Inverse Tritium Beta Decay, Relic Neutrinos, Angular Momentum Conservation, Neutrino-less Double-beta Decays

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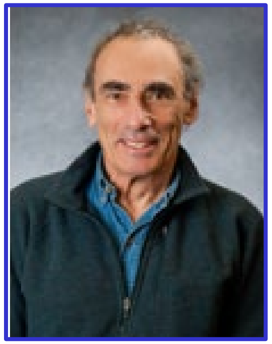
Based on papers in collaboration with Gordon Baym



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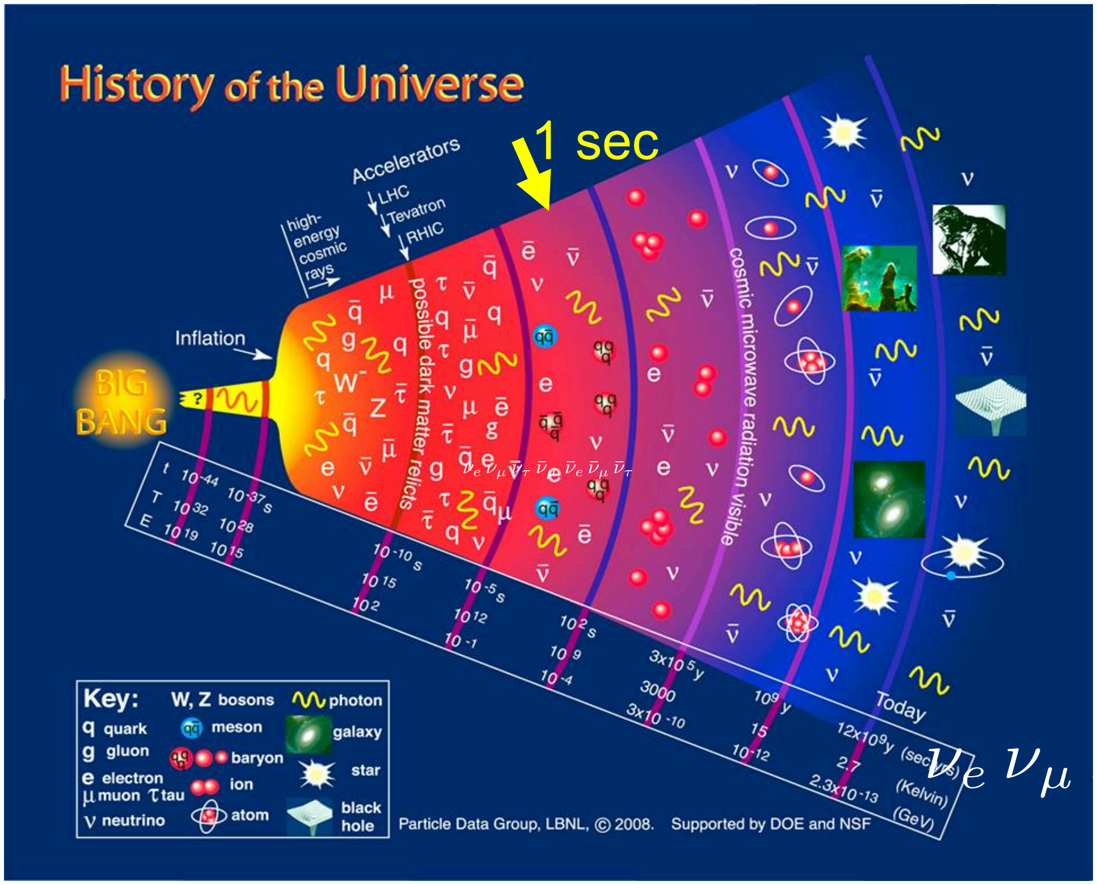
Relic neutrinos from the Big Bang forming the cosmic neutrino background (CνB)



Decoupling occurs at t ~1 sec, T ~1 MeV

CνB has never been observed !

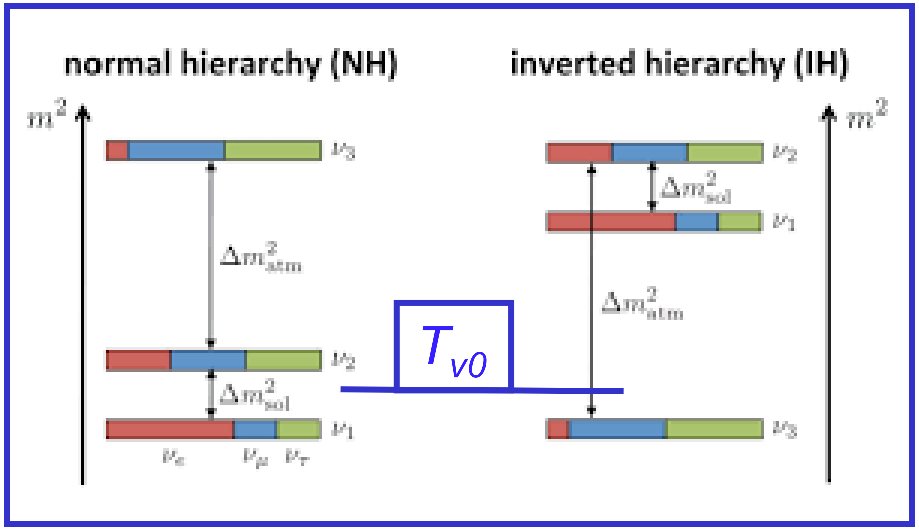
Cosmic neutrino background (CνB) versus cosmic microwave background (CMB)

	CMB	CνB	Relation
Temperature	2.73K	1.9 K (1.7 x 10^{-4} eV)	$T_\nu/T_\gamma = (4/11)^{1/3}$ $=0.714$
Decoupling at	3.8 x 10^5 years	~ 1 sec	
Density	~ 411 / cm^3	~ 336 / cm^3	$n_\nu = (9/11)\ n_\gamma$

- CνB took a snapshot of the Universe at a much earlier epoch than CMB
- At least two of the three neutrinos are non-relativistic
- ~20,000,000 of CνB inside you at this moment
- Density of CνB is ~100 times of solar neutrinos
- Produced as flavor eigenstates, now in mass eigenstates

At least 2 relic neutrino mass states are non-relativistic

(Current temperature: $T_{\nu 0} = 1.945\,\text{K} = 1.676 \times 10^{-4}\,\text{eV}$)



$$\Delta m_{21}^2 = 7.50 \times 10^{-5}\,\text{eV}^2$$

$$\Delta m_{31,N}^2 = 2.52 \times 10^{-3}\,\text{eV}^2$$

$$\Delta m_{31,I}^2 = -2.51 \times 10^{-3}\,\text{eV}^2$$

$$T_{\nu 0} = 1.945\,\text{K} = 1.676 \times 10^{-4}\,\text{eV}$$

At least two neutrino masses are larger than 100 K with $m_i \gg T_{\nu 0} = 1.945\ \text{K} = 1.676 \times 10^{-4}\ \text{eV}$

Normal Hierarchy: If $m_1 = 0$, $\beta_1 = 1,\ \beta_2 \sim 1/50,\ \beta_3 \sim 1/300$

Inverted Hierarchy: If $m_3 = 0$, $\beta_3 = 1,\ \beta_1 \sim \beta_2 \sim 1/300$

Capture of CνB on radioactive nuclei (positive Q value)

