QCD monopoles and the dual Meissner effect

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2018.4 - 2021.3 Doctor of Science, Kochi Univ.
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http://between.shinken-ad.co.jp/univ/2018/07/kochidai.html

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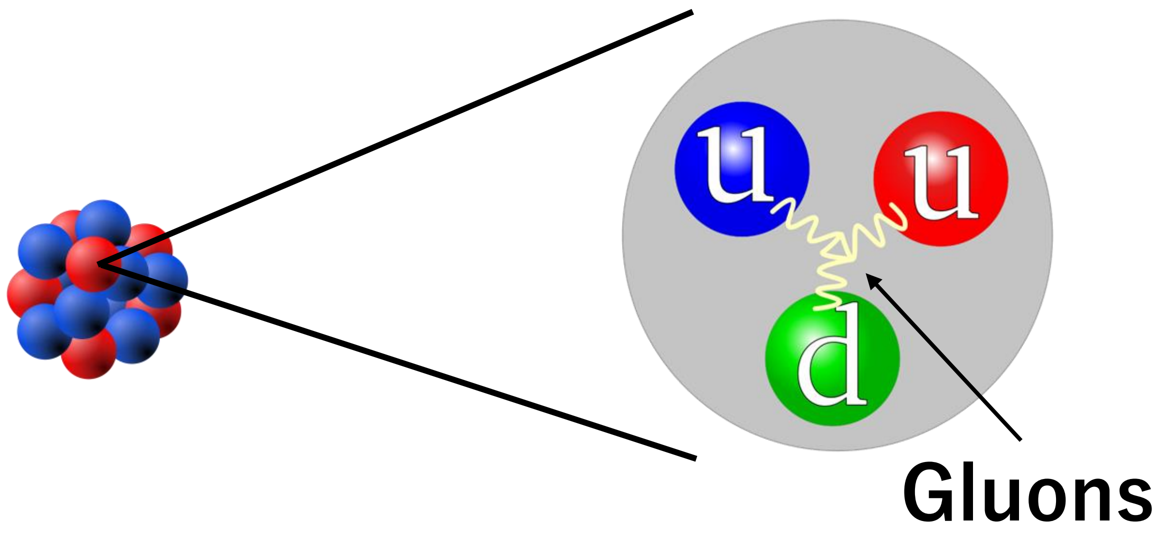
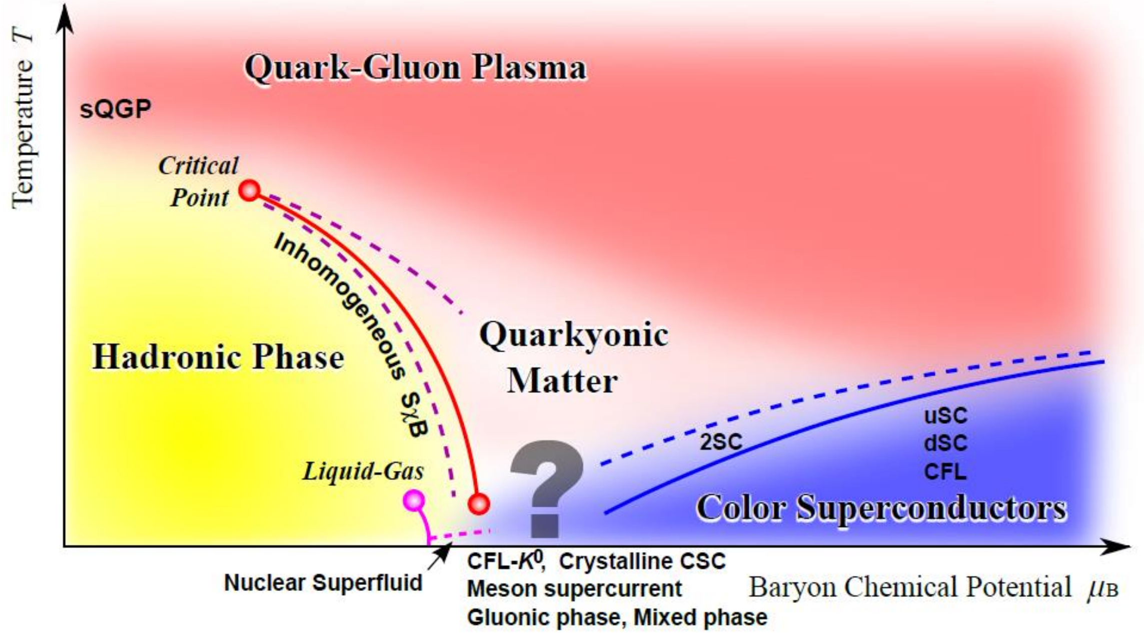
1. Introduction

1.1 Quark confinement

QCD

Hadrons





K.Fukushima,T.Hatsuda Rept.Prog.Phys.(2011)

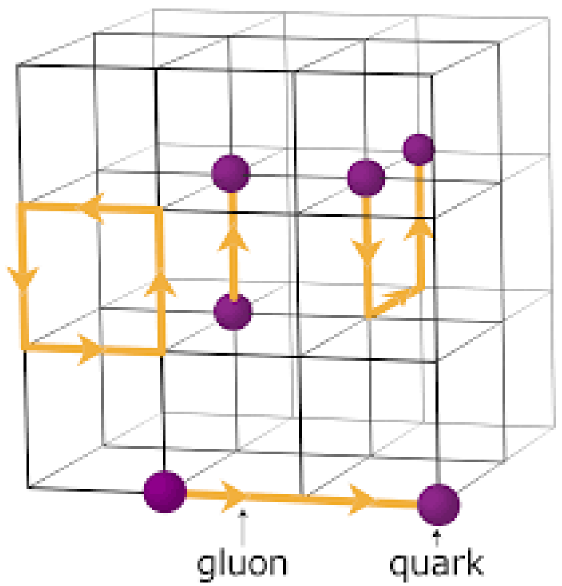
- Color confinement
- Chiral symmetry breaking

