Parton distributions from lattice QCD with physical pion mass

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Publications

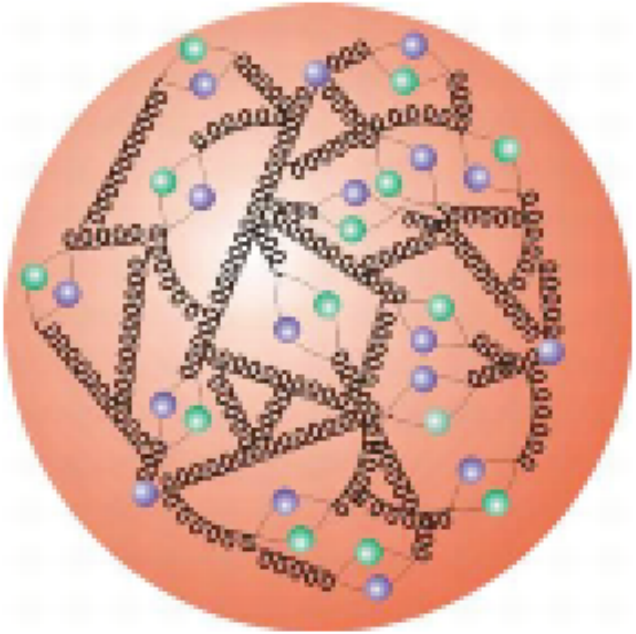
2014: 1402.1462

2016: 1603.06664, 1609.08102

2017: 1702.00008, 1706.01295, 1708.05301, 1710.01089, 1711.07916, 1711.07858, 1712.10025

