gT contribution to single-spin asymmetry in SIDIS

Hsiang-nan Li

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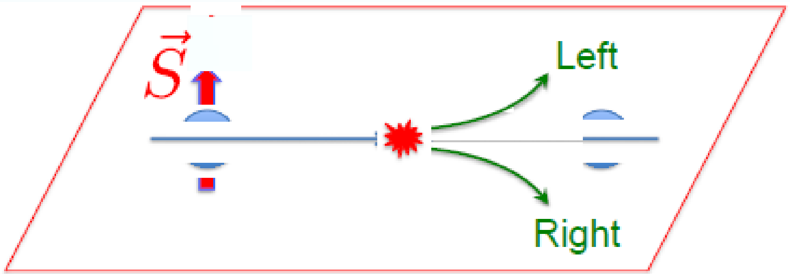
In collaboration with S. Benic, Y. Hatta, A. Kaushik

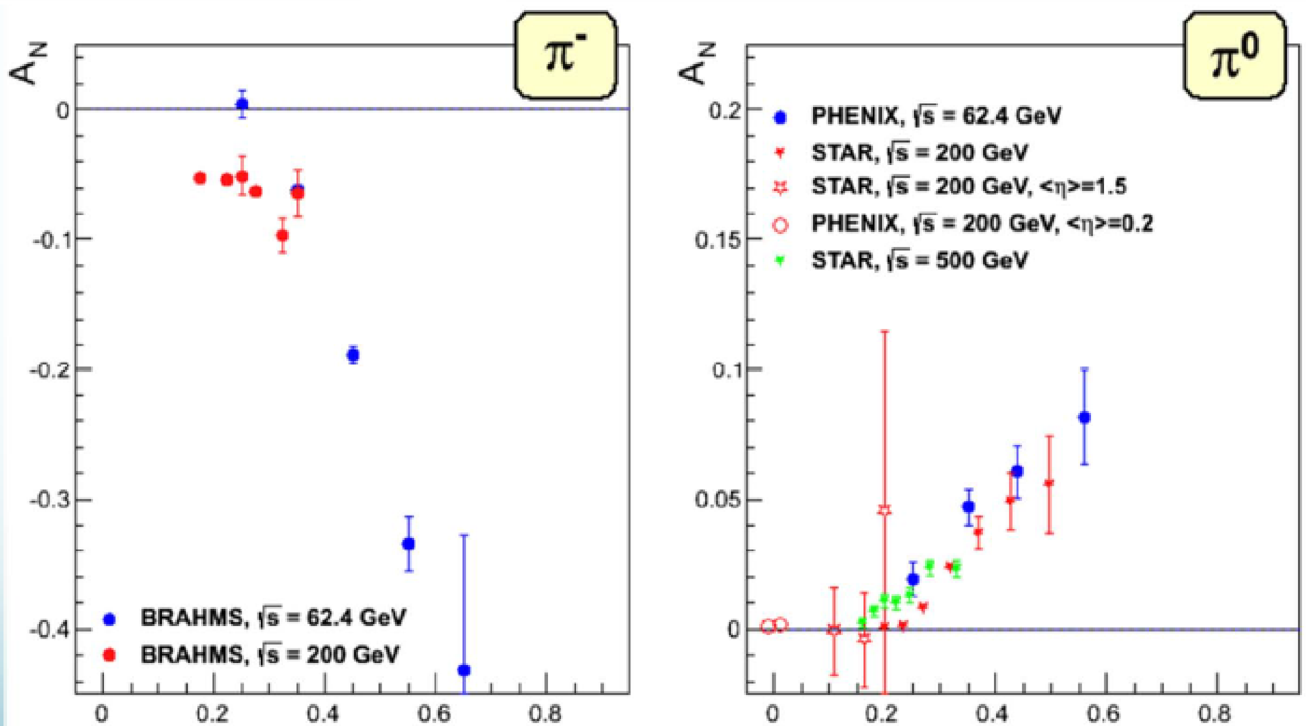
2109.05440

Single transverse spin asymmetry (SSA)

