What I do
&
what we can do together !!

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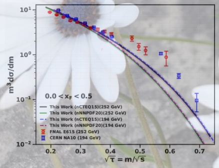
PhD (2022-2026, NIT-Jalandhar)

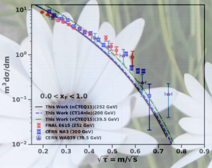
Ex-JINR Fellow

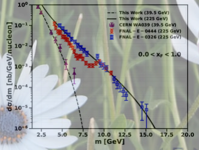
Ex-IIT Bhilai Fellow

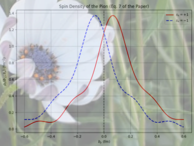
9th April, 2026

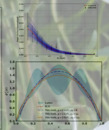
9th April, 2026











- **Introduction**
- **My Past and Ongoing Research**
- **What about Future Plans**

Who Am I ??

I am from Balasore, Odisha, India. A costal village near the bay of Bengal.

High School:- Harekrushna High School, Balasore, Odisha (2011-14)

Higher Secondary:-Siddhivinayak +2 Science Residential College, Balasore, Odisha (2014-16)

B.Sc:-Bhadrak Autonomous College, Bhadrak, Odisha (2016-19)

M.Sc:-National Institute of Technology, Jamshedpur, Jharkhand (2019-21)

PhD:-Dr. B. R. Ambedkar National Institute of Technology, Jalandhar, Punjab (2022-26)

B.Sc Project

Superconductivity and its application to Maglev train

Supervisor:- Dr. Rajat Kumar Pradhan

M.Sc Project

Proton-Oxygen elastic scattering by Taylor series expansion

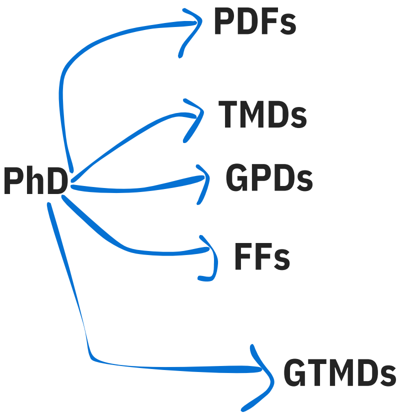
Supervisor:- Prof. Ujjwal Laha

PhD Thesis

Tomography of Light and Heavy mesons using the Light-front Dynamics

Supervisor:- Prof. Harleen Dahiya

What I did in My PhD ??



My Research areas lies

- Spin-0 (Light & Heavy Mesons)
- Spin-1 (Light Mesons)

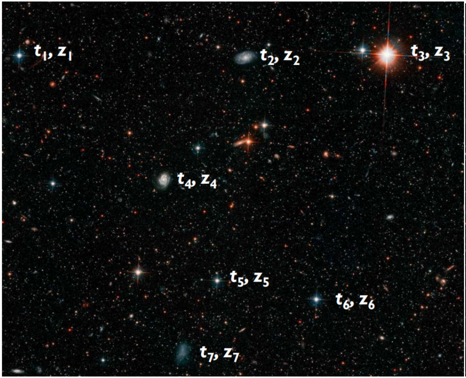
However, I have also done few work for proton TMDs at higher twist

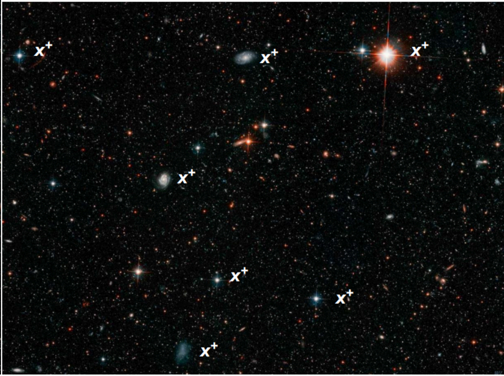
Some Results of my PhD Thesis

I have worked in different non-perturbative models for calculating the distribution functions of mesons in these years. These are

- **Light-Front Quark Model**
- **Light-Front Holographic Model (Hard Wall Model)**
- **AdS-QCD Model**
- **Nambu–Jona-Lasinio model (NJL)**
- **Quark Diquark Model**
- **Chiral SU(3) quark Mean Field Model**

Light-Front Dynamics





Instant Form **Front Form**

Image from C. Lorce Talk

Why light-front ?

