QCD group meeting

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Dilepton Angular Distributions of Drell-Yan Process in Geometric Picture and pQCD

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collaborating with

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The Drell-Yan Process

S.D. Drell and T.M. Yan, PRL 25 (1970) 316

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s \to \infty$, Q^2/s finite, Q^2 and s being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as $Q^2/s \to 1$ is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function νW_2 near threshold.

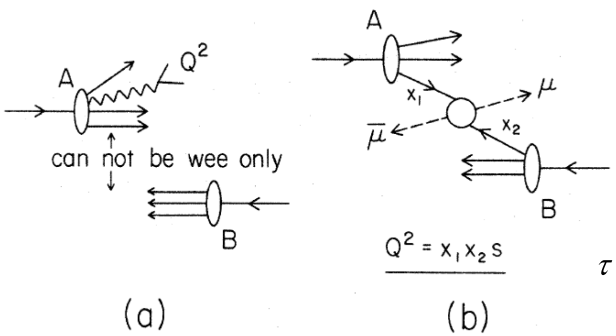


FIG. 1. (a) Production of a massive pair Q^2 from one of the hadrons in a high-energy collision. In this case it is kinematically impossible to exchange "wee" partons only. (b) Production of a massive pair by parton-antiparton annihilation.

$$\tau = \frac{Q^2}{s} = x_1 x_2$$

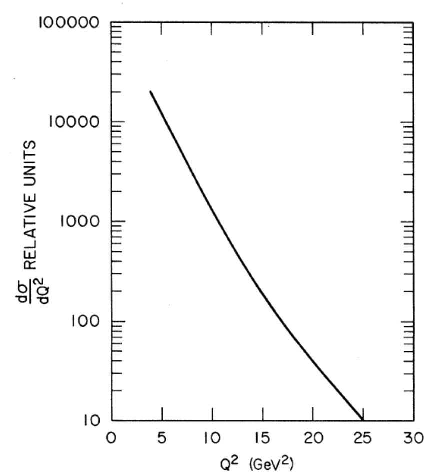
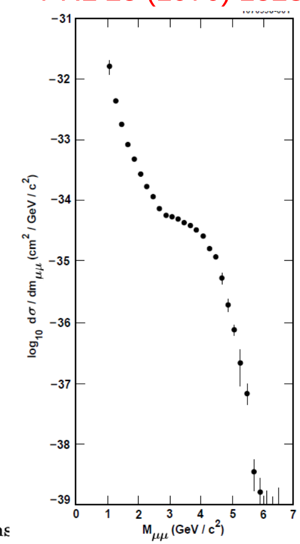


FIG. 2. $d\sigma/dQ^2$ computed from Eq. (10) assuming identical parton and antiparton momentum distributions and with relative normalization.

PRL 25 (1970) 1523



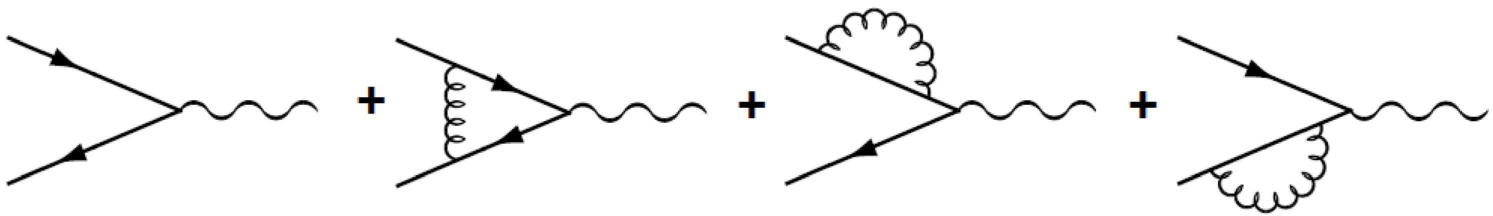
$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2}{3Q^2}\right)\left(\frac{1}{Q^2}\right)\mathfrak{F}(\tau) = \left(\frac{4\pi\alpha^2}{3Q^2}\right)\left(\frac{1}{Q^2}\right)\int_0^1 dx_1 \int_0^1 dx_2\, \delta(x_1 x_2 - \tau) \sum_a \lambda_a^{-2} F_{2a}(x_1) F_{2\bar{a}}{}'(x_2),$$

Angular Distribution in the “Naïve” Drell-Yan

VOLUME 25, NUMBER 5 PHYSICAL REVIEW LETTERS 3 AUGUST 1970

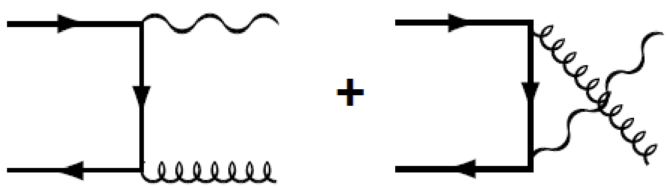
(3) The virtual photon will be predominantly transversely polarized if it is formed by annihilation of spin-$\frac{1}{2}$ parton-antiparton pairs. This means a distribution in the di-muon rest system varying as $(1+\cos^2\theta)$ rather than $\sin^2\theta$ as found in Sakurai's[10] vector-dominance model, where θ is the angle of the muon with respect to the time-like photon momentum. The model used in Fig.

Drell-Yan Process with QCD Effect



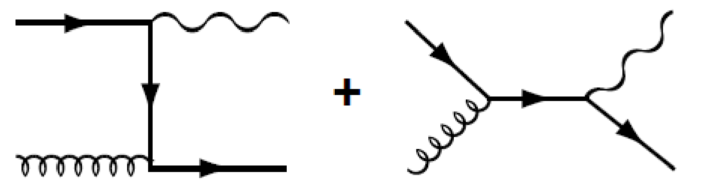
(a)

Quark-antiquark annihilation



(b)

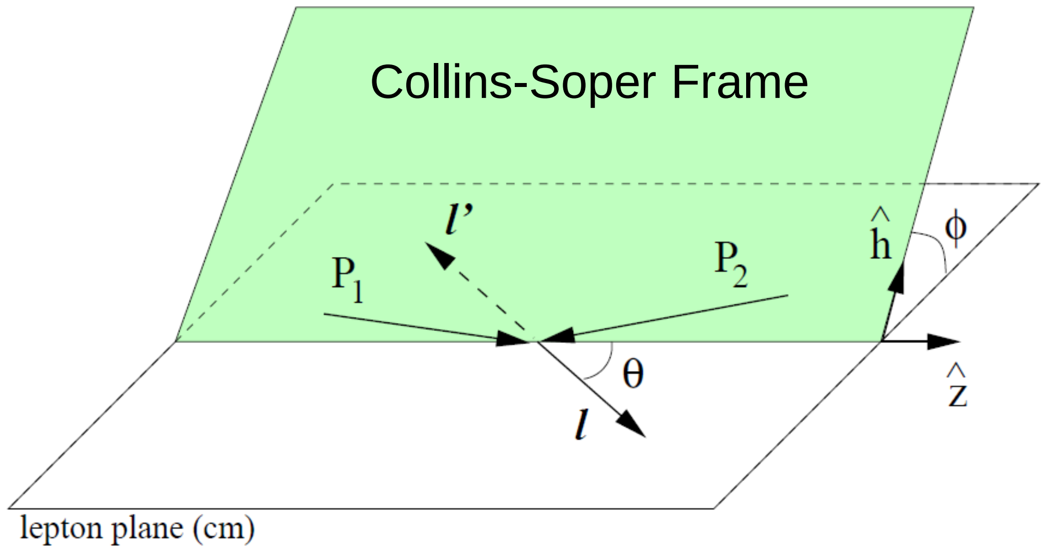
Quark-antiquark annihilation



(c)

Quark-gluon Compton scattering

Angular Distributions of Lepton Pairs



$$\lambda = \frac{2-3A_0}{2+A_0}$$

$$\mu = \frac{2A_1}{2+A_0}$$

$$\nu = \frac{2A_2}{2+A_0}$$

$$\frac{d\sigma}{d\Omega} \propto (1+\lambda\cos^2\theta+\mu\sin2\theta\cos\phi+\frac{\nu}{2}\sin^2\theta\cos2\phi)$$

$$\propto [(1+\cos^2\theta)+\frac{A_0}{2}(1-3\cos^2\theta)+A_1\sin2\theta\cos\phi+\frac{A_2}{2}\sin^2\theta\cos2\phi]$$

$q\bar{q}$ annilation parton model: $O(\alpha_s^0)$ $\lambda=1,\ \mu=\nu=0;\ A_0=A_2=0$

pQCD: $O(\alpha_s^1)$, ; $1-\lambda-2\nu=\frac{4(A_0-A_2)}{2+A_0}=0$; $A_0=A_2$

Lam-Tung Relation [PRD 18 (1978) 2447]

Angular Distributions of Lepton Pairs from Z/γ^*

$$\frac{d\sigma}{d\Omega} \propto [(1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta) + A_1 \sin 2\theta \cos\phi + \frac{A_2}{2}\sin^2\theta\cos 2\phi$$
$$+ A_3 \sin\theta\cos\phi + A_4 \cos\theta$$
$$+ A_5 \sin^2\theta \sin 2\phi + A_6 \sin 2\theta \sin\phi + A_7 \sin\theta\sin\phi]$$

$A_3, A_4 : \gamma^* / Z$ interference, sensitive to $\sin^2\theta_W$

A_5, A_6, A_7:=0, up to $O(\alpha_s^1)$

Angular Distribution

I.R. Kenyon, Rep. Prog. Phys. 45 (1982) 1261

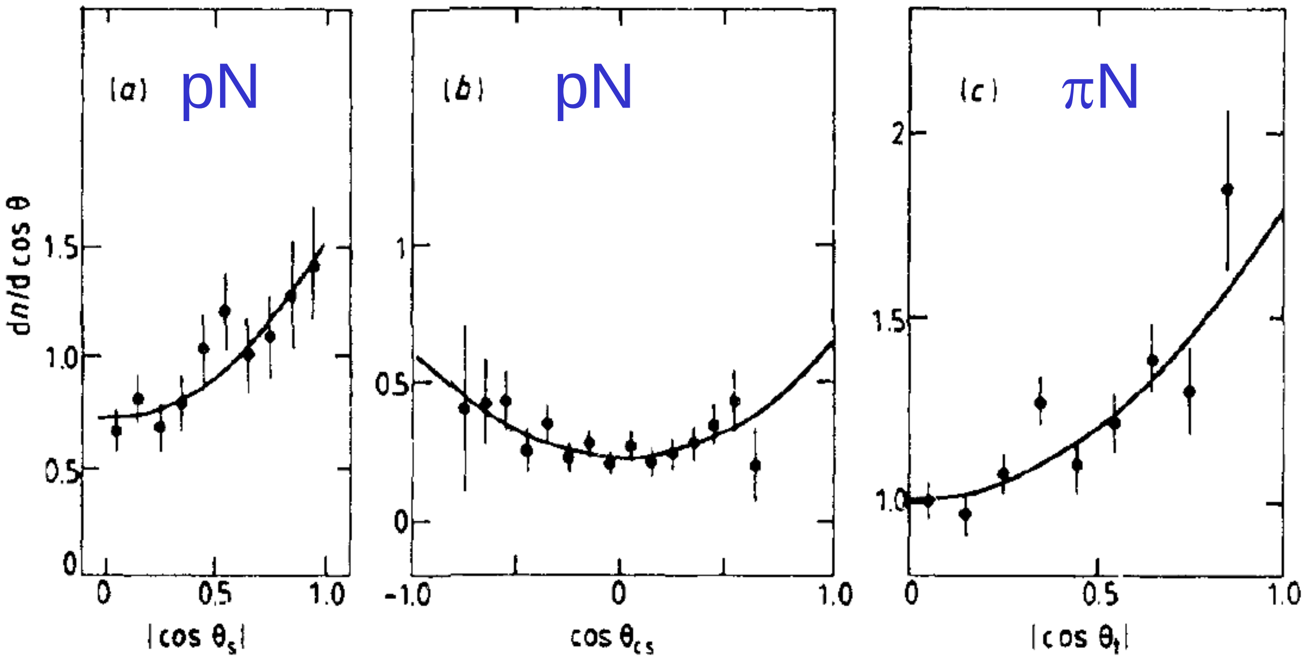


Figure 17. Measurements of the decay angular distribution of lepton pairs by Kourkoumelis *et al* (1980), Antreasyan *et al* (1980) and Badier *et al* (1980a). Fits to the form $1+\alpha \cos^2\theta$ are shown as full curves and are discussed in the text. (*a*) ISR ABCS, $4.5<M<8.7$ GeV, (*b*) ISR CHFMNP, $6<M<8$ GeV, (*c*) NA3, π^- 200 GeV, $4<M<6$ GeV, $p_t<1$ GeV.