- Consider a transversely polarized proton scatter off an unpolarized proton or electron



$$A_N \equiv \frac{L-R}{L+R} = \frac{\sigma^{\uparrow}-\sigma^{\downarrow}}{\sigma^{\uparrow}+\sigma^{\downarrow}}$$



$x_F \sim 2P_z/\sqrt{s}$ scaled longitudinal momentum

Mechanism

- There exists correlation proportional to

$$\varepsilon_{\mu\nu\rho\lambda} S_T^{\mu} p_{hT}^{\nu} \cdots$$

- To generate such term in Feynman diagram, need

$$tr\left[\gamma_5 \not{S}_T \not{p}_{hT} \cdots\right] = i\varepsilon_{\mu\nu\rho\lambda} S_T^{\mu} p_{hT}^{\nu} \cdots$$

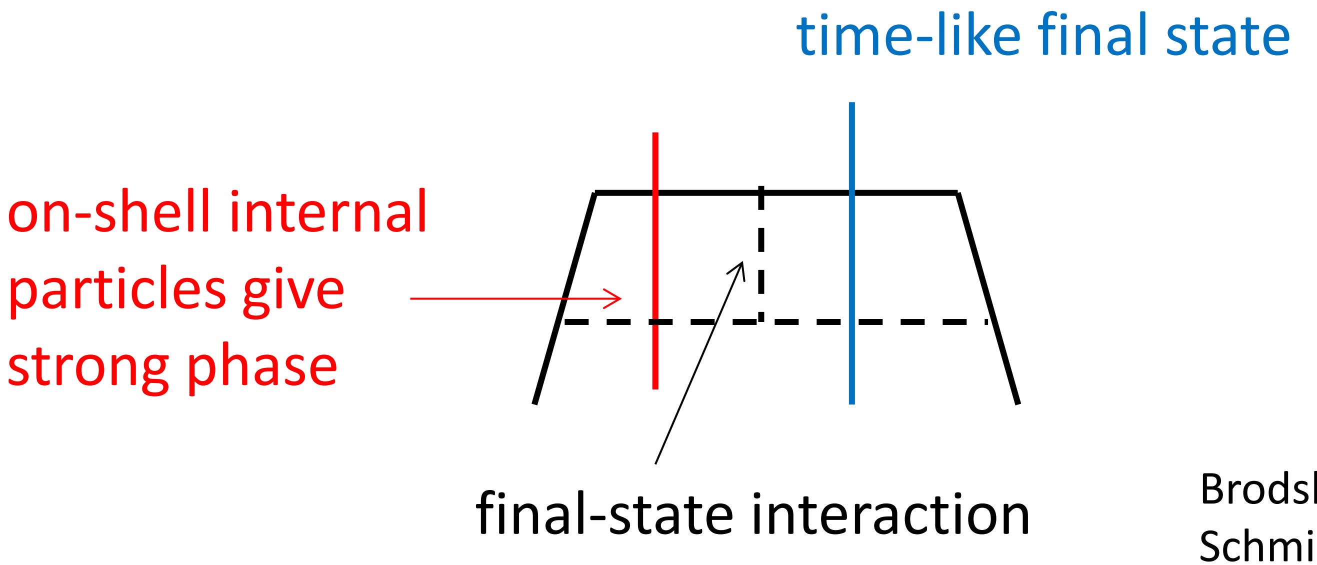
- Projector for polarized proton $\left(\not{p} + m\right)\gamma_5 \not{S}_T$
- Projector for produced hadron $\not{p}_h + m_h$
- But need strong phase to make cross section real

Phase at two loops

- Phase comes from on-shell internal particles

$$\frac{1}{k^2+i\varepsilon}=\frac{P}{k^2}-i\pi\delta(k^2)$$

- Need time-like final-state particles with gluon exchanges (FSI) between them
- Nonvanishing phase appears in box diagram



