Quantum Entanglement in Collider Physics

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Outline

- What is Quantum Entanglement (QE)?
  - What can be measured in collider experiments
- Overview of QE analyses at LHC
  - Top quark pairs
  - Higgs boson decaying to diboson
- QE as a probe to (new) physics → What can be done at EIC?
- Summary and Outlook

Quantum Entanglement (QE)

- Entanglement: The density matrix of a system cannot be written as a product of its components

$$|\psi\rangle = |a\rangle \otimes |b\rangle$$ Separable

QE = Not separable

Quantum Entanglement (QE)

- Experimental consequence: (Spin) correlations that exceed the bounds predicted theories which preassign the features *before* measurements (local hidden variables).

  → Violation of Bell's inequalities

Quantum Entanglement

Clauser-Horne-Shimony-Holt (CHSH) inequality (for an entangled pair of spin-½ particles):

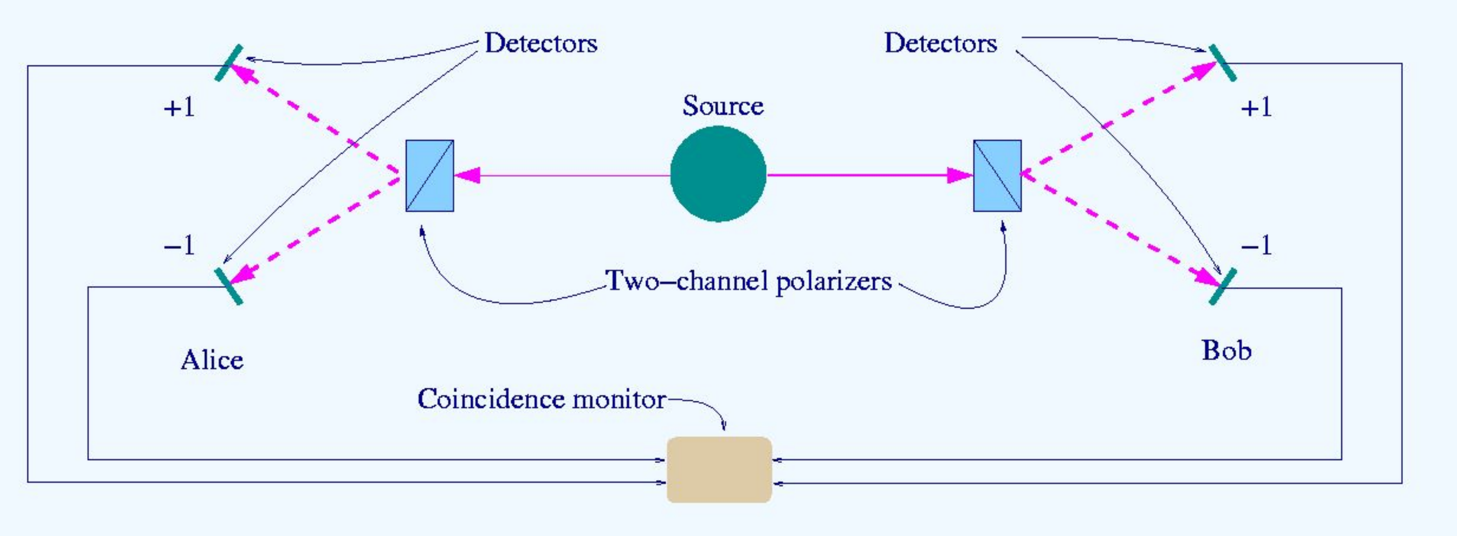
$$E(a,b) - E(a,b') + E(a',b) + E(a',b') \leq 2$$

E(a,b) = expectation value of measuring spin along a by Alice and b by Bob.
– Alice and Bob each has two choices (a, a’) or (b, b’)
– QM bound <= 2√2

Quantum Entanglement

- Experimental proof: Pairs of photons (spin-1 two-state systems)

[wikipedia]



Quantum Entanglement

The Nobel Prize in Physics 2022

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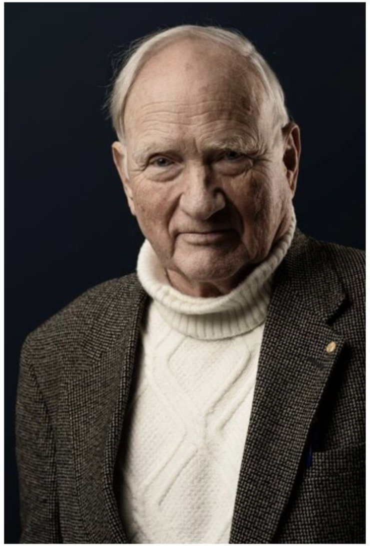
Alain Aspect



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John F. Clauser

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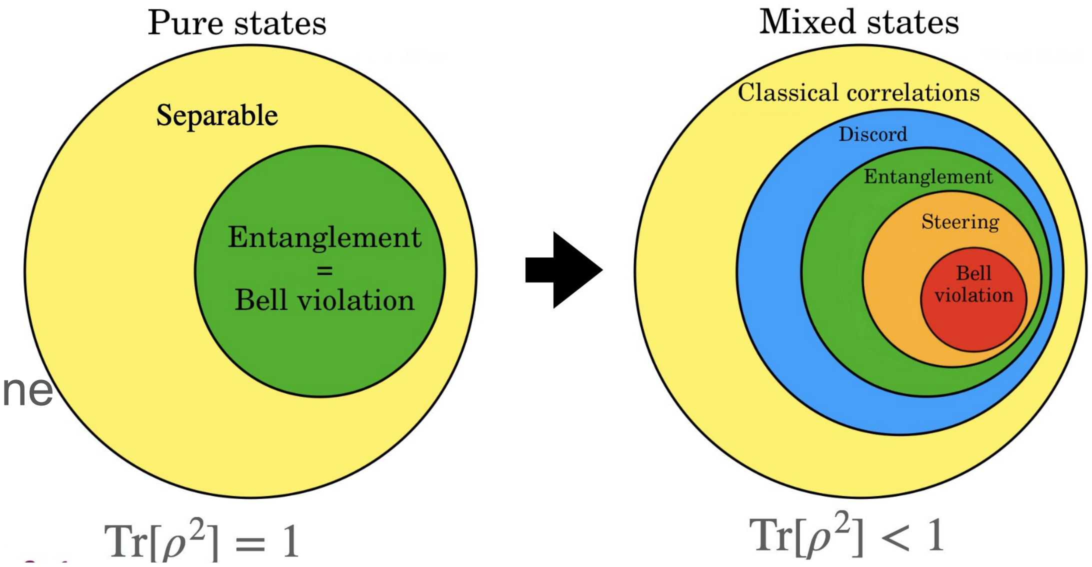
Anton Zeilinger

“for experiments with **entangled photons**, establishing the **violation of Bell inequalities** and pioneering quantum information science.”

Quantum Entanglement

- More than Bell's inequalities → Many 'jargons' for different levels of correlations

Will not go through the details, but focus on what have been done/can be done with collider data.



QE at Colliders

- Note: It is not possible to 'test' QE (exclude local-hidden-variable models) with collider experiments
  → cannot evade loopholes
  Phys. Rev. D **112**, 096008

- Current mindset (as I understand): Measure QE-inspired quantities and see how they look like.
  → do not (over)interpret the results

QE at Colliders

- So what can we do with collider data?
  - Qubits: top-pairs, di-tau, diphoton
  - Qutrits: WW, ZZ
  - From Higgs boson decays, B meson decays, or generic productions of these particles.
- Focus on tt and $H \to VV^*$ in the following

Quantum Entanglement: Top Quark Pairs

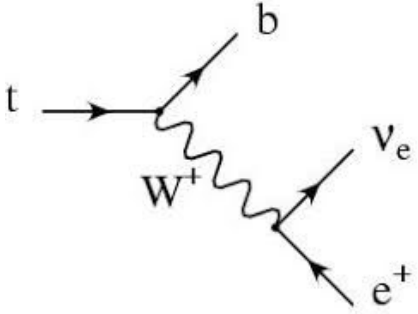
- Top-antitop pairs = pairs of spin-½ particles
  – Top decays before hadronization → spin information is preserved in angular distributions of decay products

- Spin correlations encapsulated in the density matrix of ttbar

$$\rho = \frac{\mathbb{1}_2 \otimes \mathbb{1}_2 + B_i^+ \sigma^i \otimes \mathbb{1}_2 + B_i^- \mathbb{1}_2 \otimes \sigma^i + C_{ij} \sigma^i \otimes \sigma^j}{4}$$

Quantum Entanglement: Top Quark Pairs

- Peres-Horodecki criterion: A sufficient condition of entanglement is tr[C]<-1 (at low m_{tt})



- Differential cross section of ttbar

$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\varphi} = \frac{1}{2}(1 - D\cos\varphi)$$

Angle between the two leptons in *parent top rest frame*

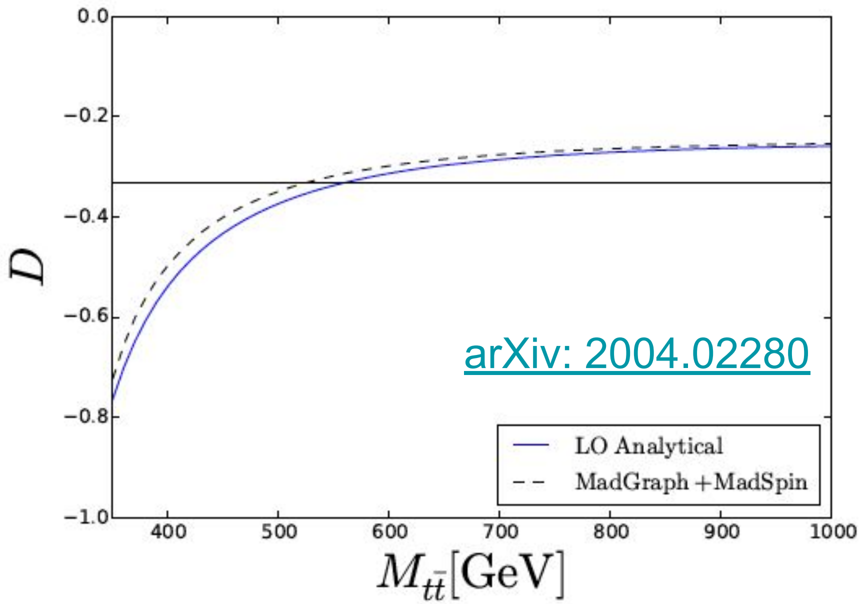
$$D = \frac{\mathrm{tr}[\mathbf{C}]}{3}$$

Quantum Entanglement: Top Quark Pairs

- Recipe: Extract D from ttbar cross section measurement

- Condition for observation of entanglement: $D < -\frac{1}{3}$

- Narrow phase space: near-threshold ttbar



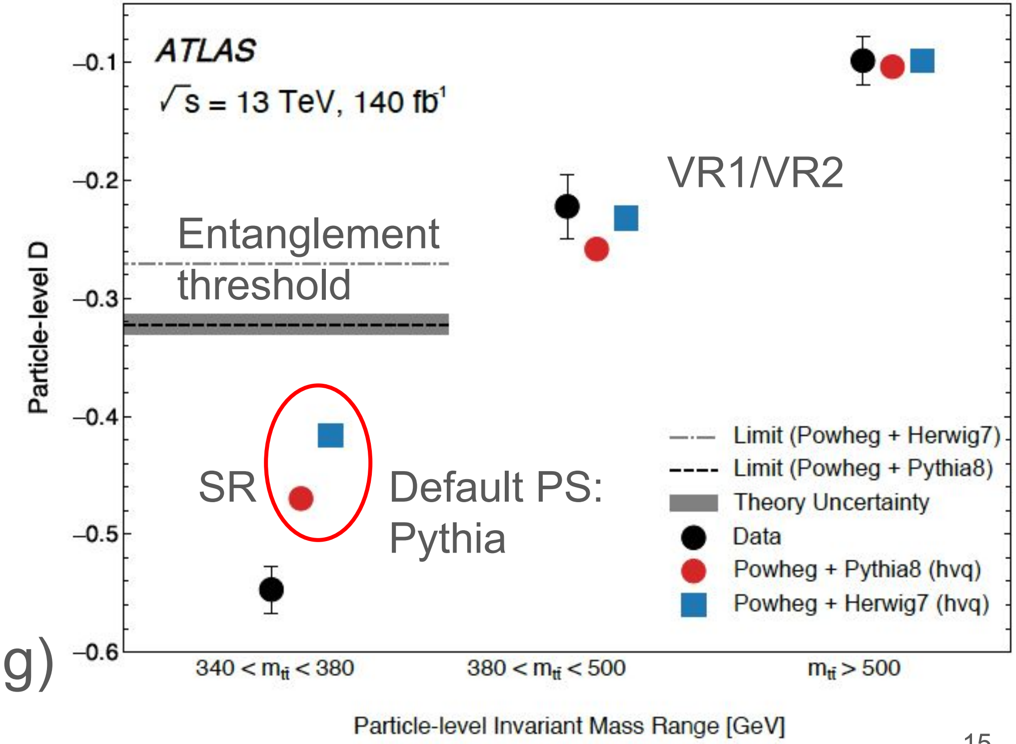
arXiv: 2004.02280

Quantum Entanglement: Top Quark Pairs

- ATLAS: First observation of ttbar entanglement at the LHC in Nov. 2023 (Nature 633, 542 (2024)) with full Run-2 data
- Event selection: eμ + >=1 b-jet
- Top rest frame reconstruction: Ellipse method (geometric approach to solve for ν kinematics; 85%) → Neutrino weighting (scan and find the most compatible solution) → simple pairing.

