Gribov-Zwanziger Formalism
A Brief Introduction

Taiwan QCD Meeting

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Presentation Outline

- Does Faddeev-Popov quantization fix the problem of gauge copies?

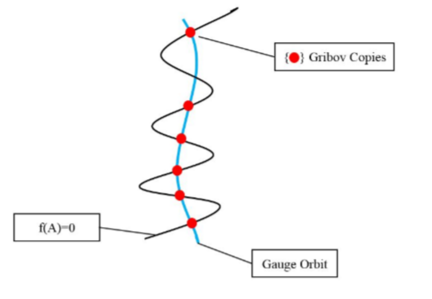
[1] Nucl. Phys., B139:1, 1978.

- Does Faddeev-Popov quantization fix the problem of gauge copies?
  - For Abelian theory and in perturbative non-Abelian theory: YES
  - For non-perturbative non-Abelian theory: NO[1]

[1] Nucl. Phys., B139:1, 1978.

- Does Faddeev-Popov quantization fix the problem of gauge copies?
  - For Abelian theory and in perturbative non-Abelian theory: YES
  - For non-perturbative non-Abelian theory: NO[1]
- What is the problem in non-perturbative non-Abelian theory?

- **GRIBOV COPIES**
  - Large copies: $\partial_\mu A_\mu = 0$;
    $A'_\mu = U A_\mu U^\dagger - i/g(\partial_\mu U)U^\dagger$; $\partial_\mu A'_\mu = 0$
  - Infinitesimal copies: $\partial_\mu A^a_\mu = 0$ &
    $\partial_\mu D^{ab}_\mu \omega^b = 0$; $A'^a_\mu = A^a_\mu + D^{ab}_\mu \omega^b$; $\partial_\mu A'^a_\mu = 0$



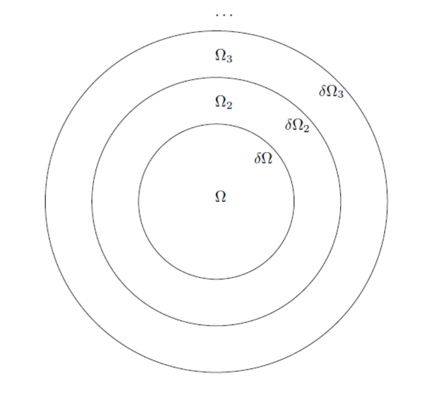
- Implications:
  - Overcouting configurations
  - Faddeev-Popov has zero-modes. So the path integral is ill defined.

$$-\partial_\mu D^{ab}_\mu = -\partial_\mu(\partial_\mu \delta^{ab} - g f^{abc} A^c_\mu) \quad (1)$$

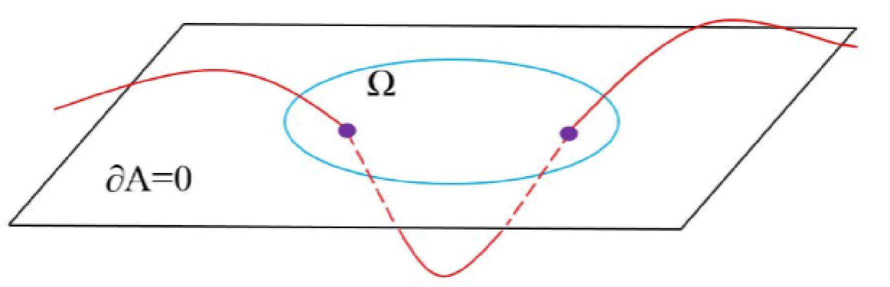
[1] Nucl. Phys., B139:1, 1978.

Gribov Region

$$\Omega = \{A^a_\mu;\ \partial_\mu A^a_\mu = 0, \quad \mathcal{M}^{ab}(A) = -\partial_\mu D^{ab}_\mu(A) > 0\}. \tag{2}$$



Fundamental Modular Region Λ (FMR)



Large copies

$$\partial_\mu A_\mu = 0$$
$$A'_\mu = U A_\mu U^\dagger - i/g(\partial_\mu U)U^\dagger$$
$$\partial_\mu A'_\mu = 0$$

Gribov-Zwanziger action

$$S = S_{YM} + S_{FP} + S_{GZ}$$

$$S_{FP} = \int d^4x \left(ib^a \, \partial_\mu A^a_\mu + \bar{c}^a \partial_\mu D^{ab}_\mu(A) c^b \right)$$

$$\begin{aligned} S_{GZ} \;\; = \;\; & \int d^4x \left[\bar{\varphi}^{ac}_\mu \partial_\nu D^{ab}_\nu \varphi^{bc}_\mu - \bar{\omega}^{ac}_\mu \partial_\nu (D^{ab}_\nu \omega^{bc}_\mu) \right. \\ & \left. - g(\partial_\nu \bar{\omega}^{an}_\mu) f^{abc} D^{bm}_\nu c^m \varphi^{cn}_\mu \right] \\ & - \gamma^2 g \int d^4x \left[f^{abc} A^a_\mu \varphi^{bc}_\mu + f^{abc} A^a_\mu \bar{\varphi}^{bc}_\mu + \frac{4}{g}(N_c^2 - 1)\gamma^2 \right] \end{aligned} \tag{3}$$

γ is the Gribov parameter, dynamically fixed by means of its gap equation[2]

$$\langle f^{abc} A^a_\mu (\varphi^{bc}_\mu + \bar{\varphi}^{bc}_\mu) \rangle = 2d(N^2 - 1)\frac{\gamma^2}{g^2} \tag{4}$$

[2] Nucl.Phys., B326:333–350, 1989.