Brodsky, Hwang, Schmidt 2002

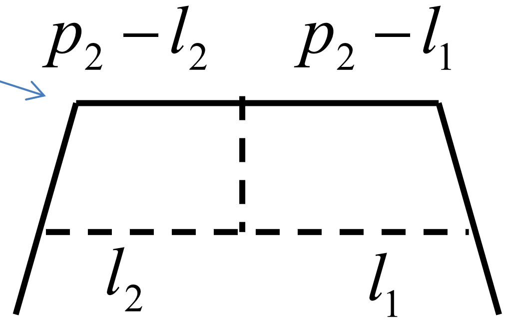
Kinematics for phase

$q = p_2 - p_1, \quad p_2 = (p_2^+, p_2^-, 0_T) \qquad p_1^+, p_2^- >> p_2^+ >> \Lambda_{QCD}$

$l_{1T} \leftrightarrow p_{hT}$

$p_2^2 > 0$

time-like



$p_1 = (p_1^+, 0, 0_T)$

$$l_1^- = l_{1T}^2 / (2 l_1^+)$$

$$l_2^- = l_{2T}^2 / (2 l_2^+)$$

$$l_1^+ = \frac{1}{2}\left(p_2^+ \pm \sqrt{p_2^{+2} - 2\frac{p_2^+}{p_2^-} l_{1T}^2}\right)$$

$$l_2^+ = \frac{1}{2}\left(p_2^+ \pm \sqrt{p_2^{+2} - 2\frac{p_2^+}{p_2^-} l_{2T}^2}\right)$$

Collinear to initial state

- Picking up plus signs, ie., (l1=+,l2=+), gluons collimate to polarized proton

$$l_{1,2}^{+} \sim O(p_2^{+}) >> l_{1T,2T} >> l_{1,2}^{-}$$

$$p_1 - l_2 \approx p_1^{+} - p_2^{+}$$

collinear

$$p_2 - l_1 \approx p_2 - l_2 \approx p_2^{-}$$

- Phase goes into Sivers function
- FSI gluon is soft

Sivers function

Sivers 1990

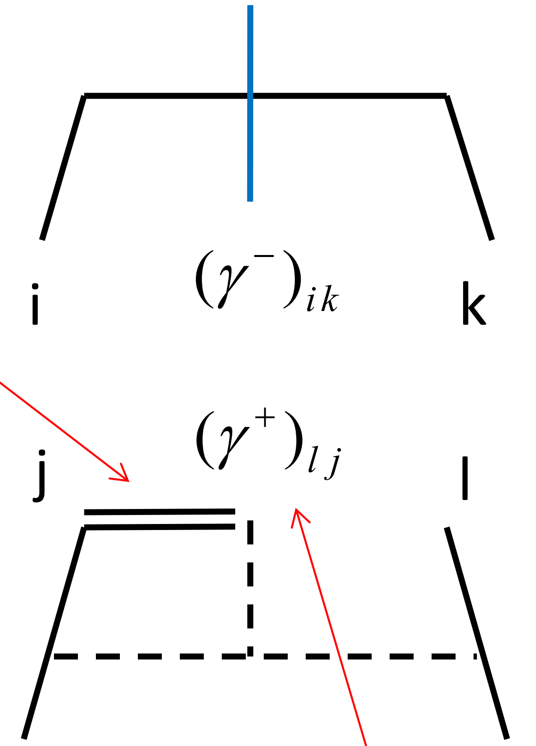
- Eikonalize outgoing quark and insert Fierz identity

$$I_{ij}I_{lk} = \frac{1}{4}I_{ik}I_{lj} + \frac{1}{4}(\gamma^\alpha)_{ik}(\gamma_\alpha)_{lj}$$

$$+ \frac{1}{4}(\gamma^5\gamma^\alpha)_{ik}(\gamma_\alpha\gamma^5)_{lj} + \frac{1}{4}(\gamma^5)_{ik}(\gamma^5)_{lj}$$

$$+ \frac{1}{8}(\gamma^5\sigma^{\alpha\beta})_{ik}(\sigma_{\alpha\beta}\gamma^5)_{lj}$$

