Internal structure of exotic hadrons by high-energy reactions

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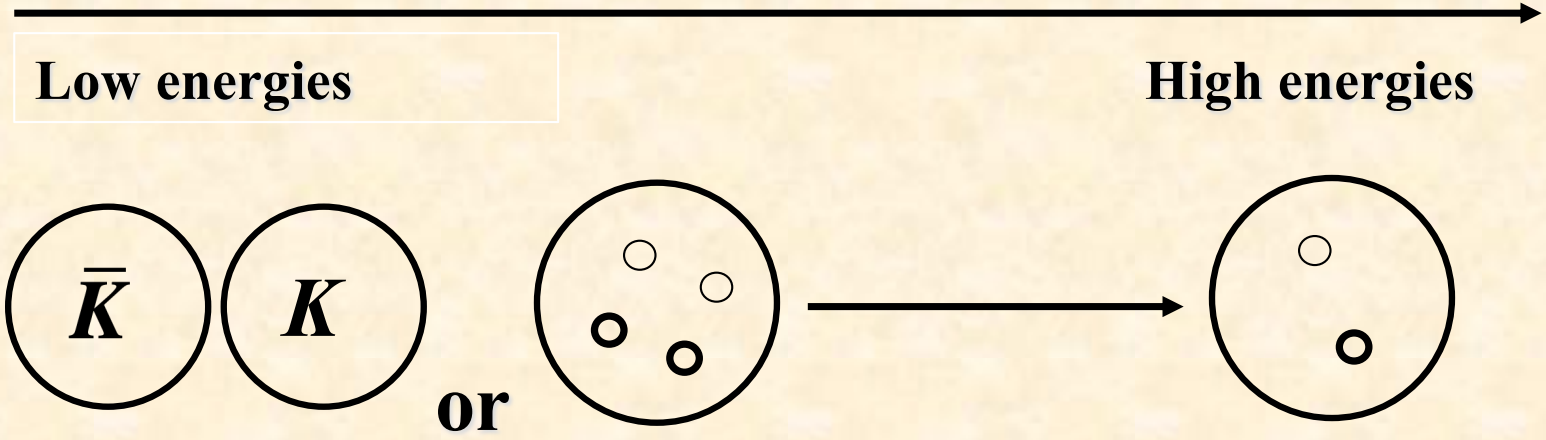
13th Circum-Pan-Pacific Symposium on High Energy Spin Physics
October 27-30, 2026, Academia Sinica, Taipei
https://indico.phys.sinica.edu.tw/event/371/

Collaborators: W.-C. Chang , M. Hirai, H. Kawamura,
M. Oka, T. Sekihara, K. Sudoh

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1. Introduction to exotic hadrons
2. Constituent-counting rule for exotic hadrons
3. Fragmentation functions of exotic hadrons
4. Summary

Suggestion from our studies:
Exotic-hadron configuration depends on the energy scale.



Constituent-counting rule for exotic hadrons

H. Kawamura, SK, T. Sekihara,
Phys. Rev. 88 (2013) 034010.
W.-C. Chang, S. Kumano, and T. Sekihara,
Phys. Rev. D 93 (2016) 034006.

Research purpose: pQCD counting rule

It is not easy to find undoubted evidence for exotic hadrons by global observables (mass, spin, parity, decay width) at low energies.

(1) Determination of internal structure of exotic hadrons by high energy processes, where quark-gluon degrees of freedom appear.

Constituent-counting rule could be used because it counts internal constituents.

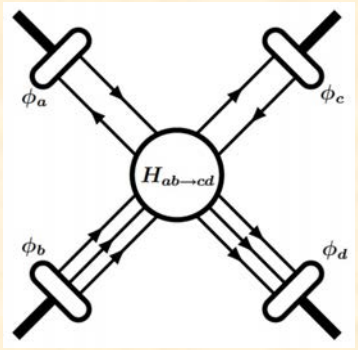
(2) Investigation on transition from hadron degrees of freedom to quark-gluon degrees of freedom for exotic hadrons.

$$\frac{d\sigma_{a+b\to c+d}}{dt} \simeq \frac{1}{16\pi s^2}\overline{\sum_{pol}}\left|M_{a+b\to c+d}\right|^2 \quad \Rightarrow \quad \frac{d\sigma_{a+b\to c+d}}{dt} = \frac{1}{s^{n-2}} f_{a+b\to c+d}(t/s)$$

consituent-counting rule

$$n = n_a + n_b + n_c + n_d$$

Constituent-counting rule in perturbative QCD: Hard exclusive processes $a + b \to c + d$



Consider the hard exclusive hadron reaction $a + b \to c + d$

$$M_{ab\to cd} = \int d[x_a]d[x_b]d[x_c]d[x_d]\phi_c([x_c])\phi_d([x_d])H_M([x_a],[x_b],[x_c],[x_d],Q^2)\phi_a([x_a])\phi_b([x_b])$$

ϕ_p = proton distribution amplitude, H_M = hard amplitude (calculated in pQCD)

Rule for estimating $M_{ab\to cd}$

(1) Feynman diagram: Draw leading and connected Feynman diagram by connecting $n/2$ quark lines by gluons.

(2) Gluon propagators: The factor $1/P^2$ is assigned for each gluon propagator.

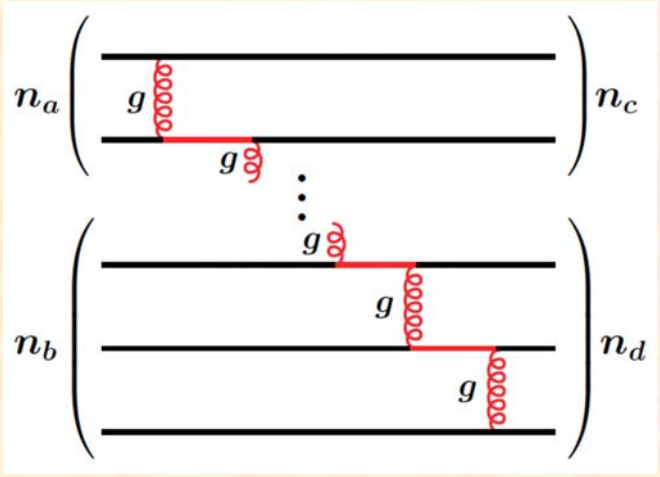
There are $n/2-1$ gluon propagators $\sim 1/(P^2)^{n/2-1}$.

(3) Quark propagators: The factor $1/P$ is assigned for each quark propagator.

There are $n/2-2$ gluon propagators $\sim 1/(P)^{n/2-2}$.

(4) External quarks: The factor $\sqrt{P}$ is assigned for each external quark.

There are n gluon propagators $\sim (\sqrt{P})^n$.



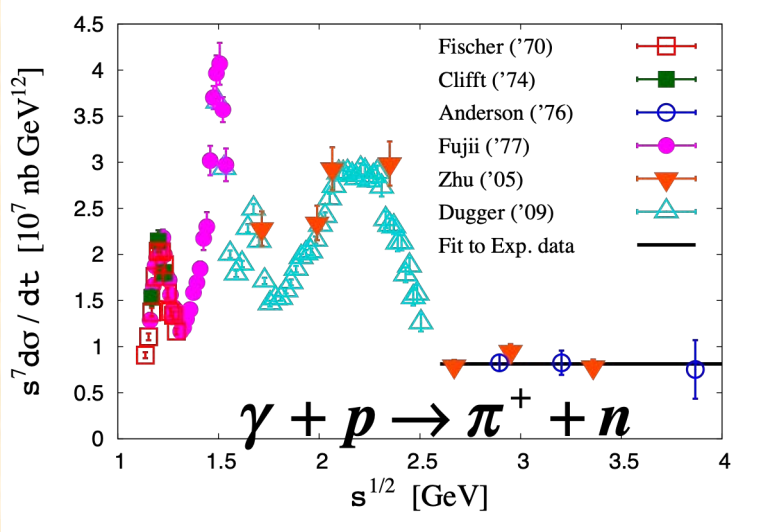
$$M_{ab\to cd} \sim \frac{1}{(P^2)^{n/2-1}}\frac{1}{(P)^{n/2-2}}(\sqrt{P})^n = \frac{(P)^{n/2}}{(P)^{n-2}(P)^{n/2-2}} = \frac{1}{(P)^{n-4}} \sim \frac{1}{s^{n/2-2}}$$

Cross section: $$\frac{d\sigma_{ab\to cd}}{dt} \simeq \frac{1}{16\pi^2}\overline{\sum_{spol}}\left|M_{ab\to cd}\right|^2 \sim \frac{1}{s^{n-2}}$$

Constituent-counting rule, Transition from hadron degrees of freedom to quark-gluon ones

Typical current situation

- **Transition from hadron d.o.f to quark d.o.f.**
- **(Looks like) Constituent-counting scaling**



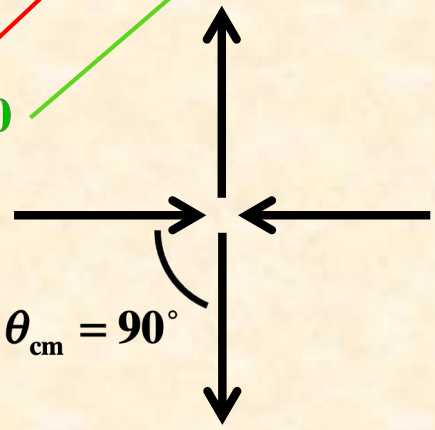
BNL experiment

C. White it et al., PRD 49 (1994) 58.

No.	Interaction	Cross section E838	Cross section E755	$n-2$ $(\frac{d\sigma}{dt} \sim 1/s^{n-2})$
1	$\pi^+ p \to p\pi^+$	132 ± 10	4.6 ± 0.3	6.7 ± 0.2
2	$\pi^- p \to p\pi^-$	73 ± 5	1.7 ± 0.2	7.5 ± 0.3
3	$K^+ p \to pK^+$	219 ± 30	3.4 ± 1.4	$8.3^{+0.6}_{-1.0}$
4	$K^- p \to pK^-$	18 ± 6	0.9 ± 0.9	≥ 3.9
5	$\pi^+ p \to p\rho^+$	214 ± 30	3.4 ± 0.7	8.3 ± 0.5
6	$\pi^- p \to p\rho^-$	99 ± 13	1.3 ± 0.6	8.7 ± 1.0
13	$\pi^+ p \to \pi^+ \Delta^+$	45 ± 10	2.0 ± 0.6	6.2 ± 0.8
15	$\pi^- p \to \pi^+ \Delta^-$	24 ± 5	≤ 0.12	≥ 10.1
17	$pp \to pp$	3300 ± 40	48 ± 5	9.1 ± 0.2
18	$\bar{p}p \to p\bar{p}$	75 ± 8	≤ 2.1	≥ 7.5