(S. Weinberg, 1962)

Tritium beta decay:

$$a)\ {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

3-body β-decay with Q-value of

$$Q_a = M({}^3\mathrm{H}) - M({}^3\mathrm{He}) - M(e^-) - M(\bar{\nu}_e)$$

Inverse tritium beta decay (ITBD):

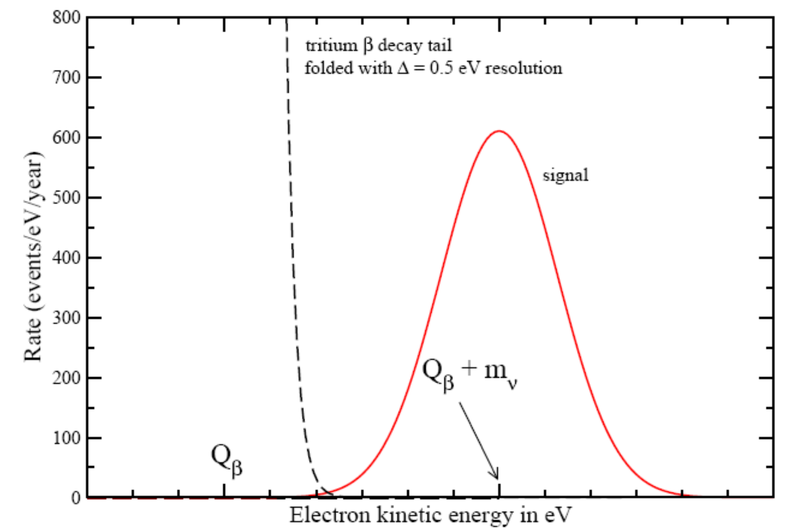
$$b)\ \nu_e + {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^-$$

2-body reaction with the Q-value of

$$Q_b = M({}^3\mathrm{H}) - M({}^3\mathrm{He}) - M(e^-) + M(\bar{\nu}_e)$$

Therefore, $Q_b = Q_a + 2M(\bar{\nu}_e)$

Positive Q value implies low-energy relic neutrinos can be captured !



Look for a mono-energetic peak beyond the endpoint of tritium beta decay

PTOLEMY experiment for this search

Helicity dependence of the ITBD

$(\nu_e + {}^3H \to {}^3He + e^-)$

- ITBD for neutrino in mass eigenstate i and helicity h:

$$\sigma_i^h = \frac{G_F^2}{2\pi v_i} |V_{ud}|^2 |U_{ei}|^2 F(Z, E_e) \frac{m({}^3He)}{m({}^3H)} E_e p_e A_i^h (\bar{f}^2 + 3\bar{g}^2)$$

- The helicity-dependent factor, A_i^h , is given as

$$A_i^{\pm} = 1 \mp \beta_i; \quad \text{where } \beta_i = v_i / c$$

- For relativistic neutrinos, $\beta_i \to 1$, we have

$$A_i^+ \to 0 \quad \text{and} \quad A_i^- \to 2$$

- For non-relativistic neutrinos, $\beta_i \to 0$, we have

$$A_i^+ \to 1 \quad \text{and} \quad A_i^- \to 1$$

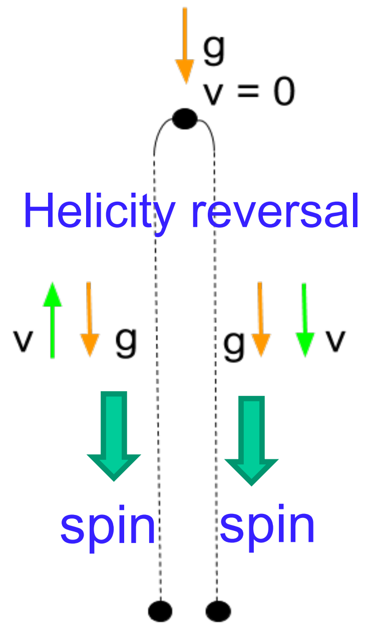
- ITBD rate depends on the helicity, h, of neutrinos

What are the helicities of relic neutrinos?

Evolution of relic neutrino helicity
(from t ~ 1 sec to t ~ 13.8 billion years)

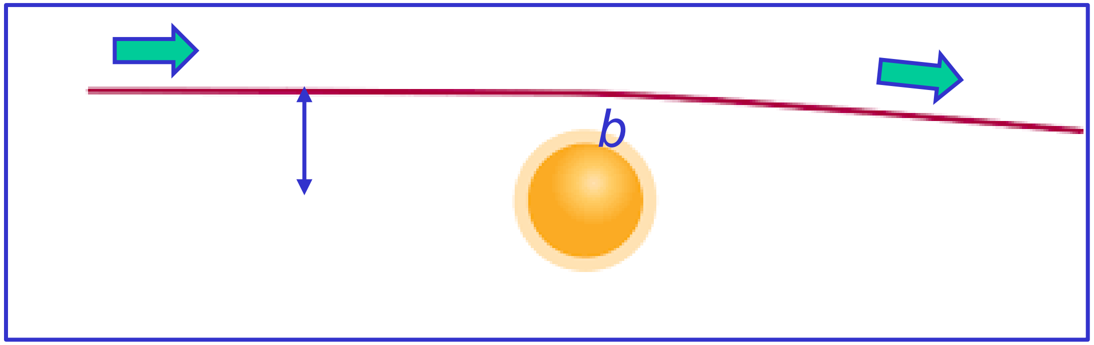
- Relic neutrinos decoupled at a temperature of ~1 MeV, and were highly relativistic. Neutrinos were produced essentially in $h = -1$ state, and antineutrinos in $h = +1$ state.
- Rotation of neutrino spin due to a transverse matter source is less than the rotation of neutrino momentum (gravitational lensing of neutrino), changing neutrino helicity.
- Dirac neutrino with non-zero magnetic moment will precess in galactic or cosmic magnetic fields, changing neutrino helicity.

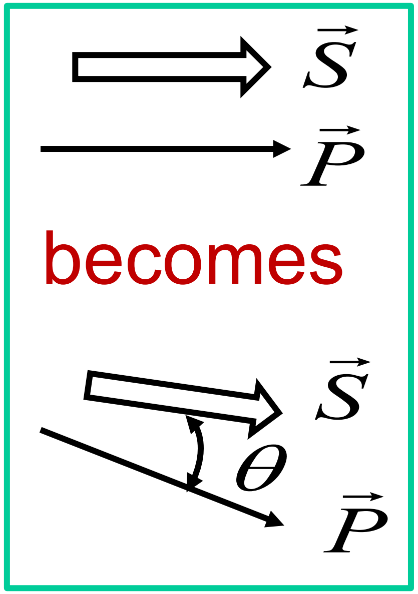
How would gravity modify the neutrino helicity?



If a low-energy neutrino with negative helicity is emitted upward from the Earth, it could fall back to the Earth having a positive helicity, affecting its weak interaction rate!

How would gravity modify the neutrino helicity?





