Smearing and extracting states in LQCD

TQCD workshop @ Academia Sinica

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Outline

- Current project: Composite Higgs Model

  Sp(4) gauge theories for BSM models on the lattice
- Lattice method
- Smearing (APE & Gaussian) technique
- Extracting excited states: variation method
- Quenched QCD as an example

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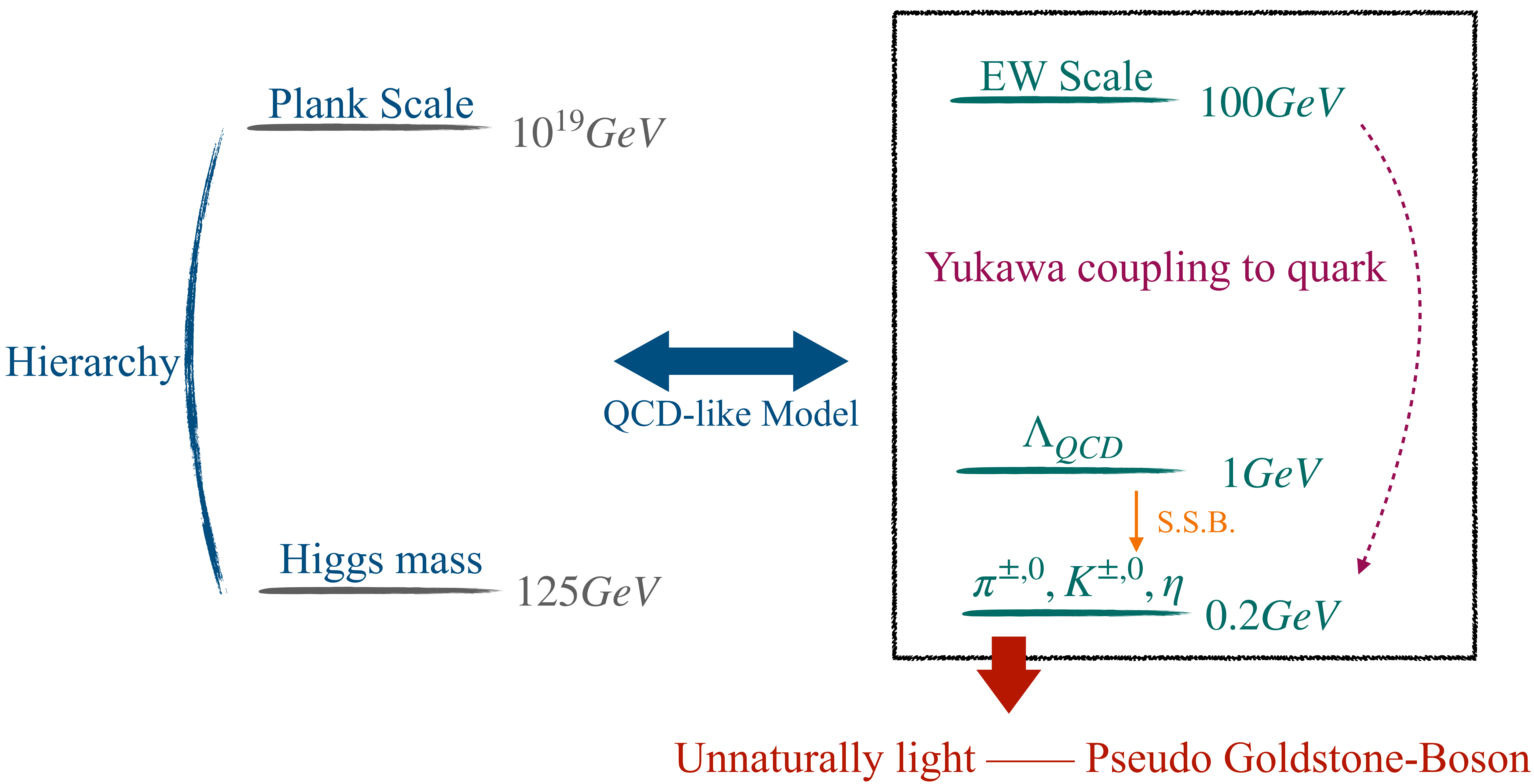
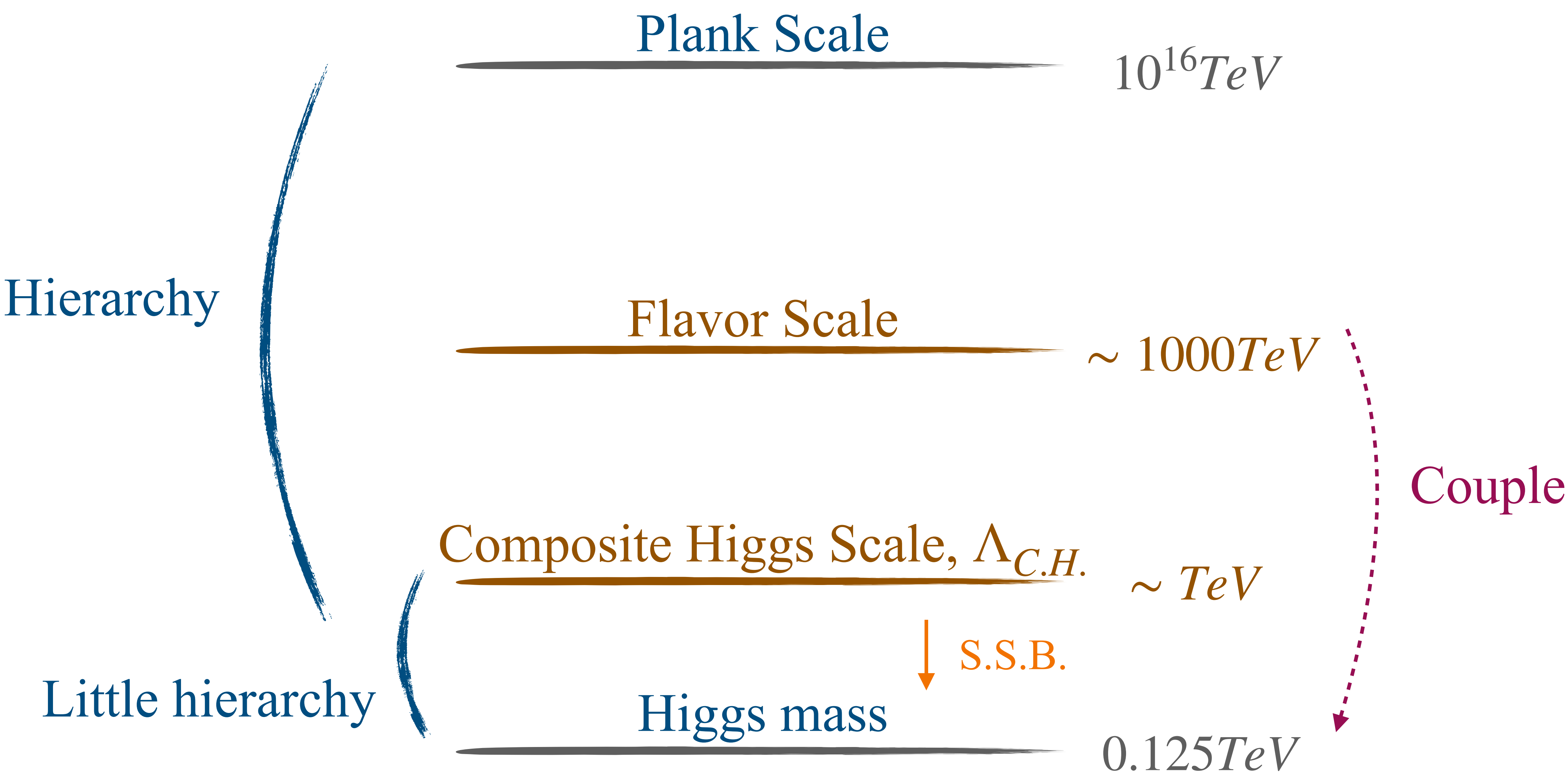
Composite Higgs Model