give dominant (twist-2) contribution



pick up $\not{p} = p^+\gamma^-$

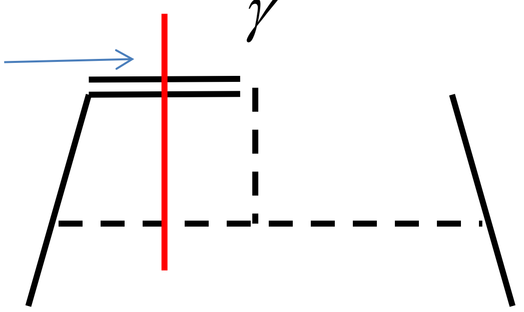
Parton transverse momentum

- Sivers function demands inclusion of parton transverse momentum

compensated by phase here

$$tr\left[\gamma_5 \not{S}_T \not{k}_T \gamma^+ \gamma^- \cdots\right] = i\varepsilon_{\mu\nu+-} S_T^\mu k_T^\nu \cdots$$

$l_{1T,2T}$



γ^+

$(\not{p}+m)\gamma_5 \not{S}_T$

- This correlation determines preferred direction of k_T for polarized proton , which then propagates into p_h

Collinear to final state

- Picking up minus signs, ie., (-,-), gluons collimate to produced hadron

$$l_{1,2}^{-} \sim O(p_2^{-}) >> l_{1T,2T} >> l_{1,2}^{+}$$

$$p_2 - l_1 \sim O(p_2^{-}), \quad p_2 - l_2 \sim O(p_2^{-})$$

collinear

$p_1 - l_2$ highly off-shell

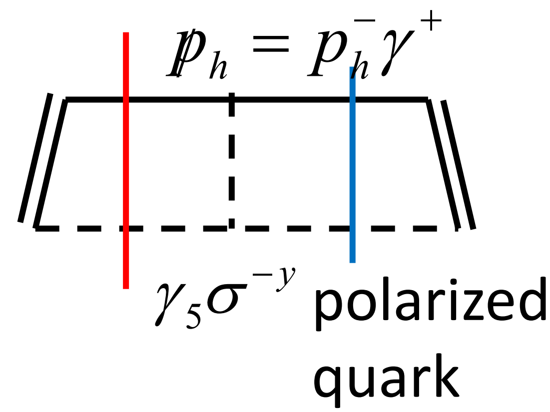
- Phase goes into Collins fragmentation function

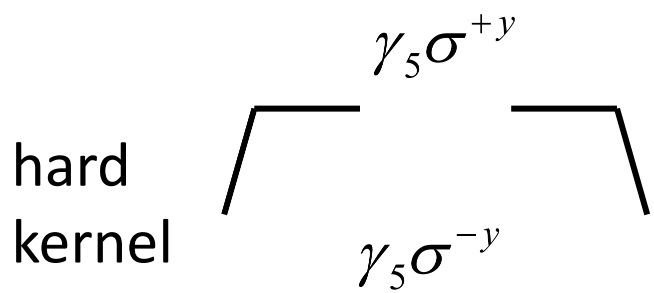
Collins 1993

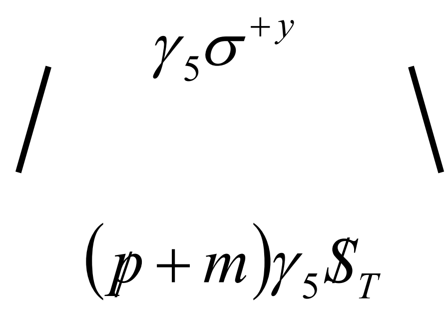
Collins function

- Eikonalize incoming quark and insert Fierz identity
- $\gamma_5\sigma^{-y}$ dominates
- Collins function demands inclusion of parton k_T
- LO hard kernel demands projector for initial state

- Transversity distribution defined







Twist-2 TMDs

$$\Phi^{[\gamma^+]} = f_1 - \frac{\epsilon_T^{\rho\sigma} p_{T\rho} S_{T\sigma}}{M} f_{1T}^{\perp},$$