The action (3) breaks the BRST (Becchi-Rouet-Stora-Tyutin) symmetry softly given by the following BRST transformation

$$\begin{aligned} sA^a_\mu &= -D^{ab}_\mu c^b\,, & sc^a &= \frac{g}{2}f^{abc}c^bc^c\,, \\ s\bar{c}^a &= b^a\,, & sb^a &= 0\,, \\ s\varphi^{ab}_\mu &= \omega^{ab}_\mu\,, & s\omega^{ab}_\mu &= 0\,, \\ s\bar{\omega}^{ab}_\mu &= \bar{\varphi}^{ab}_\mu\,, & s\bar{\varphi}^{ab}_\mu &= 0\,. \end{aligned} \tag{5}$$

Due to the presence of γ in S_{GZ}. It is easy to check this doing

$$s(S) = s(S_{GZ}) = -\gamma^2 g\int d^4x\, f^{abc}\left[A^a_\mu\omega^{bc}_\mu - (D^{am}_\mu c^m)(\bar{\varphi}^{bc}_\mu + \varphi^{bc}_\mu)\right]. \tag{6}$$

However, the BRST symmetry was restored via a definition in Capri et al., 2017[3].

[3] Phys. Rev., D95, 2017.

GZ Formalism does not Match with Lattice Simulation

Propagators	Gribov-Zwanziger framework	Lattice results since 2007
Gluons	$\mathscr{D}(p=0)=0$	$\mathscr{D}(p=0)\neq 0, <\infty$
Ghosts	$\mathscr{G}(p)\underset{p\to 0}{\sim}\frac{1}{p^4}$	$\mathscr{G}(p)\underset{p\to 0}{\sim}\frac{1}{p^2}$

Refined Gribov-Zwanziger (RGZ) action

In Landau gauge:

$$S_R = S_{YM} + S_{FP} + S_{RGZ} \tag{7}$$

$$S_{RGZ} = S_{GZ} + \frac{m^2}{2} \int d^4x A^a_\mu A^a_\mu + M^2 \int d^4x \bar{\varphi}^{ab}_\mu \varphi^{ab}_\mu \tag{8}$$

- (m^2, M^2) are dynamically determined by their own gap equations[4]:

$$\frac{\partial \Gamma}{\partial m^2} = 0 \qquad \text{and} \qquad \frac{\partial \Gamma}{\partial M^2} = 0 \; . \tag{9}$$

- Non-perturbative RGZ vacuum has been shown to have a lower vacuum energy compared to the original GZ action.
- The GZ action represents an unstable point of the effective potential
- With the formation of the condensates properly produces a minimum.

[4] Phys. Rev., D77:071501, 2008.

- How to implement these condensates on the action?
  - Via local composite operators (LCO)[5].

[5] Phys. Rev., D10:3235, 1974

[6] Phys. Rev., D14:2182, 1976; Phys. Rev., D64:085006, 2001; Annals Phys., 212:371–401, 1991; Phys. Rev., D54:1614–1625, 1996.

[7] Phys. Lett., B351:242–248, 1995

- How to implement these condensates on the action?
  - Via local composite operators (LCO)[5].
- Problems: New divergences show up and non-renormalizable theory[6]
- These problems were overcome[7]
- So, the dimension two condensates (e.g. $\langle AA \rangle$ and $\langle \bar{\varphi}\varphi \rangle$) are introduced via LCO formalism.
  - The LCO $\mathcal{O}$ are given by

$$\mathcal{O}_i = \{A_\mu A_\mu, \varphi_i^a \varphi_i^a, \bar{\varphi}_i^a \varphi_i^a, \bar{\varphi}_i^a \bar{\varphi}_i^a\}, \tag{10}$$

    here, e.g. $\varphi_i^a \varphi_i^a = \varphi_\mu^{ac} \varphi_\mu^{ac}$.
  - The operators $\mathcal{O}$ of interest are introduced to the Lagrangian via $\tau\mathcal{O}$, where τ is a source.

[5] Phys. Rev., D10:3235, 1974

[6] Phys. Rev., D14:2182, 1976; Phys. Rev., D64:085006, 2001; Annals Phys., 212:371–401, 1991; Phys. Rev., D54:1614–1625, 1996.

[7] Phys. Lett., B351:242–248, 1995

The new action is given by

$$\Sigma = S + S_{A^2} + S_{\varphi\bar{\varphi}} + S_{\text{vac}}, \tag{11}$$

whereby, S is given by (3) and we also have

$$S_{A^2} = \int d^dx \frac{1}{2}\tau A^a_\mu A^a_\mu \,, \qquad S_{\bar{\varphi}\varphi} = \int d^dx Q \bar{\varphi}^a_\mu \varphi^a_\mu \,,$$
$$S_{\text{vac}} = -\int d^dx \left(\frac{1}{2}\zeta\tau^2 + \alpha Q^2 + \chi Q\tau \right) \,.$$

τ, α and χ are called LCO parameters.

After renormalization process and using suitable Hubbard–Stratonovich transformation, for example,

$$1 = \int [\mathcal{D}\sigma] \, e^{-\frac{1}{2Z_\zeta}\int d^dx \left(\sigma + \frac{\bar{a}}{2}A^2 + \bar{b}Q + \bar{c}\tau\right)^2}.$$

The RGZ action can be found[8]:

$$S_{RGZ} = S_{GZ} + \frac{m^2}{2}\int d^4x A_\mu^a A_\mu^a + M^2 \int d^4x \bar{\varphi}_\mu^{ab}\varphi_\mu^{ab} \tag{13}$$

Now, the RGZ propagators match with Lattice simulation.

[8] Eur.Phys.J.C 79 (2019) 9, 731.

Thank You!

Questions?