Quantum Entanglement: Top Quark Pairs

- >5σ from no-entanglement Scenario ("within SM").
- Systematics dominated by signal modeling (3.2%)
- MC prediction differs with different parton shower simulation (Pythia v.s. Herwig)



Quantum Entanglement: Top Quark Pairs

- CMS: 36 fb-1 of Run-2 data Rep. Prog. Phys. 87 117801 (plots from previous PAS)

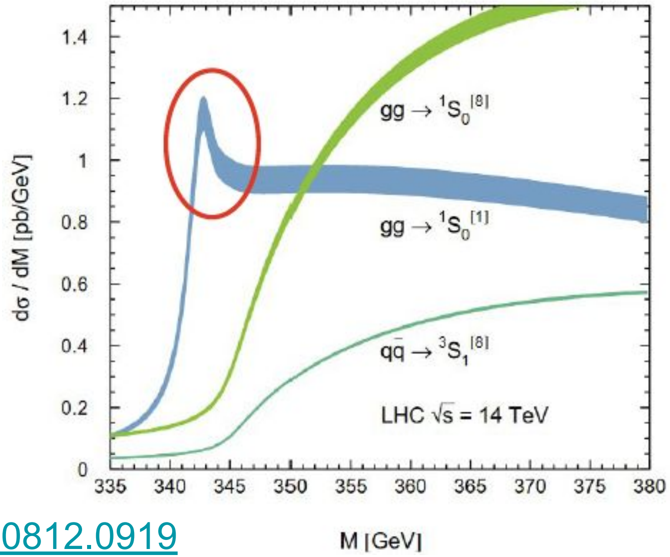
- Di-lepton (ee, eμ, μμ) channel with >=1 b-jet

- Top rest frame reconstruction: Neutrino weighting

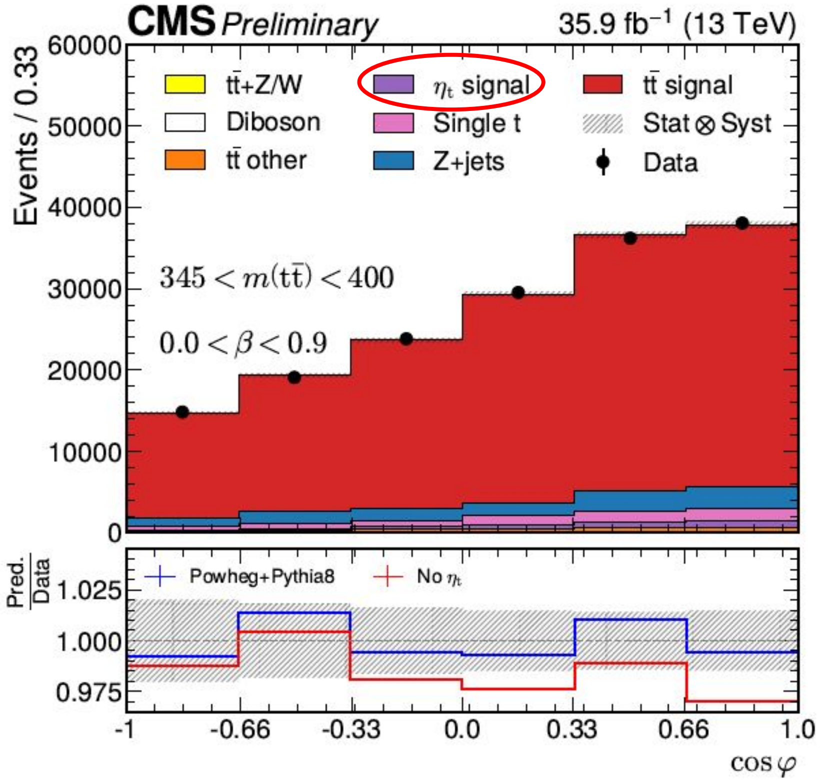
- SR: $345 < m(\mathrm{t\bar{t}}) < 400\,\mathrm{GeV}$ and $\beta = |\frac{p_z^t - p_z^{\bar{t}}}{E^t - E^{\bar{t}}}| < 0.9$

Quantum Entanglement: Top Quark Pairs

- CMS: A spin-0 toponium η_t in signal improves data modeling in SR



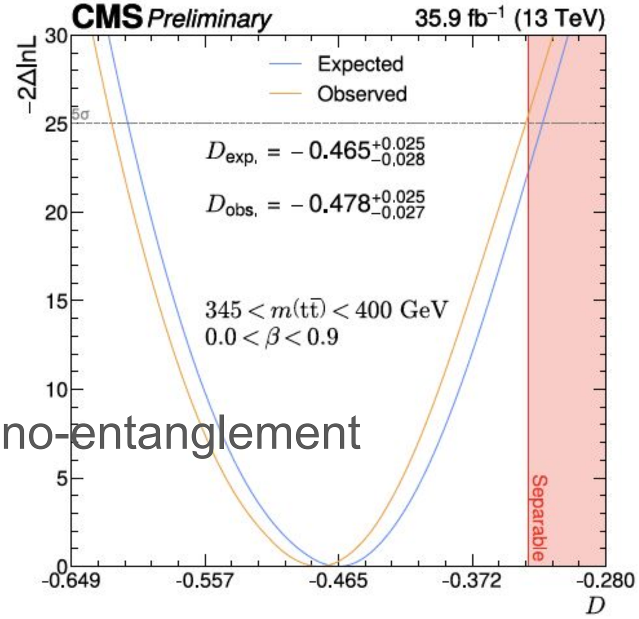
arXiv:0812.0919



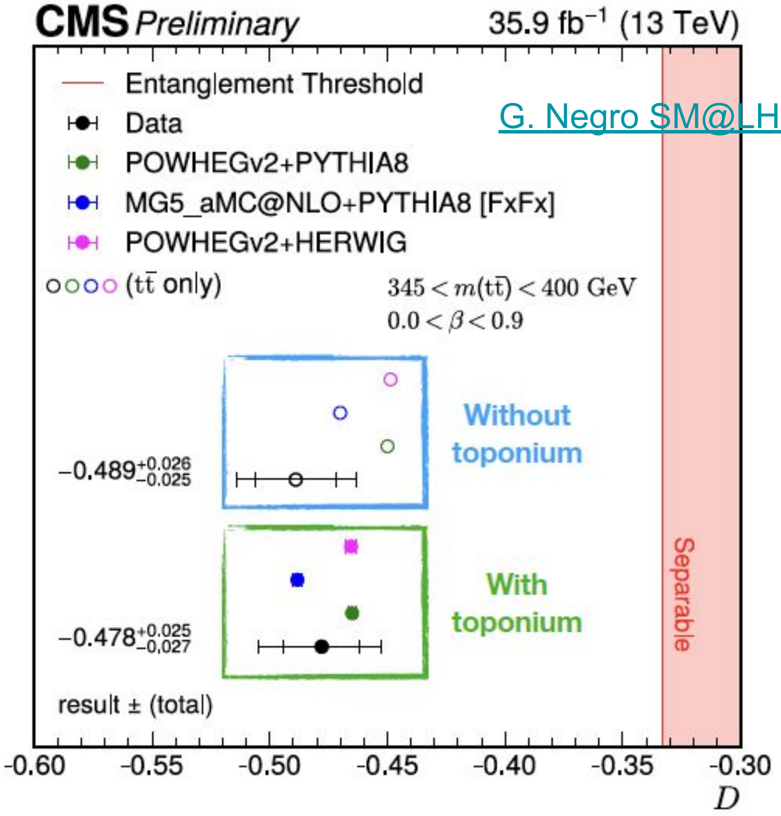
Quantum Entanglement: Top Quark Pairs

- Parton-level D from likelihood fit

G. Negro SM@LHC

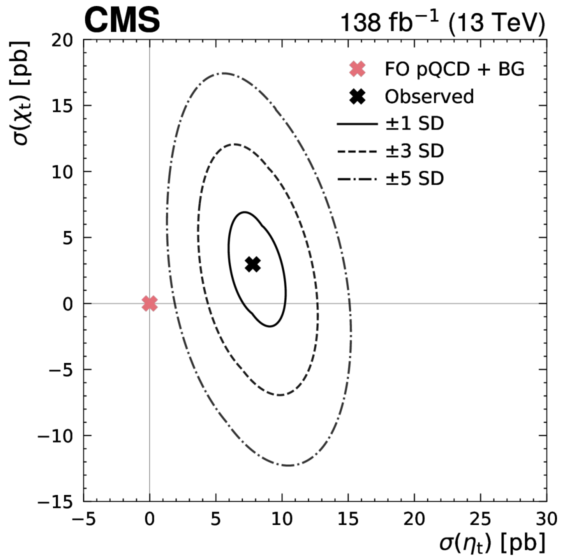


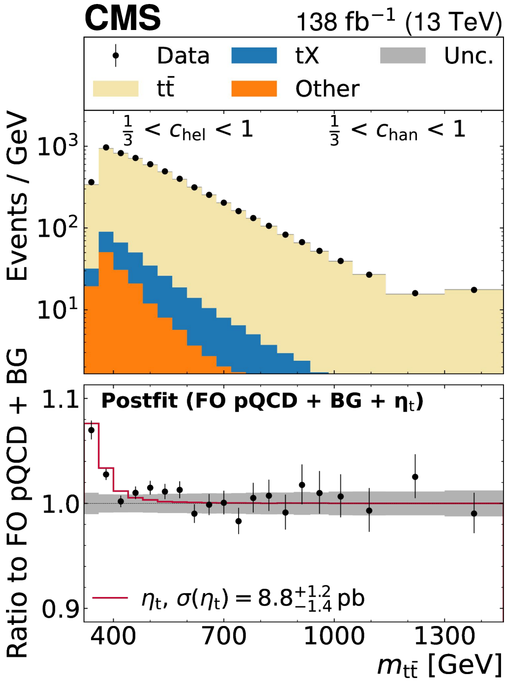
>5σ w.r.t. no-entanglement



QE surprise: Discovery of Toponium

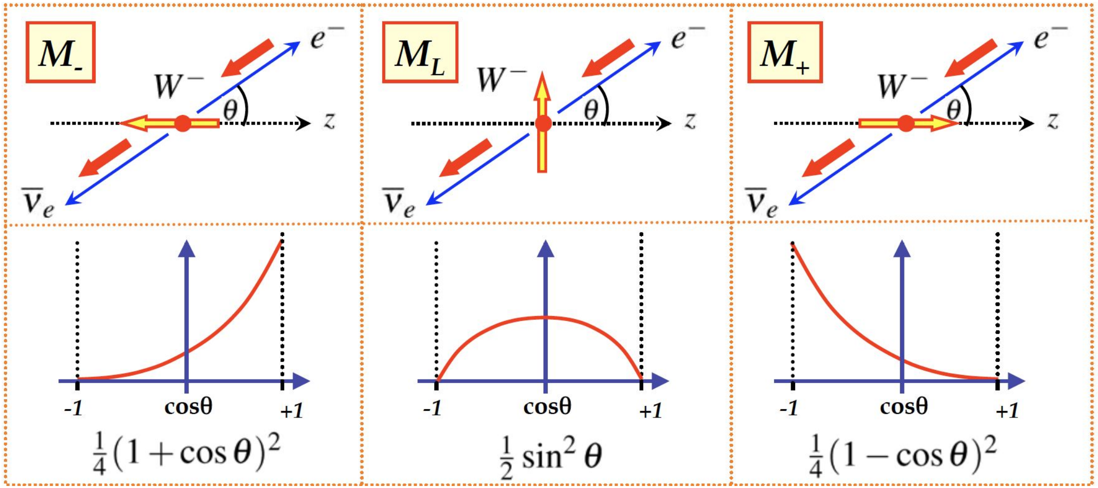
- Both ATLAS and CMS then confirm the existence of ttbar bound states near ttbar mass threshold.

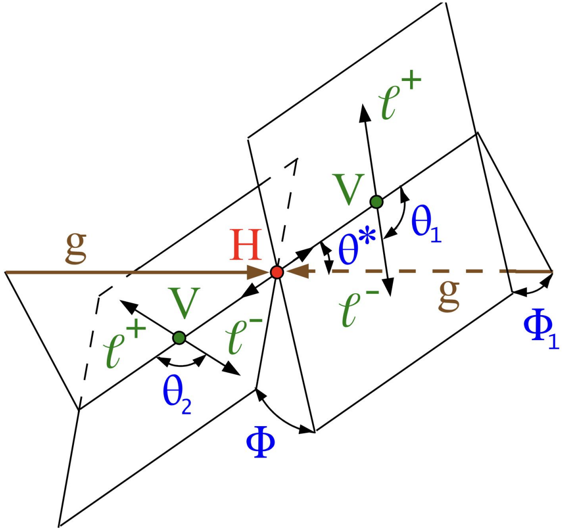




Quantum Entanglement: H→VV

- Massive vector bosons are ‘their own polarizers’ – polarization encoded in the angular distributions of the decay products *in the rest frame of the boson*.





Quantum Entanglement: H→VV

- Quantum observables can be express in terms of coefficients of the 9x9 density matrix of spin-1 pairs:

arXiv: 2302.00683

$$\rho = \frac{1}{9}[\mathbb{1} \otimes \mathbb{1}] + \sum_a f_a [T^a \otimes \mathbb{1}] + \sum_a g_a [\mathbb{1} \otimes T^a] + \sum_{ab} h_{ab} \left[T^a \otimes T^b\right]$$

3x3 Gell-Mann matrices
a, b = 1,2,...,8

- Assume Spin-0 Higgs → only 9 nonzero elements in the 9x9 density matrix

Quantum Entanglement: H→VV

arXiv:2402.07972

- Assuming full knowledge of the density matrix, can construct different QE observables.

Assume SM H→VV, the two V's are entangled iff h_44 ≠ 0 or h_16 ≠ 0

$$\rho = 2 \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{16} & 0 & 2\,h_{33} & 0 & h_{16} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & h_{44} & 0 & h_{16} & 0 & h_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Quantum Entanglement: H→VV

- The optimal Bell inequality for a qutrit is the Collins-Gisin-Linden-Massar-Popescu (CGLMP) inequality

  P(Ai=Bj+k): prob. that Ai and Bj differ by (k mod 3)

$$
\begin{aligned}
\mathcal{I}_3 \quad = \quad & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\
& - P(A_1 = B_1 - 1) - P(A_1 = B_2) - P(A_2 = B_2 - 1) - P(B_2 = A_1 - 1)
\end{aligned}
$$

Quantum Entanglement: H→WW

- $I_3 > 2$ (classical max. bound) implies entanglement (QM max ≈ 2.9149)

- Translate to physically measurable observables: directional cosines $\xi_i^{\pm} = \vec{n}_i \cdot \vec{n}_{\ell\pm}$ of the charged lepton in the W's rest frame.

Quantum Entanglement: H→WW

$\hat{\mathbf{k}}$ is the direction of the W+;

$\hat{\mathbf{p}}$ is the direction of the beam;

$\hat{\mathbf{n}}$ is perpendicular to the $\hat{\mathbf{k}}$-$\hat{\mathbf{p}}$ plane;

$\hat{\mathbf{r}} = \hat{\mathbf{k}} \times \hat{\mathbf{n}}$ is the vector orthogonal to

$\hat{\mathbf{k}}$ within the $\hat{\mathbf{k}}$-$\hat{\mathbf{p}}$ plane.