Lattice QCD

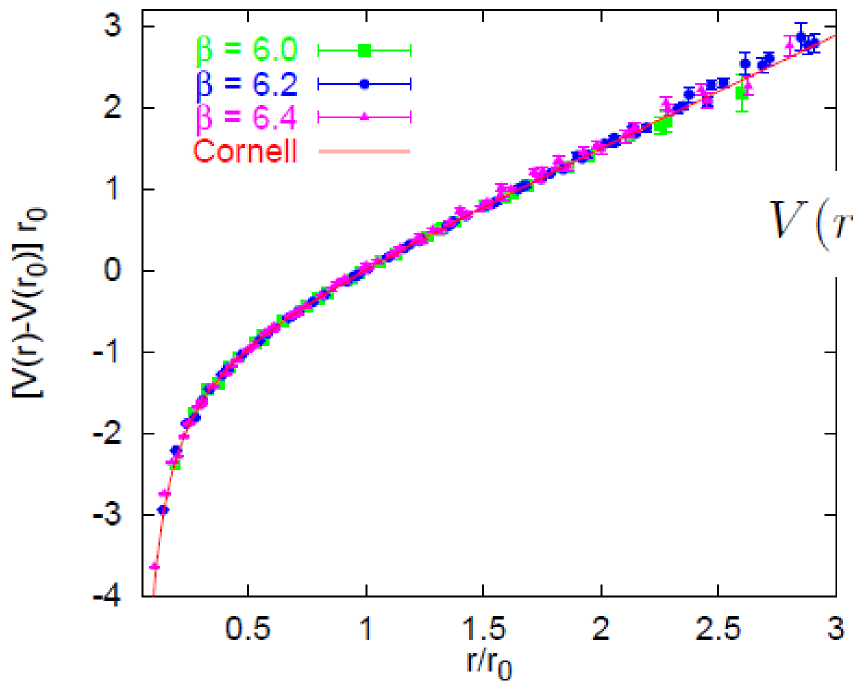
The static potential between quark and anti-quark from Wilson loop.

Link variables

$U_\mu(s)$



http://vietnam.in2p3.fr/2018/windows/transparencies/05_friday/01_morning/HEP/03_Patella.pdf



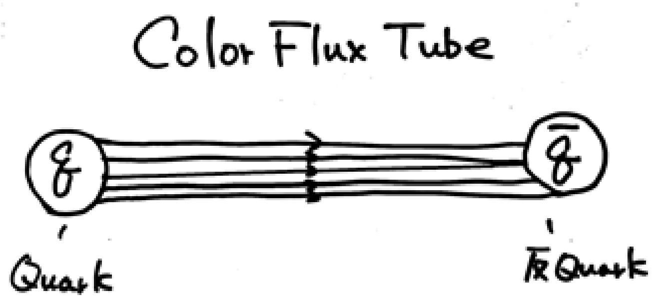
Linear potential

$V(r) = \sigma r - c/r + \mu$

Gunnar S.Bail Phys.Rept.343:1-136,2001

Monte Carlo simulation

Color flux tube

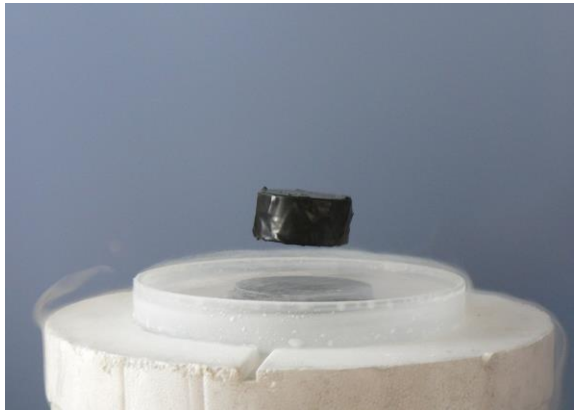


The dual Meissner effect

G. 't Hooft, in Proceedings of the EPS International, edited by A. Zichichi (Editrice Compositori, Bologna, 1976), p. 1225.

S. Mandelstam, Phys. Rep. 23, 245 (1976).

Fig 1. Magnetic flux pinning



Wikipedia

QCD		Superconductivity
Color electric flux-tube	⟺	Magnetic flux-tube
Condensation of color magnetic monopoles	⟺	Condensation of pair of electrons (Cooper pairs)

1. Introduction

1.2 Previous researches of monopole dominance

Abelian projection

G. 't Hooft, *Nucl. Phys.* **B190**, 455 (1981).

Partial gauge fixing: SU(2) → U(1)

Gauge condition: $X \to \tilde{X}(x) = V(x)X(x)V^\dagger(x) = \mathrm{diag}\{\lambda_1, \lambda_2\}$

⇒ $d\tilde{X}d^\dagger = \tilde{X} \qquad \mathrm{d} \in \mathrm{U}(1)$

Monopole currents as topological current

$$k^\nu = -\frac{1}{2g}\epsilon_{\mu\nu\rho\sigma}\epsilon_{abc}(\partial_\mu \hat{Y}^a)(\partial_\rho \hat{Y}^b)(\partial_\sigma \hat{Y}^c)$$

Dirac condition: $g g_m = 4\pi n$

Gauge fixings

- Maximal Abelian Gauge fixing (MAG)

$$R = \sum_{s,\hat{\mu}} \mathrm{Tr}\Big(\sigma_3 \widetilde{U}(s,\hat{\mu})\sigma_3 \widetilde{U}^\dagger(s,\hat{\mu})\Big)$$

A.S. Kronfeld et al., Phys. Lett. B198, (1987) 516; A.S. Kronfeld et al. , Nucl.Phys.B293, (1987) 461

- Landau Gauge fixing

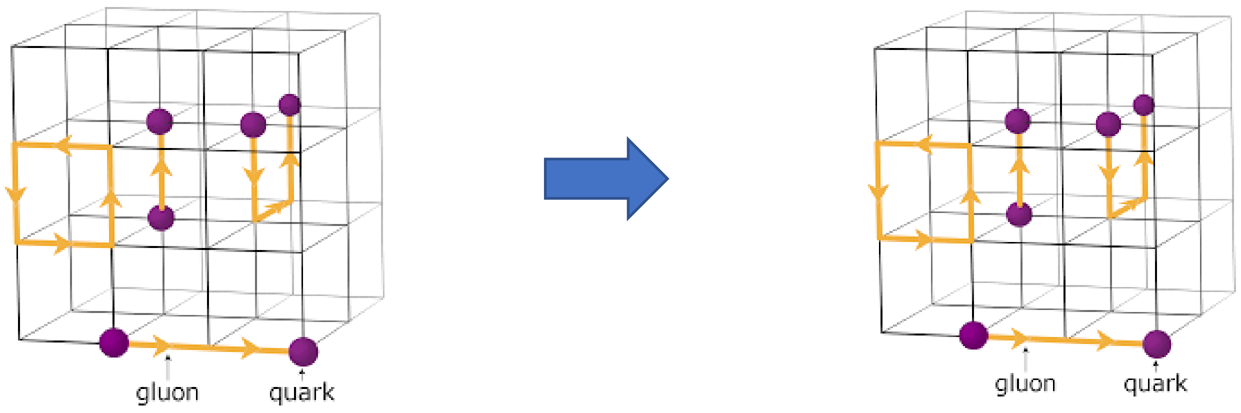
$$R = \sum_{s,\mu} \mathrm{Tr}\, U_\mu(s)$$