Composite Higgs Model



➡ Higgs boson as a bound state of new strong dynamics, which is lighter because of being a Pseudo Goldstone Boson.

Our project

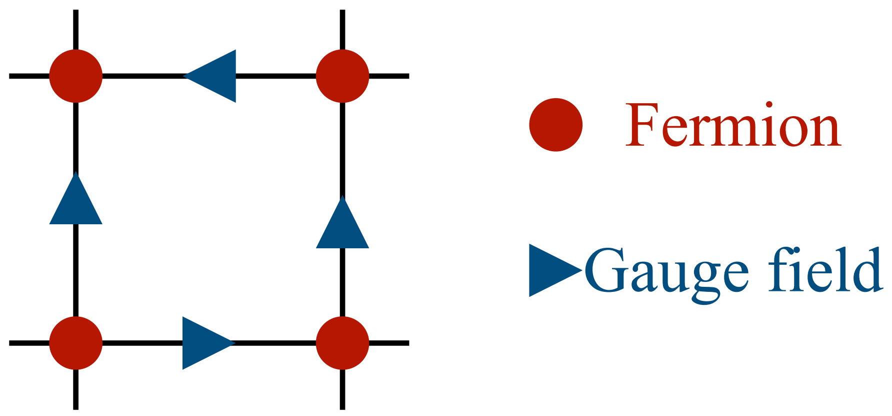
- Sp(4) gauge + 2 fundamental Dirac fermions
- Global symmetry breaking pattern: SU(4)/Sp(4)

  → 5 Goldstone boson
- Study spectrum to gain basic understanding
- Compute 4-fermion operator matrix elements

  → relevant to generating Higgs mass

Lattice Method

- Strongly coupled theory → lattice field theory
- Fermions on the grids, carrying colors, spin or flavors
- Gauge fields on the links
- Generating functional



$$Z = \int DUD\psi D\bar{\psi} e^{-S[U]} e^{-2\int d^4x \bar{\psi}(D[U]+m)\psi}$$

$$= \int DU \det(D[U]+m)^2 e^{-S[U]}$$

→ (Hybrid) Monte-Carlo simulation

Quench calculation: $\det(D[U]+m) = 1$

Lattice Method

Measuring observables

- 2-point correlation function

$$C(t) = \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0 | \, T\left[O(\vec{x},t)O^{\dagger}(0,0)\right] \, |0\rangle$$

$$\sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \langle 0 | \, \left[\bar{u}\gamma_5 d\right](\vec{x},t)\left[\bar{d}\gamma_5 u\right](0,0) \, |0\rangle$$

$$= \sum_{\vec{x}} e^{-i\vec{p}\cdot\vec{x}} \mathrm{Tr}\left[S_u(0,0;\vec{x},t)S_d^{\dagger}(0,0;\vec{x},t)\right]$$

$$S = M^{-1} q$$

$$\sum_n \frac{\langle 0 | \, O_\pi \, |n\rangle \langle n | \, O_\pi^{\dagger} \, |0\rangle}{2E_n} e^{-E_n t}$$

$$\xrightarrow{t\to\infty} \frac{1}{2m_\pi} \left| \langle 0 | \, O_\pi \, |n\rangle \right|^2 e^{-M_\pi t}$$

M is the Dirac operator calculated on a given background field.

