Quasi Parton Distribution Functions for Gluons

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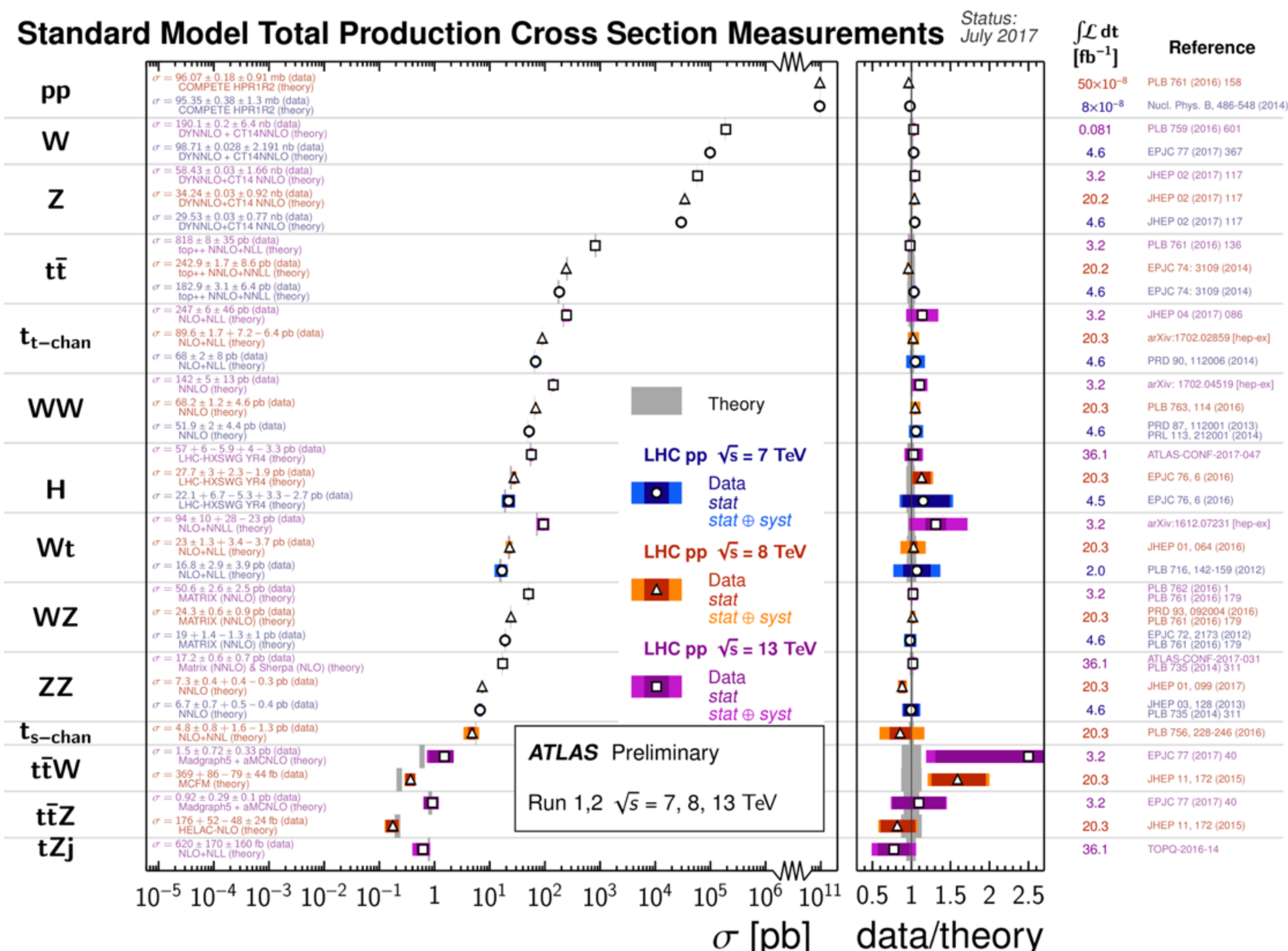
Outline

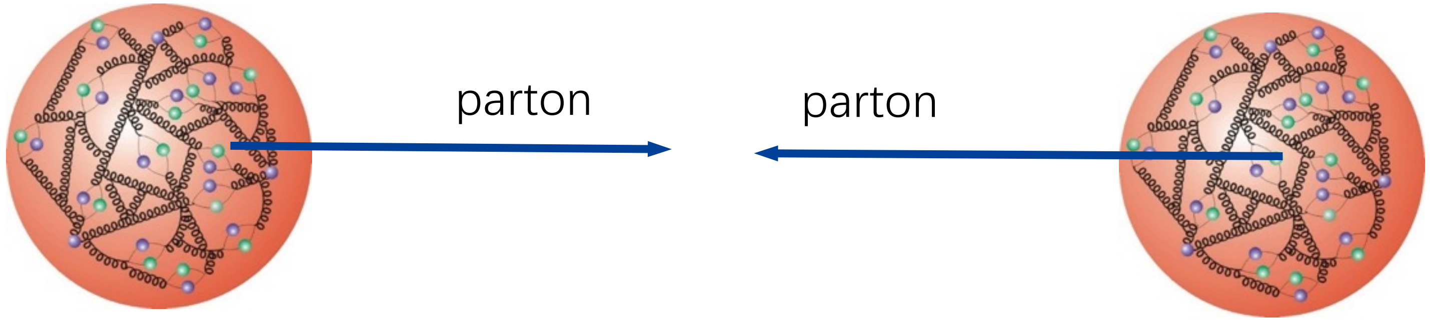
- Introduction
- Quasi Distribution and LaMET
- Gluon quasi PDF: linear divergence (Problem?)
- Auxiliary Field: new operators (Solution?)
- Summary

Success of SM



success of EW and flavor sectors but also QCD

parton distribution function



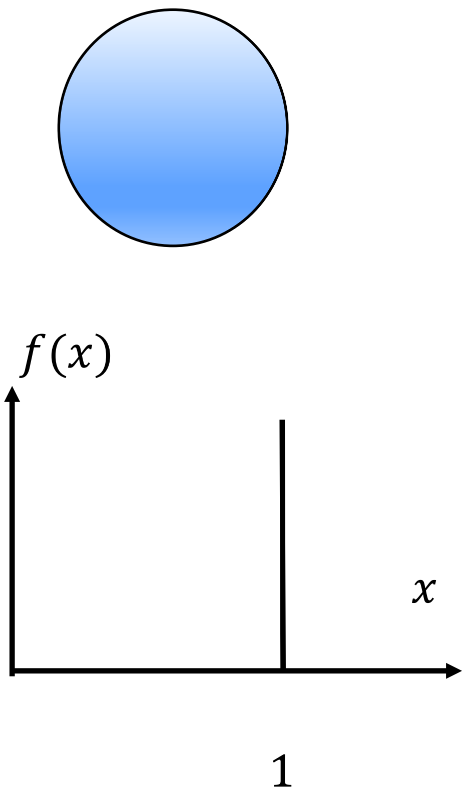
- Factorization theorems:

$$d\sigma \sim \int dx_1 dx_2 * f(x_1) * f(x_2) * C(x_1, x_2, Q)$$

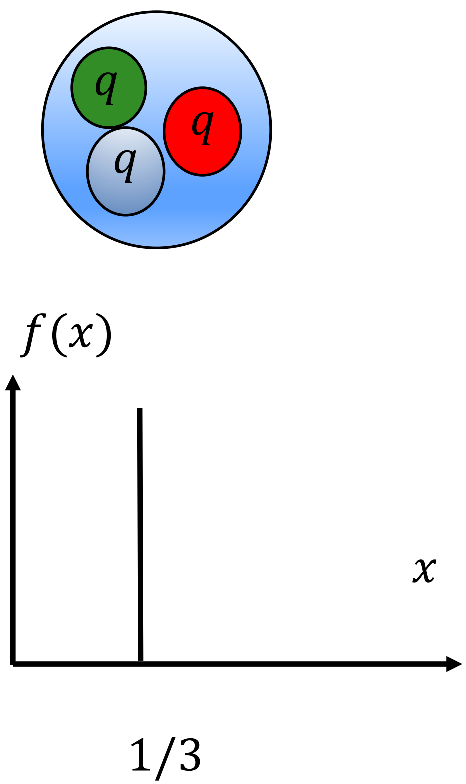
- PDF: basic inputs for particle physics at hadron colliders.