Momentum bending: $\Delta\theta_P = \frac{2MG}{b\beta^2}(1+\beta^2)$

Spin bending: $\Delta\theta_S = \frac{2MG}{b}\frac{2\gamma+1}{\gamma+1}; \quad (\gamma = 1/\sqrt{1-\beta^2})$

Helicity bending: $\theta \equiv \Delta\theta_S - \Delta\theta_P = -\frac{2MG}{b\gamma\beta^2}$

(spin bending lags momentum bending)

$\theta \to 0$ as $\beta \to 1$

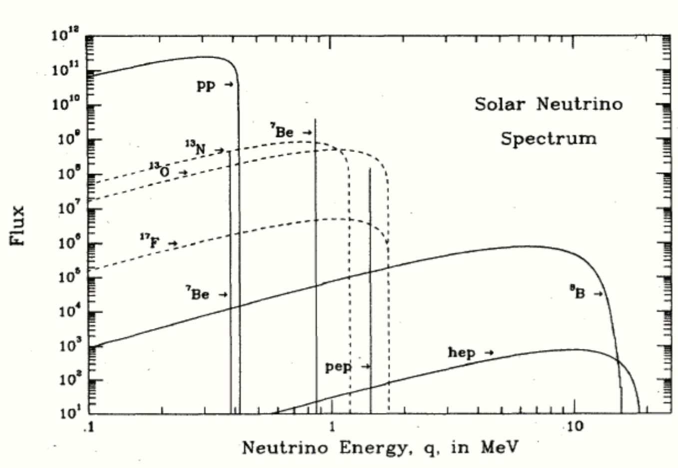
θ is large as $\beta \to 0$

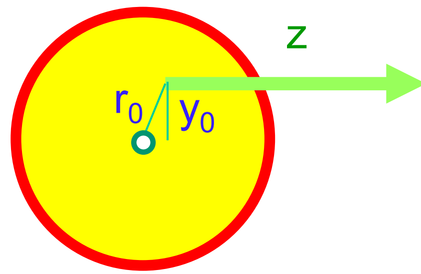
An angle θ between the spin and momentum directions means

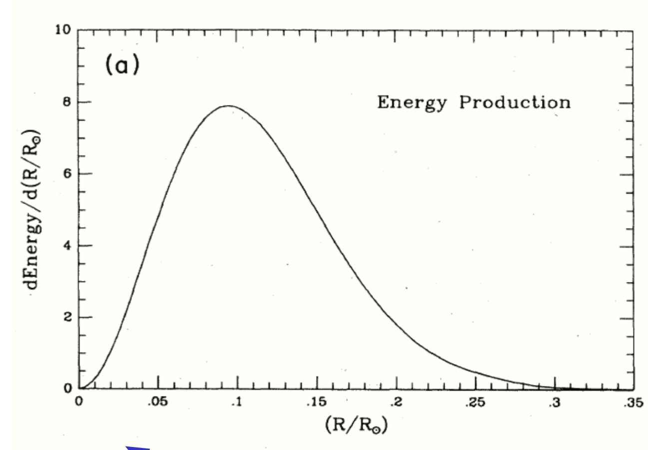
$$\left|h=+1\right\rangle \to \cos(\theta/2)\left|h=+1\right\rangle + \sin(\theta/2)\left|h=-1\right\rangle$$

Probability for $h=-1$ is $\sin^2(\theta/2)$

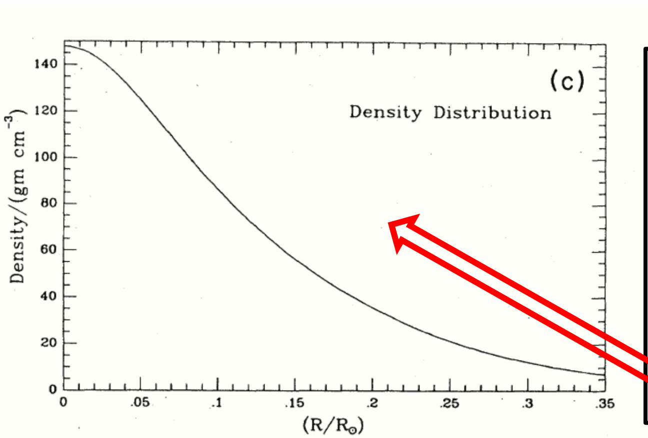
Helicity modification of solar neutrinos by Sun's gravity



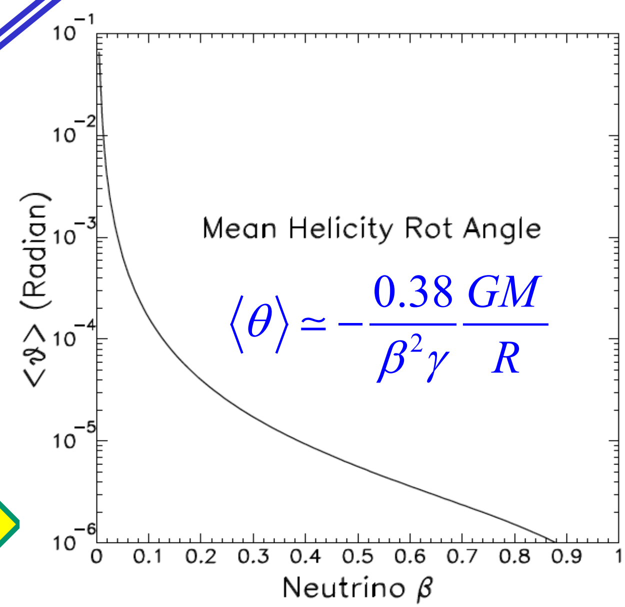




$$\theta(y_0, r_0) = -\frac{1}{\gamma\beta^2}\int_{z_0}^{\infty} dz\, \frac{GM(r)\, y_0}{r^3}$$



Averaged over spatial distribution of solar neutrino emission and mass distribution in Sun



Significant helicity modification of heavy particles with spin, e.g., dark photons, from Sun

Neutrino propagation in an expanding universe (Random walk of relic neutrinos)

Metric of expanding universe with weak gravitational inhomogeneities

$$ds^2 = a(u)^2\left[-(1+2\Phi)du^2 + (\delta_{ij}(1-2\Phi)+h_{ij})dx_i dx_j\right]$$

$a =$ scale factor (a grows from $\sim 10^{-10}$ at $T = 1$ MeV to $a = 1$ now)

$u =$ conformal time; $dt = a\ du$

$x_i =$ comoving spatial coordinates, $h_{ij} =$ gravitational waves

$\Phi =$ weak potential driven by density fluctuations

$$\nabla_x^2\Phi = 4\pi G\,(\delta\rho(x) + 3\,\delta P(x))\,a(u)^2$$

Radiation dominated era ($P = \rho/3$), down to redshift $\sim 10^4$

Matter dominated era ($P(x) \to 0$) from redshift $\sim 10^4$ to now

Gravitational spin rotation relative to momentum

For massive relic neutrinos, after including matter and dark energy

in $\bar{\rho}(a) = \rho_M / a^3 + \rho_V$:

$$\left\langle (\Delta\theta_p)^2 \right\rangle = \frac{9}{8\pi} P H_0^3 \int_0^1 \frac{da}{a^2} \left(\Omega_M a + \Omega_V a^4 \right)^{3/2} \beta \left(\frac{1}{\beta} + \beta \right)^2$$

$$\left\langle (\Delta\theta_s)^2 \right\rangle = \frac{9}{8\pi} P H_0^3 \int_0^1 \frac{da}{a^2} \left(\Omega_M a + \Omega_V a^4 \right)^{3/2} \beta^3 \left(\frac{2\gamma + 1}{\gamma + 1} \right)^2$$

$$\left\langle \theta^2 \right\rangle \equiv \left\langle (\Delta\theta_p)^2 \right\rangle - \left\langle (\Delta\theta_s)^2 \right\rangle = \frac{9}{8\pi} P H_0^3 \int_0^1 \frac{da}{a^2} \left(\Omega_M a + \Omega_V a^4 \right)^{3/2} \left(\frac{1}{\beta} - \beta \right)$$