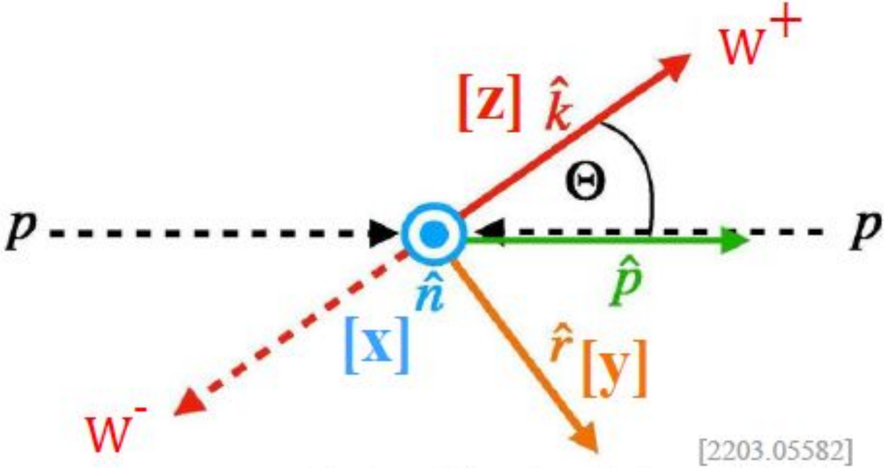
The directional cosines ξ are defined in the farther W-rest frame.

$$\{\hat{\mathbf{n}}, \hat{\mathbf{r}}, \hat{\mathbf{k}}\} \to \{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{z}}\}$$

$$\xi_i^+ = \hat{\mathbf{n}}_i \cdot \hat{\mathbf{n}}_{\ell+} \text{ and } \xi_j^- = \hat{\mathbf{n}}_j \cdot \hat{\mathbf{n}}_{\ell-}$$

$$\hat{\mathbf{k}}, \qquad \hat{\mathbf{r}} = \frac{1}{r}(\hat{\mathbf{p}} - y\hat{\mathbf{k}}), \qquad \hat{\mathbf{n}} = \frac{1}{r}(\hat{\mathbf{p}} \times \hat{\mathbf{k}}),$$

where $y = \hat{\mathbf{p}} \cdot \hat{\mathbf{k}}$ and $r = \sqrt{1-y^2}$.



[2203.05582]
Quantum information with top quarks in QCD

Quantum Entanglement: H→WW

- With ξ's and the coordinate system defined, the CGLMP inequality can be evaluated from the expectation value of the Bell operator B

arXiv: 2106.01377

$$\langle \mathcal{B}^{xy}_{\mathrm{CGLMP}} \rangle = \frac{8}{\sqrt{3}} \langle \xi^{+}_{x} \xi^{-}_{x} + \xi^{+}_{y} \xi^{-}_{y} \rangle_{av}$$
$$+ 25 \langle ((\xi^{+}_{x})^2 - (\xi^{+}_{y})^2)((\xi^{-}_{y})^2 - (\xi^{-}_{y})^2) \rangle_{av}$$
$$+ 100 \langle \xi^{+}_{x} \xi^{+}_{y} \xi^{-}_{x} \xi^{-}_{y} \rangle_{av}$$
$$\mathcal{I}_3 = \max(\langle \mathcal{B}^{xy}_{\mathrm{CGLMP}} \rangle, \langle \mathcal{B}^{yz}_{\mathrm{CGLMP}} \rangle, \langle \mathcal{B}^{zx}_{\mathrm{CGLMP}} \rangle)$$

Quantum Entanglement: H→WW

- Challenges:
  - The formula assumes full acceptance
  - Dominant WW backgrounds
  - Reconstruct the W rest frames given the existence of neutrinos

- The NTHU team is working on machine-learning methods.

- It turns out (for us) more feasible to target on the 4-vectors of the two W bosons rather than neutrinos.

QE in H→WW: Reconstruct W boson rest frames

- **P**hysics-**c**onstrained **Res**idual **regressor**: PcRes regressor
- Learned constraints by NN
  - p_x^{miss}, p_y^{miss} from $\nu\nu$ (-2 DoF)
  - $m_H \approx 125$ GeV (-1 DoF)
- Constraints put by hand
  - $m_\nu \approx 0$ GeV (-2 DoF)
  - Constrain m_W (-1 DoF)

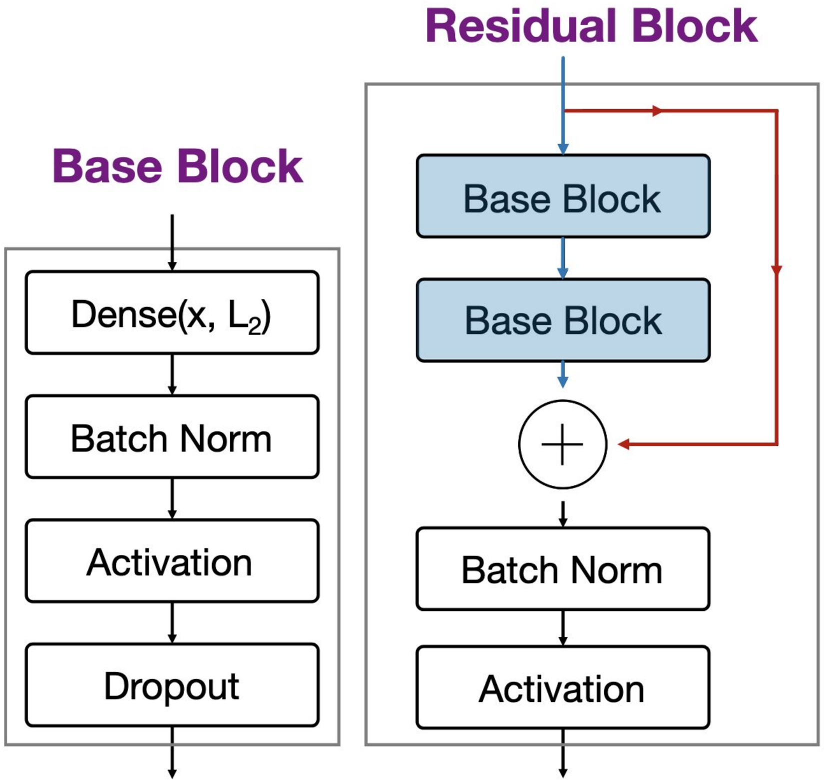
Training object	Components
ℓ_0, ℓ_1	p_x, p_y, p_z, E
E_T^{miss}	p_x, p_y

Target object	Components
W_0, W_1(*)	p_x, p_y, p_z, E m (derived from $\boldsymbol{p}^{W_{0/1}}$)

(*) $W_{0/1}$ is the W boson associated with $\ell_{0/1}$

Master thesis of Yuan-Yen Peng

QE in H→WW: Reconstruct W boson rest frames

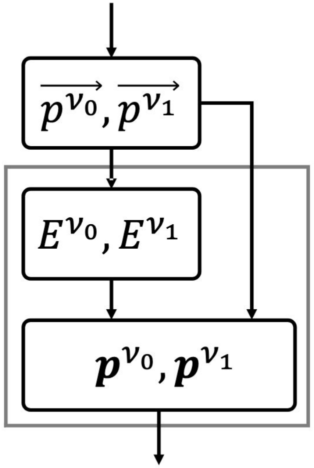


- **Shortcut: preserve the signal**
- **Residual net: train the difference**

- The refining idea helps avoid degradation
- Allow deeper training

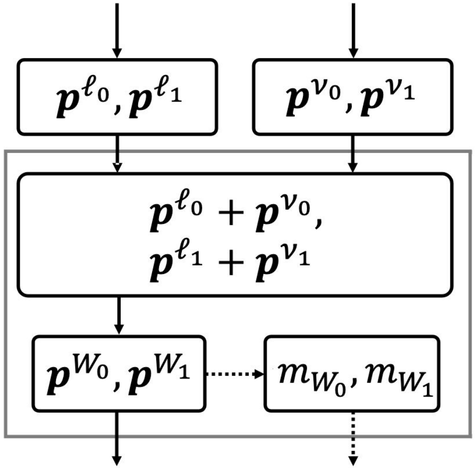
QE in H→WW: Reconstruct W boson rest frames

$\nu\nu$-layer

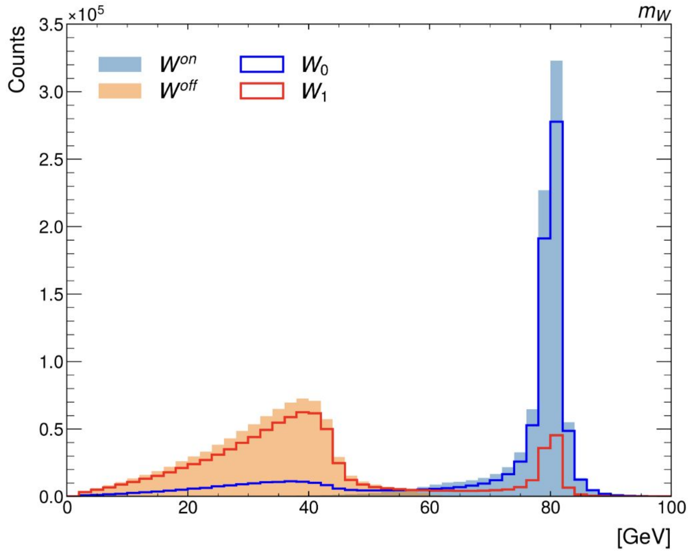


- Force a massless neutrino:
  $$E = \sqrt{||\vec{p}||^2 + 0^2}$$
- This layer ensures that all predicted W are time-like!

WW-layer



- Add up the 4-vec of the associated decay products
- Also, drive the $m_{W_{0/1}}$ as an indirect predicting object

