國立交通大學
National Chiao Tung University

Moments of the Pion's Light Cone Distribution Amplitude

THE HOPE COLLABORATION

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with
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Talk Outline

- Motivation: Exclusive processes in Perturbative QCD
- Theoretical Background
  - Heavy quark Operator Product Expansion (HOPE)
  - Numerical Simulation via Lattice QCD
- Results & further calculations

Exclusive Processes in Perturbative Quantum Chromodynamics (1980)

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Exclusive processes in perturbative quantum chromodynamics

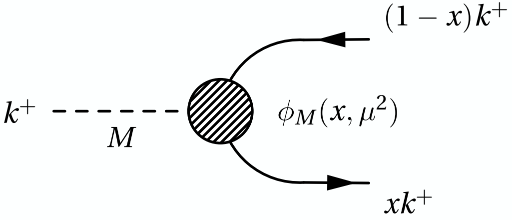
G. Peter Lepage
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

Stanley J. Brodsky
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 27 May 1980)

We present a systematic analysis in perturbative quantum chromodynamics (QCD) of large-momentum-transfer exclusive processes. Predictions are given for the scaling behavior, angular dependence, helicity structure, and normalization of elastic and inelastic form factors and large-angle exclusive scattering amplitudes for hadrons and photons. We prove that these reactions are dominated by quark and gluon subprocesses at short distances, and thus that the dimensional-counting rules for the power-law falloff of these amplitudes with momentum transfer are rigorous predictions of QCD, modulo calculable logarithmic corrections from the behavior of the hadronic wave functions at short distances. These anomalous-dimension corrections are determined by evolution equations for process-independent meson and baryon "distribution amplitudes" $\phi(x_i,Q)$ which control the valence-quark distributions in high-momentum-transfer exclusive reactions. The analysis can be carried out systematically in powers of $\alpha_s(Q^2)$, the QCD running coupling constant. Although the calculations are most conveniently carried out using light-cone perturbation theory and the light-cone gauge, we also present a gauge-independent analysis and relate the distribution amplitude to a gauge-invariant Bethe-Salpeter amplitude.

Exclusive Processes in Perturbative QCD

- Systematic analysis of inclusive processes at large energy.
- Introduce the meson light cone distribution amplitude $\phi_M(x, \mu^2)$
  - Probability amplitude for converting the meson into a (collinear) quark anti-quark pair with momentum fraction x and $(1-x)$, respectively.

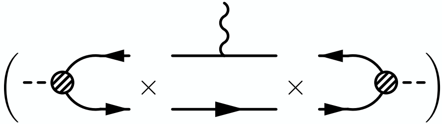


The Pion Form Factor

- Within this framework, one can calculate the pion form factor as the convolution of three factors:

$$F_\pi(Q^2) \underset{\text{large } Q^2}{=} \int_0^1 dy \int_0^1 dy \phi_{\overline{M}}(y, Q^2) T_H(x, y, Q^2) \phi_M(x, Q^2)$$

$$\underset{\text{large } Q^2}{=} \int_0^1 dy \int_0^1 dy$$



(1)

- The light cone distribution amplitudes contain all the non-perturbative information about the process.

The Light Cone Distribution Amplitude

- Can decompose in terms of Gegenbauer Polynomials

$$\phi(x, Q^2) = 6x(1-x)\sum_n a_n(Q^2)C_n^{3/2}(2x-1) \quad (2)$$

- In the limit that $Q^2 \to \infty$, $\phi(x, Q^2) \to 6x(1-x)$.
- In this decomposition, all non-perturbative information is found in the $a_n(Q^2)$: the Gegenbauer moments.
- Thus full knowledge of moments allows one to reconstruct the full LCDA.
- Process independent: Used to predict
  - Pion electromagnetic form factor, pion transition form factor, $B \to \pi\pi$, two photon processes.