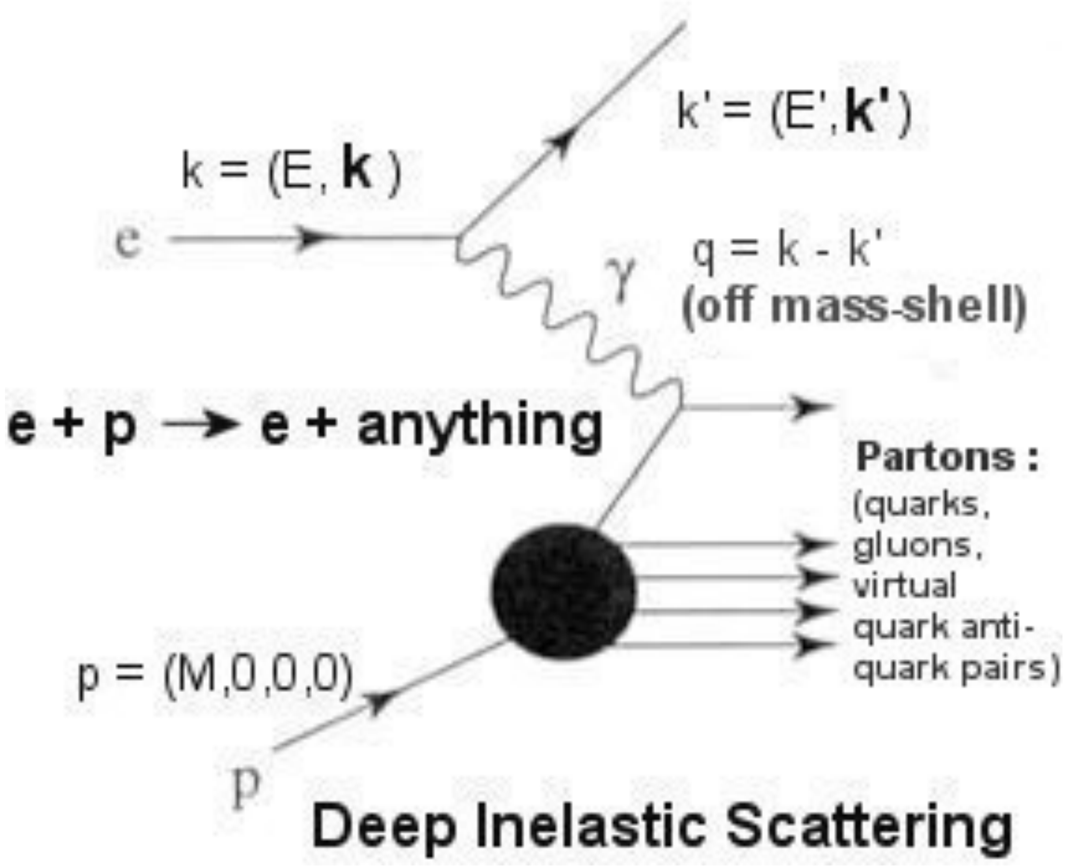
2018: 1801.03023, 1803.04393

Feynman’s Parton Model

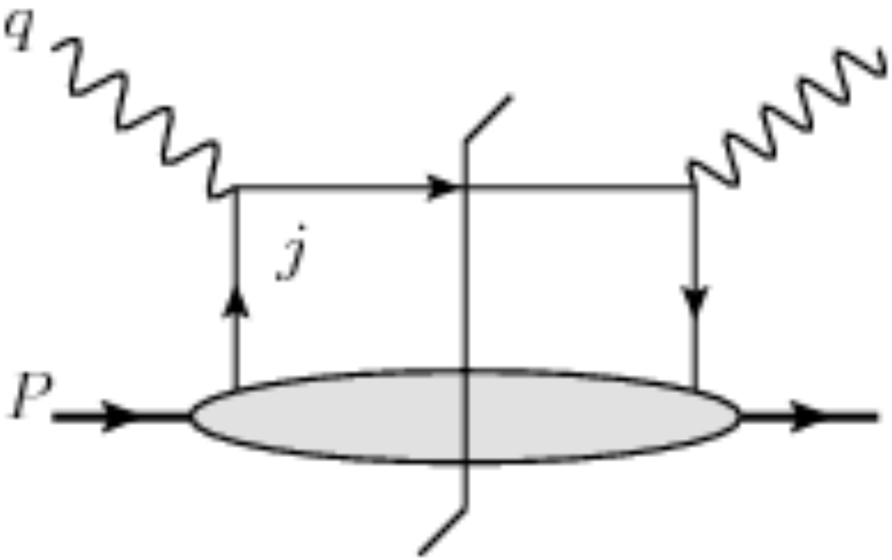


The momentum distributions of partons (quarks, antiquarks and gluons) become one dimensional distributions in the infinite momentum frame.

Measuring Parton Distributions Using DIS experiments



Parton Distribution Function (PDF) in QCD



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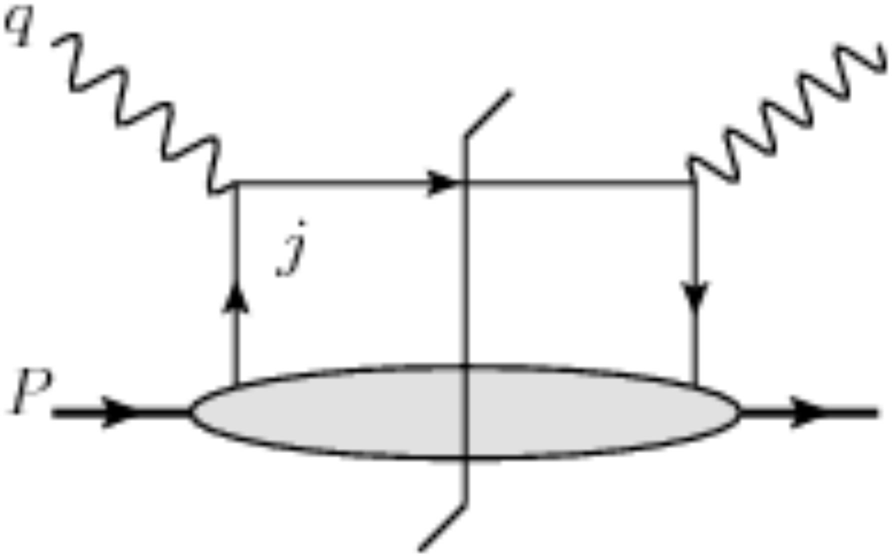

The struck parton moves on a light cone at the leading order in the twist-expansion.