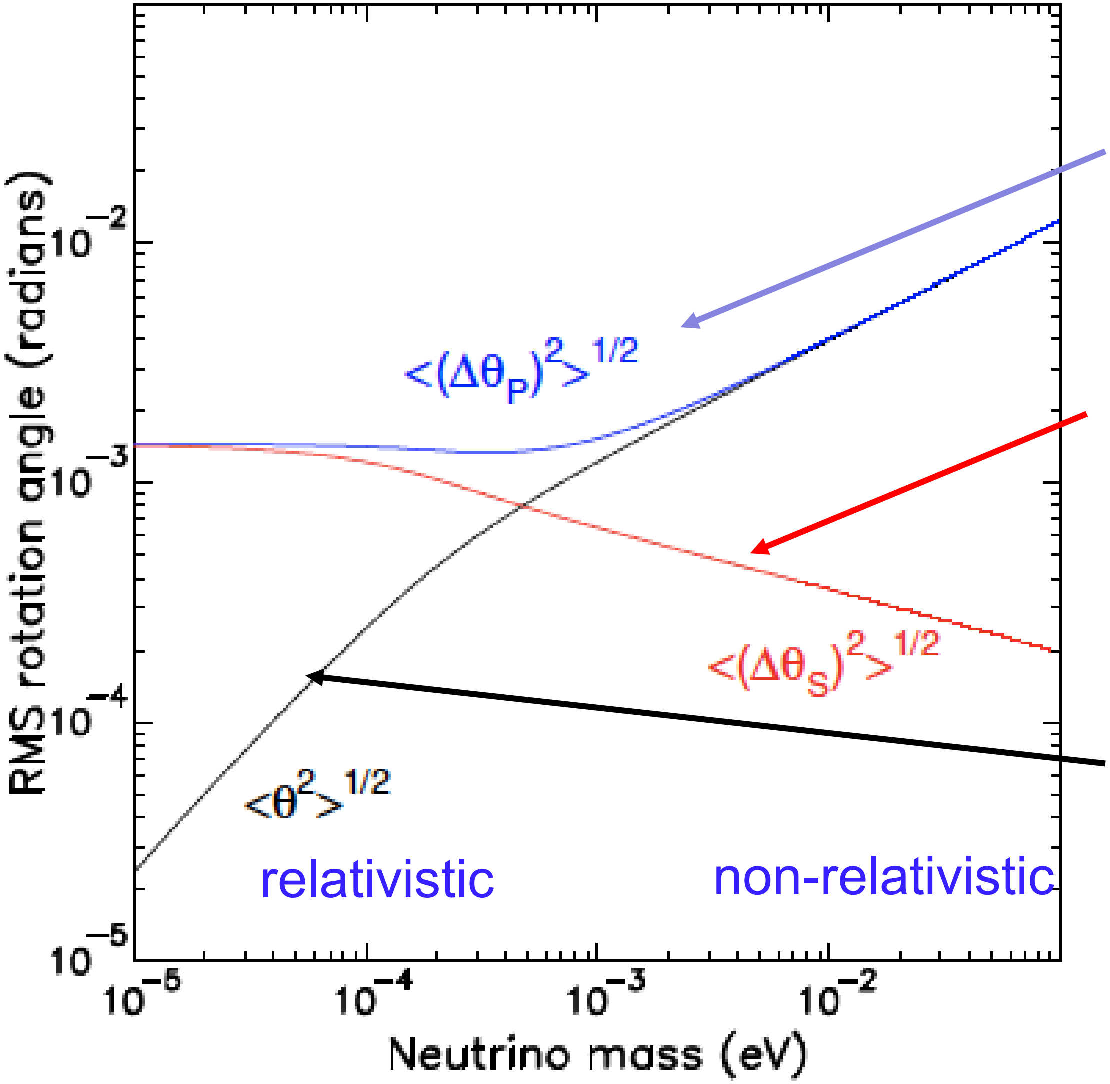
(where Ω_M = matter fraction, Ω_V =dark energy fraction)

Main effect is from matter dominated era (redshift $\sim 10^4$ to now)

(For detailed derivation, please see Baym and Peng, PRD 103 (2021))

Spin rotation relative to momentum rotation due to gravity for relic neutrino mass state

(depending on neutrino's mass)



RMS for $\Delta\theta_P$:
rotation angle for momentum

RMS for $\Delta\theta_S$:
rotation angle for spin

RMS for θ :
rotation angle for spin relative to momentum

Neutrino's spin precesses in B field, but momentum does not

(neutrinos are electrically neutral)

Magnetic fields change neutrino helicity: $h = \hat{S} \cdot \hat{p}$

Define spin in rest frame of neutrino.

Rest frame precession :

$$\frac{d\vec{S}}{d\tau} = 2\mu_\nu \vec{S} \times \vec{B}_R \qquad B_R = \text{magnetic field in rest frame}$$

In terms of "lab" frame magnetic field: $B_{\parallel R} = B_\parallel, \; B_{\perp R} = \gamma B_\perp$

Bargmann-Michel-Telegdi (BMT) equation of motion:

$$\frac{d\vec{S}_\perp}{dt} = 2\mu_\nu \left(\vec{S}_\parallel \times \vec{B}_\perp + \frac{1}{\gamma} \vec{S}_\perp \times \vec{B}_\parallel \right)$$

Apply to both galactic and cosmic magnetic fields

Neutrino spin rotation by galactic magnetic field

For uniform galactic magnetic field: $\theta_g \sim 2\mu_\nu B_g \dfrac{l_g}{\mathrm{v}}$

l_g = mean crossing distance of the galaxy

Since galactic fields are uniform only over coherence length $\Lambda_g \sim kpc$, spin direction undergoes a random walk in magnetic field

$$\left\langle \theta^2 \right\rangle_g \simeq \left(2\mu_\nu B_g \frac{\Lambda_g}{\mathrm{v}} \right)^2 \frac{l_g}{\Lambda_g}$$

Milky Way with characteristic parameters:

$$\left\langle \theta^2 \right\rangle_{MW} \sim 4\times 10^{29} m_{-2}^2 \left(\frac{\Lambda_g}{1kpc} \right) \left(\frac{B_g}{10\mu G} \right)^2 \left(\frac{\mu_\nu}{\mu_B} \right)^2$$

$\mu_\nu \sim 1.5\times 10^{-15}\mu_B \sim 10^{-4}\mu_{1T} \Rightarrow \sqrt{\left\langle \theta^2 \right\rangle} \sim 1$ helicity randomizes