Calculating the Pion's LCDA

- Only *ab-initio* method to calculate non-perturbative QCD: Lattice QCD.
- But problem: LCDA defined as

$$\phi_\pi(x,\mu^2) = \int \frac{dx^-}{(2\pi)} e^{i(2z-1)x^- p_+/2} \langle 0| \overline{q}(-x^-/2)\gamma^+ \overline{q}(x^-/2) |\pi(\mathbf{p})\rangle \tag{3}$$

  ie, on the light cone.
- Requires real time evolution: impossible in Lattice QCD.
- Number of different proposals to address this.

Options on the Market

- OPE: Moments of LCDA may be calculated via local matrix elements.

$$\langle 0| O^{\mu_0 \dots \mu_n} |\pi(\mathbf{p})\rangle = \langle \xi^n \rangle p^{\mu_0} \dots p^{\mu_n} \tag{4}$$

  - Must subtract power divergences, lot of work for single moment!
- More recently, quasi-PDF and pseudo-PDF: Relate equal time matrix elements in large P^z limit to light cone matrix elements.
  - Infinite momentum frame: Requires large boosts and matching. Obtain full x dependence!
- Feynman-Hellman Theorem: Measure energy shifts in two point function
- We pursue Heavy quark Operator Product Expansion (HOPE)

Operator Product Expansion

- Wilson: Expand a non-local operator as the sum of local operators

$$T\{A(z/2)B(-z/2)\} = \sum_n C_n(z^2) z_{\mu_1} \dots z_{\mu_n} \mathcal{O}_n^{\mu_1 \dots \mu_n}(0) \tag{5}$$

- Example of factorization.
- Relegate short distance physics to $C_n(z^2)$: perturbatively calculable!
- Non-pert. physics stored in moments $\langle \xi^n \rangle (\mu^2)$:

$$\langle 0 | \mathcal{O}_n^{\mu_1 \dots \mu_n}(0) | \pi(\mathbf{p}) \rangle = \langle \xi^n \rangle (\mu^2) p^{\mu_1} \dots p^{\mu_n} - \mathrm{Tr} \tag{6}$$

- Recall:

$$\phi(x, \mu^2) = 6x(1-x) \sum_n a_n(\mu^2) C_n^{3/2}(2x-1) \tag{7}$$

- OPE allows one to reconstruct full LCDA!

Heavy quark Operator Product Expansion

- Aim to study the matrix element $U^{\mu\nu} = (T^{\mu\nu} - T^{\nu\mu})/2$ where

$$T^{\mu\nu}(p,q) = \int d^4 z e^{iq\cdot z} \langle 0| T\{J^\mu(z/2) J^\nu(-z/2)\} |\pi(\mathbf{p})\rangle \quad (8)$$

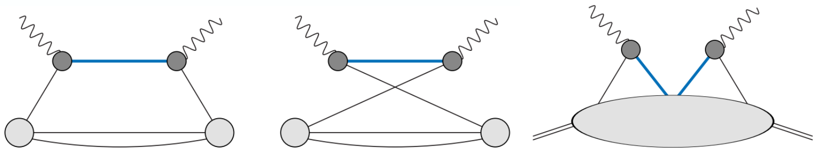
- Replace current J^μ with heavy-light current:

$$J_Q^\mu(x) = \overline{q}(x)\gamma^\mu\gamma_5 Q(x) + \overline{Q}(x)\gamma^\mu\gamma_5 q(x) \quad (9)$$

- $Q(x)$ is a fictitious heavy quark species.
- By tuning the mass ($m_Q \sim \sqrt{-q^2}$), we relegate the heavy quark effects to Wilson Coefficients.
- Moments $\langle \xi^n \rangle (\mu^2)$ unchanged.

Advantages of Heavy Quark Approach

- Use of heavy quark has a number of advantages:
- Theoretical
  - Some higher twist contributions removed (cat's ears diagram)
  - Heavy quark mass provides additional contribution to $\tilde{Q}^2 = Q^2 + m_Q^2$: Since higher twist contributions arise from eg $1/\tilde{Q}^2$, these are suppressed.
- Computational
  - Heavy quark is cheap to calculate.