Sivers function

$$\Phi^{[\gamma^+\gamma_5]} = S_L\, g_{1L} - \frac{p_T \cdot S_T}{M}\, g_{1T}\,,$$

$$\Phi^{[i\sigma^{\alpha+}\gamma_5]} = S_T^{\alpha} h_1 + S_L \frac{p_T^{\alpha}}{M} h_{1L}^{\perp} - \frac{p_T^{\alpha} p_T^{\rho} - \frac{1}{2} p_T^2 g_T^{\alpha\rho}}{M^2} S_{T\rho} h_{1T}^{\perp} - \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M} h_1^{\perp}$$

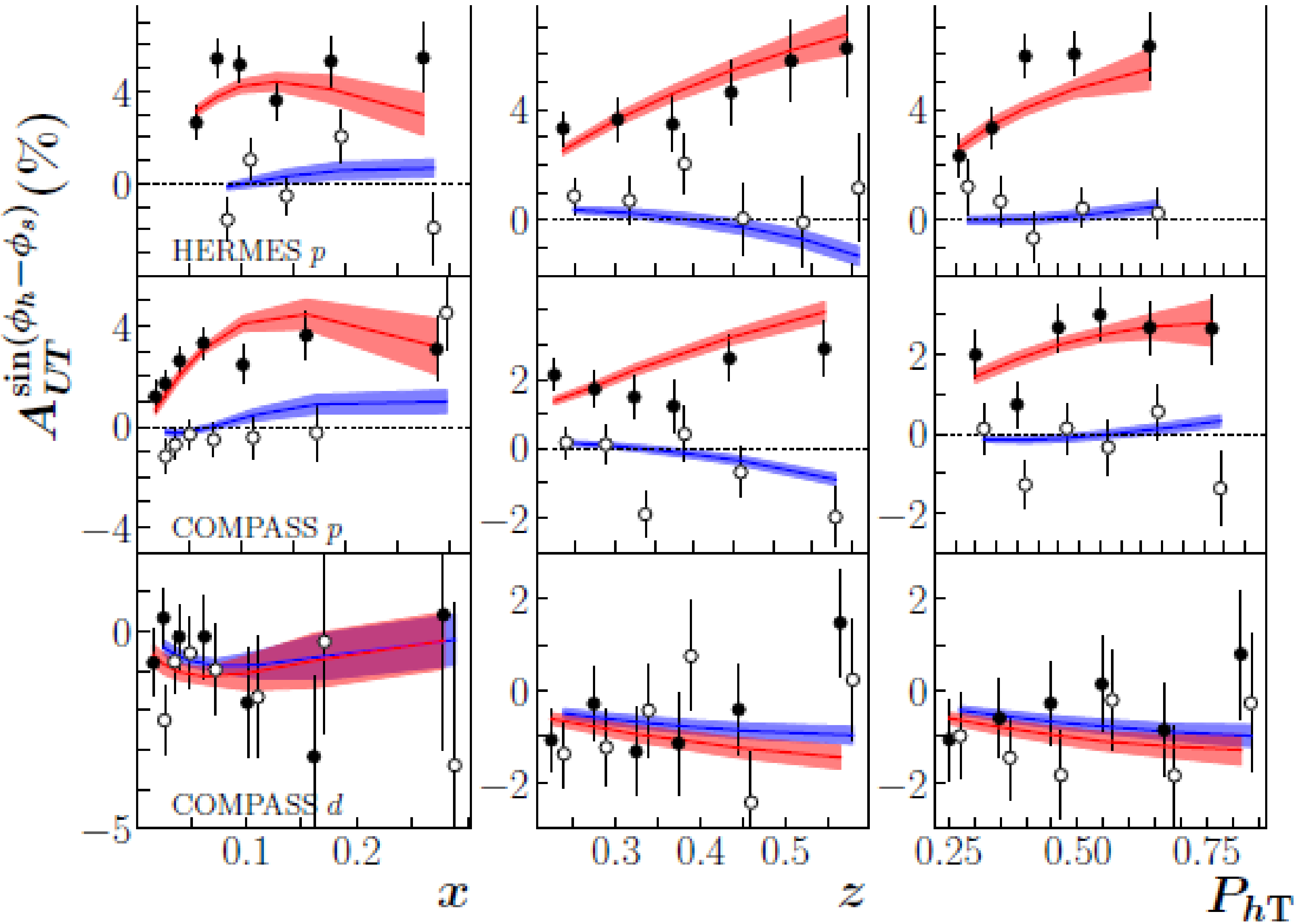
transversity function

in the case of FFs, it is Collins function

Boer, Mulders 1997
Goeke, Meta, Schlegel 2005
Bacchetta et al., 2007

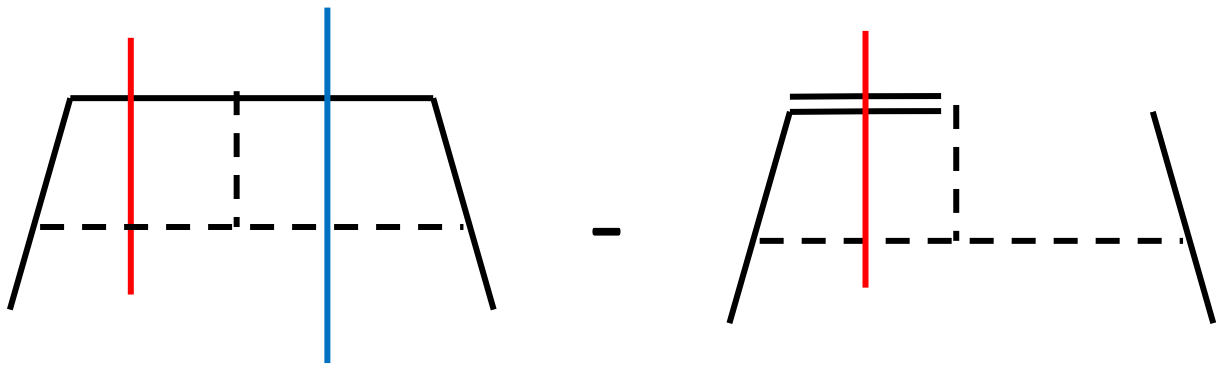
Global determination of Sivers and Collins functions from data

Cammarota et al, 2021



Phase in hard kernel

- For other sign combinations, or finite transverse momenta
- phase appears in hard kernel

$H^{(2)} =$ 

- How to extract this phase?
- Use $\gamma_5 \gamma^{\perp}$