- Effective Mass

$$M_{eff}(t) = -\ln\left[\frac{C(t+1)}{C(t)}\right]$$

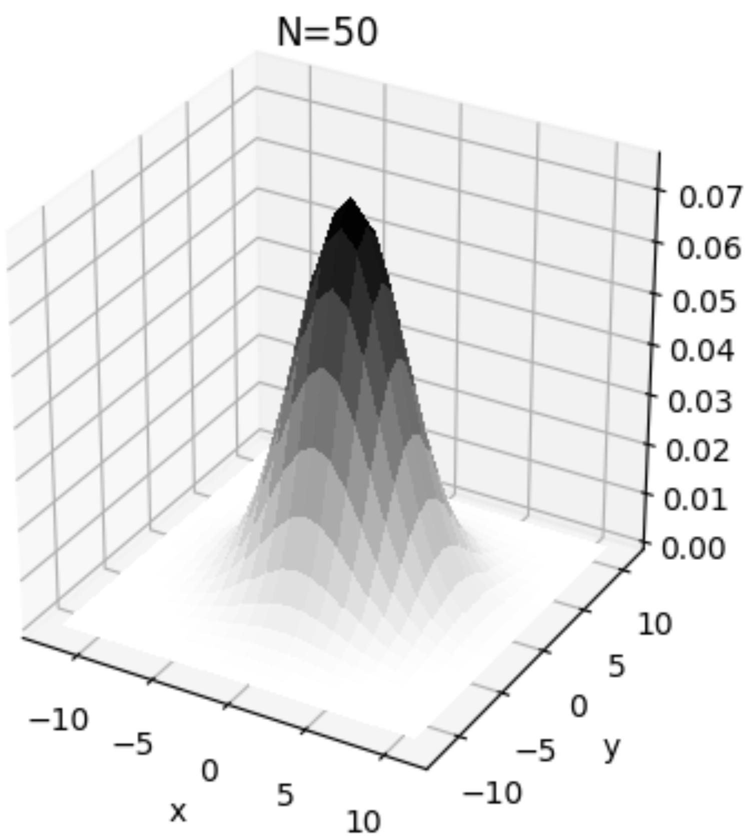
Smearing

- Wuppertal smearing (Gaussian smearing) acts on fermion field increasing the overlap of ground state.

$$q^{(n+1)}(x) = \frac{1}{1+2d\varepsilon}\left[q^{(n)}(x) + \varepsilon \sum_{\mu=\pm 1}^{\pm d} U_\mu(x) q^{(n)}(x+\hat{\mu})\right]$$