Ideal Framework to describe the hadronic structure. It can overcome many obstacles with many advantages.

Simple vacume state

Boost invariant frame

Frame independent wave functions

Hamiltonial formlism for relativistic bound state

No square root in Hamiltonial p^-

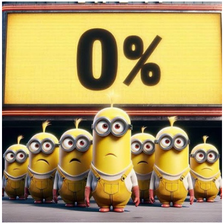
Maximum number of kinematic variables

Light-Front quark model

The Meson bound state wave function up to first principle

$$|\Psi_m\rangle = \sum |q\bar{q}\rangle\psi_{q\bar{q}} + \sum |q\bar{q}g\rangle\psi_{q\bar{q}g} + \sum |q\bar{q}gg\rangle\psi_{q\bar{q}gg} + |q\bar{q}(q\bar{q})_{sea}\rangle\psi_{q\bar{q}(q\bar{q})_{sea}} + \cdots$$

Complicated! ☹



The unpolarized quark, gluon and sea quark PDF

$$f_1^v(x) = f_{1,q\bar{q}}^v(x) + f_{1,q\bar{q}g}^v(x) + f_{1,q\bar{q}gg}^v(x) + \sum_{\{s\bar{s}\}} f_{1,q\bar{q}\{s\bar{s}\}}^v(x),$$

$$f_1^g(x) = f_{1,q\bar{q}g}^g(x) + f_{1,q\bar{q}gg}^g(x),$$

$$f_1^S(x) = 2\sum_{\{s\bar{s}\}} f_{1,q\bar{q}\{s\bar{s}\}}^S(x),$$

The Pion Form Factors

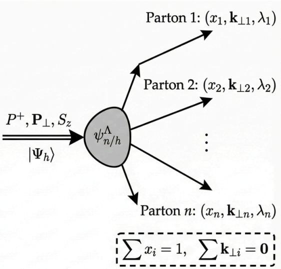
$$F_\pi(Q^2) = F_{\pi,q\bar{q}}(Q^2) + F_{\pi,q\bar{q}g}(Q^2) + F_{\pi,q\bar{q}gg}(Q^2) + \sum_{\{s\bar{s}\}} F_{\pi,q\bar{q}\{s\bar{s}\}}(Q^2),$$

Light-Front quark model

The multi-particle Fock-state can be expressed in the Light-cone frame as

$$|\Psi_h(P^+,\mathbf{P}_\perp,S_z)\rangle_\Lambda = \sum_{n,\lambda_i}\int\prod_{i=1}^{n}\frac{\mathrm{d}x_i\mathrm{d}^2\mathbf{k}_{\perp i}}{\sqrt{x_i}\,16\pi^3}16\pi^3\delta\Big(1-\sum_{i=1}^{n}x_i\Big)\delta^{(2)}\Big(\sum_{i=1}^{n}\mathbf{k}_{\perp i}\Big)$$

$$\psi^\Lambda_{n/h}(x_i,\mathbf{k}_{\perp i},\lambda_i)|n;x_iP^+,x_i\mathbf{P}_\perp+\mathbf{k}_{\perp i},\lambda_i\rangle.$$



=0 (pseudo-scalar mesons with Sz=0)
=1 (vector mesons with Sz=-1,0,+1)

Helicities of the parton

Satisfies the ortho-normality condition

$${}_{\Lambda'}\langle\Psi_h(P^{+\prime},\mathbf{P}'_\perp,S_z)|\Psi_h(P^+,\mathbf{P}_\perp,S_z)\rangle_\Lambda = 2(2\pi)^3P^+\delta_{\Lambda'\Lambda}\delta(P^+-P^{+\prime})\delta^2(P_\perp-P'_\perp)$$

$$|\Psi_h(P^+,\mathbf{P}_\perp,S_z)\rangle_\Lambda = \sum_{\lambda_1,\lambda_2}\int\frac{\mathrm{d}x\mathrm{d}^2\mathbf{k}_\perp}{\sqrt{x(1-x)}16\pi^3}\psi^\Lambda_{q\bar{q}}(x,\mathbf{k}_\perp,\lambda_1,\lambda_2)|x,\mathbf{k}_\perp,\lambda_1,\lambda_2\rangle$$

