Strangeness in the mechanical structure of the proton and hyperons

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Three main missions of the Electron-Ion Collider

1. How does the mass of the nucleon arise?
2. How does the spin of the nucleon arise?
3. How are emergent properties of dense systems of gluons?

EIC CDR (2021)

Key words

1. The fundamental structure of the nucleon
2. Gluons inside the nucleon
3. Gluons under extreme conditions

Behind the scenes: Gravitational (Energy-momentum tensor) form factors → Mechanical properties of the nucleon

Strangeness in the nucleon

- In the Scalar channel

  Lattice: σ_s = 35(13) MeV, f_{ts} ≈ 3.7% [1]. A sizable scalar response.

- In the Vector channel

  Lattice: $G_M^s(0) = -0.0179(20)$ [2]. EM contribution: $-G_M^s/3 \approx +0.0060$.

  HAPPEX-III: $G_E^s + 0.517 G_M^s = 0.003 \pm 0.014$ at $Q^2 = 0.624$ GeV² [3].

- In the Axial-vector channel

  Lattice: $\Delta S = g_A^s = -0.037(18)$, about −4% of the nucleon spin [4].

  KamLAND: $g_A^s = -0.14$ (+0.25/−0.26) [5]. Inclusive DIS: ≈ −0.085 with SU(3) input [6].

- In the antisymmetric Tensor channel

  Lattice: $g_T^s = -0.0016(12)$, consistent with zero [1]. No precise experimental value.

- **In the Gravitational (EMT) channel?**

[1] Park et al. [Jlab], [2] Alexandrou et al., arXiv:2603.26591.
[3] HAPPEX, PRL 108 (2012) 102001. [4] Alexandrou et al., PRD 111 (2025) 054505. [5] KamLAND, PRD 107 (2023) 072006.
[6] HERMES, PRD 75 (2007) 012007.

Gravitational (EMT) form factors

- Gravitational form factors of the nucleon in QCD

Kobzarev et al. 1962; Pagels, 1966

O. V. Teryaev, Front. Phys. 11 (2016)
K.-F. Liu, PRD 104 (2021)

$$\langle p'|T^{\mu\nu}(0)|p\rangle = \bar{u}(p')\left[A^a(t)\frac{P^\mu P^\nu}{M_N} + J^a(t)\frac{iP^{\{\mu}\sigma_\nu\}^\rho\Delta_\rho}{2M_N} + D^a(t)\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M_N} + M_N\bar{c}^a(t)g^{\mu\nu}\right]u(p)$$

δg^{00} δg^{0i} δg^{ij}

Weiss & Polyakov, 1999

$\sum_a A^a(0) = 1$ Mass $\sum_a J^a(0) = \frac{1}{2}$ Spin

Deformation of space(D(Druck)-term)
= mechanical properties of the nucleon

Non-conservationof EMT pieces
(cosmological constant

Pressure & Shear-force distributions (pressure anisotropy)

$$\partial_\mu T^{\mu\nu} = 0 \to \sum_{q,g}\bar{c}^{q,g} = 0$$

QCD Effective operators

QCD → Low-energy Effective quark-gluon dynamics (Essential physics: SBXB)

$\mu \sim 1\,\mathrm{GeV}$

QCD operators	Effective QCD operators
$V^\mu,\ A^\mu, T^{\mu\nu}$	$V^\mu_{\rm eff},\ A^\mu_{\rm eff},\ T^{\mu\nu}_{\rm eff}$
$\bar\psi\gamma^\alpha\nabla^{\beta_1}\dots\nabla^{\beta_n}\psi$ $G^{\alpha_1\beta}\dots F^{\beta\alpha_n}$ $G^2, G\tilde G, \dots$	?

Twist-2

June-Young Kim's Talk (11am)

Twist-2 effective operators are determined by dynamics.

(from the instanton vacuum)

Effective EMT operators

QCD operators

Effective QCD operators

$$T_q^{\mu\nu} = \frac{i}{4}\bar{\psi}_q \left(\gamma^{\{\mu} \overleftrightarrow{\mathcal{D}}^{\nu\}}\right) \psi_q \longrightarrow T^{\mu\nu}_{\chi=0}(x) = \frac{i}{4}\bar{\psi}(x)\gamma^{\{\mu} \overleftrightarrow{\partial}^{\nu\}} \lambda_0 \psi(x)$$

(Gluons integrated out)

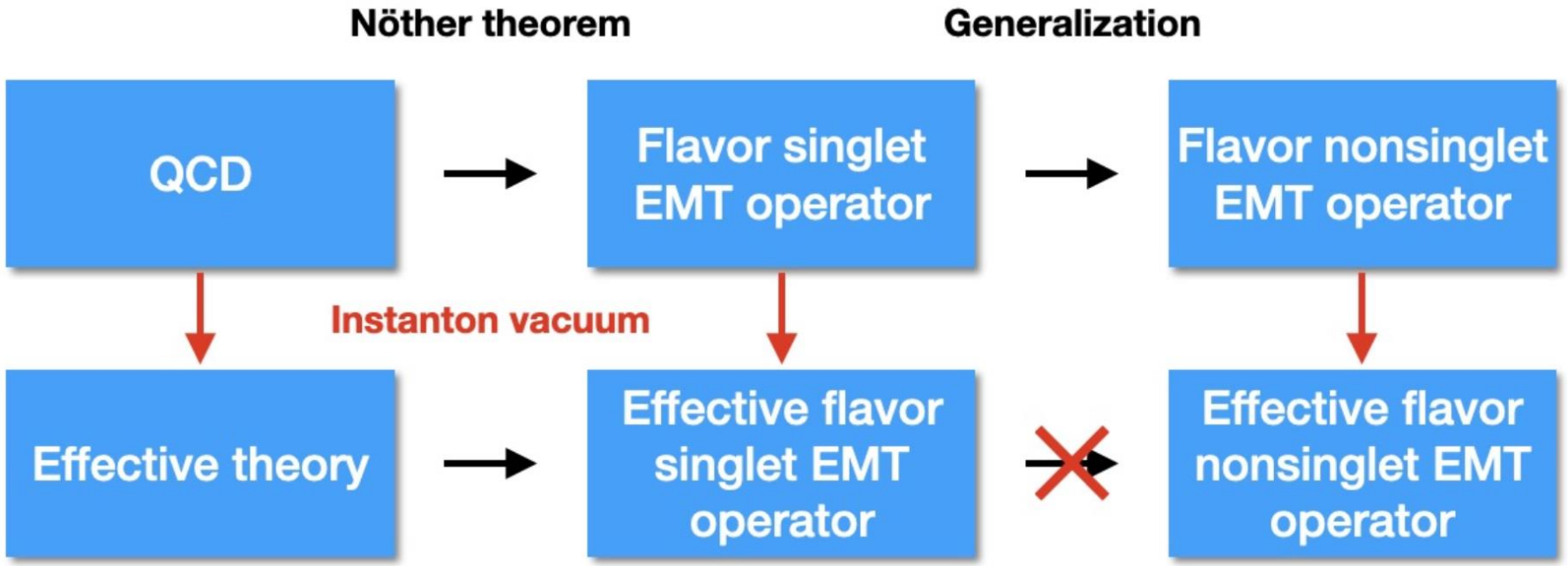
$$T_q^{\mu\nu}(x) + T_g^{\mu\nu}(x)\ [\mathrm{QCD}] \xrightarrow{\text{eff}} T^{\mu\nu}_{\chi=0}(x)$$

However…

Flavor decomposition of GFFs require both twist-2 & twist-4 EMT operators.
But we don't know yet how to derive the flavor-nonsinglet EMT currents without ambiguity.