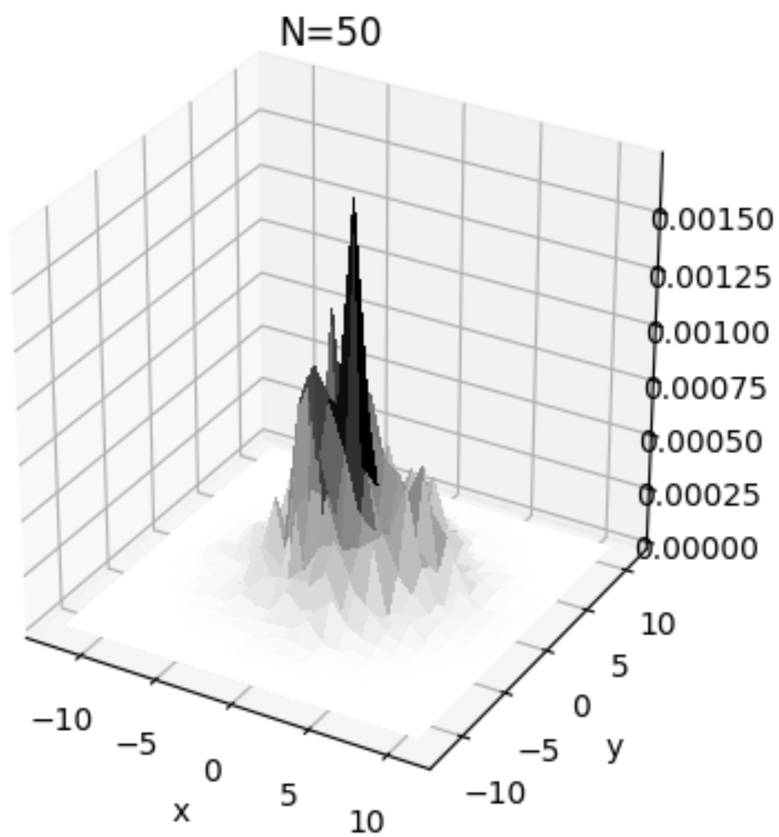
Free fermion, $\varepsilon = 0.1$

Real conf., $\varepsilon = 0.1$



Source density

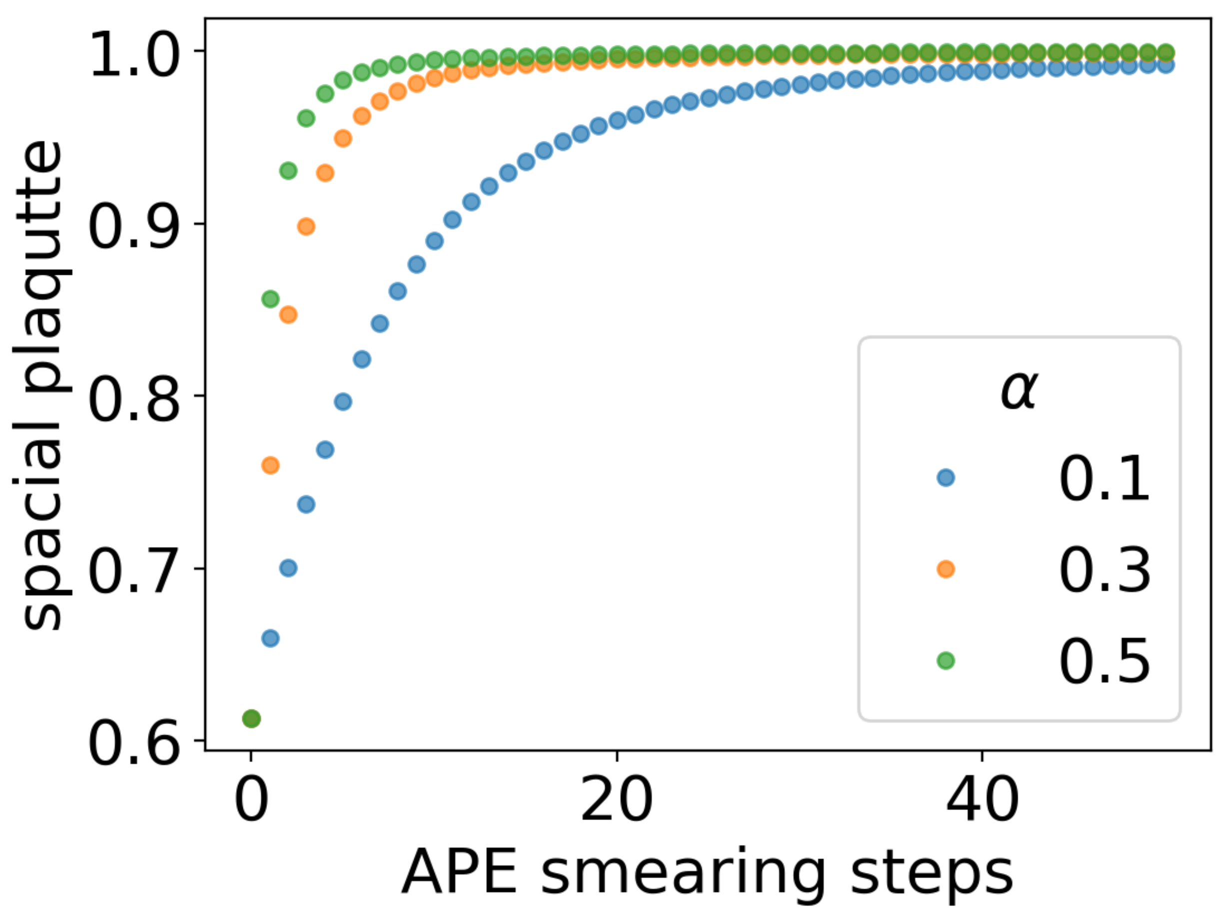
$$\rho(x) = \frac{\sum_{ab}\left|q^{ab}(x)\right|^2}{\sum_x \sum_{ab}\left|q^{ab}(x)\right|^2}$$

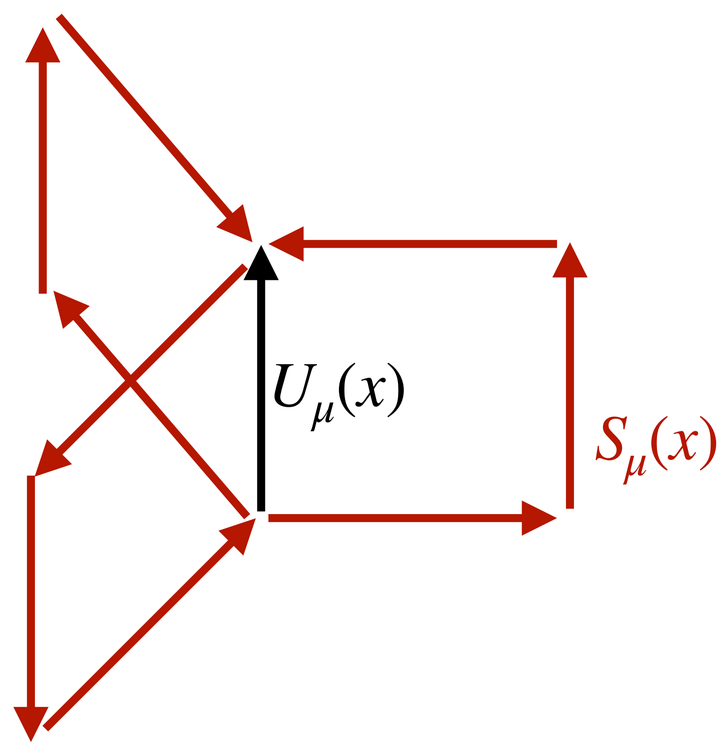


Smearing

- APE smearing averages out UV fluctuations of the gauge fields.

$$U_\mu^{(n+1)}(x) = P\left\{(1-\alpha)U_\mu^{(n)}(x) + \frac{\alpha}{6}S_\mu^{(n)}(x)\right\}, \quad S_\mu(x) = \sum_{\pm\nu\neq\mu} U_\nu(x)U_\mu(x+\hat{\nu})U_\nu^\dagger(x+\hat{\mu})$$