Cosmic magnetic field rotation of neutrino spin

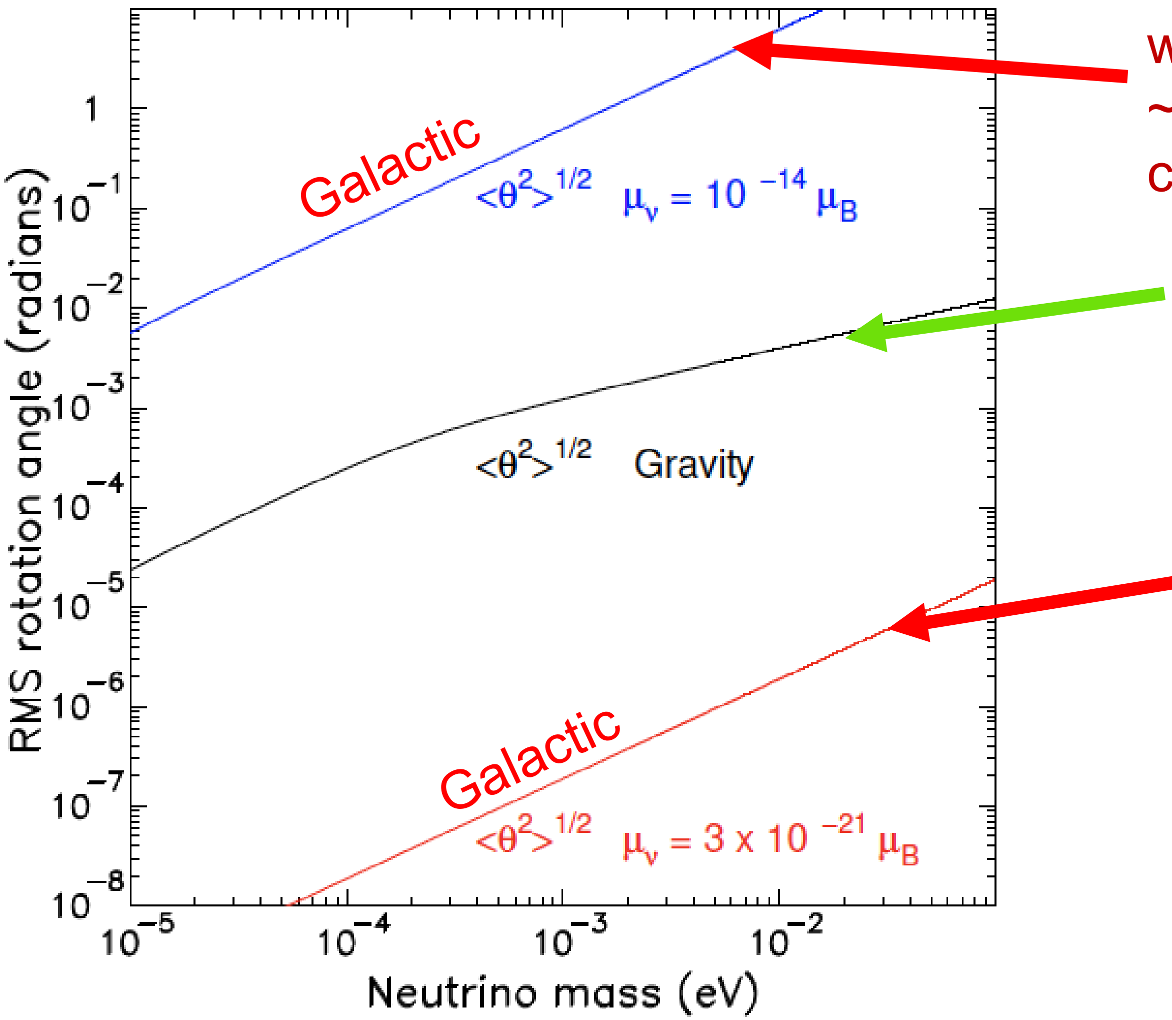
$$\left\langle \theta^2 \right\rangle_{\text{Galaxy}} \sim 4\times10^{29}\, m_{-2}^2 \left(\frac{\Lambda_g}{1kpc}\right)\left(\frac{B_g}{10\mu G}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

$$\left\langle \theta^2 \right\rangle_{\text{Cosmic}} \sim 2\times10^{27} \left(\frac{\Lambda_0}{1Mpc}\right)\left(\frac{B_0}{10^{-12}G}\right)^2\left(\frac{\mu_\nu}{\mu_B}\right)^2$$

$\Lambda_0 = $ coherence length of cosmic magnetic field

To within uncertainties in magnetic fields, coherence lengths, and neutrino masses, spin rotation in cosmic magnetic fields is comparable to that in galactic fields

Spin rotation from gravitational vs. magnetic fields



Rotation in Milky Way with magnetic moment ~100 times smaller than current upper limit

Gravitational rotation

Rotation in Milky Way with standard model magnetic moment

ITBD rate depends on the helicity, mass and type of relic neutrinos

- $\sigma_i^h = \dfrac{G_F^2}{2\pi v_i} |V_{ud}|^2 |U_{ei}|^2 \; F(Z,E_e) \dfrac{m(^3He)}{m(^3H)} E_e p_e A_i^h (\overline{f}^2 + 3\overline{g}^2)$
- Define A_{eff} as the sum of $|U_{ei}|^2 \; A_i^h$ over mass state i and helicity h:

$$A_{eff} = \sum_{i,h=\pm} |U_{ei}|^2 \langle A_i^h \rangle_T$$

- Helicity-dependent factor, A_i^h, is $A_i^{\pm} = 1 \mp \beta_i$; where $\beta_i = v_i / c$
- T denotes the thermal average over the present momentum distribution, $f(p)$, of relic neutrinos: $f(p) = [e^{p/T_0} + 1]^{-1}; T_0 = 0.1676 \; m\text{eV}$

- For Dirac type, only neutrinos (not antineutrinos) contribute

$$A_{eff,D} = \sum_{i,h=\pm} |U_{ei}|^2 \langle A_i^h \rangle_T = 1 + \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T$$

- For Majorana type, both neutrinos and antineutrinos contribute

$$A_{eff,M} = (1 + \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T) + (1 - \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T) = 2$$

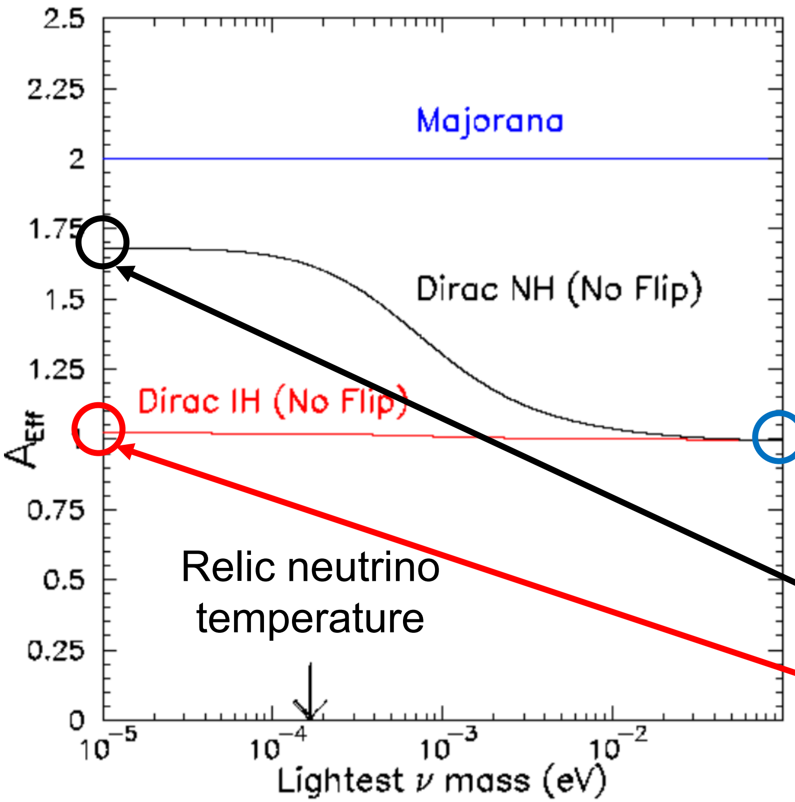
ITBD rate for Dirac neutrinos without helicity flip