Spin wave function

Meson wave function

$$\psi^\Lambda_{q\bar{q}}(x,\mathbf{k}_\perp,\lambda_1,\lambda_2) = \phi(x,\mathbf{k}^2_\perp)\mathcal{S}_\Lambda(x,\mathbf{k}_\perp,\lambda_1,\lambda_2)$$

TMDs

The quark TMDs for the hadrons can be calculated by solving the quark-quark correlation function of the hadronic tensor as

$$\Phi_q^{\Lambda[\Gamma]}(x,\mathbf{k}_\perp^2) = \int \frac{\mathrm{d}k^+\mathrm{d}k^-}{2(2\pi)^4 P^+}\delta\left(x-\frac{k^+}{P^+}\right) \times \int \mathrm{d}^4 z e^{ik\cdot z}\, {}_{\Lambda'}\langle\Psi_h(P^{+\prime},\mathbf{P}'_\perp,S_z)|\bar{\psi}(0)\Gamma\mathcal{W}(0,z)\psi(z)|\Psi_h(P^+,\mathbf{P}_\perp,S_z)\rangle_\Lambda$$

$$= \int \frac{\mathrm{d}z^-\mathrm{d}^2\boldsymbol{z}_\perp}{2(2\pi)^3}e^{ik\cdot z} \times {}_{\Lambda'}\langle\Psi_h(P^{+\prime},\mathbf{P}'_\perp,S_z)|\bar{\psi}(0)\Gamma\mathcal{W}(0,z)\psi(z)|\Psi_h(P^+,\mathbf{P}_\perp,S_z)\rangle_{\Lambda,z^+=0}.$$

There are 4 T-even and 4 T-odd TMDs for spin-0 particles up to twist-4. The T-odd TMDs are zero as the gluons are not considered in these study.

The twist expansion of TMDs up to twist-4 is expressed as

$$\Phi_q^{[\Gamma]} = \Phi_{q(twist-2)}^{[\Gamma]} + \frac{1}{Q}\Phi_{q(twist-3)}^{[\Gamma]} + \frac{1}{Q^2}\Phi_{q(twist-4)}^{[\Gamma]} + ...$$

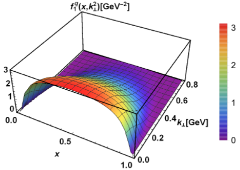
S.~Meissner, A.~Metz, M.~Schlegel and K.~Goeke, JHEP 08, 038 (2008)

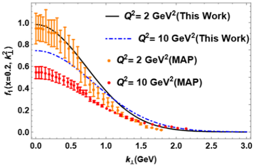
Spin-0 TMDs

Twist-2

$$\Phi_q^{[\gamma^+]}(x, \mathbf{k}_\perp^2) = f_1^q(x, \mathbf{k}_\perp^2).$$

$$= \frac{1}{2(2\pi)^3}\Big[\,|\,\psi_{q\bar{q}}^{\Lambda=0}(x, \mathbf{k}_\perp, \uparrow, \uparrow)\,|^2 + |\,\psi_{q\bar{q}}^{\Lambda=0}(x, \mathbf{k}_\perp, \downarrow, \downarrow)\,|^2 + |\,\psi_{q\bar{q}}^{\Lambda=0}(x, \mathbf{k}_\perp, \downarrow, \uparrow)\,|^2 + |\,\psi_{q\bar{q}}^{\Lambda=0}(x, \mathbf{k}_\perp, \uparrow, \downarrow)\,|^2\,\Big],$$



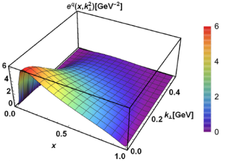


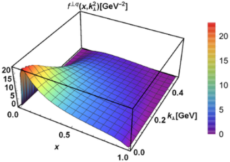
Evolved leading twist pion quark TMDs using the NLO Collins–Soper–Sterman (CSS) compared with MAP data

Twist-3

$$\Phi_q^{[1]}(x, \mathbf{k}_\perp^2) = \frac{M_{q\bar{q}}}{P^+} e^q(x, \mathbf{k}_\perp^2),$$