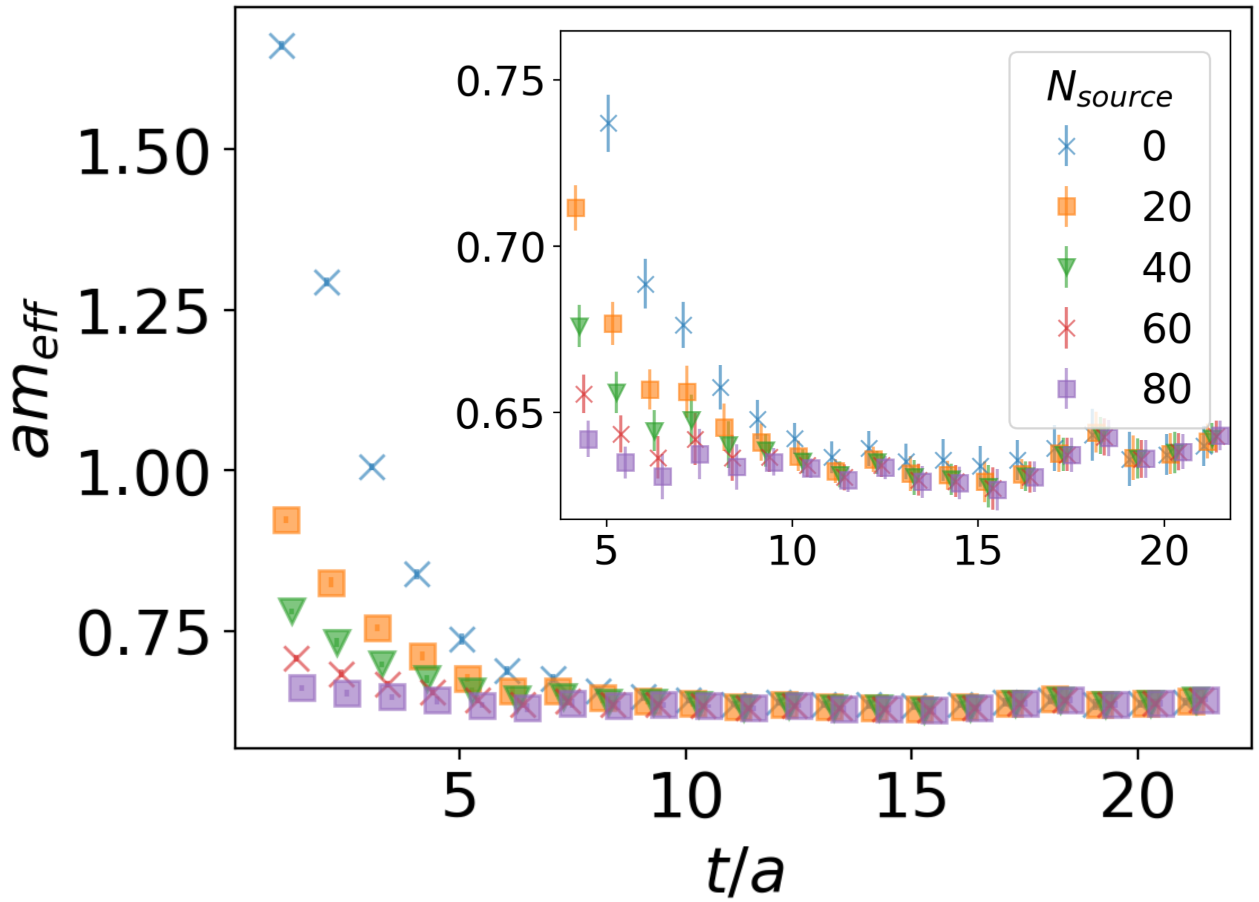




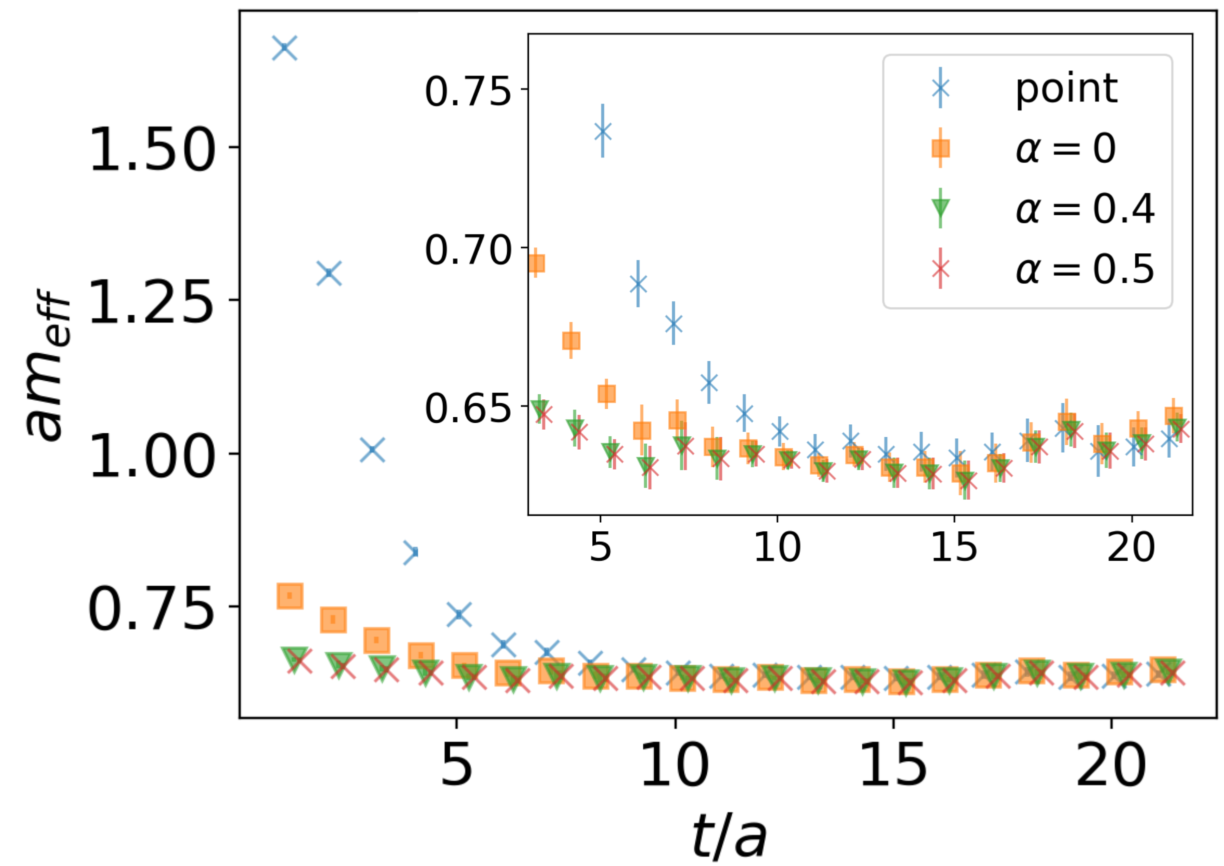
Smearing effects

Results

- Effects of Wuppertal smearing



- Effects of APE smearing



Variational Method

- General Eigenvalue problem

$$C(t_2)v_n(t_2,t_1) = \lambda_n(t_2,t_1)C(t_1)v_n(t_2,t_1) \rightarrow \lambda_n(t_2,t_1) = e^{-E_n(t_2-t_1)}$$