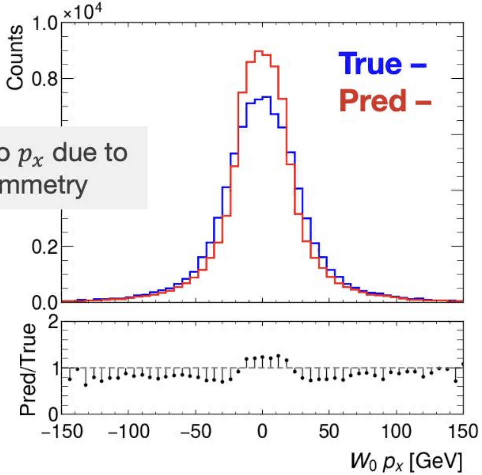


$W_{0/1}$ represents the W boson associated with leading/subleading lepton

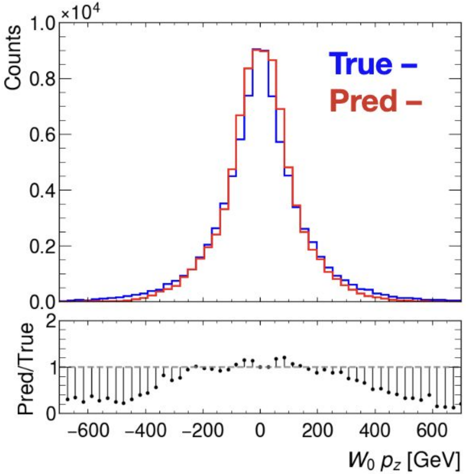
Results

p_y is similar to p_x due to the spatial symmetry

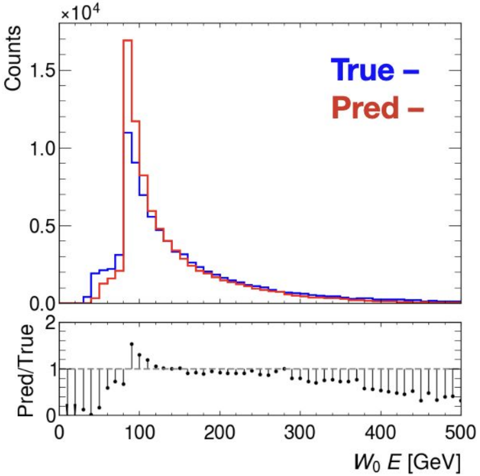
p_x



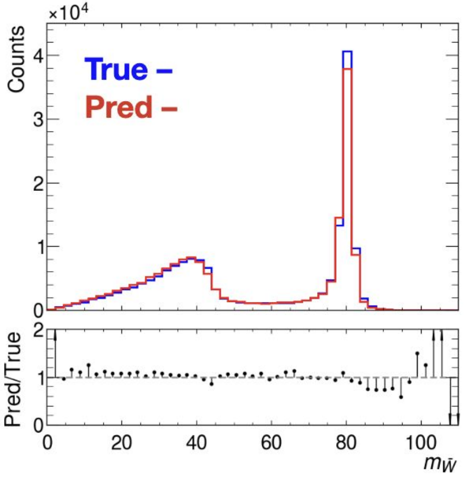
p_z



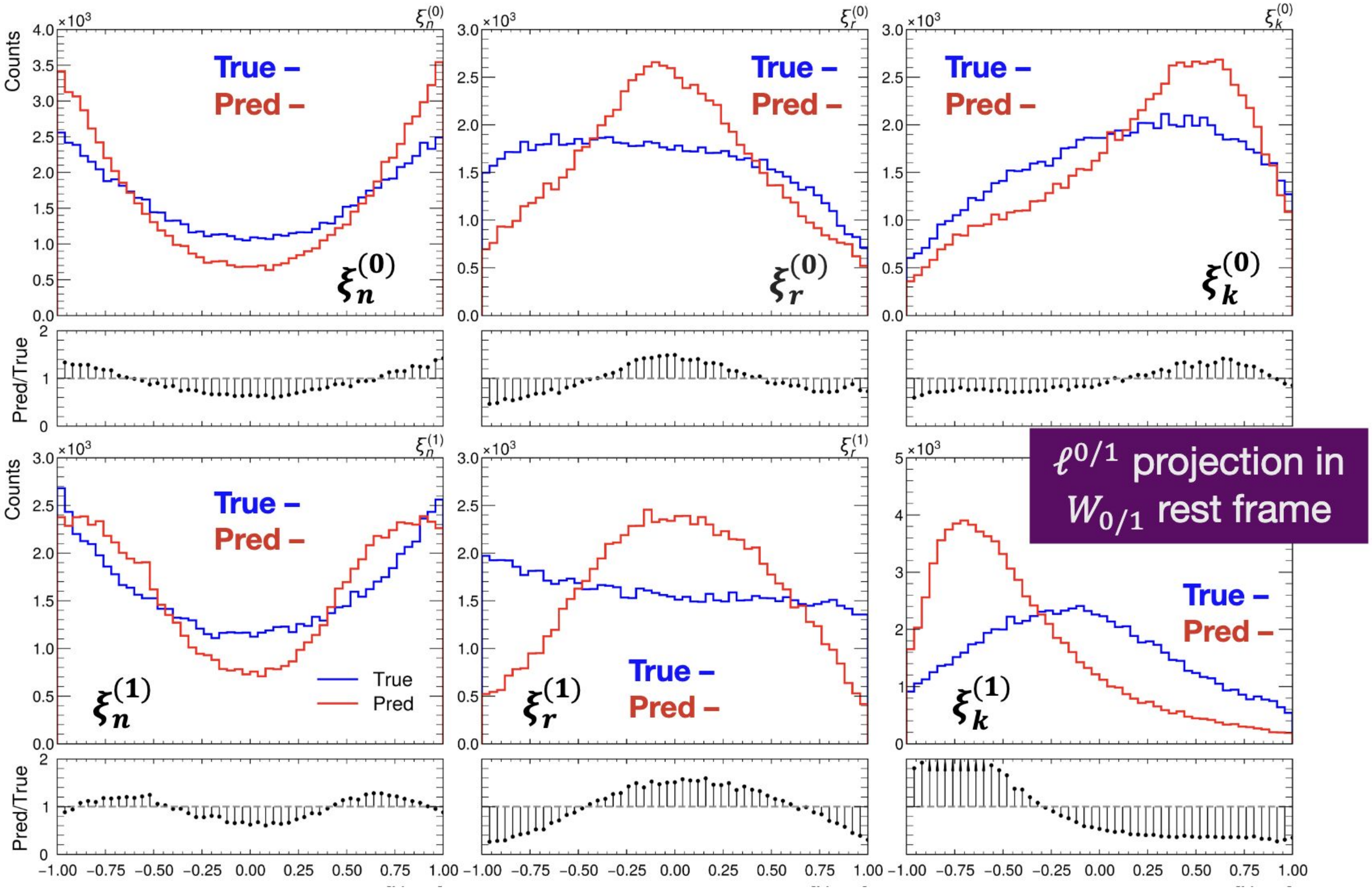
E



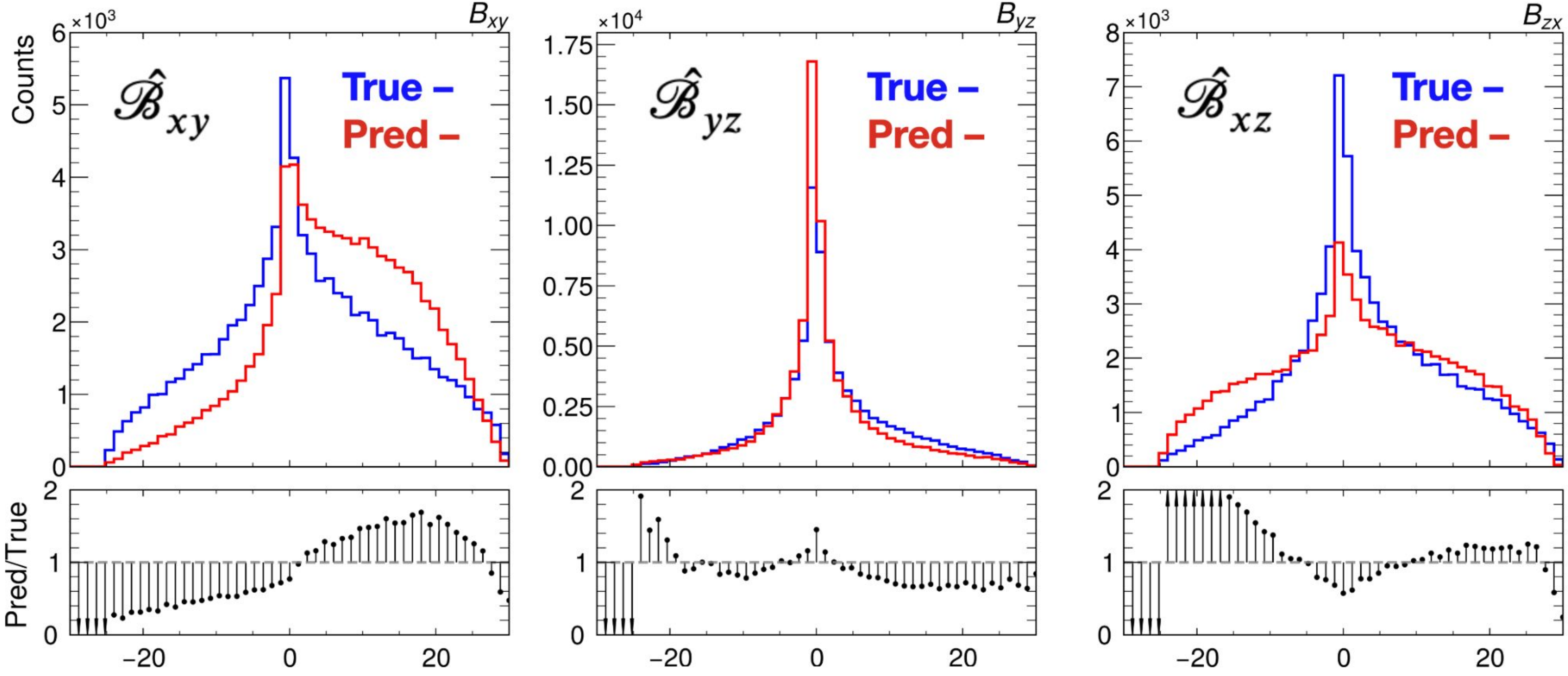
m_W



QE variables



QE variables



Quantum Entanglement: H→WW

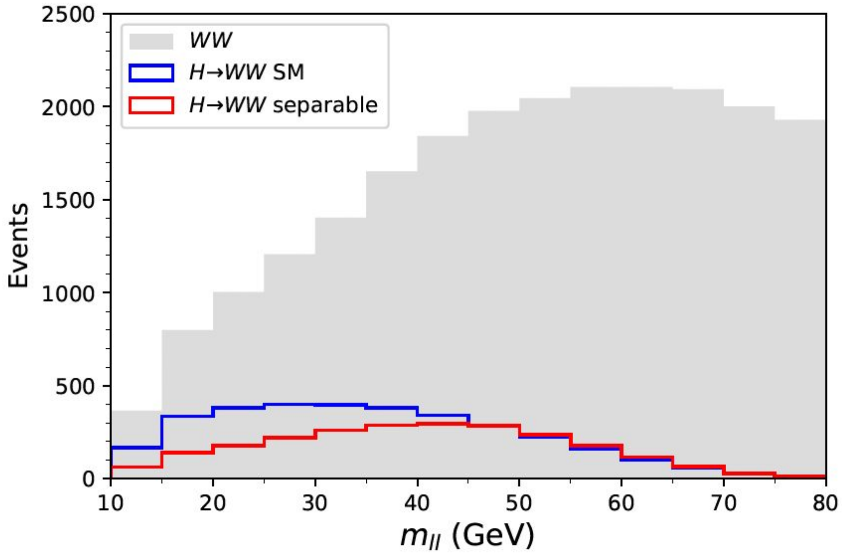
Phys. Rev. D **107**, 076016

- Another proposal: Exclude |00> state (non-entangled) (and thereby observe QE) using di-lepton distributions in the lab frame (|++> and |--> must be symmetric assuming SM Higgs)

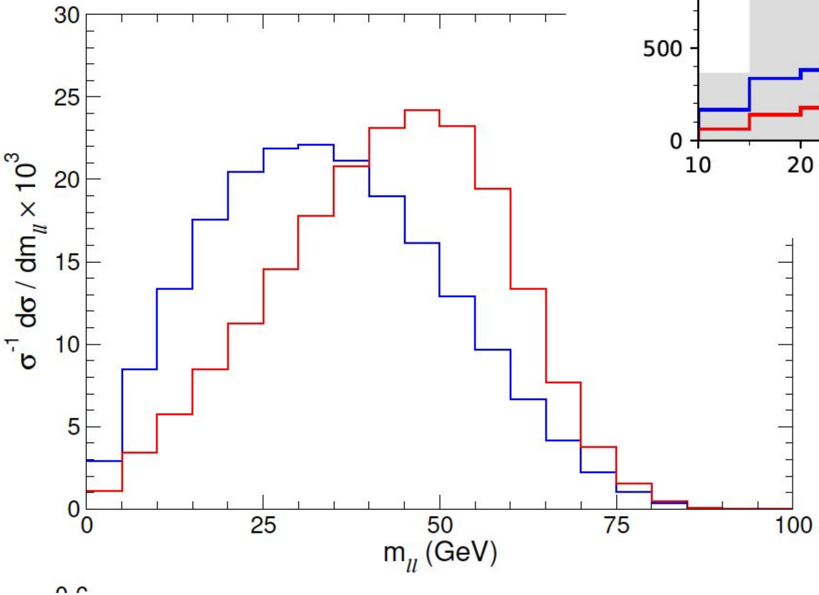
$$|\psi\rangle = \frac{a_{++}|++\rangle + a_{00}|00\rangle + a_{--}|--\rangle}{\sqrt{|a_{++}|^2 + |a_{00}|^2 + |a_{--}|^2}}$$

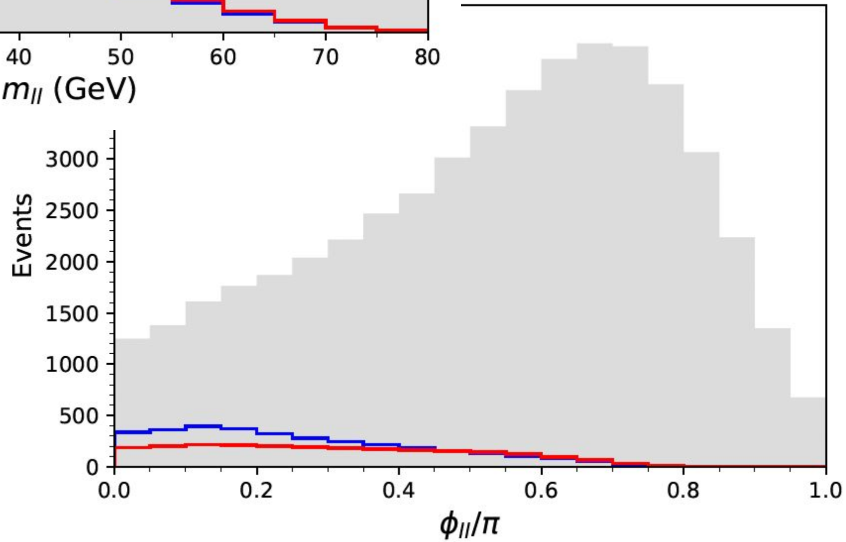
QE in H→WW

[Phys. Rev. D **107**, 076016](Phys. Rev. D 107, 076016)



Challenge: Distributions may be distorted by selection cuts





Quantum Entanglement: $H \to ZZ$

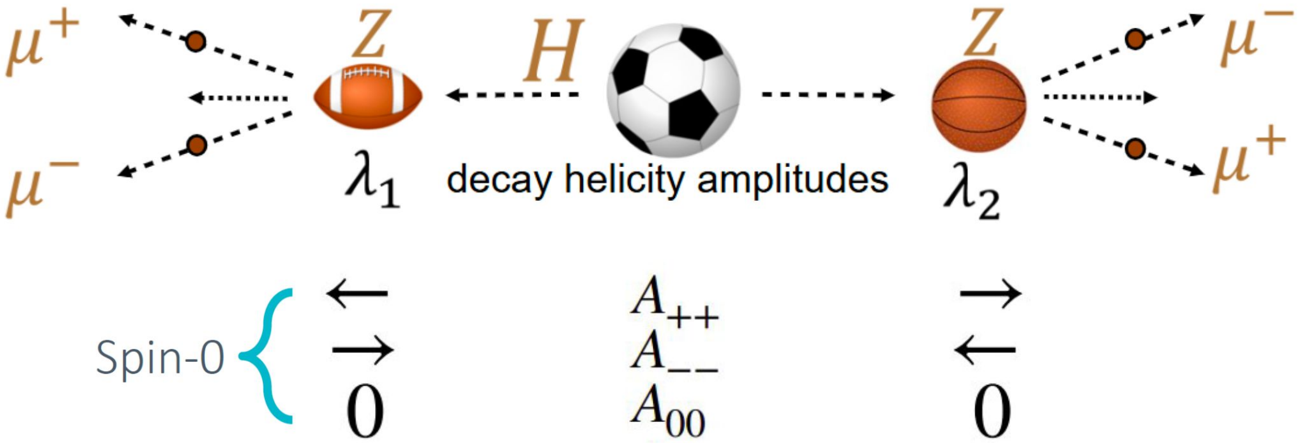
- In principle one can fully reconstruct the Z boson rest frames in $H \to ZZ^* \to 4l$ decays
  - Low stats
  - 'Projection method' assumes 100% acceptance (no cuts on kinematic phase space)

- CMS recently published the results. ATLAS results underway.

QE in H→ZZ: CMS

CMS-PAS-HIG-25-011

- CMS encloses the QE analysis in the ‘spin correlation’ studies



Higgs2025 slides

QE in H→ZZ: CMS

- Assuming ZZ from a spin-0 Higgs decay:

Prob. density as a fn. of the masses of the two Z's

$$\rho = \int \frac{dm_1 dm_2\, \mathcal{P}(m_1, m_2)}{|A_{++}|^2 + |A_{00}|^2 + |A_{--}|^2} \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{++}A^*_{++} & 0 & A_{++}A^*_{00} & 0 & A_{++}A^*_{--} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{00}A^*_{++} & 0 & A_{00}A^*_{00} & 0 & A_{00}A^*_{--} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{--}A^*_{++} & 0 & A_{--}A^*_{00} & 0 & A_{--}A^*_{--} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

QE in H→ZZ: CMS

- CMS further parametrizes ρ using two parameters
  - Average fraction of longitudinal decay:

$$f_L = \int dm_1 dm_2 \, \mathcal{P}(m_1, m_2) \frac{|A_{00}|^2}{|A_{++}|^2 + |A_{--}|^2 + |A_{00}|^2}$$

QE in H→ZZ: CMS

- CMS further parametrizes ρ using two parameters
  - Average fraction of longitudinal decay f_L

  - Assuming CP-even H and the decay conserves CP:

$$A_{++} = A_{--}.$$

$$A_{\|} = \frac{A_{++} + A_{--}}{\sqrt{2}}$$

QE in H→ZZ: CMS

- CMS further parametrizes ρ using two parameters
  - Average fraction of longitudinal decay f_L

  - Assuming CP-even H and the decay conserves CP
    – "Coherence parameter" C_ll

$$-C_{\|}\sqrt{f_L(1-f_L)} = \int dm_1 dm_2 \, \mathcal{P}(m_1, m_2) \frac{A_{\|}(m_1, m_2) A_{00}(m_1, m_2)}{|A_{\|}|^2 + |A_{00}|^2}$$

"to quantify the interference between the longitudinal and CP-even transverse polarization states"