$n-2$: $(2+3+2+3)-2=8$

$(3+3+3+3)-2=10$

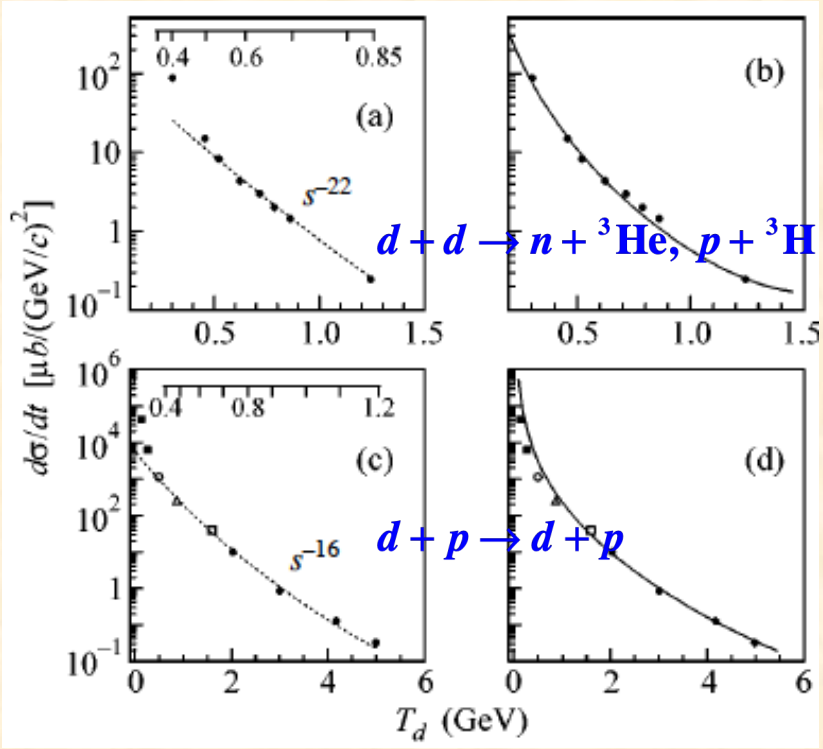


JLab: L.Y. Zhu *et al.*, PRL 91, 022003 (2003);
PRC 71, 044603 (2005);
W. Chen *et al.*, PRL 103, 012301 (2009).

see R. A. Schumacher and M. M. Sargsian,
PRC 83 (2011) 025207 for hyperon production

Constituent-counting rule for "molecular" systems

Y. N. Uzikov, JETP Lett. 81, 303 (2005).



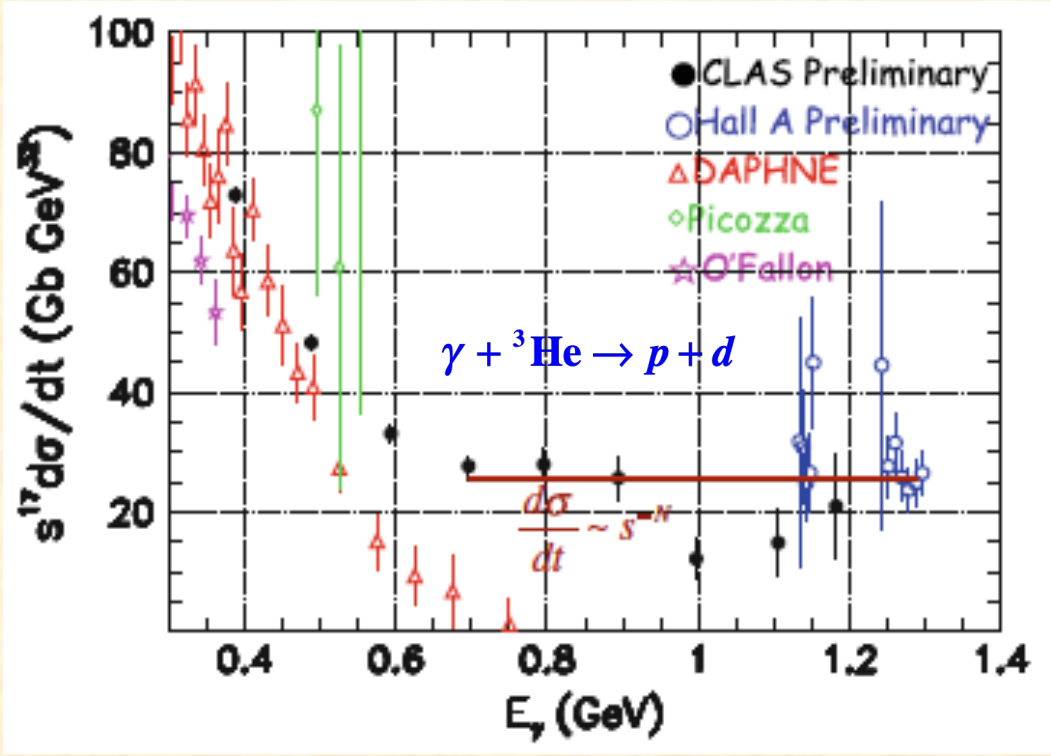
$$d + d \to n + {}^3\mathrm{He},\ p + {}^3\mathrm{H}$$

$$n - 2 = (6 + 6 + 3 + 9) - 2 = 22$$

$$d + p \to d + p$$

$$n - 2 = (6 + 3 + 6 + 3) - 2 = 16$$

Y. Ilieva, Few Body Syst. 54, 989 (2013).

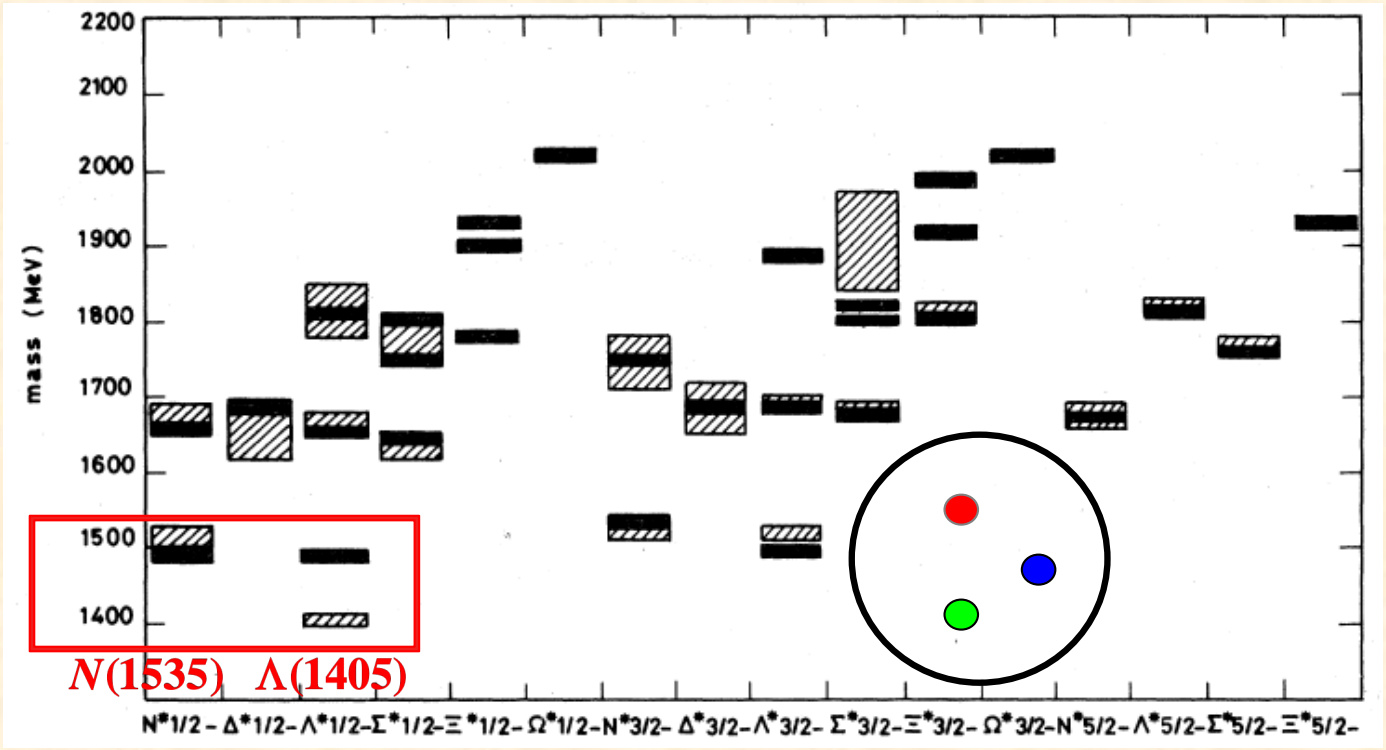


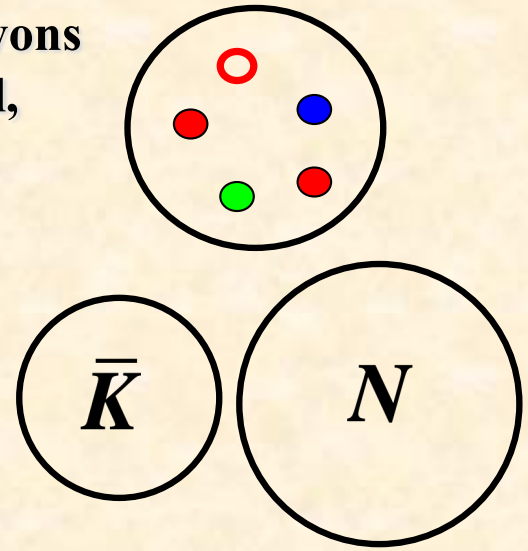
$$\gamma + {}^3\mathrm{He} \to p + d$$

$$n - 2 = (1 + 9 + 3 + 6) - 2 = 17$$

Λ(1405): exotic hadron?

Negative-parity baryons
N. Isgur and G. Karl,
PRD 18 (1978) 4187.