$$d\sigma(\Omega) \propto (1+\cos^2\theta)$$

NA10 @ CERN: Violation of LT Relation

Z. Phys. 37 (1988) 545

Lam-Tung relation: $1-\lambda-2\nu=0$

π^{-}+W

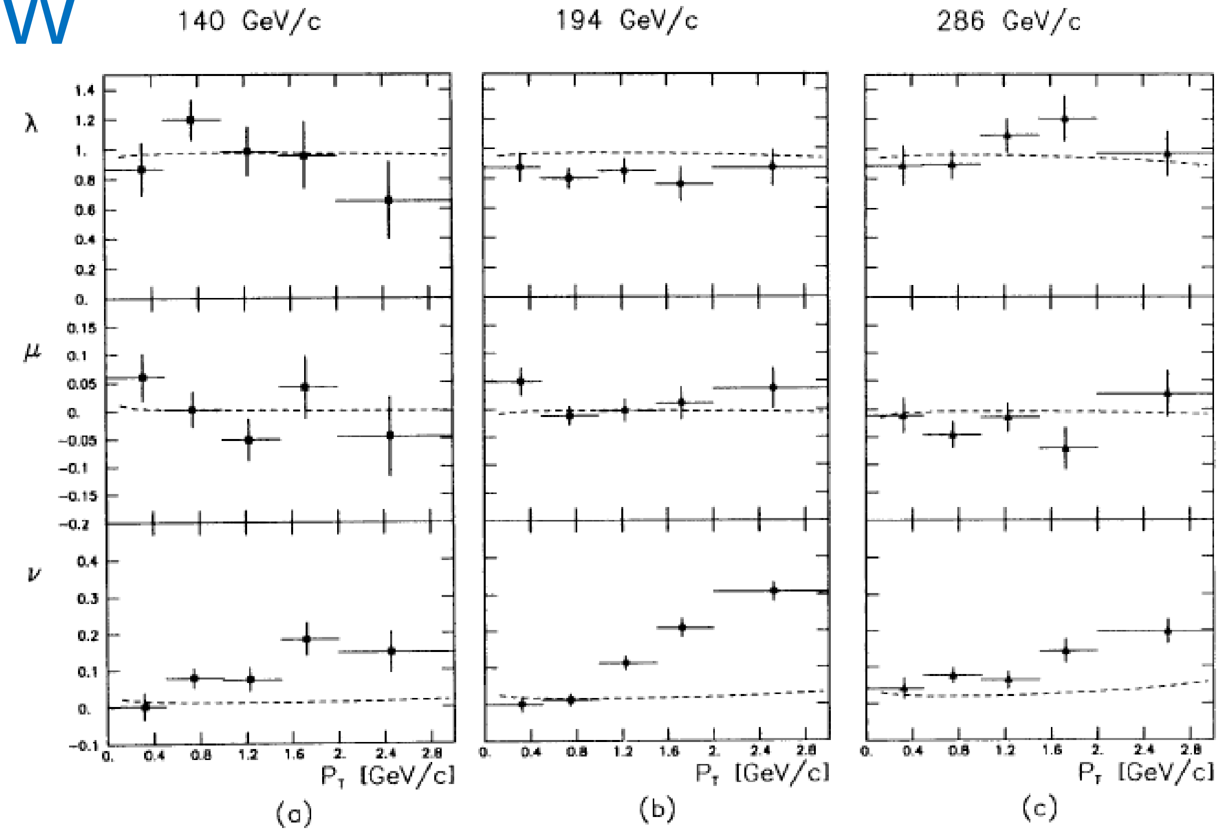


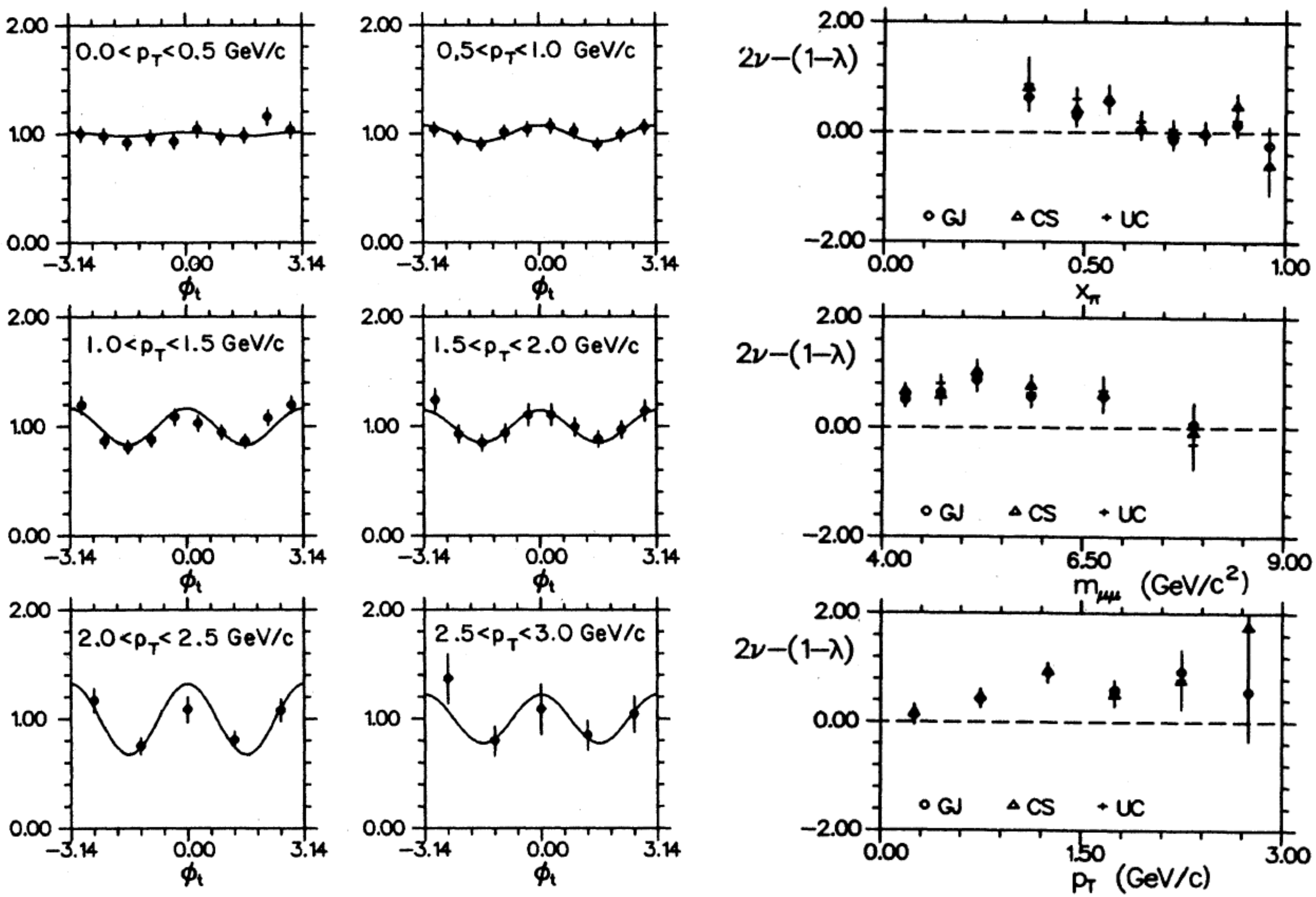
Fig. 3a–c. Parameters λ, μ, and ν as a function of P_T in the CS frame. **a** 140 GeV/c; **b** 194 GeV/c; **c** 286 GeV/c. The error bars correspond to the statistical uncertainties only. The horizontal bars give the size of each interval. The dashed curves are the predictions of perturbative QCD [3]

$\nu \neq 0$ and ν increases with p_T

E615 @ FNAL: Violation of LT Relation

PRD 39, 92 (1989)