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Effective EMT operators



$$T_q^{\mu\nu}(x) + T_g^{\mu\nu}(x)\ [\mathrm{QCD}] \xrightarrow{\mathrm{eff}} T_{\chi=0}^{\mu\nu}(x)\ [\chi\mathrm{QSM}]$$

Effective operators: $T_\chi^{\mu\nu}(x) = \frac{i}{4}\bar{\psi}(x)\gamma^{\{\mu}\overleftrightarrow{\partial}^{\nu\}}\lambda_\chi\psi(x)$

Flavor nonsinglet operators require careful derivation! (H.D. Son's Talk)

$T_3^{\mu\nu}(x)\ \ T_8^{\mu\nu}(x)\ \ T_{1\pm 2i}^{\mu\nu}(x)\ \ T_{4\pm 5i}^{\mu\nu}(x)\ \ T_{6\pm 7i}^{\mu\nu}(x)$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Gravitational form factors: Twist-2 case

$$T_a^{\mu\nu} = \bar{T}_a^{\mu\nu} + \hat{T}_a^{\mu\nu}$$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Twist-4 EMT current: $\hat{T}_a^{\mu\nu} = \frac{1}{4} g^{\mu\nu} T^{\alpha}_{a,\alpha}$

Twist-2 EMT current: $\bar{T}_a^{\mu\nu} = T_a^{\mu\nu} - \frac{1}{4} g^{\mu\nu} T^{\alpha}_{a,\alpha}$

- Twist-2 baryon matrix elements

$$\langle B(p', J_3')|\bar{T}^a_{\mu\nu}(0)B(p, J_3)\rangle = \bar{u}(p', J_3')\Bigg[A^a_B(t)\frac{P_\mu P_\nu}{M_B} + J^a_B(t)\frac{iP_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{2M_B} + D^a_B(t)\frac{\Delta_\mu\Delta_\nu - tg^{\mu\nu}}{4M_B}$$
$$- g_{\mu\nu}\left\{\frac{t}{8M_B}J^a_B(t) - \frac{3t}{16M_B}D^a_B(t) + \frac{M_B}{4}\left(1 - \frac{t}{4M_B^2}\right)A^a_B(t)\right\}\Bigg]u(p, J_3)$$

- Twist-4 baryon matrix elements

$$\langle B(p', J_3')|\hat{T}^a_{\mu\nu}(0)|B(p, J_3)\rangle = \bar{u}(p', J_3')\Bigg[g_{\mu\nu}\Big\{M_B\bar{c}^a_B(t) + \frac{t}{8M_B}J^a_B(t)$$
$$- \frac{3t}{16M_B}D^a_B(t) + \frac{M_B}{4}\left(1 - \frac{t}{4M_B^2}\right)A^a_B(t)\Big\}\Bigg]u(p, J_3)$$

cbar form factor comes from the **twist-4** operator!

Possible regularization and renormalization dependence

MV Polyakov, HD Son, JHEP (2018)
Y Hatta, A Rajan, K Tanaka, JHEP (2018)

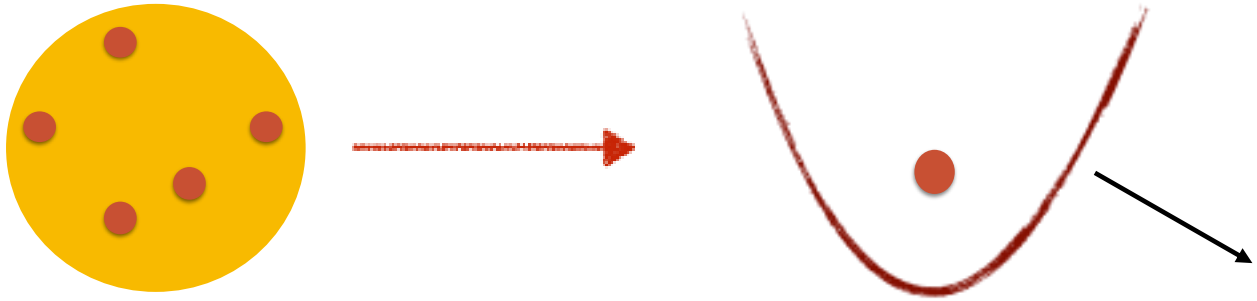
J.Y. Kim, in preparation

Mean fields

Given action $S[\phi]$,

$$\left.\frac{\delta S}{\delta \phi}\right|_{\phi=\phi_0} = 0$$: Solution of this saddle-point equation ϕ_0

This classical solution is regarded as a mean field.



Mean-field potential is produced by all other particles.

- Nuclear shell model
- Landau-Ginzberg theory
- Quark potential models for baryons

Pion mean fields: Chiral quark-soliton model

* Baryons as a state of Nc quarks bound by mesonic mean fields. E. Witten (1979)

Effective chiral action:

$$S_{\rm eff}[\pi^a] = -N_c {\rm Trlog}\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right)$$

D. Diakonov & V. Petrov (1986)

D. Diakonov, V. Petrov, P. Pobylitsa (1988)

M : Dynamical quark mass – chiral symmetry & its spontaneous breakdown

* Key point: Hedgehog Ansatz

$$\pi^a(\boldsymbol{r}) = \begin{cases} n^a F(r),\ n^a = x^a/r, & a = 1,\ 2,\ 3 \\ 0, & a = 4,\ 5,\ 6,\ 7,\ 8. \end{cases} \qquad U_o = \begin{pmatrix} e^{i\boldsymbol{n}\cdot\boldsymbol{\tau}P(r)} & 0 \\ 0 & 1 \end{pmatrix}$$

- Current quark masses

$$\hat{m} = {\rm diag}(m_u, m_d, m_s)$$

Ch. Christov, HChK, K. Goeke et al. PPNP (1996)

D. Diakonov hep-ph/9802298

Pion mean fields: Chiral quark-soliton model

- Rotational & Translational zero modes

$$\int \mathcal{D}U\,\mathcal{F}[U(\boldsymbol{x})] \to \int d^3\boldsymbol{X} \int \mathcal{D}A\,\mathcal{F}\left[TAU_{\rm cl}(R\boldsymbol{x})A^\dagger T^\dagger\right]$$

