j\to h}(z, p_T)$$



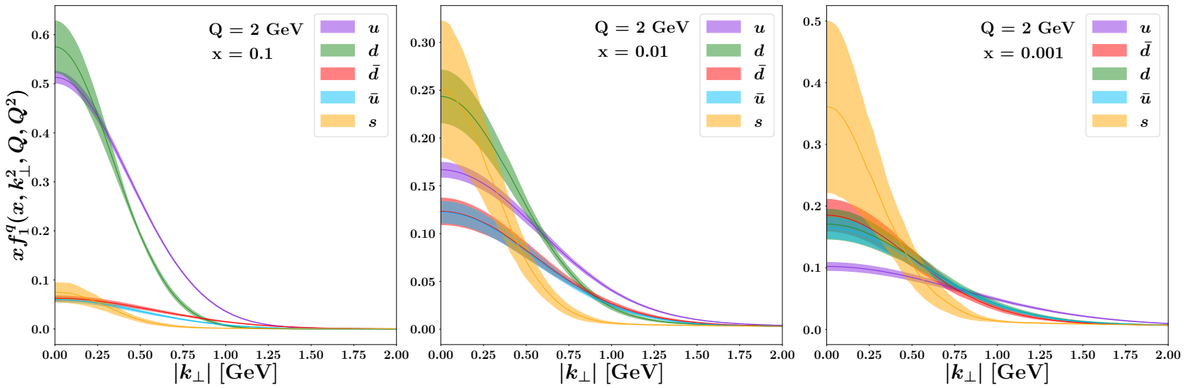
P_{hT} is defined in the photon-hadron frame

[Figure from JHEP10(2019)122]

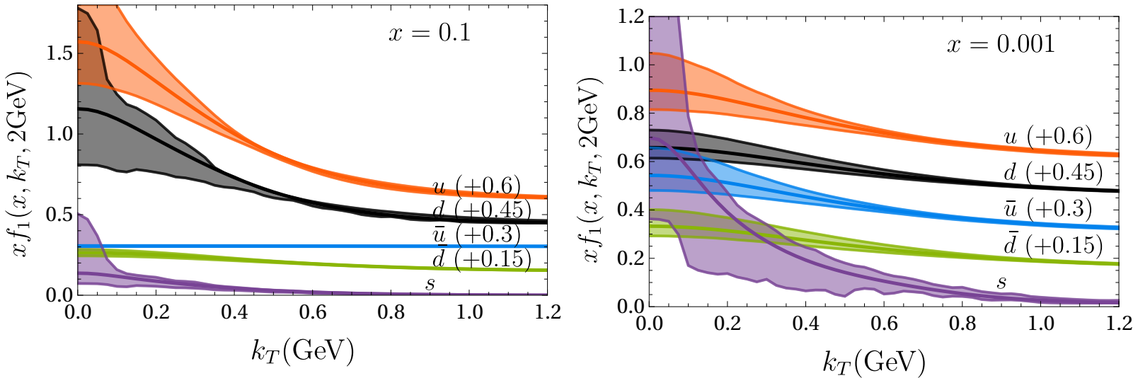
Leading Twist TMD PDFs



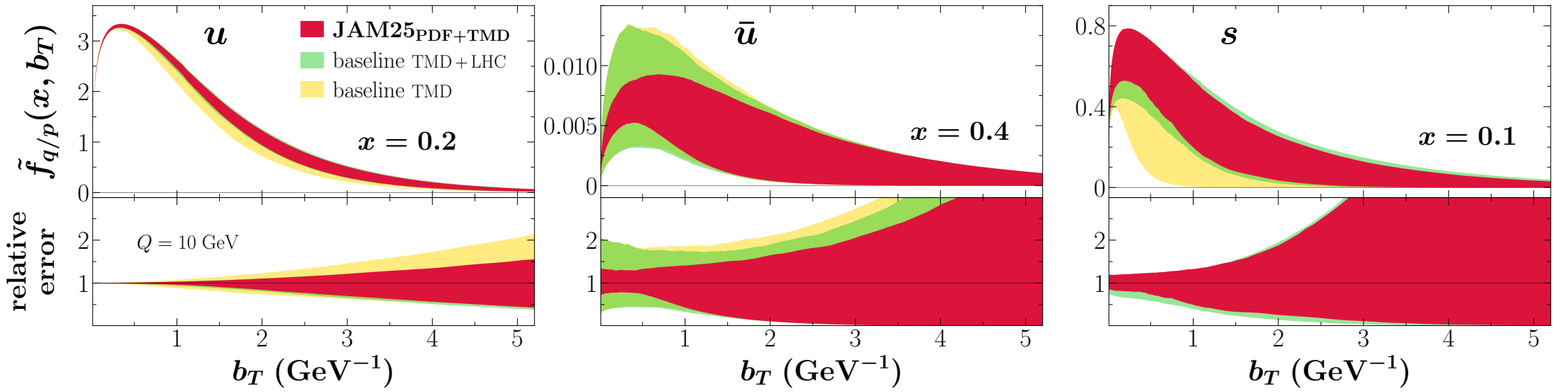
Unpolarized TMD PDFs



A. Bacchetta *et al.* (MAP Collaboration), JHEP 08 (2024) 232.



V. Moos, I. Scimemi, A. Vladimirov, P. Zurita, JHEP 11 (2025) 134.



P.C. Barry *et al.* (JAM Collaboration), PRL 137 (2026) 171902.

Longitudinal Double Spin Asymmetry

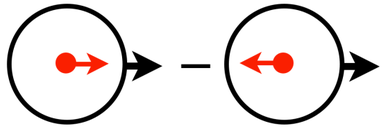
Longitudinal DSA in SIDIS

$$A_{LL} \equiv \frac{\sigma_{++} - \sigma_{+-} + \sigma_{--} - \sigma_{-+}}{\sigma_{++} + \sigma_{+-} + \sigma_{--} + \sigma_{-+}} = \frac{\sqrt{1-\varepsilon^2} F_{LL}\left(x, z, P_{hT}^2, Q^2\right)}{F_{UU}\left(x, z, P_{hT}^2, Q^2\right)}$$

In TMD region:

$$F_{LL}\left(x, z, P_{hT}^2, Q^2\right) \sim g_{1L}(x, k_T^2) \otimes D_1(z, p_T^2)$$

$$F_{UU}\left(x, z, P_{hT}^2, Q^2\right) \sim f_1(x, k_T^2) \otimes D_1(z, p_T^2)$$

g_{1L} 

(both collinear and TMD)

Several global analyses of collinear helicity
but TMD helicity was missing for a long time!

P_{hT} dependent DSA measurements

JLab CLAS: proton (NH_3) target — CLAS Collaboration, Phys. Lett. B 782 (2018) 662.

COMPASS: deuteron (D_2) targets — COMPASS Collaboration, Eur. Phys. J. C 78 (2018) 952.

HERMES: proton (H_2) and deuteron (D_2) targets — HERMES Collaboration, Phys. Rev. D 99 (2019) 112001.

Experimental Data Sets

SIDIS Longitudinal DSA data:

Collaboration	Lepton beam	Target	Detected hadron	Data points
HERMES	27.6 GeV $e^{\pm}$	H_2	π^+	80
			π^-	80
		D_2	π^+	80
			π^-	80
			K^+	79
			K^-	78
CLAS	6 GeV e^-		π^0	21

Target and hadron mass corrections:

$$x_S \equiv -\frac{q^+}{P^+} = \frac{2x}{\gamma^2}\left[\sqrt{1+\gamma^2\left(\frac{q_T^2}{Q^2}\right)}-1\right] \qquad z_S \equiv \frac{P_h^-}{q^-} = z\frac{x_S}{x}\frac{1+\sqrt{1-\varsigma^2}}{2\left(1-q_T^2/Q^2\right)} \qquad \varsigma = \frac{\gamma M_h}{zQ}$$

Low transverse momentum selection: $\delta = \frac{q_T}{Q} < 0.5$

TMD Evolution

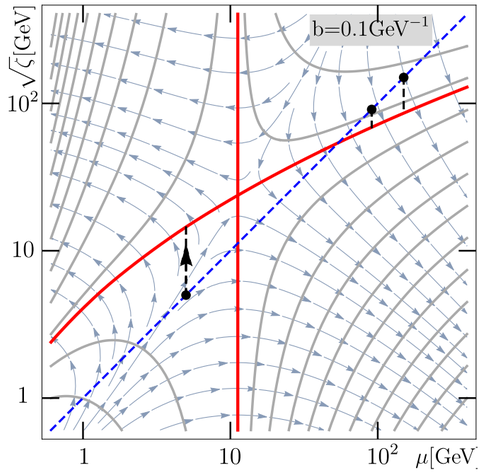
Evolution equations

$$\mu^2\frac{dF(x,b;\mu^2,\zeta)}{d\mu^2}=\frac{\gamma_F(\mu,\zeta)}{2}F(x,b;\mu^2,\zeta) \qquad -\zeta\frac{d\gamma_F(\mu,\zeta)}{d\zeta}=\mu\frac{d\mathscr{D}(\mu,b)}{d\mu}=\Gamma_{\rm cusp}(\mu)$$

$$\zeta\frac{dF(x,b;\mu^2,\zeta)}{d\zeta}=-\mathscr{D}(\mu,b)F(x,b;\mu^2,\zeta) \qquad \gamma_F(\mu,\zeta)=\Gamma_{\rm cusp}(\mu)\ \ln\frac{\mu^2}{\zeta}-\gamma_V(\mu)$$

$$F\left(x,b;\mu_f,\zeta_f\right)=\exp\left[\int_P\left(\gamma_F(\mu,\zeta)\frac{d\mu}{\mu}-\mathscr{D}(\mu,b)\frac{d\zeta}{\zeta}\right)\right]F\left(x,b;\mu_i,\zeta_i\right)$$

ζ-prescription



I. Scimemi, A. Vladmirov, JHEP 06 (2020) 137.

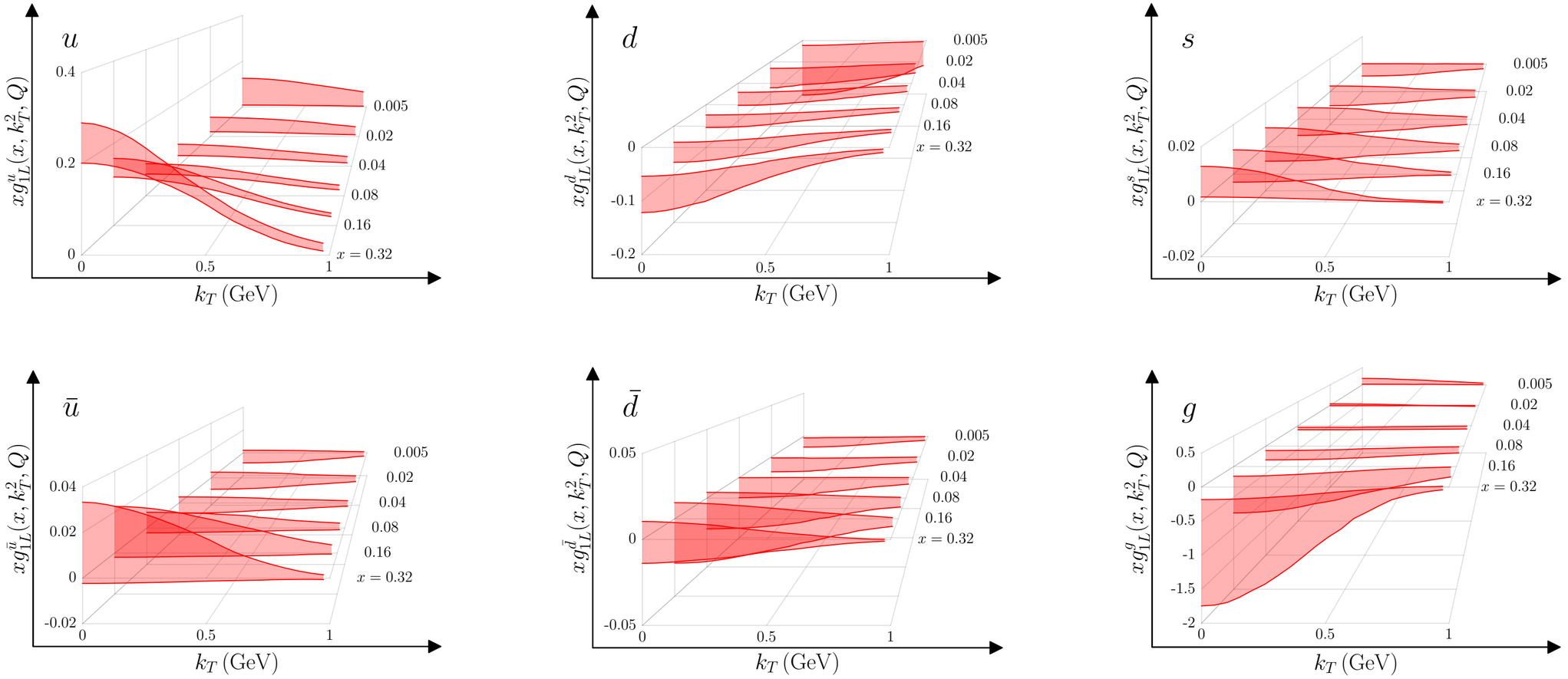
equipotential lines: $\dfrac{d\ln\zeta_\mu(\mu,b)}{d\ln\mu^2}=\dfrac{\gamma_F\left(\mu,\zeta_\mu(\mu,b)\right)}{2\mathscr{D}(\mu,b)}$

$$\mathscr{D}\left(\mu_0,b\right)=0,\quad \gamma_F\left(\mu_0,\zeta_\mu\left(\mu_0,b\right)\right)=0$$

$$F\left(x,b;Q,Q^2\right)=\left(\frac{Q^2}{\zeta_Q(b)}\right)^{-\mathscr{D}(Q,b)}F(x,b),\quad \mu_f^2=\zeta_f=Q^2$$

Extracted TMD Helicity Distributions

NNLO+NNLL



K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902;

updated analysis: K. Yang, TL, B.-Q. Ma, P. Sun, Y. Zhao, Y. Zhou, arXiv:2610.31290.

Transverse Momentum Dependent Polarization

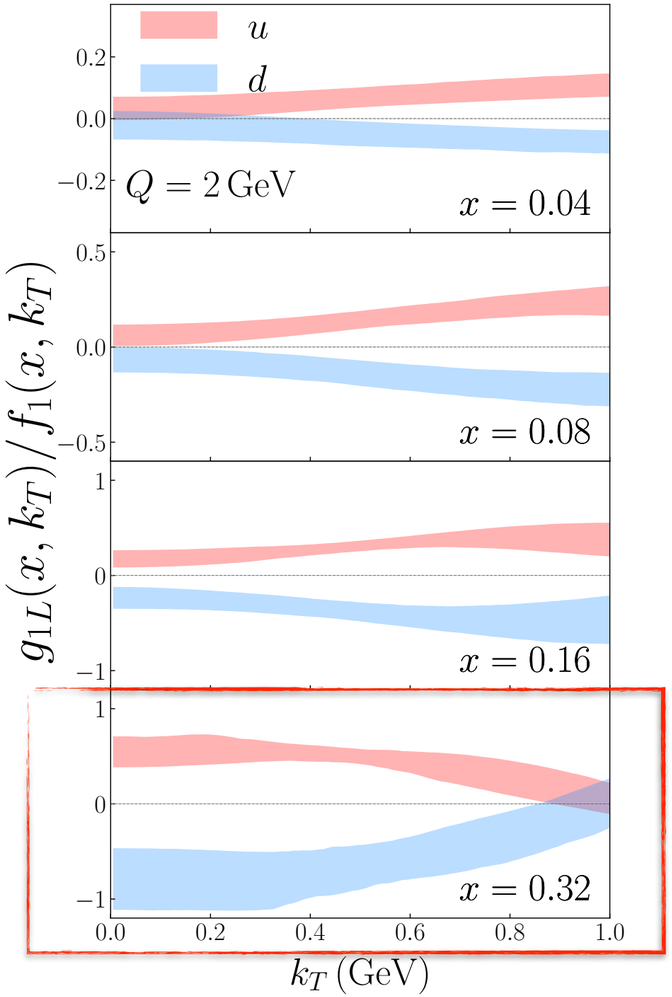
$g_{1L}(x, k_T^2)$ gives the absolute number density difference between spin-parallel and spin-antiparallel quarks.

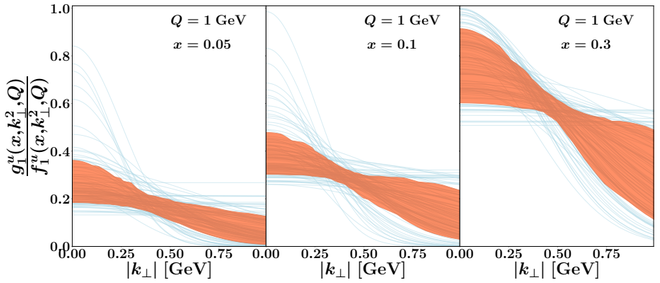
The ratio $g_{1L}(x, k_T^2)/f_1(x, k_T^2)$ measures the polarization rate of quarks.

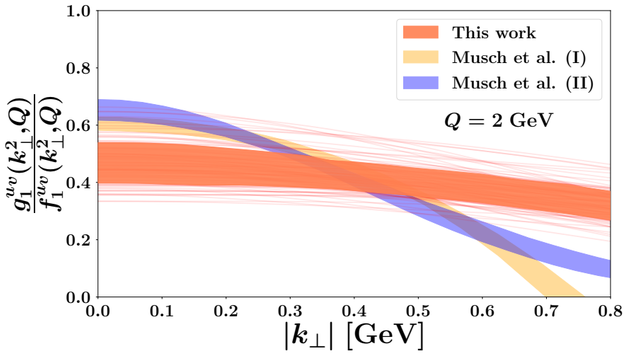
- At large x, where valence components dominate, the polarization decreases with increasing k_T
  Qualitatively consistent with kinetic Wigner rotation effects
- At low x, where the valence component is no longer adequate, distributions are highly driven by complex QCD dynamics

K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. Lett. 134 (2025) 121902; arXiv:2409.08110.

updated analysis: K. Yang, TL, B.-Q. Ma, P. Sun, Y. Zhao, Y. Zhou, arXiv:2610.31290.

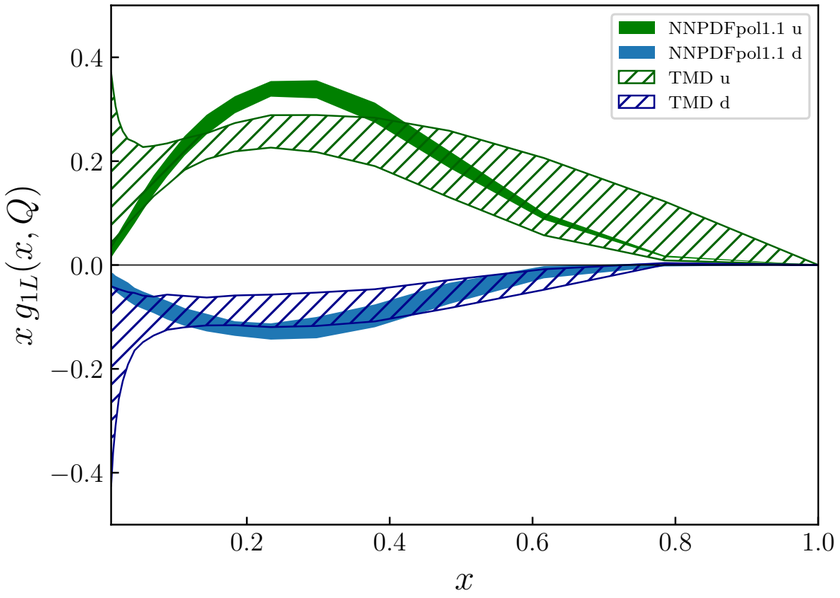


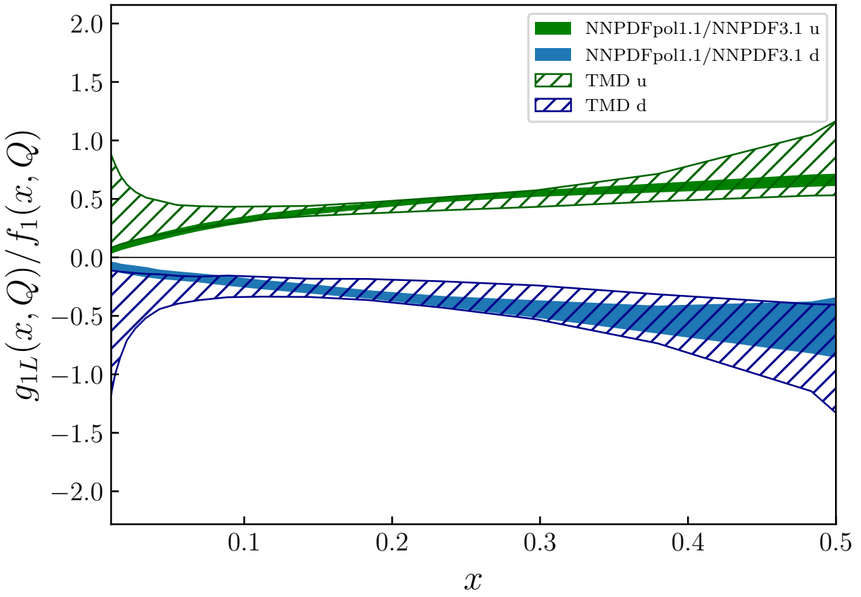




A. Bacchetta *et al.* (MAP Collaboration), Phys. Rev. Lett. 134 (2019) 121901; arXiv:2409.18078.

TMD vs. Collinear Helicity Distributions

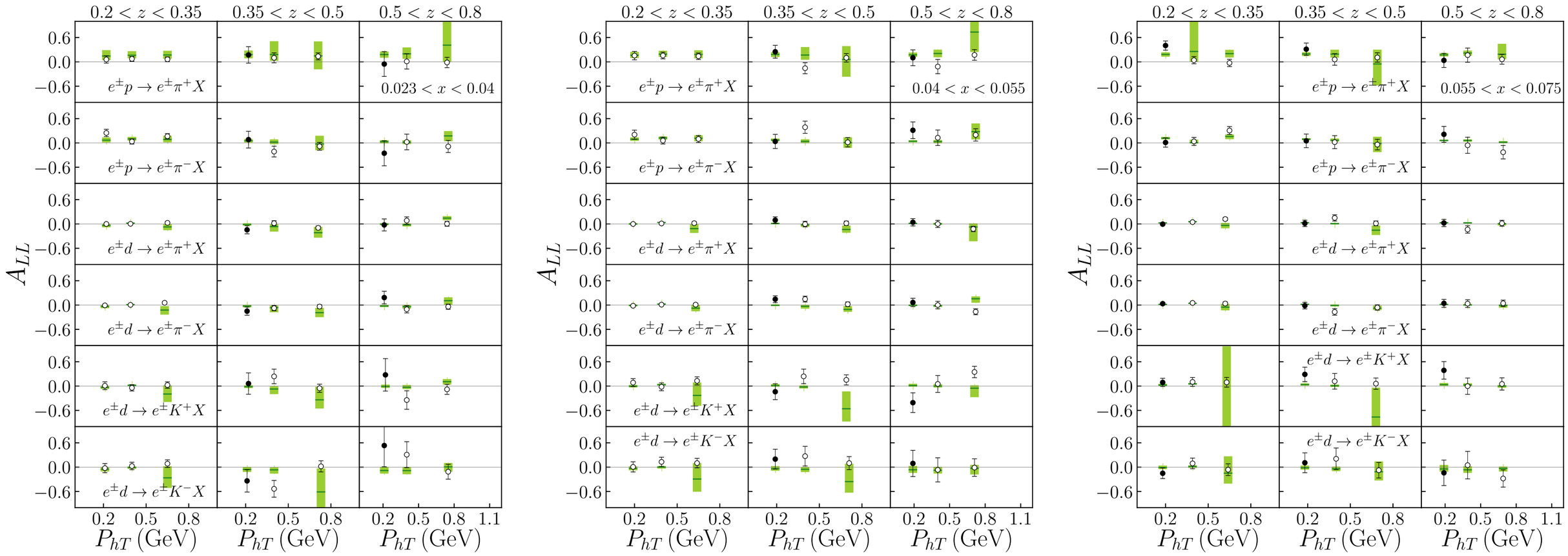




Transverse momentum integrated distributions: $g_{1L}(x,Q) = \pi \int_0^{Q^2} \mathrm{d}k_T^2\; g_{1L}\left(x, k_T^2, Q, Q^2\right)$

K. Yang, TL, B.-Q. Ma, P. Sun, Y. Zhao, Y. Zhou, arXiv:2610.31290.

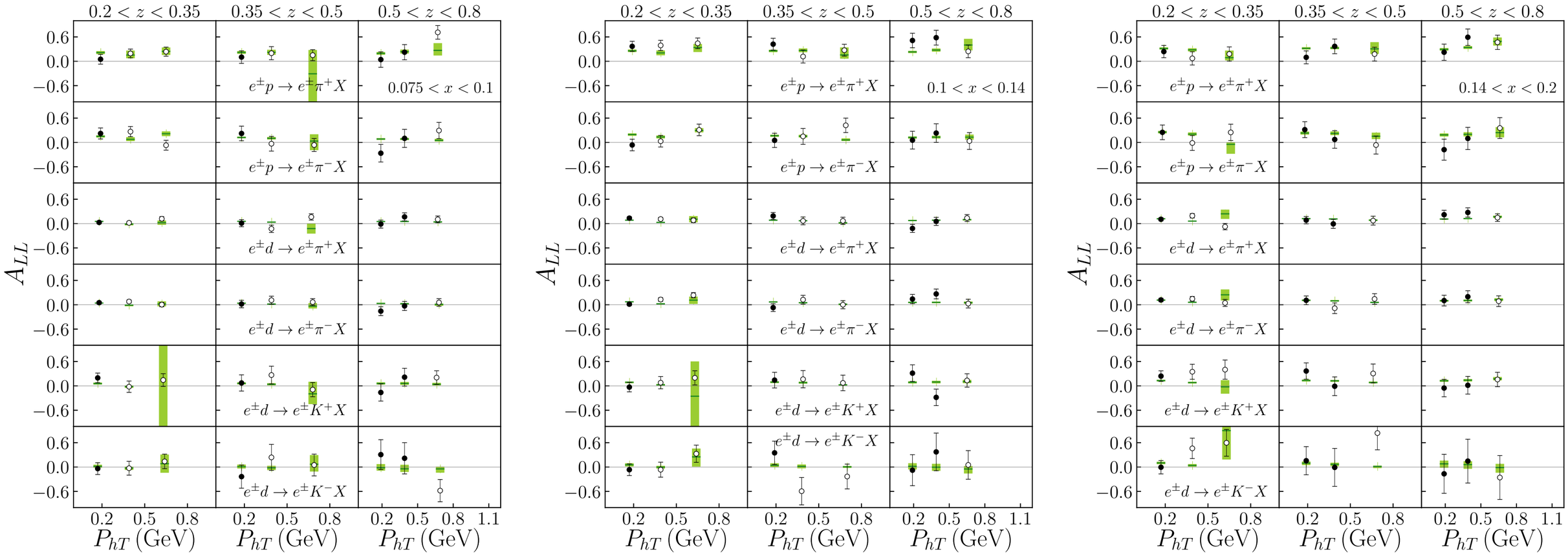
Comparison with Data



Data: HERMES Collaboration, Phys. Rev. D 99 (2019) 112001.