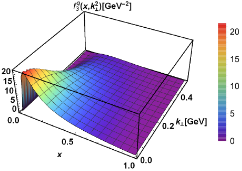
$$\Phi_q^{[\gamma^j]}(x, \mathbf{k}_\perp^2) = \frac{k_\perp^j}{P^+} f^{\perp q}(x, \mathbf{k}_\perp^2),$$





Twist-4

$$\Phi_q^{[\gamma^-]}(x, \mathbf{k}_\perp) = \frac{M_{q\bar{q}}^2}{(P^+)^2} f_3^q(x, \mathbf{k}_\perp^2).$$



S.~Puhan, et.al, JHEP 02, 075 (2024)

Positivity Constraints on spin-0 TMDs

$$f_1^q(x, \mathbf{k}_\perp^2) \geq 0, \quad f^{\perp q}(x, \mathbf{k}_\perp^2) \geq 0,$$
$$e^q(x, \mathbf{k}_\perp^2) \geq 0, \quad f_3^q(x, \mathbf{k}_\perp^2) \geq 0.$$

Spin-1 TMDs

Spin-1 leading twist TMDs

Quark / Hadron	U (γ^+) T-even	U (γ^+) T-odd	L ($\gamma^+\gamma_5$) T-even	L ($\gamma^+\gamma_5$) T-odd	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-even	T ($i\sigma^{i+}\gamma_5 / \sigma^{i+}$) T-odd
U	f_1					$[h_1^\perp]$
L			g_{1L}		$[h_{1L}^\perp]$	
T		$f_{1T}^\perp$	g_{1T}		$[h_1], [h_{1T}^\perp]$	
LL	f_{1LL}					$[h_{1LL}^\perp]$
LT	f_{1LT}			g_{1LT}		$[h_{1LT}], [h_{1LT}^\perp]$
TT	f_{1TT}			g_{1TT}		$[h_{1TT}], [h_{1TT}^\perp]$

Tensor TMDs

- There are total 72 TMDs present for the case of spin-1 system upto twist-4.
- At the leading twist 9 are T-even and 9 are T-odd TMDs.

Twist-2 TMD: $f_{1LL},\ f_{1LT},\ f_{1TT},\ g_{1LT},\ g_{1TT},$
$h_{1LL}^\perp,\ h_{1LT},\ h_{1LT}^\perp,\ h_{1TT},\ h_{1TT}^\perp,$

Twist-3 TMD: $f_{LL}^\perp,\ e_{LL},\ f_{LT},\ f_{LT}^\perp,\ e_{1T},\ e_{1T}^\perp,\ f_{TT},\ f_{TT}^\perp,$
$e_{TT},\ e_{TT}^\perp,\ g_{LL}^\perp,\ g_{LT},\ g_{LT}^\perp,\ g_{TT},\ g_{TT}^\perp,$
$h_{1L},\ h_{LT},\ h_{LT}^\perp,\ h_{TT},\ h_{TT}^\perp,$

Extra For Spin-1 Particles than Spin-1/2 Nucleons

S. Kumano and Q. T. Song, PLB 826 (2022)

Spin-1 TMDs

For the case spin-1 mesons, the polarization of mesons plays an important role while calculating the DFs. The leading twist quark TMDs can be calculated from the quark-quark correlation matrix as

$$\Phi_q^{\Lambda[\Gamma]}(x,\mathbf{k}_\perp^2) = \varepsilon_\Lambda^{*\mu}(P^*)\langle\Gamma\rangle_{\lambda_1\lambda_2}^{\mu\nu}(x,\mathbf{k}_\perp^2)\varepsilon_\Lambda^{\nu}(P) = \langle\Gamma\rangle_{\lambda_1\lambda_2}^{\Lambda}(x,\mathbf{k}_\perp)$$