- Collective Hamiltonian & Wavefunctions in flavor SU(3) symmetry

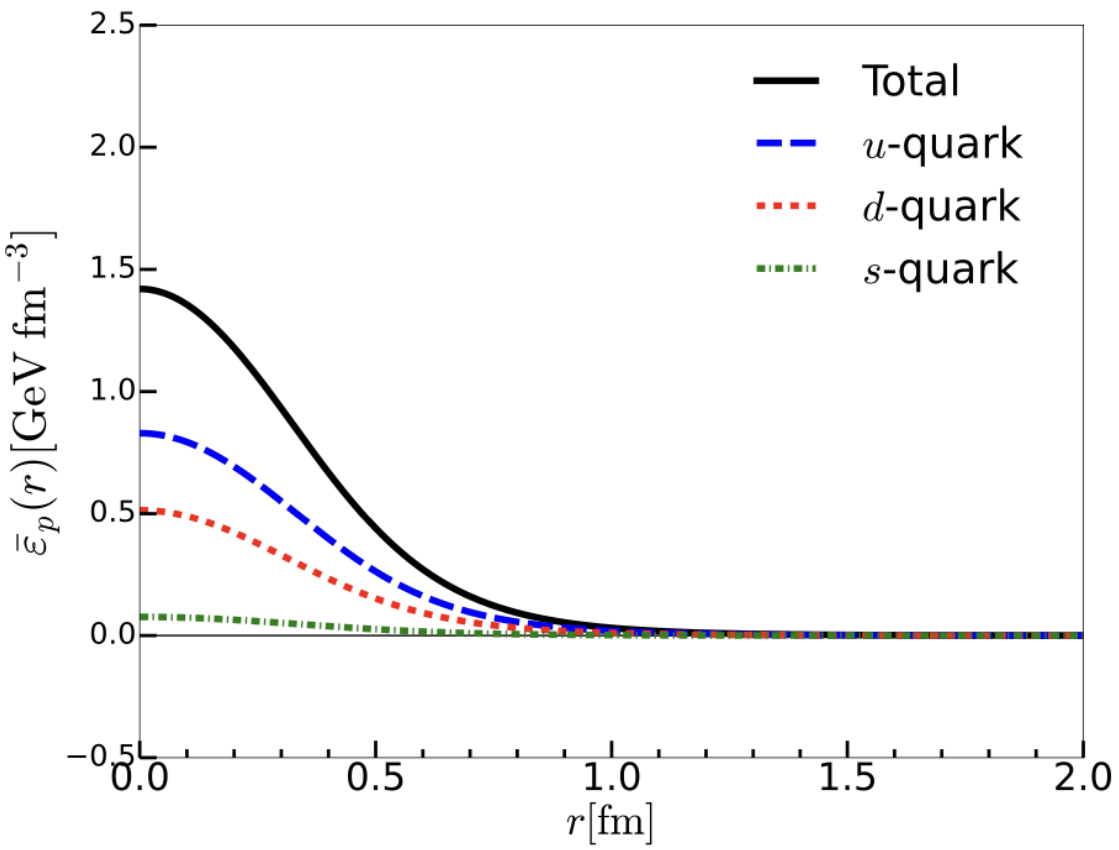
$$H_{\rm coll} = M_{\rm sol} + \frac{1}{2I_1}\sum_{i=1}^{3}\hat{J}_i^2 + \frac{1}{2I_2}\sum_{p=4}^{7}\hat{J}_p^2$$

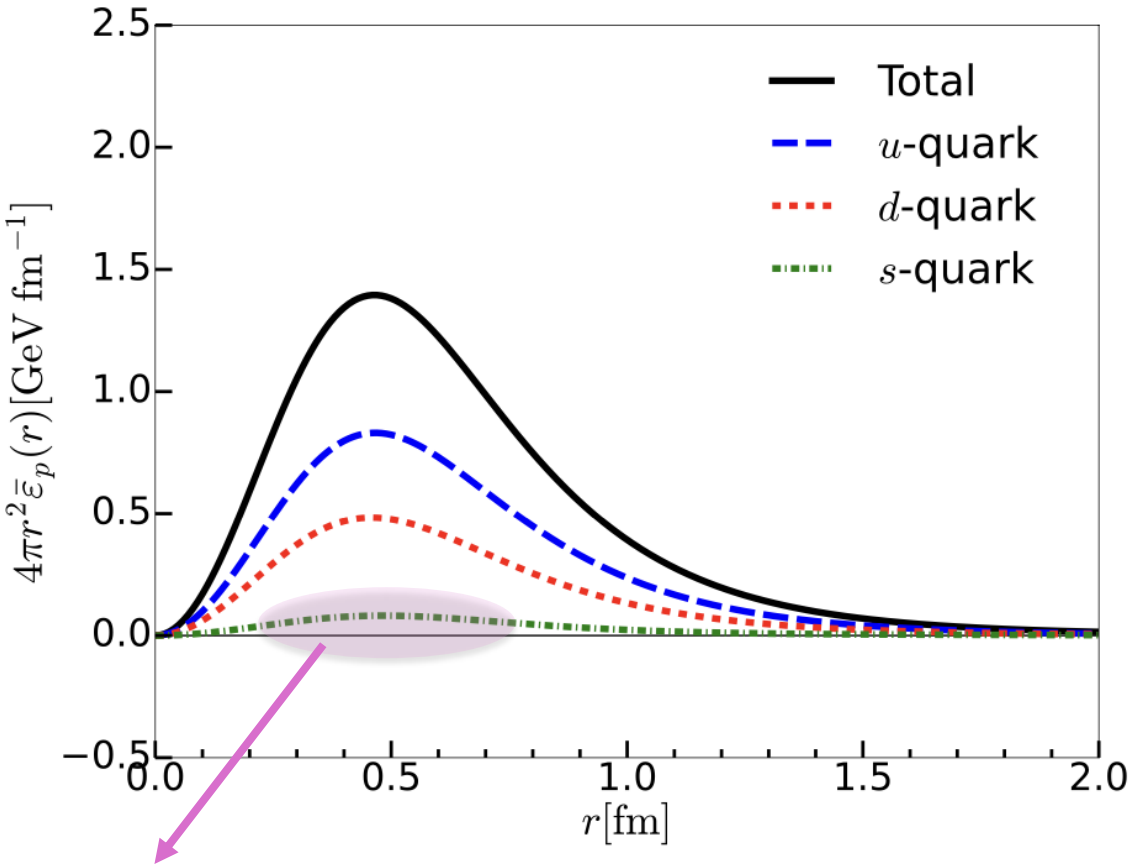
$$\Psi^{(\mu)}_{(YTT_3)(Y_RJJ_3)}(A) = \sqrt{\dim(\mu)}(-1)^{J_3-Y_R/2}D^{(\mu)*}_{(YTT_3)(Y_RJ-J_3)}(A)$$

Ch. Christov, HChK, K. Goeke et al. PPNP (1996)

D. Diakonov hep-ph/9802298

Mass distributions





Contribution from the s quark is negligible.