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P \right\rangle$$

Current Status of Proton PDFs

How do momentum and spin distribute among partons?

- Exp: 1d mom. dist. largely mapped out (up to parameterizations of the functional forms); largest sys. uncertainty in Higgs production.

  improve 1d(spin)+3d: BNL, JLab, J-PARC,
  COMPASS, GSI, EIC, LHeC, ...
- Theory: Only first few moments could be computed directly from QCD!!!

PDFs from QCD---why is it so hard?

- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P\rangle$$

- Non-perturbative, infinite dof, need lattice QCD
- Euclidean lattice: light cone operators cannot be distinguished from local operators

$$t^2 - \mathbf{r}^2 = 0$$

$$-t_E^2 - \mathbf{r}^2 = 0$$

- Moments of PDF given by local twist-2 operators (twist = dim - spin); limited to first few moments but carried out successfully

$$a_n = \int_{-1}^{1} dx\, x^{n-1} q(x) \text{ and } q(-x) = -\bar{q}(x)$$

Beyond the first few moments

- Smeared sources: Davoudi & Savage
- Gradient flow: Monahan & Orginos
- Current-current correlators: K.-F. Liu & S.-J. Dong; Braun & Müller; Detmold & Lin; QCDSF
- Xiangdong Ji (Phys. Rev. Lett. 110 (2013) 262002): quasi-PDF: computing the x -dependence directly. (variation: pseudo-PDF, Radyushkin)

Ji’s idea

- Quark PDF in a proton: $(\lambda^2 = 0)$

$$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{ix\xi^- P^+} \langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(\xi^-\lambda) \right| P \rangle$$

- Boost invariant in the z-direction, rest frame OK
- Quark bilinear op. always on the light cone
- What if the quark bilinear is slightly away from the light cone (space-like) in the proton rest frame?

- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.

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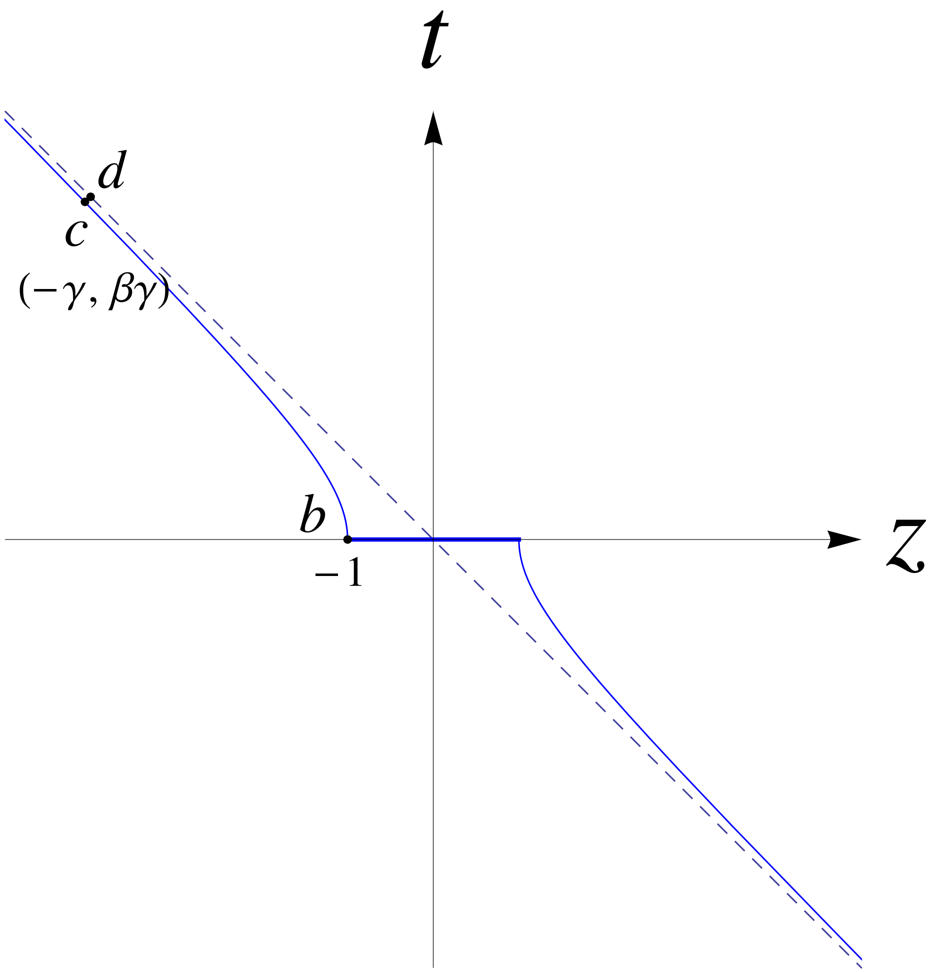