K. Yang, TL, B.-Q. Ma, P. Sun, Y. Zhao, Y. Zhou, arXiv:2610.31290.

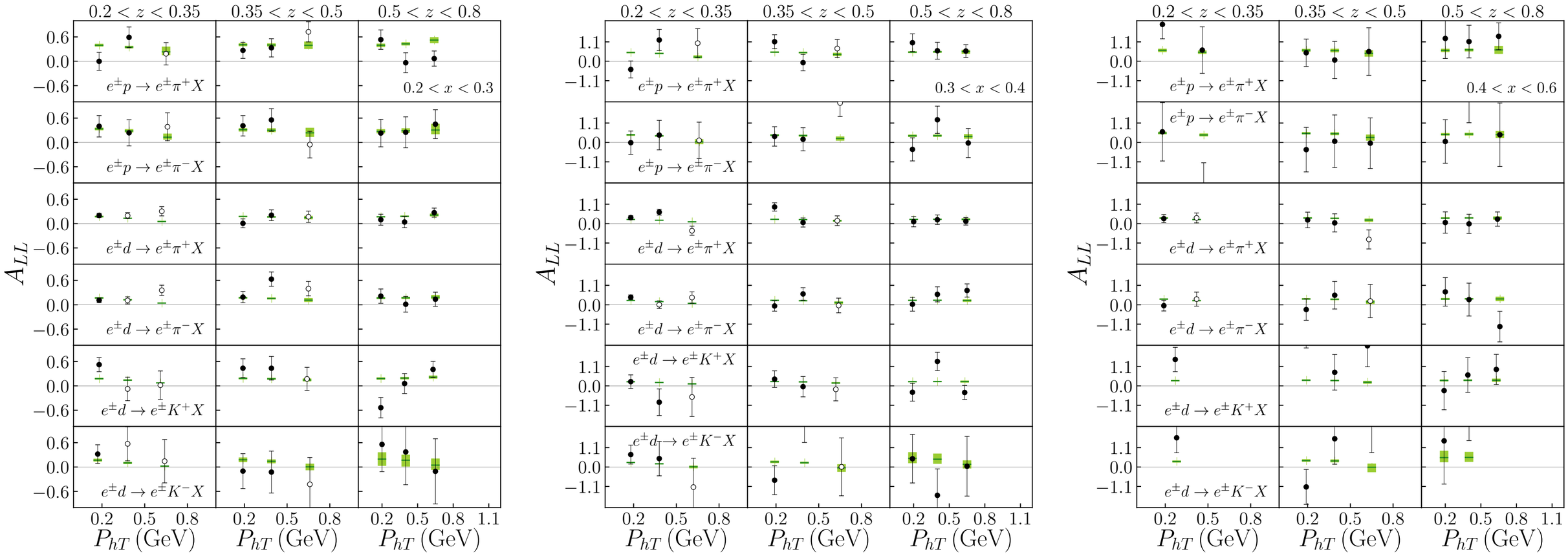
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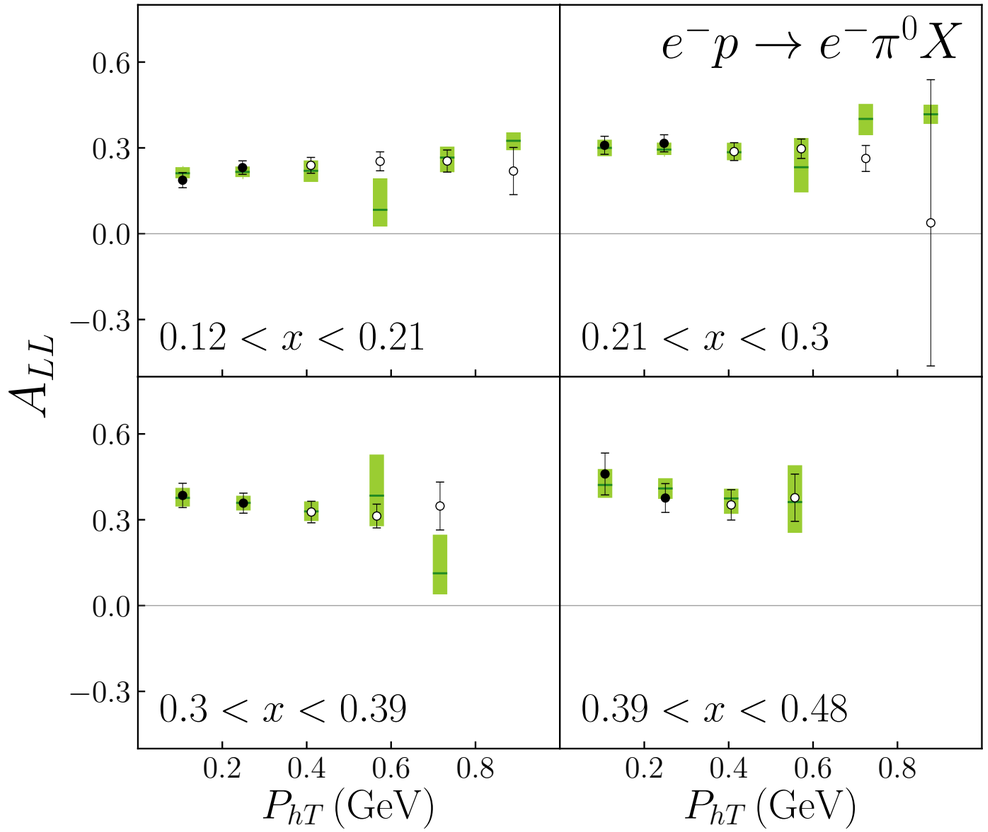
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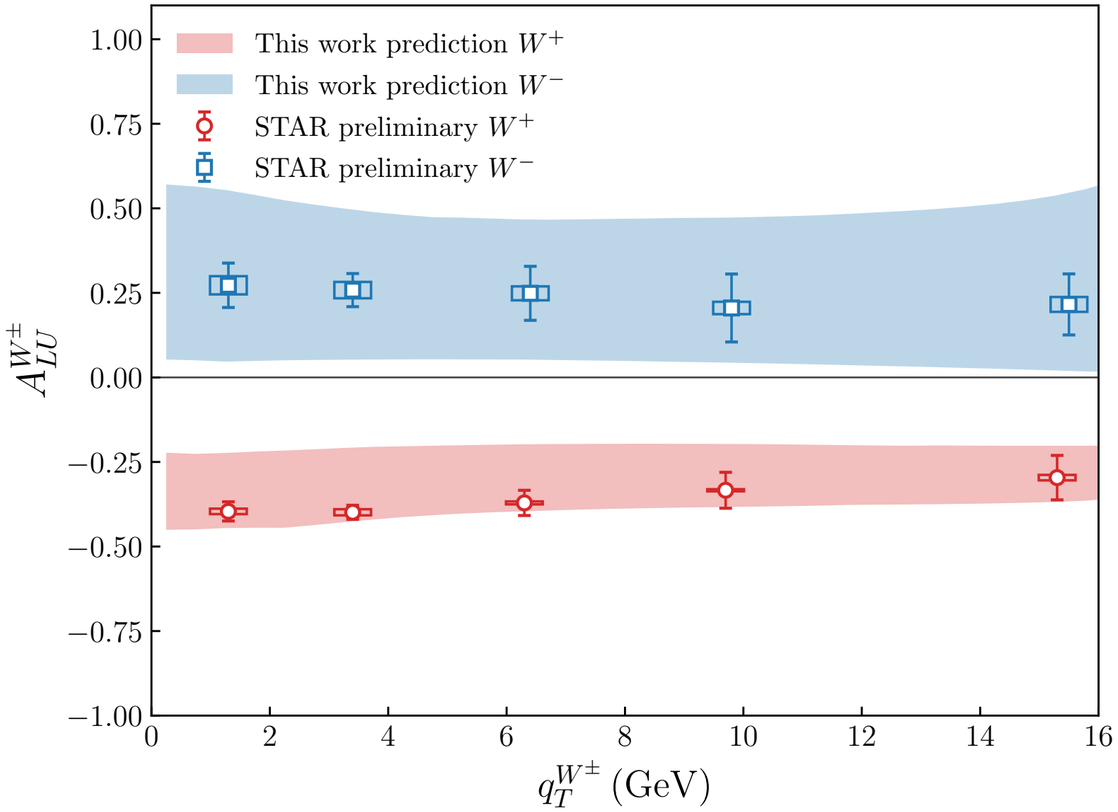
Comparison with Data



Data: CLAS Collaboration, Phys. Lett. B 782 (2018) 662.

K. Yang, TL, B.-Q. Ma, P. Sun, Y. Zhao, Y. Zhou, arXiv:2610.31290.

Prediction of $W^{\pm}$ SSA $A_{LU}^{W^{\pm}}$ in pp Collisions



Uncertainty bands propagated from extracted TMD helicity from SIDIS.

K. Yang, TL, B.-Q. Ma, P. Sun, Y. Zhao, Y. Zhou, arXiv:2610.31290.

Azimuthal Modulation of Longitudinal DSA

Azimuthal modulation of A_{LL} in SIDIS

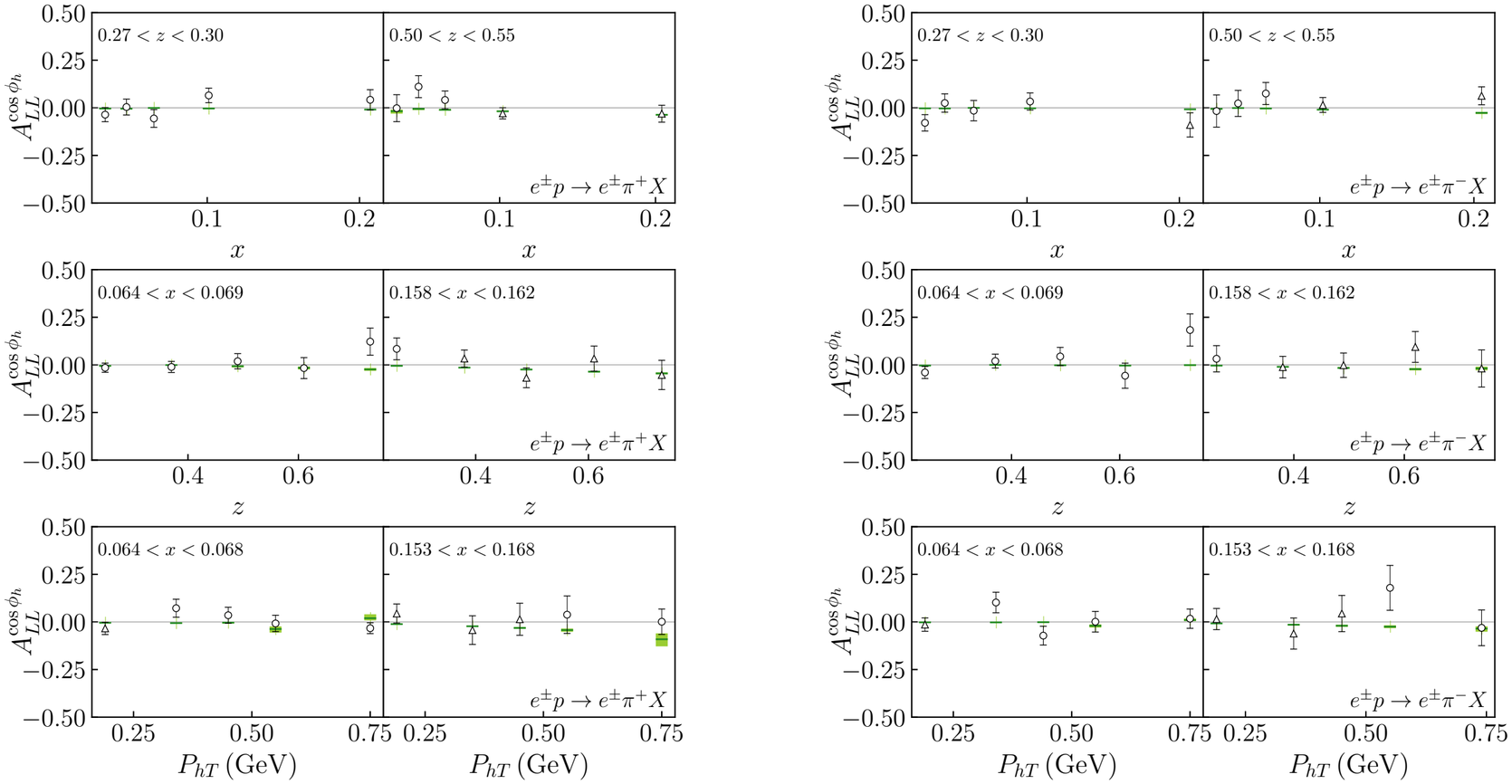
$$F_{LL}^{\cos\phi_h} = \frac{2M}{Q}\Bigg\{ -z^2 M_h \mathscr{B}_1\left[xe_L, H_1^{\perp(1)}\right] + \frac{z^2 M_h^2}{M}\mathscr{B}_1\left[g_{1L}, \frac{\tilde{D}^{\perp(1)}}{z}\right]$$
$$-M\mathscr{B}_1\left[xg_L^{\perp(1)}, D_1\right] - M_h\mathscr{B}_1\left[h_{1L}^{\perp(1)}, \frac{\tilde{E}}{z}\right]\Bigg\}$$

up to subleading power in $1/Q$

Relation from QCD equation of motion: $xg_L^{\perp} = x\tilde{g}_L^{\perp} + g_{1L} + \frac{m}{M}h_{1L}^{\perp}$

Neglecting twist-three contributions (Wandzura-Wilczek approximation): $F_{LL}^{\cos\phi_h} \approx -\frac{2M^2}{Q}\mathscr{B}_1\left[g_{1L}^{(1)}, D_1\right]$

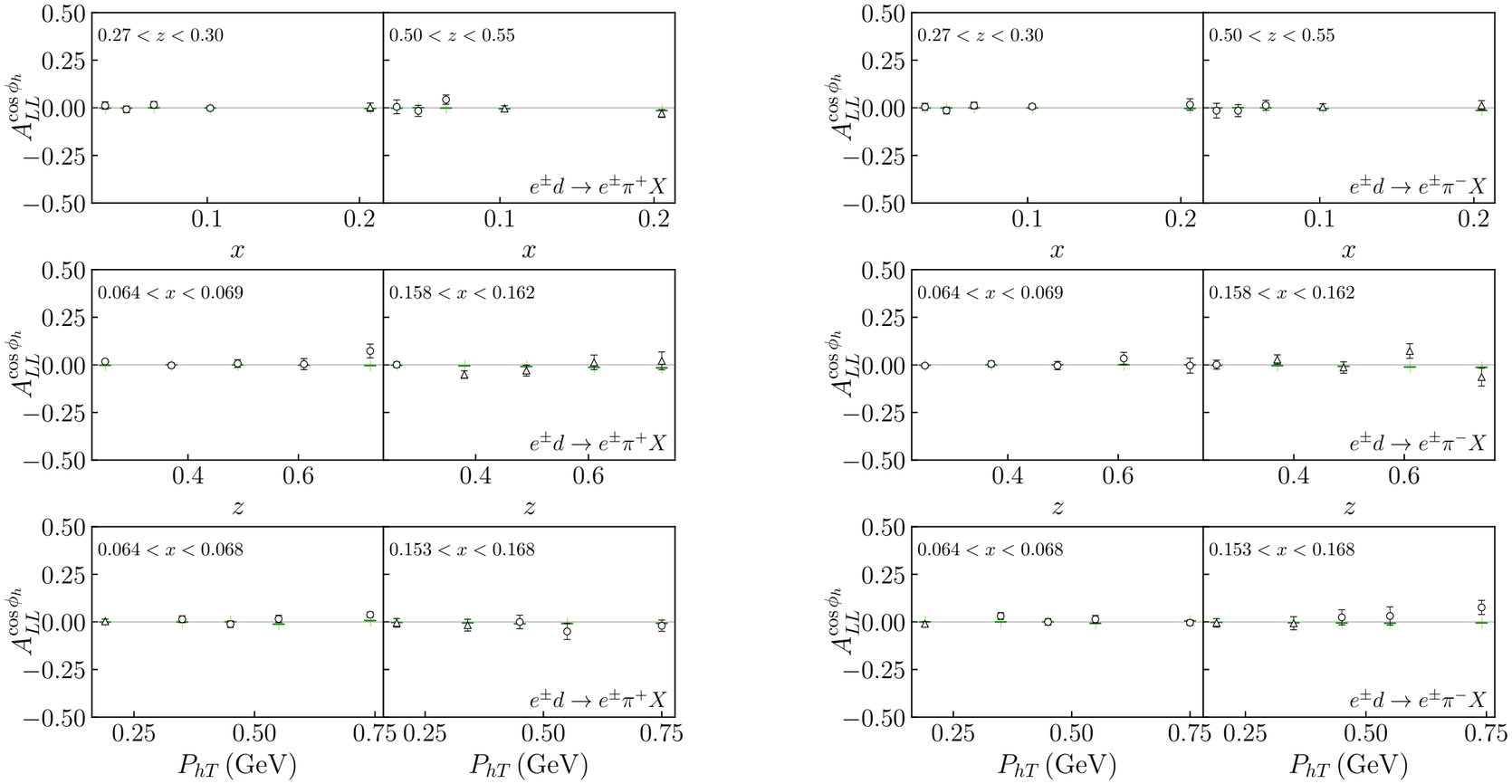
Comparison with $A_{LL}^{\cos\phi_h}$ Data



Data: HERMES Collaboration, Phys. Rev. D 99 (2019) 112001.

K. Yang, TL, B.-Q. Ma, P. Sun, Y. Zhao, Y. Zhou, arXiv:2610.31290.

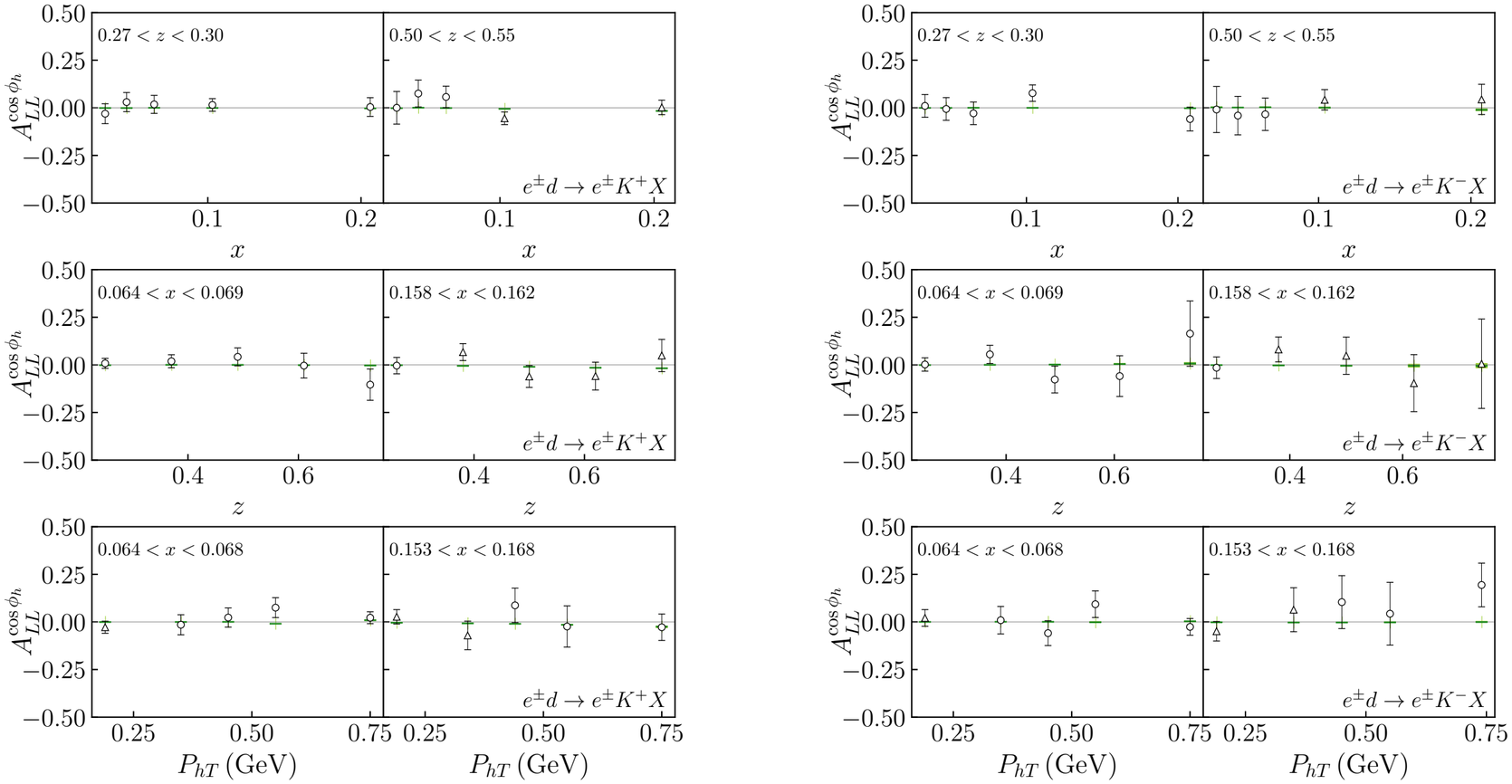
Comparison with $A_{LL}^{\cos\phi_h}$ Data



Data: HERMES Collaboration, Phys. Rev. D 99 (2019) 112001.