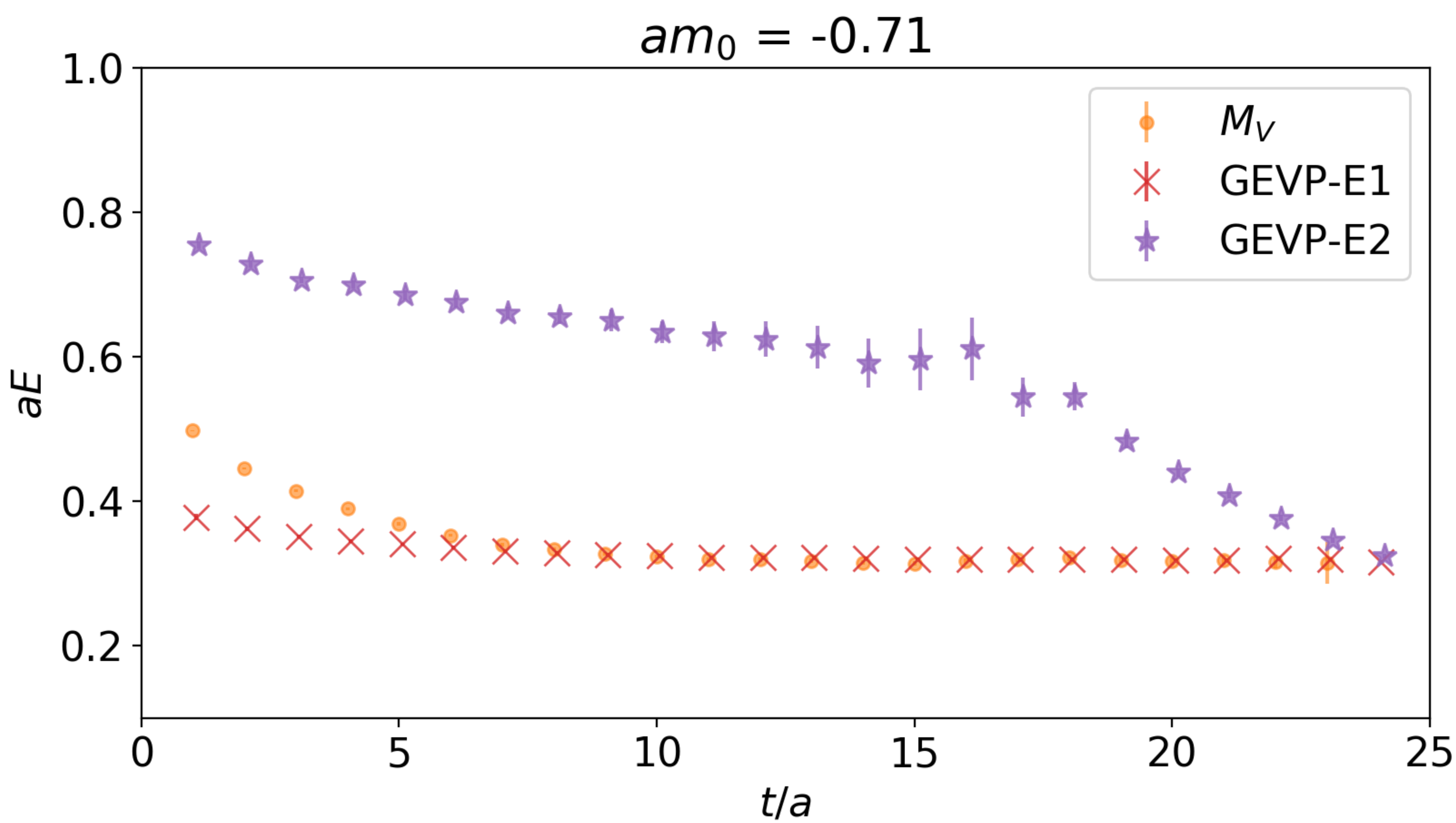
The matrix $C(t)$ is constructed by different interpolating operators. Here, we vary the operators by the number of steps of Wuppertal smearing at source and sink with fixed step size ε.

source **sink**

$$C(t) = \begin{pmatrix} c_\varepsilon^{(N_1,N_1)}(t) & c_\varepsilon^{(N_1,N_2)}(t) & \dots & c_\varepsilon^{(N_1,N_n)}(t) \\ c_\varepsilon^{(N_2,N_1)}(t) & c_\varepsilon^{(N_2,N_2)}(t) & & \\ \vdots & & \ddots & \\ c_\varepsilon^{(N_n,N_1)}(t) & c_\varepsilon^{(N_n,N_2)}(t) & & c_\varepsilon^{(N_n,N_n)}(t) \end{pmatrix}$$

Example: SU(3) gauge theory

- Lattice: $48 \times 24 \times 24 \times 24$, $\beta = 6.2$ $\longrightarrow$ $a^{-1} \approx 2.73(5)GeV$
- 280 configurations
- Wilson fermion