- For Majorana type, both neutrinos and antineutrinos contribute

$$A_{eff,M} = (1+\sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T) + (1-\sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T) = 2$$

- For Dirac type, only neutrinos (not antineutrinos) contribute

$$A_{eff,D} = \sum_{i,h=\pm} |U_{ei}|^2 \langle A_i^h \rangle_T = 1+\sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T$$



- For Dirac neutrinos without helicity flip ($\cos\theta_i = 1$)

$$A_{eff,D} = 1+\sum_i |U_{ei}|^2 \langle \beta_i \rangle_T$$

- If all neutrinos are non-relativistic, $\beta_i \to 0$, then

$$A_{eff,D} = 1$$

- If the lightest neutrino is relativisitic, then

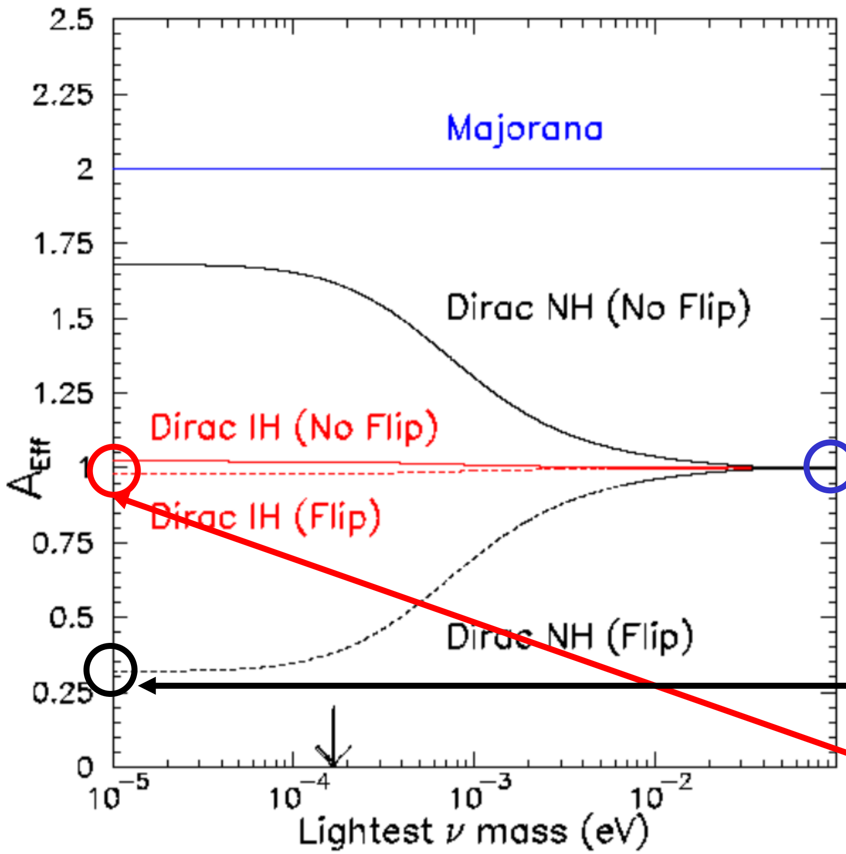
$A_{eff,D} = 1+|U_{e1}|^2 = 1.68$ for normal mass hierarchy

$A_{eff,D} = 1+|U_{e3}|^2 = 1.02$ for inverted mass hierarchy

ITBD rate for Dirac neutrinos with helicity flip

- For Dirac type, only neutrinos (not antineutrinos) contribute

$$A_{eff,D} = \sum_{i,h=\pm} |U_{ei}|^2 \langle A_i^h \rangle_T = 1 + \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T$$



- Dirac neutrinos with helicity flip ($\cos\theta_i = -1$)

$$A_{eff,D} = 1 - \sum_i |U_{ei}|^2 \langle \beta_i \rangle_T$$

- If all neutrinos are non-relativistic, $\beta_i \to 0$,

$$A_{eff,D} = 1$$

- If the lightest neutrino is relativisitic,

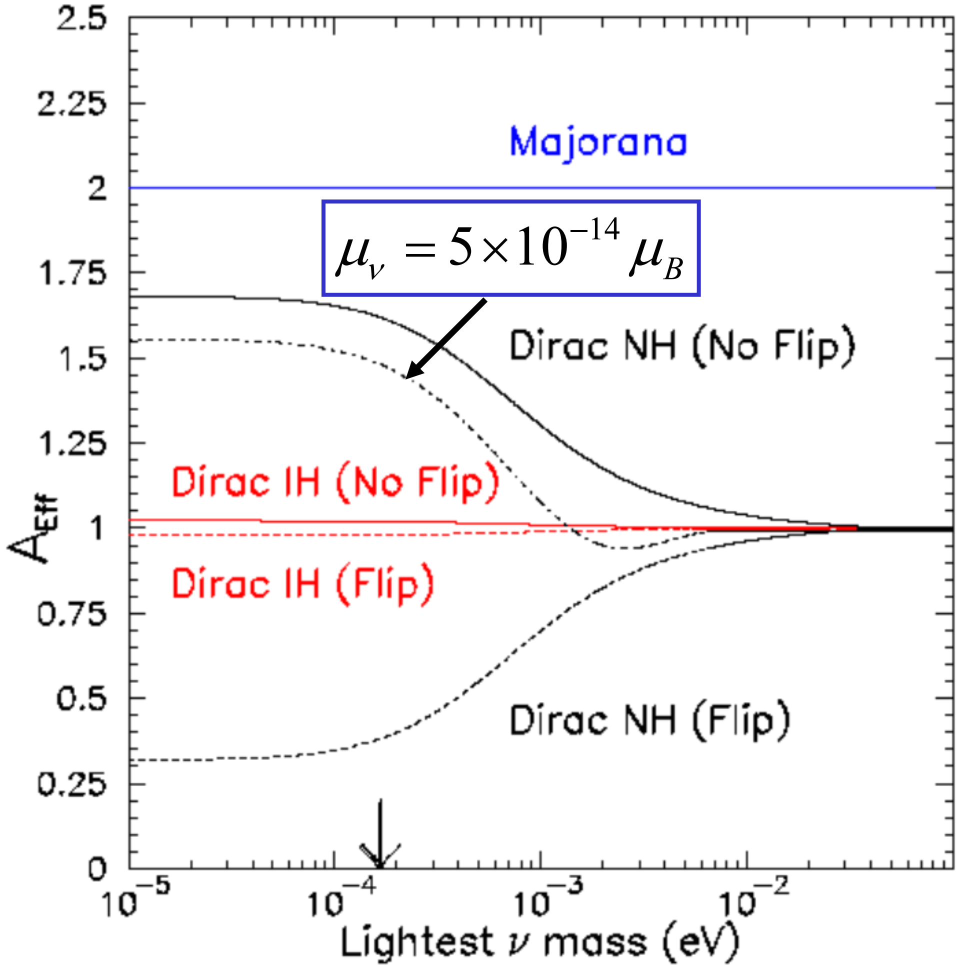
$$A_{eff,D} = 1 - |U_{e1}|^2 = 0.32 \quad \text{normal hierarchy}$$

$$A_{eff,D} = 1 - |U_{e3}|^2 = 0.98 \quad \text{inverted hierarchy}$$

ITBD rate for Dirac neutrinos with partial helicity flip

- For Dirac type, only neutrinos (not antineutrinos) contribute

$$A_{eff,D} = \sum_{i,h=\pm} |U_{ei}|^2 \langle A_i^h \rangle_T = 1 + \sum_i |U_{ei}|^2 \langle \beta_i \cos\theta_i \rangle_T$$