252-GeV $\pi^- + W$

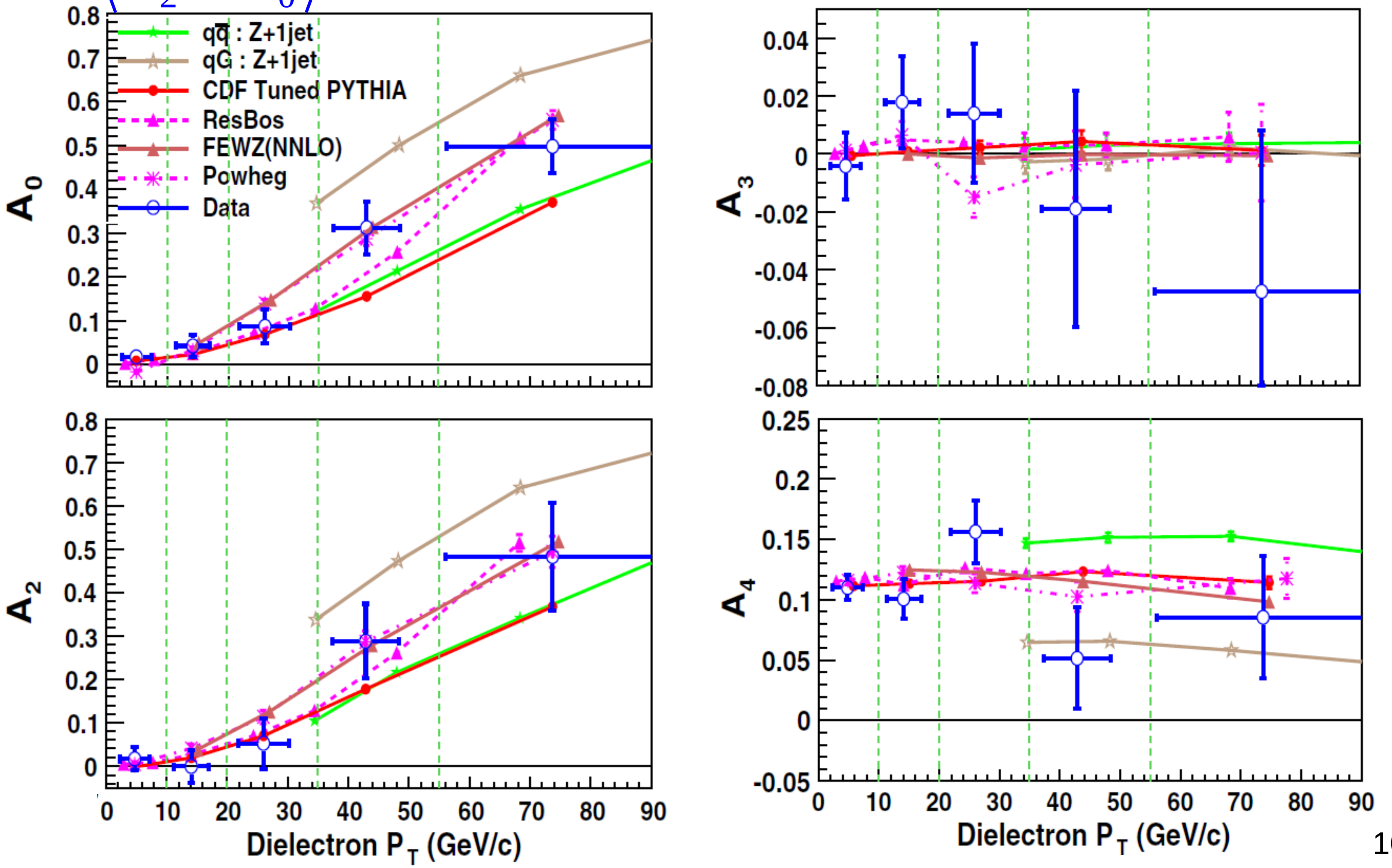


$\cos 2\phi$ modulation at large p_T $\qquad 1-\lambda-2\nu \neq 0$

Angular Distributions of Z Production

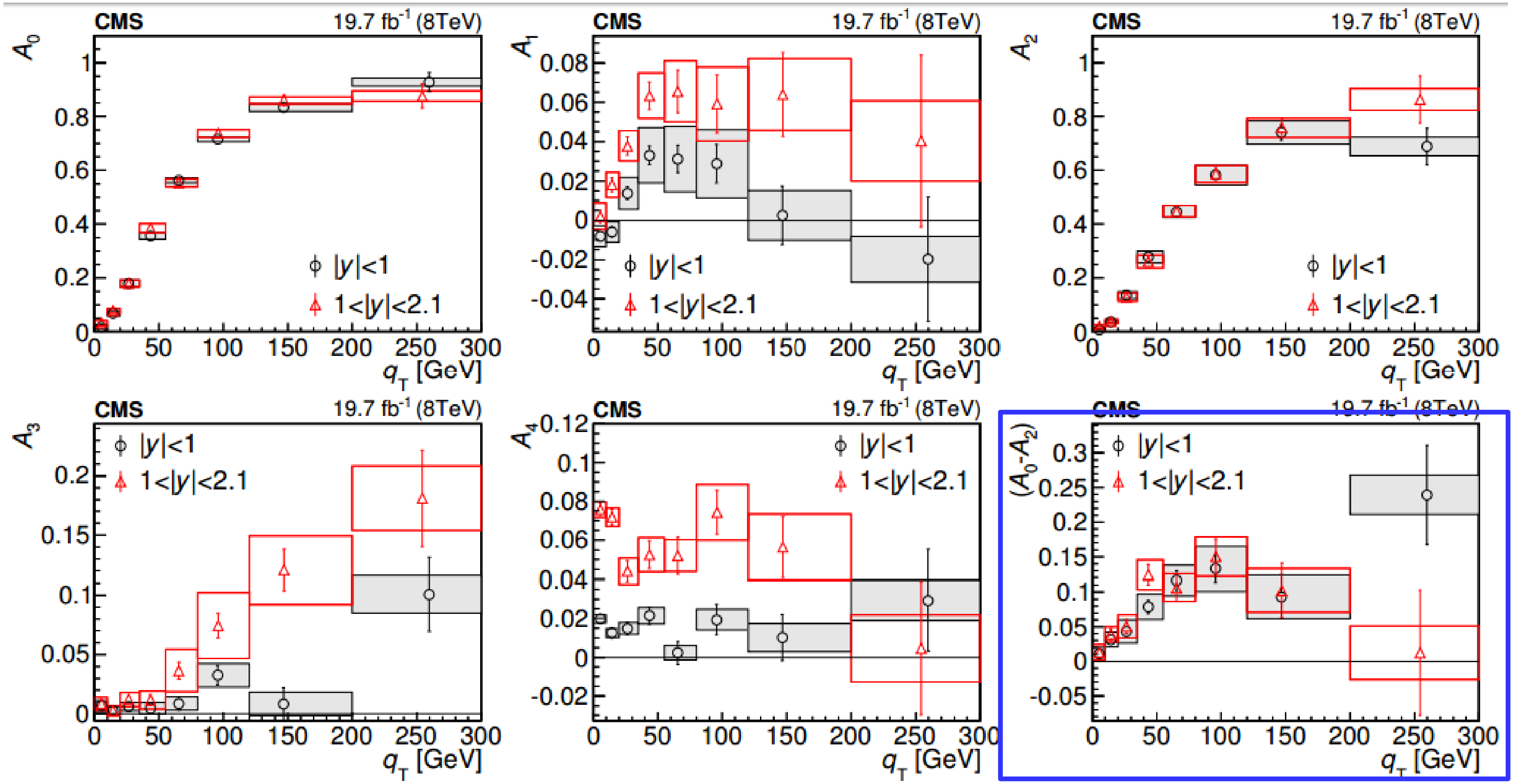
CDF, PRL 106, 241801 (2011)

$$\langle A_2 - A_0 \rangle = 0.02 \pm 0.02$$



Angular Distributions of Z Production

CMS, PLB750, 154 (2015)



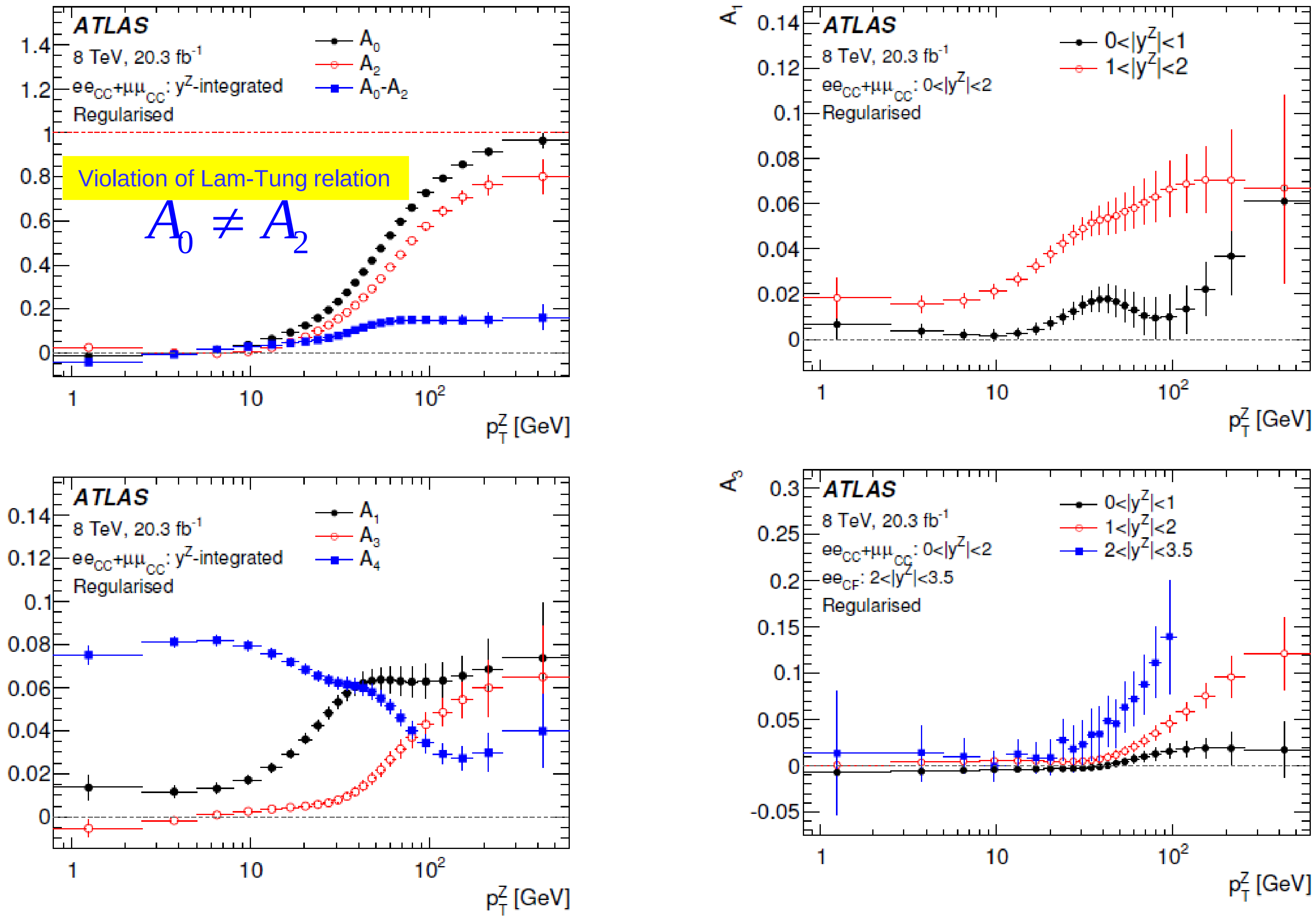
$$\frac{d^2\sigma}{d\cos\theta^* d\phi^*} \propto \Big[(1+\cos^2\theta^*) + A_0\frac{1}{2}(1-3\cos^2\theta^*) + A_1\sin(2\theta^*)\cos\phi^* + A_2\frac{1}{2}\sin^2\theta^*\cos(2\phi^*)$$
$$+A_3\sin\theta^*\cos\phi^* + A_4\cos\theta^* + A_5\sin^2\theta^*\sin(2\phi^*) + A_6\sin(2\theta^*)\sin\phi^* + A_7\sin\theta^*\sin\phi^*\Big].$$

Violation of Lam-Tung relation

$A_0 \neq A_2$

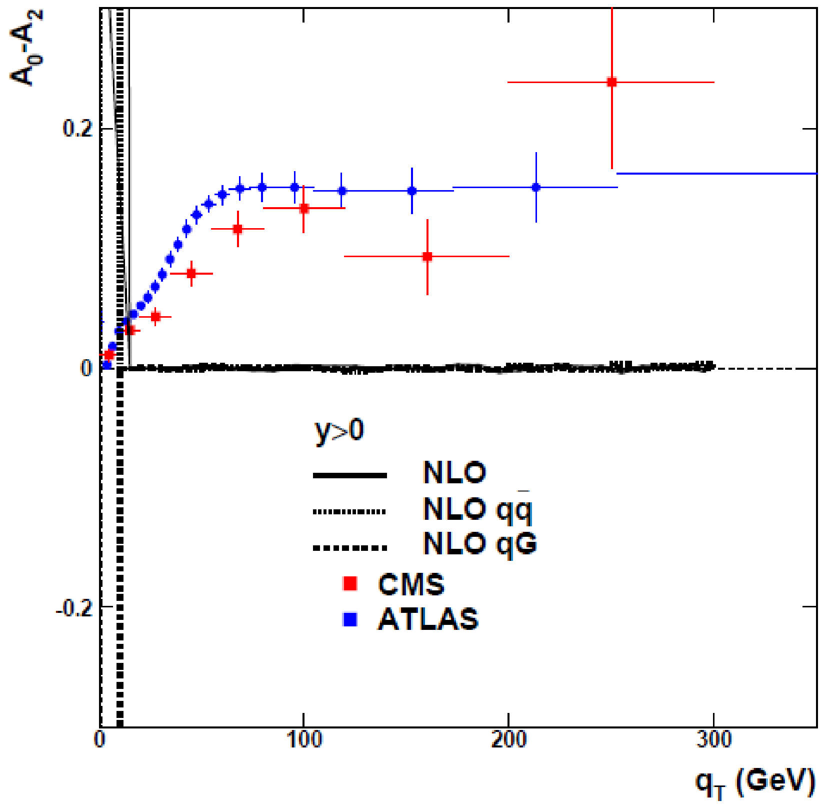
Angular Distributions of Z Production

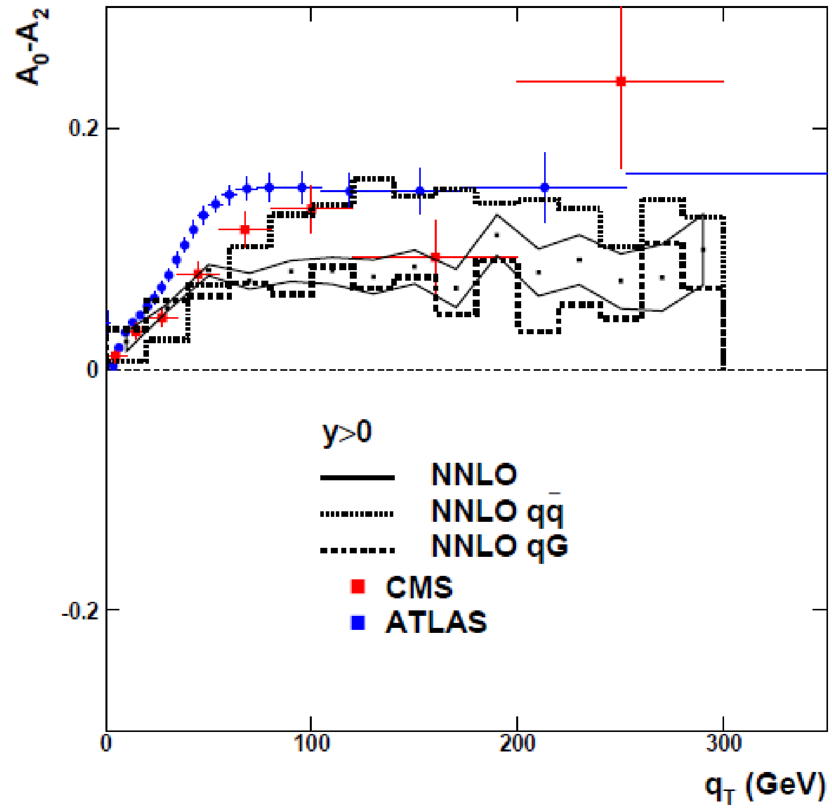
ATLAS, JHEP08, 159 (2016)