Our proposal:

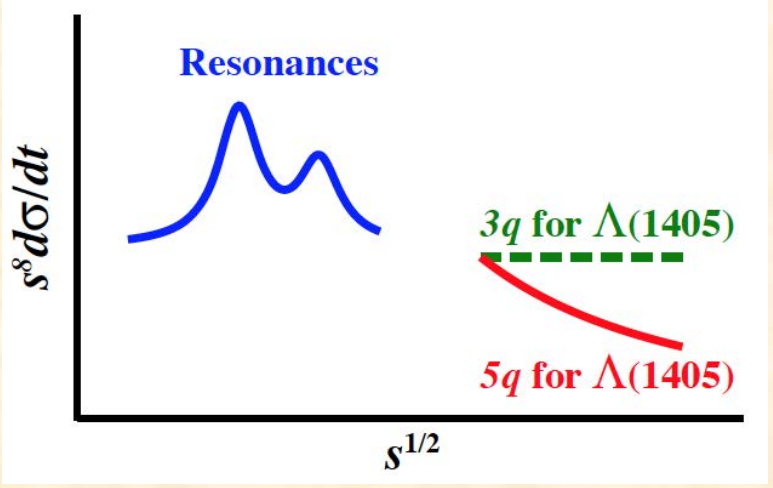
Exotic hadron production

$\pi^- + p \to K^0 + \Lambda(1405)$: J-PARC, COMPASS?

$\gamma + p \to K^+ + \Lambda(1405)$: JLab, EICs

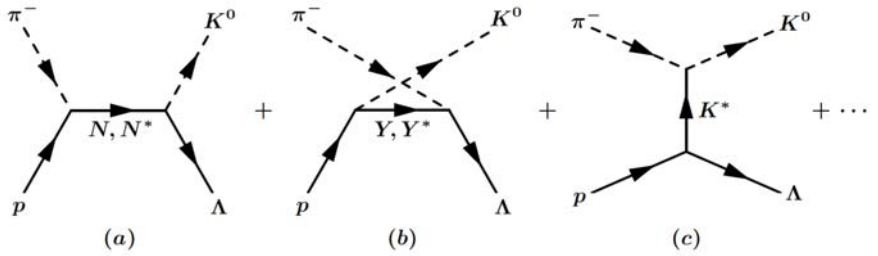
Most spectra agree with the ones by a 3*q*-picture

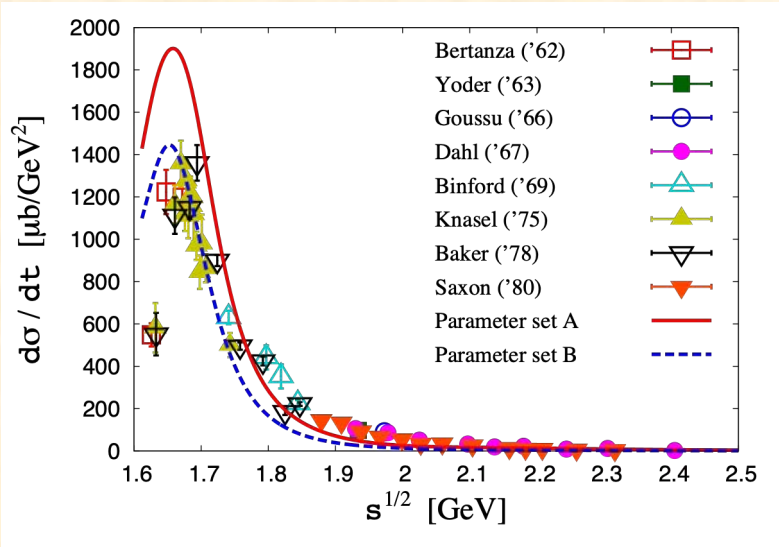
- **Only Λ(1405) deviates from the measurement.**
- **Difficult to understand the small mass of Λ(1405) in comparison with *N*(1535).**
  → $\bar{K}N$ molecure or penta-quark ($qqqq\bar{q}$)?



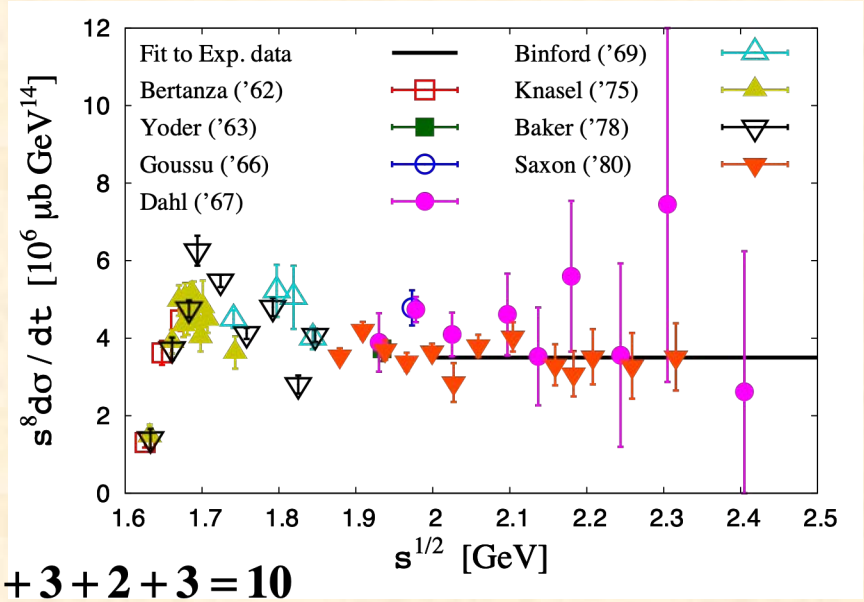
Ordinary-hadron production $\pi^- + p \to K^0 + \Lambda$ as a reference

At low energies





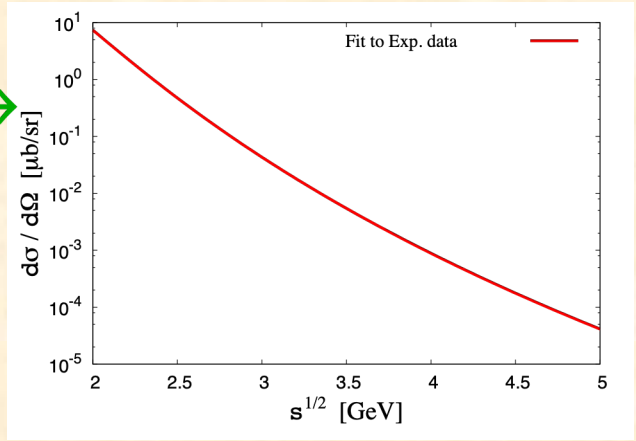
From low to higher energies



$$n = 2 + 3 + 2 + 3 = 10$$

$$\frac{d\sigma_{ab\to cd}}{dt} = \frac{\text{const}}{s^{n-2}}, \quad n = 10.1 \pm 0.6, \text{ encouraging!}$$

Our prediction at high energies →



Exotic-hadron production $\pi^- + p \to K^0 + \Lambda(1405)$

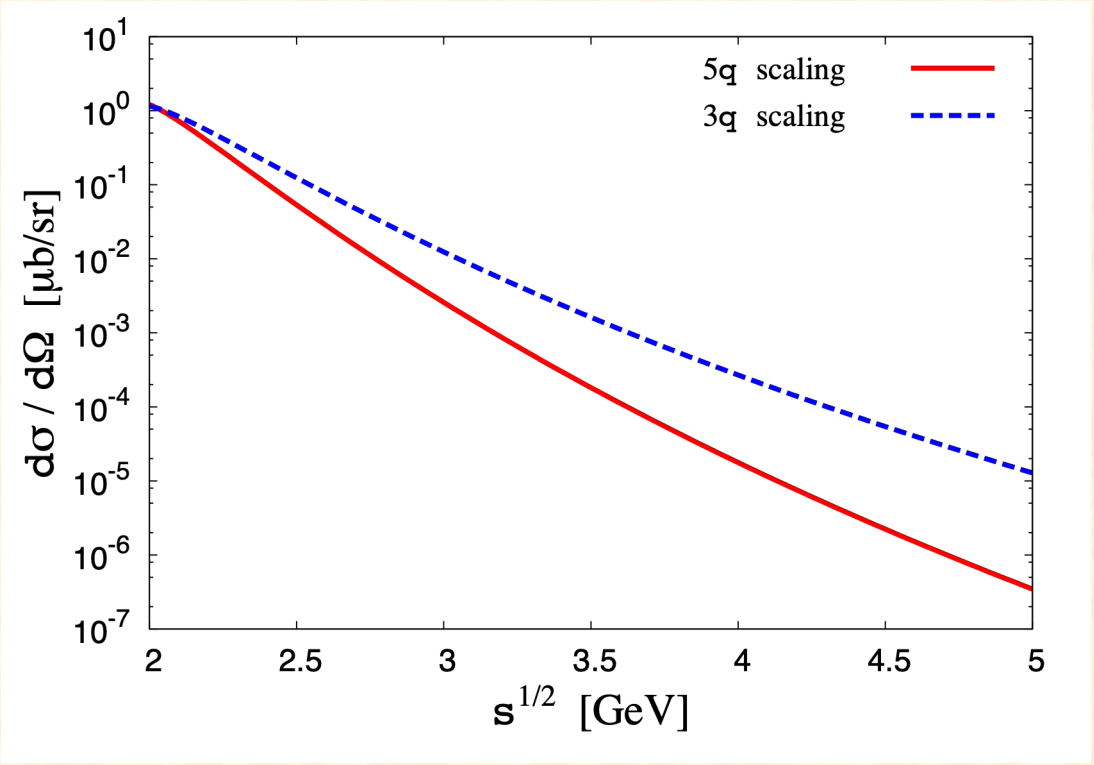
Theoretical and experimental situation is not as good as the one for the ground Λ.