- For Dirac with NH, ITBD rate is modified even with a modest μ_ν of $5\times10^{-14}\mu_B$
- For Dirac with IH $A_{eff,D} \simeq 1$ insensitive to μ_ν
- For Majorana neutrinos $A_{eff,M} = 2$, independent of μ_ν

Baym and Peng, PRL 126, 191803 (2021)

Summary on Relic Neutrinos

- Relic neutrino helicities could be modified by gravity and magnetic fields. The cosmic neutrino background can probe cosmic magnetic fields and density fluctuations
- Detection rate of relic neutrinos via the ITBD reaction is sensitive to the Dirac/Majorana nature of neutrino, and to the lightest mass of neutrinos and the mass hierarchy
- For Dirac neutrino with normal hierarchy, the ITBD rate also depends on neutrino helicity, which is sensitive to neutrino magnetic moment
- Detection of relic neutrinos can reveal fundamental properties of neutrinos and the Early Universe

Macroscopic neutrinoless double beta decay: Long range quantum coherence

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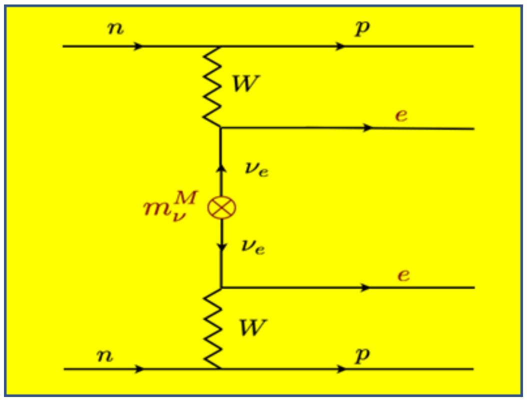
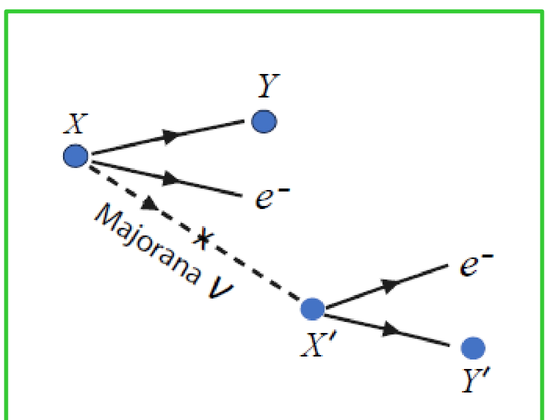


0νDBD (neutrinoless double-beta decay)

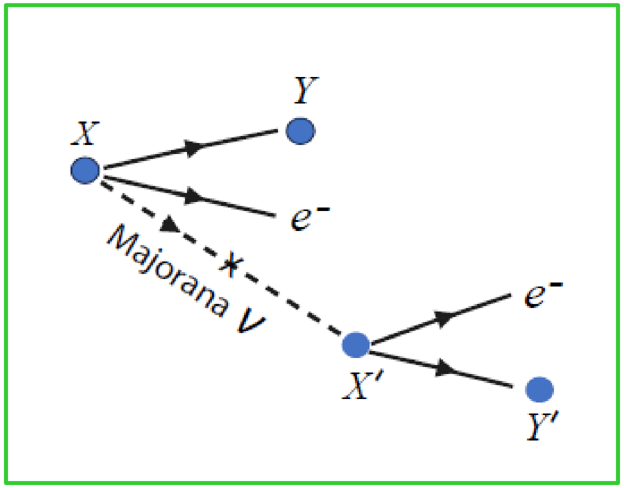
$$(A, Z) \to (A, Z+2) + e^- + e^-.$$



MDBD (macroscopic double-beta decay)

$$X + X' \to Y + Y' + e^- + e^-,$$

Consider tritium beta decay followed by inverse tritium beta decay



$$^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^- + \bar{\nu}_e$$

$$\nu_e + {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + e^-$$

(if neutrino is Majorana)

$$^3\mathrm{H} + {}^3\mathrm{H} \rightarrow {}^3\mathrm{He} + {}^3\mathrm{He} + e^- + e^-$$

MDBD (macroscopic double-beta decay)

- Majorana neutrino travels a macroscopic distance before being captured by another radiative nucleus

Similarities and differences between 0νDBD and MDBD

- Similarities
  - Both processes can proceed only if neutrinos are Majorana types and are massive.
  - Both processes imply lepton number non-conservation and are evidence for BSM physics
- Differences
  - All beta-decay nuclei could be candidates for MDBD
  - No matrix-element uncertainties for MDBD
  - Rate is linear (~N) for 0νDBD, but ~$N^{4/3}$ for MDBD (where N is the number of nuclei)
  - Sources of backgrounds are different (no irreducible 2νDBD for MDBD)

Comparing the rates of MDBD with 0νDBD

Table 1
Expected yields, Y, for MDBD and 0νDBD for a 100 g-yr exposure using various nuclei. We assume the Majorana neutrino effective mass $\bar{m} = 0.1$ eV. The uncertainties in the predicted $T_{1/2}$ and yield for 0νDBD reflect the range of uncertainties of the nuclear matrix elements adopted by Ref. [7] to relate the current experimental limits on $T_{1/2}$ to values of $\bar{m}$. For MDBD we assume 100 g spherical sources of tritium (and neutrons, for illustration) with density 1 g/cm^3 and ^{11}C with density 2.2 g/cm^3.