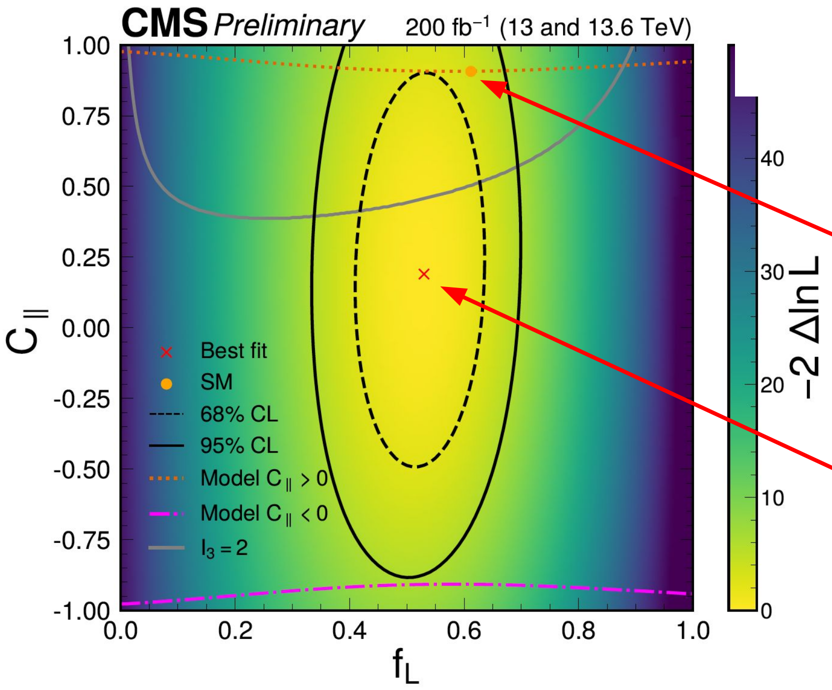
QE in H→ZZ: CMS

- CMS further parametrizes ρ using two parameters
  - Average fraction of longitudinal decay f_L
  - (Assuming CP-even H and the decay conserves CP) Coherence parameter C_ll

$$\rho = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-f_L)/2 & 0 & -C_{\parallel}\sqrt{f_L(1-f_L)/2} & 0 & (1-f_L)/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_{\parallel}\sqrt{f_L(1-f_L)/2} & 0 & f_L & 0 & -C_{\parallel}\sqrt{f_L(1-f_L)/2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & (1-f_L)/2 & 0 & -C_{\parallel}\sqrt{f_L(1-f_L)/2} & 0 & (1-f_L)/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

QE in H→ZZ: CMS

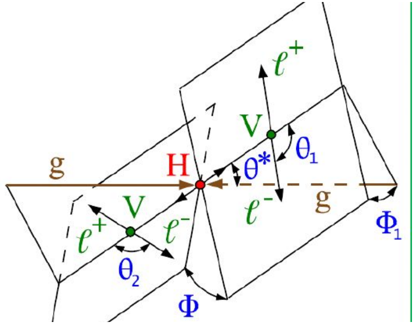
Parameter	Scenario	Observed 68% CL	Observed 95% CL	Expected 68% CL	Expected 95% CL
I_3	fix $C_{\parallel}$	$2.69^{+0.03}_{-0.08}$	$[2.52, 2.75]$	2.60 ± 0.08	$[2.39, 2.71]$
I_3	float all	$1.60^{+0.70}_{-0.68}$	$[0.28, 2.87]$	$2.60^{+0.21}_{-0.68}$	$[1.26, 2.87]$



I_3>2 (violates CGLMP inequality)

I_3 < 2

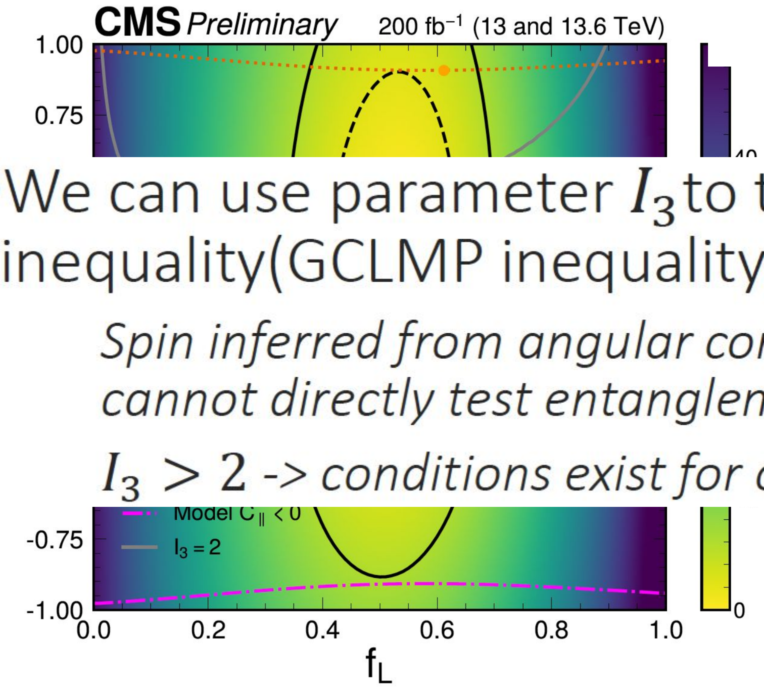
- **Observables in Fit:**
  - D_{bkg}
  - $\cos\theta_1$
  - $\cos\theta_2$
  - ϕ



D_{bkg}: Matrix-Element based discriminator for separating $H \to 4l$ from background

QE in H→ZZ: CMS

Parameter	Scenario	Observed 68% CL	Observed 95% CL	Expected 68% CL	Expected 95% CL
I_3	fix $C_{\parallel}$	$2.69^{+0.03}_{-0.08}$	$[2.52, 2.75]$	2.60 ± 0.08	$[2.39, 2.71]$
I_3	float all	$1.60^{+0.70}_{-0.68}$	$[0.28, 2.87]$	$2.60^{+0.21}_{-0.68}$	$[1.26, 2.87]$



- We can use parameter I_3 to test if ***conditions exist for*** Bell-type inequality(GCLMP inequality) measurement
- *Spin inferred from angular correlations (Assume Quantum Mechanics) so we cannot directly test entanglement/non-locality*
- $I_3 > 2$ *-> conditions exist for observing non-locality*

QE as a guide?

- Instead of making sure QE ‘exists’, using QE as a ‘principle’ to study/explain collider data. Two examples are
    - Max Entanglement in QCD processes e.g. hadronization
    - Min Entanglement and emergent symmetry in low energy QCD

Max Entanglement in hadronization

[Phys. Rev. Lett. **134**, 111902](#)

Entanglement as a probe of hadronization

Jaydeep Datta,[1,*] Abhay Deshpande,[1,2,†] Dmitri E. Kharzeev,[3,4,‡] Charles Joseph Naïm,[1,§] and Zhoudunming Tu[2,¶]

[1] *Center for Nuclear Frontiers in Nuclear Science, Department of Physics and Astronomy, Stony Brook University, New York 11794-3800, USA*
[2] *Department of Physics, Brookhaven National Laboratory, Upton, New York 11973-5000, USA*
[3] *Center for Nuclear Theory, Department of Physics and Astronomy, Stony Brook University, New York 11794-3800, USA*
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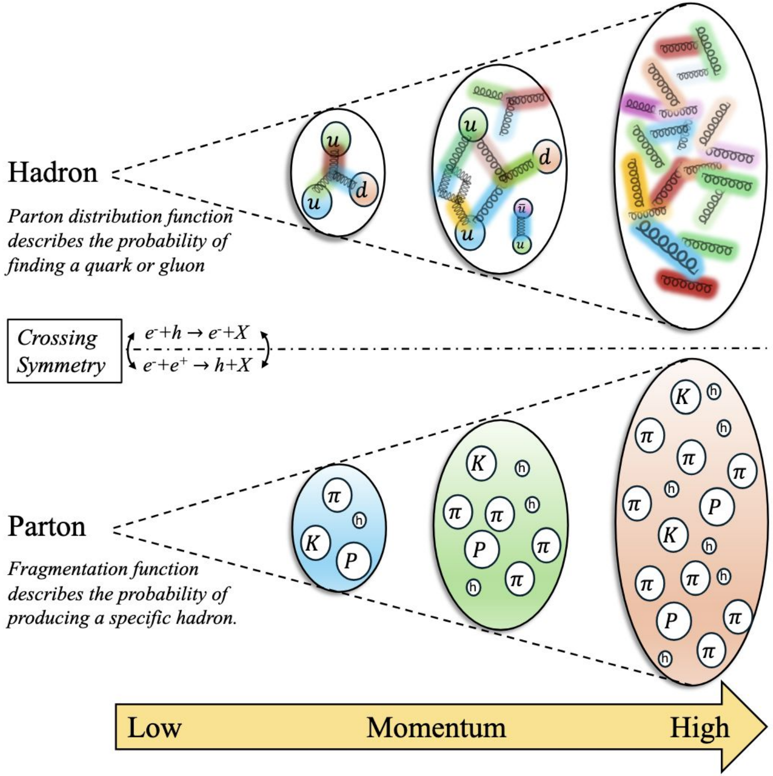
(Dated: March 20, 2025)

Recently, it was discovered that the proton structure at high energies exhibits maximal entanglement. This leads to a simple relation between the proton's parton distributions and the entropy of hadrons produced in high-energy inelastic interactions, that has been experimentally confirmed. In this Letter, we extend this approach to the production of jets. Here, the maximal entanglement predicts a relation between the jet fragmentation function and the entropy of hadrons produced in jet fragmentation. We test this relation using the ATLAS Collaboration data on

Really?

Max Entanglement in hadronization

[Phys. Rev. Lett. **134**, 111902]



Max entanglement → ~equal prob. for each microstate of n-parton (→ hadrons)

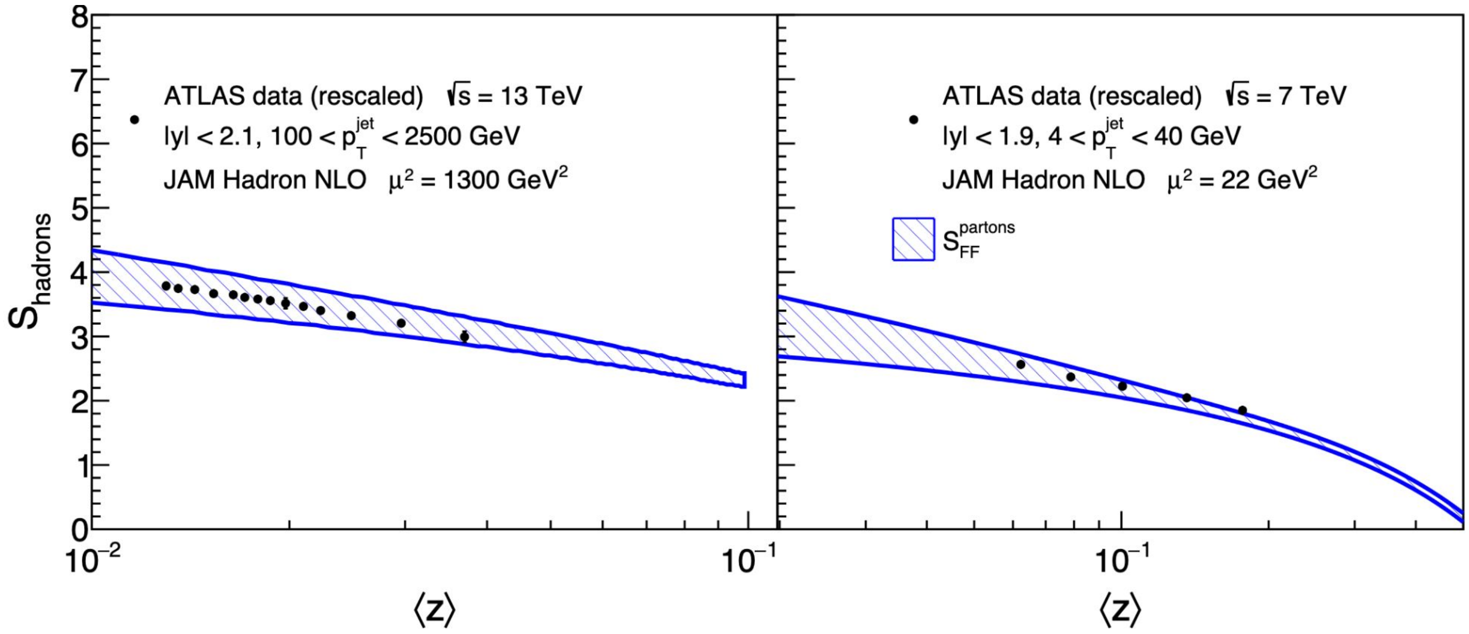
Proposed a relation between the PDF and the entropy of the produced hadrons (S~ln Ñ, Ñ avg number of partons from PDF)

PDF and the fragmentation functions (FF) are related (s/t/u channel of the 'same' process)

Entropy S evaluated from FF (of q or g) = S from ∑ Pn ln(Pn), Pn= Prob. of n charged hadrons

Max Entanglement in hadronization

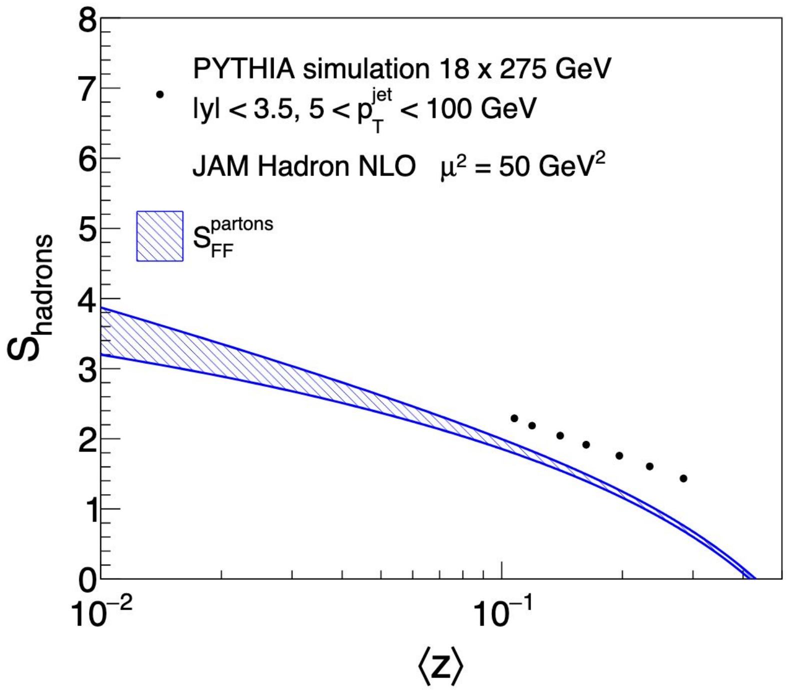
[Phys. Rev. Lett. **134**, 111902]



Comparison of S evaluated from FF (using MC), and the ATLAS results on the charged particle multiplicity distribution in jets. <z> is the average parton jet fraction (for data by converting PT_jet to <z> using MC).

Max Entanglement in hadronization

Phys. Rev. Lett. **134**, 111902



Not so well for EIC (by MC)?