- A new contribution to SSA

2-parton twist-3 TMDs

$$\Phi^{[i\gamma_5]} = \frac{M}{P^+}\left[S_L\, e_L - \frac{p_T\cdot S_T}{M}\, e_T\right], \qquad \Phi^{[1]} = \frac{M}{P^+}\left[e - \frac{\epsilon_T^{\rho\sigma}\, p_{T\rho}\, S_{T\sigma}}{M}\, e_T^{\perp}\right]$$

$$\Phi^{[\gamma^\alpha]} = \frac{M}{P^+}\Bigg[-\epsilon_T^{\alpha\rho} S_{T\rho}\, f_T - S_L\, \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, f_L^{\perp} - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\, \epsilon_{T\rho\sigma} S_T^\sigma\, f_T^{\perp} + \frac{p_T^\alpha}{M} f^{\perp}\Bigg]$$

$$\Phi^{[\gamma^\alpha\gamma_5]} = \frac{M}{P^+}\Bigg[S_T^\alpha\, g_T + S_L\, \frac{p_T^\alpha}{M}\, g_L^{\perp} - \frac{p_T^\alpha p_T^\rho - \frac{1}{2}\, p_T^2\, g_T^{\alpha\rho}}{M^2}\, S_{T\rho}\, g_T^{\perp} - \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M}\, g^{\perp}\Bigg]$$