- Then one can find a frame where the quark bilinear is of equal time but the proton is moving.
- Analogous to HQET: need power corrections & matching---LaMET (Large Momentum Effective Theory)

Review: Ji's LPDF (LaMET)

$$\begin{aligned} \widetilde{q}(x,\mu^2,P^z) &= \int \frac{dz}{4\pi} e^{-ixzP^z} \left\langle P \left| \overline{\psi}(0)\lambda\cdot\gamma\Gamma\psi(z\lambda) \right| P \right\rangle \\ &\equiv \int \frac{dz}{2\pi} e^{-ixzP^z} h(zP^z)P^z \end{aligned}$$

$$\lambda^\mu = (0,0,0,1)$$

- Taylor expansion yields

$$\overline{\psi}\lambda\cdot\gamma\Gamma\left(\lambda\cdot D\right)^n\psi = \lambda_{\mu_1}\lambda_{\mu_2}\cdots\lambda_{\mu_n}O^{\mu_1\cdots\mu_n}$$

op. symmetric but not traceless

$$\left(\lambda_{\mu_1}\lambda_{\mu_2} - g_{\mu_1\mu_2}\lambda^2/4\right)$$

Review: Ji's LPDF (LaMET)

$$\left\langle P \left| O^{(\mu_1 \cdots \mu_n)} \right| P \right\rangle = 2a_n P^{(\mu_1} \cdots P^{\mu_n)}$$

- LHS: trace, twist-4 $\mathcal{O}(\Lambda_{QCD}^2/(P^z)^2)$ corrections, parametrized in this work
- RHS: trace $\mathcal{O}(M^2/(P^z)^2)$
- One loop matching $\alpha_s \ln P^z$, OPE

$$\tilde{q}(x, \Lambda, P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}, \frac{\Lambda}{P_z}\right) q(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}^2}{P_z^2}, \frac{M^2}{P_z^2}\right) + \ldots .$$

RG of Wilson Coefficient

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{M^2}{P_z^2}\right) + \ldots .$$

Xiong, Ji, Zhang, Zhao (GPD: Ji, Schafer, Xiong, Zhang; Xiong, Zhang) Factorization (Ma, Qiu; Li; OPE: Izubuchi, Ji, Jin, Stewart, Zhao), Linear divergence & LPT (Ishikawa, Ma, Qiu, Yoshida; JWC, Ji, Zhang; Xiong, Luu, Meissner; Rossi, Testa; Constantinou et al.; Wang, Zhao, Zhu), RI (Monahan & Orginos; Yong & Stewart; Constantinou et al.), NPR(Constantinou et al.; LP3; Ji, Zhang, Zhao; Ishikawa, Ma, Qiu, Yoshida; Green, Jansen, Steffens), E vs. M spaces (Carlson et al.; Briceno et al.)

Lattice Setup (isovector proton PDF)

- Lattice: $64^3 \times 96$

  $a = 0.09 \text{ fm} \qquad L \approx 5.8 \text{ fm}$

- Fermions: MILC highly improved staggered quarks (HISQ) Clover (valence)

  $N_f = 2+1+1 \qquad M_\pi \approx 135 \text{ MeV}$

- Gauge fields/links: hypercubic (HYP) smearing (one step), 309 config.