Min Entanglement and Emergent Symmetry

Phys. Rev. Lett. 122, 102001(2019)

Entanglement Suppression and Emergent Symmetries of Strong Interactions

Silas R. Beane,[1] David B. Kaplan,[2] Natalie Klco,[1,2] and Martin J. Savage[2]

[1] *Department of Physics, University of Washington, Seattle, WA 98195-1560, USA*
[2] *Institute for Nuclear Theory, University of Washington, Seattle, WA 98195-1550, USA*

(Dated: December 10, 2018 - 1:30)

Entanglement suppression in the strong interaction S-matrix is shown to be correlated with approximate spin-flavor symmetries that are observed in low-energy baryon interactions, the Wigner $SU(4)$ symmetry for two flavors and an $SU(16)$ symmetry for three flavors. We conjecture that dynamical entanglement suppression is a property of the strong interactions in the infrared, giving rise to these emergent symmetries and providing powerful constraints on the nature of nuclear and hypernuclear forces in dense matter.

Entanglement power in s-wave nucleon-nucleon scattering
Function of two phase shifts for $^3S_1$ and $^1S_0$ channels

D. Kaplan
Lattice 2019

$$\hat{\mathbf{S}} = \frac{1}{4}\left(3e^{i2\delta_1} + e^{i2\delta_0}\right)\hat{\mathbf{1}} + \frac{1}{4}\left(e^{i2\delta_1} - e^{i2\delta_0}\right)\hat{\boldsymbol{\sigma}}\cdot\hat{\boldsymbol{\sigma}}$$

$$\mathcal{E}(\hat{\mathbf{S}}) = \frac{1}{6}\sin^2\left(2(\delta_1 - \delta_0)\right)$$

Entanglement power of S

Vanishes when: 1. $\delta_0 = \delta_1$ ⇐ SU(4)$_{\text{Wigner}}$ symmetry → $\begin{pmatrix} p\uparrow \\ p\downarrow \\ n\uparrow \\ n\downarrow \end{pmatrix}$

or: 2. $\delta_{0,1} = 0$ or $\pi/2$ ⇐ conformal symmetry

Look at the low energy EFTs for $p_{cm} < m_\pi/2$:

$$\mathcal{L}_6 = -\tfrac{1}{2}C_S(N^\dagger N)^2 - \tfrac{1}{2}C_T(N^\dagger\vec{\sigma}N)^2$$

$$^1S_0: \quad \bar{C}_0 = (C_S - 3C_T)$$

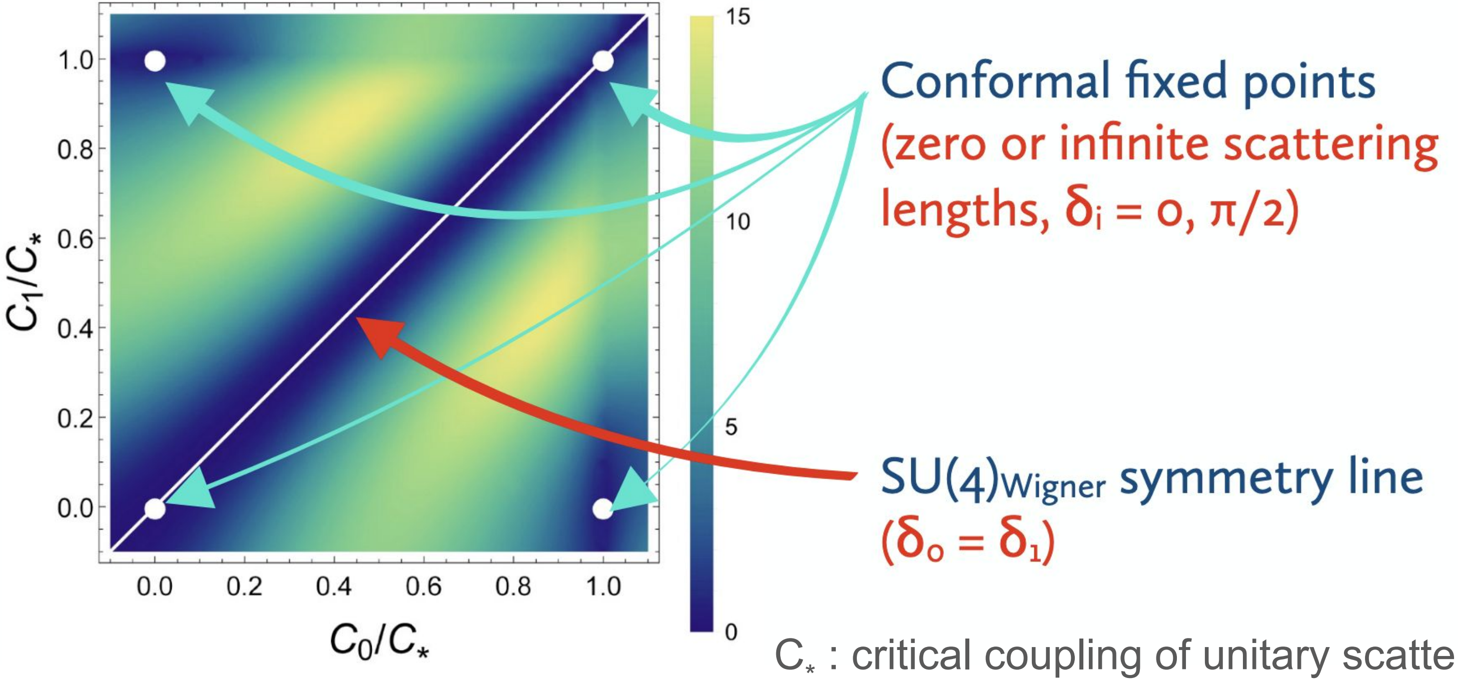
$$^3S_1: \quad \bar{C}_1 = (C_S + C_T)$$

Fit C_0, C_1 to scattering lengths

D. Kaplan
Lattice 2019

Min Entanglement and Emergent Symmetry

$$\mathcal{E}(\hat{\mathbf{S}}) = \frac{1}{6}\ \sin^2\left(2(\delta_1 - \delta_0)\right)$$



C_* : critical coupling of unitary scattering

Min Entanglement and Emergent Symmetry

D. Kaplan
Lattice 2019

Minimal entanglement in scattering occurs at points of enhanced symmetry

Real world: Fit C_0, C_1 to scattering lengths

(Not sure how the numerical values are obtained. Maybe from one of the Ref.)

➪ $C_T/C_S = 0.08 \ldots$ ~ SU4 symmetric

➪ $C_0 = .94\, C_\star$,

➪ $C_1 = 1.35\, C_\star \ldots$ ~ pretty close to conformal

Summary Outlook

- QE offers new directions of analyses in collider experiments

- QE offers insights into phenomenology and/or new physics

- The merit of QE studies is under discussion:
  - What can we learn (besides the discovery of toponium)?

Backup

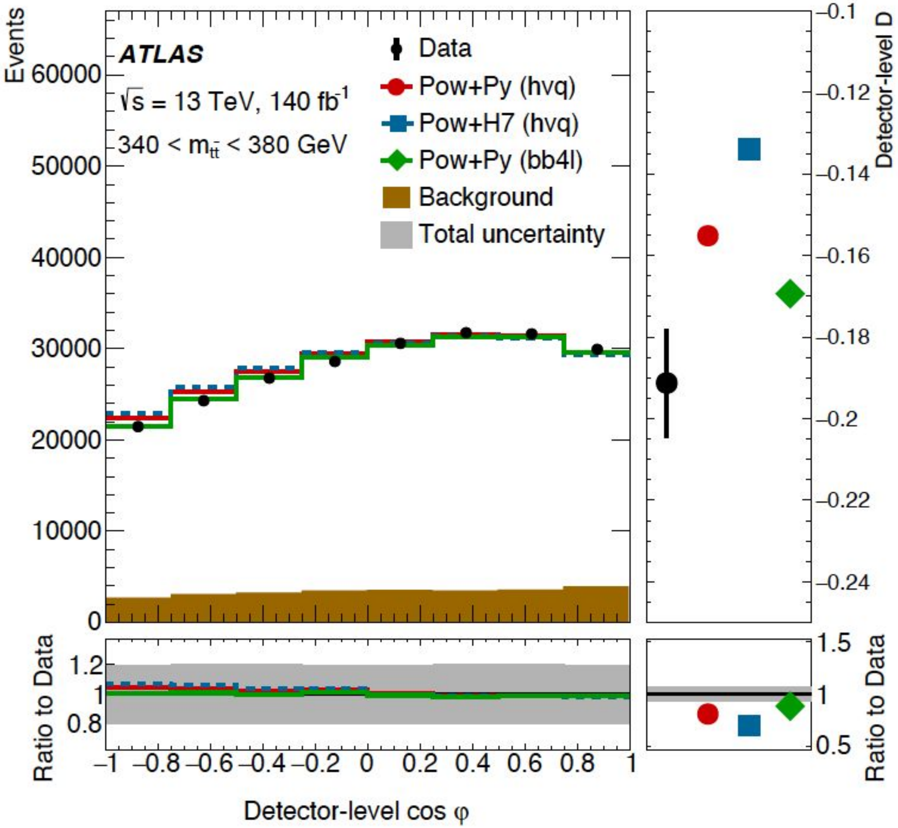
Quantum Entanglement: Top Quark Pairs

- Signal region:

  $$340 < m_{t\bar{t}} < 380\,\text{GeV}$$

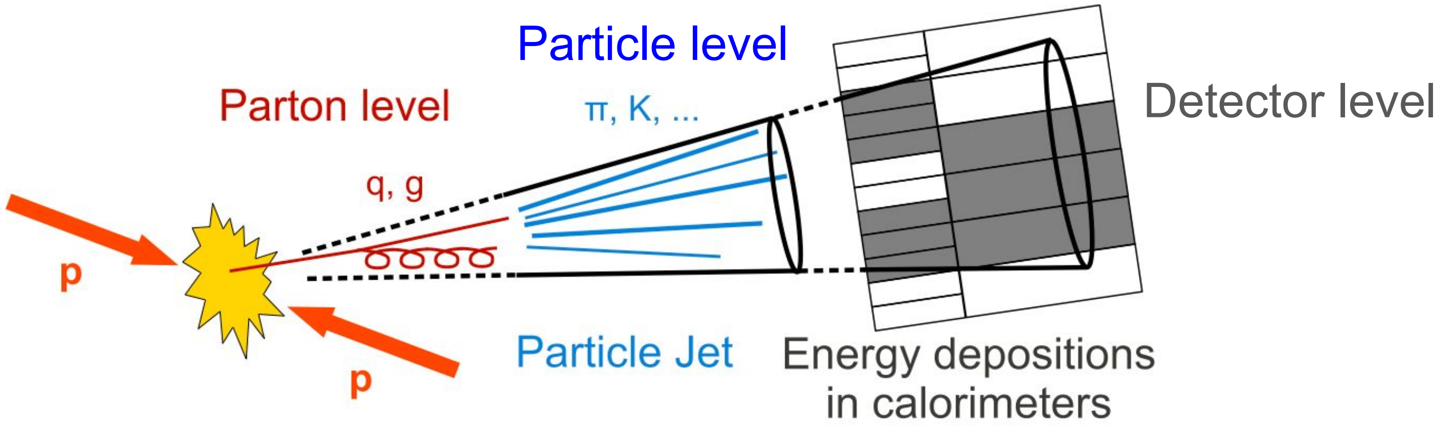
- Main backgrounds: Single top, Z+jets

- Plot: Detector-level cosφ and D



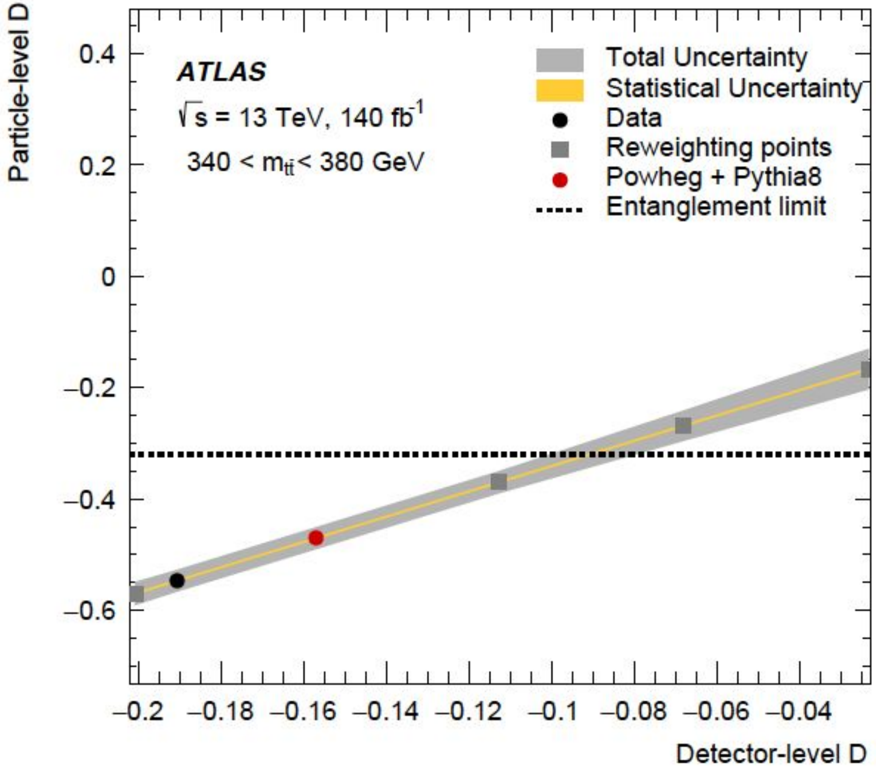
Quantum Entanglement: Top Quark Pairs

- The shape of the distribution is distorted by the detector response and event selection. → Need to correct this.