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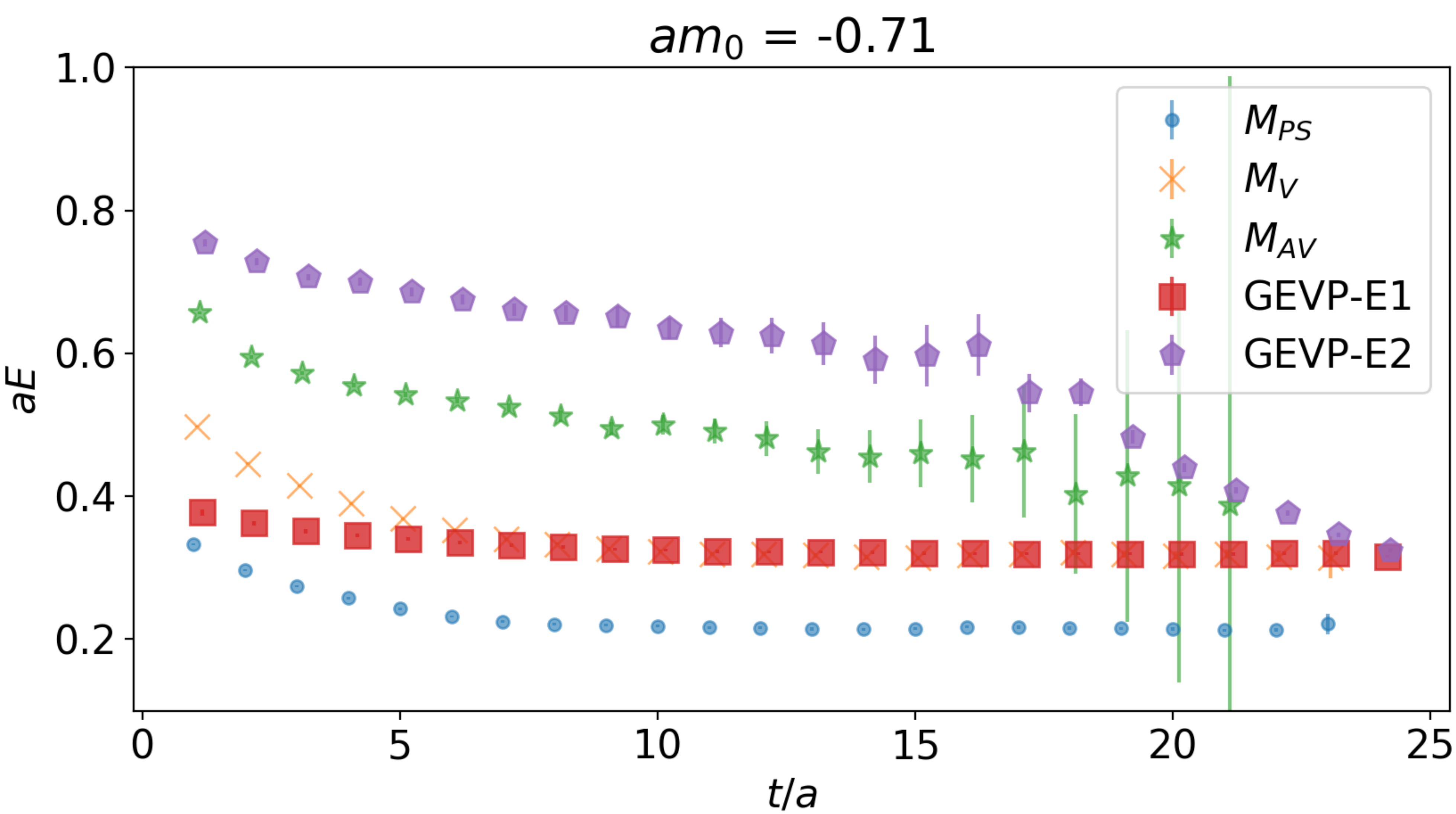
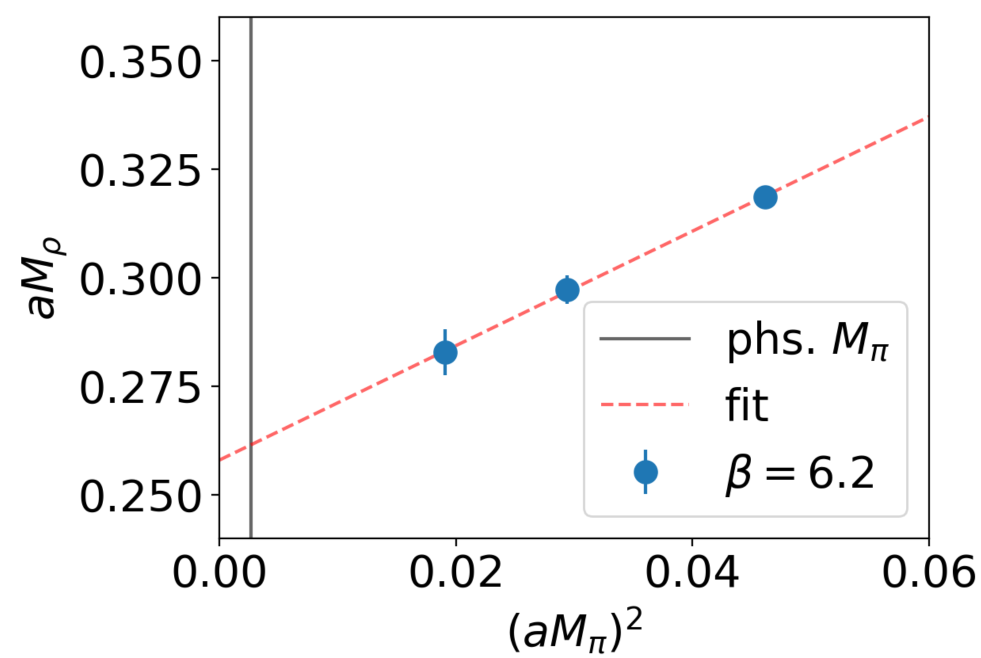