$n = 2+3+2+3 = 10$ if $\Lambda(1405)$ = three-quark state

$= 2+3+2+5 = 12$ if $\Lambda(1405)$ = five-quark state (including $\bar{K}N$ molecule)

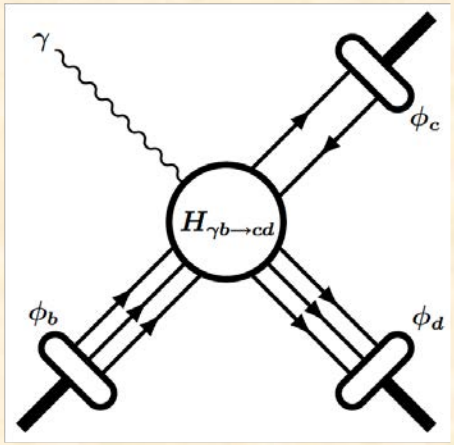
$$\frac{d\sigma_{ab\to cd}}{dt} = \frac{\text{const}}{s^{n-2}}, \quad n = 10 \text{ or } 12$$

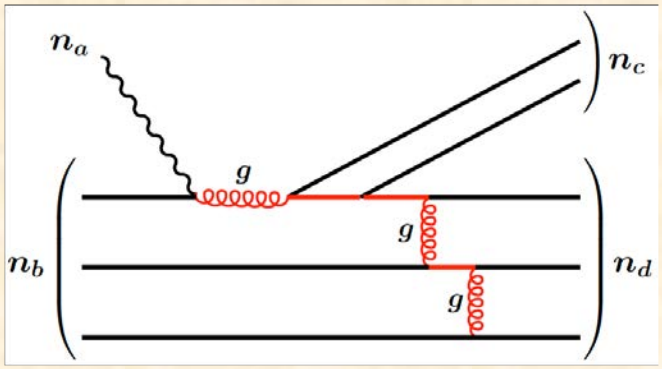
Our prediction at high energies

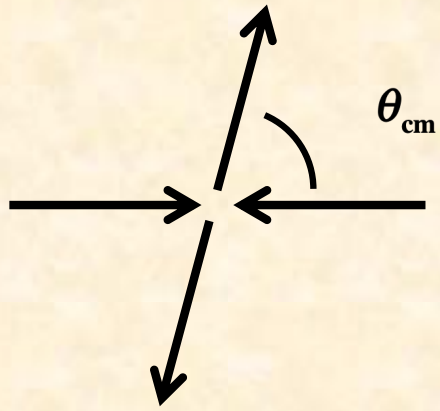


Hard production of hyperons

JLab hyperon productions







5 bins: $-0.25 < \cos\theta_{cm} < -0.15$, ⋯, $0.15 < \cos\theta_{cm} < 0.25$

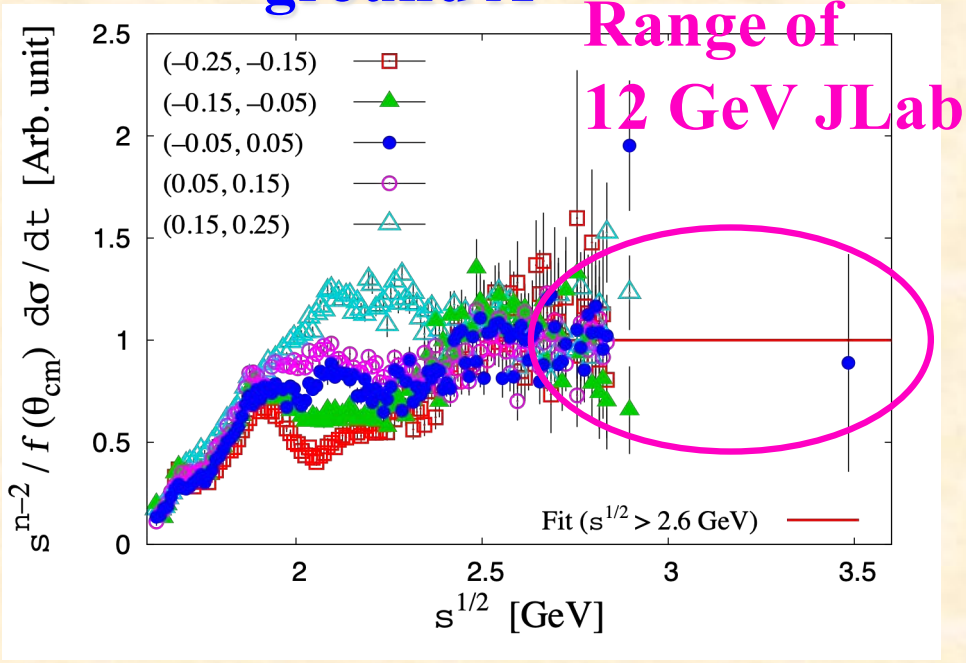
4 bins: $-0.20 < \cos\theta_{cm} < -0.10$, ⋯, $0.10 < \cos\theta_{cm} < 0.20$

...

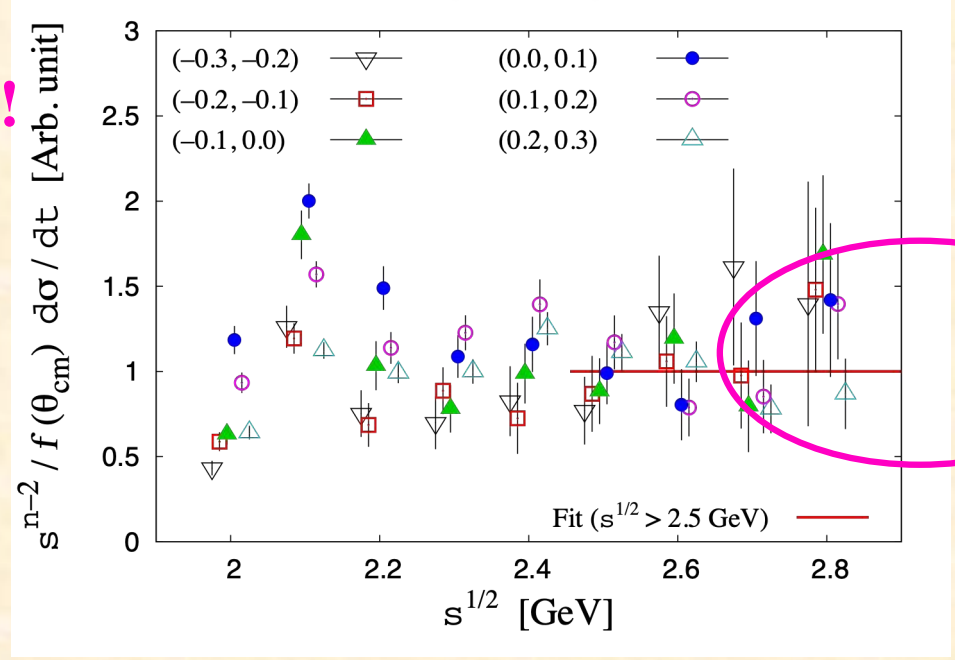
1 bin: $-0.05 < \cos\theta_{cm} < +0.05$

ground Λ

Range of 12 GeV JLab!

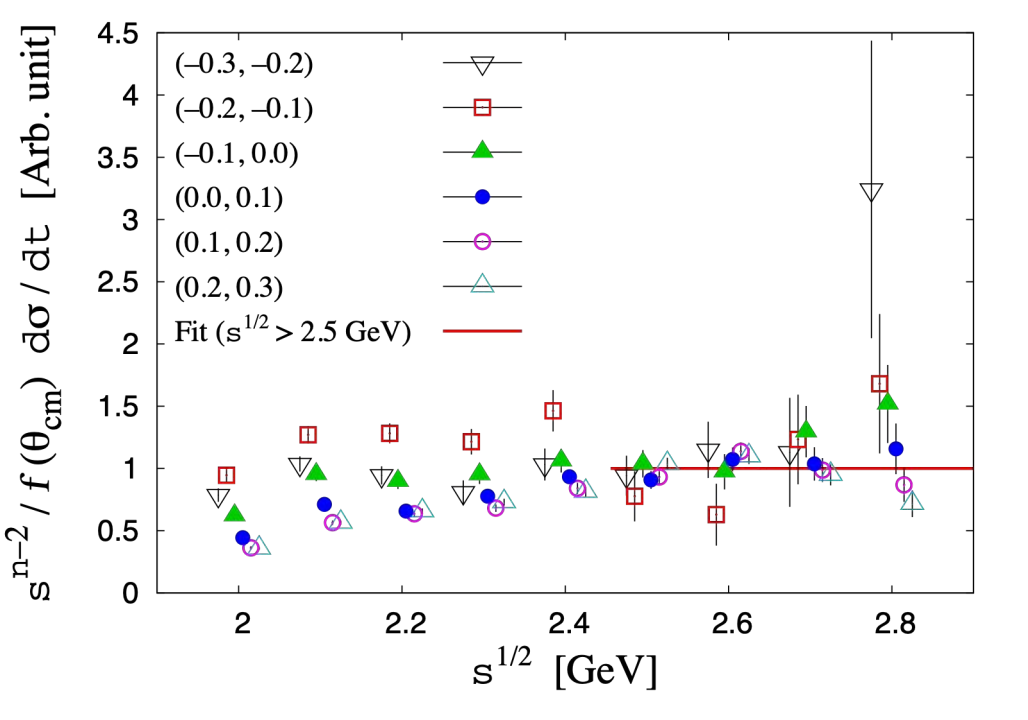


Λ(1405)