Quantum Entanglement: Top Quark Pairs

- A calibration curve is built for converting detector-level D to particle-level D
- Alternative hypotheses of D from reweighted MC
- Uncertainty band includes stat+syst



Summary of ATLAS Vs. CMS

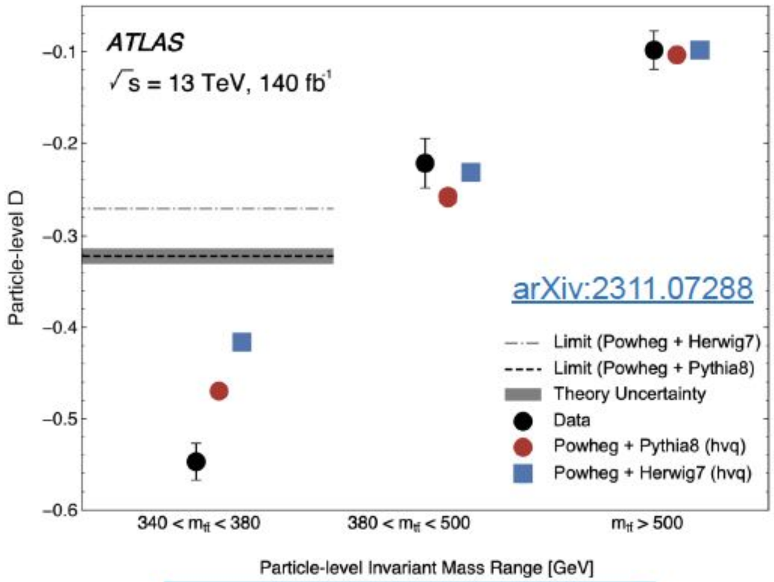
Y. Afik ATL-PHYS-SLIDE-2024-093

Analysis Method	ATLAS	CMS
Dataset	Full Run 2 (140.0 fb^{-1})	2016 (35.9 fb^{-1})
$t\bar{t}$ decay	Di-lepton ($e\mu$)	Di-lepton ($e\mu/ee/\mu\mu$)
Main selections	$340 < M_{t\bar{t}} < 380$ GeV	$345 < M_{t\bar{t}} < 400$ GeV, $\beta_{t\bar{t}} < 0.9$
$t\bar{t}$ reconstruction	Ellipse method	Neutrino weighting
Corrected to	Particle-level	Parton-level
Fit type	No fit, calibration curve	Template fit
Alternative hypothesis D	Reweighing	Mixing samples with and without spin correlation
Threshold effects	Neglected	Considered
Dominant systematic	Top decay, PDF, Recoil, FSR, Scales, NNLO	JES, Toponium, ISR
Nominal MC	PowhegBox+Pythia	PowhegBox+Pythia
Alternative MC	PowhegBox+Herwig, $bb4\ell$	PowhegBox+Herwig, MG5_aMC@NLO [FxFx]
Expected D	-0.470 ± 0.002 [stat.] ± 0.018 [syst.]	$-0.465^{+0.016}_{-0.017}$ [stat.]$^{+0.019}_{-0.022}$ [syst.]
Observed D	-0.547 ± 0.002 [stat.] ± 0.021 [syst.]	-0.478 ± 0.017 [stat.]$^{+0.018}_{-0.021}$ [syst.]
Significance	$>> 5\sigma$	$> 5\sigma$

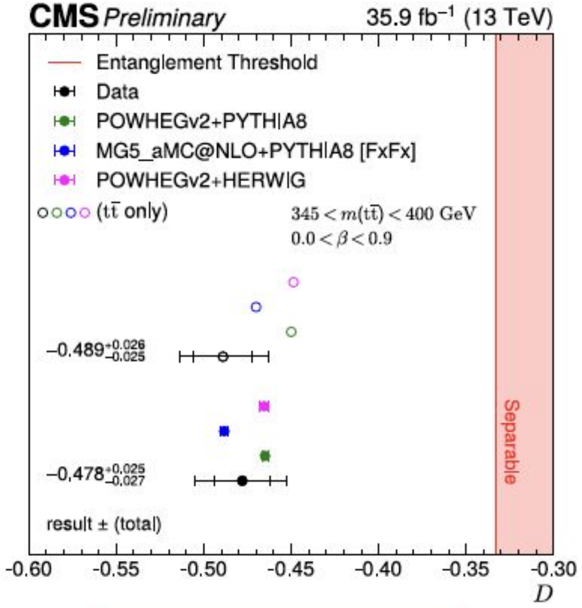
Table: Main differences between the ATLAS and CMS analyses.

Quantum Entanglement: Top Quark Pairs

- ATLAS v.s. CMS: Similar results, different syst considered.



ATLAS: limit of D = −1/3 is folded from parton to particle-level



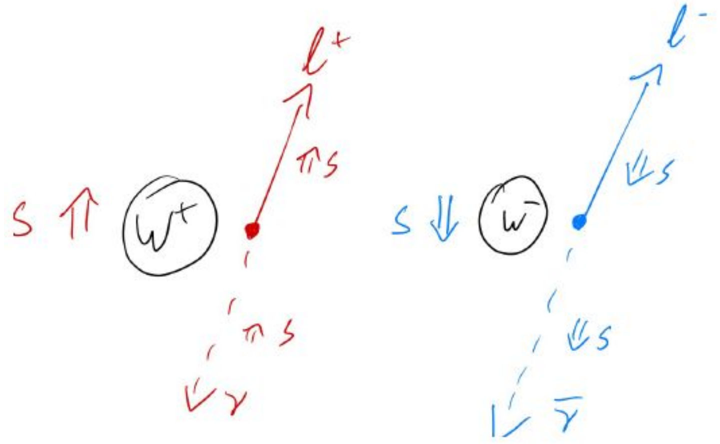
CMS: limit of D = −1/3 is shown at parton-level

G. Negro SM@LHC

Quantum Entanglement: H→VV

- Higgs bosons are spin-0 → decaying diboson pairs form entangled 3-outcome (spin-1) pairs, i.e. ‘Qutrits’.

- H → WW* → lvlv decays:
  Spins of W bosons are encoded in the directions of charge leptons *in the rest frames of the corresponding W bosons*.



Illustrated by A. Barr

Quantum Entanglement: H→VV

- Two examples for entanglement observables:
  – (lower bound of) Concurrence C_2
  – Bell inequality for pairs of spin-1 particles I_3

- Concurrence: $C_2[\rho] = 2\max\left(0,\ \mathrm{Tr}\,[\rho^2] - \mathrm{Tr}\,[(\rho_A)^2],\ \mathrm{Tr}\,[\rho^2] - \mathrm{Tr}\,[(\rho_B)^2]\right)$

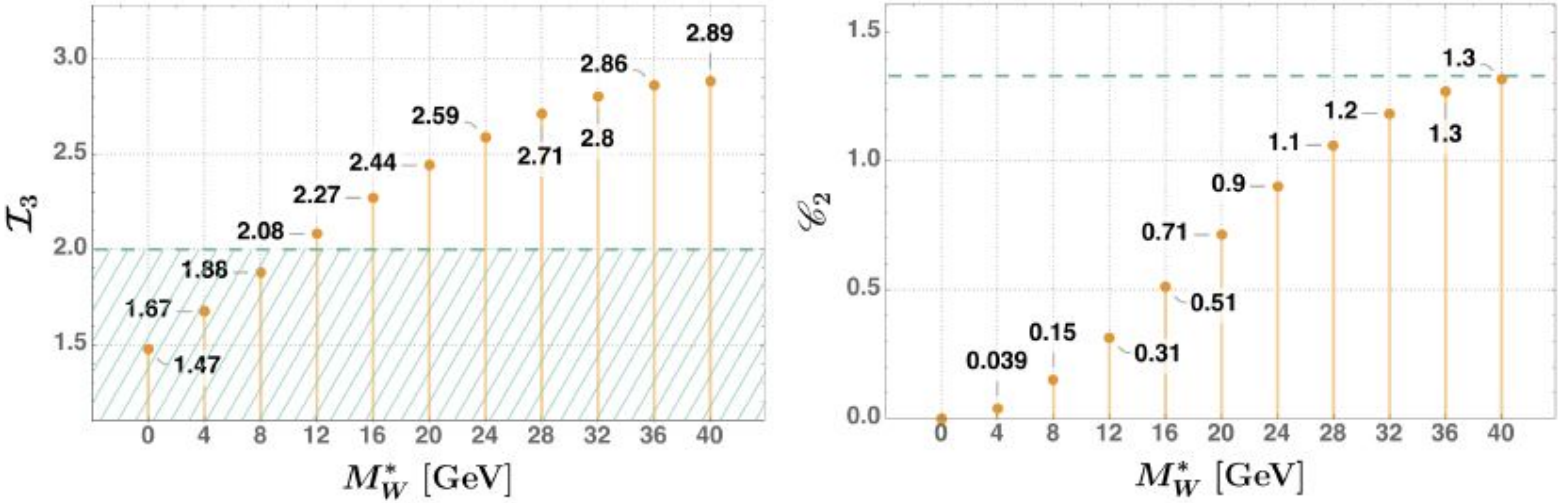
Reduced density matrix

arXiv:2402.07972

Quantum Entanglement: H→WW

- Can optimize the sensitivity of the Bell operator B by rotation (finding optimal axes)

[arXiv: 2302.00683](https://arxiv.org/abs/2302.00683)



QE in H→WW: Reconstruct W boson rest frames

- **MAE** (**M**ean **A**bsolute **E**rror): track predicted $\boldsymbol{p}^{W_{0/1}}$

$$\mathrm{MAE} = \frac{1}{N}\sum_{i=1}^{N} |x_i - y_i| \,,$$

where x_i and y_i are predictions and truth labels.

- **MMD** (**M**aximum **M**ean **D**iscrepancy): track derived $m_{W_{0/1}}$
  - ∵ Hard to directly evaluate ∴ Fits the distributions
  - Non-parametrical, and computationally efficient

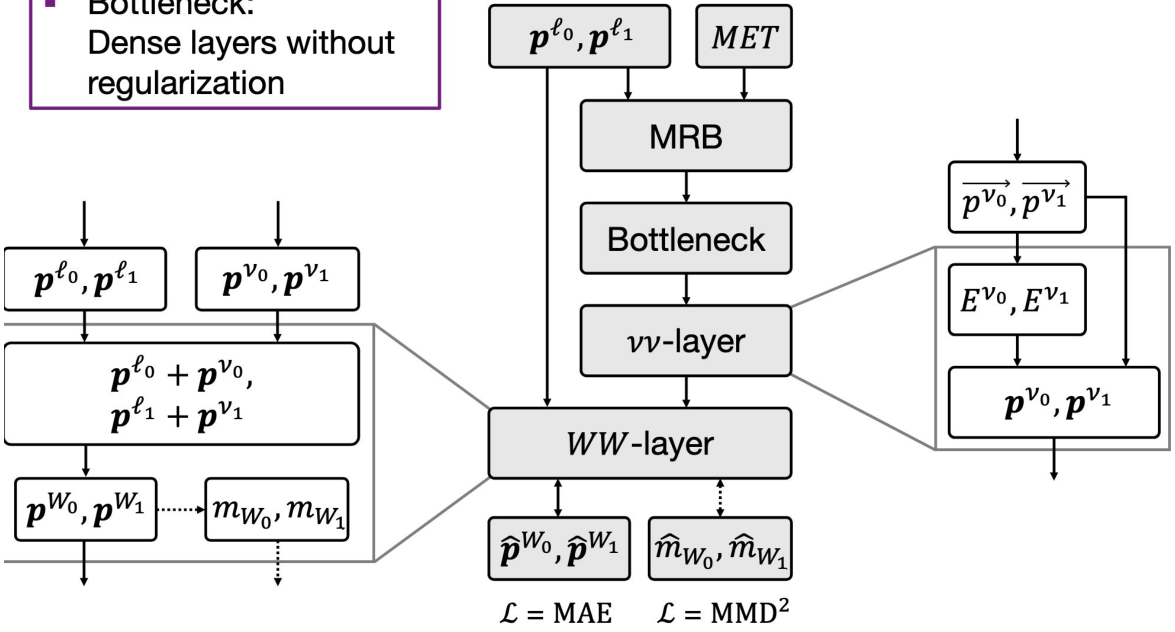
$$\mathrm{MMD}^2 \approx \sum_{i}\sum_{i\neq j} k(x_i, x_j) - 2\sum_{i}\sum_{j} k(x_i, y_j) + \sum_{i}\sum_{i\neq j} k(y_i, y_j)\,,$$

where $k(\cdot)$ is Gaussian kernel $= \exp\left(-\frac{|x_i - y_i|^2}{2\sigma^2}\right)$.

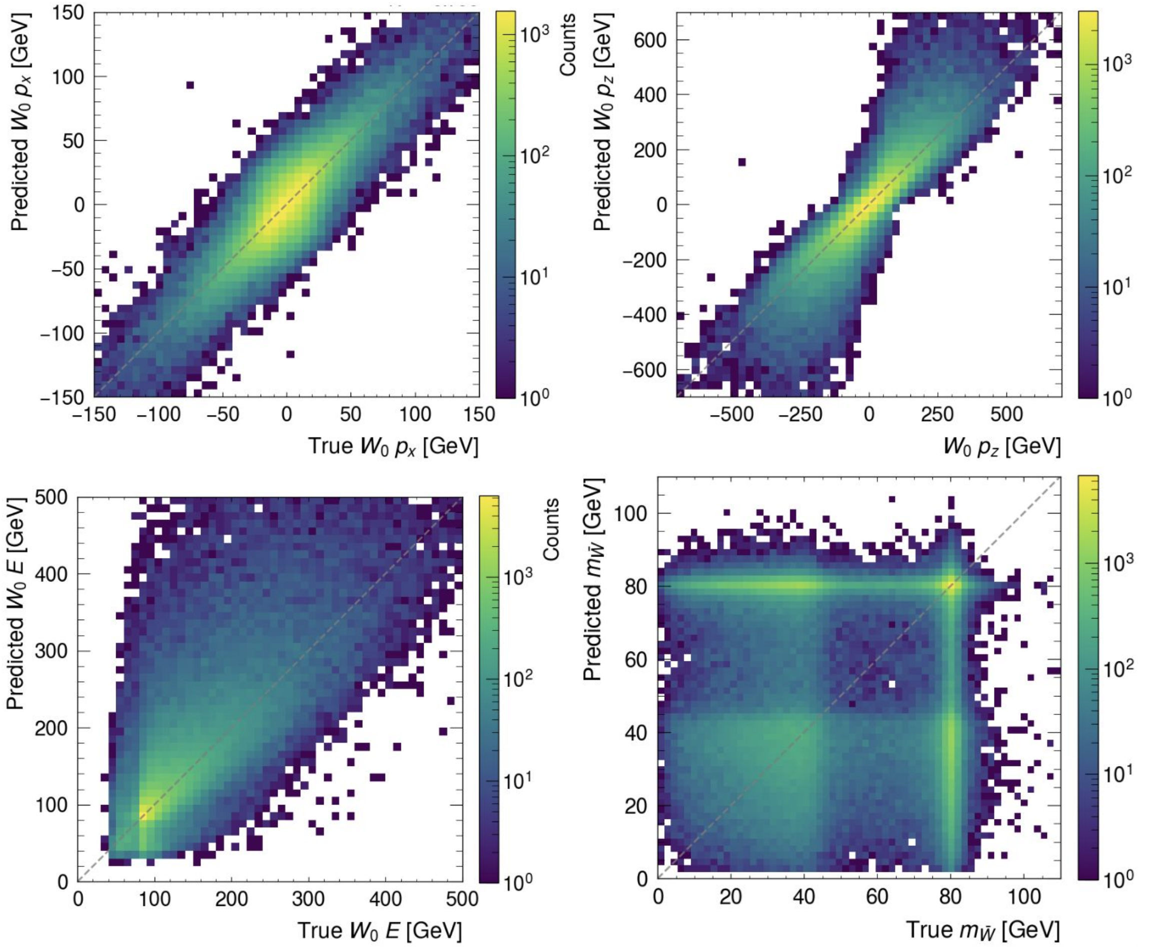
QE in H→WW: Reconstruct W boson rest frames