- Maximal center gauge, Polyakov loop gauge, …

Abelian projection on lattice

$U_\mu(s)$

$$u_\mu(s) = \begin{pmatrix} e^{i\theta_\mu(s)} & 0 \\ 0 & e^{-i\theta_\mu(s)} \end{pmatrix}$$



$$U_\mu(s) = \begin{pmatrix} \sqrt{1-|c_\mu(s)|^2} & -c_\mu(s)^* \\ c_\mu(s) & \sqrt{1-|c_\mu(s)|^2} \end{pmatrix} \begin{pmatrix} e^{i\theta_\mu(s)} & 0 \\ 0 & e^{-i\theta_\mu(s)} \end{pmatrix}$$
$$\equiv C_\mu(s) u_\mu(s)$$

U(1) gauge field : $\theta_\mu(s) = \tan^{-1} \dfrac{U_\mu^3(s)}{U_\mu^0(s)}$

The definition of monopoles on the lattice

T. A. DeGrand and D. Toussaint, Phys. Rev. D22, 2478 (1980)

Monopole currents on the lattice, $\theta_\mu \in [-\pi, \pi]$

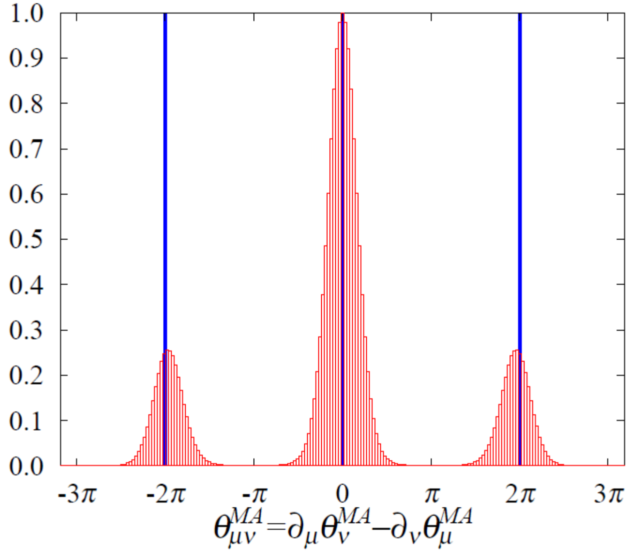
$$k_\nu(s) = \frac{1}{4\pi}\epsilon_{\mu\nu\rho\sigma}\partial_\mu\bar{\Theta}_{\rho\sigma}(s+\hat{\nu}) = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\mu n_{\rho\sigma}(s+\hat{\nu}) \in \mathbb{Z}$$

$$\Theta_{\mu\nu}(s) = \theta_\mu(s) + \theta_\nu(s+\hat{\mu}) - \theta_\mu(s+\hat{\nu}) - \theta_\nu(s) = \bar{\Theta}_{\mu\nu}(s) + 2\pi n_{\mu\nu}(s)$$

Current conservation law: $\partial'_\mu k_\mu(s) = 0$

Number of Dirac strings: $n_{\mu\nu} \in \{-2, -1, 0, 1, 2\}$

Maximal Abelian Gauge



T. Suzuki, K. Ishiguro, Y. Mori, and T. Sekido, AIP Conf. Proc. **756**, 172 (2005), arXiv:hep-lat/0410039.

Monopole dominance on MAG

H. Shiba and T. Suzuki, Phys. Lett. B351, 519 (1995).

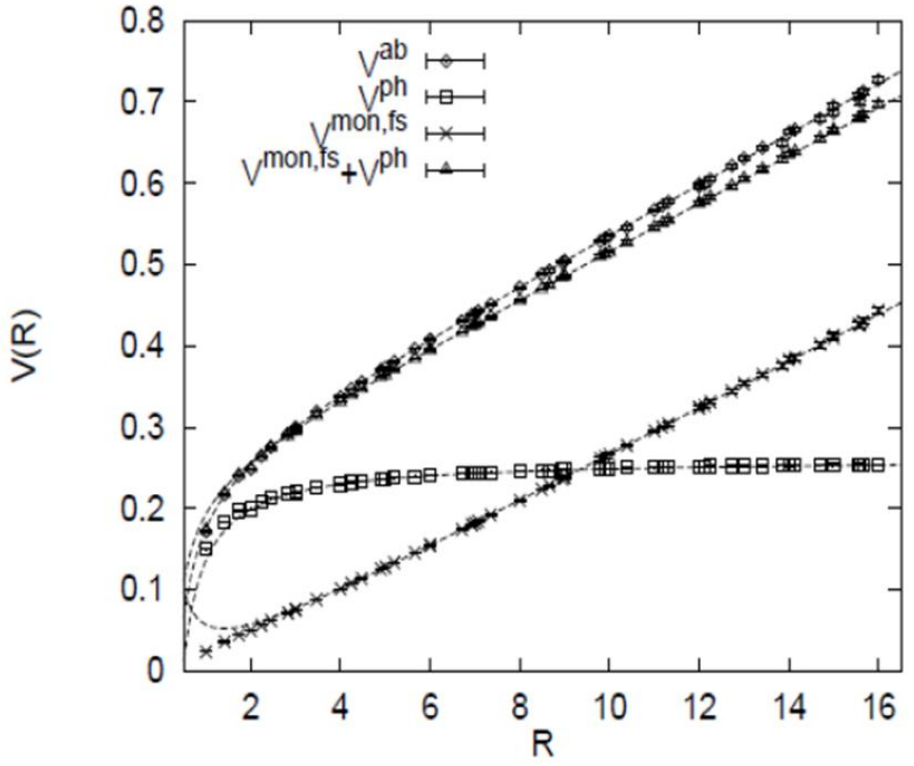
G. S. Bali, V. Bornyakov, M. Muller-Preussker and K. Schilling, Phys. Rev. D54, 2863 (1996).

$$W_A = W_{mon} W_{ph}$$

$$W_{mon} = \exp\{2\pi i \sum k_\beta(s) D(s-s') \frac{1}{2} \epsilon_{\alpha\beta\rho\sigma} \partial_\alpha M_{\rho\sigma}(s')\}$$

$$W_{ph} = \exp\{-i \sum \partial' \bar{\theta}_{\mu\nu}(s) D(s-s') J_\nu(s')\}$$

The potentials between quark and anti-quark



Potential function: $V(r) = \sigma r - c/r + \mu$

String tension

Monopole dominance: $\frac{\sigma_A}{\sigma_{nonA}} \sim \frac{\sigma_{mono}}{\sigma_{nonA}} \sim 0.8$

SU(2) gauge theory without gauge fixing

Tsuneo Suzuki, Masayasu Hasegawa, Katsuya Ishiguro, Yoshiaki Koma, and Toru Sekido, Physical Review D, Vol. 80, No. 5, pp. 1-11, 2009.