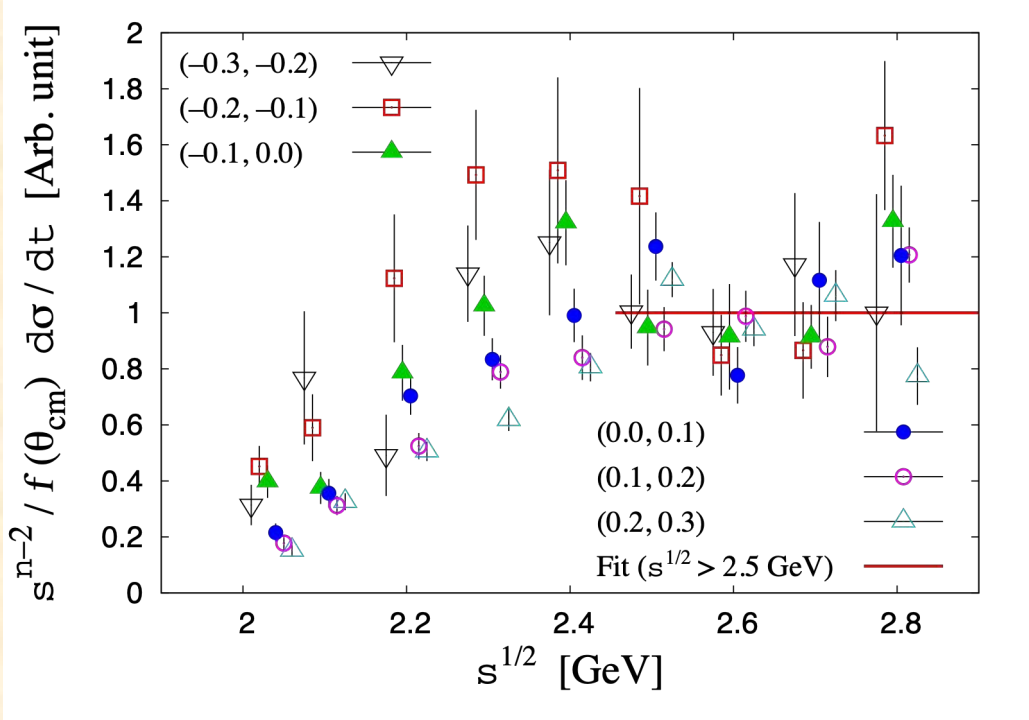


Hyperon productions

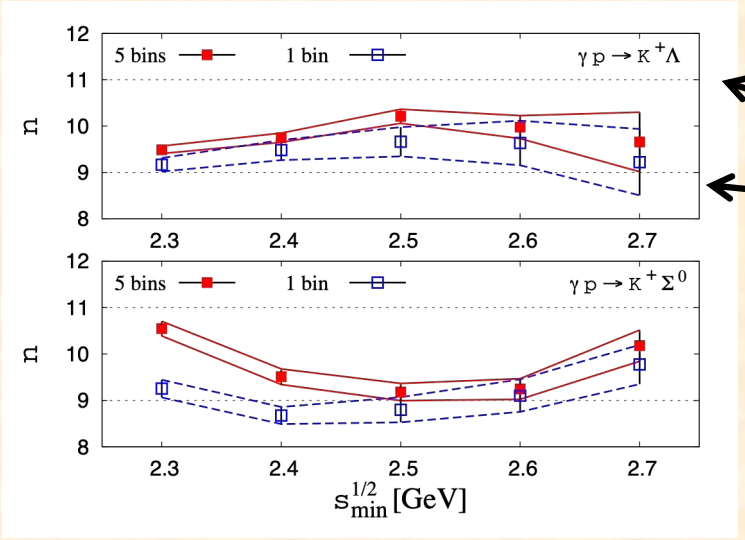
$\Sigma^0(1385)$



$\Lambda(1520)$

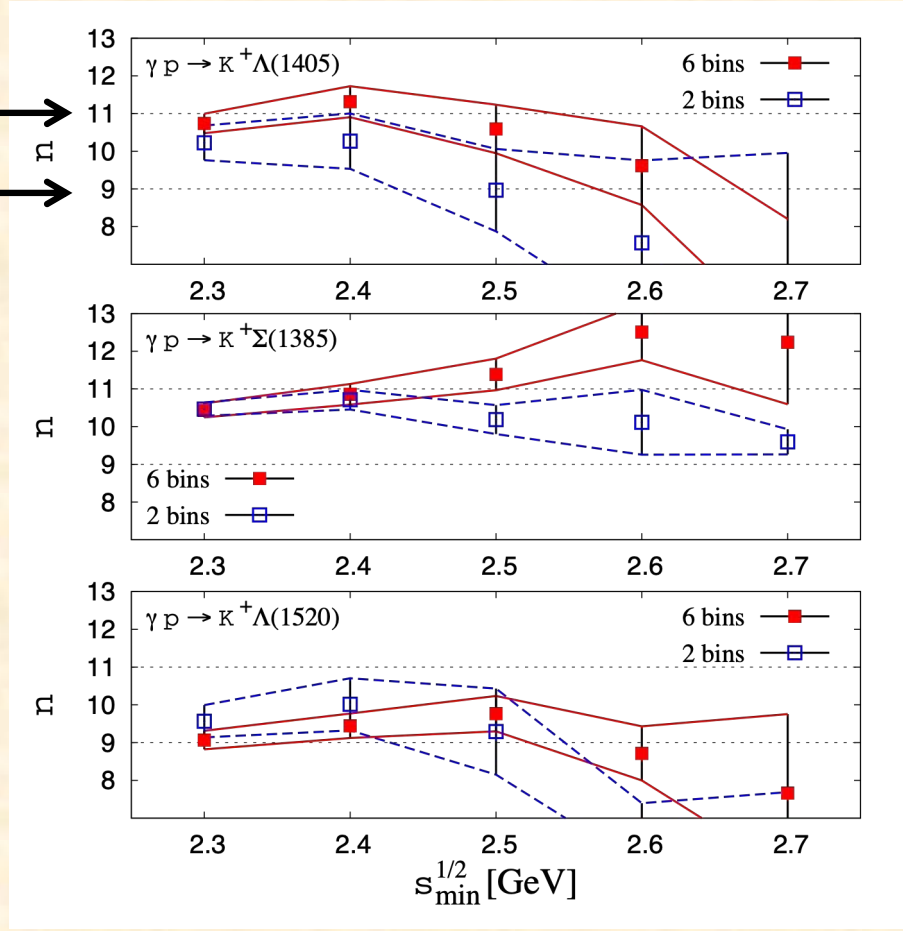


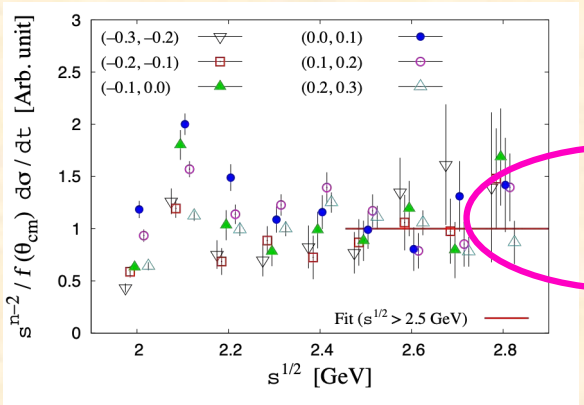
JLab hyperon productions including Λ(1405)



$n_\Lambda = 5$

$n_\Lambda = 3$





Range of
12 GeV JLab!

EIC

- Λ. Λ(1520) and Σ seem to be consistent with ordinary baryons with $n = 3$.
- Λ(1405) looks penta-quark at low energies but $n \sim 3$ at high energies???
- Σ(1385) : $n = 5$???

→ In order to clarify the nature of Λ(1405) $\left[qqq,\ \bar{K}N,\ qqqq\bar{q}\right]$, the JLab 12-GeV and EIC experiments play an important role!

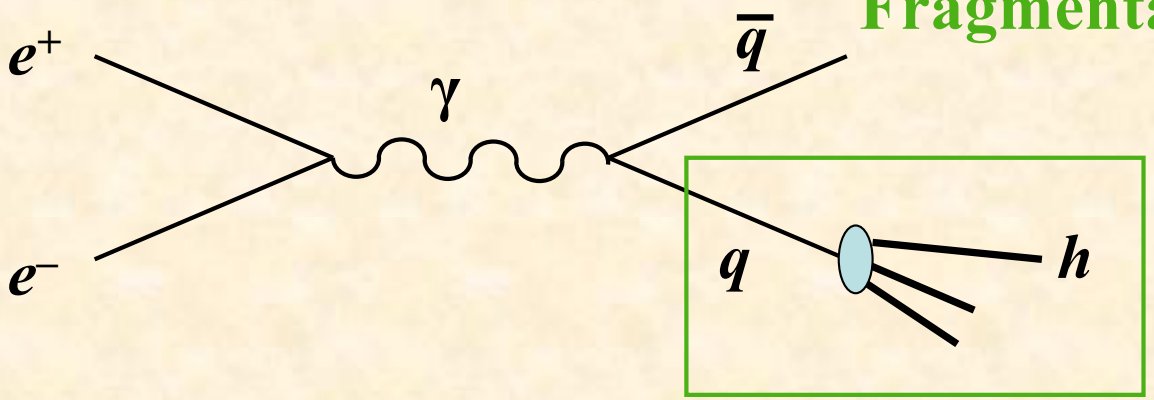
W.-C. Chang, SK, T. Sekihara, PRD 93 (2016) 034006.

Exotic hadrons by fragmentation functions

M. Hirai, SK, M. Oka, K. Sudoh,
Phys. Rev. D77 (2008) 017504;

SK, Phys. Rev. D 114 (2026) L071501.

Fragmentation Function



Fragmentation: hadron production from a quark, antiquark, or gluon

Fragmentation function is defined by

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

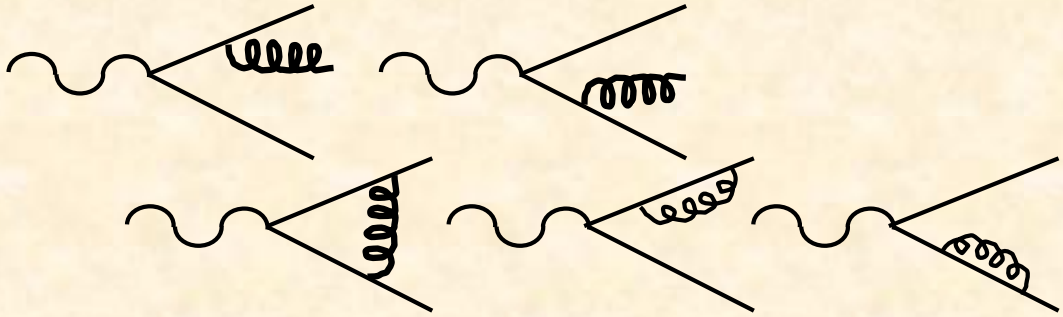
$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q}$$

(Energy fraction = hadron energy scaled to the beam energy)

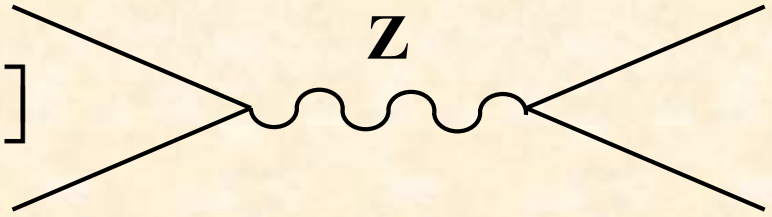
σ_{tot} = total hadronic cross section

$$\sigma_{tot} = \sum_q \sigma_0^q(s)\left[1 + \boxed{\frac{\alpha_s(s)}{\pi}}\right]$$

Higher-order correction



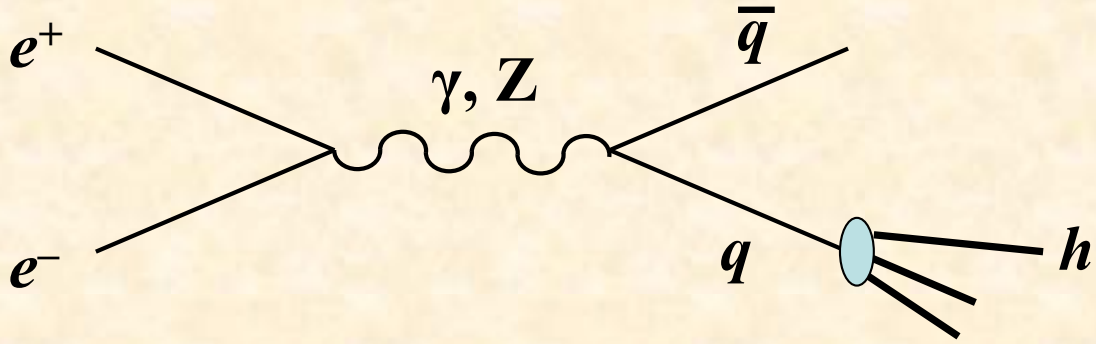
$$\sigma_0^q(s) = \frac{4\pi\alpha^2}{s}\left[e_q^{\,2} + (\gamma\text{-}Z \text{ interference}) + (Z \text{ term})\right]$$