LHC: $A_0 - A_2$, NLO vs. NNLO

p+p at $\sqrt{s}$= 8 TeV

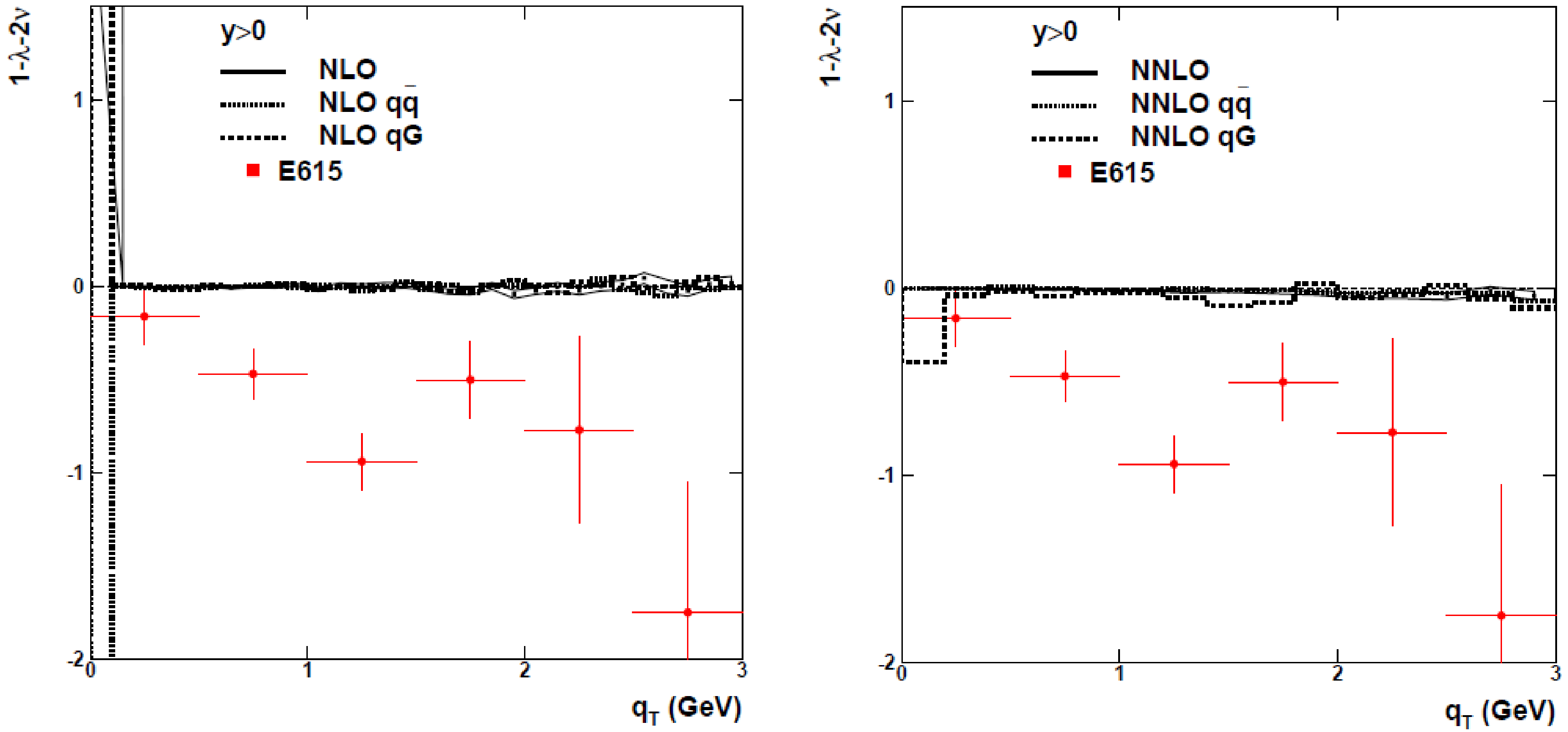




$A_0 - A_2$ can be described by fixed-order NNLO QCD.

E615: $1 - \lambda - 2\nu$, NLO vs. NNLO

252-GeV π^-+W



$1 - \lambda - 2\nu$ cannot be described by fixed-order NLO and NNLO QCD.

Theoretical Interpretations of Lam-Tung Violation in pion-induced DY

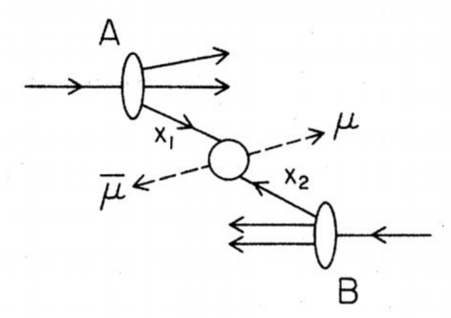
	Boer-Mulders Function	QCD chromo-magnetic effect	Glauber gluon
Origin of effect	Hadron	QCD vacuum	Pion specifically
Quark-flavor dependence	Yes	No	No
Hadron dependence	Yes	No	Yes
Large P_T limit	0	Nonzero	0
Reference	PRD 60, 014012 (1999)	Z. Phy. C 60,697 (1993)	PLB 726, 262 (2013)

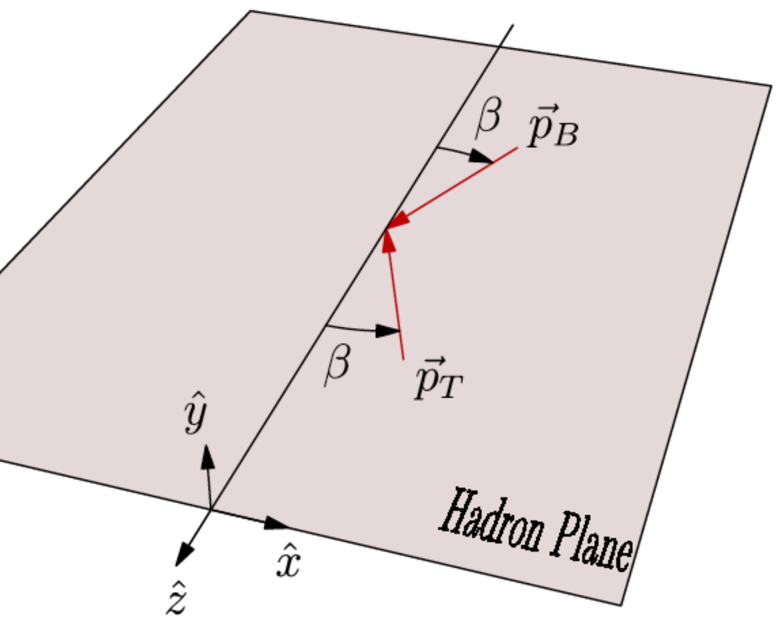
Measurements with different beams $\pi^{\pm}, p, K^{\pm}, \bar{p}$ over wide kinematical ranges would help differentiating the origin.

Observations and Interpretations of Lam-Tung Violation

- Lam-Tung Violation: $(1-\lambda-2\nu \neq 0\ ;\ A_0 \neq A_2)$
  - Small q_T:
    - Intrinsic partonic transverse momentum k_T
    - TMD Boer-Mulders functions (Boer 1999) or non-pert. effects
  - Large q_T:
    - Hard multi-gluon radiation ($O(\alpha_s^2)$ or higher)
- A simple and intuitive interpretation of Lam-Tung Violation: the geometric picture
  - *J.C. Peng, W.C. Chang, R.E. McClellan, O. Teryaev Phys. Lett. B 758, 394 (2016).*
  - *W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev Phys. Rev. D 96, 054020 (2017).*

How is the angular distribution expression derived?





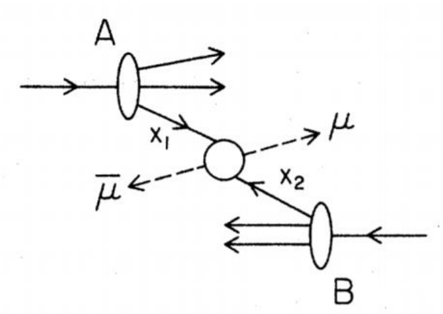
Collins-Soper (γ^*/Z rest) Frame

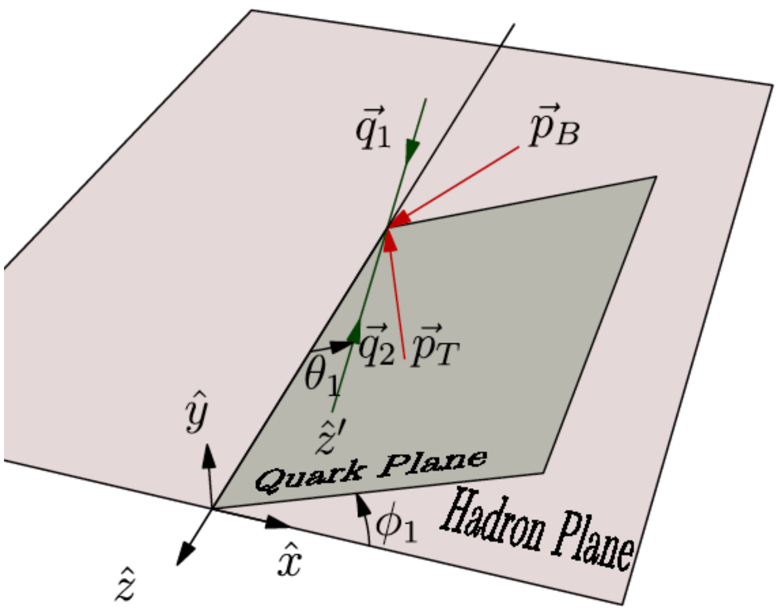
<u>Define three planes in the Collins-Soper frame</u>

1) Hadron Plane ($\vec{P}_B \times \vec{P}_T$)

- Contains the beam $\vec{P}_B$ and target $\vec{P}_T$ momenta
- Angle β satisfies the relation $\tan\beta = q_T / Q$

How is the angular distribution expression derived?





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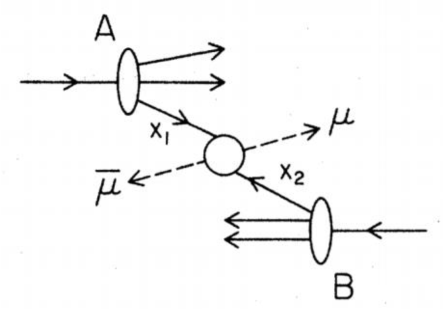
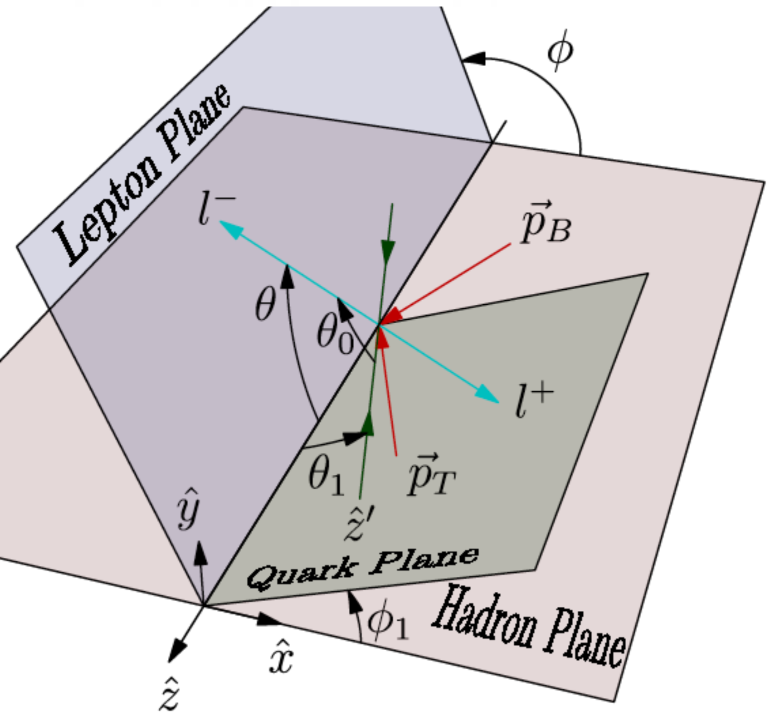
2) Quark Plane ($\hat{z}' \times \hat{z}$)

- q and $\bar{q}$ have head-on collision along the $\hat{z}'$ axis
- $\hat{z}'$ axis has angles θ_1 and φ_1 in the C-S frame

How is the angular distribution expression derived?