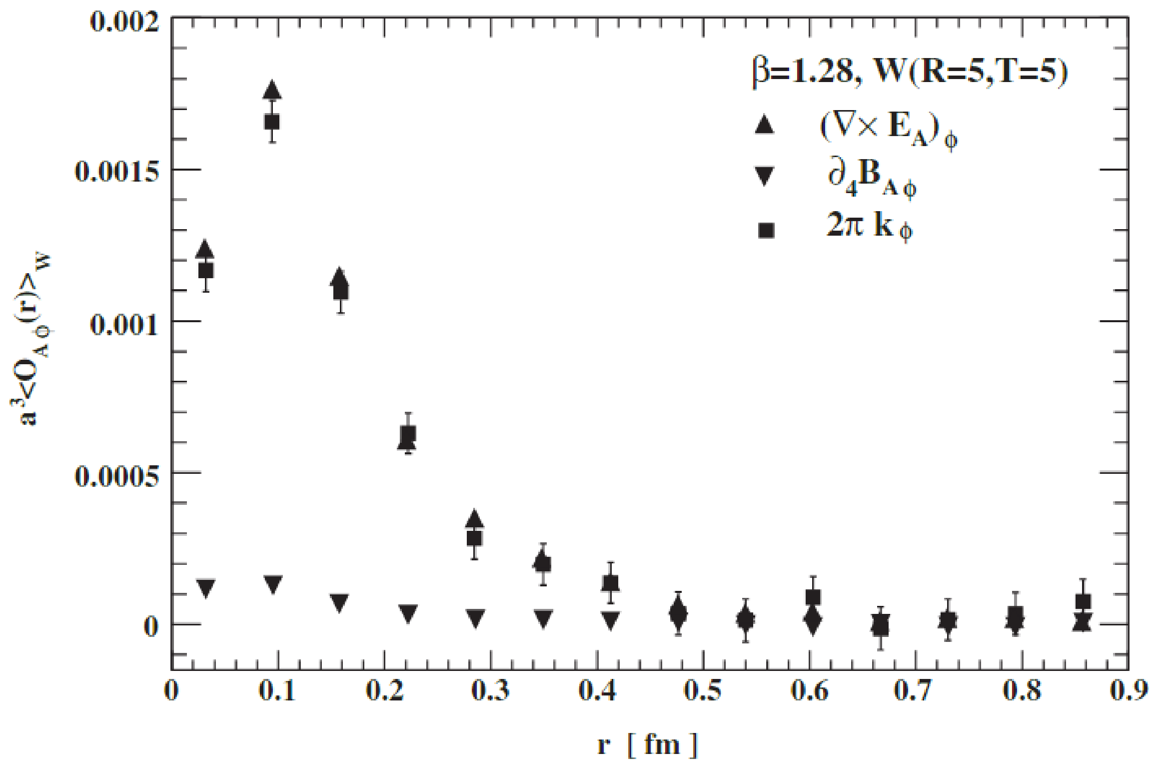
TABLE III. Best fitted values of the string tension σa^2, the Coulombic coefficient c, and the constant μa for the potentials $V_{\rm NA}$, V_A, $V_{\rm mon}$, and $V_{\rm ph}$.

$24^3 \times 4$	σa^2	c	μa	FR (R/a)	$\chi^2/N_{\rm df}$
$V_{\rm NA}$	0.181(8)	0.25(15)	0.54(7)	3.9–8.5	1.00
V_A	0.183(8)	0.20(15)	0.98(7)	3.9–8.2	1.00
$V_{\rm mon}$	0.183(6)	0.25(11)	1.31(5)	3.9–6.7	0.98
$V_{\rm ph}$	$-2(1) \times 10^{-4}$	0.010(1)	0.48(1)	4.9–9.4	1.02
$24^3 \times 6$					
$V_{\rm NA}$	0.072(3)	0.49(6)	0.53(3)	4.0–9.0	0.99
V_A	0.073(4)	0.41(7)	1.09(3)	3.7–10.9	1.00
$V_{\rm mon}$	0.073(4)	0.44(10)	1.41(4)	3.9–9.3	1.00
$V_{\rm ph}$	$-1.7(3) \times 10^{-4}$	0.0131(1)	0.4717(3)	5.1–9.4	0.99
$36^3 \times 6$					
$V_{\rm NA}$	0.072(3)	0.48(9)	0.53(3)	4.6–12.1	1.03
V_A	0.073(2)	0.47(6)	1.10(2)	4.3–11.2	1.03
$V_{\rm mon}$	0.073(3)	0.46(7)	1.43(3)	4.0–11.8	1.01
$V_{\rm ph}$	$-1.0(1) \times 10^{-4}$	0.0132(1)	0.4770(2)	6.4–11.5	1.03
$24^3 \times 8$					
$V_{\rm NA}$	0.0415(9)	0.47(2)	0.46(8)	4.1–7.8	0.99
V_A	0.041(2)	0.47(6)	1.10(3)	4.5–8.5	1.00
$V_{\rm mon}$	0.043(3)	0.37(4)	1.39(2)	2.1–7.5	0.99
$V_{\rm ph}$	$-6.0(3) \times 10^{-5}$	0.0059(3)	0.466 49(6)	7.7–11.5	1.02

100%

The dual Ampere's law

$$({\rm rot}E^a)_\phi = \partial_t B^a_\phi + 2\pi k^a_\phi$$



Ideas of QCD monopoles

- **Partial Gauge Fixing**

G. 't Hooft, *Nucl. Phys.* **B190**, 455 (1981).

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- **The violation of non-Abelian Bianchi identity**

T.Suzuki,K.Ishiguro,V.Bornyakov,Phys.Rev.D97:034501 (2018)
T.Suzuki,Phys.Rev.D97,034509 (2018)

The violation of non-Abelian Bianchi identities(VNABI)

T.Suzuki,K.Ishiguro,V.Bornyakov,Phys.Rev.D97:034501 (2018)
T.Suzuki,Phys.Rev.D97,034509 (2018)

$$D_\nu G^*_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} D_\nu G_{\rho\sigma}$$

⇓

$$D_\nu G^*_{\mu\nu} = \partial_\nu f^*_{\mu\nu} = k_\mu$$

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\frac{\lambda^a}{2}$$

(Suppose a gauge field has a line Singularity)

$$\begin{aligned}[D_\rho, D_\sigma] &= [\partial_\rho - igA_\rho, \partial_\sigma - igA_\sigma] \\ &= -ig(\partial_\rho A_\sigma - \partial_\sigma A_\rho - ig[A_\rho, A_\sigma]) + [\partial_\rho, \partial_\sigma] \\ &= -igG_{\rho\sigma} + \underline{[\partial_\rho, \partial_\sigma]}\end{aligned}$$

On the continuum theory, the violation of non-Abelian Bianchi identity leads to Abelian monopole currents for each color.

We define these monopole currents on the lattice!