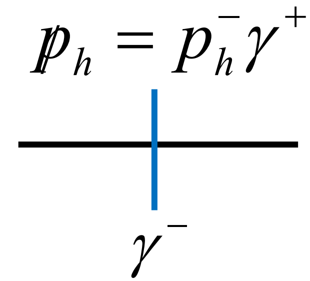
$$\Phi^{[i\sigma^{\alpha\beta}\gamma_5]} = \frac{M}{P^+}\left[\frac{S_T^\alpha p_T^\beta - p_T^\alpha S_T^\beta}{M}\, h_T^{\perp} - \epsilon_T^{\alpha\beta}\, h\right],$$

$$\Phi^{[i\sigma^{+-}\gamma_5]} = \frac{M}{P^+}\left[S_L\, h_L - \frac{p_T\cdot S_T}{M}\, h_T\right],$$

Boer, Mulders 1997
Goeke, Meta, and Schlegel 2005
Bacchetta et al., 2007

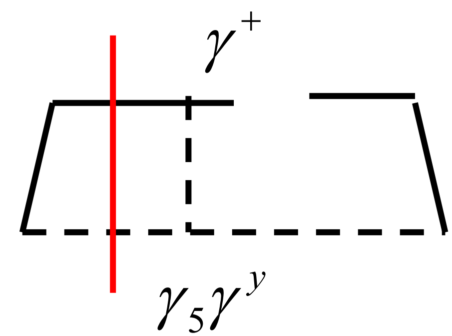
Factorization of new contribution

- Unpolarized twist-2 fragmentation function

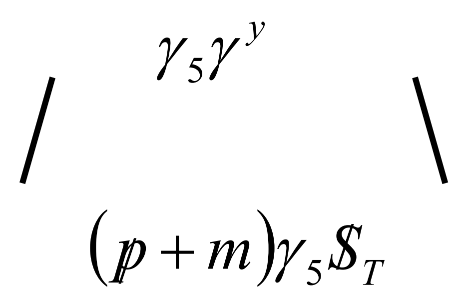


- Polarized quark scattered into direction preferred by correlation

$$tr\left[\gamma_5 \gamma^y \not{p}_{hT} \gamma^+ \gamma^- \cdots\right] = i\varepsilon_{yx+-} p_{hT}^x \cdots$$



- 2-parton twist-3 TMD or PDF g_T



gT contribution

- Focus on pT > 1 GeV, use (collinear) PDFs
- gT, twist-3, but related to twist-2 helicity PDF

$$\sim \langle \bar{\psi} g F \psi \rangle$$

$$g_T(x) = \int_x^1 \frac{dx'}{x'} \Delta q(x') + (\text{genuine twist three})$$

neglected under Wandzura-Wilczek approximation

$$\mathcal{O}(\alpha_s^2)$$

$$\frac{d\Delta\sigma}{dP_{hT}} \sim g_T(x) \otimes H \otimes D_1(z)$$

unpolarized FF

- gT well constrained, contribution is in fact not numerically important at low pT