Nucleus	$T_{1/2}$ for $\bar{m} = 0.1$ eV	Yield per 100 g-yr
^{3}H (MDBD)	–	2.3×10^{-7}
n (MDBD)	–	3.4×10^{-2}
^{11}C (MDBD)	–	5.1×10^{-5}
^{76}Ge (0νDBD) [25,24]	$1.1 \times 10^{26} < T_{1/2} < 6.0 \times 10^{26}$ yr	$9.1 \times 10^{-4} < Y < 5.0 \times 10^{-3}$
^{136}Xe (0νDBD) [26–28]	$3.0 \times 10^{25} < T_{1/2} < 5.6 \times 10^{26}$ yr	$5.5 \times 10^{-4} < Y < 1.0 \times 10^{-2}$
^{130}Te (0νDBD) [29]	$1.8 \times 10^{25} < T_{1/2} < 2.0 \times 10^{26}$ yr	$1.6 \times 10^{-3} < Y < 1.8 \times 10^{-2}$
^{82}Se (0νDBD) [30,31]	$3.2 \times 10^{25} < T_{1/2} < 2.3 \times 10^{26}$ yr	$2.3 \times 10^{-3} < Y < 1.6 \times 10^{-2}$
^{100}Mo (0νDBD) [32]	$1.4 \times 10^{25} < T_{1/2} < 4.3 \times 10^{25}$ yr	$9.7 \times 10^{-3} < Y < 3.0 \times 10^{-2}$

The MDBD rates are somewhat lower than the 0νDBD rates, and cannot be a viable alternative process

Angular momentum conservation in Inverse Beta Decays and other processes

PNAS RESEARCH ARTICLE | PHYSICS

Understanding the puzzle of angular momentum conservation in beta decay and related processes

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Contributed by Gordon Baym; received August 17, 2024; accepted October 20, 2024; reviewed by Sanjay Reddy and Jörg Schmiedmayer

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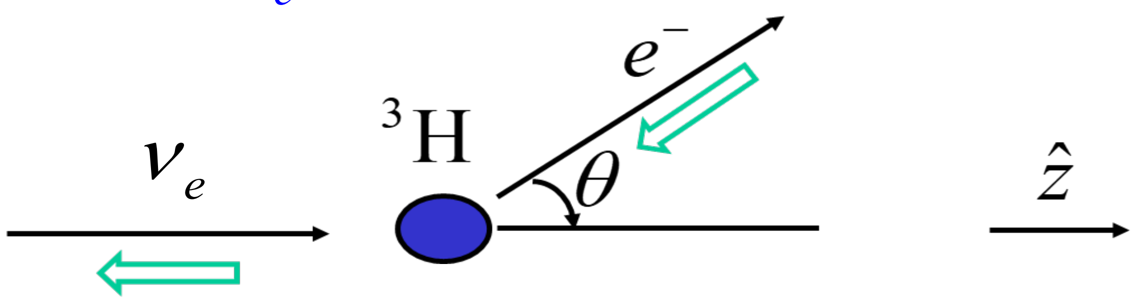
Consider the Inverse Beta Decay:

$$\nu_e + {}^3\mathrm{H} \rightarrow e^- + {}^3\mathrm{He}$$

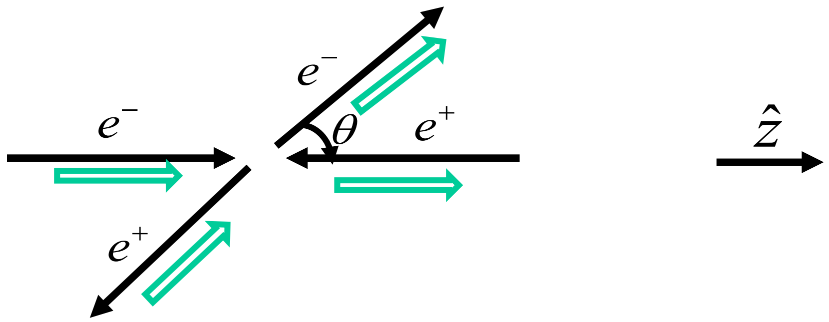


The initial state contains a ν_e with $S_z = -1/2$, the final state now contains an e$^-$ with $S_z \neq -1/2$

How is angular momentum conserved in this process?

Angular momentum conservation in another reaction

$$e^- + e^+ \rightarrow e^- + e^+$$

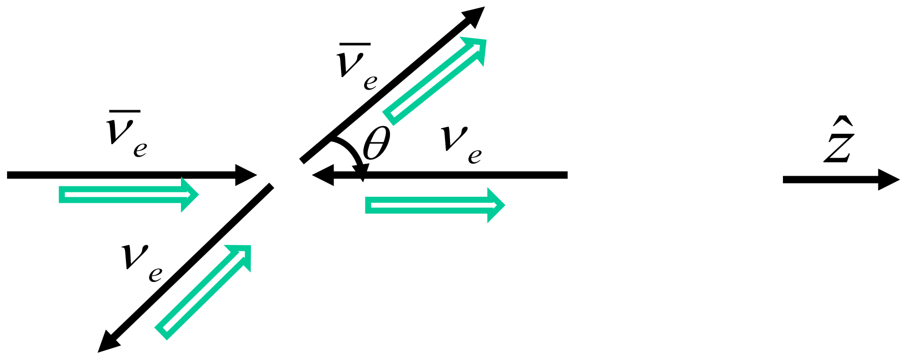


The initial state has $J_z = S_z = 1$,

the final state has $J_z = S_z \neq 1$ ($\vec{J}$ is now along the $\hat{\theta}$ direction)

How is angular momentum conserved in this process?

Similarly for the $\bar{\nu}_e + \nu_e \rightarrow \bar{\nu}_e + \nu_e$



Consider the inverse tritium beta decay reaction

$$\nu_e + {}^3\mathrm{H} \to e^- + {}^3\mathrm{He}$$

For simplicity, consider massless ν_e and e^- :

The incoming ν_e has a negative helicity with a plane wave and a Dirac spinor:

$$\psi_\nu(r) \sim e^{i\vec{P}\cdot\vec{r}}(0,1,0,-1)^T$$

The outgoing e^- is a spherical wave of the form

$$\psi_e^0(r) = (\varphi, \chi)^T; \text{ where } \varphi = h_0(pr)\ (0,1)^T, \quad h_0(pr)\ = ie^{ipr}/pr$$

$$\chi = \frac{1}{ip}\vec{\sigma}\cdot\vec{\nabla}\varphi = h_0(pr)(\sin\theta e^{-i\phi}, -\cos\theta)^T$$

Hence, $\psi_e^0(r) = h_0(pr)(0,1,\sin\theta e^{-i\phi}, -\cos\theta)^T$

$V-A$ interaction projects out $\gamma_5 = -1$ state with $(1-\gamma_5)/2$ operator;

$$\psi_e(\vec{r}) = \frac{h_0(pr)}{2}(-\sin\theta e^{-i\phi}, 1+\cos\theta, \sin\theta e^{-i\phi}, -(1+\cos\theta))^T$$

Consider the inverse tritium beta decay reaction

$$\nu_e + {}^3\text{H} \rightarrow e^- + {}^3\text{He}$$

$$\psi_e(\vec{r}) = \frac{h_0(pr)}{2}(-\sin\theta e^{-i\phi}, 1+\cos\theta, \sin\theta e^{-i\phi}, -(1+\cos\theta))^T$$

$$S_z\psi_e(\vec{r}) = \frac{h_0(pr)}{4}(-\sin\theta e^{-i\phi}, -1-\cos\theta, \sin\theta e^{-i\phi}, (1+\cos\theta))^T$$

$$L_z\psi_e(\vec{r}) = \frac{h_0(pr)}{2}(\sin\theta e^{-i\phi}, 0, -\sin\theta e^{-i\phi}, 0)^T; \quad \text{note } L_z = -i\partial/\partial\phi$$

ψ_e is not an eigenstate of S_z or L_z

$$J_z = S_z + L_z$$

$$J_z\psi_e(\vec{r}) = \frac{h_0(pr)}{4}(-\sin\theta e^{-i\phi}, 1+\cos\theta, \sin\theta e^{-i\phi}, -(1+\cos\theta))^T$$

ψ_e is indeed an eigenstate of J_z with eigenvalue of -1/2 (just like the incoming ν_e)

More details can be found in: PNAS 121, e2416768121 (2024)

谢谢