PDF and Proton Structure

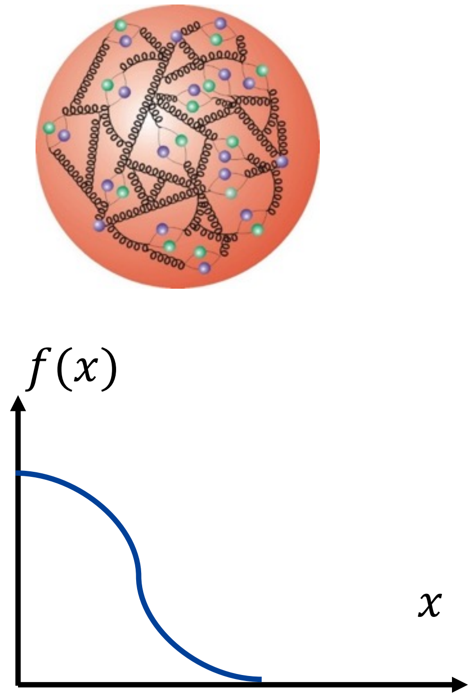
Elastic



3 free quarks



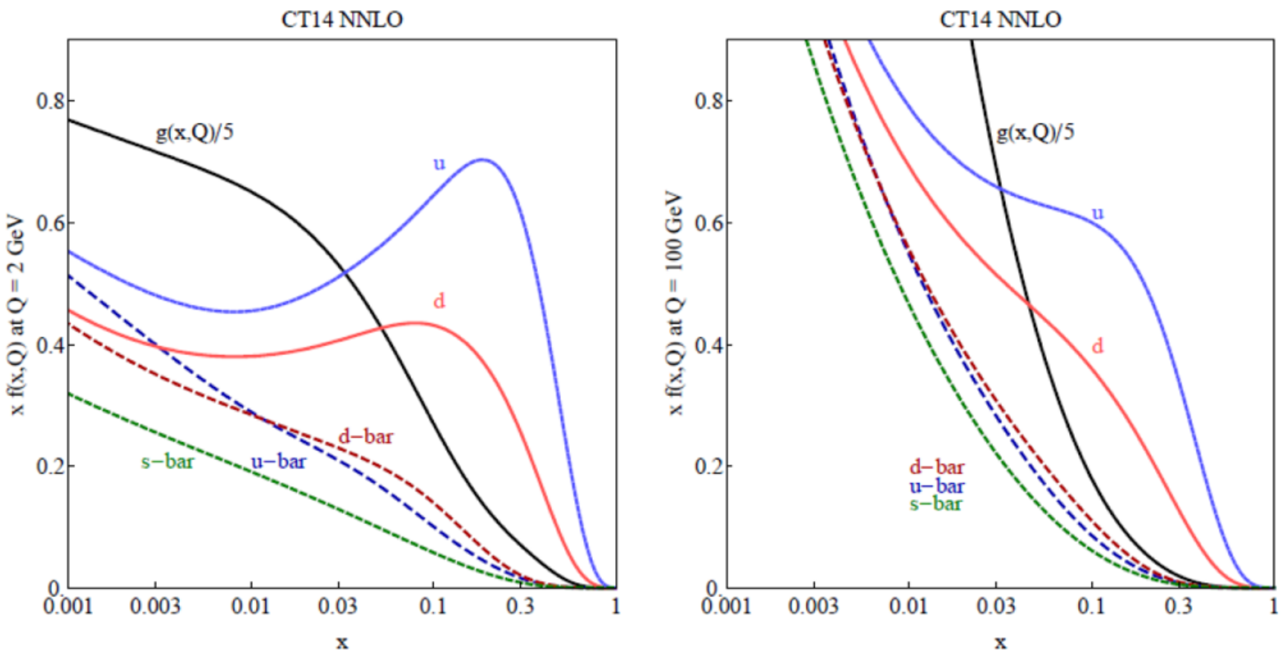
Quarks and Gluons
Many body problem



PDF from data

CTEQ (Dulat *et al.* arxiv: 1506.07443)

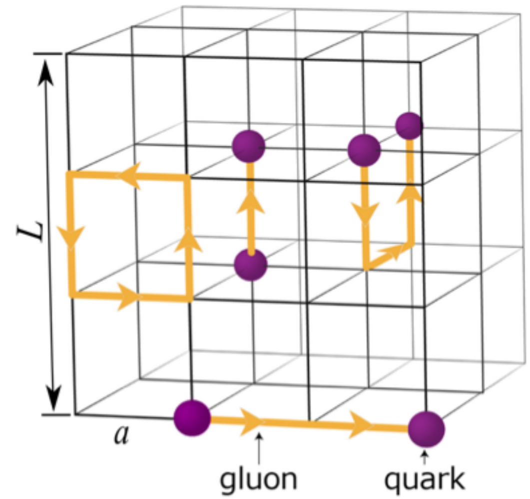
NNPDF, MSTW⋯



first principle?

Lattice QCD

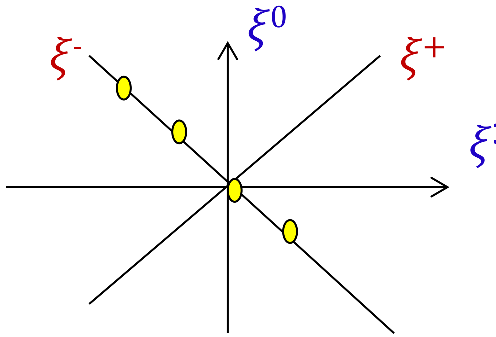
- Numerical simulation in discretized Euclidean space-time
- Finite volume (L should be large)
- Finite lattice spacing (a should be small)



PDF on Lattice

- PDF can be formulated as the matrix elements of the boost-invariant light-cone correlations
  -> time-dependence

$$f_{j/H}(\xi) = \int \frac{dw^-}{2\pi}\, e^{-i\xi P^+ w^-} \langle P|\overline{\psi}_j(0, w^-, \mathbf{0}_T)\, \frac{\gamma^+}{2}\, \psi_j(0)|P\rangle_c.$$



- One can form local moments to get rid of the time-dependence
  - $\langle x^n \rangle = \int f(x) x^n dx$ →matrix elements of local operators
  - However, one can only calculate lowest few moments in practice.
  - Higher moments quickly become noisy.