2. Results

2.1 The dual Meissner effect in SU(3) gauge theory

Tsuneo Suzuki, Atsuki Hiraguchi and Katsuya Ishiguro,
Proceedings of LATTICE2021, 26th-30th July 2021, Zoom/Gather@MIT,
arXiv: 2110.14702

Purpose

Monopole dominance and the dual Meissner effect due to the VNABI in SU(3) without gauge fixing

For example,

The GL parameter

$$\kappa = \frac{\lambda}{\xi}$$

→ **The type of the vacuum**

Methods

Wilson action 24^3 or 40^3 x Nt

- β =5.6(Nt=4), β =5.75(Nt=6) at 0.8Tc
- APE and HYP smearings, random gauge transformations
- No gauge fixing

The connected correlation

P.Cea, L.Cosmai, F.Cuteri, A.Papa, EPJ Web Conf. 175 (2018) 12006

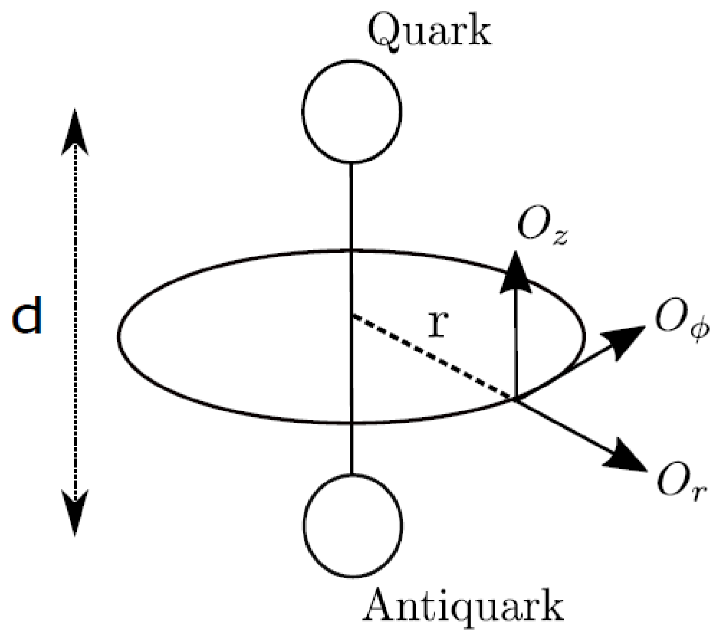
$$\rho_{conn}(O(r)) = \frac{\langle \mathrm{Tr}(PLO(r)L^\dagger)\,\mathrm{Tr}\,P^\dagger \rangle}{\langle \mathrm{Tr}\,P\,\mathrm{Tr}\,P^\dagger \rangle} - \frac{1}{3}\frac{\langle \mathrm{Tr}\,P\,\mathrm{Tr}\,P^\dagger\,\mathrm{Tr}\,O(r) \rangle}{\langle \mathrm{Tr}\,P\,\mathrm{Tr}\,P^\dagger \rangle}$$

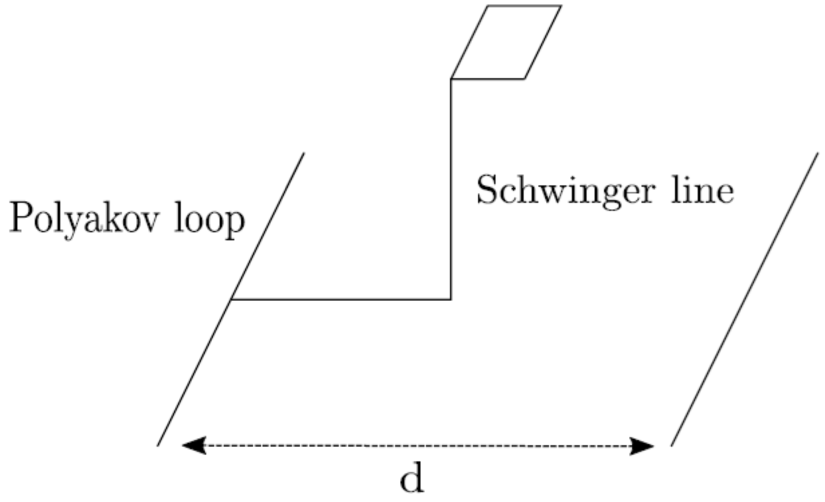
Flux-tube profiles at finite temperature

P.Cea, L.Cosmai, F.Cuteri, A.Papa, EPJ Web Conf. 175 (2018) 12006

The connected correlation

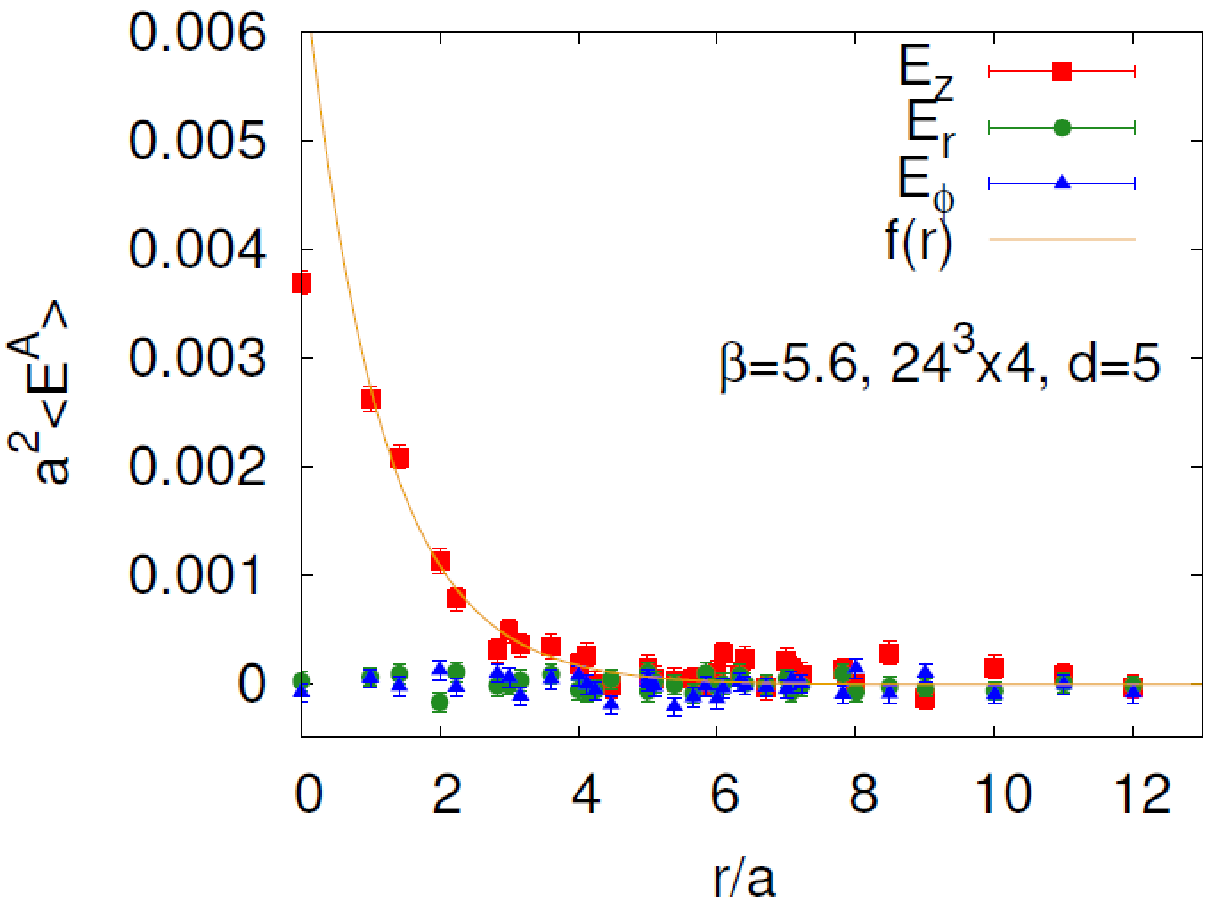
$$\rho_{conn}(O(r)) = \frac{\langle \mathrm{Tr}(P(0) L O(r) L^\dagger) \mathrm{Tr} P(d)^\dagger \rangle}{\langle \mathrm{Tr} P(0) \mathrm{Tr} P(d)^\dagger \rangle} - \frac{1}{3} \frac{\langle \mathrm{Tr} P(0) \mathrm{Tr} P(d)^\dagger \mathrm{Tr} O(r) \rangle}{\langle \mathrm{Tr} P(0) \mathrm{Tr} P(d)^\dagger \rangle}$$