Example: SU(3) gauge theory

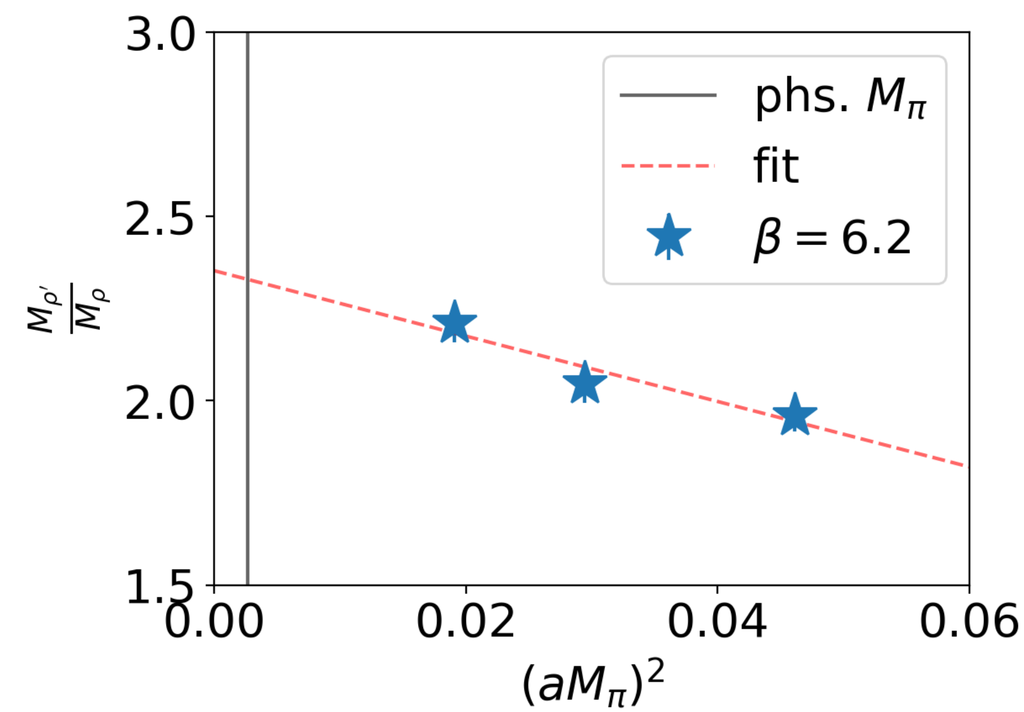
am_0 / ch	-0.71	-0.72	-0.727
PS	0.2148(12)	0.1715(17)	0.1382(18)
V	0.3186(23)	0.2973(33)	0.2828(53)
E1	0.3206(20)	0.2986(24)	0.2822(31)
E2	0.628(14)	0.611(16)	0.624(17)
E2/E1	1.961(45)	2.045(51)	2.212(54)
AV	0.463(30)	0.466(16)	0.438(37)

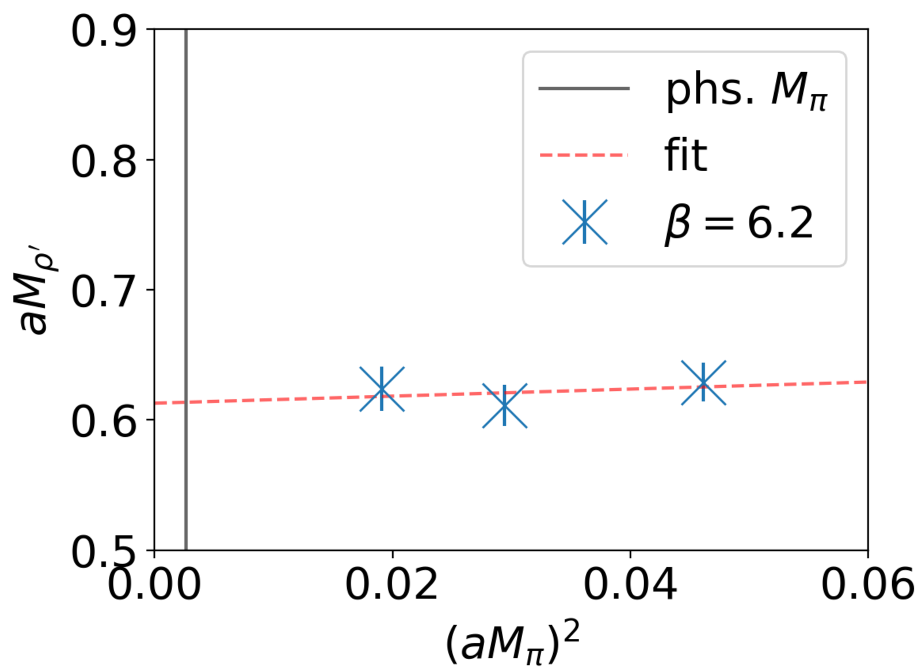
SU(3) gauge theory example

$a^{-1} \approx 2.73(5) GeV$



M_ρ at physical M_π
approximates 715MeV
physical M_ρ is roughly 770 MeV





$M_{\rho'}$ at physical M_π
approximates 1665 MeV
physical $M_{\rho'}$ is roughly 1450/1700 MeV

Summary

- Both Wuppertal smearing and APE smearing bring the plateau of the effective mass earlier.
- The variational method extracts the excited states by solving the general eigenvalue problem.
- In this study, we measured the ρ mass at 715 MeV and ρ' at 1665 MeV