HOPE: Final Equation

- Also summed target mass effects: Improve agreement at sub-asymptotic scales.

$$U^{\mu\nu}(p,q) = \frac{2if_\pi \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2} \sum_{\text{even}} \frac{\mathcal{C}_n^{(2)}(\eta)}{n+1} \zeta^n \underbrace{C_n(\tilde{Q}^2/\mu^2, \tilde{Q}^2/m_Q)}_{\text{hard}} \underbrace{\langle \xi^n \rangle}_{\text{soft}} \tag{10}$$

- where $\eta = p \cdot q/\sqrt{p^2 q^2}$, $\zeta = \sqrt{p^2 q^2}/\tilde{Q}^2$

Roadmap to the Pion's LCDA

$$T^{\mu\nu}(p,q) = \int d^4z e^{iq\cdot z} \langle 0| T\{J^{\mu}(z/2)J^{\nu}(-z/2)\} |\pi(\mathbf{p})\rangle \qquad (11)$$

Pert. Calculable Wilson Coefficents $C_n(\tilde{Q}^2)$

HOPE: $U^{\mu\nu}(p,q) = \frac{2f_\pi \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2} \sum_n C_n(\tilde{Q}^2) a_n(\mu^2) \omega^n$

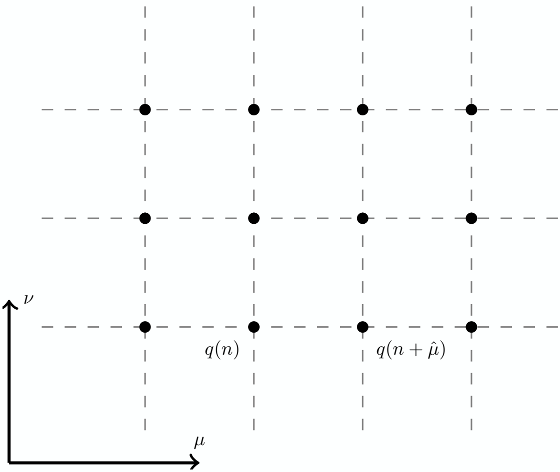
Simulate $T^{\mu\nu}(p,q;a)$ on lattice at finite a

Extrapolate $T^{\mu\nu}(p,q;a)$ to the continuum

Fit continuum $U^{\mu\nu}(p,q)$ to HOPE formula. Extract $a_n(\mu^2)$.

Lattice QCD

- Construct Euclidean lattice with $n_x \times n_y \times n_z \times n_t$ vertices separated by distance a: UV regulator.
- Choose suitable boundary conditions
- Must relate Euclidean space finite volume matrix elements to Minkowski space continuum matrix elements.



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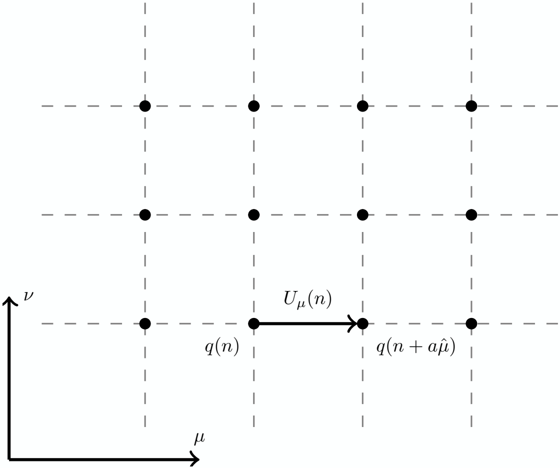

Lattice Details

- Currently, quenched calculation: Proof of principle.
- Simulate at constant physical volume: $L = 1.9$ fm.
- Pion mass $m_\pi = 0.55$ GeV
- Continuum extrapolation:
  - $32^3 \times 64$, $m_\pi L \sim 5$
  - $40^3 \times 80$
  - $48^3 \times 96$
  - $64^3 \times 128$
- Two heavy quark masses to study heavy quark dependence.