Operators:
- non-Abelian plaquette
- Abelian plaquette
- Monopole currents

Abelian color electric fields at $\beta = 5.6$



Fitting function :

$$f(r) = c_1 \exp(-\frac{r}{\lambda}) + c_0$$

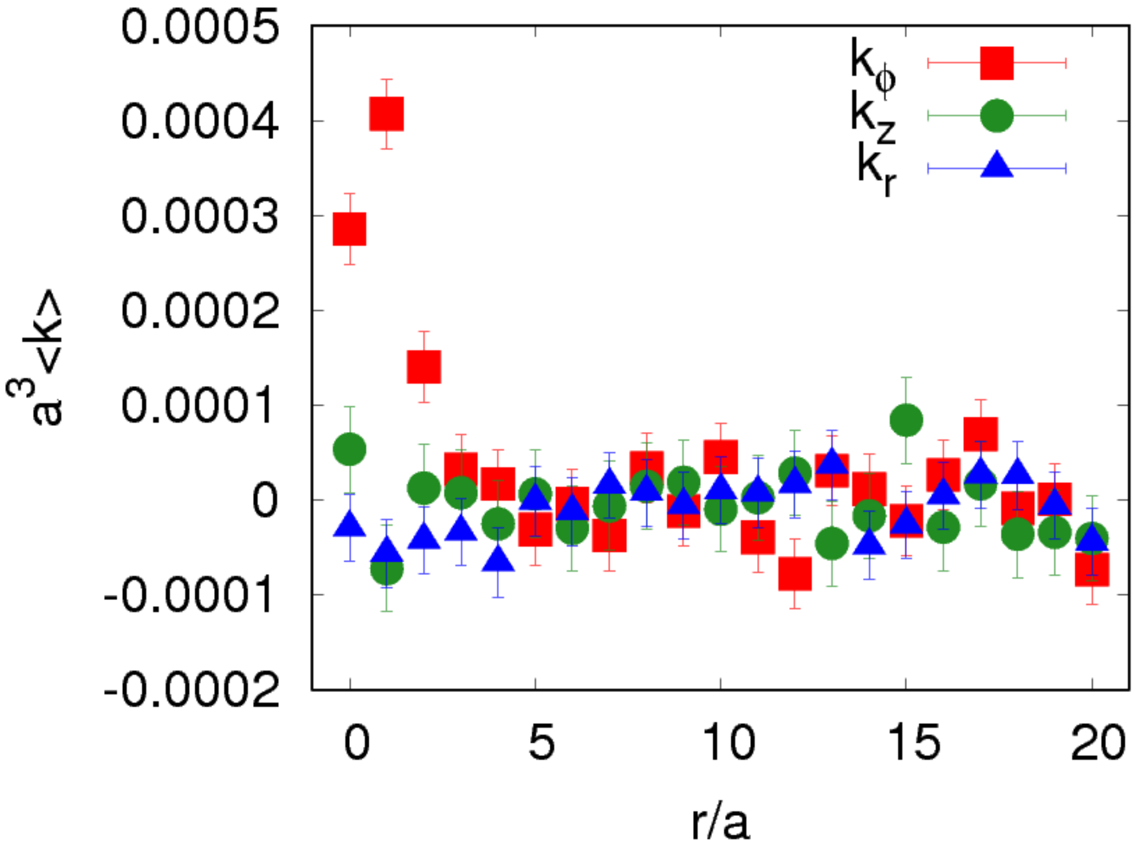
The penetration length at β =5.6

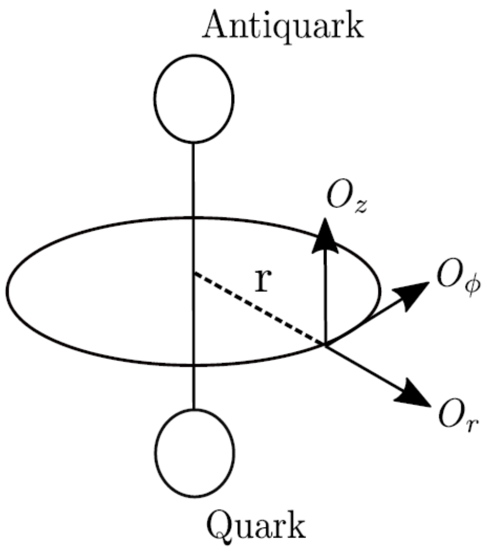
Fitting function : $$f(r) = c_1 \exp(-\frac{r}{\lambda}) + c_0$$

Table 5: The penetration length λ at $\beta = 5.6$ on $24^3 \times 4$ lattices.