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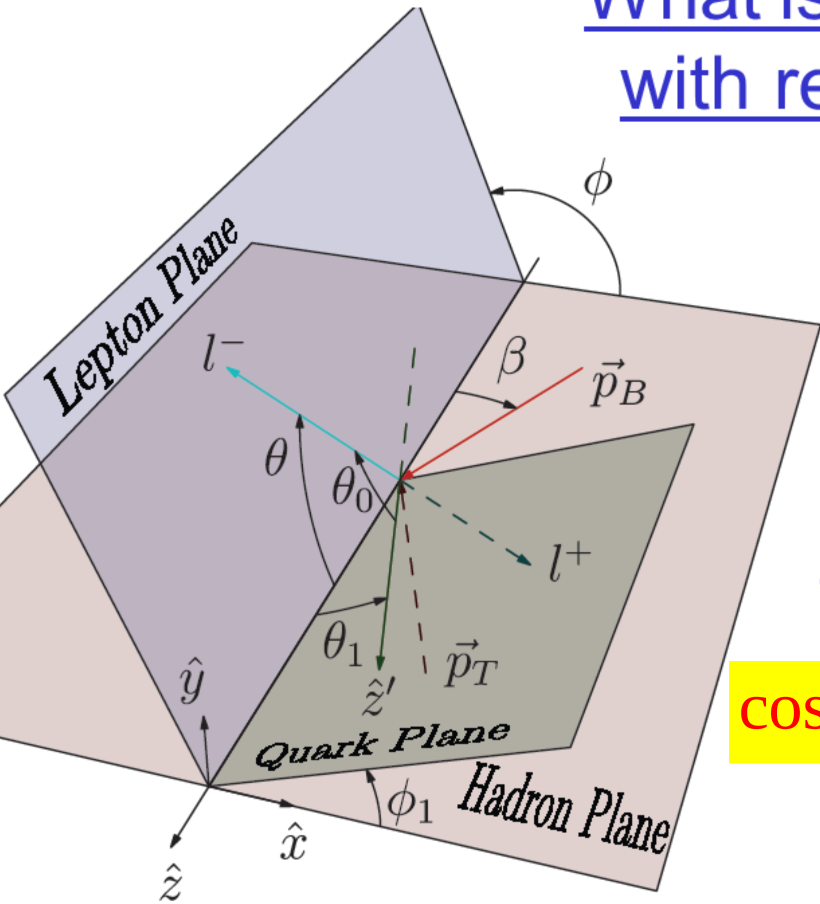
3) Lepton Plane $(\overrightarrow{l^-} \times \hat{z})$

- l^- and l^+ are emitted back-to-back with equal $|\vec{P}|$
- l^- is emitted at angle θ and φ in the C-S frame

How is the angular distribution expression derived?



What is the lepton angular distribution with respect to the $\hat{z}'$ (natural) axis?

$$\frac{d\sigma}{d\Omega} \propto 1 + a\cos\theta_0 + \cos^2\theta_0$$

a: forward-backward asymmetry coefficient

How to express the angular distribution in terms of θ and Φ?

$$\cos\theta_0 = \cos\theta\cos\theta_1 + \sin\theta\sin\theta_1\cos(\phi - \phi_1)$$

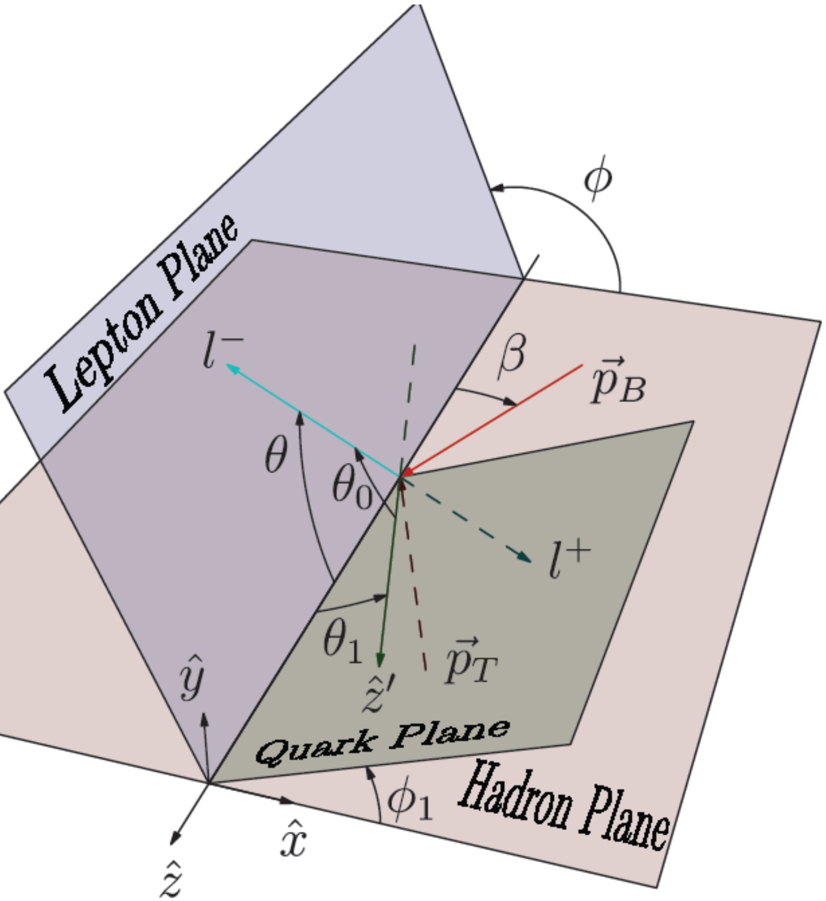
How is the angular distribution expression derived?

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{\sin^2\theta_1}{2}(1-3\cos^2\theta)$$
$$+ (\frac{1}{2}\sin 2\theta_1 \cos\phi_1)\sin 2\theta\cos\phi$$
$$+ (\frac{1}{2}\sin^2\theta_1 \cos 2\phi_1)\sin^2\theta\cos 2\phi$$
$$+ (a\sin\theta_1\cos\phi_1)\sin\theta\cos\phi + (a\cos\theta_1)\cos\theta$$
$$+ (\frac{1}{2}\sin^2\theta_1 \sin 2\phi_1)\sin^2\theta\sin 2\phi$$
$$+ (\frac{1}{2}\sin 2\theta_1 \sin\phi_1)\sin 2\theta\sin\phi$$
$$+ (a\sin\theta_1\sin\phi_1)\sin\theta\sin\phi.$$

$$\frac{d\sigma}{d\Omega} \propto (1+\cos^2\theta) + \frac{A_0}{2}(1-3\cos^2\theta)$$
$$+ A_1\sin 2\theta\cos\phi$$
$$+ \frac{A_2}{2}\sin^2\theta\cos 2\phi$$
$$+ A_3\sin\theta\cos\phi + A_4\cos\theta$$
$$+ A_5\sin^2\theta\sin 2\phi$$
$$+ A_6\sin 2\theta\sin\phi$$
$$+ A_7\sin\theta\sin\phi$$

$A_0 - A_7$ are entirely described by θ_1, φ_1 and a.

Angular distribution coefficients $A_0 - A_7$



$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

Some implications of the angular distribution coefficients $A_0 - A_7$

$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

$$A_3 = a\langle \sin\theta_1 \cos\phi_1 \rangle$$

$$A_4 = a\langle \cos\theta_1 \rangle$$

$$A_5 = \frac{1}{2}\langle \sin^2\theta_1 \sin 2\phi_1 \rangle$$

$$A_6 = \frac{1}{2}\langle \sin 2\theta_1 \sin\phi_1 \rangle$$

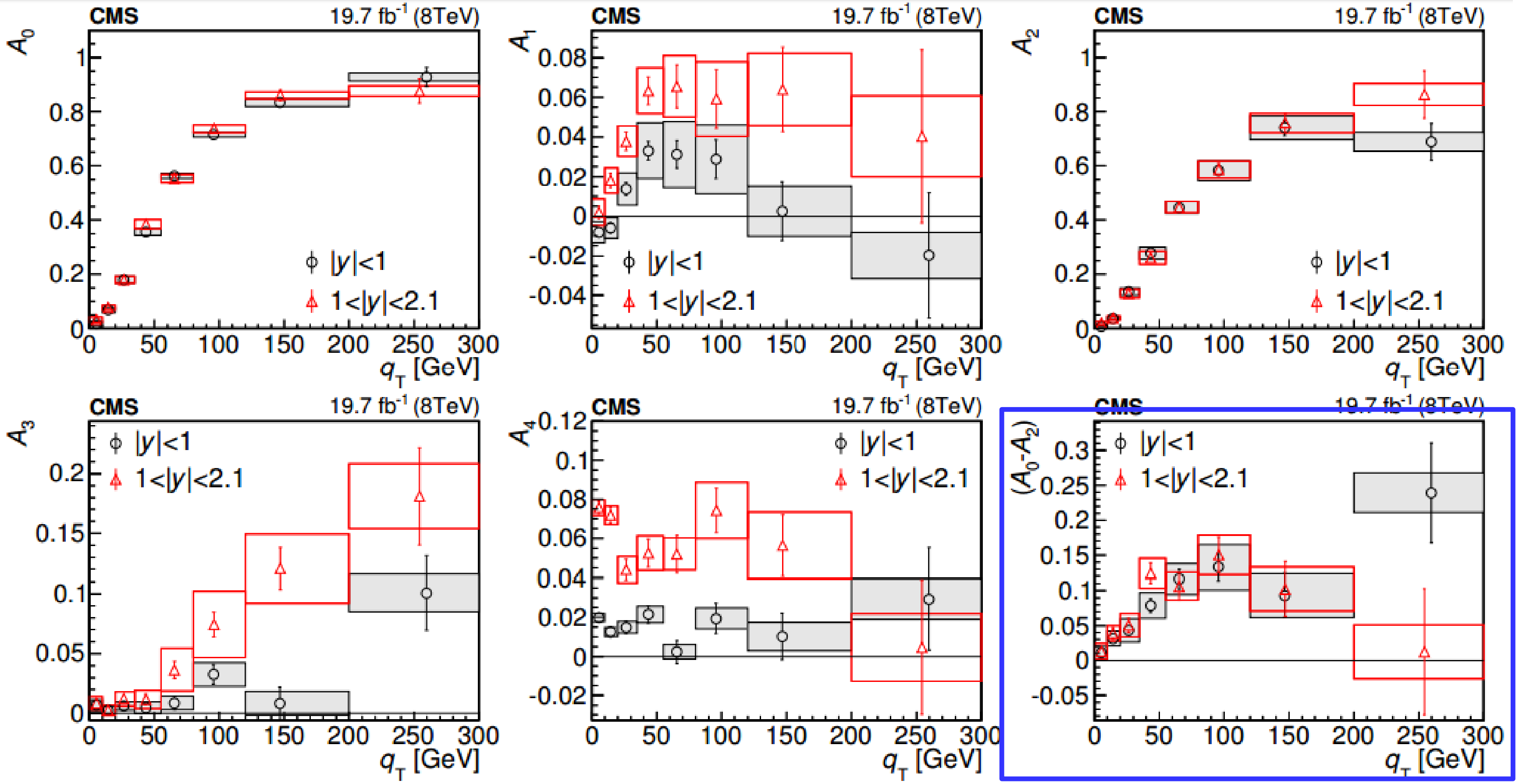
$$A_7 = a\langle \sin\theta_1 \sin\phi_1 \rangle$$

$$\tan\theta_1 = q_T / Q$$

- $A_0 \geq A_2$ (or $1-\lambda-2\nu \geq 0$)
- Lam-Tung relation ($A_0 = A_2$) is satisfied when $\phi_1 = 0$.
- Forward-backward asymmetry, a, is reduced by a factor of $\langle \cos\theta_1 \rangle$ for A_4.
- A_5, A_6, A_7 are odd functions of ϕ_1 and must vanish from symmetry consideration.
- A_0, A_2 and A_3 increase with q_T monotonically while A_4 decreases with q_T monotonically.
- A_1 ($\propto \langle \sin 2\theta \rangle$) first increases with q_T, reaching a maximum, and then decrease.
- Some equality and inequality relations among $A_0 - A_7$ can be obatined.