Quasi Distribution

X.D.Ji, Phys. Rev. Lett. 110 (2013) 262002

Many progresses on quasi distributions

- One loop matching for quark (Xiong, Ji, Zhang, Zhao,2013)
- Renormalization (Ji,Zhang,2014)
- Quasi GPD (Ji ,Schafer, Xiong ,Zhang, 2015)
- Quasi TMD and soft factor subtraction (Ji,Sun,Xiong,Yuan,2015)
- “Lattice cross section” approach (Ma, Qiu, 2014)
- Lattice calculation (Lin, Chen, Cohen,Ji, 2014; Chen, Cohen, Ji, Lin , Zhang ,2016)
- Quasi distribution amplitude of Heavy Quarkonia (Jia, Xiong,2015)
- Non-dipolar Wilson line (Li,2016)
- diquark spectator model (Gamberg, Kang, Vitev, Xing)
- Matching continuum to lattice (T. Ishikawa, Y.Q. Ma, J.W. Qiu, S.Yoshida, 2016)
- 2017…
- 2018…

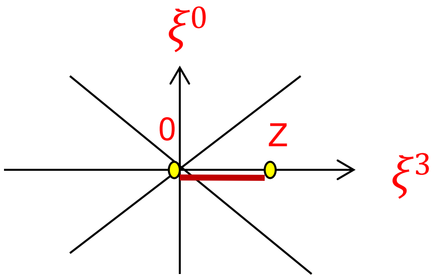
See the talk by J.W.Chen

上海交通大学 SHANGHAI JIAO TONG UNIVERSITY

A Euclidean quasi-distribution

- Consider space correlation in a large momentum P in the z-direction.

$$\tilde{q}(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$



  - Fields separated along the z-direction
  - Gauge-link/Wilson line along the z-direction
  - The matrix element depends on the momentum P.

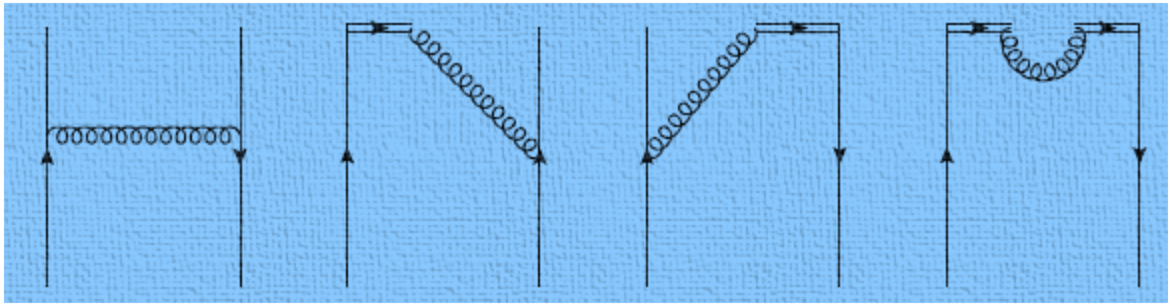
A Euclidean quasi-distribution

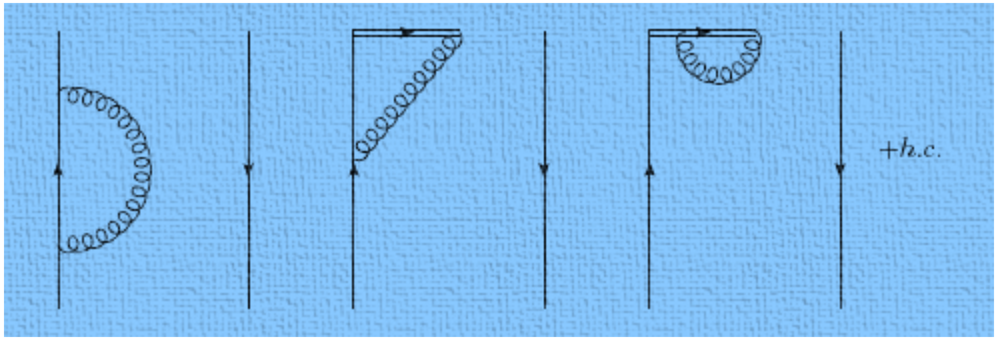
- Matching onto Light-cone PDF:

$$\tilde{q}(x,\mu^2,P^z)=\int_{-1}^{1}\frac{dy}{|y|}Z\left(\frac{x}{y},\frac{\mu}{P^z}\right)q(y,\mu^2)+\mathcal{O}\left(\Lambda^2/(P^z)^2,M^2/(P^z)^2\right)\ ,$$

- Quasi pdfs: finite but large p^z, from "full theory"
- Light-cone pdfs : $p^z \to \infty$
- Z: matching coefficient, the difference of the UV physics, can be calculated in perturbation theory.

$$Z(x,\mu/P^z)=\delta(x-1)+\frac{\alpha_s}{2\pi}Z^{(1)}(x,\mu/P^z)+\ldots$$




+h.c.

Linear power UV divergence

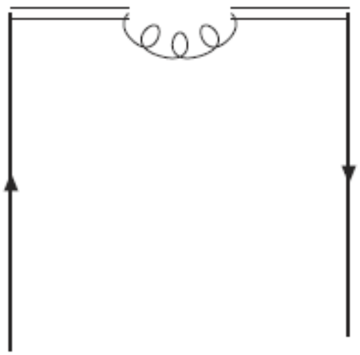
$$Z^{(1)}\left(\xi,\frac{P^z}{\Lambda}\right)=\frac{\alpha_S C_F}{2\pi}\begin{cases}\left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi}{\xi-1}+1+\frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, & \xi>1,\\ \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{(P^z)^2}{\Lambda^2}+\left(\frac{1+\xi^2}{1-\xi}\right)\ln\left[4\xi(1-\xi)\right]-\frac{2\xi}{1-\xi}+1+\frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, & 0<\xi<1,\\ \left(\frac{1+\xi^2}{1-\xi}\right)\ln\frac{\xi-1}{\xi}-1+\frac{1}{(1-\xi)^2}\frac{\Lambda}{P^z}, & \xi<0.\end{cases}$$

$$+\delta(1-\xi)\frac{\alpha_S C_F}{2\pi}\int dy\begin{cases}-\frac{1+y^2}{1-y}\ln\frac{y}{y-1}-1-\frac{\Lambda}{(1-y)^2P^z} & y>1\\ -\frac{1+y^2}{1-y}\ln\frac{(P_z^2)}{\Lambda^2}-\frac{1+y^2}{1-y}\ln\left[4y(1-y)\right]+\frac{4y^2-2y}{1-y}+1-\frac{\Lambda}{(1-y)^2P^z} & 0<y<1\\ -\frac{1+y^2}{1-y}\ln\frac{y-1}{y}+1-\frac{\Lambda}{(1-y)^2P^z} & y<0,\end{cases}\quad($$

- Free of IR divergences
- Linear power UV divergence

Linear power UV divergence

- Linear UV divergence in quasi quark pdf



$$= \frac{\alpha_S C_F}{2\pi} \begin{cases} \frac{1}{1-x} + \frac{\Lambda}{(1-x)^2 p^z} , & x > 1 , \\ -\frac{1}{1-x} + \frac{\Lambda}{(1-x)^2 p^z} , & 0 < x < 1 , \\ -\frac{1}{1-x} + \frac{\Lambda}{(1-x)^2 p^z} , & x < 0 . \end{cases}$$

- The linear divergence comes from n^2 term

$$\sim \int dl^0 dl_\perp dl^z \delta(l^z - (1-x)p^z) \frac{n^2}{l^2 (l^z + i\epsilon)(l^z - i\epsilon)}$$

- Light-cone PDF: $n^2=0$, No linear power UV divergence.