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Simulation Details

- Construct ratio $R^{\mu\nu}(\mathbf{p}, \mathbf{q}, \tau)$ of three-point to two-point functions

$$R^{\mu\nu}(\mathbf{p}, \mathbf{q}, \tau) = \int d^3x e^{i\mathbf{q}\cdot\mathbf{x}} \langle 0| T\{J^\mu(\frac{x}{2})J^\nu(\frac{-x}{2})\} |\pi(\mathbf{p})\rangle \propto \frac{C^{\mu\nu}(\mathbf{p}, \mathbf{q}, \tau)}{C_\pi(\mathbf{p}, \tau)} \quad (12)$$

- Performing a Fourier transform gives

$$T^{\mu\nu}(p, q) = \int d\tau e^{iq_4\tau} R^{\mu\nu}(\mathbf{p}, \mathbf{q}, \tau) \quad (13)$$

- We construct the anti-symmetric combination

$$U^{\mu\nu}(p, q) = \frac{1}{2}(T^{\mu\nu}(p, q) - T^{\nu\mu}(p, q)) \quad (14)$$

Kinematic Tricks

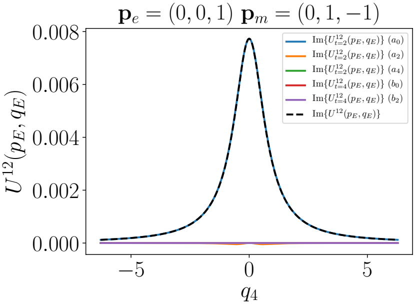
- OPE proportional to

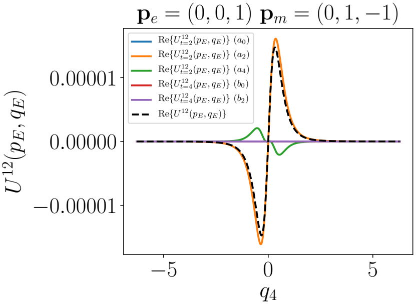
$$U^{\mu\nu}(p,q) \sim \sum_{n=0}^{\infty} \langle \xi^n \rangle \, \omega^n, \qquad \omega = \frac{2p \cdot q}{\tilde{Q}^2} = \frac{1}{x} \tag{15}$$

- Physical region $\omega > 1$, $x < 1$. However, on the lattice, we access $\omega < 1$.
- Thus only a few moments contribute (kinematic suppression).
- By increasing $2p \cdot q$ while keeping $\tilde{Q}^2$ fixed lets us enhance the contribution from higher moments.
- But! Requires boosted pion: increase excited states, require momentum smearing to access large momenta
- We can optimize the kinematics.

Kinematic Tricks

- Split matrix element into real and imaginary parts
- In certain kinematics, Im part has no a_0 contribution: starts at a_2.





Roadmap to the Pion's LCDA

$$T^{\mu\nu}(p,q) = \int d^4z e^{iq\cdot z} \langle 0| T\{J^{\mu}(z/2)J^{\nu}(-z/2)\} |\pi(\mathbf{p})\rangle \tag{16}$$

Pert. Calculable Wilson Coefficents $C_n(\tilde{Q}^2)$

↓

HOPE: $U^{\mu\nu}(p,q) = \frac{2f_\pi \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2} \sum_n C_n(\tilde{Q}^2) a_n(\mu^2)\omega^n$

Simulate $T^{\mu\nu}(p,q;a)$ on lattice at finite a

↓

Extrapolate $T^{\mu\nu}(p,q;a)$ to the continuum

↓

Fit continuum $U^{\mu\nu}(p,q)$ to HOPE formula. Extract $a_n(\mu^2)$.