The different quark TMDs can be calculated by taking the different Gamma matrix as

$$\langle\gamma^+\rangle_{\lambda_1\lambda_2}^{\Lambda}(x,\mathbf{k}_\perp) = \frac{1}{2}\mathrm{Tr}\left[\gamma^+\Phi_q^{\Lambda[\gamma^+]}(x,\mathbf{k}_\perp^2)\right] \equiv \varepsilon_\Lambda^{*\mu}(P^*)\langle\gamma^+\rangle_{\lambda_1\lambda_2}^{\mu\nu}(x,\mathbf{k}_\perp^2)\varepsilon_\Lambda^{\nu}(P)$$
$$\equiv f_1(x,\mathbf{k}_\perp^2) + S_{LL}f_{1LL}(x,\mathbf{k}_\perp^2) + \frac{S_{LT}.\mathbf{k}_\perp}{M_\rho}f_{1LT}(x,\mathbf{k}_\perp^2) + \frac{\mathbf{k}_\perp.S_{TT}.\mathbf{k}_\perp}{M_\rho^2}f_{1TT}(x,\mathbf{k}_\perp^2)$$

$$\langle\gamma^+\gamma_5\rangle_{\lambda_1\lambda_2}^{\Lambda}(x,\mathbf{k}_\perp) = \frac{1}{2}\mathrm{Tr}\left[\gamma^+\gamma_5\Phi_q^{\Lambda[\gamma^+\gamma_5]}(x,\mathbf{k}_\perp^2)\right] \equiv \varepsilon_\Lambda^{*\mu}(P^*)\langle\gamma^+\gamma_5\rangle_{\lambda_1\lambda_2}^{\mu\nu}(x,\mathbf{k}_\perp^2)\varepsilon_\Lambda^{\nu}(P)$$
$$\equiv \Lambda\left[S_L g_{1L}(x,\mathbf{k}_\perp^2) + \frac{\mathbf{k}_\perp.S_T}{M_\rho}g_{1T}(x,\mathbf{k}_\perp^2)\right]$$

$$\langle\gamma^+\gamma^{i_1}\gamma_5\rangle_{\lambda_1\lambda_2}^{\Lambda}(x,\mathbf{k}_\perp) = \frac{1}{2}\mathrm{Tr}\left[\gamma^+\gamma^{i_1}\gamma_5\Phi_q^{\Lambda[\gamma^+\gamma^{i_1}\gamma_5]}(x,\mathbf{k}_\perp^2)\right] \equiv \varepsilon_\Lambda^{*\mu}(P^*)\langle\gamma^+\gamma^{i_1}\gamma^5\rangle_{\lambda_1\lambda_2}^{\mu\nu}(x,\mathbf{k}_\perp^2)\varepsilon_\Lambda^{\nu}(P)$$
$$\equiv \Lambda\left[S_T^{i_1}h_1(x,\mathbf{k}_\perp^2) + \frac{S_L\mathbf{k}_\perp^{i_1}}{M_\rho}h_{1L}(x,\mathbf{k}_\perp^2) + \frac{\left(2\mathbf{k}_\perp^{i_1}\mathbf{k}_\perp.S_T - S_T^{i_1}\mathbf{k}_\perp\right)}{2M_\rho^2}h_{1T}(x,\mathbf{k}_\perp^2)\right]$$

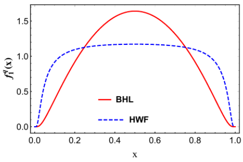
with tensor polarizations

$$S_{LL} = (3\Lambda^2-2)\Big(\frac{1}{6}-\frac{S_L^2}{2}\Big),$$
$$S_{LT}^{i_1} = (3\Lambda^2-2)\,S_L S_T^{i_1},$$
$$S_{TT}^{i_1j_1} = (3\Lambda^2-2)\Big(S_T^{i_1}S_T^{j_1}-\frac{S_T^2}{2}\delta^{i_1j_1}\Big)$$

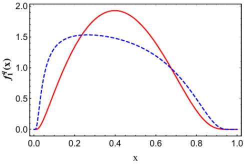
S.~Puhan, S.~Sharma, N.~Kumar and H.~Dahiya,Phys.Rev.D 113 (2026) 3, 036030