$$A_p^0(0) = 1, \quad A_p^3(0) = 0.25, \quad A_p^8(0) = 0.47, \quad [\mathrm{SU}(3)]$$

$$A_p^0(0) = 1, \quad A_p^3(0) = 0.24 \qquad\qquad [\mathrm{SU}(2)]$$

Momentum fractions

$$A_p^u(0) = 0.59, \quad A_p^d(0) = 0.35, \quad A_p^s(0) = 0.06, \quad [\mathrm{SU}(3)]$$
$$A_p^u(0) = 0.62, \quad A_p^d(0) = 0.38, \quad [\mathrm{SU}(2)]$$

- These numbers can be understood as the second Mellin moments of the PDFs. We list the predictions of the proton momentum fraction carried by the u-, d-, and s-quarks:

$$[\langle x \rangle_u : \langle x \rangle_d : \langle x \rangle_s] = [59\% : 35\% : 6\%]$$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Spin of the proton

$$J_p^0 = 0.50, \quad J_p^3 = 0.58, \quad J_p^8 = 0.22, \quad [\mathrm{SU}(3)].$$
$$J_p^0 = 0.50, \quad J_p^3 = 0.55, \quad [\mathrm{SU}(2)]$$

Strange quark contribution is negligible.

$$J_p^u = 0.52, \quad J_p^d = -0.06, \quad \boxed{J_p^s = 0.04,} \quad [\mathrm{SU}(3)].$$
$$J_p^u = 0.53, \quad J_p^d = -0.03, \quad [\mathrm{SU}(2)]$$

$$J = \frac{1}{2}\sum_q \Delta q + \sum_q L^q + J_g$$: Ji's relation

X. Ji, PRL 78 (1997)

$J_g \approx 0$ Suppressed by the instanton packing fraction (Balla et al. (1998)).

$$J = \frac{1}{2}\sum_q \Delta q + \sum_q L^q \quad \Rightarrow \quad \frac{1}{2} = \frac{1}{2}\sum_q \Delta q + \sum_q L^q = 0.23 + 0.27$$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Problem of naïve decomposition

- Decomposition of the isotriplet J

$$J_p^{u-d} = L_p^{u-d} + S_p^{u-d} + \boxed{\delta J_p^{u-d}}$$

M. Wakamatsu & H. Nakakoji, PRD 71 (2005)

Violation of Ji's sum rule (X.D. Ji, PRL 78 (1997))

- Origin of δJ_p^{u-d} : role of gluons

- The second moment of the chiral-odd **twist-3** quark distribution

$$\int_{-1}^{1} dx x\, e^{u+d}(x) = \frac{m}{M_N} N_c + \boxed{\frac{M}{M_N}\beta}$$

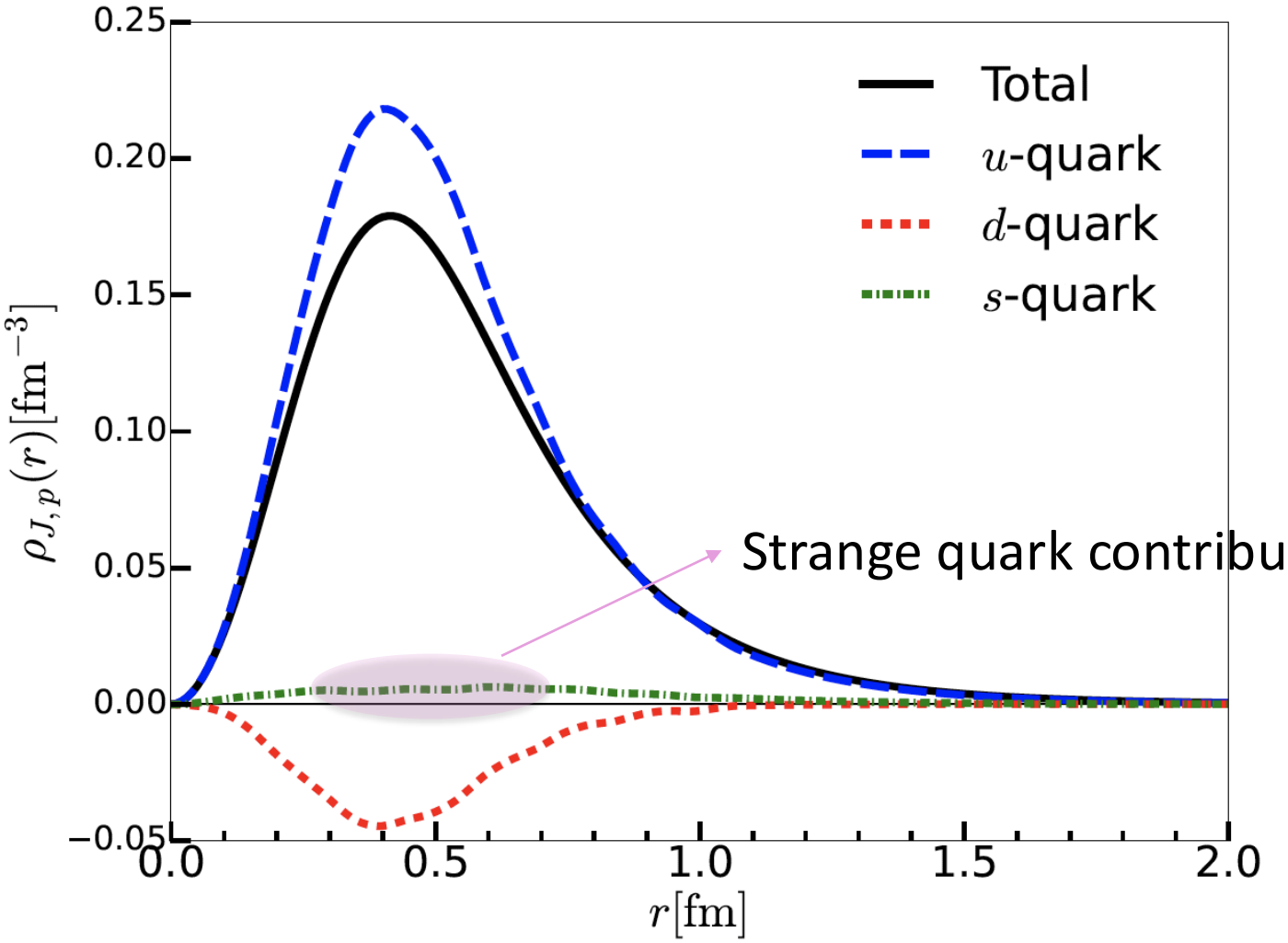
P. Schweitzer, PRD 67 (2005)
Ohnishi & M. Wakamatsu, PRD 69 (2004)

This makes the second moment deviate from QCD.

- Indication: If the covariant derivatives had been used, these discrepancies would have been resolved.
- Spin-orbit correlations are also very important to consider (JY Kim's talk).

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Angular momentum distribution



Strange quark contribution is negligible.