Continuum Extrapolation

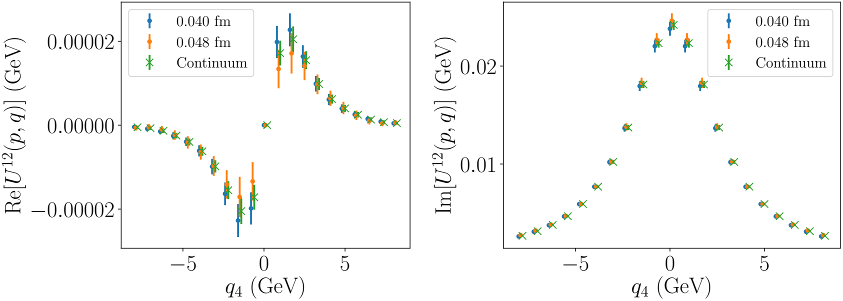
- Calculate $T^{\mu\nu}(p, q; a)$ for fixed lattice spacing a. By keeping physical volume fixed and reducing a, we approach continuum limit.
- $T^{\mu\nu}(p, q; a)$ product of conserved currents: Renormalization is simple.
- Thus we extrapolate with a functional form

$$T^{\mu\nu}(p, q; a) = T^{\mu\nu}(p, q; 0) + aT^{\mu\nu}_{(1)}(p, q) + a^2 T^{\mu\nu}_{(2)}(p, q) + \dots \quad (17)$$

- $T^{\mu\nu}(p, q; 0)$ is our continuum limit.

Continuum Extrapolation

- Using $L = 40$, $L = 48$ data



Roadmap to the Pion's LCDA

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Extrapolate $T^{\mu\nu}(p,q;a)$ to the continuum

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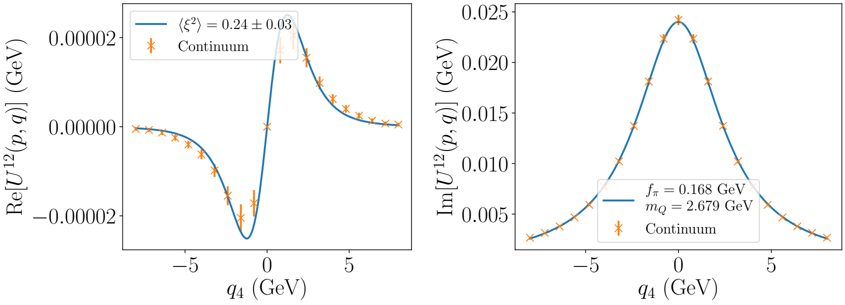
Fit continuum $U^{\mu\nu}(p,q)$ to HOPE formula. Extract $a_n(\mu^2)$.

Results (Preliminary!)

- Currently Wilson coefficents are unity: $\sim 10\%$ systematic error. To be corrected in full analysis.

$$U^{\mu\nu}(p,q) = \frac{2if_\pi \epsilon^{\mu\nu\alpha\beta} q_\alpha p_\beta}{\tilde{Q}^2}\left[1 + \frac{1}{3}\mathcal{C}_2^{(2)}(\eta) C_2(\tilde{Q}^2/\mu^2, \tilde{Q}^2/m_Q) a_2(\mu^2)\xi^2\right] \tag{19}$$



Comparison with Other Results

f_π	0.167 $\pm$ 0.003 GeV
$\langle \xi^2 \rangle$	0.24 $\pm$ 0.03
m_Q	2.68 $\pm$ 0.02 GeV

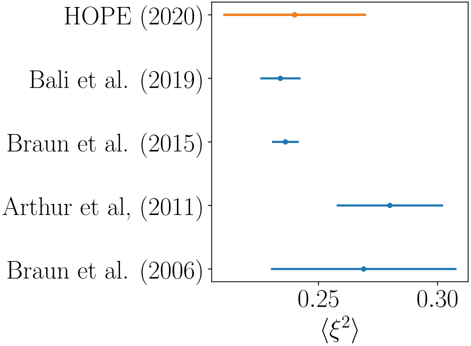


Figure 1: Other calculations at $\overline{\text{MS}}$ $\mu = 2$ GeV. We have neglected the chiral extrapolation.