PDFs for spin-0 Mesons

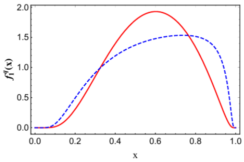
Pion u-quark PDFs

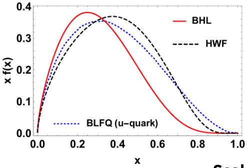


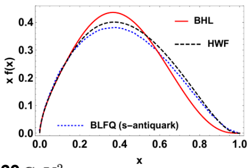
Kaon u-quark PDFs



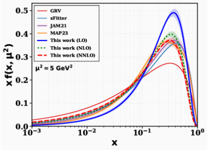
Kaon s-antiquark PDFs

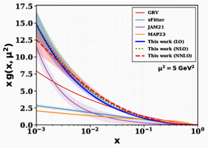


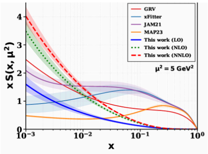




Scale= 20 GeV^2







All the evolutions have been carried out through DGLAP evolutions using the HOPPET.

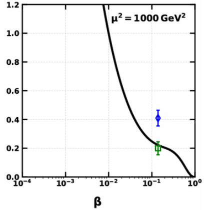
Structure Functions

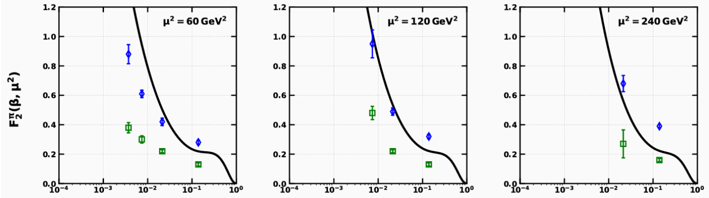
For the case of pion, there is only a single structure functions, compared to the three for the case of nucleons. The structure functions can be calculated from the PDFs at NLO as

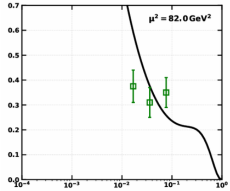
$$F_2^\pi(\beta,\mu^2) = \sum_q e_q^2\,\beta\Big\{ f(\beta,\mu^2) + \bar{f}(\beta,\mu^2) + \frac{\alpha_s(\mu^2)}{2\pi} \times \Big[C_q \otimes \big(f(\beta,\mu^2)+\bar{f}(\beta,\mu^2)\big) + 2\,C_g \otimes g(\beta,\mu^2) \Big] \Big\}$$

LO terms

NLO terms





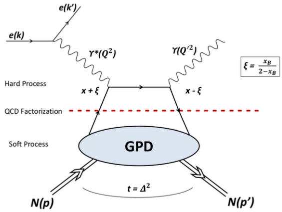


DESY-HERA-Data of Leading-neutron production data of 2002 & 2009

Generalized Parton Distribution Function

PARTON DISTRIBUTION FUNCTIONS AND GDA (PION)

MECHANICAL PROPERTIES (FORCE AND PRESSURE)



ELASTIC FORM FACTORS
GRAVIATIONAL FORM FACTORS
MAGNETIC MOMENTS
QUADRAPOLE MOMENTS

Why GPDs ??

DVCS AND DVMP SCATTERING CROSS SECTION

STRUCTURE FUNCTION

SPIN DENSITIES
ANGULAR MOMENTUM
SUM RULE

PHYSICAL PROPERTIES (CHARGE RADII)

MANY MORE

Spin-0 GPDs

The quark-quark correlator for GPDs can be expressed as

$$\Phi_q^{\Lambda[\Gamma]}(x,\xi,-\Delta_\perp^2) \;=\; \int \frac{dz^-}{2(2\pi)} e^{ik\cdot z}\, {}_{\Lambda'}\langle \Psi_h(P^{+\prime},\mathbf{P}'_\perp,S_z)|\bar{\Psi}(0)\Gamma \mathcal{W}(0,z)\psi(z)|\Psi_h(P^+,\mathbf{P}_\perp,S_z)\rangle_{\Lambda,z^+,z_\perp=0}$$

Momentum transfer: $\Delta = P' - P$, with $t = \Delta^2$.

Skewness: $\xi = \frac{P^+ - P'^+}{P^+ + P'^+}$, representing the difference in longitudinal momentum fractions.