Variable z

$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}$$

note $s = Q^2$



- Hadron energy / Beam energy
- Hadron energy / Primary quark or antiquark energy

Fragmentation functions

A fragmentation process occurs from quarks, antiquarks, and gluons, so that F^h is expressed by their contributions:

$$F^h(z,Q^2) = \sum_i \int_z^1 \frac{dy}{y} C_i\left(\frac{z}{y},Q^2\right) D_i^h(y,Q^2)$$

$C_i\left(\frac{z}{y},Q^2\right)$: Calculated in perturbative QCD

$D_i^h(y,Q^2)$: Non-perturbative (determined from experiments)

$C_i(z,Q^2)$ = coefficient function

$D_i^h(z,Q^2)$ = fragmentation function of hadron h from a parton i

Energy sum rule, Favored and disfavored functions

$D_i^h\left(z,Q^2\right)=$ probability to find the hadron h from a parton i with the energy fraction z

Energy conservation: $\sum_h \int_0^1 dz\; z\, D_i^h\left(z,Q^2\right)=1$

$h=\pi^+,\ \pi^0,\ \pi^-,\ K^+,\ K^0,\ \bar{K}^0,\ K^-,\ p,\ \bar{p},\ n,\ \bar{n},\ \cdots$

Quark model: $\pi^+(u\bar{d}),\ \pi^0((u\bar{u}-d\bar{d})/2),\ \pi^-(\bar{u}d),$

$K^+(u\bar{s}),\ K^0(d\bar{s}),\ \bar{K}^0(\bar{d}s),\ K^-(\bar{u}s),$

$p(uud),\ \bar{p}(\bar{u}\bar{u}\bar{d}),\ n(udd),\ \bar{n}(\bar{u}\bar{d}\bar{d}),\ \cdots$

Favored fragmentaion

(from a quark which exists in a naive quark mode)

for example $D_u^{\pi^+},\ D_{\bar{d}}^{\pi^+}$

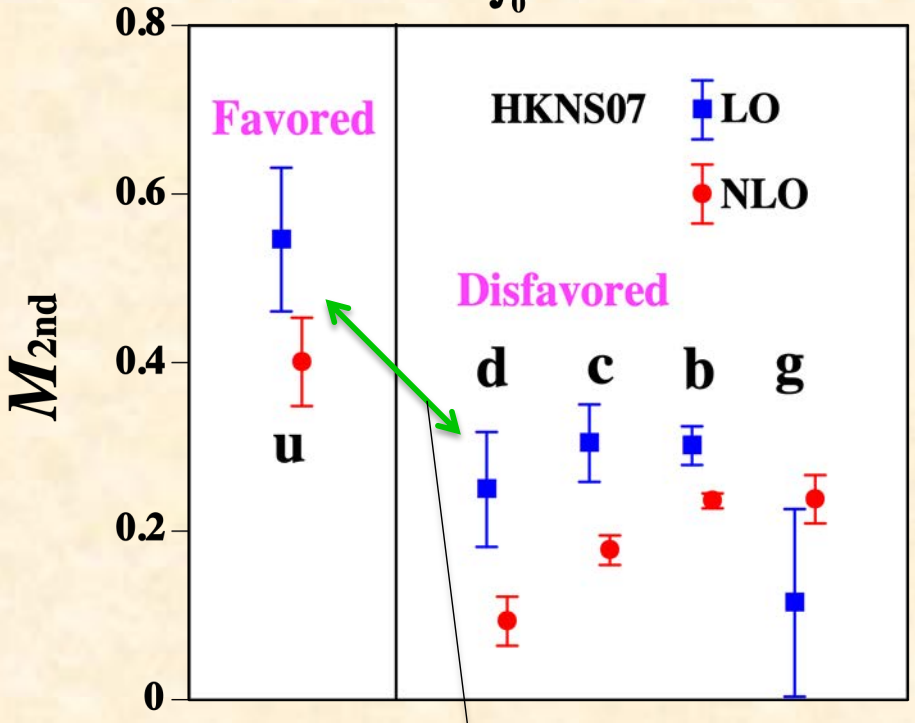
Disfavored fragmentaion

(from a quark which doe not exist in a naive quark mode)

for example $D_d^{\pi^+},\ D_{\bar{u}}^{\pi^+},\ D_s^{\pi^+}$

2nd moments of pion fragmentation functions

$$M_{2\mathrm{nd}} = \int_0^1 dz\, z\, D_i^{\pi^+}(z)$$



2nd moments of
M. Hirai, SK, T.-H. Nagai, K. Sudoh,
PRD 75 (2007) 094009.

The situation is the same in the recent NLO analysis:

$M^{\pi^+}_{u,\bar{d}} = 0.481 \gg M^{\pi^+}_{s,\bar{s}} = 0.234,\ M^{\pi^+}_{d,\bar{u}} = 0.119,\ M^{\pi^+}_{g} = 0.102$

at $Q^2 = (1.4)^2$ GeV^2 in J. Gao *et al.*, PRD 112, 054045 (2025).

There are distinct differences between
the favored and disfavored 2nd moments.

→ It could be used for exotic-hadron studies.

Criteria for determining f_0 structure by its fragmentation functions

M. Hirai, S. Kumano, M. Oka, K. Sudoh, PRD 77 (2008) 017504.

Possible configurations of $f_0(980)$

(1) ordinary u,d - meson $\quad \frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$

(2) strange meson, $\quad s\bar{s}$

(3) tetraquark $(K\bar{K})$, $\quad \frac{1}{\sqrt{2}}(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})$

(4) glueball $\quad gg$

Contradicts with experimental widths

$$\Gamma_{\rm theo}(f_0\to\pi\pi)=500-1000\ {\rm MeV} \gg \Gamma_{\rm exp}=40-100\ {\rm MeV}$$

$$\Gamma_{\rm theo}(f_0\to\gamma\gamma)=1.3-1.8\ {\rm keV} \gg \Gamma_{\rm exp}=0.205\ {\rm keV}$$

Contradicts with lattice-QCD estimate

$$m_{\rm lattice}(f_0)=1600\ {\rm MeV} \gg m_{\rm exp}=980\ {\rm MeV}$$

There could difference in fragmentation functions for f_0 depending on its internal structure.

- Favored and disfavored fragmentation functions
- 2nd moments and functional forms

$s\bar{s}$ picture for $f_0(980)$

$$M \equiv \int_0^1 zD(z)\,dz \quad \text{(2nd moment)}$$

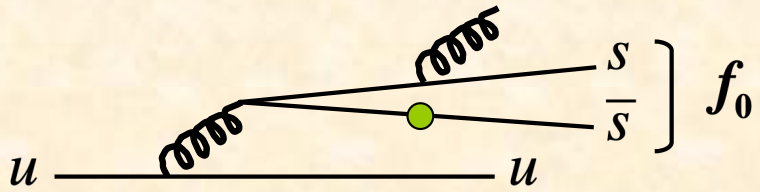
2nd moment: $M(u) < M(s) \lesssim M(g)$

Peak of function: $z_{max}(u) < z_{max}(s)$

More energy is transferred to f_0 from the parent s.

Naive estimates!

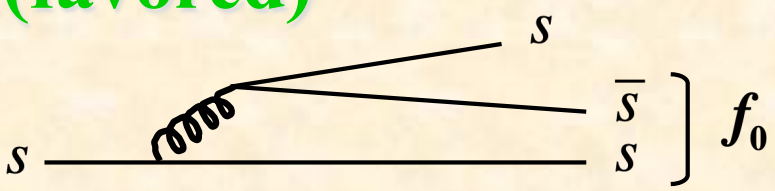
u (disfavored)



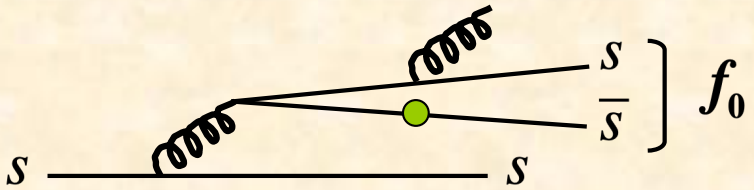
O (g^3)

+ one O (g^3) term of gluon radiation from the antiquark

s (favored)



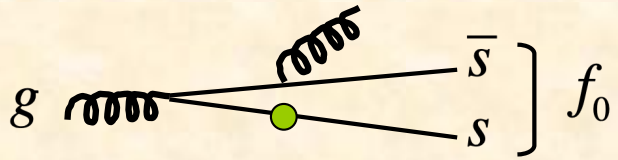
O (g^2)



O (g^3)

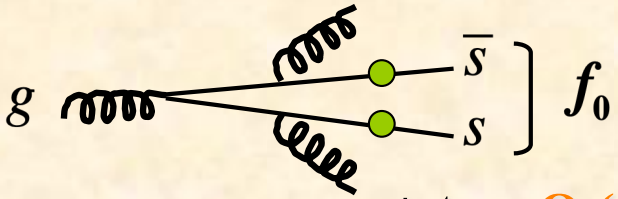
+ one O (g^3) term of gluon radiation from the antiquark

g



O (g^2)

+ one O (g^2) term of gluon radiation from the quark



O (g^3)