K. Yang, TL, B.-Q. Ma, P. Sun, Y. Zhao, Y. Zhou, arXiv:2610.31290.

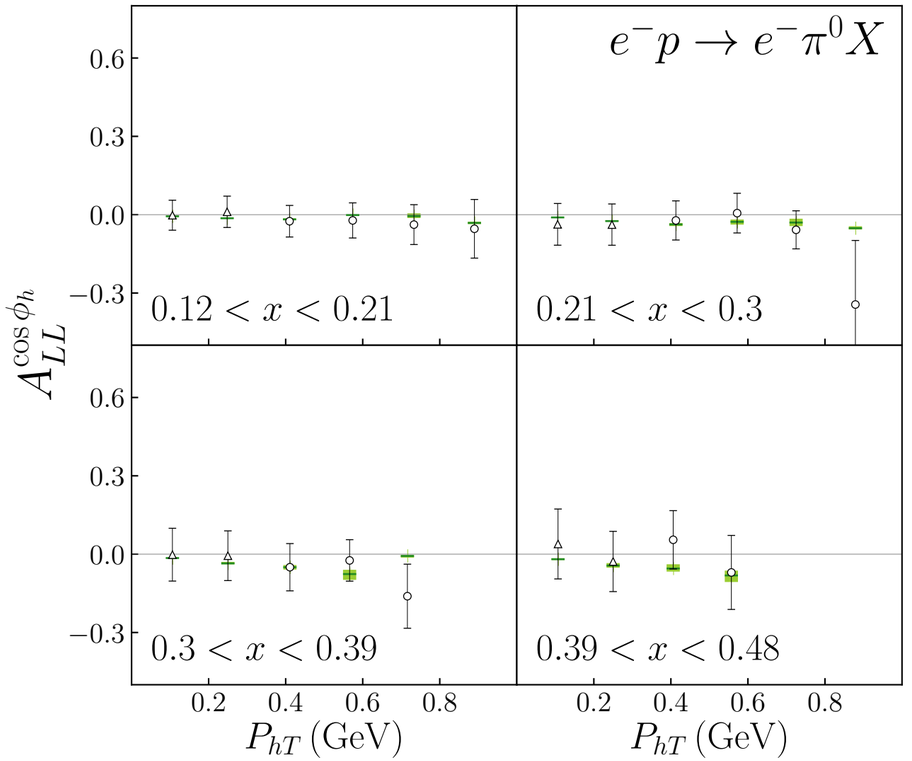
Comparison with $A_{LL}^{\cos\phi_h}$ Data



Data: HERMES Collaboration, Phys. Rev. D 99 (2019) 112001.

K. Yang, TL, B.-Q. Ma, P. Sun, Y. Zhao, Y. Zhou, arXiv:2610.31290.

Comparison with $A_{LL}^{\cos\phi_h}$ Data

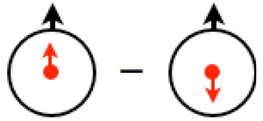


Data: CLAS Collaboration, Phys. Lett. B 782 (2018) 662.

K. Yang, TL, B.-Q. Ma, P. Sun, Y. Zhao, Y. Zhou, arXiv:2610.31290.

Transversity Distribution

Transversity distribution

h_1  (Collinear & TMD)

A transverse counter part to the longitudinal spin structure: helicity g_{1L}, but NOT the same.

Chiral-odd:

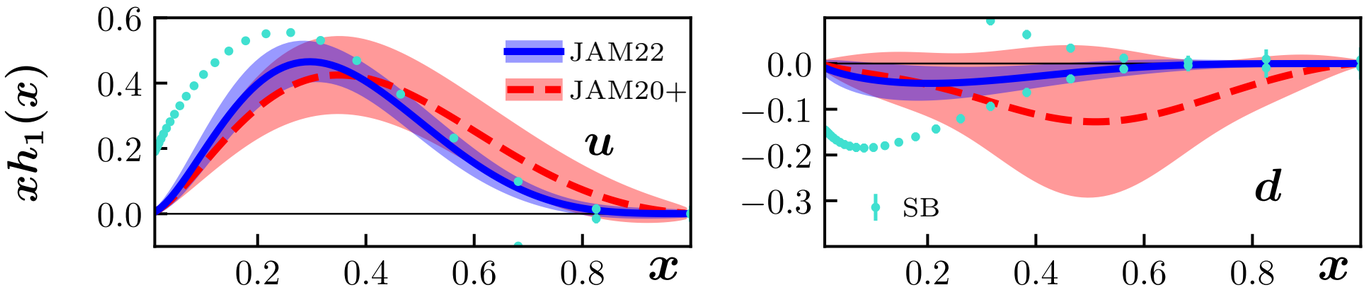
No mixing with gluons
Valence dominant
Couple to another chiral-odd function.

Effect in SIDIS:

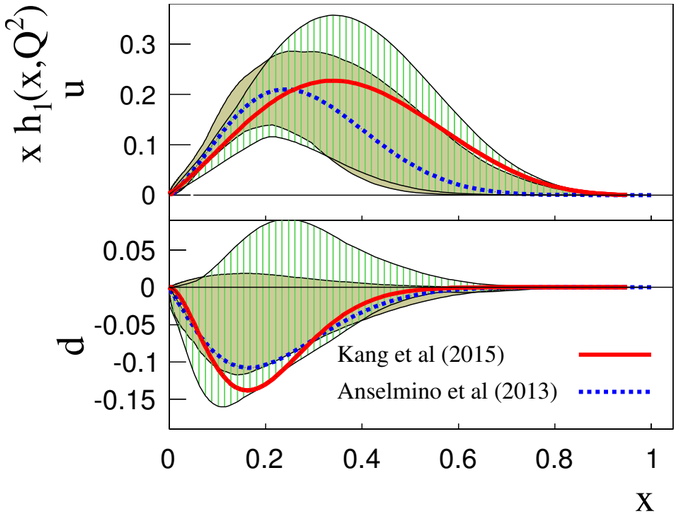
transverse single spin asymmetry (Collins asymmetry)

$$A_{UT}^{\sin(\phi_h+\phi_S)} \sim h_1(x,k_T^2) \otimes H_1^{\perp}(z,p_T^2)$$

Phenomenological extractions



JAM Collaboration, PRD 106, 034014 (2022).



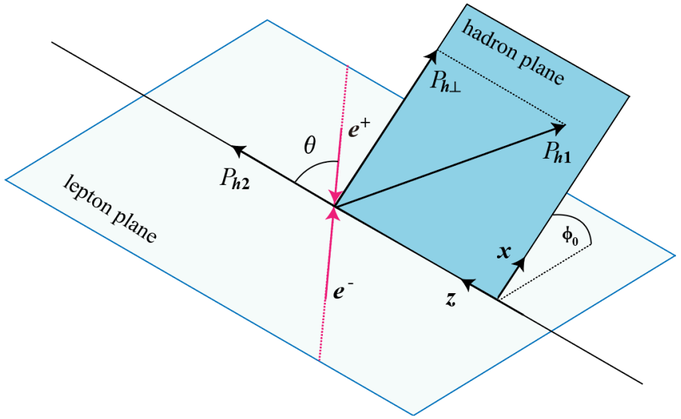
Z.-B. Kang, A. Prokudin, P. Sun, F. Yuan, PRD 93, 014009 (2016).

Assuming vanishing transverse polarization of sea quarks in early analysis!

Complementary Process

Semi-inclusive e^+e^- annihilation:

$$e^+e^- \to h_1 h_2 X$$



$$\frac{\mathrm{d}^5\sigma}{\mathrm{d}z_1\,\mathrm{d}z_2\,\mathrm{d}^2\boldsymbol{P}_{h\perp}\,\mathrm{d}\cos\theta} = \frac{3\pi\alpha^2}{2Q^2} z_1^2 z_2^2 \left[\left(1+\cos^2\theta\right) F_{UU}^{h_1 h_2} + \sin^2\theta \cos\left(2\phi_0\right) F_{\text{Collins}}^{h_1 h_2} \right]$$

In TMD region: h_1 and h_2 are near back-to-back, $P_{hT} \ll Q$

$$F_{\text{Collins}}^{h_1 h_2} \sim H_1^{\perp h_1} \otimes H_1^{\perp h_2}$$

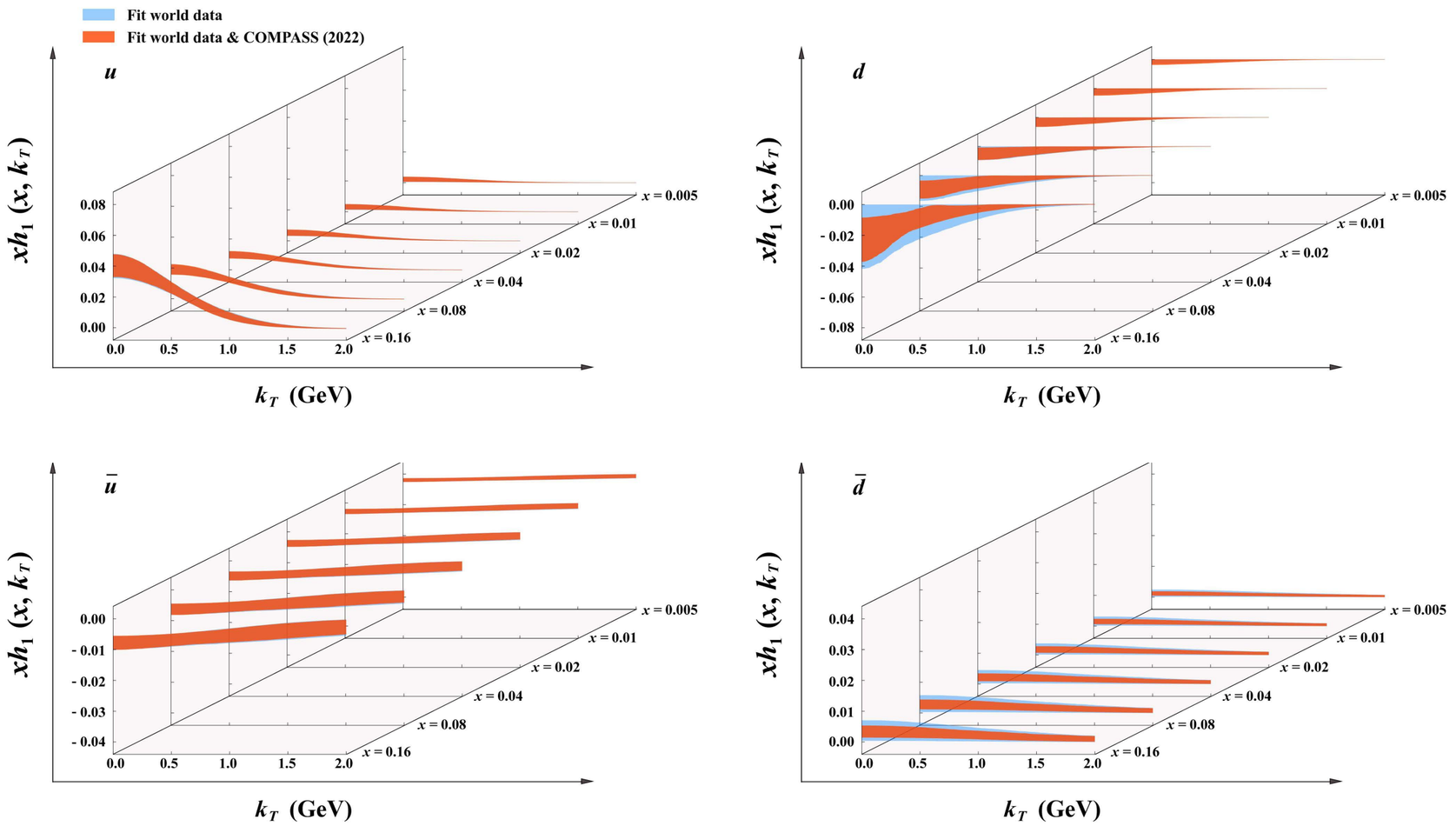
Experimental measurements:

Belle: $\sqrt{s} = 10.58\,\text{GeV}$ Phys. Rev. D 78 (2008) 032011; 86 (2012) 039905(E).

BaBar: $\sqrt{s} = 10.6\ \text{GeV}$ Phys. Rev. D 90 (2014) 052003; Phys. Rev. D 92 (2015) 111101.

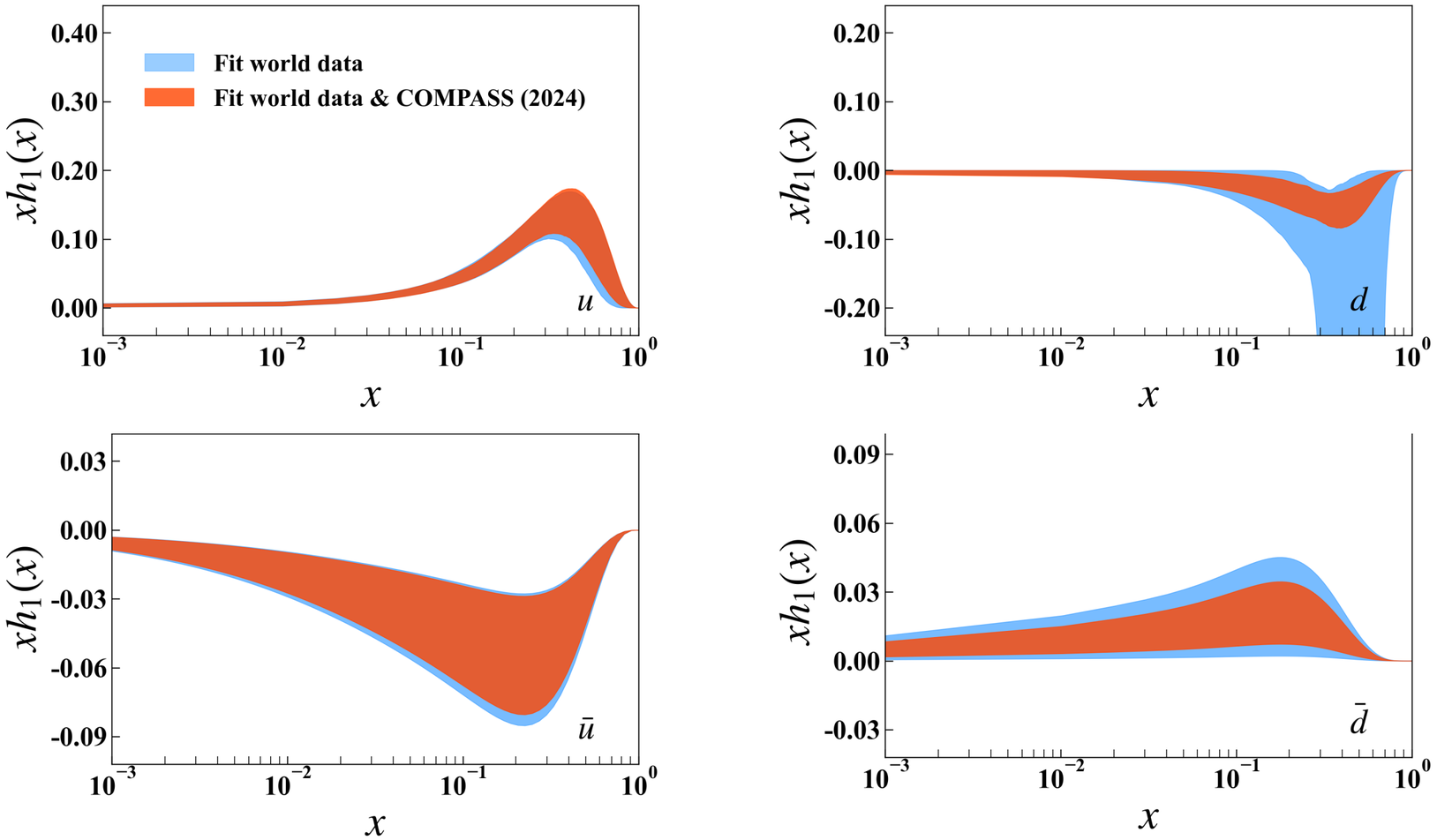
BESIII: $\sqrt{s} = 3.68\ \text{GeV}$ Phys. Rev. Lett. 116 (2016) 042001.

Transversity Distributions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, PRD 109 (2024) 056002; arXiv:2412.18324.

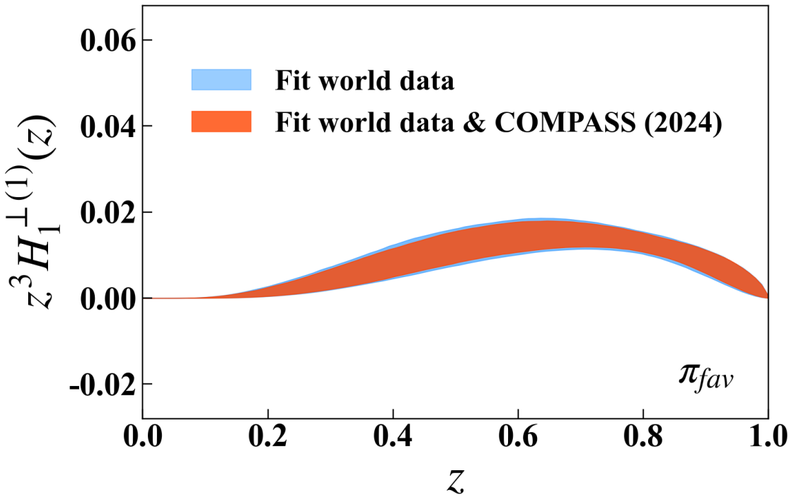
Transversity Distributions

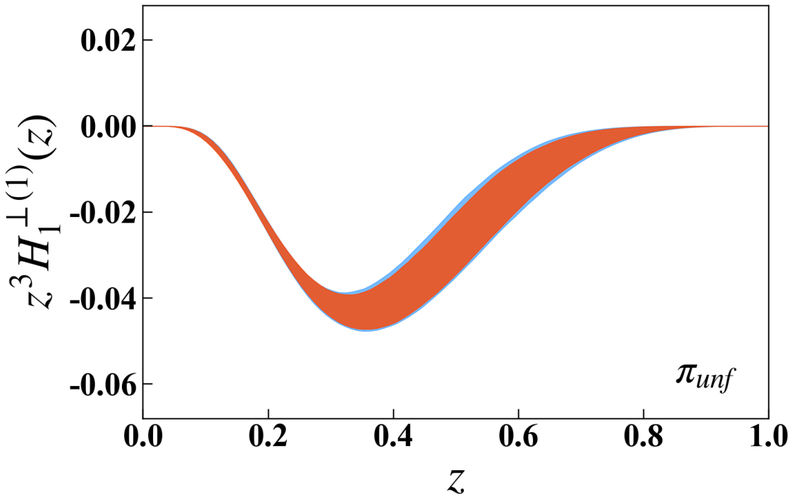


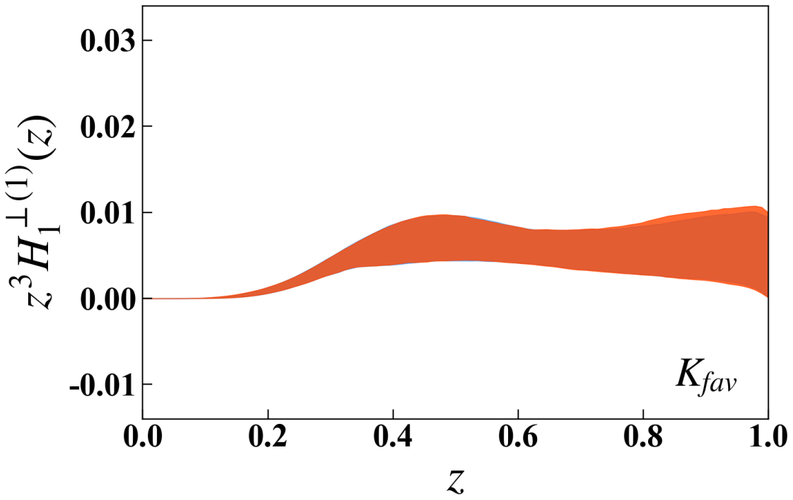
Find $\bar{u}$ favors *negative* transverse polarization

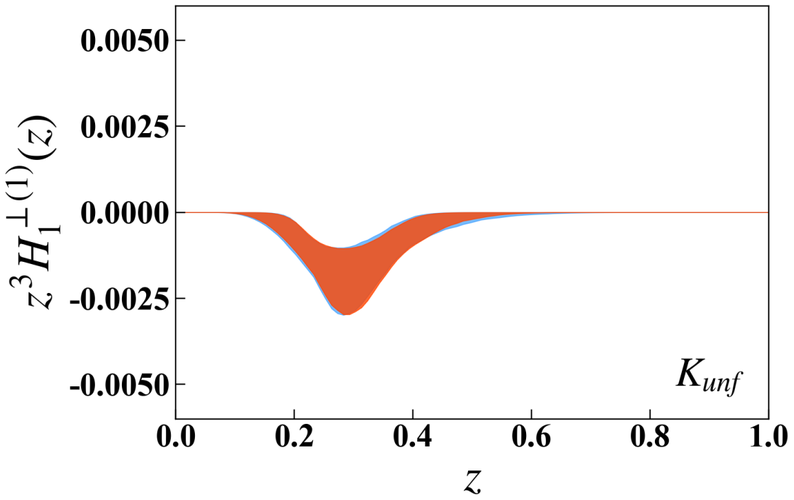
C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, PRD 109 (2024) 056002; arXiv:2412.18324.

Collins Fragmentation Functions









C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, PRD 109 (2024) 056002; arXiv:2412.18324.

Collinear Transversity Distributions in Dihadron SIDIS

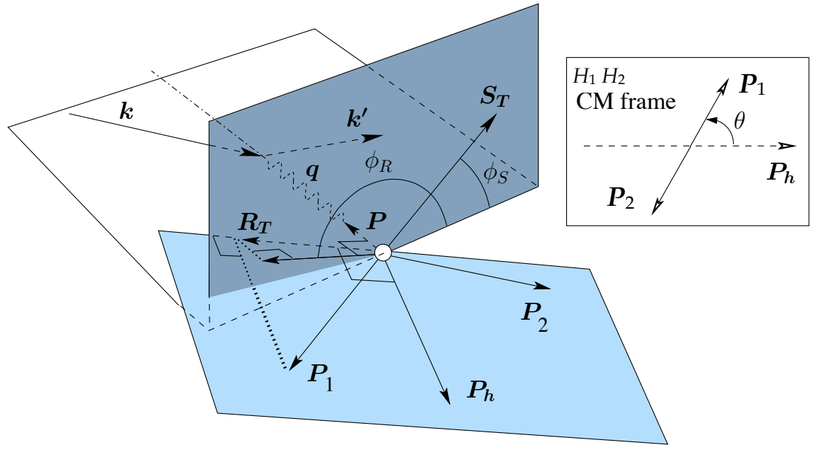
Dihadron SIDIS

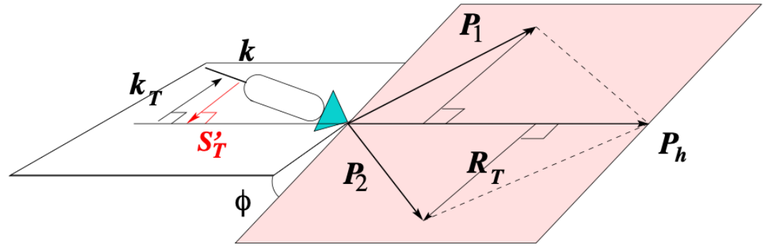
$$l + N \to l' + h_1 h_2 + X$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B\,\mathrm{d}y\,\mathrm{d}\phi_S\,\mathrm{d}z\,\mathrm{d}\phi_R\,\mathrm{d}M_h^2\,\mathrm{d}\cos\theta} =$$

$$\frac{\alpha^2}{x_B y Q^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\left\{F_{UU} + S_T\,\varepsilon\,\sin(\phi_R+\phi_S)F_{UT}^{\sin(\phi_R+\phi_S)}\right\}$$

$$F_{UT}^{\sin(\phi_R+\phi_S)} \sim h_1 \otimes H_1^{\sphericalangle}$$





h_1: collinear transversity distribution function

$H_1^{\sphericalangle}$: a chiral-odd dihadron fragmentation function

describing the correlation between quark transverse polarization and the azimuthal orientation of the hadron pair plane

M. Radici, R. Jakob, and A. Bianconi, Phys. Rev. D 65 (2002) 074031;
A. Bacchetta, A. Courtoy, M. Radici, JHEP 03 (2013) 119.