There are total 8 GPDs for the case of spin-0 mesons. Out of which I have discussed only scalar, vector and tensor quark GPDs of pion

Vector GPDs (Chiral even in nature)

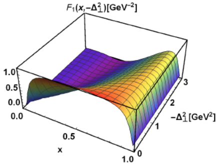
$$F_1(x,\xi,-\Delta_\perp^2) \;=\; \Phi_q^{[\gamma^+]}(x,\xi,-\Delta_\perp^2)$$

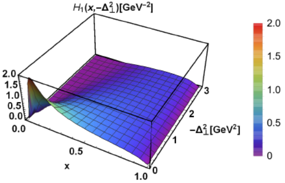
Tensor GPDs (Chiral odd in nature)

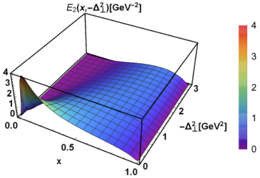
$$H_1(x,\xi,-\Delta_\perp^2) \;=\; -\frac{\iota M_{q\bar{q}}}{\epsilon^{ij}\Delta_\perp^i}\Phi_q^{[\iota\sigma^{j+}\gamma_5]}(x,\xi,-\Delta_\perp^2),$$

Scalar GPDs (Chiral odd in nature)

$$E_2(x,\xi,-\Delta_\perp^2) \;=\; \frac{P^+}{M_{q\bar{q}}}\Phi_q^{[1]}(x,\xi,-\Delta_\perp^2).$$







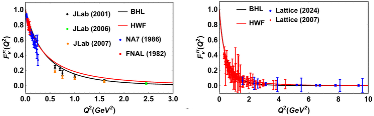
S.~Puhan and H.~Dahiya, Phys. Rev. D 111, 114039 (2025)
S.~Puhan, S.~Sharma, N.~Kumar and H.~Dahiya,'doi:10.1093/ptep/ptaf100

Spin-0 FFs

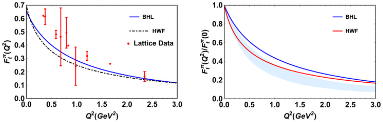
Respective Form Factors

S.~Puhan and H.~Dahiya, Phys. Rev. D 111, 114039 (2025)

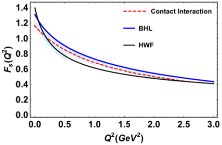
$$F_V^q(Q^2) = \int dx F_1(x, \xi = 0, -\Delta_\perp^2),$$



$$F_T^q(Q^2) = \int dx H_1(x, \xi = 0, -\Delta_\perp^2),$$ **Tensor FFs (Spin structure)**



$$F_S^q(Q^2) = \int dx E_2(x, \xi = 0, -\Delta_\perp^2).$$ **Scalar FFs**



The shaded curve are from Lattice results.

➡ **The respective charge radii of each FFs have been calculated via**

$$\langle r_S^2 \rangle = \frac{-6}{F_S(0)} \frac{\partial F_S(Q^2)}{\partial Q^2}\Big|_{Q^2\to 0} \quad \langle r_V^2 \rangle = \frac{-6}{F_V(0)} \frac{\partial F_V(Q^2)}{\partial Q^2}\Big|_{Q^2\to 0} \quad \langle r_T^2 \rangle = \frac{-6}{F_T(0)} \frac{\partial F_T(Q^2)}{\partial Q^2}\Big|_{Q^2\to 0}$$

	$\sqrt{\langle r_S^2 \rangle}$	$\sqrt{\langle r_V^2 \rangle}$	$\sqrt{\langle r_T^2 \rangle}$
BHL (This work)	0.528	0.558	0.567
HWF (This work)	0.771	0.604	0.769
LQCD (Alexandrou et al., 2022)	0.482	0.539	0.679
CI-MRL (Wang et al., 2022)	0.434	0.558	0.583
Ref. (Cui et al., 2021)	-	0.640	-
NA7 (Amendolia et al., 1984, 1986)	-	0.657, 0.662	
JLQCD (Aoki et al., 2016)	-	0.677	-
PDG (Zyla et al., 2020)	-	0.659	-
LQCD (Gülpers et al., 2014)	0.635	-	-
Ref. (Gao et al., 2017)	-	0.65	-