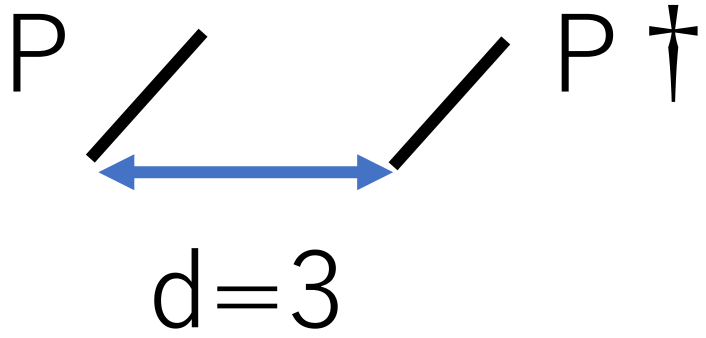
d	λ/a	c_1	c_0	χ^2/N_{df}
3	0.91(1)	0.0100(2)	-0.000002(8)	1.31628
4	1.10(6)	0.0077(4)	-0.00005(4)	0.972703
5	1.09(8)	0.0068(6)	-0.00001(4)	0.995759
6	1.1(1)	0.0055(8)	-0.00008(7)	0.869692

Monopole current



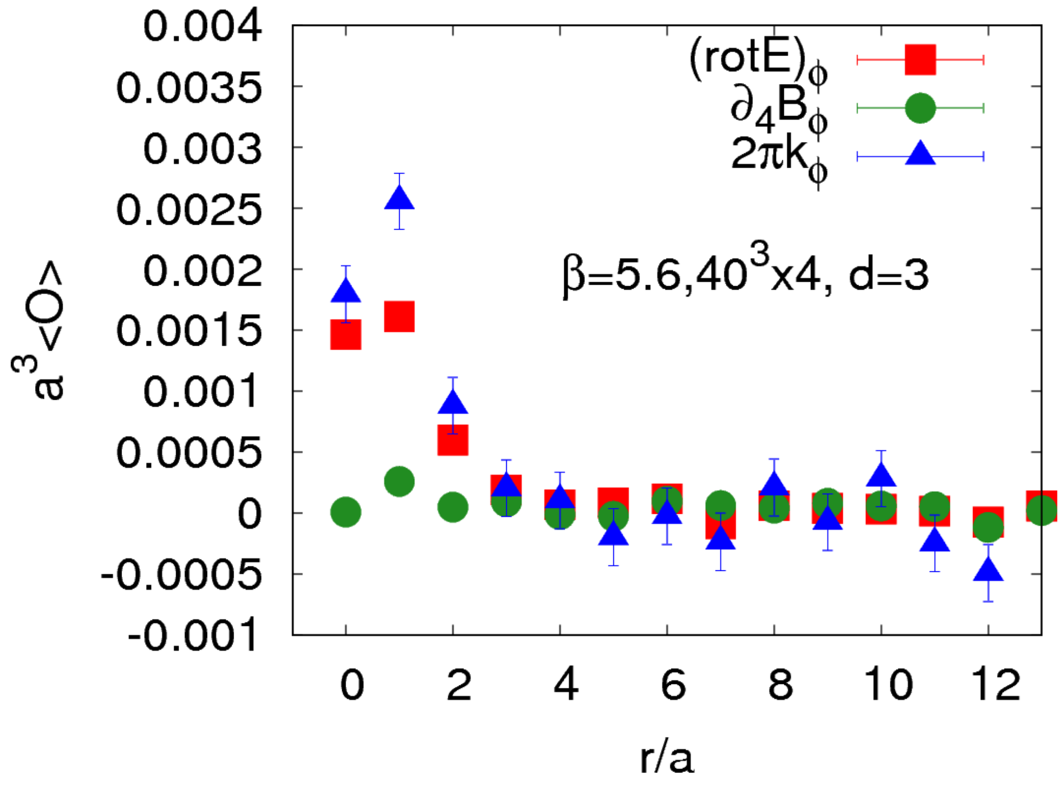


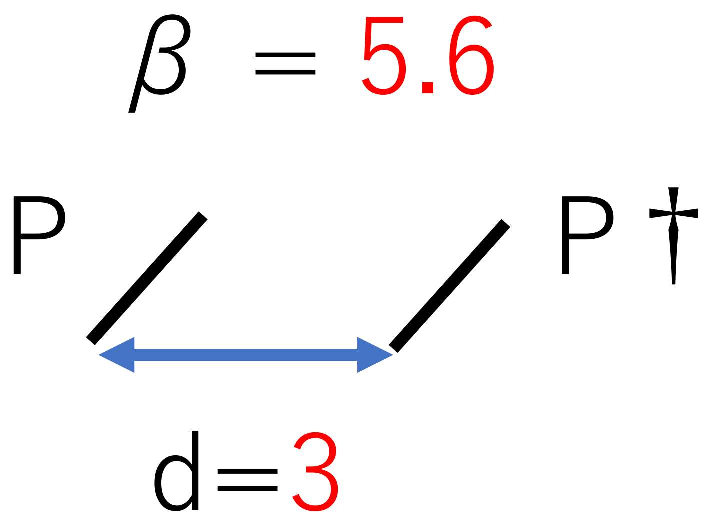
$\beta = 5.6$



The dual Ampere's law in SU(3)

$$(\mathrm{rot}E^a)_\phi = \partial_t B^a_\phi + 2\pi k^a_\phi$$





Coherence length

M. N. Chernodub, Katsuya Ishiguro, Yoshihiro Mori, Yoshifumi Nakamura, M. I. Polikarpov, Toru Sekido, Tsuneo Suzuki, and V. I. Zakharov. *Phys. Rev. D*, Vol. 72, p. 074505, Oct 2005.

T.Suzuki,M.Hasegawa,K.Ishiguro,Y.Koma,T.Sekido,Phys.Rev.D80:054504(2009)

$$\langle k^2(r)\rangle_{q\bar{q}} = \frac{\langle \mathrm{Tr}\, P(0)\, \mathrm{Tr}\, P^\dagger(d) \sum_{\mu,a} k^a_\mu(r) k^a_\mu(r)\rangle}{\langle \mathrm{Tr}\, P(0)\, \mathrm{Tr}\, P^\dagger(d)\rangle} - \langle \sum_{\mu,a} k^a_\mu(r) k^a_\mu(r)\rangle$$