+ two O (g^3) terms of gluon radiation from the quark or antiquark

Naive judgment

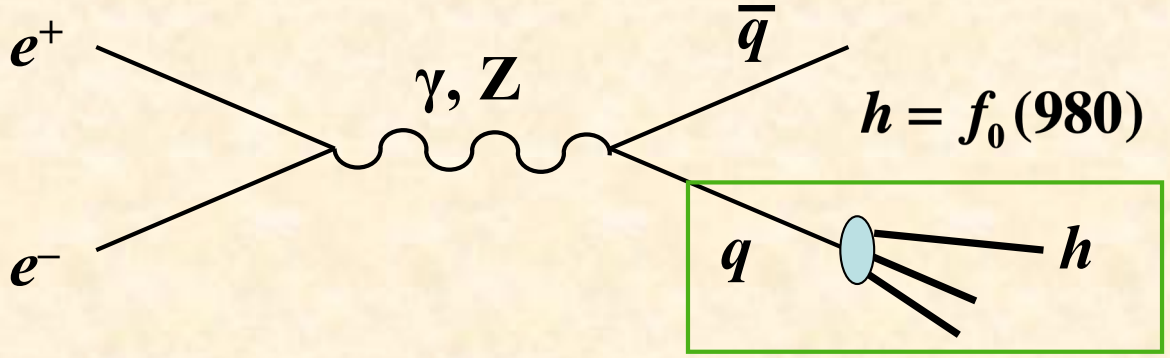
Type	Configuration	2nd Moment	Peak z
Nonstrange $q\bar{q}$	$(u\bar{u}+d\bar{d})/\sqrt{2}$	$M(s)<M(u)<M(g)$	$z_{\max}(s)<z_{\max}(u)$
Strange $q\bar{q}$	$s\bar{s}$	$M(u)<M(s)\lesssim M(g)$	$z_{\max}(u)<z_{\max}(s)$
Tetraquark	$(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})/\sqrt{2}$	$M(u)=M(s)\lesssim M(g)$	$z_{\max}(u)=z_{\max}(s)$
$K\bar{K}$ Molecule	$(K^+K^-+K^0\bar{K}^0)/\sqrt{2}$	$M(u)=M(s)\lesssim M(g)$	$z_{\max}(u)=z_{\max}(s)$
Glueball	gg	$M(u)=M(s)<M(g)$	$z_{\max}(u)=z_{\max}(s)<z_{\max}(g)$

Since there is no difference between $D_u^{f_0}$ and $D_d^{f_0}$ in the models, they are assumed to be equal. On the other hand, $D_s^{f_0}$ and $D_g^{f_0}$ are generally different from them, so that they should be used for finding the internal structure. Therefore, simple and "model-independent" initial functions are

$$D_u^{f_0}(z,Q_0^2)=D_{\bar{u}}^{f_0}(z,Q_0^2)=D_d^{f_0}(z,Q_0^2)=D_{\bar{d}}^{f_0}(z,Q_0^2),\quad D_s^{f_0}(z,Q_0^2)=D_{\bar{s}}^{f_0}(z,Q_0^2),$$

$$D_g^{f_0}(z,Q_0^2),\qquad D_c^{f_0}(z,m_c^2)=D_{\bar{c}}^{f_0}(z,m_c^2),\qquad D_b^{f_0}(z,m_b^2)=D_{\bar{b}}^{f_0}(z,m_b^2).$$

Fragmentation functions for $f_0(980)$



$$z \equiv \frac{E_h}{\sqrt{s}/2} = \frac{2E_h}{Q} = \frac{E_h}{E_q}, \quad s = Q^2$$

$$F^h(z,Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \to hX)}{dz}$$

σ_{tot} = total hadronic cross section

$$F^h(z,Q^2) = \sum_i \int_z^1 \frac{dy}{y} C_i\left(\frac{z}{y}, Q^2\right) D_i^h(y,Q^2)$$

Initial functions

$$D_u^{f_0}(z,Q_0^2) = D_d^{f_0}(z,Q_0^2) = N_u^{f_0} z^{\alpha_u^{f_0}} (1-z)^{\beta_u^{f_0}}$$

$$D_s^{f_0}(z,Q_0^2) = N_s^{f_0} z^{\alpha_s^{f_0}} (1-z)^{\beta_s^{f_0}}$$

$$D_g^{f_0}(z,Q_0^2) = N_g^{f_0} z^{\alpha_g^{f_0}} (1-z)^{\beta_g^{f_0}}$$

$$D_c^{f_0}(z,m_c^2) = N_c^{f_0} z^{\alpha_c^{f_0}} (1-z)^{\beta_c^{f_0}}$$

$$D_b^{f_0}(z,m_b^2) = N_b^{f_0} z^{\alpha_b^{f_0}} (1-z)^{\beta_b^{f_0}}$$

- $D_q^{f_0}(z,Q_0^2) = D_{\bar{q}}^{f_0}(z,Q_0^2)$
- $Q_0 = 1$ GeV
  $m_c = 1.43$ GeV
  $m_b = 4.3$ GeV

$$N = M \frac{\Gamma(\alpha+\beta+3)}{\Gamma(\alpha+2)\Gamma(\beta+1)}, \quad M \equiv \int_0^1 zD(z)dz$$

Recent experimental progress for f_0

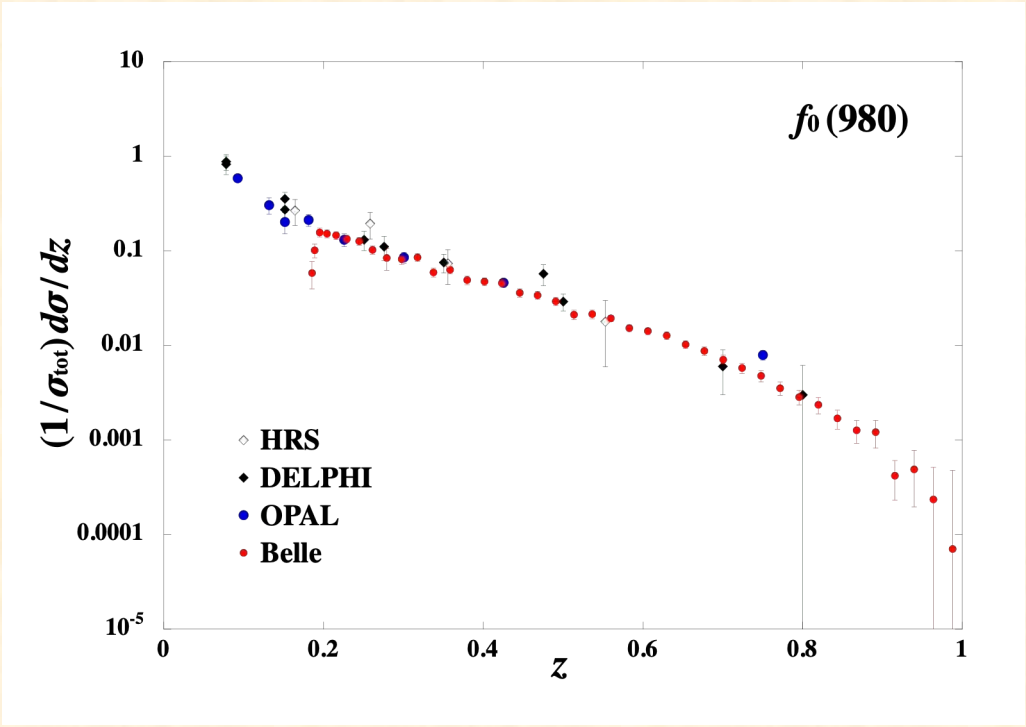
Total number of data: 23 (before Belle) → 63

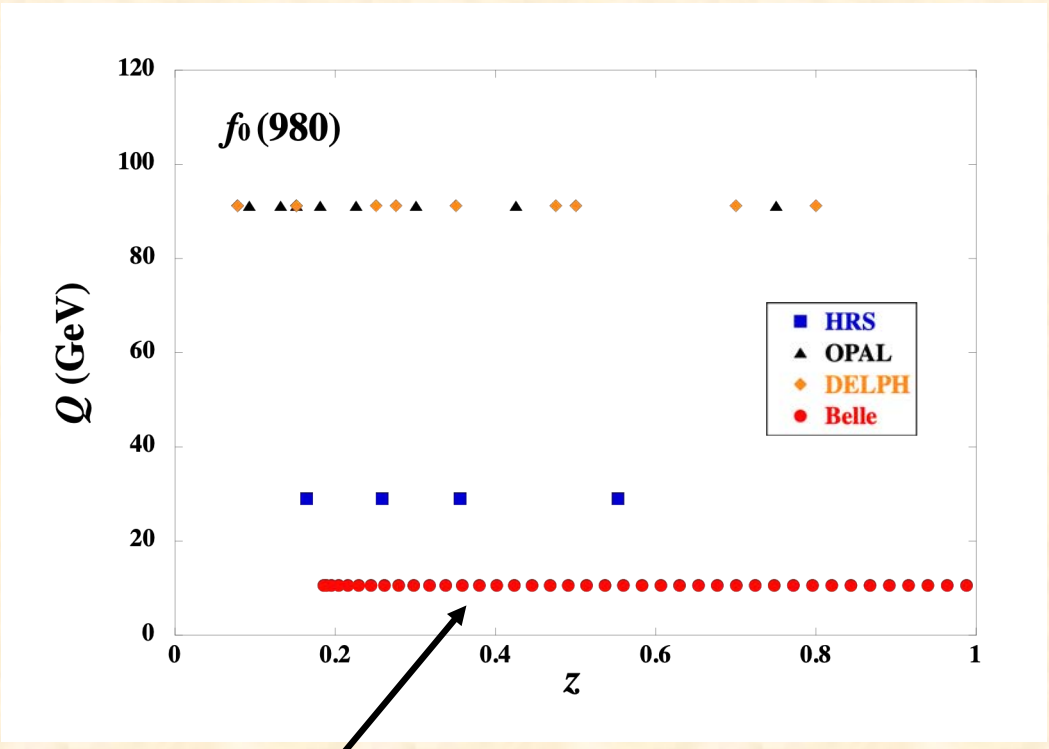
Exp. collaboration	$\sqrt{s}$ (GeV)	# of data
HRS	29	4
OPAL	91.2	8
DELPHI	91.2	11
Belle	10.58	40

R. Seidl *et al.*, (Belle Collaboration), Phys. Rev. D 111 (2025) 052003.

Advantages:

- accurate
- wide range of z
- Q^2 evolution can be probed.