GFFs for Spin-0 Case

For the case of Spin-0 mesons, there are total three GFFs, which can be calculated from energy-momentum tensors as

$$\langle p'|\hat{T}^a{}_{\mu\nu}(0)|p\rangle = 2\left[P^\mu P^\nu - \frac{P\cdot\Delta}{\Delta^2}\left(\Delta^\mu P^\nu + \Delta^\nu P^\mu - P\cdot\Delta\, g^{\mu\nu}\right)\right]A^a(t) + \frac{1}{2}\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right)D^a(t) + \left(2m_\pi^2 g^{\mu\nu}\right)\bar{c}^a(t)$$

From GPDs

$$\int_{-1}^{1} \mathrm{d}x\, x H^{q,g}(x,\xi,t) = A^{q,g}(t) + \xi^2 D^{q,g}(t)$$

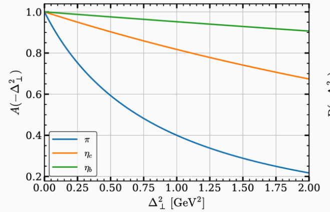
The helicity flip GPDs is absent for spin-0 mesons. The B and J GFFs also zero

GFFs for pion

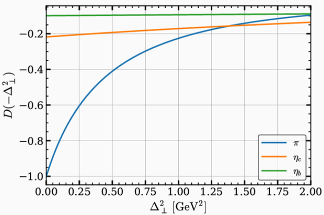
There are very less studies for the case of GFFs of pion in theoretical models as well as lattice simulations but no experimental studies. Few of the theoretical models are

- Pion GDAs, PRD 97, (2018)
- Holographic light-front QCD, PRD 109 (2024)
- QCD instanton vacuum, PRD 110 (2024)
- Top-down holographic QCD, PRD 110 (2024)
- lattice QCD, PRD 108 (2023)
- Nonlocal quark model, ArXiv:2507.06025
- Dispersion relations, Arxiv:2507.18690

Mechanical Distributions



Mass form factor $A(t)$



D-term form factor $D(t)$

The internal stress tensor

$$S^{ij}(z_\perp) = \frac{1}{4P^+}\int \frac{d^2\vec{\Delta}_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp\cdot\vec{z}_\perp}(\Delta_\perp^i\Delta_\perp^j - \vec{\Delta}_\perp^2\delta^{ij})\, D(-\vec{\Delta}_\perp^2)$$
$$= \left(\frac{z_\perp^i z_\perp^j}{z_\perp^2} - \frac{1}{2}\delta^{ij}\right) S(z_\perp) + \delta^{ij} P(z_\perp)$$

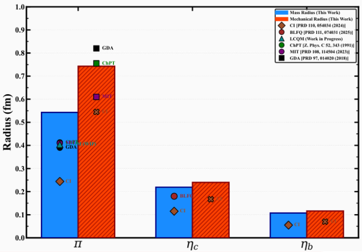
$$\langle r^2_{mech}\rangle = \frac{6\,D(0)}{\int_0^\infty dt\,(-t)\,D(t)} \frac{dD(t)}{dt}\bigg|_{t=0} \qquad \langle r^2_{mass}\rangle = \frac{-6}{A(0)}\frac{dA(t)}{dt}\bigg|_{t=0}$$

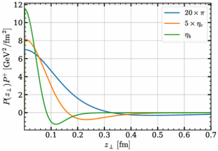
The effective normal force density on a 1D surface is

$$S^{ij}(z_\perp)n^j = \mathcal{F}^i(z_\perp) = \frac{z_\perp^i}{z_\perp}\left(P(z_\perp) + \frac{1}{2}S(z_\perp)\right)$$

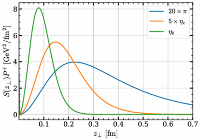
Shear and pressure from $\tilde{D}$:

$$S(z_\perp) = -z_\perp \frac{d}{dz_\perp}\left(\frac{1}{z_\perp}\frac{d\tilde{D}}{dz_\perp}\right)$$
$$P(z_\perp) = \frac{1}{2z_\perp}\frac{d}{dz_\perp}\left(z_\perp \frac{d\tilde{D}}{dz_\perp}\right)$$





Pressure Distributions



Shear Distributions

Thank You