Mechanical properties

Stability condition in SU(3)

$$\int d^3r\, p_p^{u+d+s}(r) = 0$$

Twist-2 case

$$\int d^3r\, \bar{p}_p^{u+d+s}(r) = \frac{1}{4} M_N \neq 0\,!$$

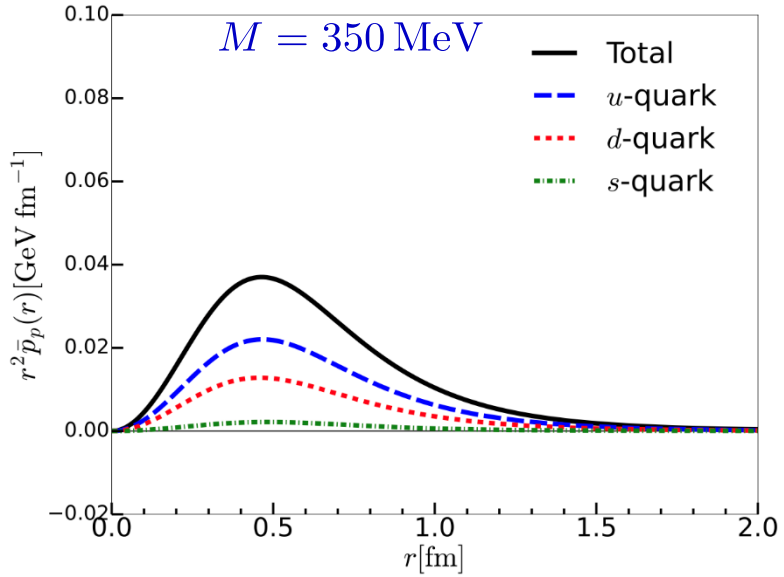
Twist-4 contribution

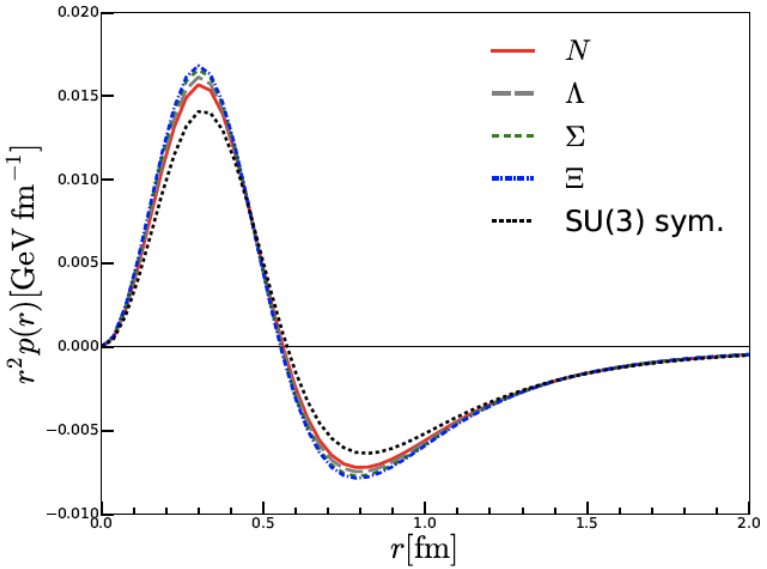
$$\int d^3r\, \hat{p}_p^{u+d+s}(r) = -\frac{1}{4} M_N$$

Both quark and gluon contribution should be considered.

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Mechanical properties

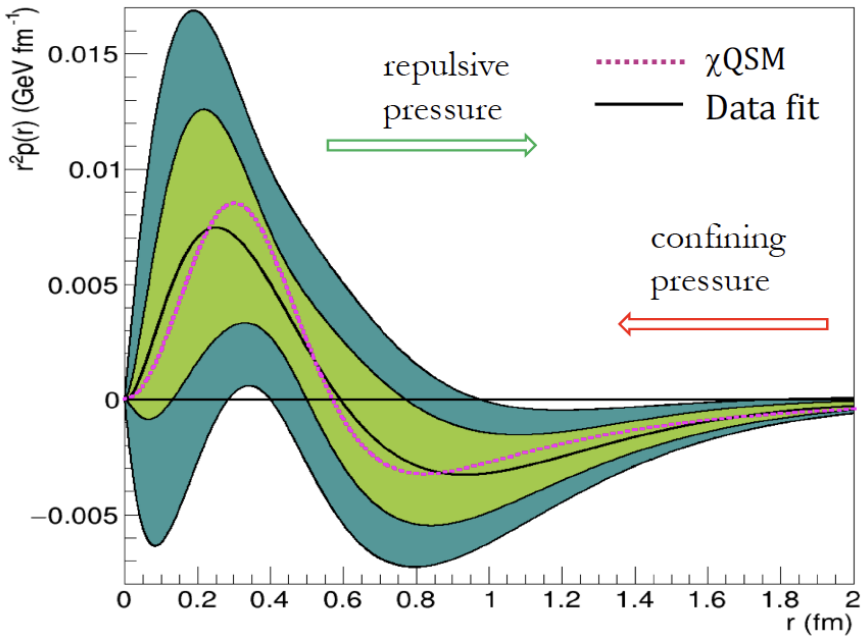




H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)

Twist-2 part of the pressure density.
No nodal point.

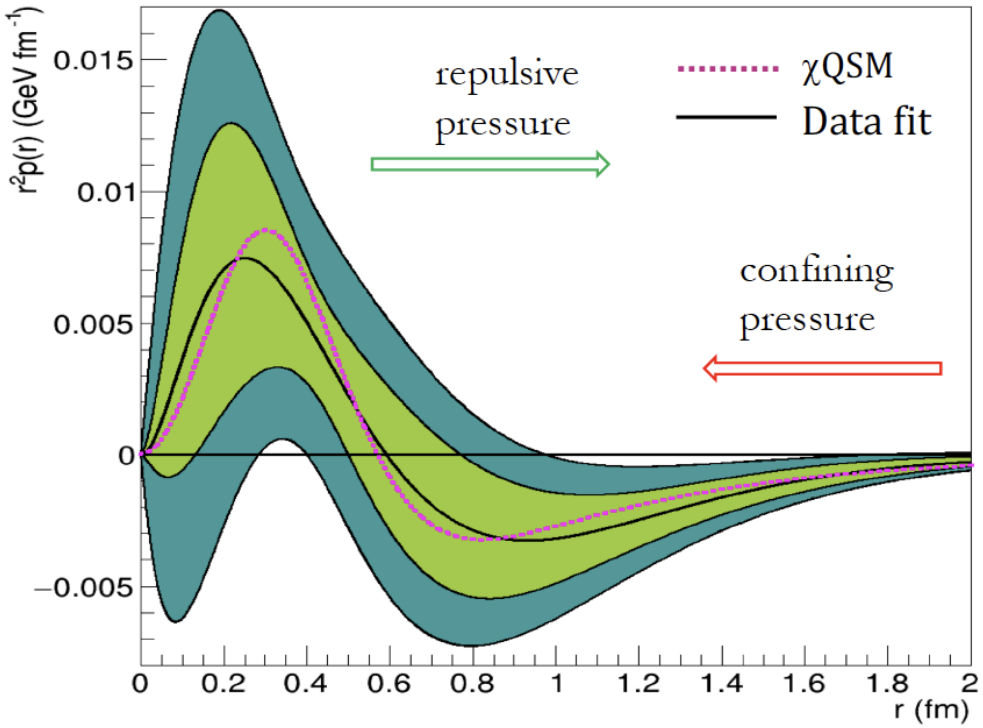
$M = 350\,\mathrm{MeV}$



V.B., L. Elouadrhiri, F.X. Girod, Nature 557 (2018) 7705, 396

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Flavor blindness in SU(3)



V.B., L. Elouadrhiri, F.X. Girod, Nature 557 (2018) 7705, 396

$D^{u-d}(0) = 0.29$ in SU(2)

$D^{u-d}(0) = 0.062$ in SU(3)

The flavor blindness is only valid in SU(3)!