- $P^z = n\frac{2\pi}{L}$ = 2.2, 2.4, 3.0 GeV (n = 10,12,14)

(high momentum smearing: Bali, Lang, Musch, Schafer)

Non-Perturbative Renormalization + Matching

$$\tilde{q}(x,\Lambda,P_z) = \int \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P_z},\frac{\Lambda}{P_z}\right) q(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{P_z^2},\frac{M^2}{P_z^2}\right) + \ldots.$$

- NPR (RI/MOM scheme), γ_t $p^2 = -\mu_R^2$
  Landau gauge $p_z = p_z^R$
- RI/MOM to $\overline{\rm MS}$ performed at one loop

Sensitivity on p_z^R

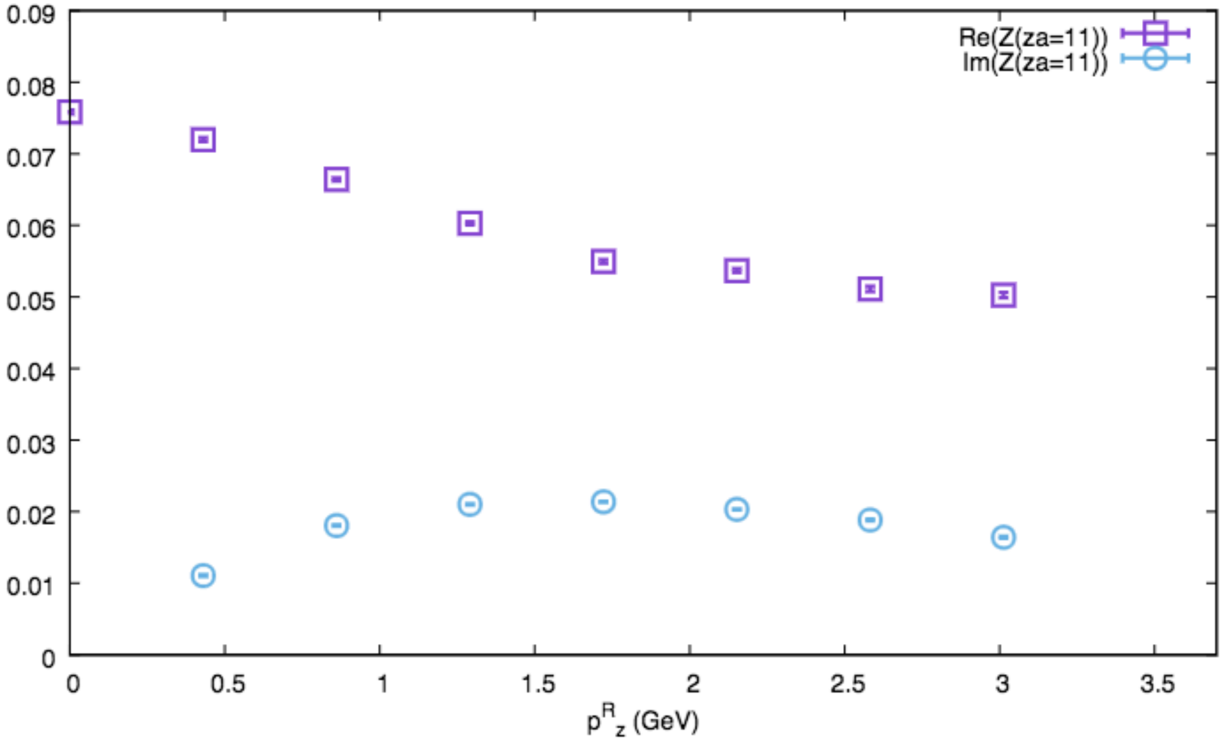
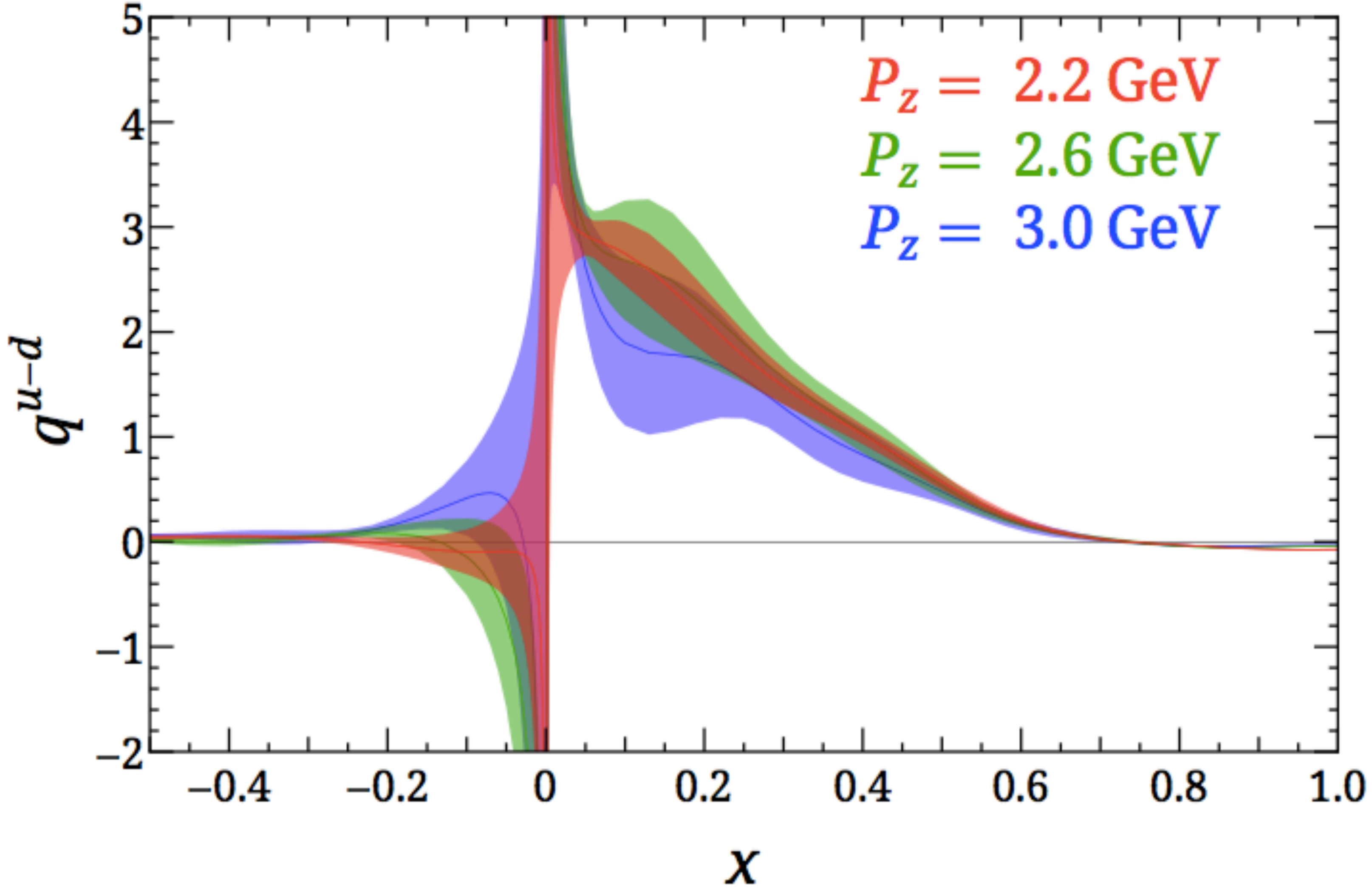


FIG. 1. The values of $Z(z)$ (the inverse of the renormalization constant) at $z = 11a \approx 1.0$ fm as a function of p_z^R. Note that $Z(z)$ becomes stable at large p_z^R.