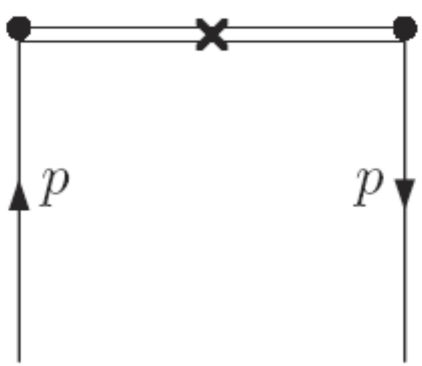
Linear power UV divergence

- Wilson line self energy renormalization (J.W.Chen, X. Ji, J.H.Zhang,2016)

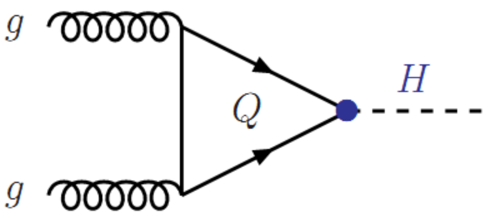
$$L^{\rm ren}(z,0) = \mathcal{Z}_Z^{-1} e^{-\delta m|z|} L(z,0),$$

$$\delta m = -\frac{\alpha_s C_F}{2\pi}(\pi\Lambda)$$

- Linear UV divergence is removed by the “Counter term diagram”



Gluon PDF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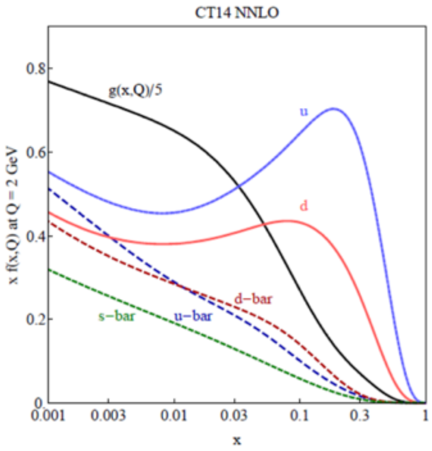

CT14 NNLO
g(x,Q)/5
u
d
d−bar
u−bar
s−bar
x f(x,Q) at Q = 2 GeV
x
0.8
0.6
0.4
0.2
0
0.001
0.003
0.01
0.03
0.1
0.3
1

gluon distribution

- Definition of quasi and light-cone gluon distirbution

$$f_{g/H}(x,\mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-i\xi^- x P^+} \langle P|G^+_{\ i}(\xi^-)W(\xi^-,0;L_{n^+})G^{i+}(0)|P\rangle$$

$$\tilde{f}_{g/H}(x,P^z) = \int \frac{dz}{2\pi x P^z} e^{izxP^z} \langle P|G^z_{\ i}(z)W(z,0;L_{n^z})G^{iz}(0)|P\rangle$$

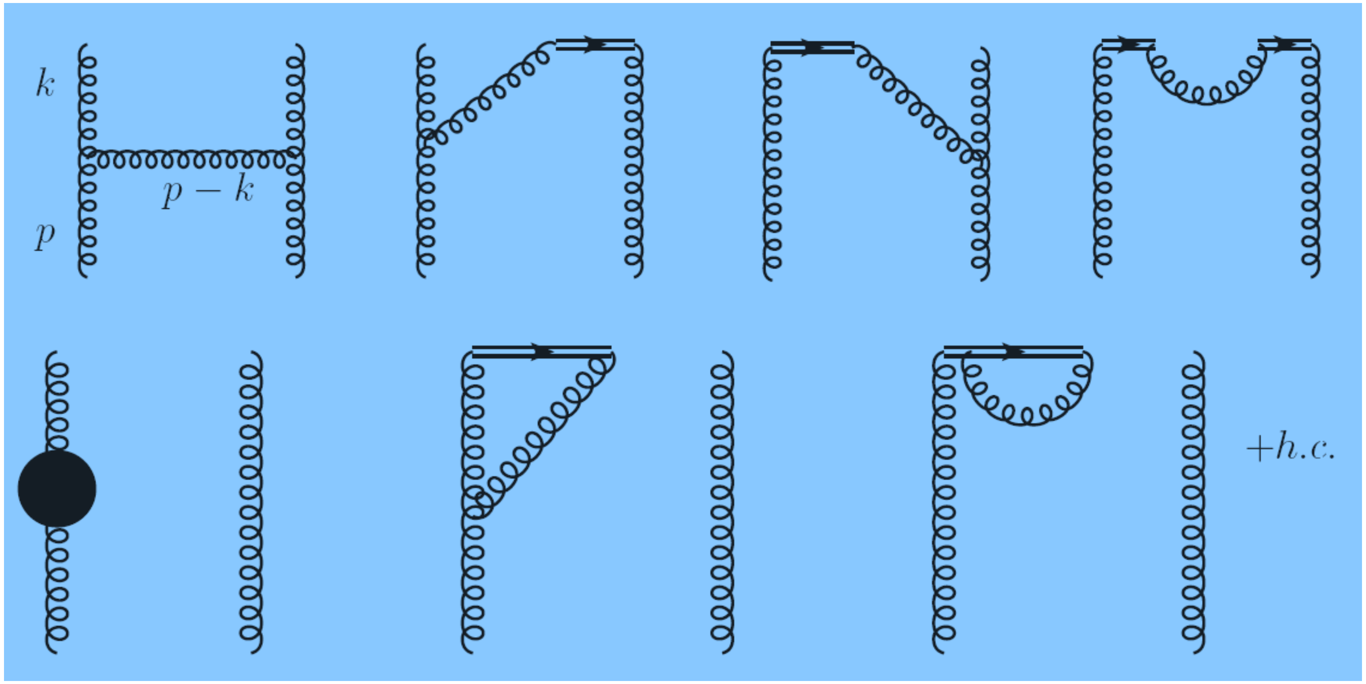
- Field Strength Tensor: G
- i sums over the transverse directions (i=1,2)
- W(z1,z2, C) is a Wilson line along contour C.

Matching for gluon quasi distribution

- Tree level: $\delta(1-x)$

(Wei Wang, Rui-lin Zhu and Shuai Zhao)

- One loop level:



Non-Abelian term has been absorbed

Matching for gluon quasi distribution

- Feynman gauge
- UV Regularization Scheme
  - ✓ First Dimensional Regularization (DR): No Power divergence;
  - ✓ UV cut-off on transverse momentum
- The collinear divergence is regularized by introducing a small gluon mass

In dimensional regularization (DR) scheme:

$$\tilde{g}^{(1)}(x,p_z)=\frac{\alpha_s N_c}{\pi}\begin{cases}\frac{(1-x+x^2)^2}{x(1-x)}\ln\frac{x}{x-1}+x-\frac{3}{4}+\frac{3}{2x}, & x>1\\ \frac{(1-x+x^2)^2}{x(1-x)}\ln\frac{4x(1-x)p_z^2}{(1-x+x^2)m^2} & \\ +\frac{9x}{4}+x^3+\frac{x}{2(x-1)}-\frac{3x}{2(1-x+x^2)}, & 0<x<1\\ \frac{(1-x+x^2)^2}{x(1-x)}\ln\frac{x-1}{x}-x+\frac{3}{4}-\frac{3}{2x}, & x<0\end{cases}$$

$$g^{(1)}(x,\mu)=\frac{\alpha_s N_c}{\pi}\begin{cases}0, & x>1,\quad x<0\\ \frac{(1-x+x^2)^2}{x(1-x)}\ln\frac{\mu^2}{m^2(1-x+x^2)} & \\ +\frac{x+7}{2}-\frac{1}{x(1-x)}-\frac{3}{2(1-x+x^2)}, & 0<x<1\end{cases}$$