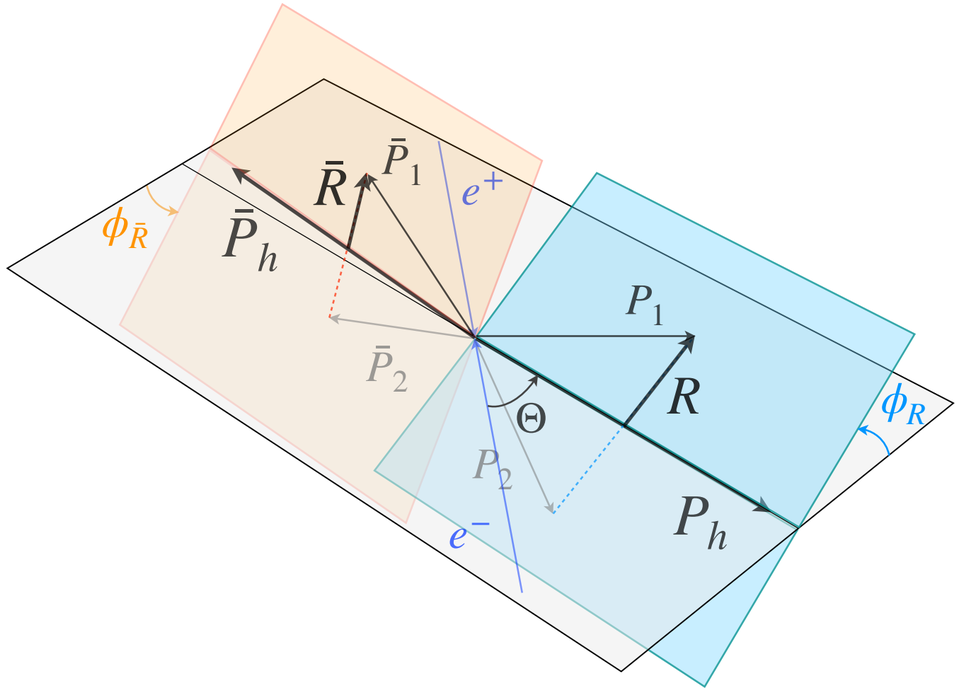
Complimentary Processes

Artru-Collins asymmetry in e^+e^- annihilation

$$e^+ + e^- \to h_1 h_2 + h_1' h_2' + X$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\,\mathrm{d}\bar{z}\,\mathrm{d}M_h^2\,\mathrm{d}\bar{M}_h^2\,\mathrm{d}\cos\Theta\,\mathrm{d}\cos\theta\,\mathrm{d}\cos\bar{\theta}\,\mathrm{d}\phi_R\,\mathrm{d}\phi_{\bar{R}}}$$

$$= \frac{3\alpha^2}{8\pi^2 Q^2}\left[\left(1+\cos^2\Theta\right)F_{UU}^{1} + \sin^2\Theta\,\cos\left(\phi_R+\phi_{\bar{R}}\right)F_{UU}^{\cos(\phi_R+\phi_{\bar{R}})} + \cdots\right]$$

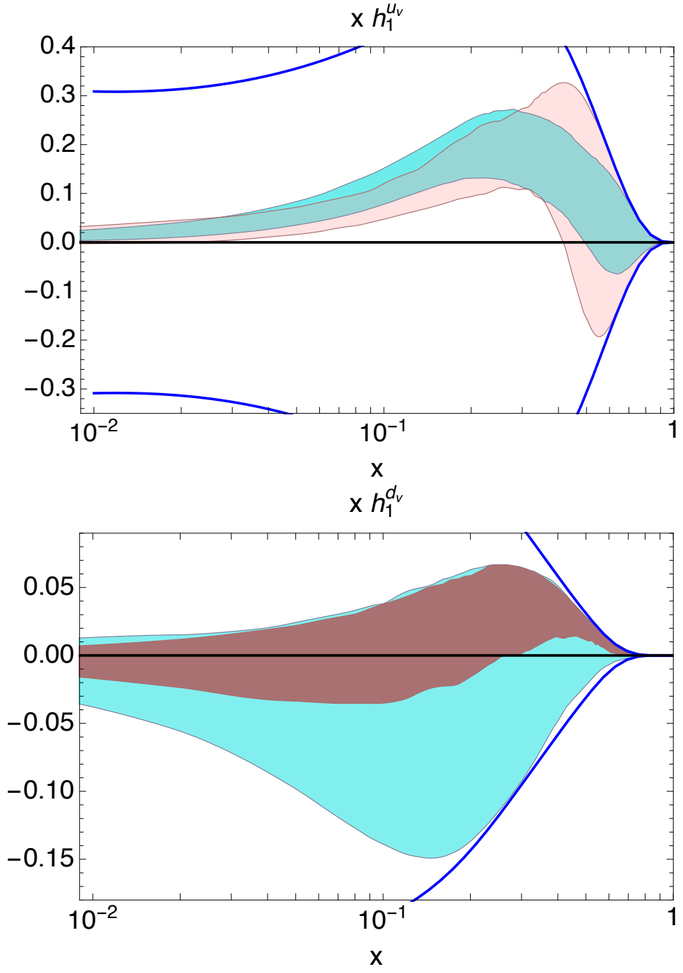


$$F_{UU}^{1} \sim D_1^{q\to h_1 h_2} \otimes D_1^{\bar{q}\to h_1' h_2'}$$

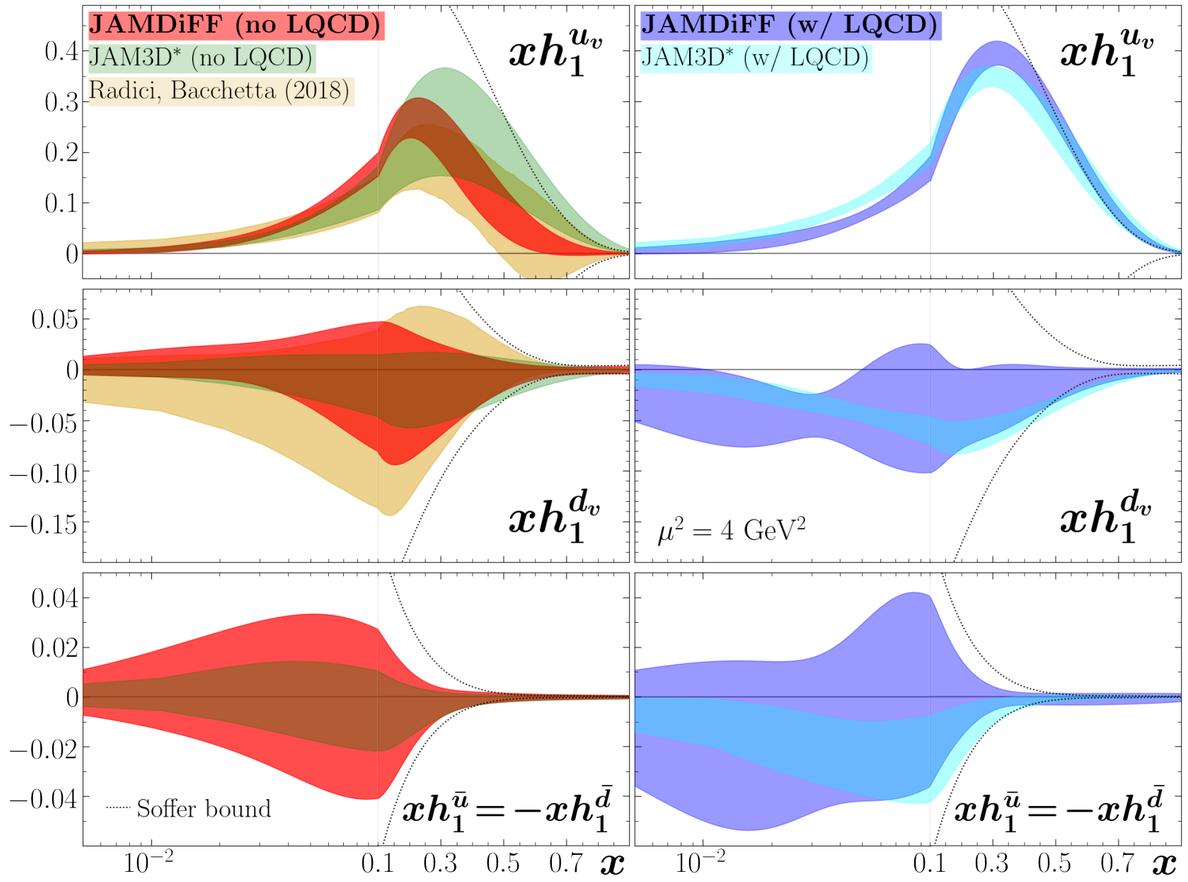
$$F_{UU}^{\cos(\phi_R+\phi_{\bar{R}})} \sim H_1^{\sphericalangle\, q\to h_1 h_2} \otimes H_1^{\sphericalangle\, \bar{q}\to h_1' h_2'}$$

X. Artru, J.C. Collins, Z. Phys. C 69 (1996) 277; D. Boer, R. Jakob, M. Radici, Phys. Rev. D 67 (2003) 094003.

Collinear Transversity Distributions

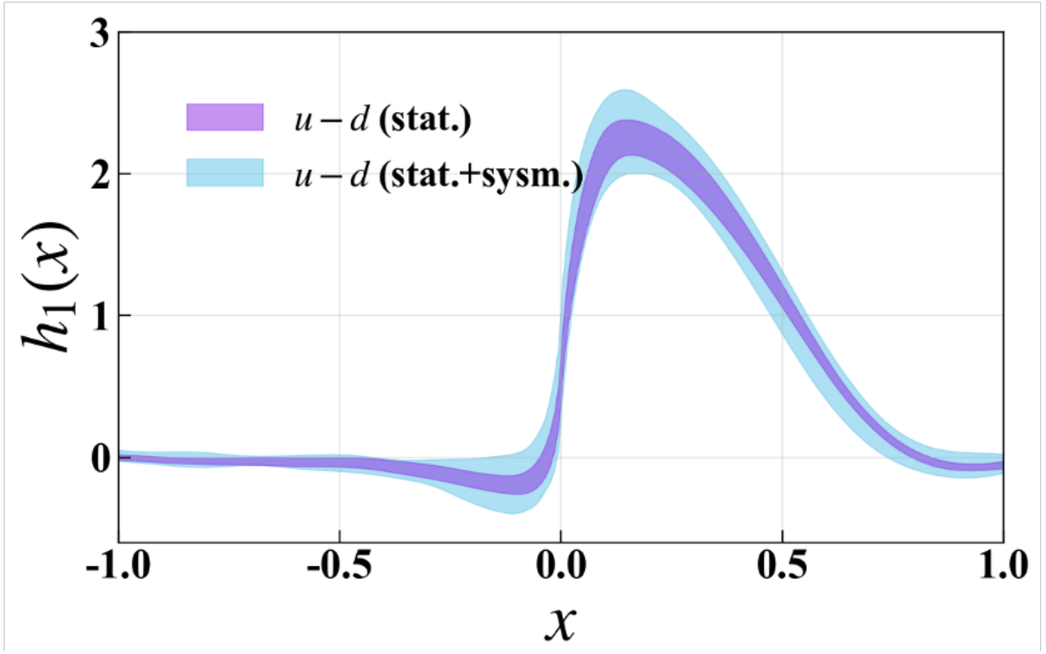


M. Radici and A. Bacchetta, Phys. Rev. Lett. 120 (2018) 192001.

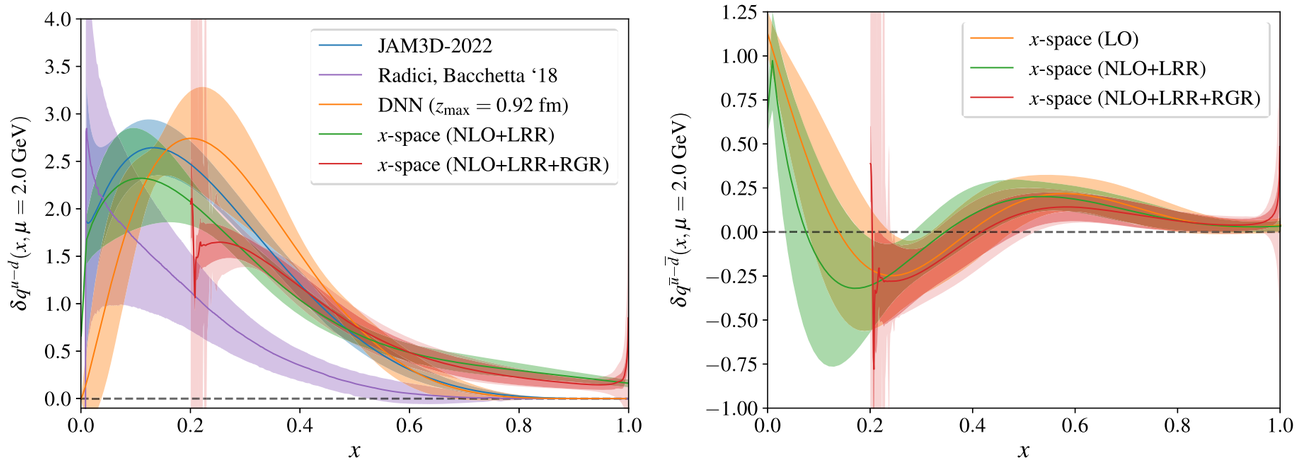


C. Cocuzza *et al.* (JAM Collaboration), Phys. Rev. Lett. 132 (2024) 091901; Phys. Rev. D 109 (2024) 034024.

Transversity Distribution from Lattice QCD



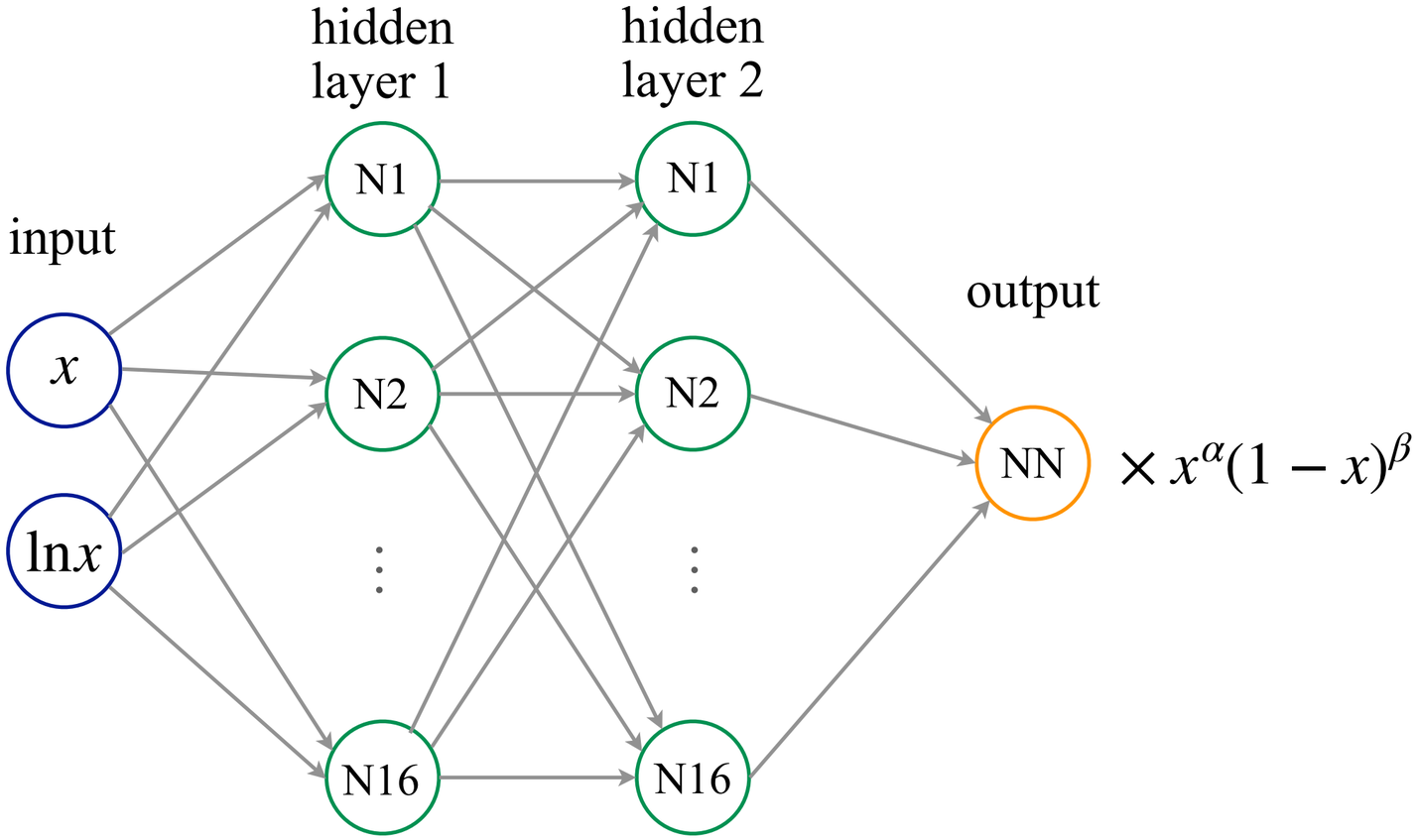
Lattice Parton Collaboration, Phys. Rev. Lett. 131 (2023) 261901.



X. Gao *et al.*, Phys. Rev. D 109 (2024) 054506.

NN Parametrization

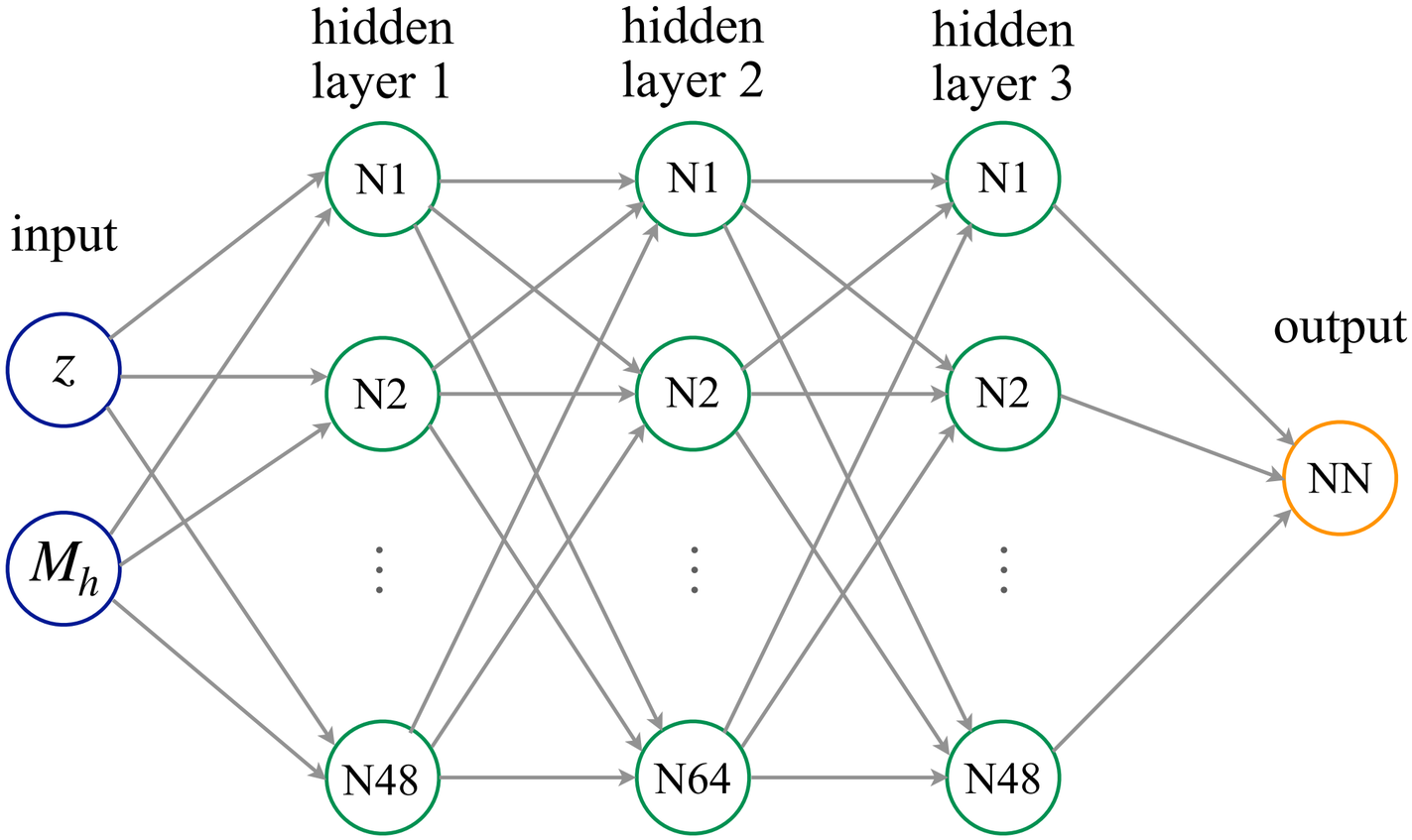
Parametrization of transversity distributions h_1:



tested with various architectures: number of layers, number of neurons, activation functions, optimization algorithms

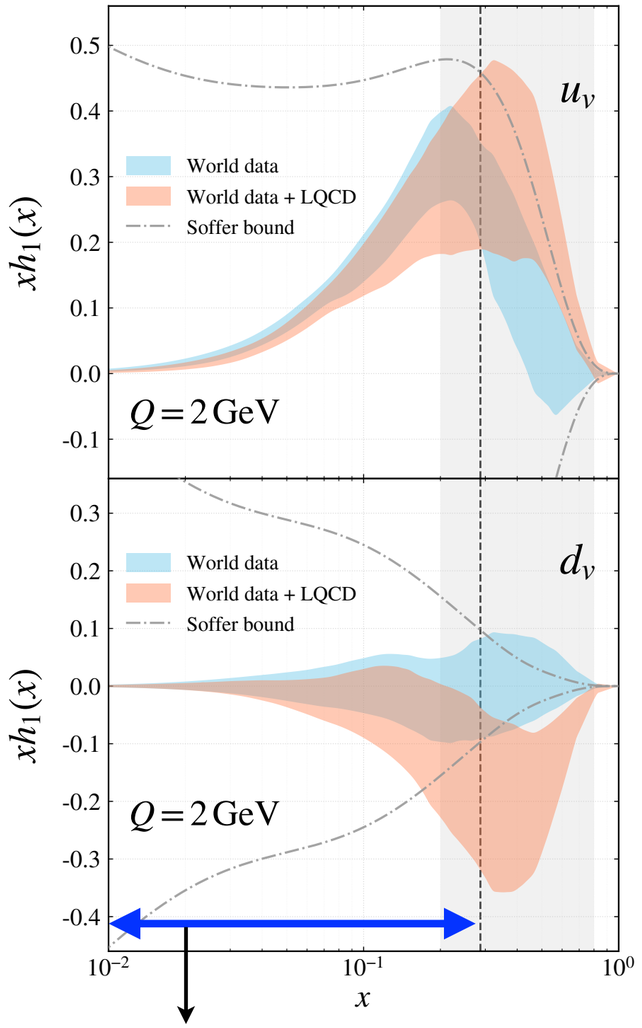
NN Parametrization

Parametrization of dihadron fragmentation functions D_1 and $H_1^{\sphericalangle}$