The strange quarks should essentially be considered in the proton!

Burkert et al. assumed the flavor blindness.

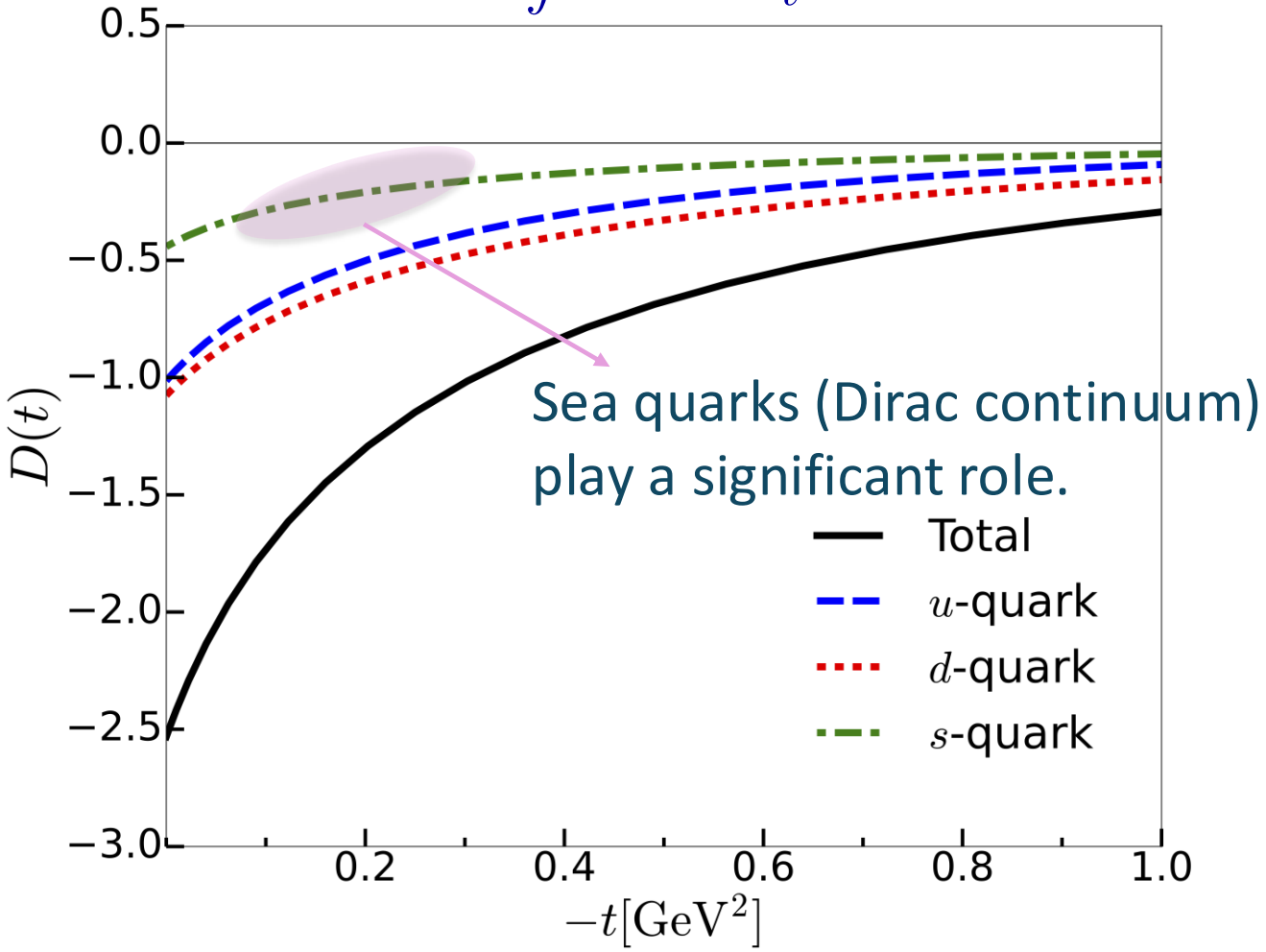
$D^{u-d}(0) \approx 0$

Lattice QCD arrives at a similar conclusion.
(D. Hackett et al. 2310.08484)

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Flavor-decomposed D-term form factors

$$D_B^{\chi}(t)\delta_{J_3' J_3} = 4M_{\rm sol}\int d^3r \frac{j_2(r\sqrt{-t})}{t} s_B^{\chi}(r)$$



Similar situation in the EM transitions of the delta isobar

Y Hatta, M Strikman, PLB (2021)

$$-1.3 < D_s < 0.4$$

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

The strange-quark contributions are essential for flavor blindness!

Mechanical radius

$$\langle r^2 \rangle_{\rm mech} = \frac{\int d^3r\ r^2\ \left[\frac{2}{3}s(r) + p(r)\right]}{\int d^3r\ \left[\frac{2}{3}s(r) + p(r)\right]} = \boxed{\frac{6D}{\int_{-\infty}^{0} dt\ D(t)}}$$

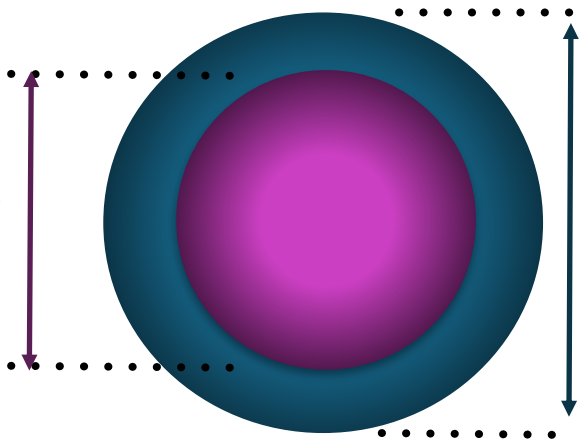
$$\sqrt{\langle r^2 \rangle}_{\rm mech} = 0.69\,{\rm fm} \qquad \sqrt{\langle r^2 \rangle}_{\rm mech} = (0.63 \pm 0.06 \pm 0.13)\,{\rm fm}$$