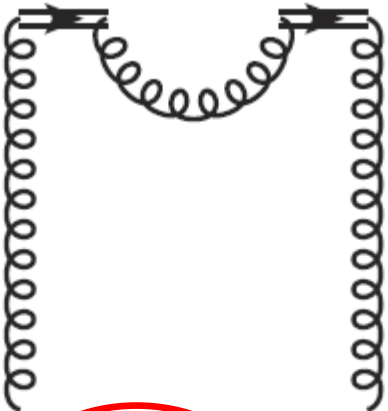
Matching for gluon quasi distribution

$$Z_g\left(\xi, \frac{\mu}{p^z}\right) = \delta(\xi - 1) + \frac{\alpha_s}{\pi} Z_g^{(1)}\left(\xi, \frac{\mu}{p^z}\right) + \cdots$$

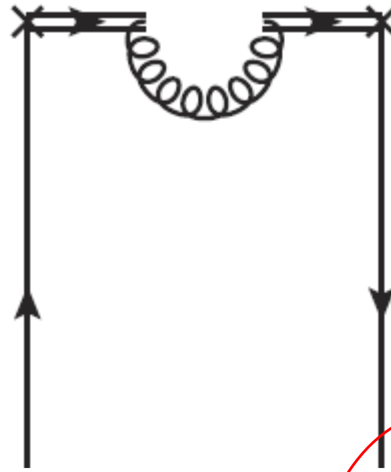
$$Z_g^{(1)}\left(\xi, \frac{\mu}{p^z}\right) = \frac{C_A}{2}\begin{cases} \frac{2(1-\xi^2+\xi^2)^2}{\xi(1-\xi)}\ln\frac{\xi}{\xi-1} + 2\xi - \frac{3}{2} + \frac{3}{\xi}, & \xi > 1, \\ \frac{2(1-\xi+\xi^2)^2}{\xi(1-\xi)}\ln\frac{4\xi(1-\xi)p_z^2}{\mu^2} & \\ + \frac{1}{(1-\xi)} + \frac{3(1-\xi)}{1-\xi+\xi^2} + \frac{2}{\xi} - 6 + \frac{7\xi}{2} + 2\xi^3, & 0 < \xi < 1 \\ -\frac{2(1-\xi^2+\xi^2)^2}{\xi(1-\xi)}\ln\frac{\xi}{\xi-1} - 2\xi + \frac{3}{2} - \frac{3}{\xi}, & \xi < 0, \end{cases}$$

P^z evolution equations: same as DGLAP for light-cone pdf !

Linear Power Divergence

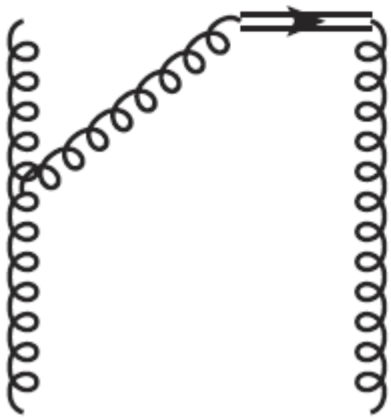


$$= \frac{x^2}{(1-x)^2}\frac{\Lambda}{P^z} + \frac{\alpha_s N_c}{2\pi}\begin{cases} \frac{x}{1-x}, & x > 1 \\ \frac{x}{x-1}, & 0 < x < 1 \\ \frac{x}{x-1}, & x < 0 \end{cases}$$

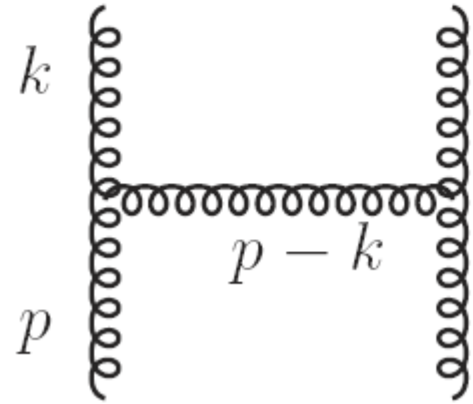


$$= \frac{\alpha_S C_F}{2\pi}\begin{cases} \frac{1}{1-x} + \frac{\Lambda}{(1-x)^2 p^z}, & x > 1 \\ -\frac{1}{1-x} + \frac{\Lambda}{(1-x)^2 p^z}, & 0 < x \\ -\frac{1}{1-x} + \frac{\Lambda}{(1-x)^2 p^z}, & x < 0 \end{cases}$$

Linear Power Divergence



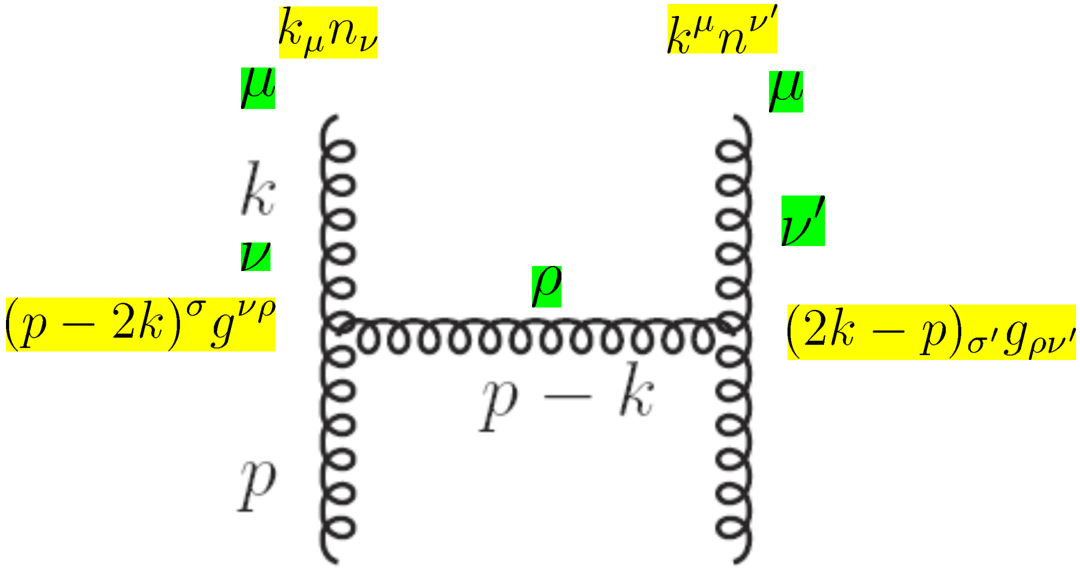
$$= \frac{x}{1-x}\frac{\Lambda}{P^z}$$



$$= \frac{\Lambda}{P^z}$$

All real diagrams have linear UV divergence
even the diagram without any eikonal line !