New Belle data

Global analysis result

Type	Configuration	2nd Moment	Peak z
$q\bar{q}$ Nonstrange	$(u\bar{u}+d\bar{d})/\sqrt{2}$	$M(s)<M(u)<M(g)$	$z_{\max}(s)<z_{\max}(u)$
$q\bar{q}$ Strange	$s\bar{s}$	$M(u)<M(s)\lesssim M(g)$	$z_{\max}(u)<z_{\max}(s)$
Tetraquark	$(u\bar{u}s\bar{s}+d\bar{d}s\bar{s})/\sqrt{2}$	$M(u)=M(s)\lesssim M(g)$	$z_{\max}(u)=z_{\max}(s)$
$K\bar{K}$ Molecule	$(K^+K^-+K^0\bar{K}^0)/\sqrt{2}$	$M(u)=M(s)\lesssim M(g)$	$z_{\max}(u)=z_{\max}(s)$
Glueball	gg	$M(u)=M(s)<M(g)$	$z_{\max}(u)=z_{\max}(s)<z_{\max}(g)$

$M(u)=M(d)=0.0033,\quad M(s)=0.0194,\quad M(g)=0.0123$

- $M(u)<M(s)\lesssim M(g)$
- $z_{\max}(u)<z_{\max}(s)$

$\rightarrow f_0 \simeq s\bar{s}$

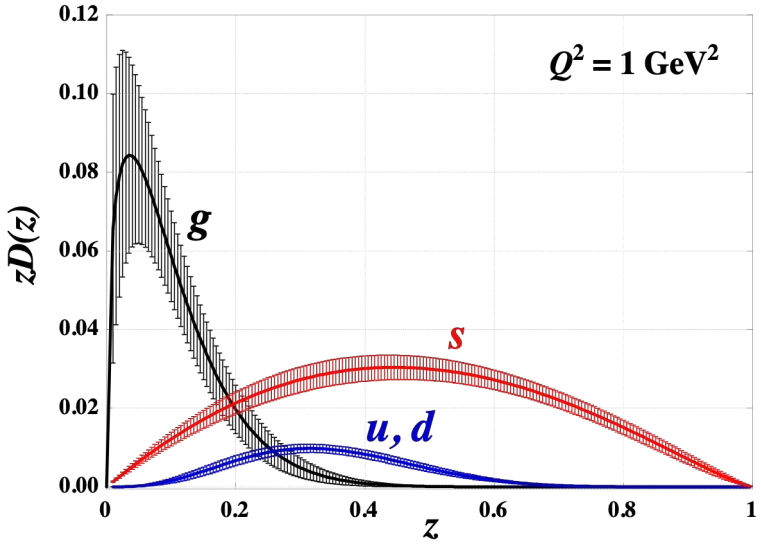
$f_0 = K\bar{K}$ (or tetra quark) at low energies ???

$\rightarrow f_0 = s\bar{s}$ at high energies

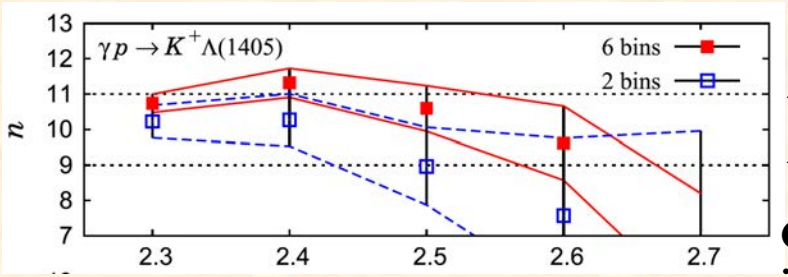
as implied for $\Lambda(1405)$ in 2016

by using constituent counting rule

in perturbative QCD.



SK, PRD114 (2026) L071501.



$\leftarrow n_\Lambda = 5$

$\leftarrow n_\Lambda = 3$

Constituent counting rule in perturbative QCD

W.-C. Chang, SK, T. Sekihara, PRD 93 (2016) 034006.

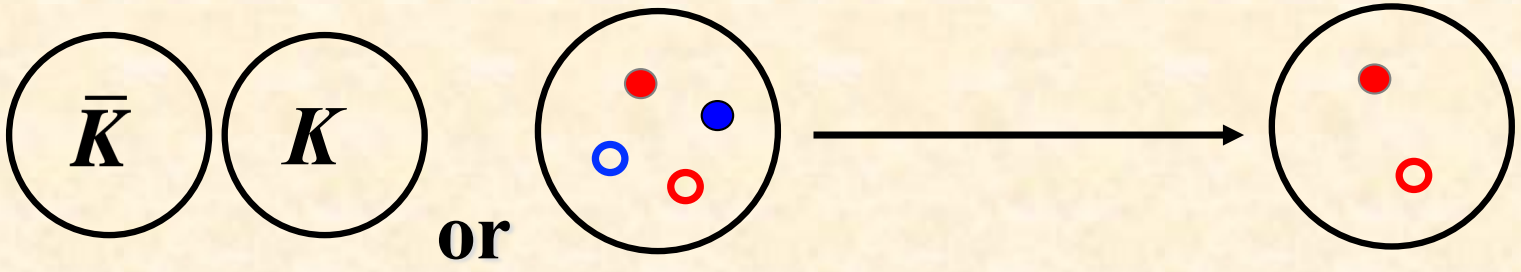
New view of exotic hadrons from the $f_0(980)$ analysis

Internal configurations change depending on the energy scale.

Experimental confirmation is needed.
→ We may look at E or p dependence of high-energy reaction cross sections.

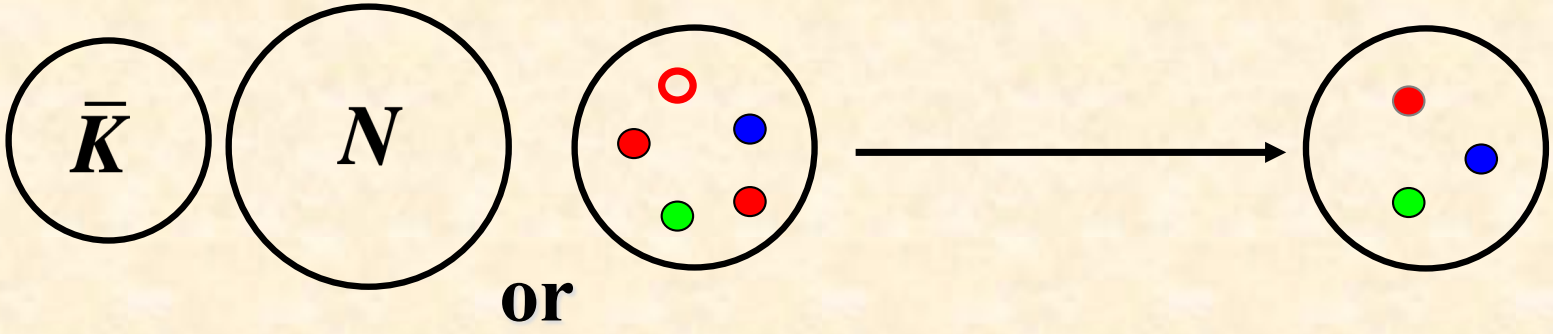
Energy scale

Low energies → High energies



SK (2026)

In the same way for baryons such as Λ(1405)



Chang, SK, Sekihara (2016)

Summary

In general, it is not easy to find internal structure of exotic-hadron candidates by global observables, such as mass, spin, …

High energies = Quark and gluon degrees of freedom.

It could be appropriate to use high-energy processes for determination of internal configurations for exotic-hadron candidates.

- **Constituent-counting rule**
- **Fragmentation functions**

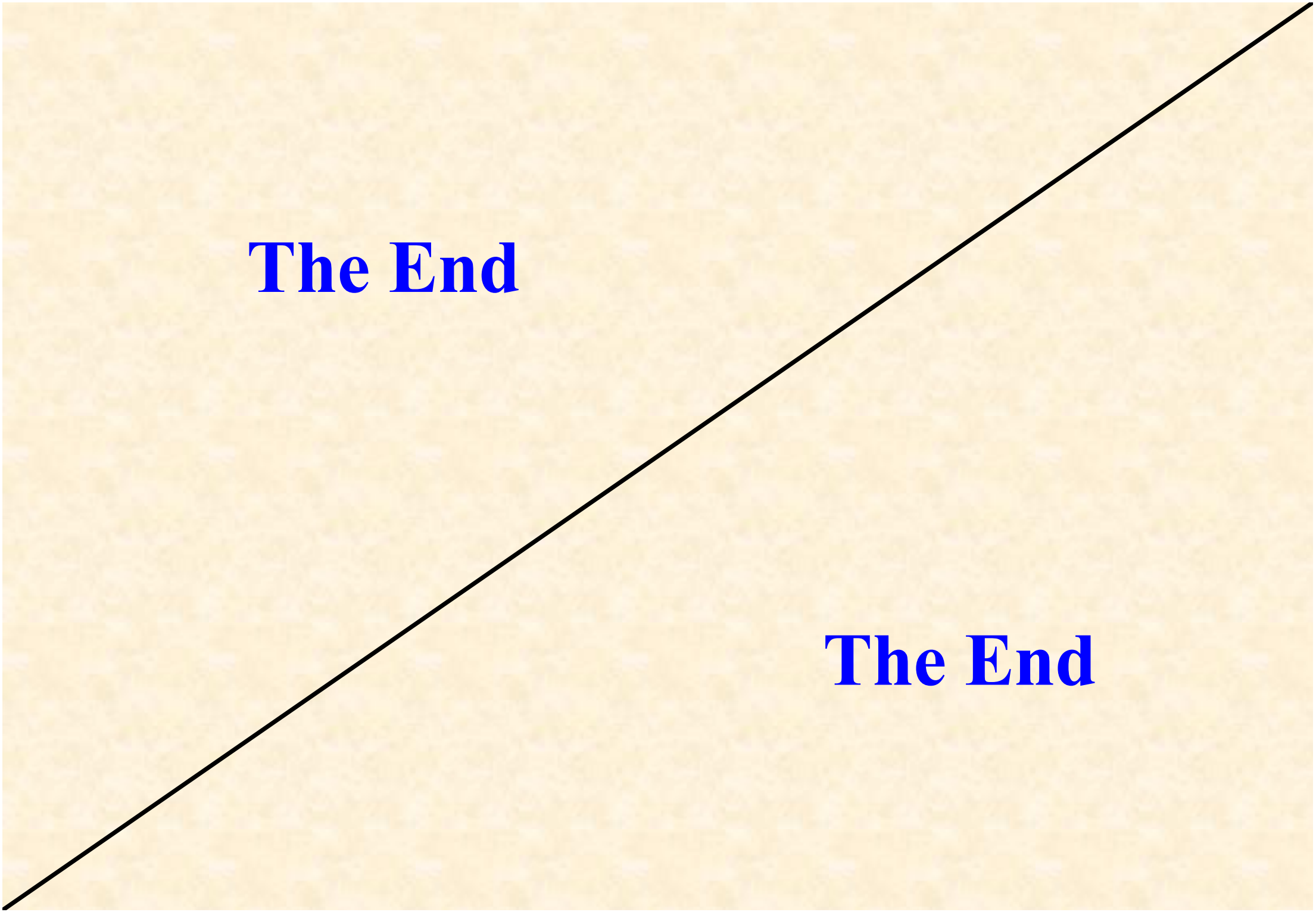
Possible new idea of exotic hadrons:

The internal configuration change depending on the energy, which should be confirmed experimentally by look at *E* or *p* dependencies.

Experimental projects

JLab, J-PARC, KEK-B, AMBER, EIC, EicC, HIAF, …

The End



The End