V. Burkert et al. (2022)

$\sqrt{\langle r^2 \rangle_{\rm mech}} = 0.73\,{\rm fm}$ in SU(3)

$$\sqrt{\langle r^2 \rangle_{\rm mech}} < \sqrt{\langle r^2 \rangle_{\rm ch}}$$

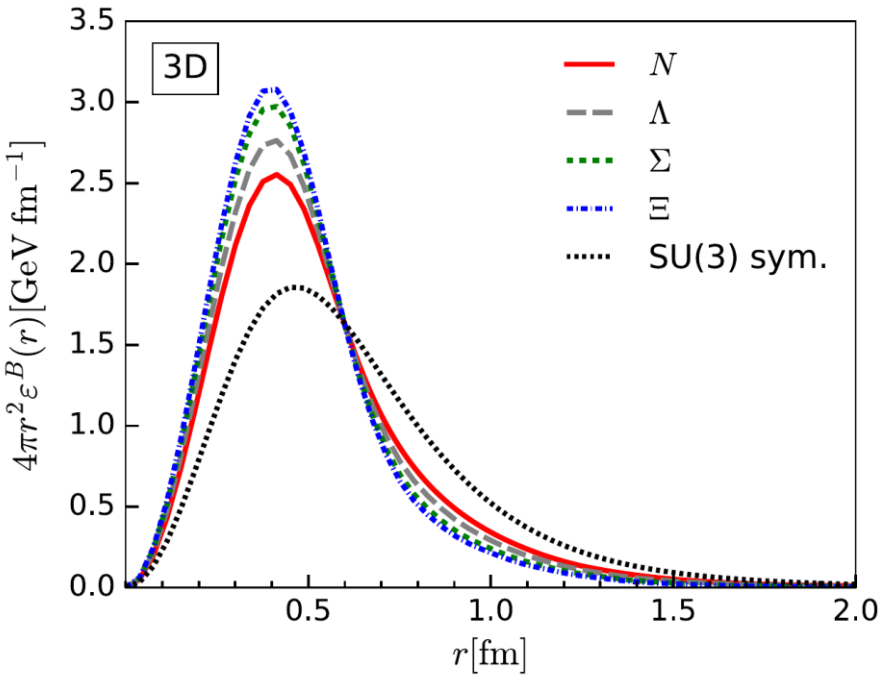
Mechanical radius
$\sim 0.6\,{\rm fm}$

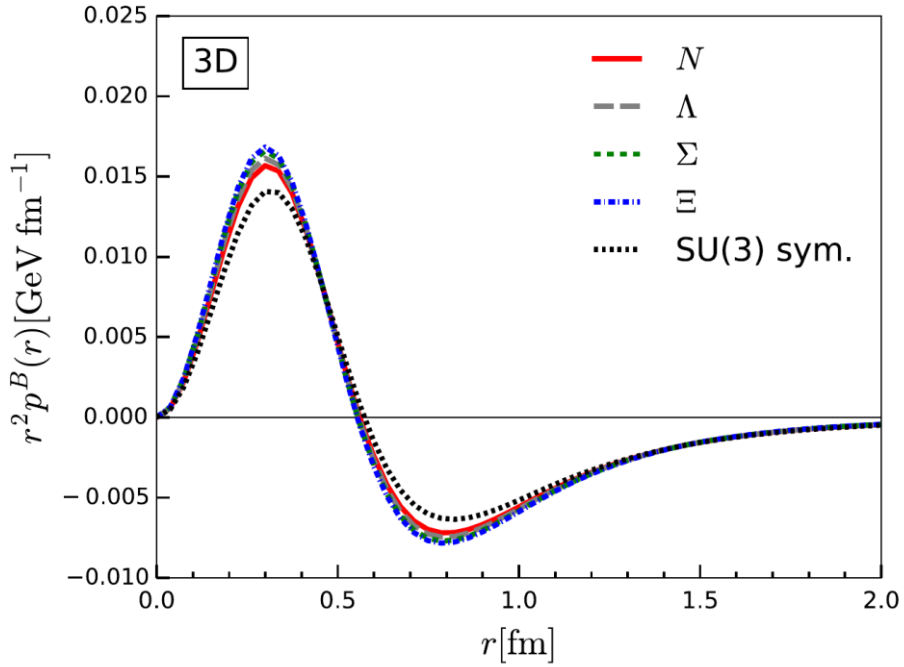


Charge radius $\sim 0.8\,{\rm fm}$

H.W. Won, J.-Y. Kim, HChK, PRD 108 (2023)

Hyperons: mass and pressure distributions





$$\langle r_\varepsilon^2 \rangle_N > \langle r_\varepsilon^2 \rangle_\Lambda > \langle r_\varepsilon^2 \rangle_\Sigma > \langle r_\varepsilon^2 \rangle_\Xi \ : \ 0.31 > 0.26 > 0.20 > 0.17 \ \mathrm{fm}^2$$

Heavier hyperons are more compact.

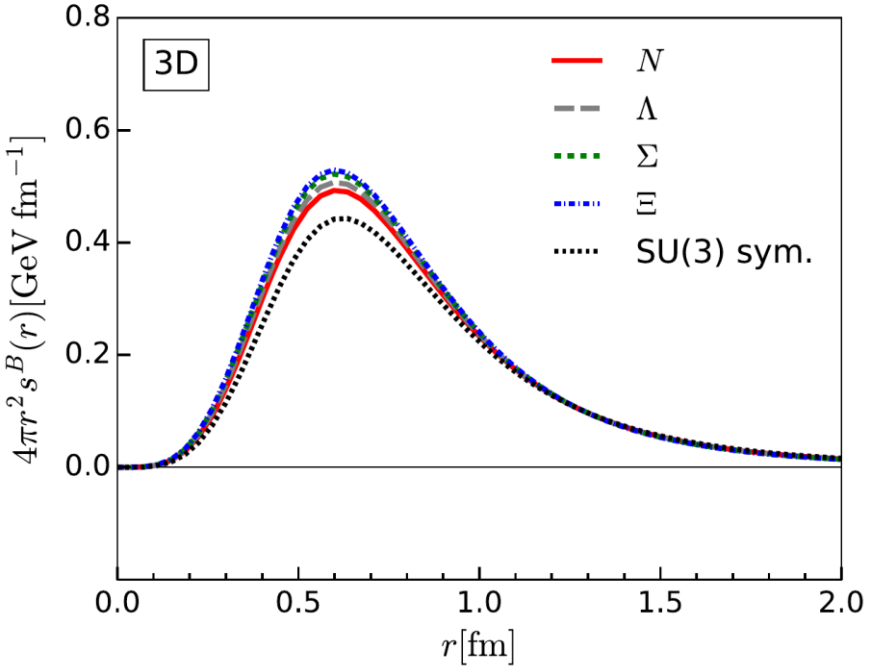
$$p^N(0) < p^\Lambda(0) < p^\Sigma(0) < p^\Xi(0) \ : \ 0.42 < 0.44 < 0.45 < 0.46 \ \mathrm{GeV\,fm}^{-3}$$