Angular Distributions of Z Production

CMS, PLB750, 154 (2015)



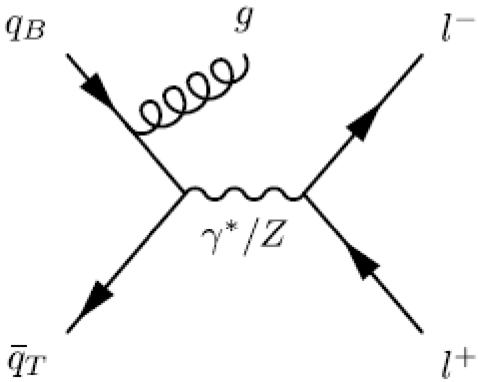
$$\frac{d^2\sigma}{d\cos\theta^* d\phi^*} \propto \Big[(1+\cos^2\theta^*) + A_0\frac{1}{2}(1-3\cos^2\theta^*) + A_1\sin(2\theta^*)\cos\phi^* + A_2\frac{1}{2}\sin^2\theta^*\cos(2\phi^*) + A_3\sin\theta^*\cos\phi^* + A_4\cos\theta^* + A_5\sin^2\theta^*\sin(2\phi^*) + A_6\sin(2\theta^*)\sin\phi^* + A_7\sin\theta^*\sin\phi^*\Big].$$

Violation of Lam-Tung relation

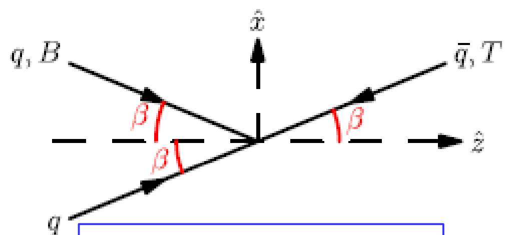
$A_0 \neq A_2$

θ_1 and ϕ_1 at NLO(α_s^1): $q\bar{q} \to \gamma^*/Z g$

Collins-Soper (γ^*/Z rest) Frame

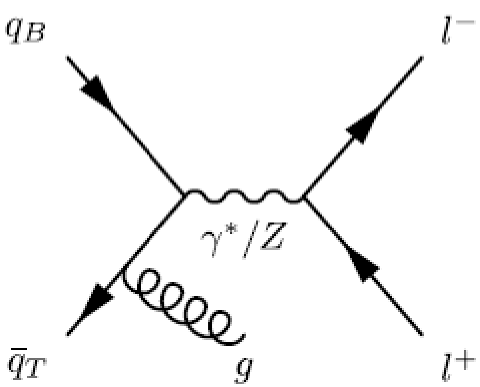


(a)

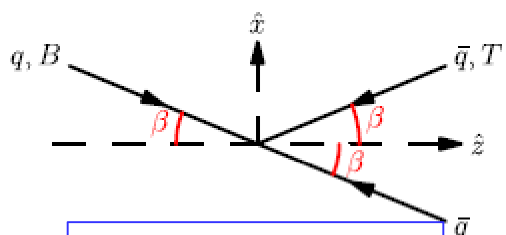


$$\theta_1 = \beta;\ \phi_1 = 0$$

(b)



(c)



$$\theta_1 = \beta;\ \phi_1 = \pi$$

(d)

$$A_0^{q\bar{q}} = \frac{q_T^2}{Q^2 + q_T^2} \text{(Collins 1979)}$$

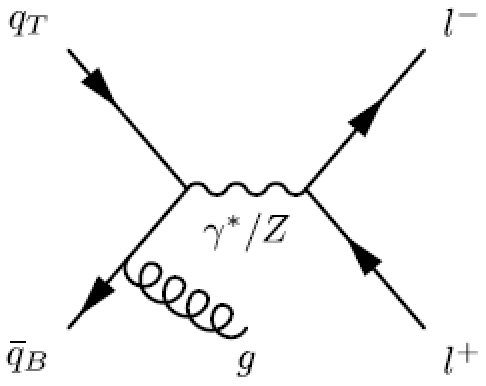
$$A_0 = A_2 = \left\langle \sin^2\theta_1 \right\rangle = \frac{q_T^2}{Q^2 + q_T^2} > 0$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}$$

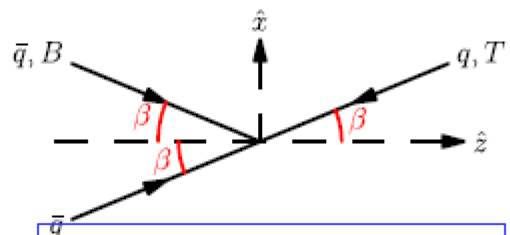
$$\nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

θ_1 and ϕ_1 at NLO(α_s^1): $q\bar{q} \rightarrow \gamma^*/Zg$

Collins-Soper (γ^*/Z rest) Frame

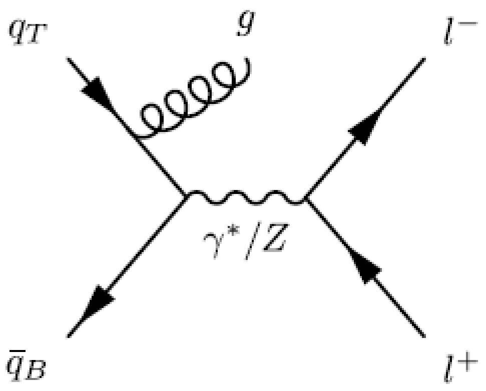


(a)

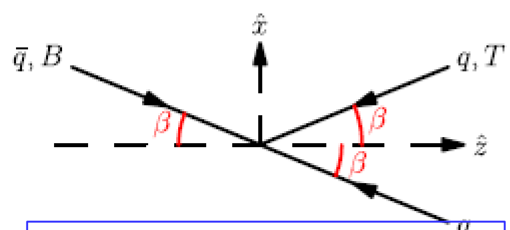


$\theta_1 = \pi - \beta;\ \phi_1 = 0$

(b)



(c)



$\theta_1 = \pi - \beta;\ \phi_1 = \pi$

(d)

$$A_0^{q\bar{q}} = \frac{q_T^2}{Q^2 + q_T^2} \text{ (Collins 1979)}$$

$$A_0 = A_2 = \langle \sin^2\theta_1 \rangle = \frac{q_T^2}{Q^2 + q_T^2} > 0$$

$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2}$$

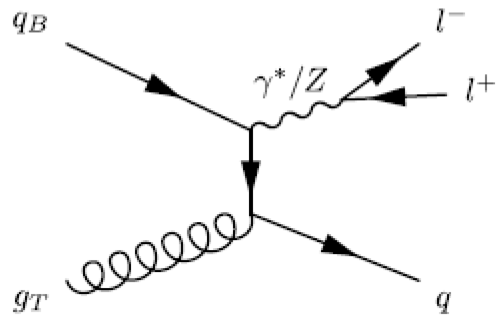
$$\nu = \frac{2A_2}{2 + A_0} = \frac{2q_T^2}{2Q^2 + 3q_T^2}$$

θ_1 and ϕ_1 at NLO(α_s^1): $qg \to \gamma^*/Zq$

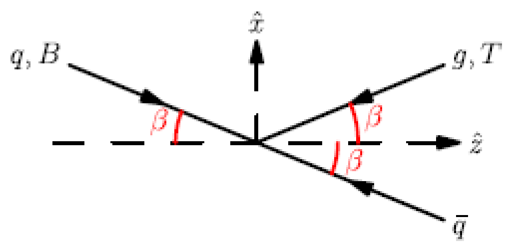
Collins-Soper (γ^*/Z rest) Frame

$$\theta_1 = \beta;\ \phi_1 = 0$$

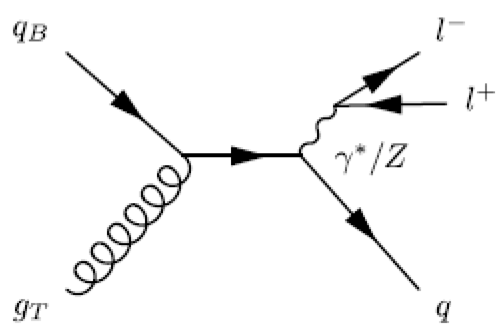
$$A_0^{qg} = \frac{5q_T^2}{Q^2 + 5q_T^2} \text{ (Thews 1979)}$$



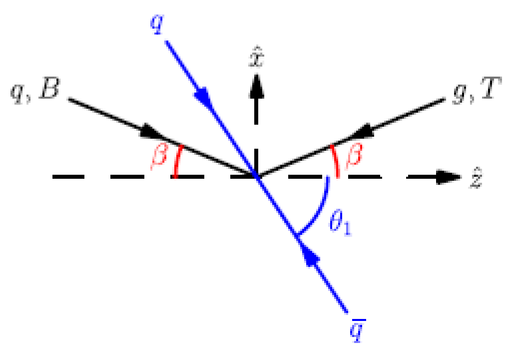
(a)



(b)



(c)



(d)

$$\theta_1 > \beta;\ \phi_1 = 0$$

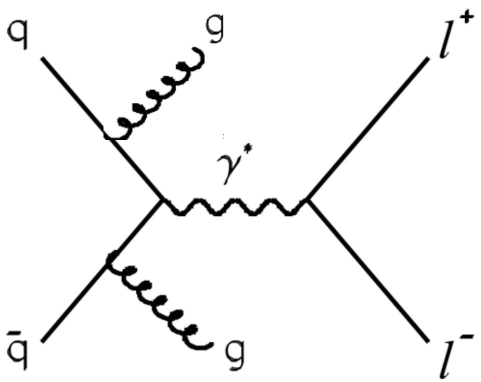
$$A_0 = A_2 = \left\langle \sin^2\theta_1 \right\rangle \approx \frac{5q_T^2}{Q^2 + 5q_T^2} > 0$$

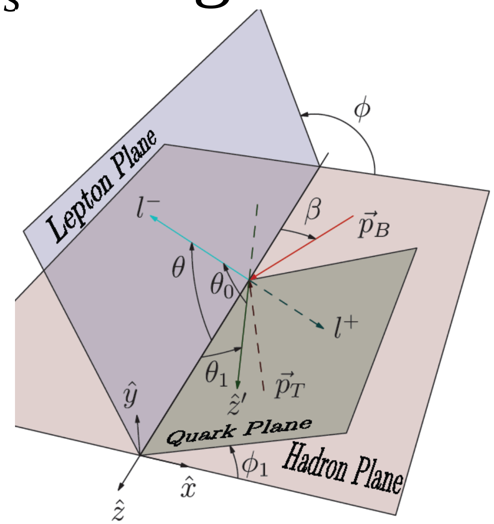
$$\lambda = \frac{2 - 3A_0}{2 + A_0} = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2}$$

$$\nu = \frac{2A_2}{2 + A_0} = \frac{10q_T^2}{2Q^2 + 15q_T^2}$$

Origins of the non-coplanarity

1) Processes at order α_s^2 or higher





$$A_0 = \langle \sin^2\theta_1 \rangle$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle$$

$$A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

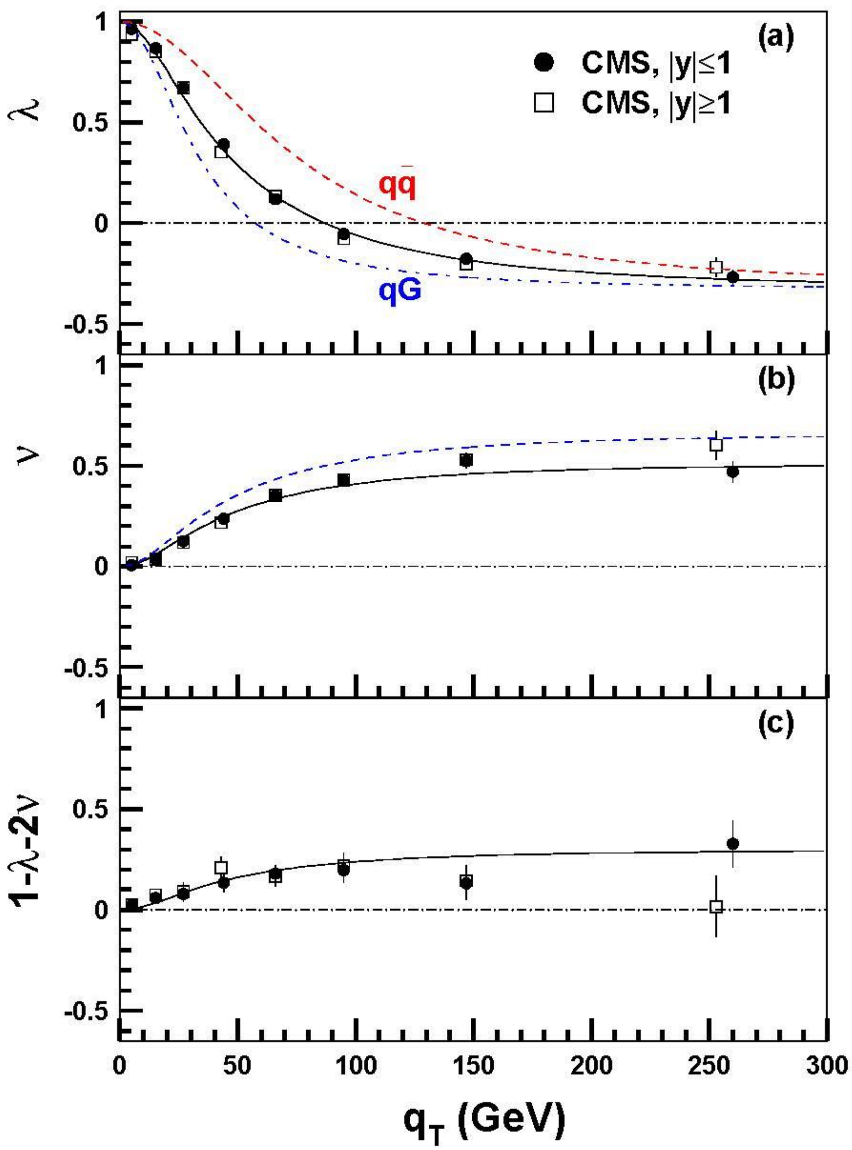
$q-\bar{q}$ axis is not sitting on the hadron plane: $\phi_1 \neq 0$