Linear Power Divergence



$\propto n^2$

- Light-cone: $n^2=0$, no linear power divergence;

- Quasi: $n^2=-1$, the integral contributes a linear power divergence!
- dk_0 d^2k_T*k^4/(k^6)

Rethinking on the definition of quasi gluon PDF

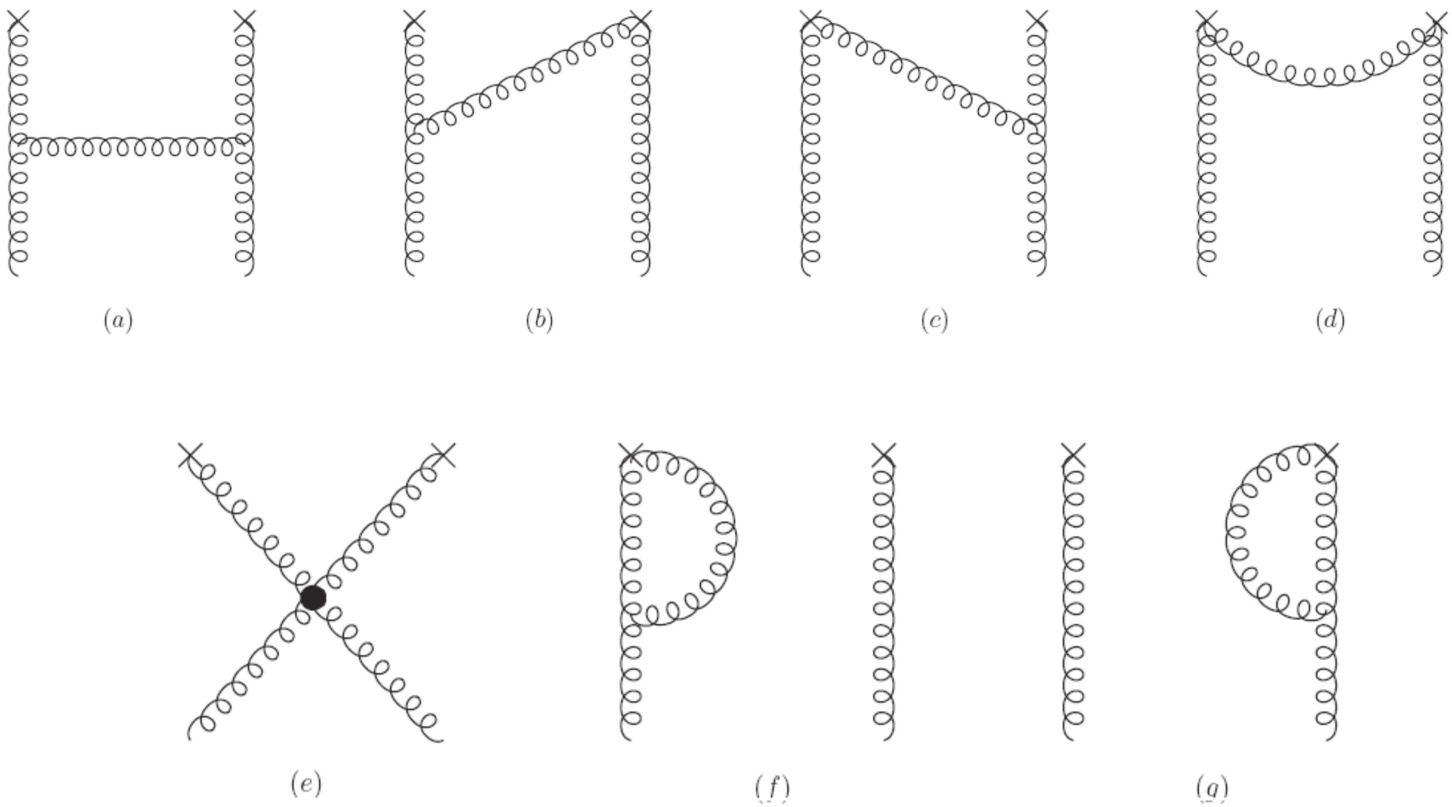
$$\tilde{f}_{g/H}(x,P^z) = \int \frac{dz}{2\pi x P^z} e^{izxP^z} \langle P|G^z{}_i(z)W(z,0;L_{n^z})G^{iz}(0)|P\rangle$$

- A definition of quasi gluon distribution might be

$$\tilde{f}_{g/H}(x,\mu) = \int \frac{dz}{2\pi x P^z} e^{izxP^z} \langle P|G^z{}_\mu(z)W(z,0;L_{n^z})G^{\mu z}(0)|P\rangle \ ,$$

with μ sums over ALL the directions: **μ=(0,1,2).**
Large pz limit, consistent

One loop diagrams without any eikonal line



$$x\tilde{f}^{(1)}_{g/g}(x,P^z,\Lambda)\Big|_{\mathrm{Fig.3}(a)} = \frac{\alpha_s C_A}{2\pi}\begin{cases} \left(2x^3-3x^2+2x-2\right)\ln\frac{x-1}{x}+2x^2-\frac{5x}{2}+3+\frac{\Lambda}{P^z}, & x>1 \\ \left(2x^3-3x^2+2x-2\right)\ln\frac{(1-x+x^2)m_g^2}{4x(1-x)(P^z)^2}+\frac{(2x-1)(4x^4-6x^3+10x^2-5x+2)}{2(1-x+x^2)} \\ +\frac{\Lambda}{P^z}, & 0<x<1 \\ -\left(2x^3-3x^2+2x-2\right)\ln\frac{x-1}{x}-2x^2+\frac{5x}{2}-3+\frac{\Lambda}{P^z}. & x<0 \end{cases}$$

$$x\tilde{f}^{(1)}_{g/g}(x,P^z,\Lambda)\Big|_{\mathrm{Fig.3}(b,c)} = \frac{\alpha_s C_A}{4\pi}\begin{cases} x(x+1)\ln\frac{x-1}{x}+2x-1-\frac{\Lambda}{P^z}, & x>1 \\ x(x+1)\ln\frac{(1-x+x^2)m_g^2}{4x(1-x)(P^z)^2}+2x(x-1)+1-\frac{\Lambda}{P^z}, & 0<x<1 \\ -x(x+1)\ln\frac{x-1}{x}-2x+1-\frac{\Lambda}{P^z}, & x<0 \end{cases}$$

$$x\tilde{f}^{(1)}_{g/g}(x,P^z,\Lambda)\Big|_{\mathrm{Fig.3}(d)} = \frac{\alpha_s C_A}{2\pi}\begin{cases} 1-x+\frac{\Lambda}{P^z}, & x>1 \\ x-1+\frac{\Lambda}{P^z}, & 0<x<1 \\ x-1+\frac{\Lambda}{P^z}. & x<0 \end{cases}$$

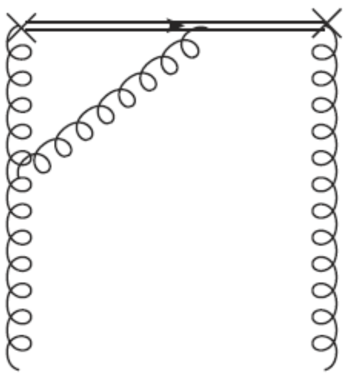
$$x\tilde{f}^{(1)}_{g/g}(x,P^z,\Lambda)\Big|_{\mathrm{Fig.3}(e)} = \frac{\alpha_s C_A}{2\pi}\begin{cases} \frac{x}{2}-\frac{\Lambda}{P^z}, & x>1 \\ \frac{x}{2}-\frac{\Lambda}{P^z}, & 0<x<1 \\ -\frac{x}{2}-\frac{\Lambda}{P^z}. & x<0 \end{cases}$$

No Linear divergence in Virtual diagram

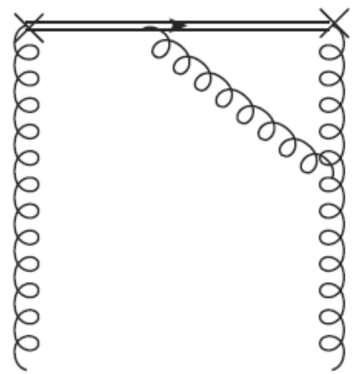
$$x\tilde{f}^{(1)}_{g/g}(x,P^z,\Lambda)\Big|_{\mathrm{Fig.3}(f,g)} = \delta(1-x)\frac{\alpha_s C_A}{4\pi}\int dy \begin{cases} (y+1)\ln\dfrac{y-1}{y}, & x>1 \\ (y+1)\ln\dfrac{(1-y+y^2)m_g^2}{4y(1-y)(P^z)^2}, & 0<x<1 \\ -(y+1)\ln\dfrac{y-1}{y}. & x<0 \end{cases}$$

- Linear divergences cancel between these diagrams!
- The total result is free of linear UV divergence for the diagrams without any Wilson line!