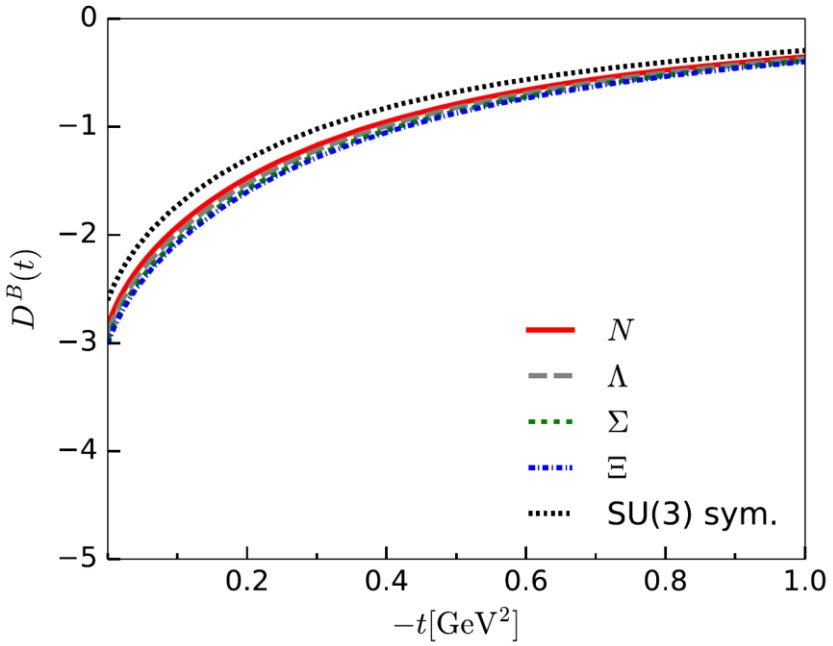
The core pressure grows with strangeness.

Linear m_s corrections, m_s = 180 MeV

H.-Y. Won, J.-Y. Kim, HChK, PRD 106 (2022)

Hyperons: D-terms and mechanical radii





$s^B(r) > 0, \qquad \frac{2}{3}s^B(r) + p^B(r) > 0 \quad \text{for all } B = N, \Lambda, \Sigma, \Xi$ Local stability holds with SU(3) breaking.

$$D^N(0) = -2.84, \quad D^\Lambda(0) = -2.91, \quad D^\Sigma(0) = -2.98, \quad D^\Xi(0) = -3.01$$

$$\langle r^2_{\rm mech}\rangle_N = 0.52, \quad \langle r^2_{\rm mech}\rangle_\Lambda = 0.51, \quad \langle r^2_{\rm mech}\rangle_\Sigma = 0.50, \quad \langle r^2_{\rm mech}\rangle_\Xi = 0.49 \text{ fm}^2$$

Heavier hyperon: larger |D|, smaller mechanical radius.

H.-Y. Won, J.-Y. Kim, HChK, PRD 106 (2022)

Summary and Conclusions

- The gravitational form factors convey information on fundamental properties of the nucleon such as the mass, spin, and D-terms.

- Strange quarks carry little of the proton mass, spin, and momentum (about 6%), but they are essential for the flavor blindness of the D-term: $D^{u-d} \approx 0$ only in SU(3).

- With flavor SU(3) symmetry breaking, heavier hyperons become more compact: larger |D| and smaller mechanical radii, while the local stability is preserved.

- Chiral quark-soliton model provides a valuable effective theory for describing the mechanical structure of the proton and hyperons.

Outlook: Studying gluonic operators will shed light on effective quark-gluon dynamics inside a nucleon.

Though this be madness,
yet there is method in it.

Hamlet Act 2, Scene 2

by Shakespeare

Many thanks to J-Y. Kim and H-Y. Won for wonderful collaborations!

Thank you very much for the attention!

Backup Slides

Energy-momentum tensor operator

Hilbert-Einstein Action (Hilbert, 1915)

$$S = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\ R + \int d^4x\sqrt{-g}L_M$$

Changing the metric in the long-wave approximation

$$g^{\mu\nu} = \eta^{\mu\nu} + \delta g^{\mu\nu}(\boldsymbol{r}) \qquad \lambda_{\mathrm{grav}} \gg M_N^{-1}$$

we find the EMT that characterizes the response of the nucleon to the static change of the space-time metric:

Energy-Momentum Tensor (EMT):

$$T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g_{\mu\nu}}$$

Energy-momentum tensor operator

Bellifante-Rosenfeld type QCD EMT Current

$$T_q^{\mu\nu} = \frac{i}{4}\bar{\psi}_q\left(\gamma^{\{\mu}\overleftrightarrow{\mathcal{D}}^{\nu\}}\right)\psi_q, \qquad T_g^{\mu\nu} = -F^{\mu\rho,a}F^{\nu,a}_{\rho} + \frac{1}{4}g^{\mu\nu}F^{\lambda\rho,a}F^{a}_{\lambda\rho}$$

Quark part

Gluon part

$$\overleftrightarrow{\mathcal{D}}^{\mu} = \overleftrightarrow{\partial}^{\mu} - 2igA^{\mu} \qquad \overleftrightarrow{\partial}^{\mu} = \overrightarrow{\partial}^{\mu} - \overleftarrow{\partial}^{\mu} \qquad a^{\{\mu}b^{\nu\}} = a^{\mu}b^{\nu} + a^{\nu}b^{\mu}$$

Conservation of the EMT current

$$\partial_\mu T_q^{\mu\nu} \neq 0 \qquad \partial_\mu T_g^{\mu\nu} \neq 0 \qquad \partial_\mu(T_q^{\mu\nu} + T_g^{\mu\nu}) = 0$$

Effective EMT operators from the instanton vacuum

A_μ → Effective quark operators → Pion mean fields

Dilute instanton medium

$$\pi^2 \left(\frac{\bar{\rho}}{\bar{R}}\right)^4 \approx 0.1$$

Bosonization

D. Diakonov et al. NPB 461 (1996)
J. Balla et al. NPB 510 (1998)

All low-energy theorems are satisfied (chiral anomaly, trace anomaly, etc).

- **The chiral-even twist-2 local operator** is generated by expanding the non-local vector current, which measures the vector GPDs, with respect to the space-time distance:

$$O_q^{\mu\nu_1\ldots\nu_n} := \bar{\psi}(x)\gamma^{\{\mu}\overleftrightarrow{D}^{\nu_1}\overleftrightarrow{D}^{\nu_2}\ldots\overleftrightarrow{D}^{\nu_n\}}\lambda_\chi\psi(x) - \text{traces},$$

$$O_g^{\mu\nu_1\ldots\nu_n} := -F^{\{\mu\rho,a}\overleftrightarrow{D}^{\nu_1}\ldots\overleftrightarrow{D}^{\nu_{n-1}}F_\rho^{\nu_n\},a} - \text{traces}$$

Twist-2 gluon operators are suppressed with respect to the packing fraction.

$$\bar{T}_\chi^{\mu\nu}(x) = \frac{i}{4}\bar{\psi}(x)\gamma^{\{\mu}\overleftrightarrow{\partial}^{\nu\}}\lambda_\chi\psi(x) - \text{traces}, \quad \bar{T}_g^{\mu\nu}(x) = 0$$

H-Y. Won, HChK, J.-Y. Kim, JHEP 05 (2024)