Conclusion

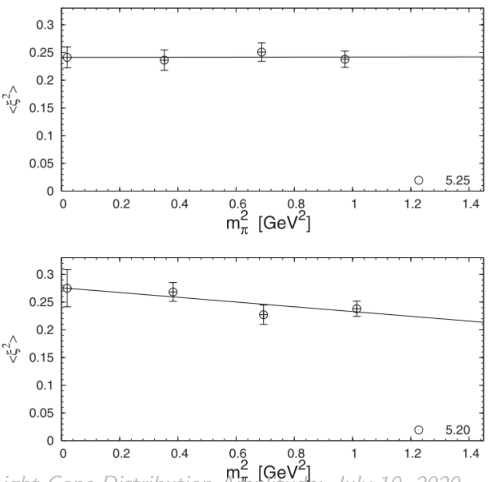
- Pion light cone distribution amplitude important for exclusive measurements at high energies: process independent
- We have used the Heavy quark Operator Product Expansion to extract the second Mellin Moment.
  - Require Wilson Coefficients to accurately extract value.
- Higher momentum will allow us to extract higher moments.
  - Requires more sophisticated momentum smearing.

Thanks

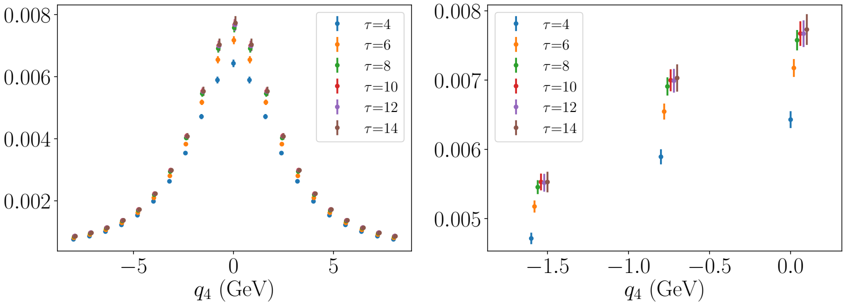
Spare Slides

Chiral Extrapolation

- From Braun et al., (2006): Observable fairly flat in m_π.
- We expect chiral extrapolation to be small.



Excited State Analysis



- $L = 32$. Vary first current insertion time.

The Parton Distribution Function on the Lattice

- On the lattice, we must calculate the parton distribution function via the moments

$$a_n(\mu^2) = \int_0^1 dx x^{n-1} f(x, \mu^2)$$

- Related to local matrix elements as

$$\langle p | \mathcal{O}^{\mu_1 \dots \mu_n} | p \rangle = a_n(\mu^2) p^{\mu_1} \dots p^{\mu_n} - \mathrm{Tr}$$

- In principle, can reconstruct the x dependence if we know all moments, ie via inverse Mellin Transform

Equations

$$C_\pi(x_E) = \langle 0| \, T\{\mathcal{O}_{\pi^+}(x_E)\mathcal{O}^\dagger_{\pi^+}(0)\} \, |0\rangle \tag{20}$$

$$C_3^{\mu\nu}(x_E, y_E) = \langle 0| \, T\{J^\mu(x_E)J^\nu(y_E)\mathcal{O}^\dagger_\pi(0)\} \, |0\rangle \tag{21}$$

$$T^{\mu\nu}(p_E, q_E) = \int dY_4 e^{-iY_4 \cdot q_4} \frac{C_3^{\mu\nu}(x_4, \mathbf{p}_1, y_4, \mathbf{p}_2)}{C_\pi((x_4+y_4)/2), \mathbf{p}_2 + \mathbf{p}_2)} \sqrt{Z_\pi(\mathbf{p}_1 + \mathbf{p}_2)} \tag{22}$$