2) Intrinsic k_T from interacting partons

$$A_0 = \langle \sin^2\theta_1 \rangle ; A_2 = \langle \sin^2\theta_1 \cos 2\phi_1 \rangle$$

When $\phi_1 \neq 0$, $A_0 \neq A_2$

Compare with CMS data on Lam-Tung relation



$$\lambda = \frac{2Q^2 - q_T^2}{2Q^2 + 3q_T^2} \qquad \nu = \frac{2q_T^2}{2Q^2 + 3q_T^2} \qquad \text{for } q\bar{q} \rightarrow ZG$$

$$\lambda = \frac{2Q^2 - 5q_T^2}{2Q^2 + 15q_T^2} \qquad \nu = \frac{10q_T^2}{2Q^2 + 15q_T^2} \qquad \text{for } qG \rightarrow Zq$$

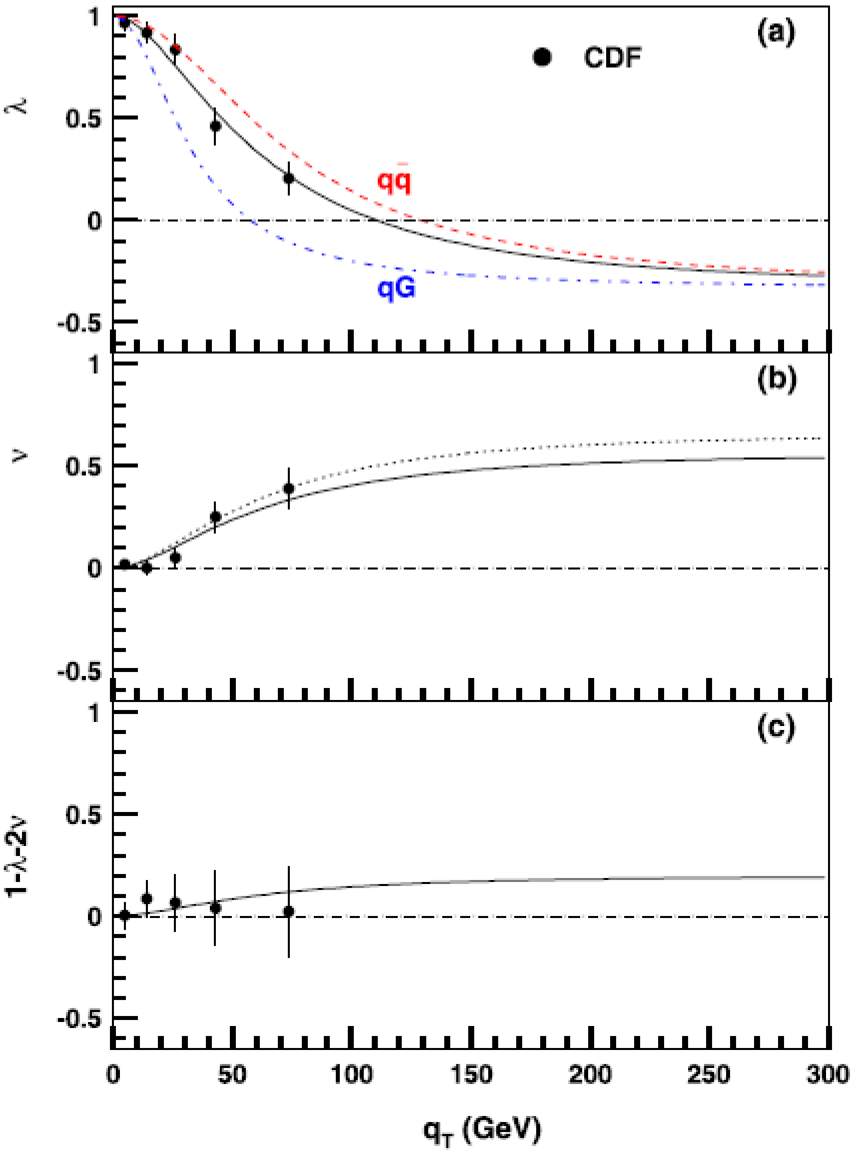
Solid curves correspond to a mixture of 58.5% qG and 41.5% $q\bar{q}$ processes, and $\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.77$

Violation of Lam-Tung relation is well described

J.C. Peng, W.C. Chang, R.E. McClellan, O. Teryaev
Phys. Lett. B758, 394 (2016)

Compare with CDF data

(Z production in $p + \bar{p}$ collision at 1.96 TeV)



Solid curves correspond to a mixture of 27.5% qG and 72.5% $q\bar{q}$ processes, and

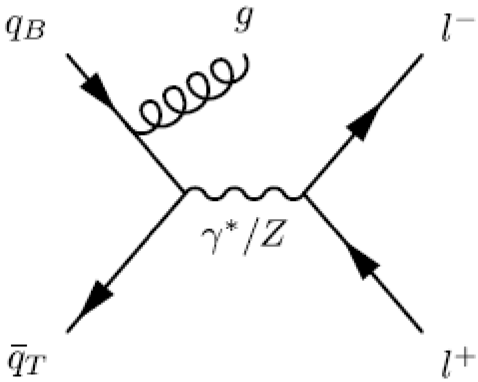
$$\langle \sin^2\theta_1 \cos 2\phi_1 \rangle / \langle \sin^2\theta_1 \rangle = 0.85$$

Violation of Lam-Tung relation is not ruled out

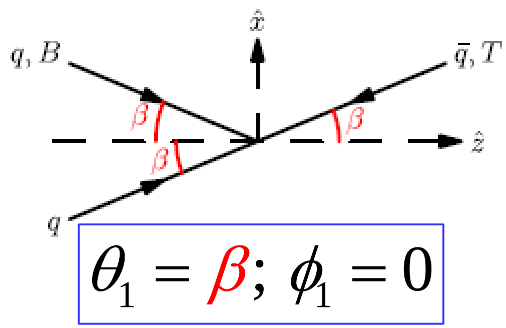
J.C. Peng, W.C. Chang, R.E. McClellan, O. Teryaev
Phys. Lett. B758, 394 (2016)

θ_1 and ϕ_1 at NLO(α_s^1): $q\bar{q} \to \gamma^*/Z\,g$

Collins-Soper (γ^*/Z rest) Frame



(a)

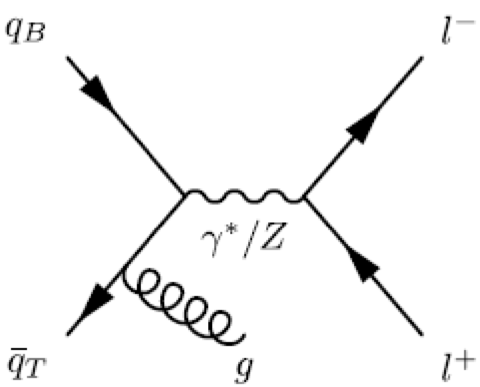


(b)

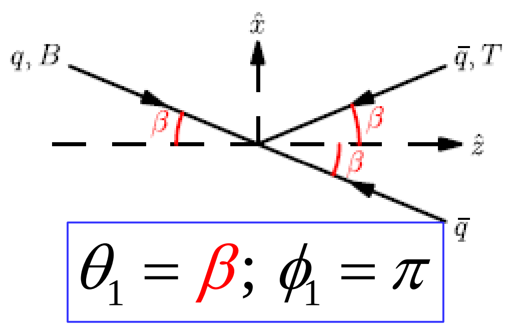
$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle = \frac{q_T Q}{Q^2 + q_T^2} > 0$$

$$A_3 = \langle a \sin\theta_1 \cos\phi_1 \rangle = a\,\frac{q_T}{\sqrt{Q^2 + q_T^2}} > 0$$

$$A_4 = \langle a \cos\theta_1 \rangle = a\,\frac{Q}{\sqrt{Q^2 + q_T^2}} > 0$$



(c)



(d)

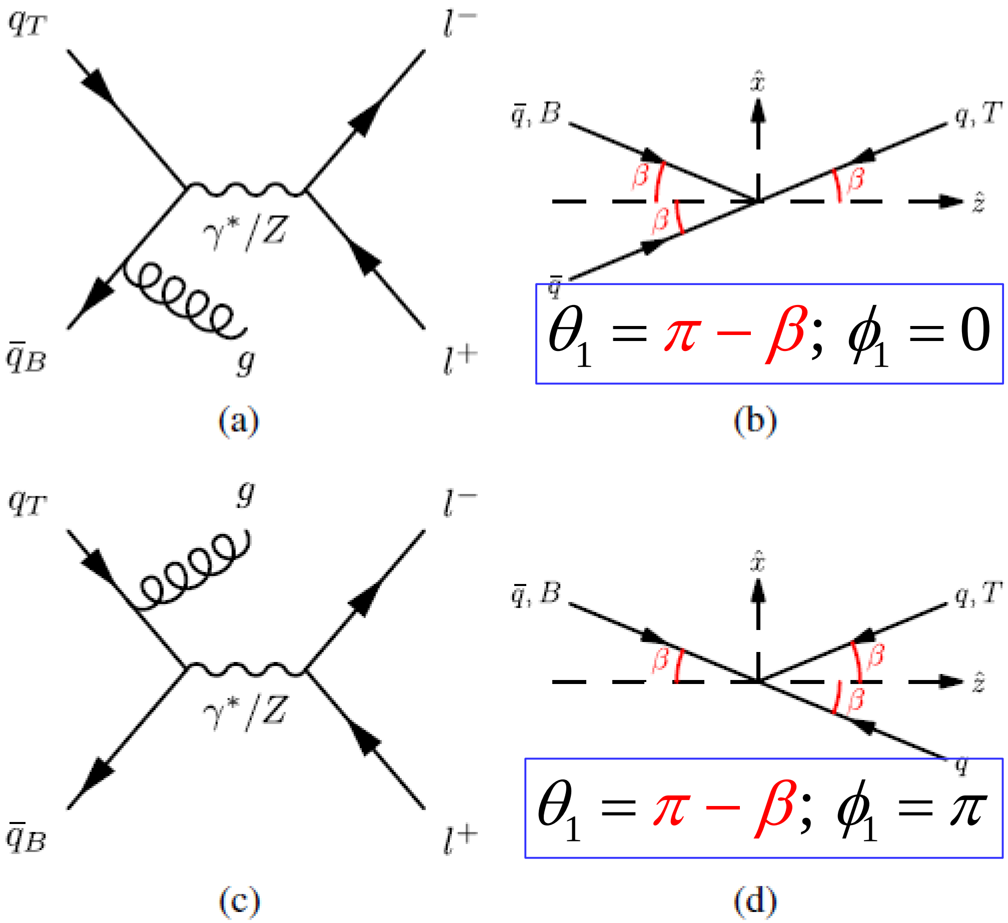
$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle = \frac{q_T Q}{Q^2 + q_T^2} < 0$$

$$A_3 = \langle a \sin\theta_1 \cos\phi_1 \rangle = a\,\frac{q_T}{\sqrt{Q^2 + q_T^2}} < 0$$

$$A_4 = \langle a \cos\theta_1 \rangle = a\,\frac{Q}{\sqrt{Q^2 + q_T^2}} > 0$$

θ_1 and ϕ_1 at NLO(α_s^1): $q\bar{q} \to \gamma^*/Zg$

Collins-Soper (γ^*/Z rest) Frame



(b): $\theta_1 = \pi - \beta;\ \phi_1 = 0$

(d): $\theta_1 = \pi - \beta;\ \phi_1 = \pi$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle = \frac{q_T Q}{Q^2 + q_T^2} < 0$$

$$A_3 = \langle a \sin\theta_1 \cos\phi_1 \rangle = a\frac{q_T}{\sqrt{Q^2 + q_T^2}} > 0$$

$$A_4 = \langle a \cos\theta_1 \rangle = a\frac{Q}{\sqrt{Q^2 + q_T^2}} < 0$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle = \frac{q_T Q}{Q^2 + q_T^2} > 0$$