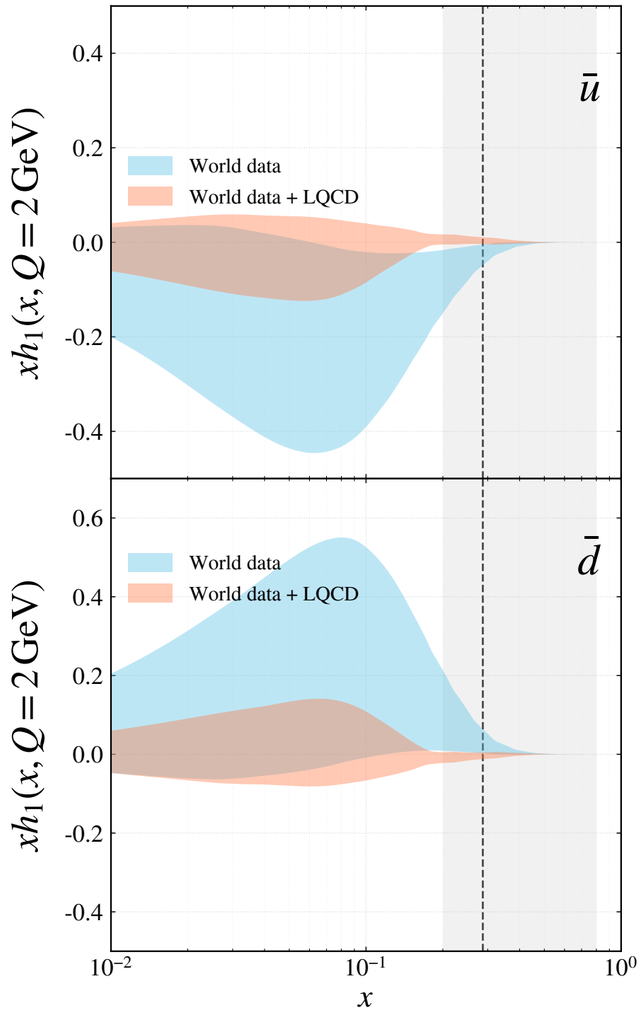


tested with various architectures: number of layers, number of neurons, activation functions, optimization algorithms

A Combined Analysis of Transversity

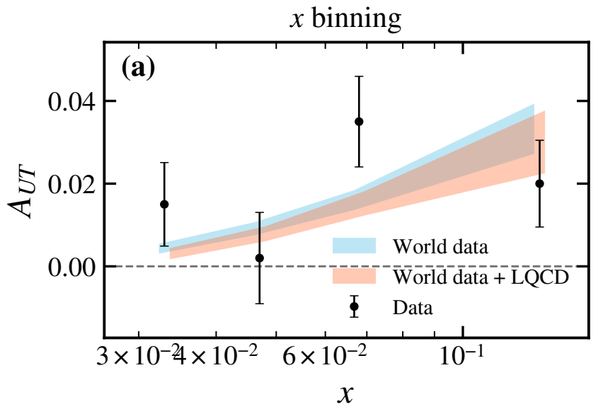


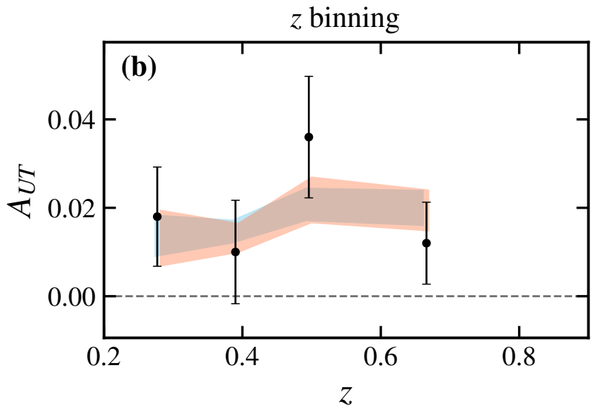
directly covered by SIDIS data

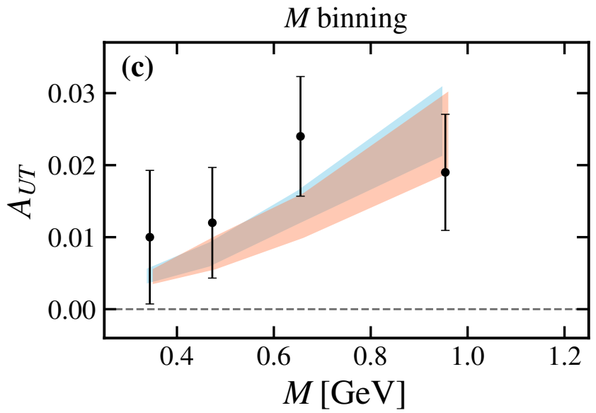


H. Dong, TL, P. Sun, Y. Zhao, axXiv:2610.xxxxx

Comparison with Data







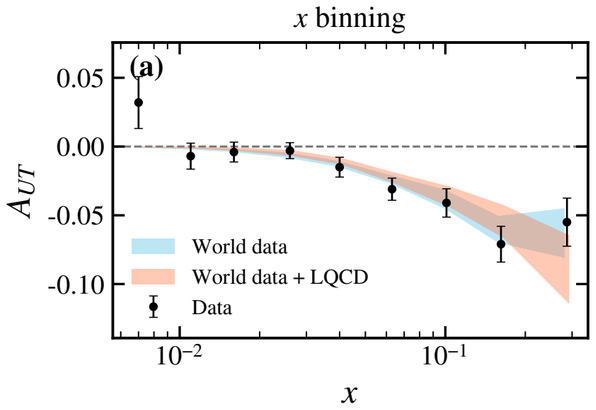
HERMES proton target

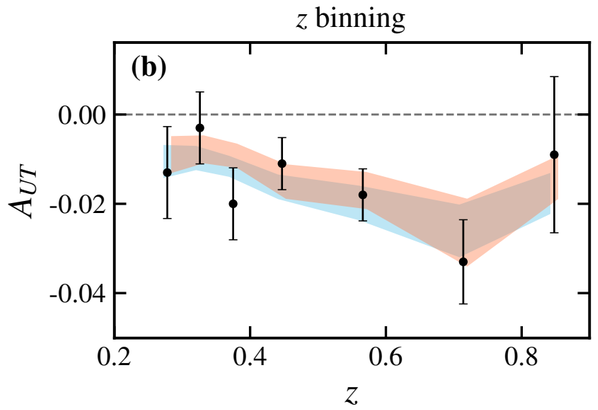
$\chi^2/N = 0.933$ (world data fit)
$\chi^2/N = 0.887$ (world data + lattice)

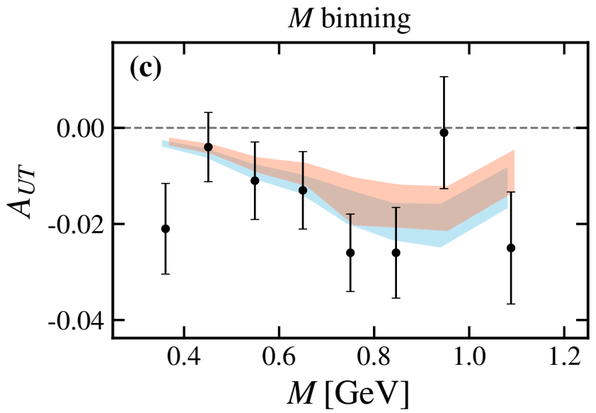
Data: HERMES Collaboration, JHEP 06 (2008) 017.

H. Dong, TL, P. Sun, Y. Zhao, axXiv:2610.xxxxx

Comparison with Data



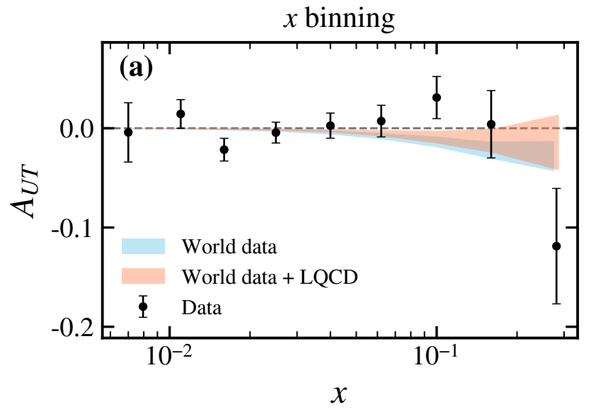


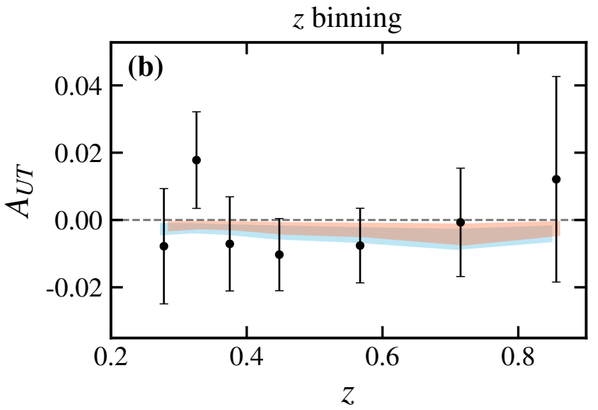


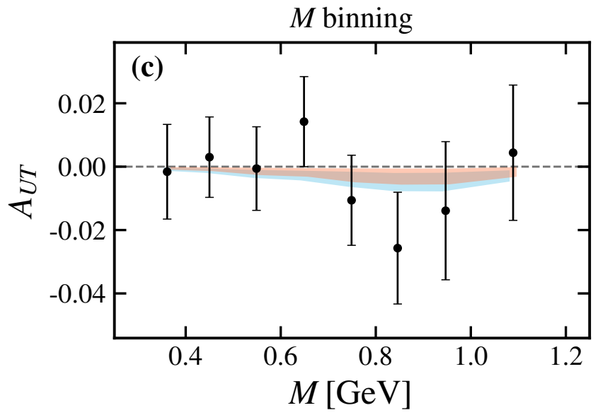
COMPASS proton target

$\chi^2/N = 0.767$ (world data fit)
$\chi^2/N = 1.003$ (world data + lattice)

Data: COMPASS Collaboration, Phys. Lett. B 736 (2014) 124.







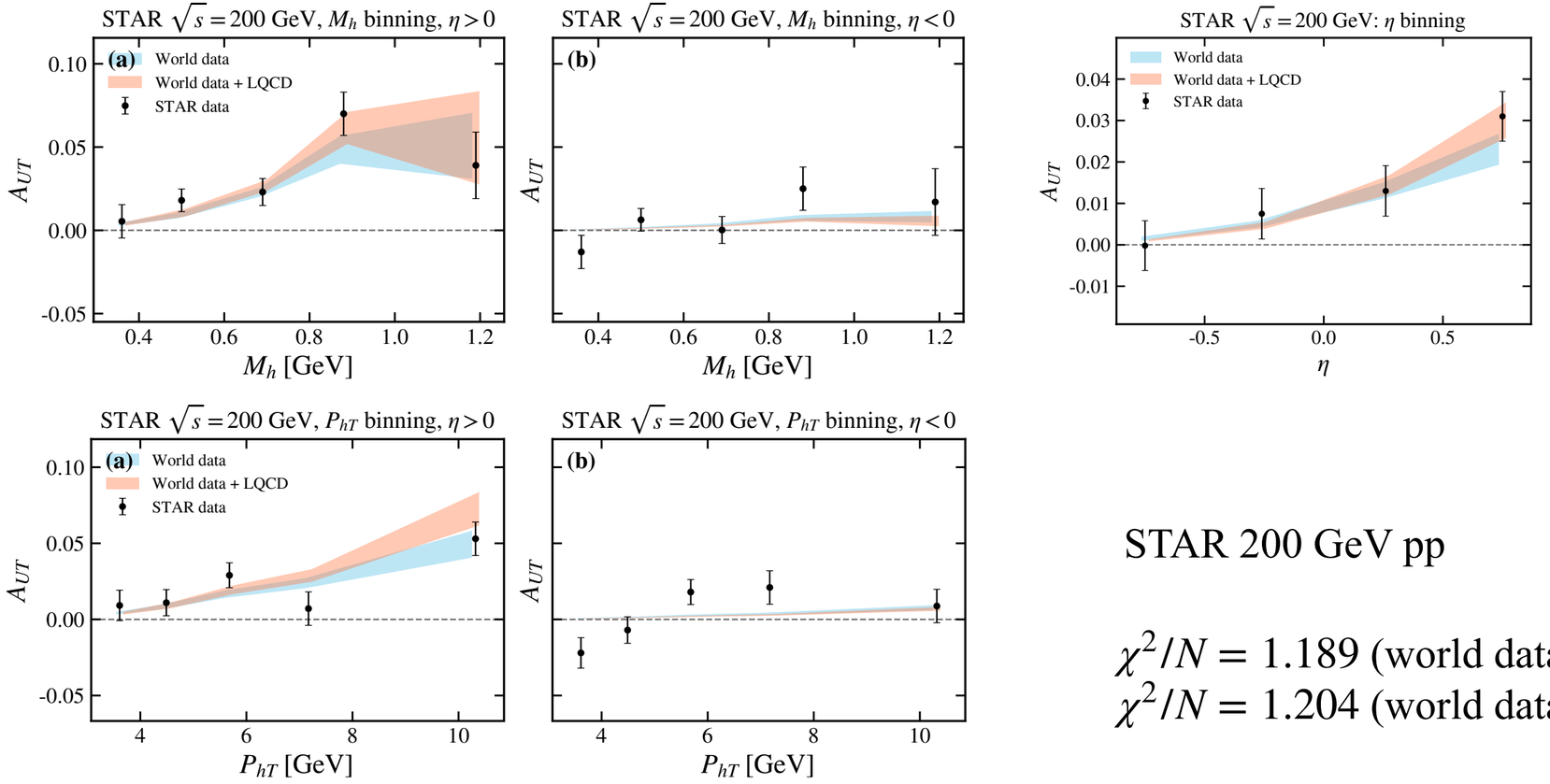
COMPASS deuteron target

$\chi^2/N = 0.808$ (world data fit)
$\chi^2/N = 0.791$ (world data + lattice)

Data: COMPASS Collaboration, Phys. Lett. B 845 (2023) 138155.

H. Dong, TL, P. Sun, Y. Zhao, axXiv:2610.xxxxx

Comparison with Data



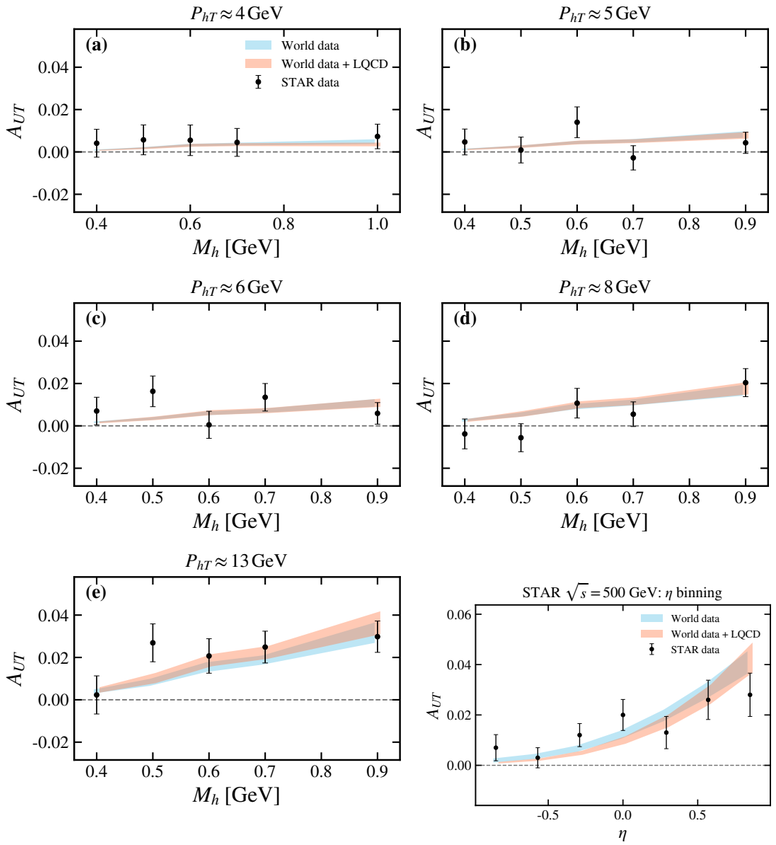
STAR 200 GeV pp

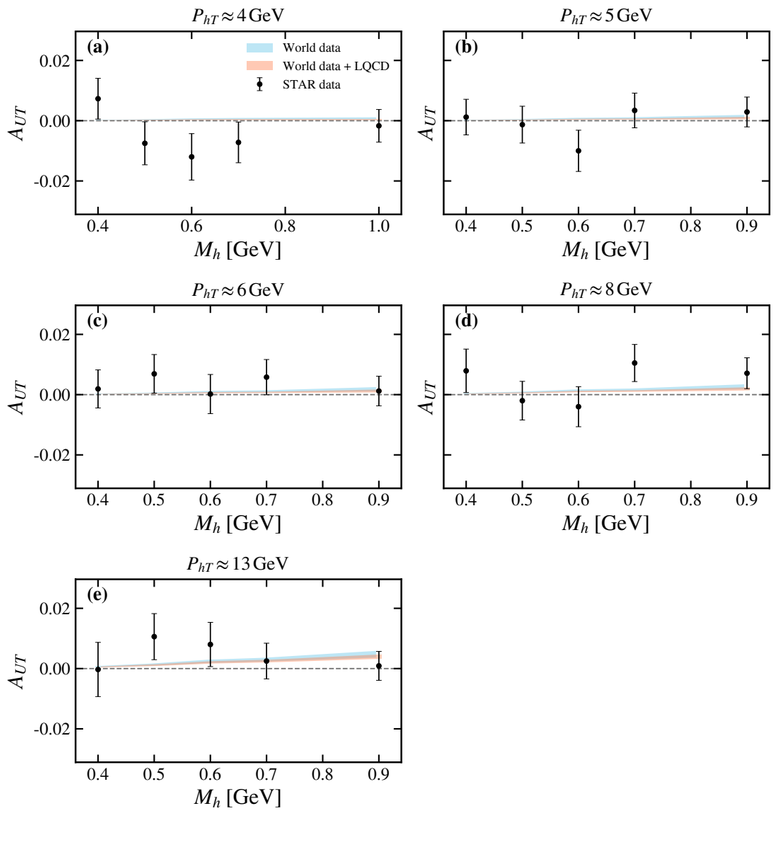
$\chi^2/N = 1.189$ (world data fit)
$\chi^2/N = 1.204$ (world data + lattice)

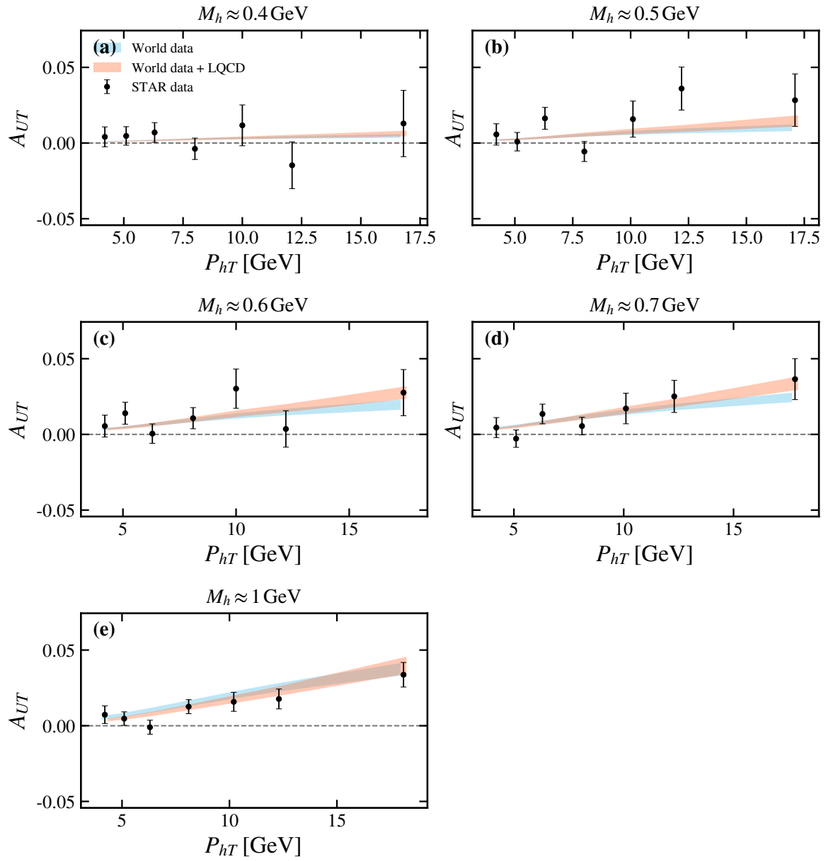
Data: STAR Collaboration, Phys. Rev. Lett. 115 (2015) 242501.

H. Dong, TL, P. Sun, Y. Zhao, axXiv:2610.xxxxx

Comparison with Data







STAR 500 GeV pp

$\chi^2/N = 0.896$ (world data fit)
$\chi^2/N = 0.867$ (world data + lattice)

Data: STAR Collaboration, Phys. Lett. B 780 (2018) 332.

H. Dong, TL, P. Sun, Y. Zhao, axXiv:2610.xxxxx

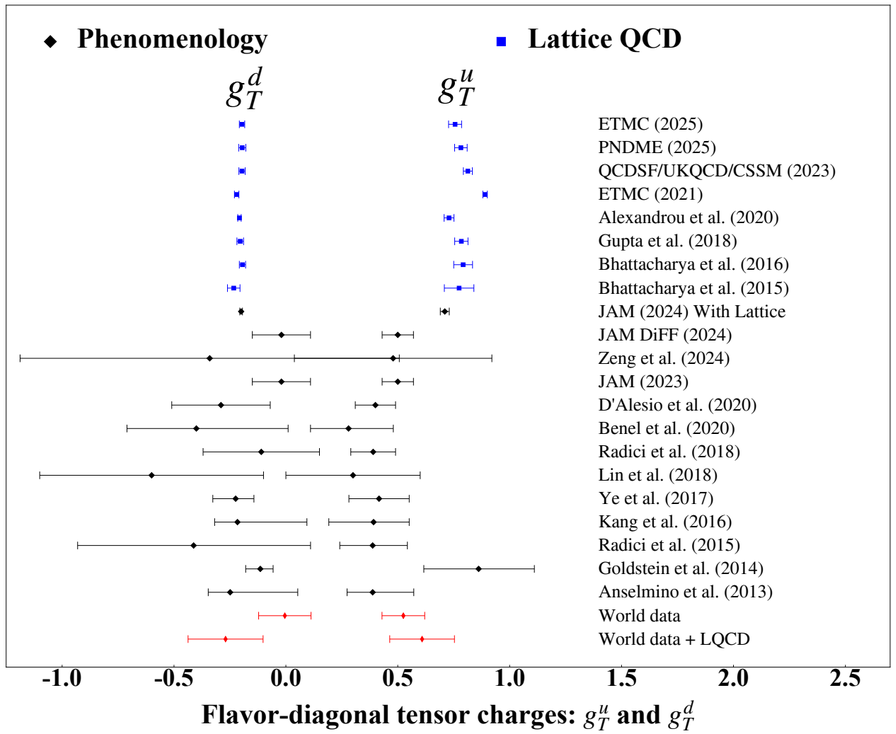
Tensor Charge

Tensor charge

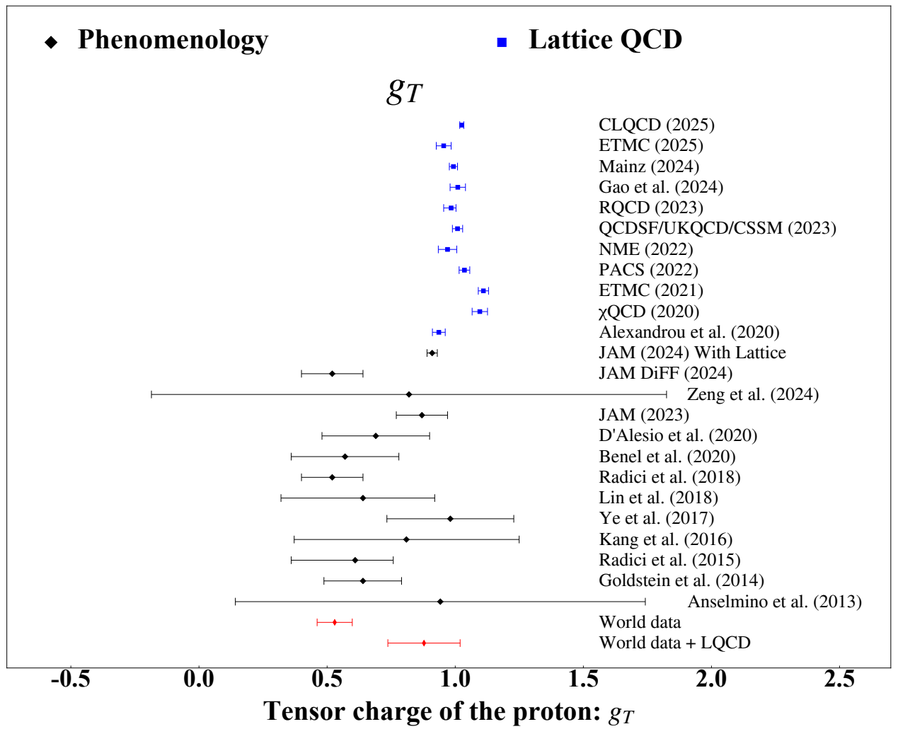
$$\langle P,S|\bar{\psi}^q i\sigma^{\mu\nu}\gamma_5\psi^q|P,S\rangle = g_T^q \bar{u}(P,S) i\sigma^{\mu\nu}\gamma_5 u(P,S)$$

$$g_T^q = \int_0^1 [h_1^q(x) - h_1^{\bar{q}}(x)]\, dx$$

- A fundamental QCD quantity: matrix element of local operators.
- Moment of the transversity distribution: valence quark dominant.
- Calculable in lattice QCD.