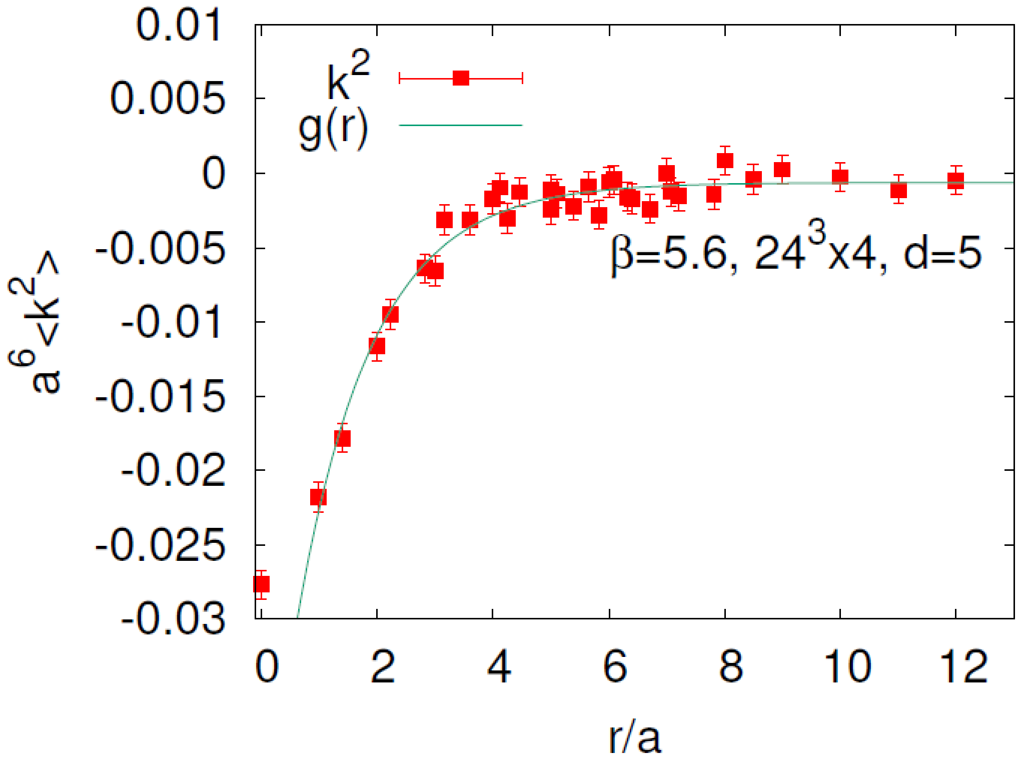
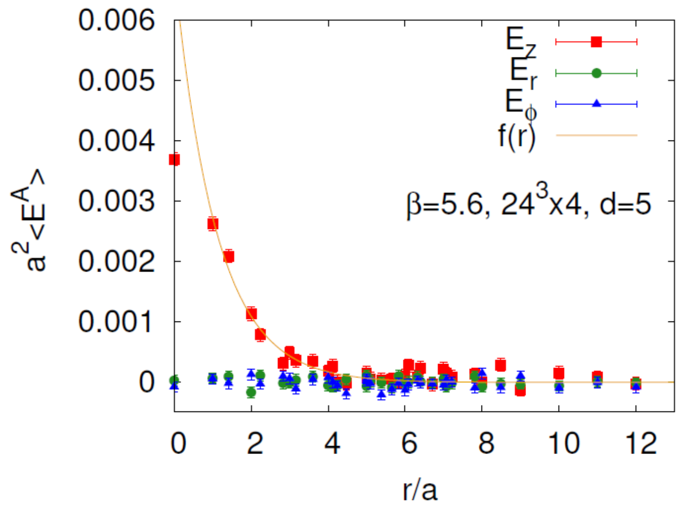
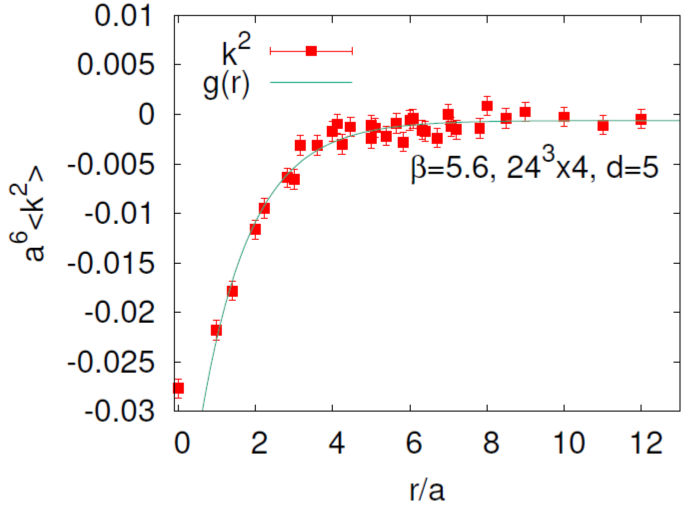


Table 6: The coherence length $\xi/\sqrt{2}$ at $\beta = 5.6$ on $24^3 \times 4$ lattices.

d	$\xi/\sqrt{2}a$	c_1'	c_0'	χ^2/N_{df}
3	1.04(6)	-0.050(3)	0.0001(2)	0.997362
4	1.17(7)	-0.052(3)	-0.0003(2)	1.01499
5	1.3(1)	-0.047(3)	-0.0006(3)	0.99758
6	1.1(1)	-0.052(8)	-0.0013(5)	1.12869

The type of vacuum in SU(3) gauge theory





Fitting function :

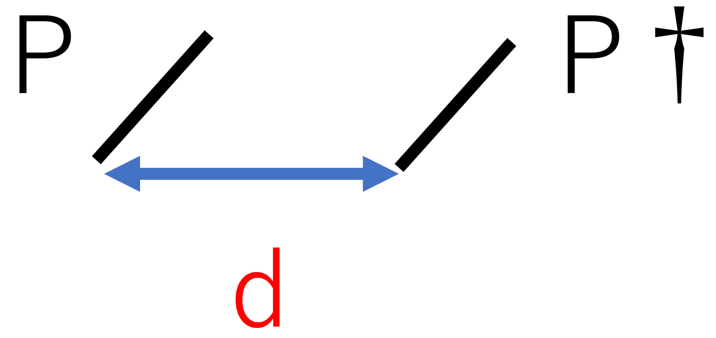
$$f(r) = c_1 \exp(-\frac{r}{\lambda}) + c_0$$

$$g(r) = c_1' \exp(-\frac{\sqrt{2}r}{\xi}) + c_0'$$

λ: Penetration length

ξ: Coherence length

$\beta = 5.6$



The type of vacuum in SU(3) gauge theory

Table 7: The Ginzburg-Landau parameters at $\beta = 5.6$ on $24^3 \times 4$ lattice.

d	$\sqrt{2}\kappa$
3	0.87(5)
4	0.93(7)
5	0.83(9)
6	0.9(2)

$$\sqrt{2}\kappa < 1$$

Type I or the border

We need a huge number of statistics at large β and large distance d.

3. Summary

1. We can define monopoles in gauge theory using "Partial gauge fixing"

2. The violation of non-Abelian Bianchi identity ("VNABI") is one of the definition of monopoles in gauge theory.

3. We confirm "VNABI" contribute to Monopole dominance and the dual Meissner effect in SU(3) gauge theory without gauge fixing at one gauge coupling constant.

3. Future works

1. To evaluate Monopole dominance and the dual Meissner effect at other gauge coupling constants in SU(3), we need much more number of statistics.

2. The relation between Abelian monopoles and Center vortexes is not clear.

3. Investigating the contribution of QCD monopoles for other non-perturbative properties of QCD is important.