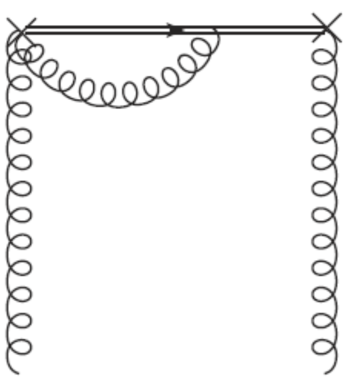
Diagrams with Wilson line(s)



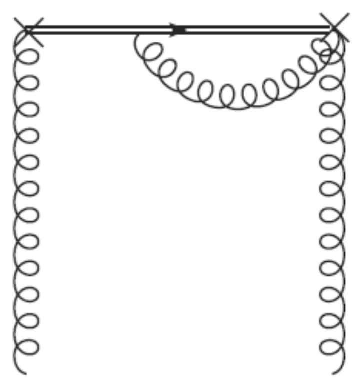
(a)



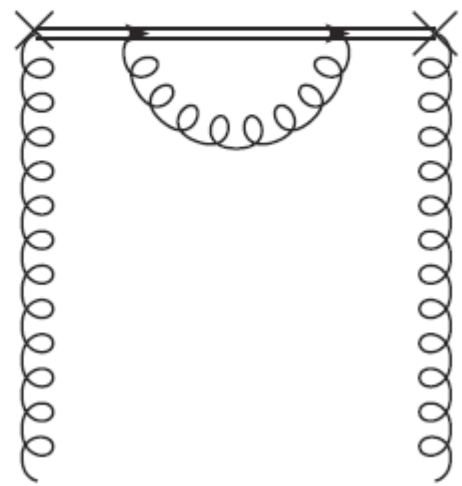
(b)



(c)



(d)



$$x\tilde{f}^{(1)}_{g/g}(x,P^z,\Lambda)\Big|_{\rm Fig.4(a,b)} = \frac{\alpha_s C_A}{4\pi}\begin{cases} \left[\frac{x(x+1)}{1-x}\ln\frac{x}{x-1}+\frac{1-2x}{1-x}+\frac{\Lambda}{(1-x)P^z}\right]_S, & x>1 \\ \left[-\frac{x(x+1)}{1-x}\ln\frac{m_g^2(1-x+x^2)}{4x(1-x)(P^z)^2}-\frac{2x^2-2x+1}{1-x}+\frac{\Lambda}{(1-x)P^z}\right]_S, & 0<x<1 \\ \left[-\frac{x(x+1)}{1-x}\ln\frac{x}{x-1}-\frac{1-2x}{1-x}+\frac{\Lambda}{(1-x)P^z}\right]_S, & x<0 \end{cases}$$

$$x\tilde{f}^{(1)}_{g/g}(x,P^z,\Lambda)\Big|_{\rm Fig.4(c,d)} = \frac{\alpha_s C_A}{2\pi}\begin{cases} \left[-1-\frac{\Lambda}{(1-x)P^z}\right]_S, & x>1 \\ \left[1-\frac{\Lambda}{(1-x)P^z}\right]_S, & 0<x<1 \\ \left[1-\frac{\Lambda}{(1-x)P^z}\right]_S, & x<0 \end{cases}$$

$$x\tilde{f}^{(1)}_{g/g}(x,P^z,\Lambda)\Big|_{\rm Fig.5} = \frac{\alpha_s C_A}{2\pi}\begin{cases} \left[\frac{1}{1-x}+\frac{\Lambda}{(1-x)^2P^z}\right]_S, \\ \left[\frac{1}{x-1}+\frac{\Lambda}{(1-x)^2P^z}\right]_S, \\ \left[\frac{1}{x-1}+\frac{\Lambda}{(1-x)^2P^z}\right]_S. \end{cases}$$

→ Wilson line self energy

Diagrams with Wilson line(s)

- Linear divergence exists in each diagram involving one Wilson line.
- The linear divergences do not cancel between these diagrams.
- Not all the linearly divergent diagram can be explained as Wilson line self energy.

Renormalizing the linear divergence in auxiliary field formalism

Auxiliary field formalism

Gervais and Neveu, 1980

- Wilson line

$$W(z_1, z_2, L_n) = P\exp\left[-ig\int_0^1 d\lambda\ \dot{x}(\lambda)\cdot A(x(\lambda))\right]$$

$$= 1 + \sum_{k=1}^{\infty}\frac{(ig)^k}{k!}\ P\int_C \langle A_{\mu_1}(x_1)\cdots A_{\mu_k}(x_k)\rangle\, dx_1^{\mu_1}\cdots dx_k^{\mu_k}$$

- Introducing auxiliary field Z:

$$W(z_1, z_2; C) = \langle \mathcal{Z}(\lambda_1)\bar{\mathcal{Z}}(\lambda_2)\rangle_z$$

- Heavy quark: static Wilson line

Gauge invariant operators in auxiliary field formulism

- Quark bilinear

$$\bar{\psi}_i(z_2)U_{ij}(z_2, z_1; C)\psi_j(z_1) = \langle (\bar{\psi}(z_1)\mathcal{Z}(\lambda_1))(\bar{\mathcal{Z}}(\lambda_2)\psi(z_2))\rangle_z,$$

- Gluonium operator

$$G^a_{\mu\nu}(z_1)W_{ab}(z_1, z_2; C)G^b_{\rho\sigma}(z_2) = \langle \left(G^a_{\mu\nu}(z_1)\mathcal{Z}_a(\lambda_1)\right)\left(\bar{\mathcal{Z}}_b(\lambda_2)G^b_{\rho\sigma}(z_2)\right)\rangle_z$$

$$= \langle \Omega^{(1)}_{\mu\nu}(z_1)\bar{\Omega}^{(1)}_{\rho\sigma}(z_2)\rangle_z$$

$$\Omega^{(1)}_{\mu\nu}(z_1) = G^a_{\mu\nu}(z_1)\mathcal{Z}(\lambda_1)_a$$

Gauge invariant non-local operators $\longrightarrow$ pairs of gauge invariant composite local operators

Operator mixing

Three operators with the same quantum number

$$\Omega^{(1)}_{\mu\nu} = G^a_{\mu\nu}\mathcal{Z}_a,$$

$$\Omega^{(2)}_{\mu\nu} = \Omega^{(1)}_{\mu\alpha}\frac{\dot{x}_\alpha\dot{x}_\nu}{\dot{x}^2} - \Omega^{(1)}_{\nu\alpha}\frac{\dot{x}_\alpha\dot{x}_\mu}{\dot{x}^2},$$

$$\Omega^{(3)}_{\mu\nu} = |\dot{x}|^{-2}(\dot{x}_\mu A^a_\nu - \dot{x}_\nu A^a_\mu)(D\mathcal{Z})_a,$$