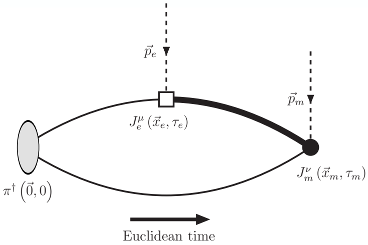
where we identify

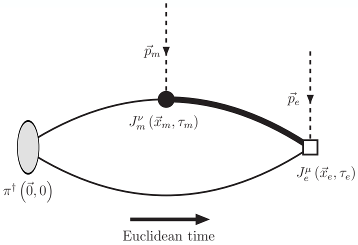
$$p_E = (iE_\pi(\mathbf{p_1} + \mathbf{p}_2), \mathbf{p}_1 + \mathbf{p}_2) \tag{23}$$

$$q_E = (q_4, (\mathbf{p}_1 - \mathbf{p}_2)/2) \tag{24}$$

3-Point Function Calculation

- Utilize a sequential source: Fix momentum insertion at $\mathbf{p}_e$



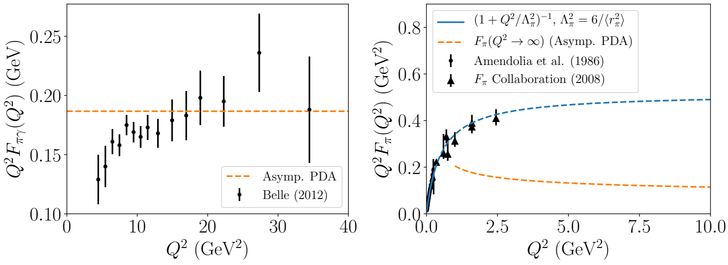


Predictions

- Leads to predictions about the asymptotic forms of various exclusive processes:

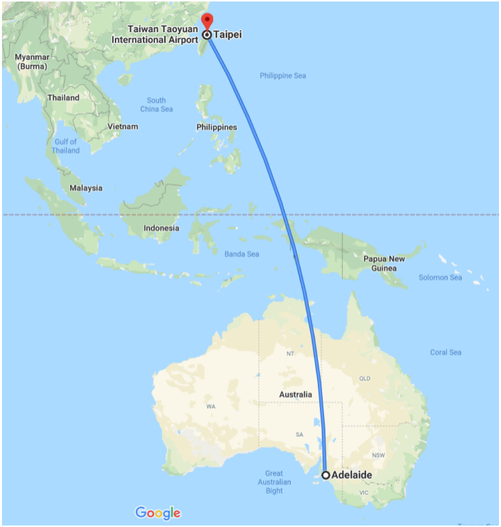
$$Q^2 F_{\gamma\pi}(Q^2) = \sqrt{2} f_\pi \left(\sum_n a_n(Q^2) \right) + \dots \tag{25}$$

$$Q^2 F_\pi(Q^2) = 16\pi f_\pi^2 \alpha_S(Q^2) \left(\sum_n a_n(Q^2) \right)^2 + \dots \tag{26}$$



- Also $B \to \pi\pi$, two photon processes: many observables of interest!

Where Am I From?



THE UNIVERSITY
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Particle Physics at Adelaide

- Special Research Centre for the Subatomic Structure of Matter (CSSM)
  - numerical simulations of fundamental quantum theories (such as QCD)
  - effective field theory calculations
  - building models that capture the essential degrees of freedom in complex systems
- ARC Centre of Excellence for Particle Physics at the Terascale (CoEPP)
  - Supersymmetry (SUSY)
  - Dark matter
  - LHC / ATLAS data analysis