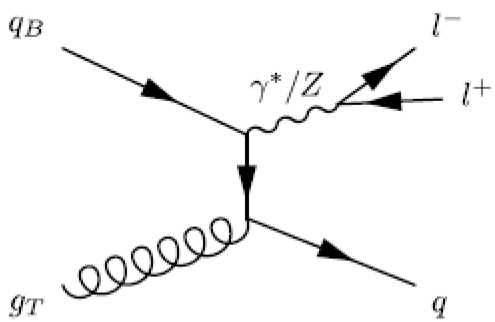
$$A_3 = \langle a \sin\theta_1 \cos\phi_1 \rangle = a\frac{q_T}{\sqrt{Q^2 + q_T^2}} < 0$$

$$A_4 = \langle a \cos\theta_1 \rangle = a\frac{Q}{\sqrt{Q^2 + q_T^2}} < 0$$

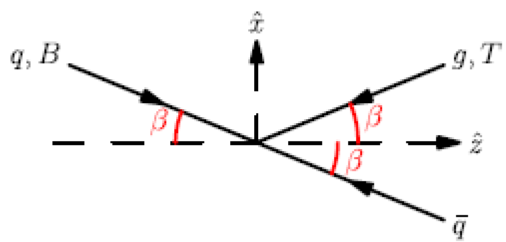
θ_1 and ϕ_1 at NLO(α_s^1): $qg \to \gamma^*/Zq$

Collins-Soper (γ^*/Z rest) Frame

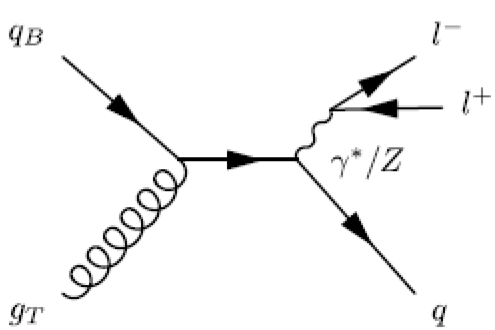
$$\theta_1 = \beta;\ \phi_1 = 0$$



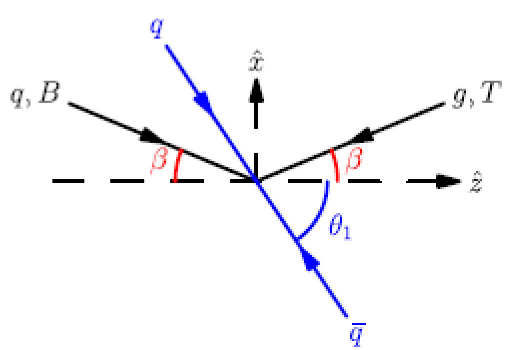
(a)



(b)



(c)



(d)

$$\theta_1 > \beta;\ \phi_1 = 0$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle = \frac{\sqrt{5}q_T Q}{Q^2 + 5q_T^2} > 0$$

$$A_3 = \langle a \sin\theta_1 \cos\phi_1 \rangle = a\frac{\sqrt{5}q_T}{\sqrt{Q^2 + 5q_T^2}} > 0$$

$$A_4 = \langle a \cos\theta_1 \rangle = a\frac{Q}{\sqrt{Q^2 + 5q_T^2}} > 0$$

$$A_1 = \frac{1}{2}\langle \sin 2\theta_1 \cos\phi_1 \rangle = \frac{\sqrt{5}q_T Q}{Q^2 + 5q_T^2} > 0$$

$$A_3 = \langle a \sin\theta_1 \cos\phi_1 \rangle = a\frac{\sqrt{5}q_T}{\sqrt{Q^2 + 5q_T^2}} > 0$$

$$A_4 = \langle a \cos\theta_1 \rangle = a\frac{Q}{\sqrt{Q^2 + 5q_T^2}} > 0$$

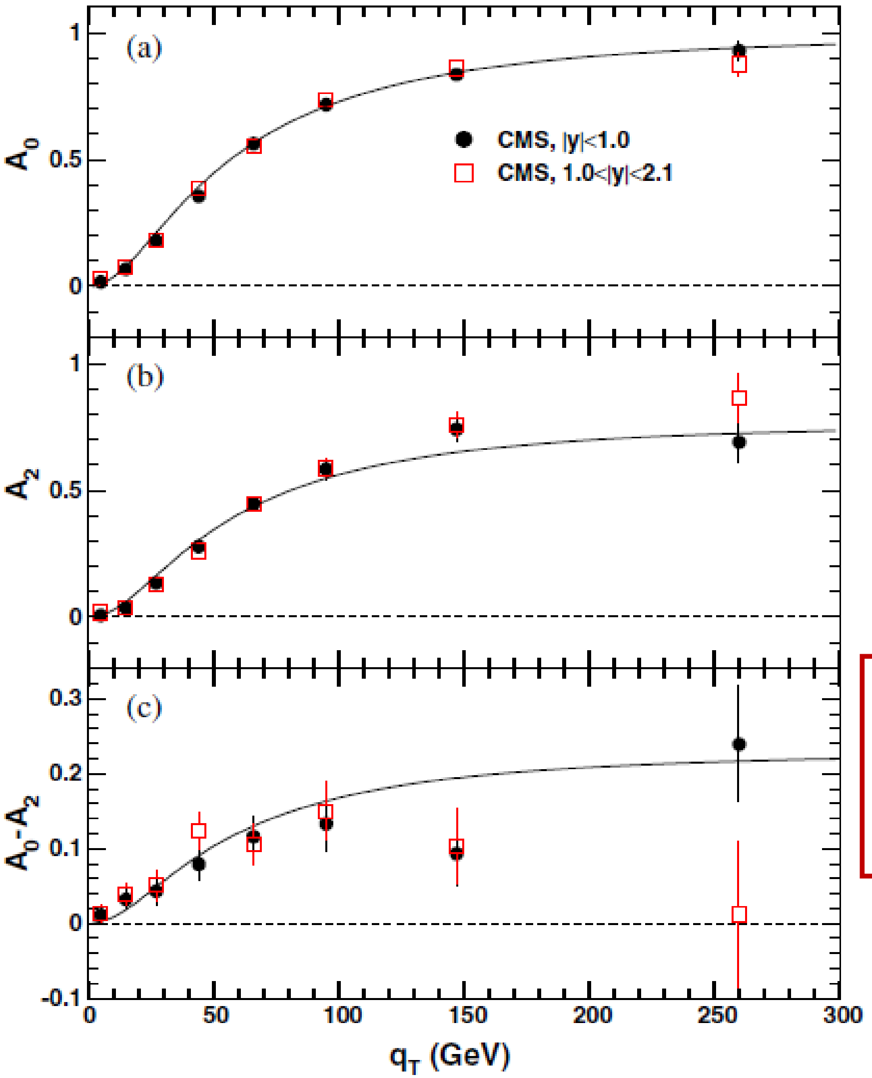
Rapidity Dependence of A_i

TABLE I. Angles θ_1 and ϕ_1 for four cases of gluon emission in the $q-\bar{q}$ annihilation process at order-α_s. The signs of A_0 to A_4 for the four cases are also listed.

Case	Gluon emitted from	θ_1	ϕ_1	A_0	A_1	A_2	A_3	A_4
1	Beam quark	β	0	+	+	+	+	+
2	Target antiquark	β	π	+	−	+	−	+
3	Beam antiquark	$\pi-\beta$	0	+	−	+	+	−
4	Target quark	$\pi-\beta$	π	+	+	+	−	−

A cancelation effect leads to a strong rapidity (y) dependence of A_1, A_3 and A_4.

Compare with CMS data on Lam-Tung relation



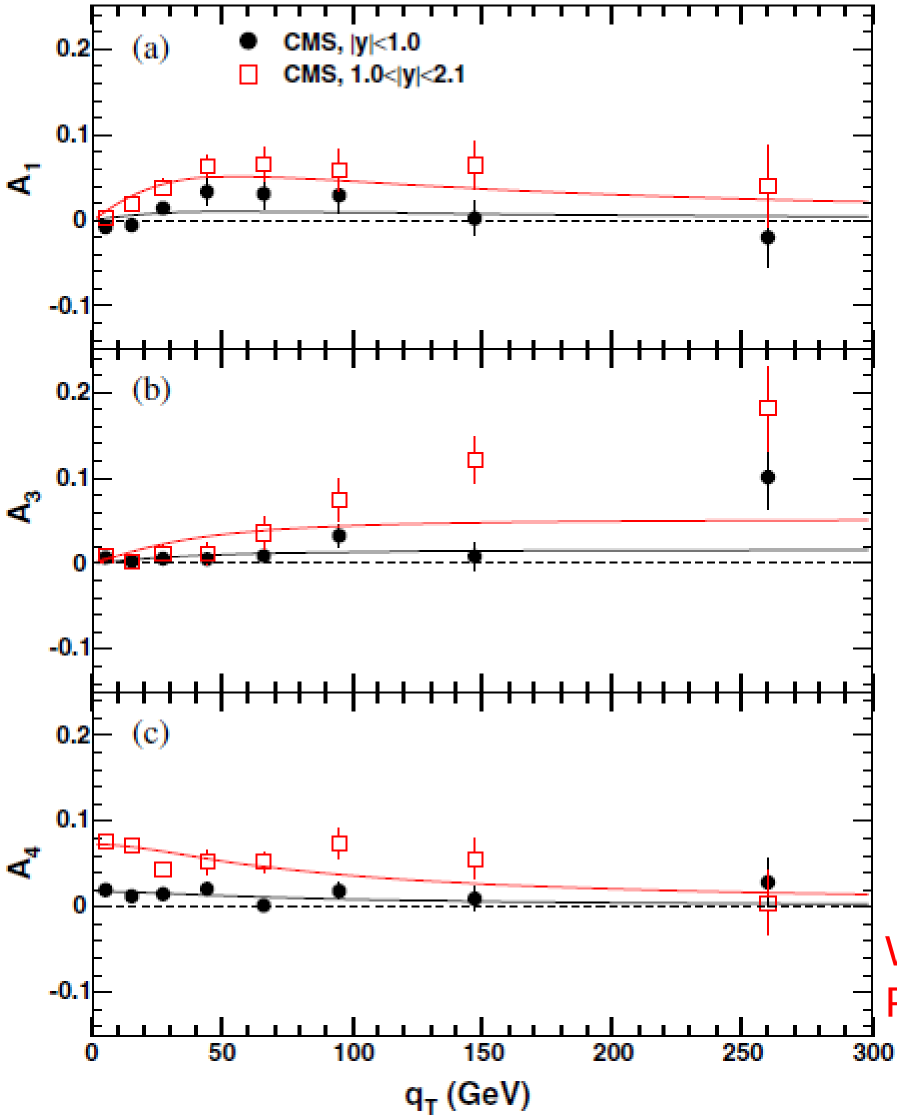
$$A_{0,2} = f \frac{q_T^2}{Q^2 + q_T^2} + (1 - f) \frac{5q_T^2}{Q^2 + 5q_T^2}$$

$$f = 0.415$$

Weak Rapidity dependence of A_0 and A_2.

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev
Phys. Rev. D 96, 054020 (2017)

Compare with CMS data on Lam-Tung relation



$$A_1 = r_1 \left[f \frac{q_T Q}{Q^2 + q_T^2} + (1 - f) \frac{\sqrt{5} q_T Q}{Q^2 + 5 q_T^2} \right]$$

$$A_3 = r_3 \left[f \frac{q_T}{\sqrt{Q^2 + q_T^2}} + (1 - f) \frac{\sqrt{5} q_T}{\sqrt{Q^2 + 5 q_T^2}} \right]$$

$$A_4 = r_4 \left[f \frac{Q}{\sqrt{Q^2 + q_T^2}} + (1 - f) \frac{Q}{\sqrt{Q^2 + 5 q_T^2}} \right]$$

Rapidity of A_1, A_3 and A_4 are well described

W.C. Chang, R.E. McClellan, J.C. Peng, O. Teryaev
Phys. Rev. D 96, 054020 (2017)

Summary

- The lepton angular coefficients A_0-A_7 are described in terms of the polar and azimuthal angles of the $\mathrm{q}-\bar{\mathrm{q}}$ axis.
- The striking q_T dependence of A_0 (or λ) can be well described by the mis-alignment of the $\mathrm{q}-\bar{\mathrm{q}}$ axis and the CS z-axis, i.e. **finite $\boldsymbol{\theta_1}$**.
- Violation of the Lam-Tung relation ($A_0 \neq A_2$) is described by the non-coplanarity of the $\mathrm{q}-\bar{\mathrm{q}}$ axis and the hadron plane, i.e. **finite $\boldsymbol{\phi_1}$**.
- Many salient features of the data could be nicely interpreted within the framework of geometric picture.