insensitive to $\mu_R = 2.3$ and 3.7 GeV

P_z Dependence



LP3 result

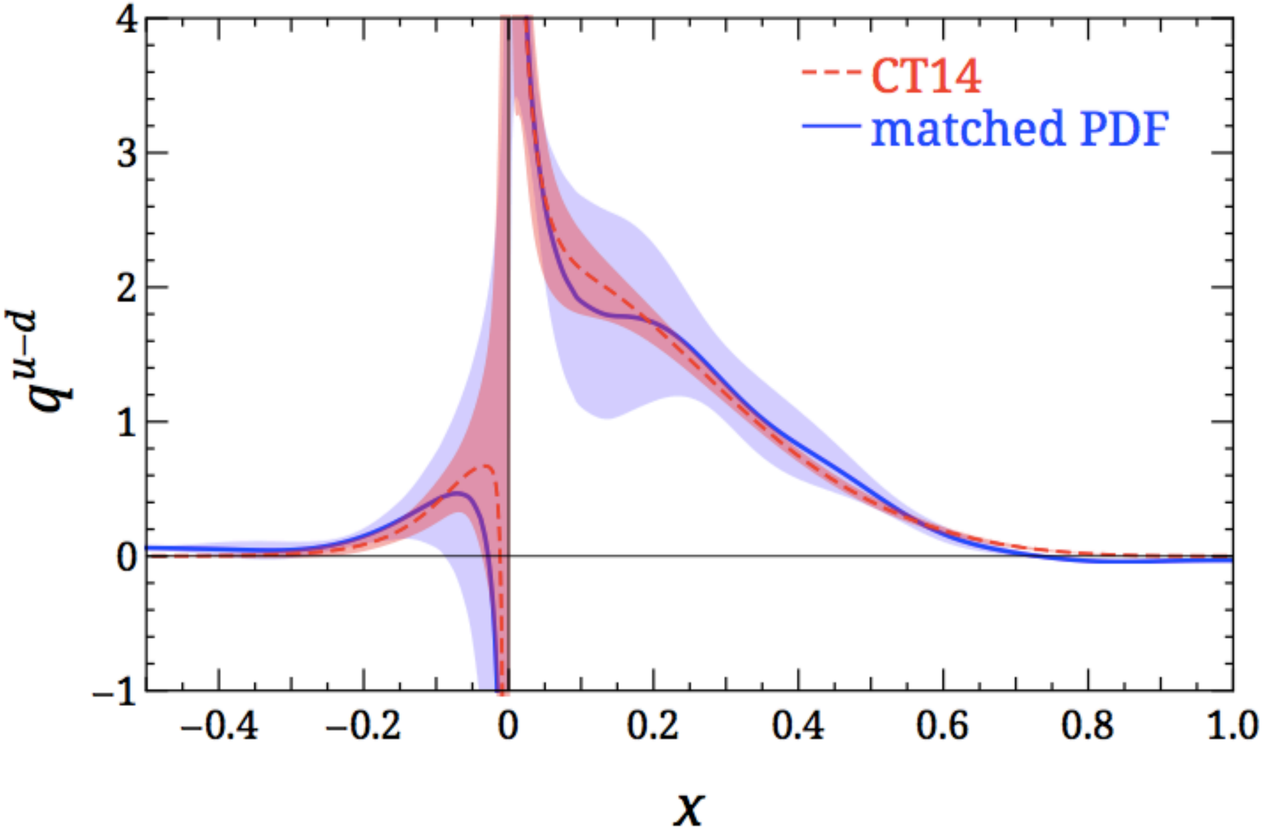
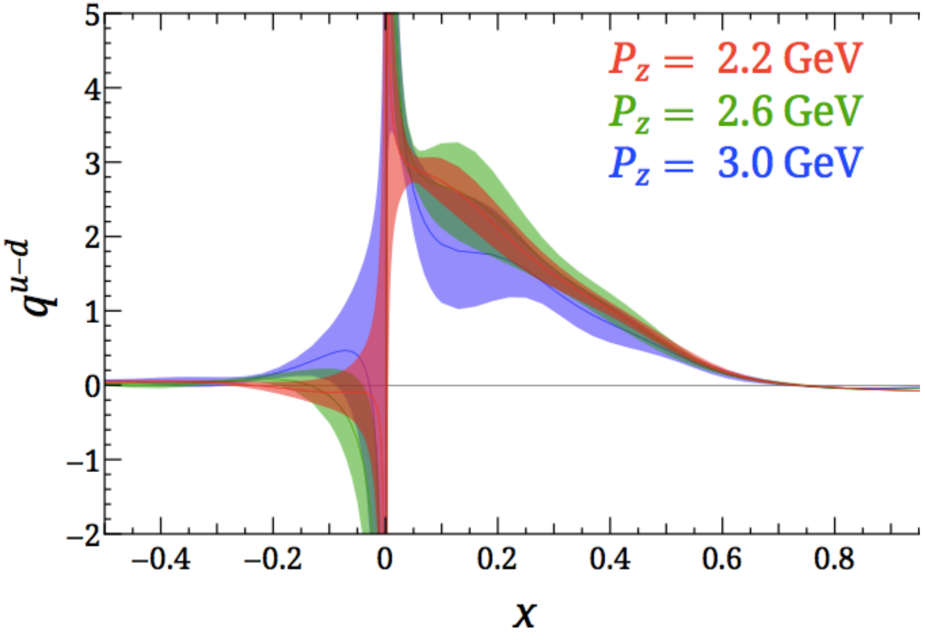
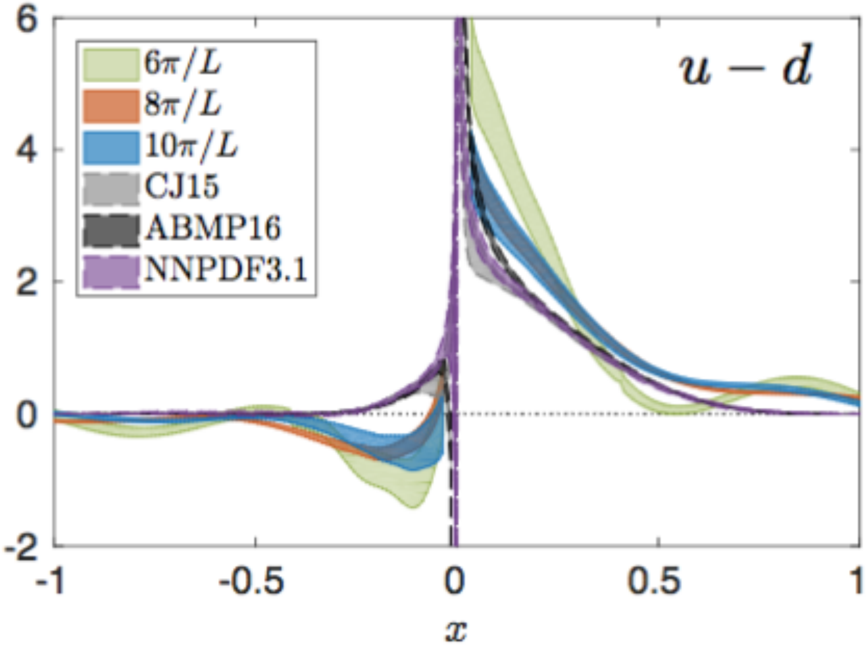


FIG. 4. Our final PDF renormalized at 3 GeV and compared with CT14 [61] which is consistent with NNPDF3.1 distribution [62] and CJ15 [63]. Our results agree nicely with the global-analysis PDF.

Compared with ETMC (1803.02685)



LP3



ETMC

Diff: momentum, a, $M_\pi L$, excited state, matching, continuum extrapolation, derivative method

$$\tilde{q}(x) = \int_{-z_{\max}}^{+z_{\max}} dz \frac{e^{ixP_z z}}{ix} \partial_z \tilde{h}^R(z)$$

Outlook

- Further tests (non-singlet): higher twist, truncation, p_z^R dependence

  Know whether it works within 3 years (~20%)?

- Singlet PDF's: s, c, b and gluons

  Additional 3-5 yrs?

- If it works, complimentary to exp.: PDF (isov. sea, small and large x's, non-valence partons), DA, GPD, TMD …

Backup slides