Flavor-diagonal tensor charges: g_T^u and g_T^d



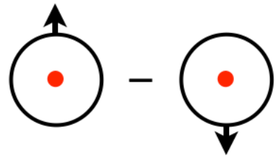
Tensor charge of the proton: g_T

H. Dong, TL, P. Sun, Y. Zhao, axXiv:2610.xxxxx

The Sivers Function

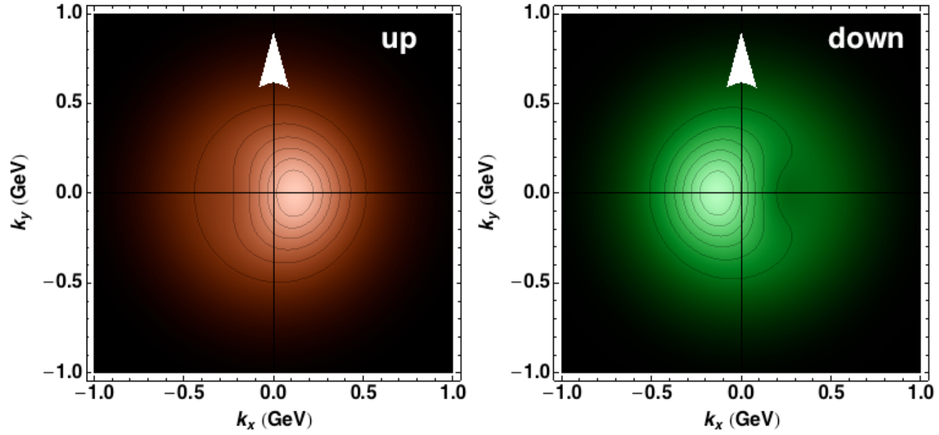
Sivers TMD distribution function

$$\frac{\epsilon_{ij} k_T^i S_T^j}{M} f_{1T}^{\perp}(x, k_T^2)$$



A naive T-odd distribution function

Transverse momentum distribution
distorted by nucleon transverse spin



[Figure from A. Bacchetta]

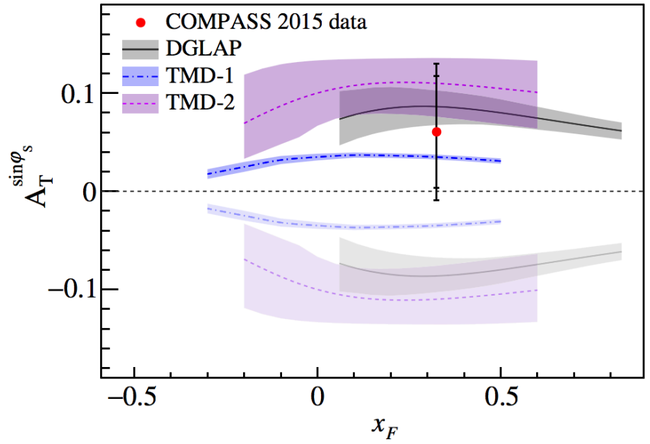
Effect in SIDIS:
transverse single spin asymmetry (Sivers asymmetry)

$$A_{UT}^{\sin(\phi_h - \phi_S)} \sim f_{1T}^{\perp} \otimes D_1$$

sizable Sivers asymmetry observed
by HERMES, COMPASS, JLab

Sign change prediction:

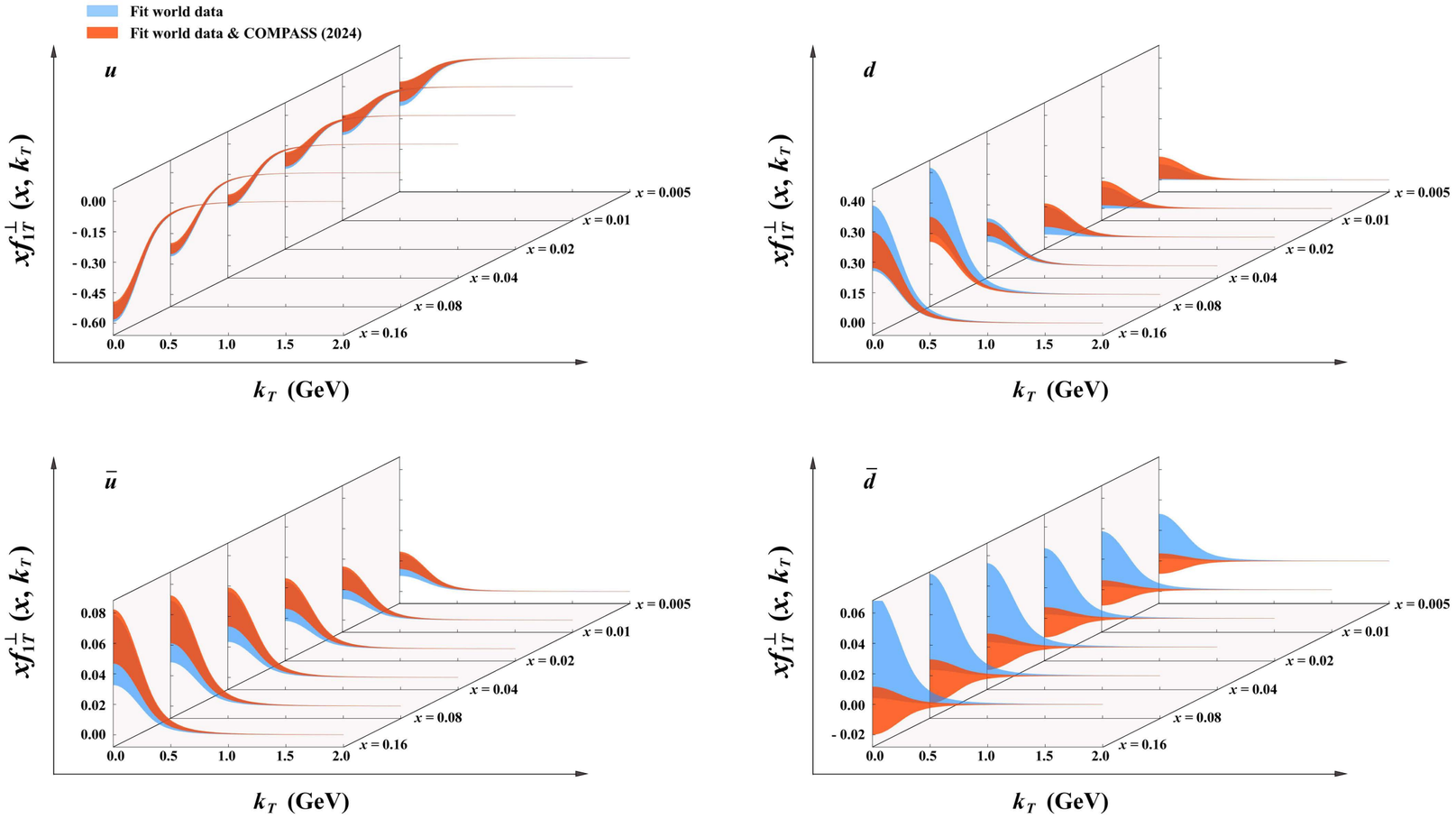
$$f_{1T}^{\perp}(x, k_T^2)\,|_{\text{SIDIS}} = -\, f_{1T}^{\perp}(x, k_T^2)\,|_{\text{DY}}$$



COMPASS Collaboration, PRL 119, 112002 (2017).

Extracted Sivers Functions

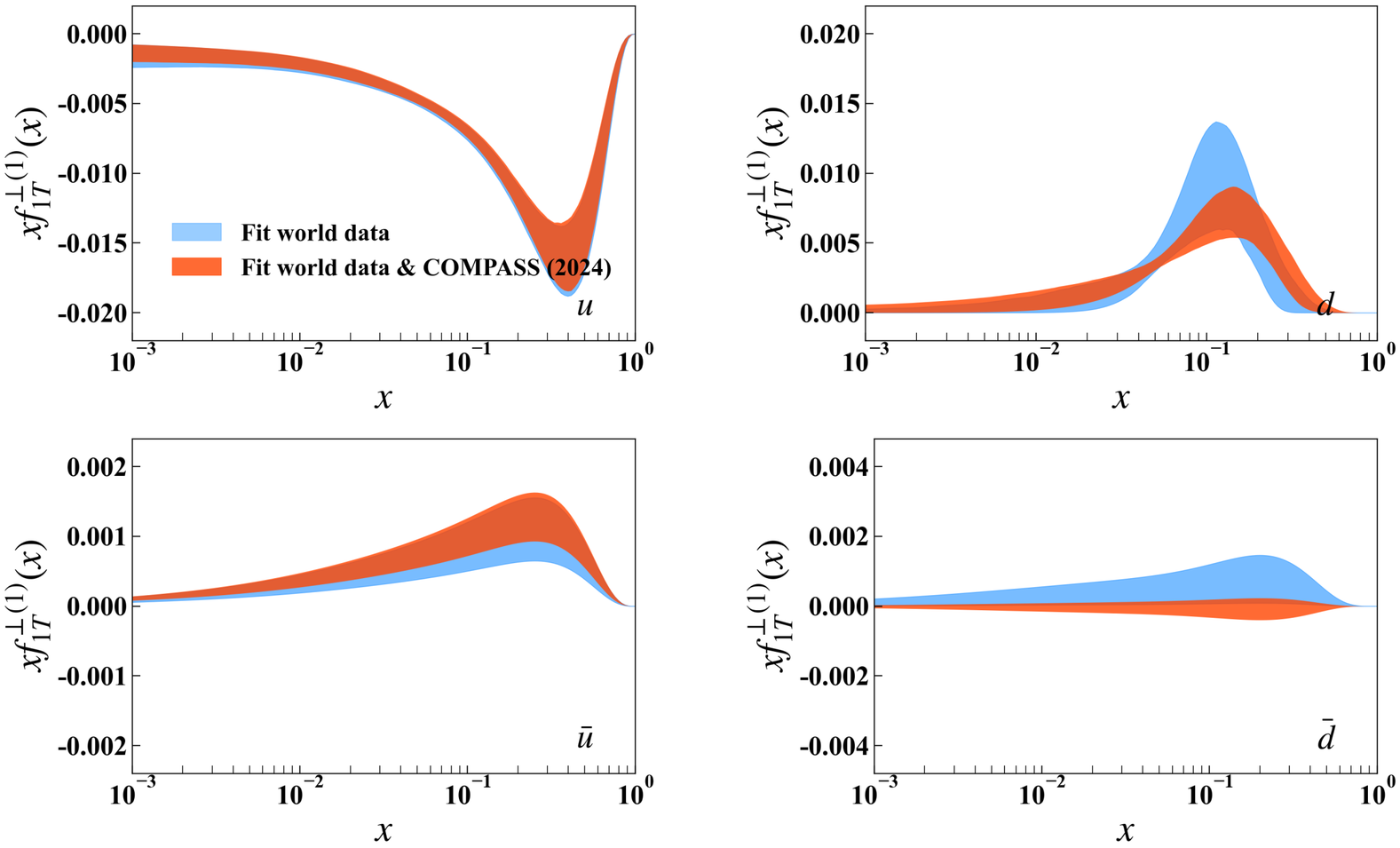
Global analysis of SIDIS, Drell-Yan, $W^{\pm}/Z^0$ production data



C. Zeng, TL, P. Sun, Y. Zhao, PRD 106 (2022) 094039; arXiv:2412.18324.

Extracted Sivers Functions

Global analysis of SIDIS, Drell-Yan, $W^{\pm}/Z^0$ production data

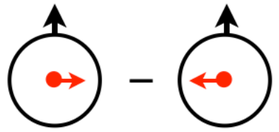


C. Zeng, TL, P. Sun, Y. Zhao, PRD 106 (2022) 094039; arXiv:2412.18324.

Double Spin Asymmetry and Worm-gear

Trans-helicity worm-gear distribution

$$\frac{k_T \cdot S_T}{M} g_{1T}^{\perp}(x, k_T^2)$$



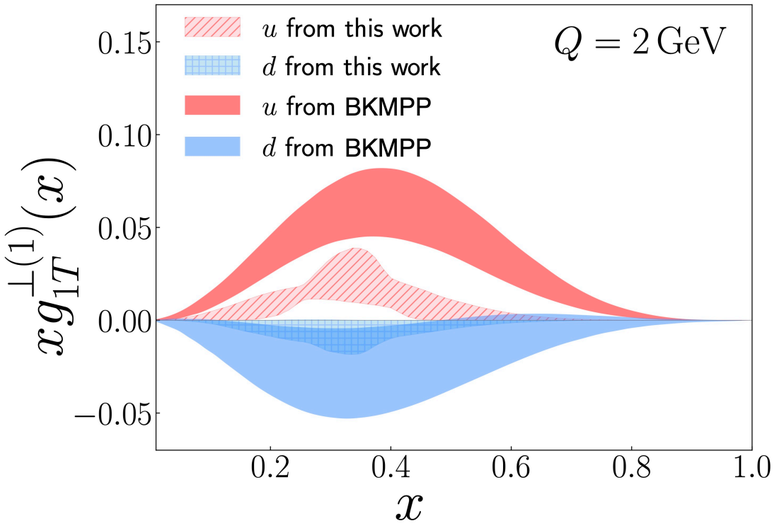
- Longitudinally polarized quark density in a transversely polarized nucleon
- Overlap between wave functions differing by one unit of orbital angular momentum

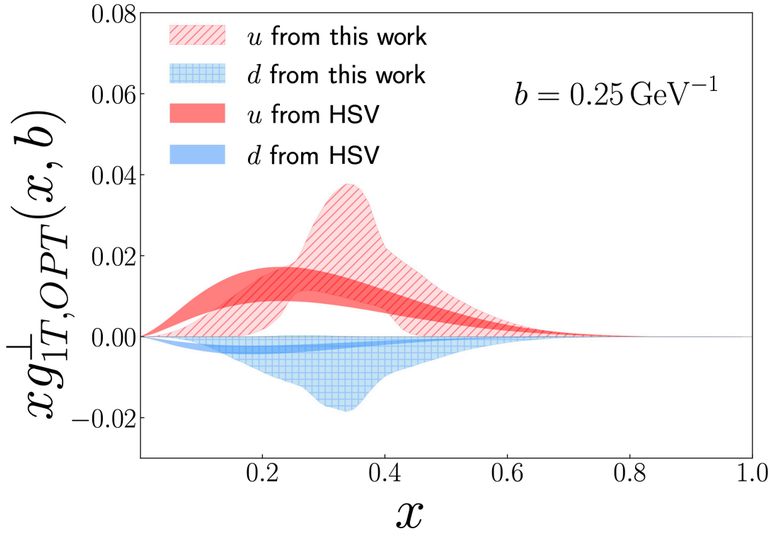


Effect in SIDIS:
A longitudinal-transverse double spin asymmetry

$$A_{LT}^{\cos(\phi_h - \phi_S)} \sim g_{1T}^{\perp} \otimes D_1$$

Phenomenological extraction



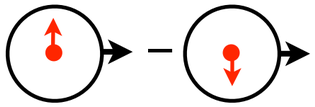


K. Yang, TL, P. Sun, Y. Zhao, B.-Q. Ma, Phys. Rev. D 110 (2024) 034036.

Longitudinal SSA and Worm-gear

Longi-transversity worm-gear distribution

$$\frac{S_L k_T^{\alpha}}{M} h_{1L}^{\perp}(x, k_T^2)$$



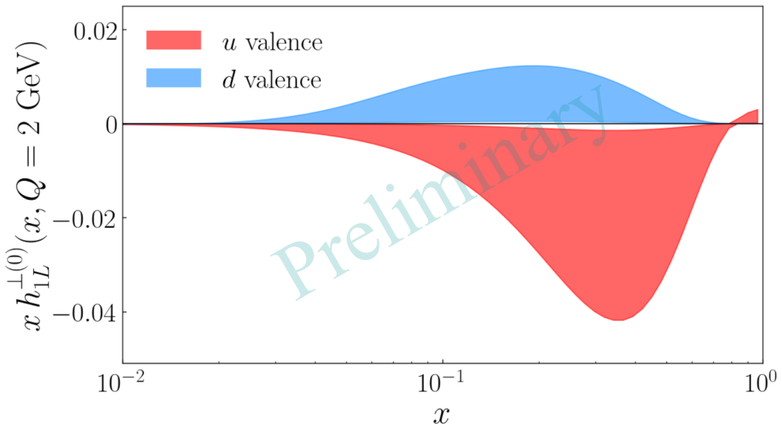
- Transversely polarized quark density in a longitudinally polarized nucleon
- Overlap between wave functions differing by one unit of orbital angular momentum

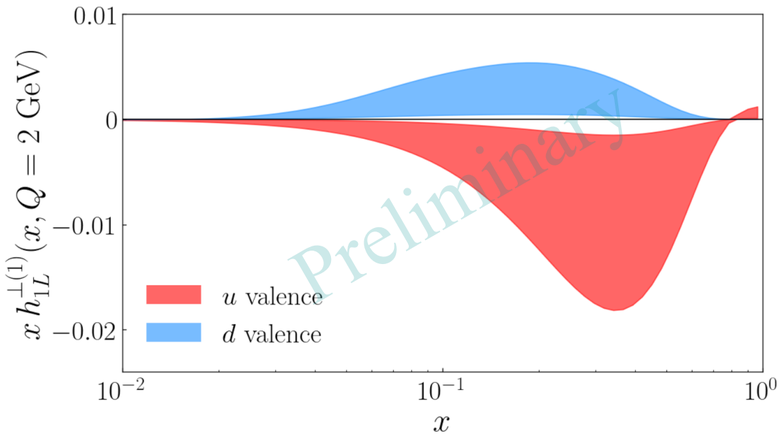


Effect in SIDIS:
A longitudinal target single spin asymmetry

$$A_{UL}^{\sin 2\phi_h} \sim h_{1L}^{\perp} \otimes H_1^{\perp}$$

First extraction





W. Chen, K. Yang, TL, P. Sun, Y. Zhao, in preparation, 2026

Summary

- Nucleon spin structure is still not well understood
- Rich information is contained in TMDs
  - helicity: quark polarization has nontrivial dependence on transverse momentum;
  - transversity: sea quarks may have nonzero transverse polarization, suggest intrinsic sea;
  - Sivers: quark transverse momentum is distorted by the nucleon transverse spin;
  - worm-gears: correlation between quark and nucleon longitudinal/transverse and transverse/longitudinal polarizations
  - …
- Polarized SIDIS and pp collisions are main processes to study polarized TMDs
- Electron-positron annihilation is important to constrain nucleon spin structures and to understand the role of spin in hadronization process
- Lattice QCD can provide complementary information to determine the distributions, especially when lacking precise data
- Opportunities from existing experiments at JLab12, BESIII, BelleII, and future facilities, EIC, EicC, STCF to understand nucleon spin structures and fragmentation functions.

Thank you!

Backup

Imposing a Bound?

Question:

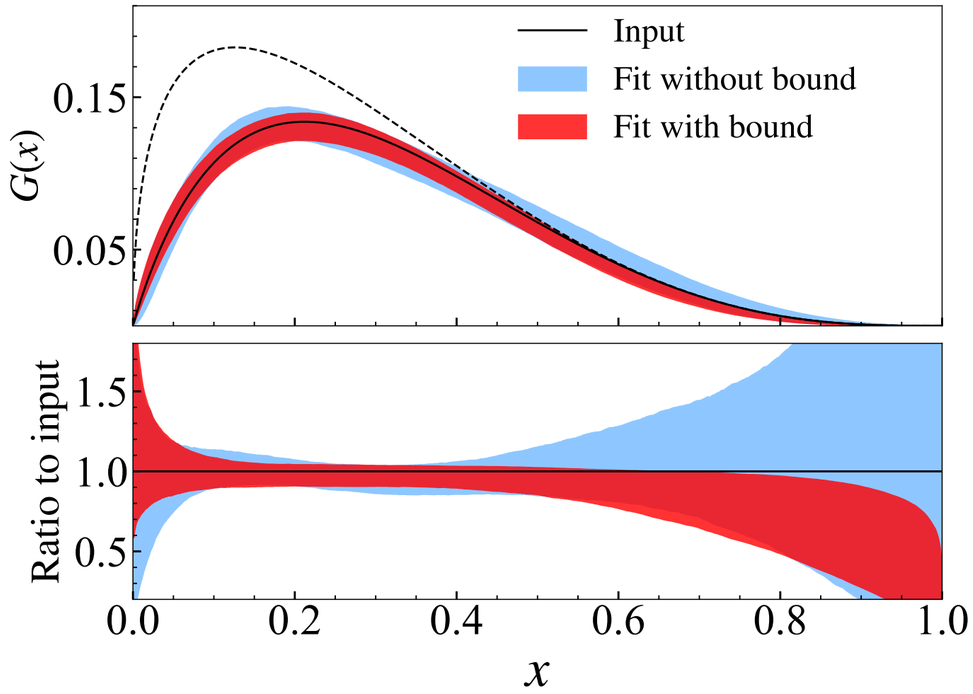
Whether we should impose the bound $|g_1(x)| \leq f_1(x)$ in the fit?
Similar situations in many other quantities.

A toy model test:

Input some "pol" function $G(x)$
and "unpol" function $F(x)$
Generate asymmetry "data"

Fit the "data" with or without
imposing the bound $G(x) \leq F(x)$

Imposing a bound in the fit may bias the fit result!