- MRB: **M**ulti-**R**esidual-**B**locks
- Bottleneck: Dense layers without regularization

Main structure

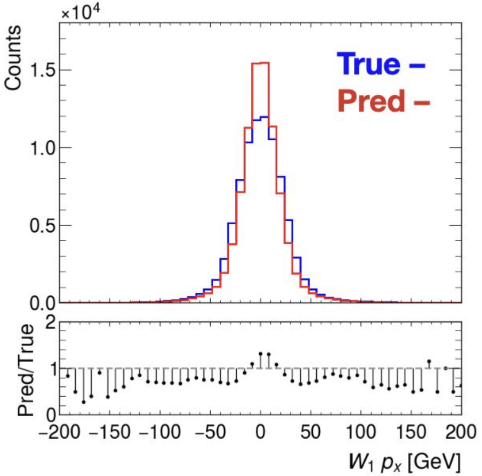


Results

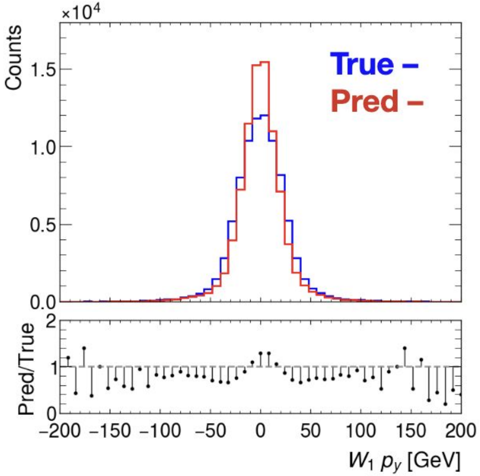

Predicted W_0 p_x [GeV]
True W_0 p_x [GeV]
Counts
Predicted W_0 p_z [GeV]
W_0 p_z [GeV]
Predicted W_0 E [GeV]
True W_0 E [GeV]
Counts
Predicted $m_{\bar{W}}$ [GeV]
True $m_{\bar{W}}$ [GeV]

Results

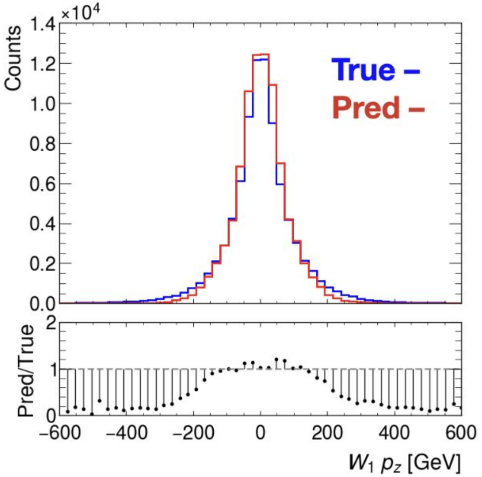
p_x



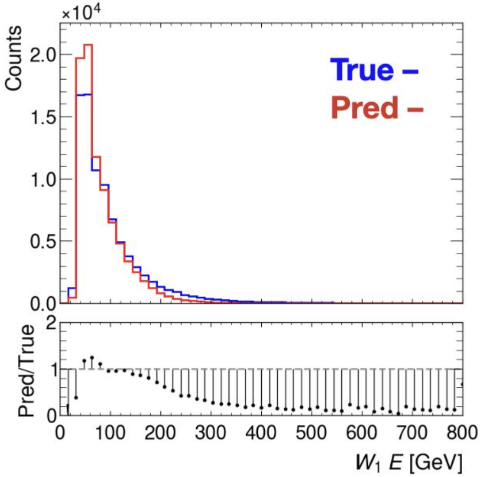
p_y



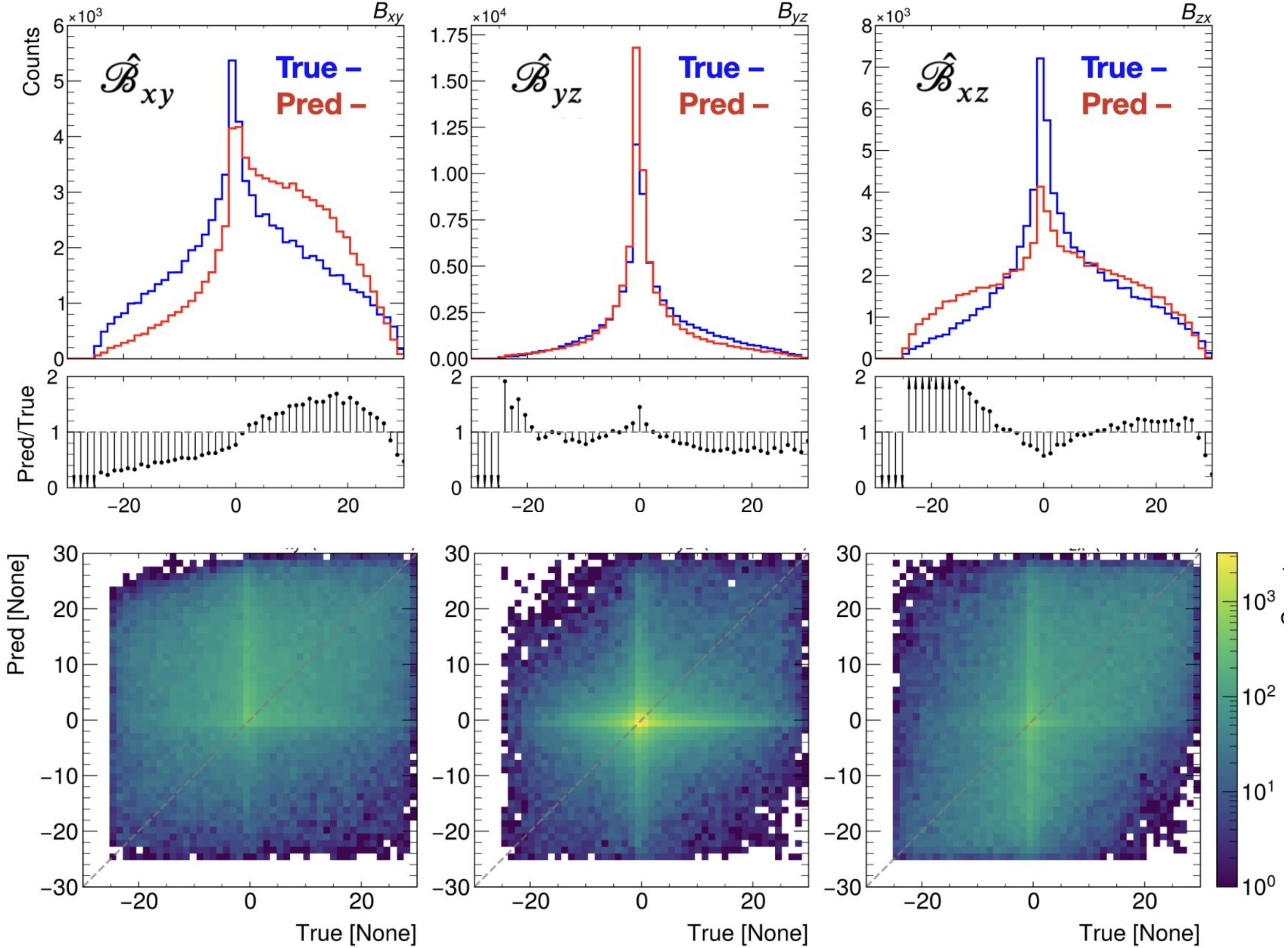
p_y



E



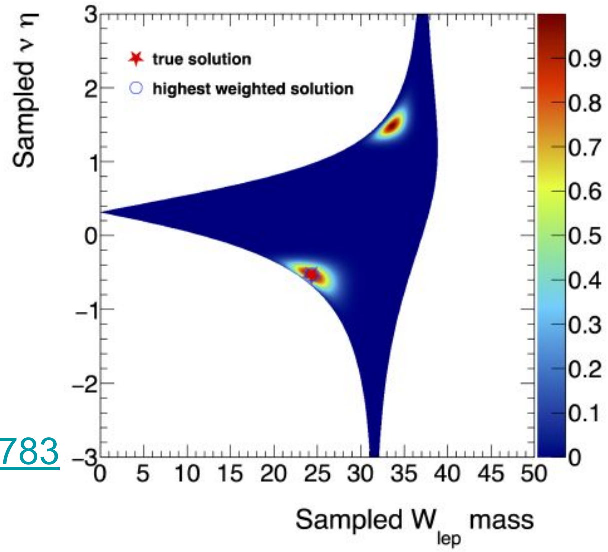
QE variables



Quantum Entanglement: H→WW

- Semi-leptonic decays H→WW* → qqlv, in particular, qq=cs.
  – Down-type quark plays the same role as charged leptons

- Reconstruct (off-shell) W rest frame using neutrino weighting



arXiv:2307.13783

Quantum Entanglement: H→ZZ

arXiv: 2302.00683

- H→ZZ* is like H→WW*, but Z→ll is a mixture of E&W interactions:

$$-i\frac{g}{\cos\theta_W}\left[g_L(1-\gamma^5)\gamma_\mu + g_R(1+\gamma^5)\gamma_\mu\right]Z^\mu$$

Couples to both left- and right-chiral leptons

$$g_L = -1/2 + \sin^2\theta_W \text{ and } g_R = \sin^2\theta_W$$

- Modified Wigner p-functions: $\tilde{\mathfrak{p}}^n = \sum_m \mathfrak{a}^n_m \mathfrak{p}^m_+$

a_m = combinations of g_L and g_R

Quantum Entanglement: H→WW

- If the qutrits are entangled then
  $C_2 > 0$, $I_3 > 2$

- Relation to coefficients of the density matrix: arXiv:2402.07972
  $C_2 =$

$$2\max\left[-\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,;\right.$$
$$\left.-\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2,\ 0\right]$$

$$I_3 = 4\left(h_{44} + h_{55}\right) - \frac{4\sqrt{3}}{3}\left[h_{61} + h_{66} + h_{72} + h_{77} + h_{11} + h_{16} + h_{22} + h_{27}\right]$$

Quantum Entanglement: H→WW

- Can extract the coefficients of the density matrix from the distributions of charged leptons in the rest frame of the parent W bosons:

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega^{+}\,d\Omega^{-}}$$

dΩ in the rest frame of W+ or W-

(*assuming* we can 'reconstruct' the rest frames of W bosons)

Quantum Entanglement: H→WW

- Extract the coefficients of the density matrix arXiv:2402.07972

$$f_a = \frac{1}{2\sigma}\int \frac{d\sigma}{d\Omega^+}\,\mathfrak{p}_+^a\, d\Omega^+$$

$$g_a = \frac{1}{2\sigma}\int \frac{d\sigma}{d\Omega^-}\,\mathfrak{p}_-^a\, d\Omega^-$$

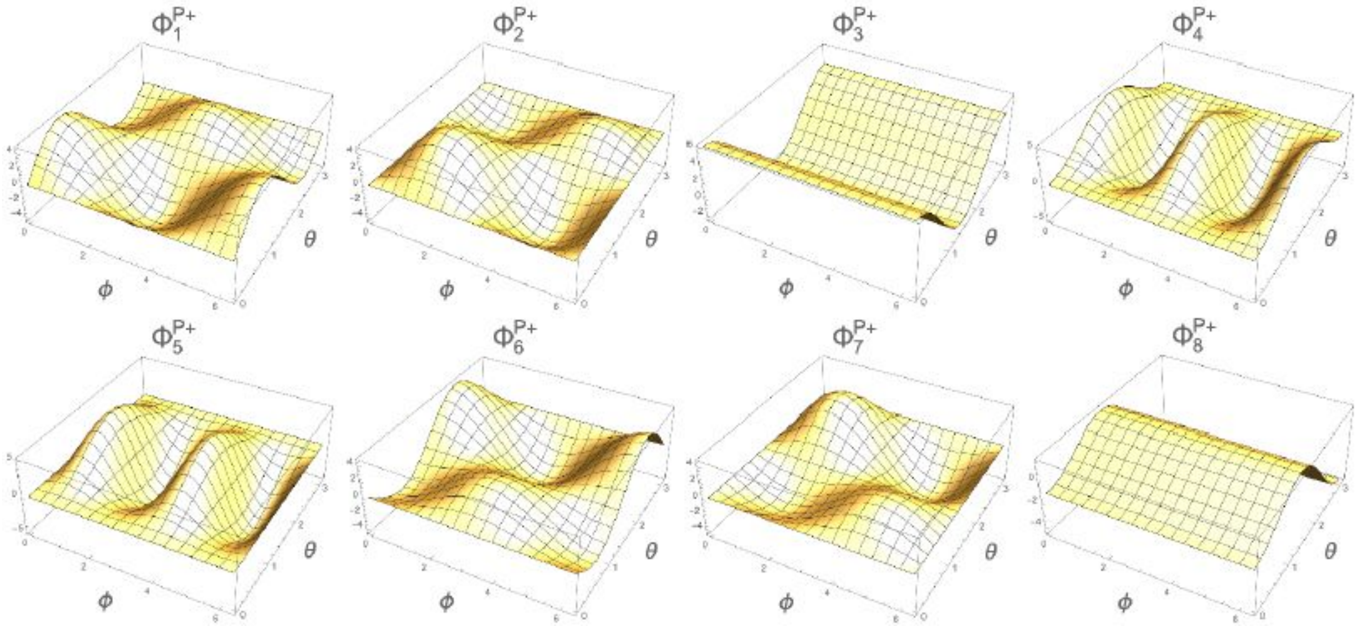
$$h_{ab} = \frac{1}{4\sigma}\int\int \frac{d\sigma}{d\Omega^+\, d\Omega^-}\,\mathfrak{p}_+^a\,\mathfrak{p}_-^b\, d\Omega^+ d\Omega^-$$

P’s : Wigner P functions (‘projectors’ of density matrices)

Quantum Entanglement: H→WW

- Wigner P functions

$$\Phi_1^{P\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\cos\phi$$
$$\Phi_2^{P\pm} = \sqrt{2}(5\cos\theta \pm 1)\sin\theta\sin\phi$$
$$\Phi_3^{P\pm} = \frac{1}{4}(\pm 4\cos\theta + 15\cos 2\theta + 5)$$
$$\Phi_4^{P\pm} = 5\sin^2\theta\cos 2\phi$$
$$\Phi_5^{P\pm} = 5\sin^2\theta\sin 2\phi$$
$$\Phi_6^{P\pm} = \sqrt{2}(\pm 1 - 5\cos\theta)\sin\theta\cos\phi$$
$$\Phi_7^{P\pm} = \sqrt{2}(\pm 1 - 5\cos\theta)\sin\theta\sin\phi$$
$$\Phi_8^{P\pm} = \frac{1}{4\sqrt{3}}(\pm 12\cos\theta - 15\cos 2\theta - 5)$$



arXiv:2209.13990