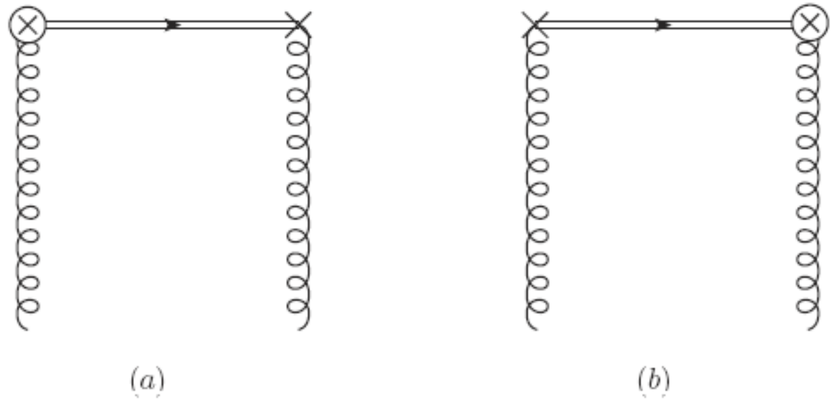
$$(D\mathcal{Z})_a = \partial_\lambda\mathcal{Z}_a + gf_{abc}A_b\mathcal{Z}_c.$$

Dorn, Robaschik, Wieczorek,1981; Dorn,1986

Under renormalization,

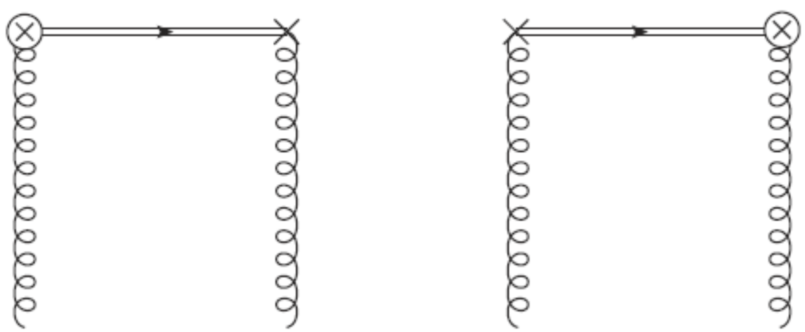
$$\Omega^{(1)}_{\mu\nu} = c_1\Omega^{(1)(r.)}_{\mu\nu} + c_2\Omega^{(2)(r.)}_{\mu\nu} + c_3\Omega^{(3)(r.)}_{\mu\nu}.$$

- $\Omega_{z\mu}^{(3)}$ will contribute linear divergence.



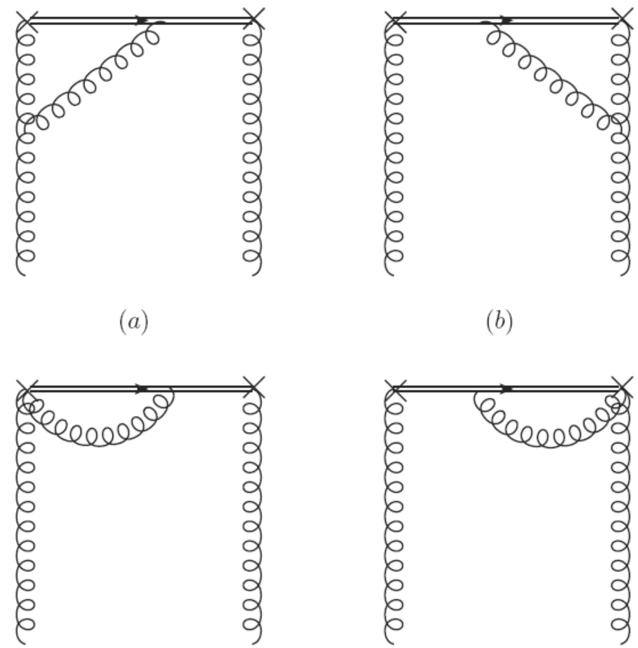
- The same type of divergence!

$$\frac{-\delta m}{\pi P^z}\left(\frac{1}{1-x}\right)_s \qquad \delta m = -\frac{\alpha_s C_A}{2\pi}(\pi\Lambda)$$



(a) (b)

$+$



(a) (b) (c) (d)

$= O(\Lambda^0)$

$$c_3 = 1/2.$$

The improved quasi gluon distribution

- With the discussions above, we propose a modified definition of quasi gluon distribution as

$$x\tilde{f}^{(r.)}_{g/H}(x,P^z,\Lambda)=\int\frac{dz}{2\pi P^z}e^{ixzP^z-\delta m|z|}\left\langle P\left|\left(\Omega^{(1)z}{}_{\mu}+\frac{1}{2}\Omega^{(3)z}{}_{\mu}\right)(z)\left(\Omega^{(1)\mu z}+\frac{1}{2}\bar{\Omega}^{(3)\mu z}\right)(0)\right|P\right\rangle$$

- δm and matrix element of Ωs can be determined on lattice non-perturbatively.
- Matching equation

$$x\tilde{f}^{(r.)}_{g/H}(x,P^z,\Lambda)=\int_0^1\frac{dy}{y}Z_{gi}\left(\frac{x}{y},\frac{\Lambda}{P^z}\right)yf_{i/H}(y,\Lambda)$$

Matching Coefficient: New Results

$$Z_{gg}\left(\xi,\frac{\Lambda}{P^z}\right)=\sum_{i=0}^{\infty}\left(\frac{\alpha_s}{2\pi}\right)^n Z_{gg}^{(n)}\left(\xi,\frac{\Lambda}{P^z}\right)$$

- At one loop level,

$$Z_{gg}^{(1)}\left(\xi,\frac{\Lambda}{P^z}\right)=C_A\begin{cases}(2\xi^3-2\xi^2+4\xi-1)\ln\frac{\xi-1}{\xi}+\left[\frac{\xi+1}{\xi-1}\ln\frac{\xi-1}{\xi}\right]_S+2\xi^2-2\xi+3, & \xi>1\\ (2\xi^3-2\xi^2+4\xi-1)\ln\frac{\Lambda^2}{4\xi(1-\xi)(P^z)^2}+\left[\frac{\xi+1}{\xi-1}\ln\frac{\Lambda^2}{4\xi(1-\xi)(P^z)^2}\right]_S \\ +\left[\frac{2\xi^2}{\xi-1}\right]_S+2\xi^3+\xi+1, & 0<\xi<1\\ -(2\xi^3-2\xi^2+4\xi-1)\ln\frac{\xi-1}{\xi}-\left[\frac{\xi+1}{\xi-1}\ln\frac{\xi-1}{\xi}\right]_S-2\xi^2+2\xi-3. & \xi<0\end{cases}$$

Free of linear UV divergence.

Free of collinear divergence.

Summary

- Quasi distribution: useful tools for parton physics
- Analysis of gluon quasi distribution at 1-loop
- Linear power divergence: cannot be removed as quark
- Auxiliary Field Formalism: new operators
- More on power divergence subtraction: Renormalizability? Lattice perturbation theory?
- Lattice QCD calculation: additional 3-5 years

Thank you very much!