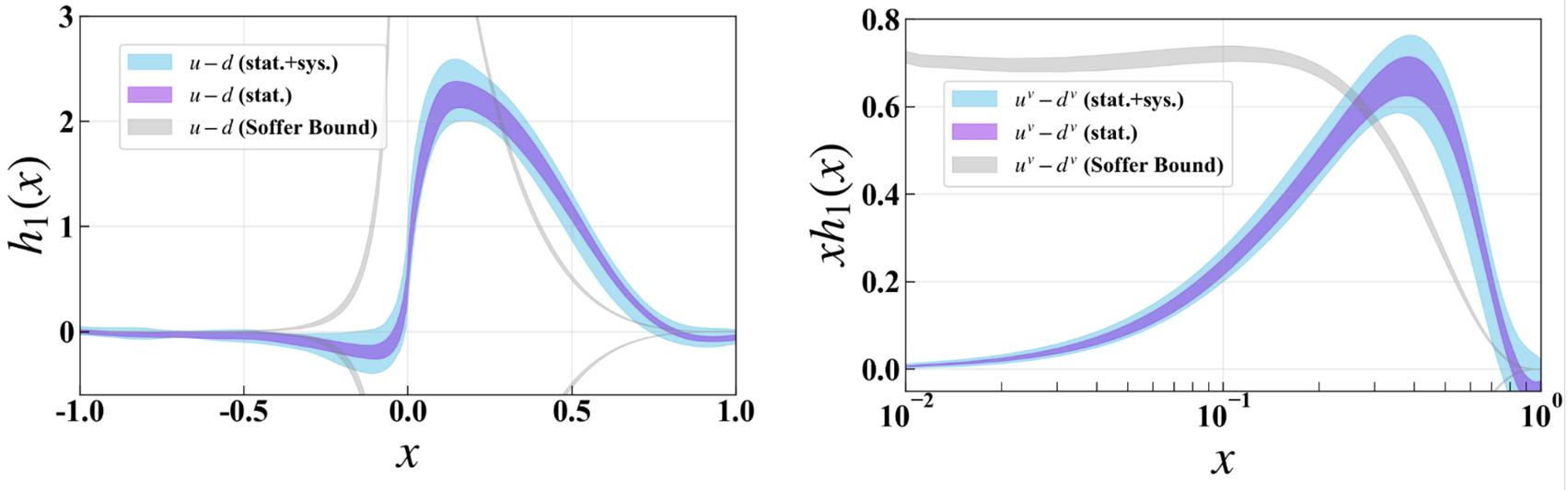


About Soffer's Bound

Soffer's bound on transversity distributions

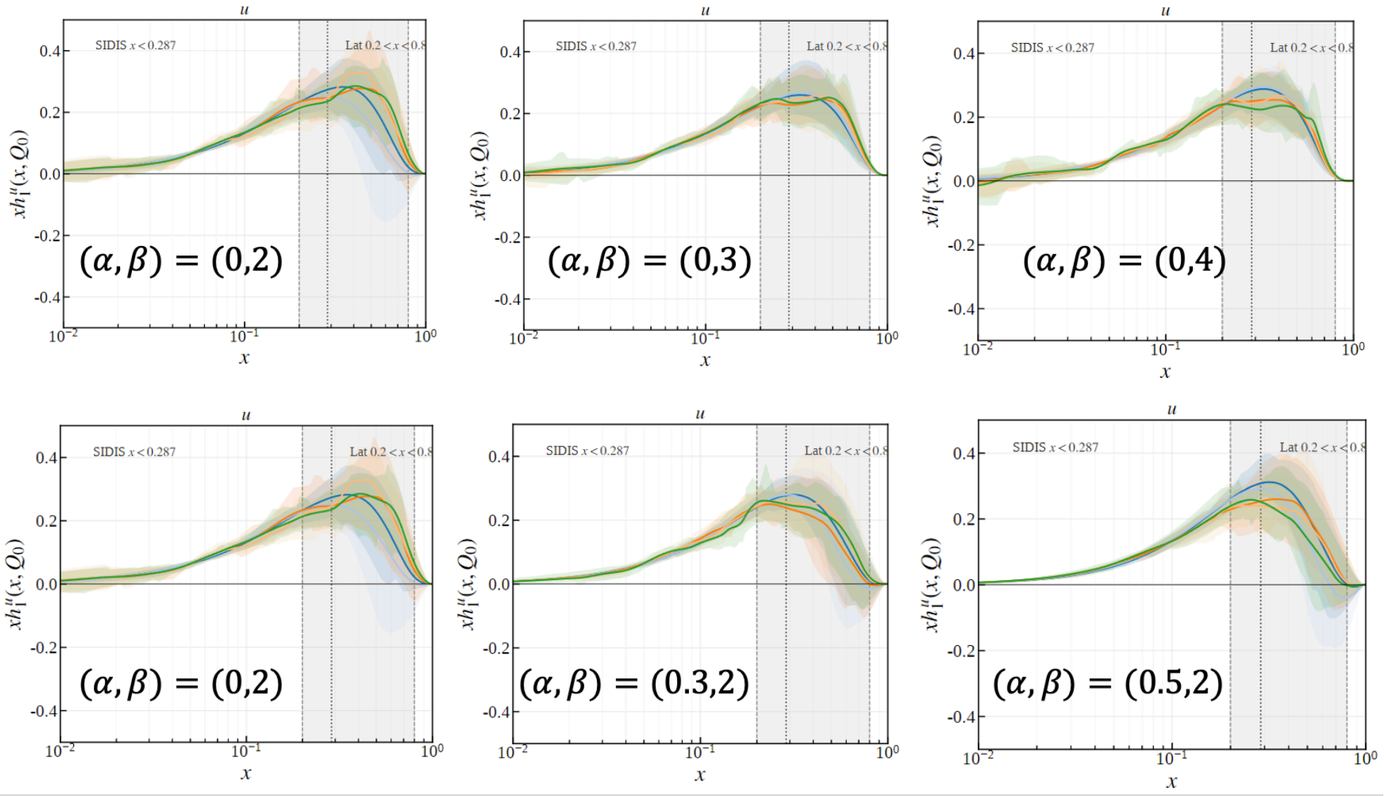
$$|h_1(x)| \leq \frac{1}{2}[f_1(x) + g_1(x)]$$

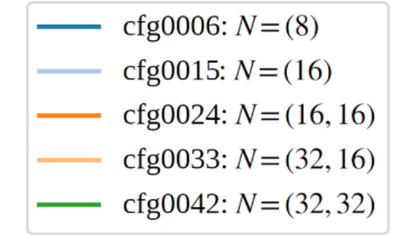
It is broken in existing lattice QCD calculations



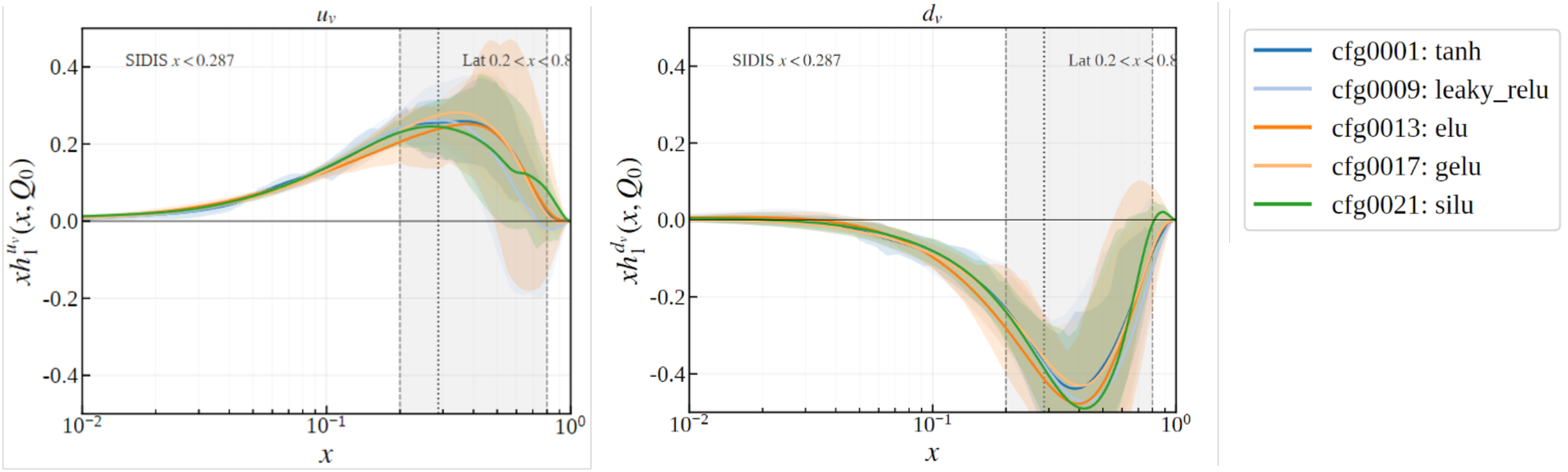
Lattice results from: LPC Collaboration, Phys. Rev. Lett. 131 (2023) 261901.

NN Hyperparameters



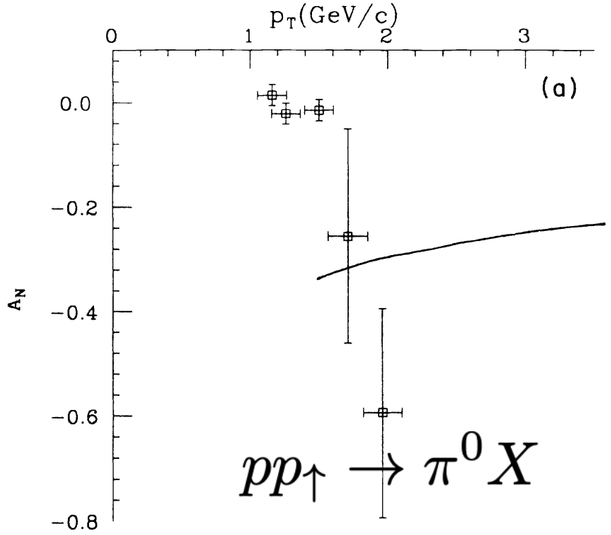


NN Hyperparameters

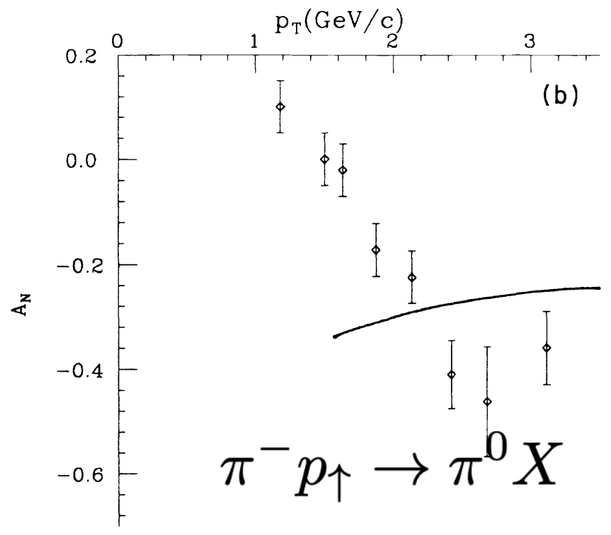


The Sivers Function: Early Story

Transverse single spin asymmetry observed in experiments



Data: J. Antille *et al*., Phys. Lett B94 (1980) 523.



Data: 7th Symposium on High Energy Spin Physics (1986).

D. Sivers proposed to explain such SSA a new distribution function

Sivers function $\Delta^N G_{a/p(\uparrow)}(x, \mathbf{k}_T; \mu^2)$ D. Sivers, Phys. Rev. D 41 (1990) 83.

However it was soon shown this function was T-odd and prohibited by QCD

J. Collins, Nucl. Phys. B 396 (1993) 161.

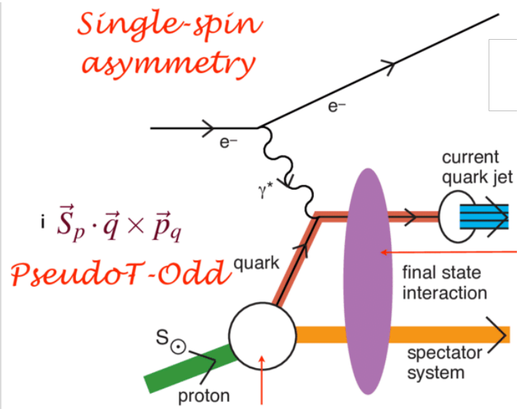
For the next decade, the "Sivers effect" was thought to vanish.

The Sivers Function: Early Story

Until an explicit model calculation showing …

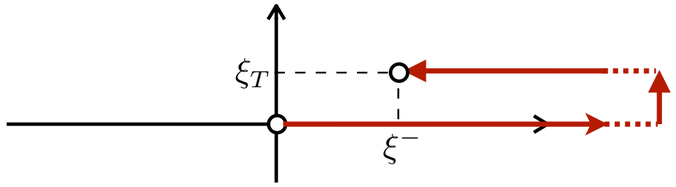
nonzero Sivers effects exist at leading twist due to final-state interactions

S.J. Brodsky, D.S. Hwang, I. Schmidt, Phys. Lett. B 530 (2002) 99.



Sivers function can exist due to nontrivial gauge link

$$\Phi_{ij}(x,p_T) = \int \frac{\mathrm{d}\xi^- \,\mathrm{d}^2\xi_T}{(2\pi)^3}\, e^{\mathrm{i}p\cdot\xi}\, \langle P|\bar{\psi}_j(0)\,\mathcal{U}^{n_-}_{(0,+\infty)}\,\mathcal{U}^{n_-}_{(+\infty,\xi)}\,\psi_i(\xi)|P\rangle\Big|_{\xi^+=0}$$



J.C. Collins, Phys. Lett. B 536 (2002) 43.

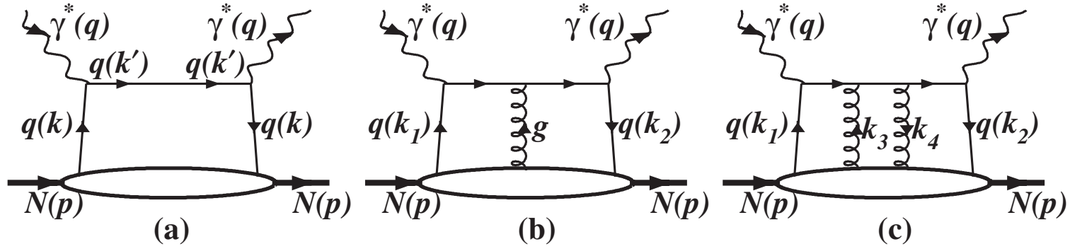
[Figure from A. Bacchetta]

This gauge link effect cannot be removed by choosing light-cone gauge $A^+ = 0$

X. Ji and F. Yuan, Phys. Lett. B 543 (2002) 66.

Collinear expansion

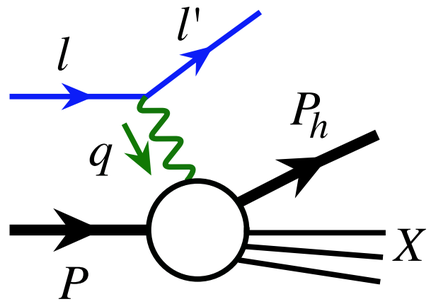
Z.T. Liang and X.N. Wang,
Phys. Rev. D 75 (2007) 094002.

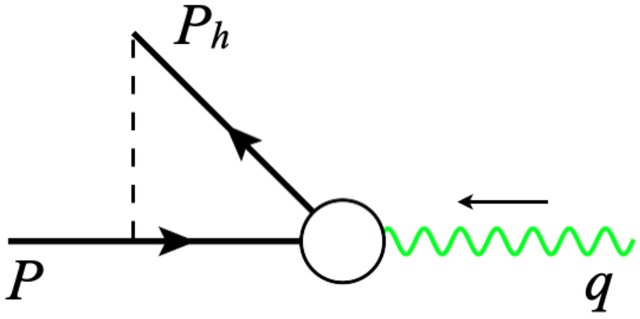


Semi-inclusive Deep Inelastic Scattering

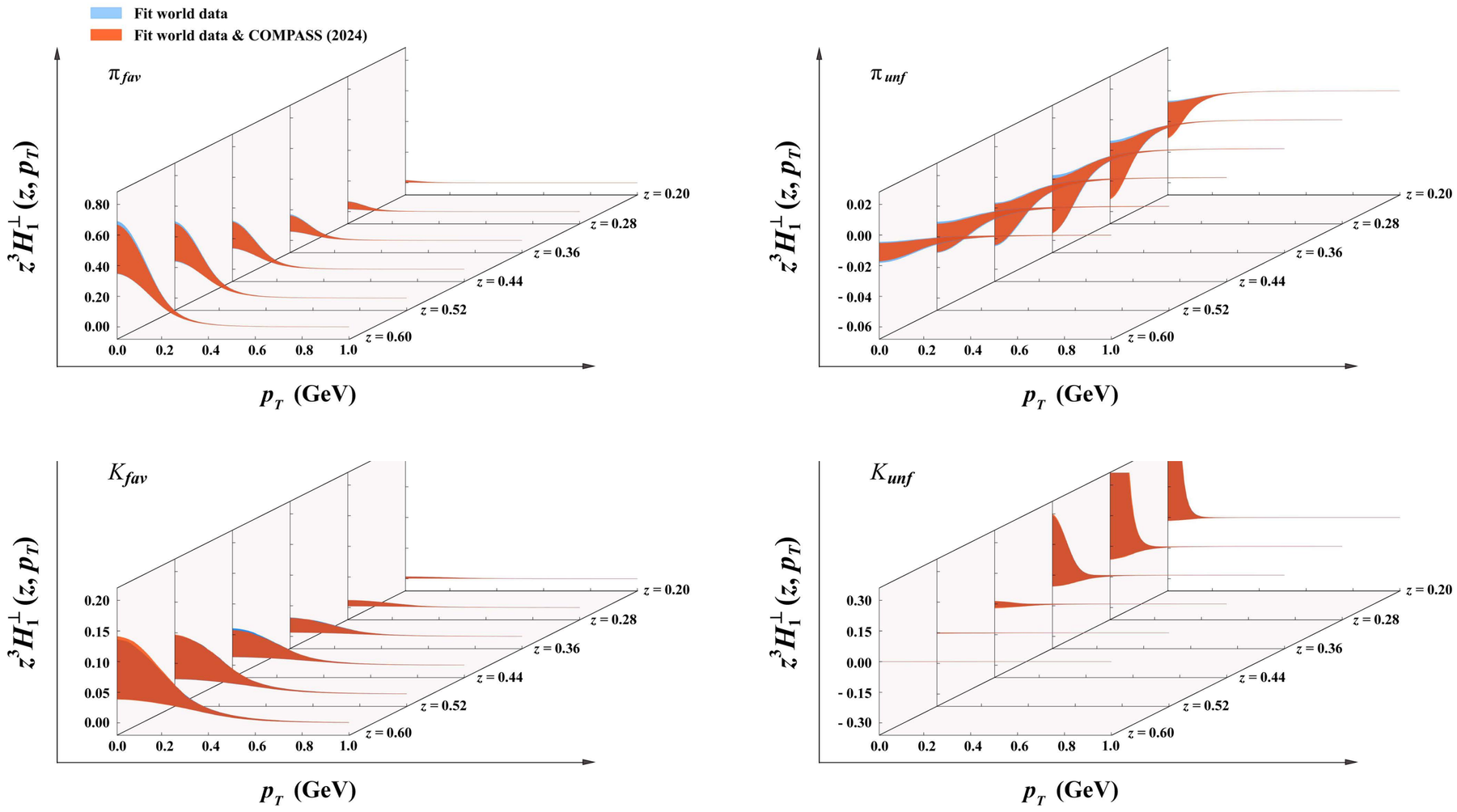
Semi-inclusive DIS: a final state hadron (P_h) is identified

- enable us to explore the emergence of color neutral hadrons from colored quarks/gluons
- flavor dependence by selecting different types of observed hadrons: pions, kaons, …
- a large momentum transfer Q provides a short-distance probe
- an additional and adjustable momentum scale P_{hT}
- multidimensional imaging of the nucleon





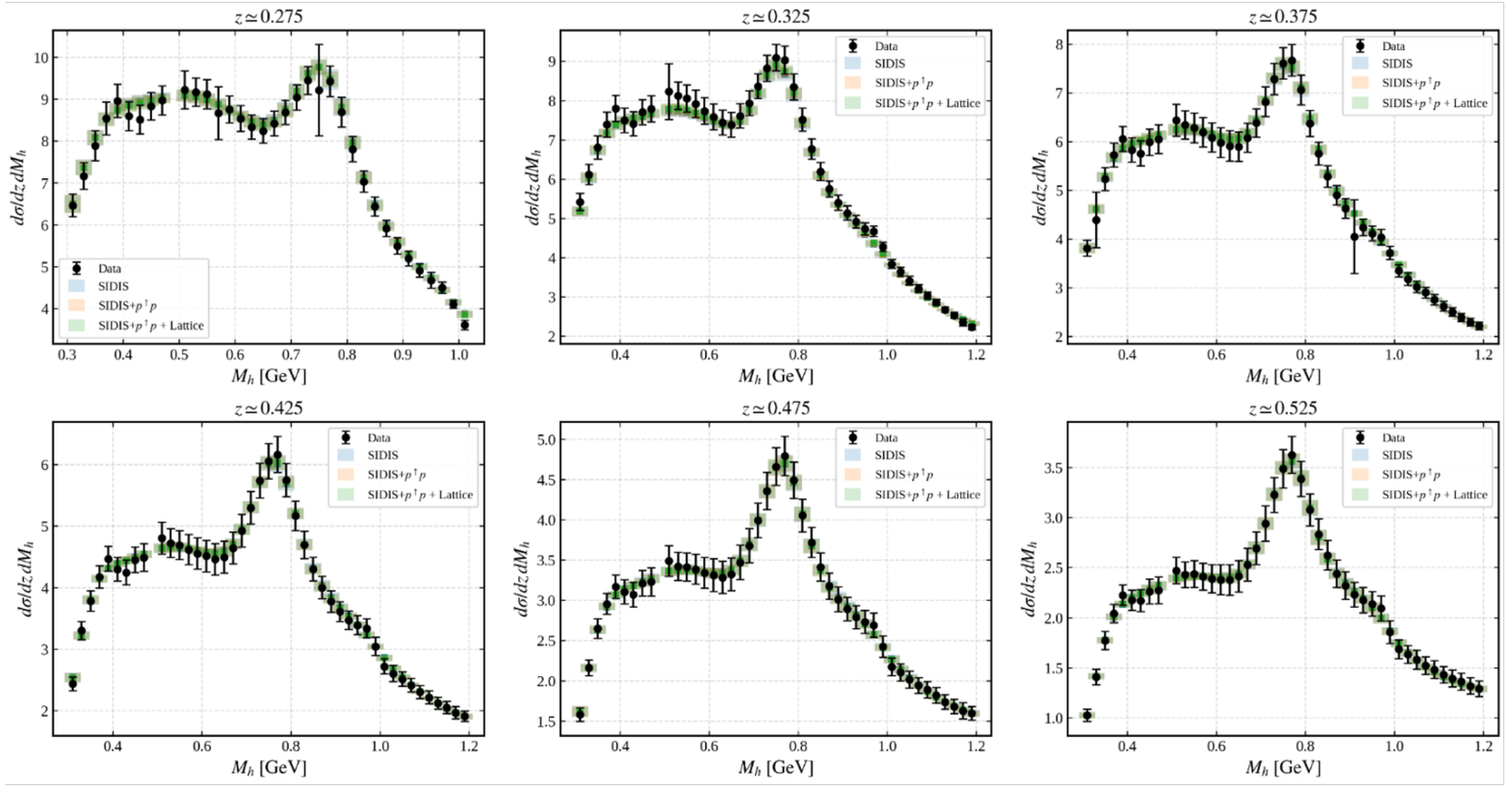
Collins Fragmentation Functions



C. Zeng, H. Dong, TL, P. Sun, Y. Zhao, arXiv:2412.18324

Comparison with Data

Comparison with Belle cross section data

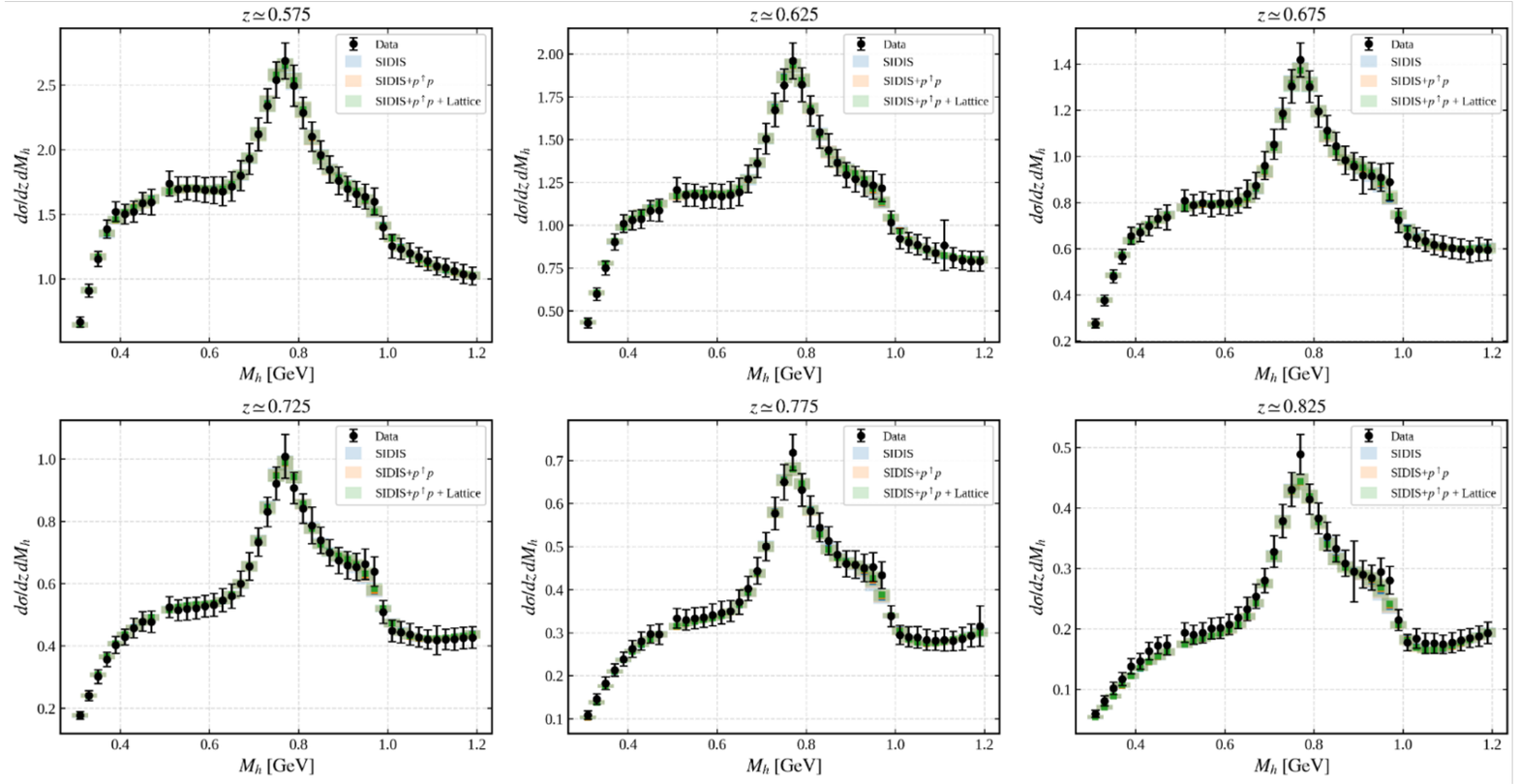


Data: Belle Collaboration, Phys. Rev. D 96 (2017) 032005.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with Belle cross section data

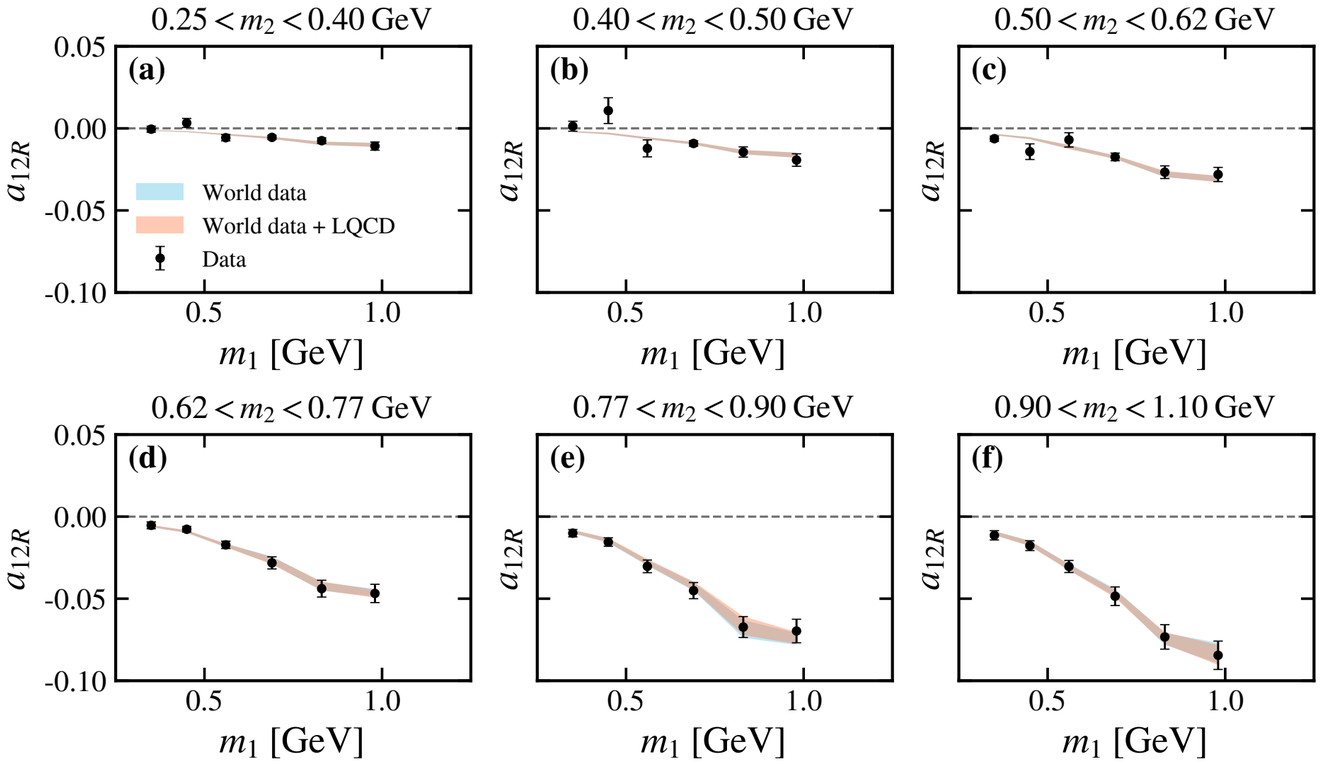


Data: Belle Collaboration, Phys. Rev. D 96 (2017) 032005.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with Belle asymmetry data

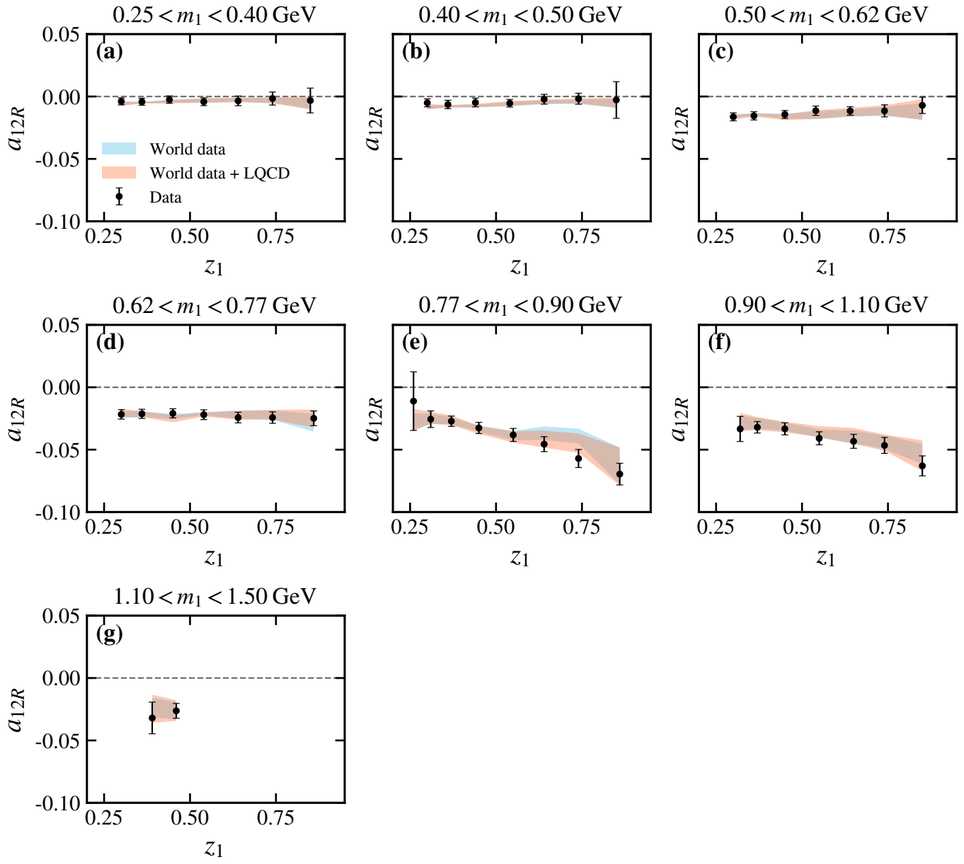


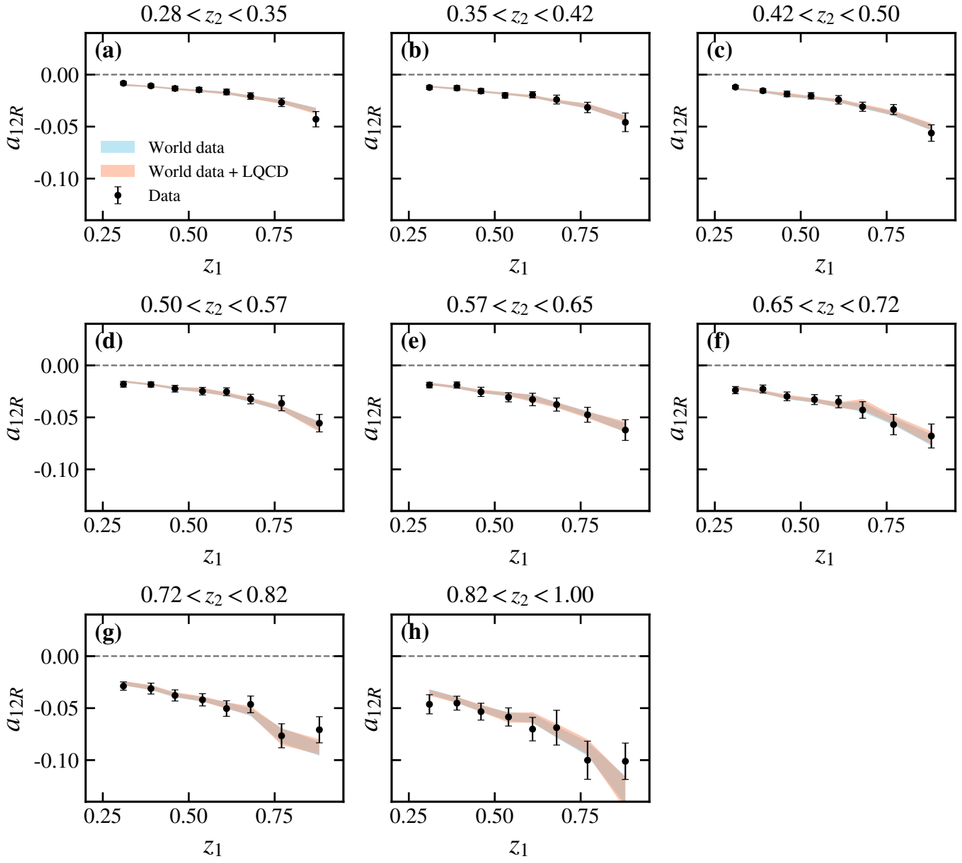
Data: Belle Collaboration, Phys. Rev. Lett. 107 (2011) 072004.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Comparison with Data

Comparison with Belle asymmetry data





Data: Belle Collaboration, Phys. Rev. Lett. 107 (2011) 072004.

H. Dong, TL, P. Sun, Y. Zhao, in preparation, 2026

Lepton-Hadron Deep Inelastic Scattering

Inclusive DIS at a large momentum transfer:

- dominated by the scattering of the lepton off an active quark/parton
- not sensitive to the dynamics at a hadronic scale ~ 1/fm
- collinear factorization:

$$\sigma \propto H(Q) \otimes f_{i/P}(x, \mu^2)$$

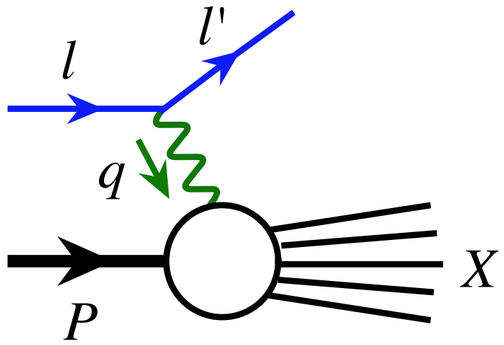
- overall corrections suppressed by $1/Q^n$
- indirectly "see" quarks, gluons and their dynamics
- predictive power relies on

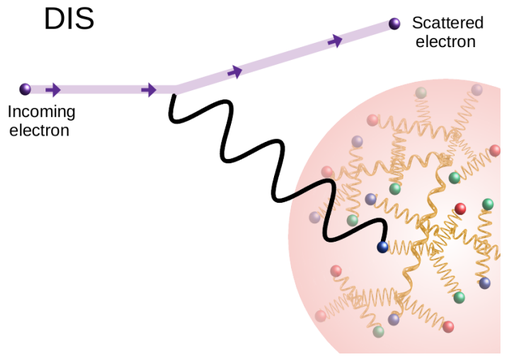
— precision of the probe

— universality of $f_{i/P}(x, \mu^2)$

$$Q \gg \Lambda_{\mathrm{QCD}}$$

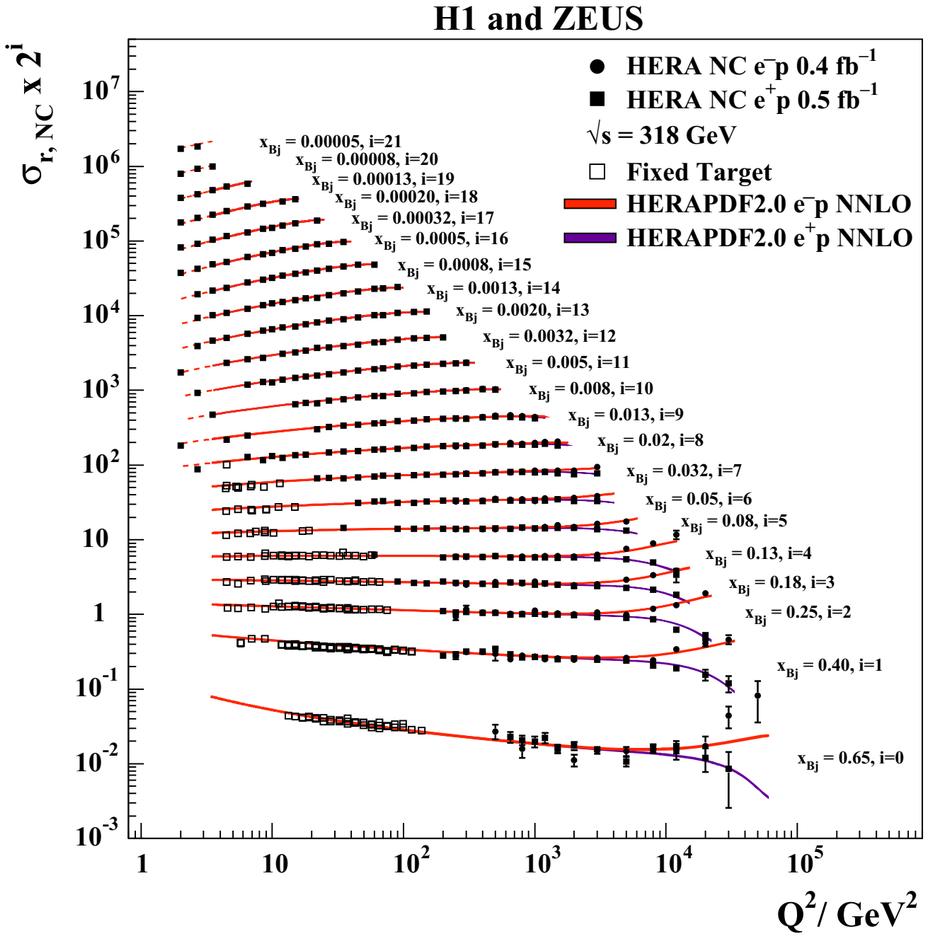
Modern "Rutherford" experiment.



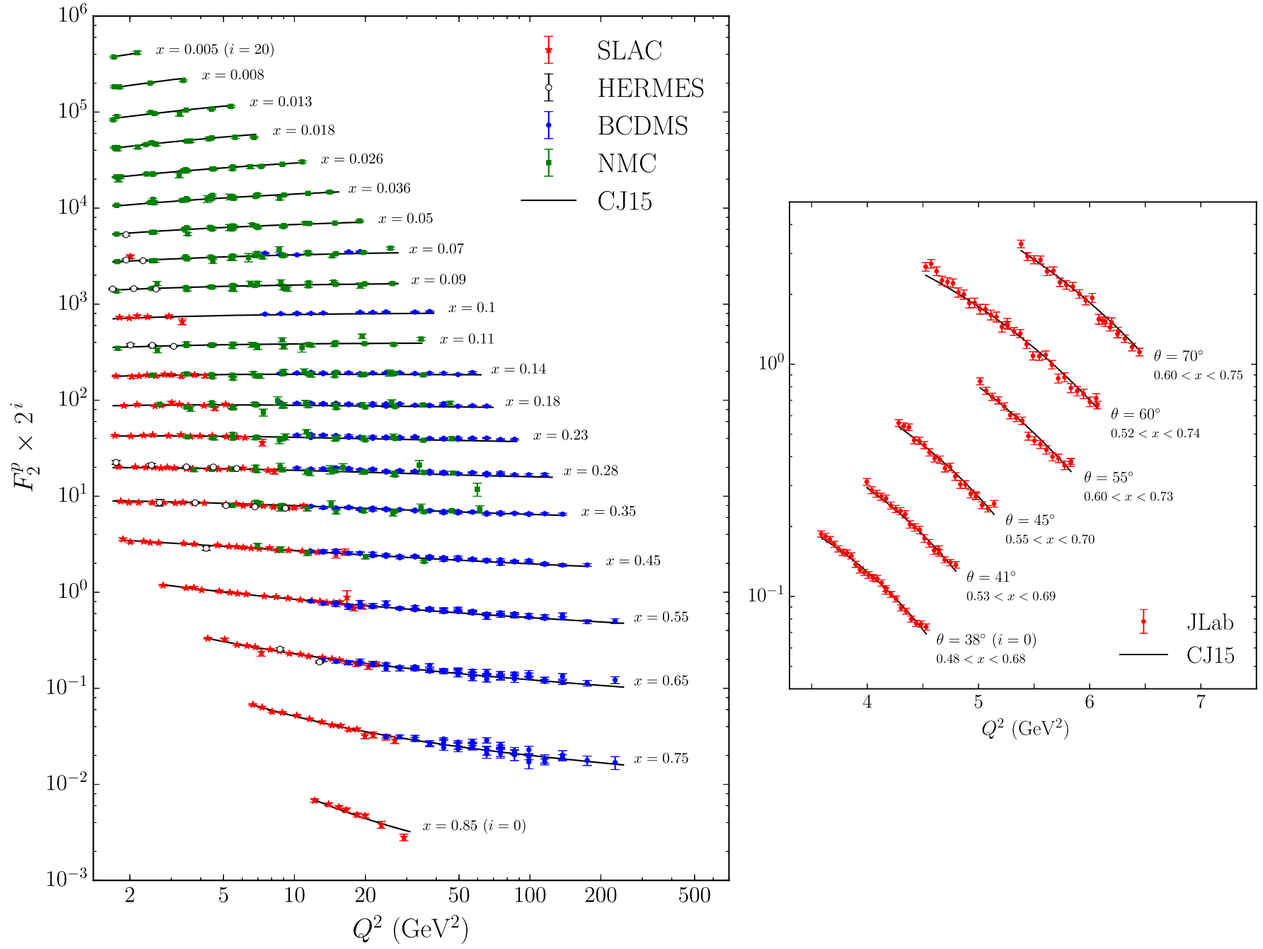


[Figure from DESY-21-099]

Lepton-Hadron Deep Inelastic Scattering

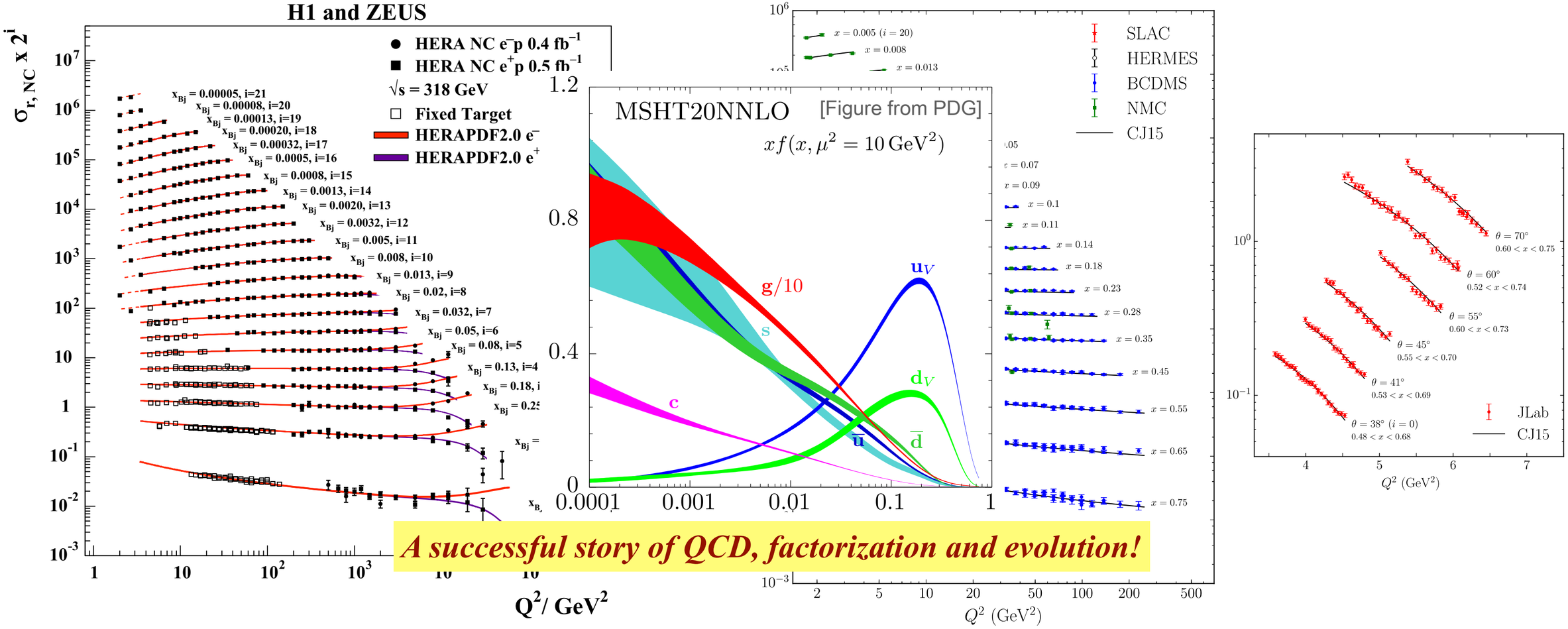


H. Abramowicz *et al.*, EPJC 78, 580 (2015).



A. Accardi *et al.*, PRD 93, 114017 (2016).

Lepton-Hadron Deep Inelastic Scattering



A successful story of QCD, factorization and evolution!

H. Abramowicz *et al.*, EPJC 78, 580 (2015).

A. Accardi *et al.*, PRD 93, 114017 (2016).