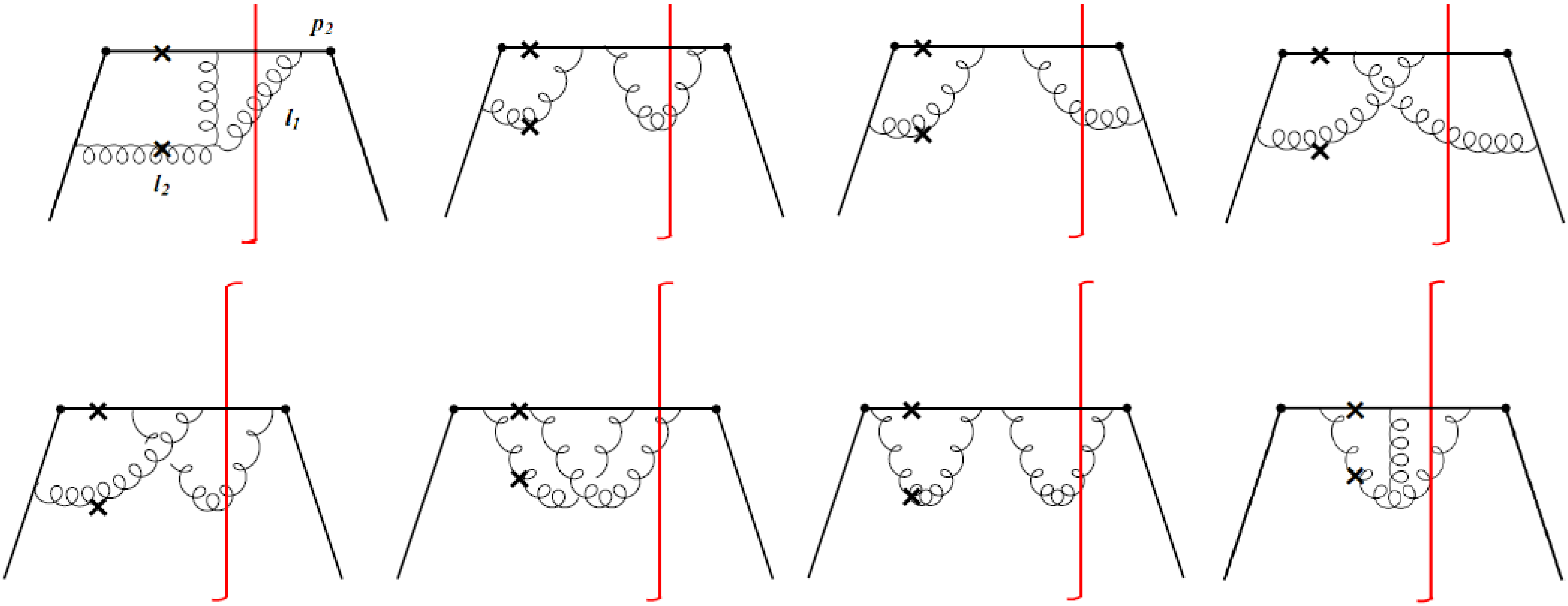
Hard kernel

- Hadronic tensor $W_{\mu\nu} = \sum_{a=q,\bar{q},g} \int \frac{dz}{z^2} D_1^a(z) w_{\mu\nu}^a$

$$w_{\mu\nu} \approx \frac{M_N}{2} \int dx g_T(x) S_T^\alpha \left(\frac{\partial}{\partial k_T^\alpha} \mathrm{Tr}[\gamma_5 \not{k} S_{\mu\nu}^{(0)}(k)] \right)_{k=xP}$$

↑ WW

derivative causes complexity

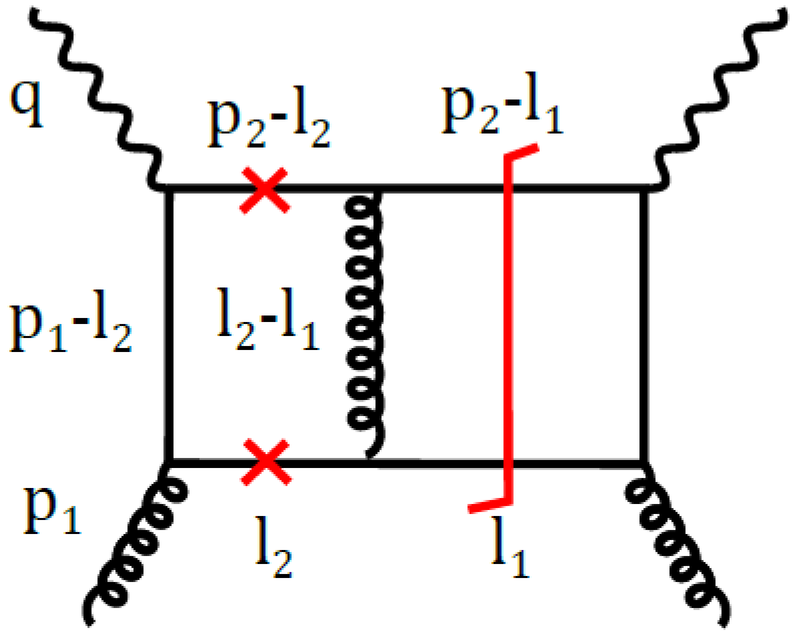


Gluon-initiated channel

- Include gluon-initiated contribution at the same order

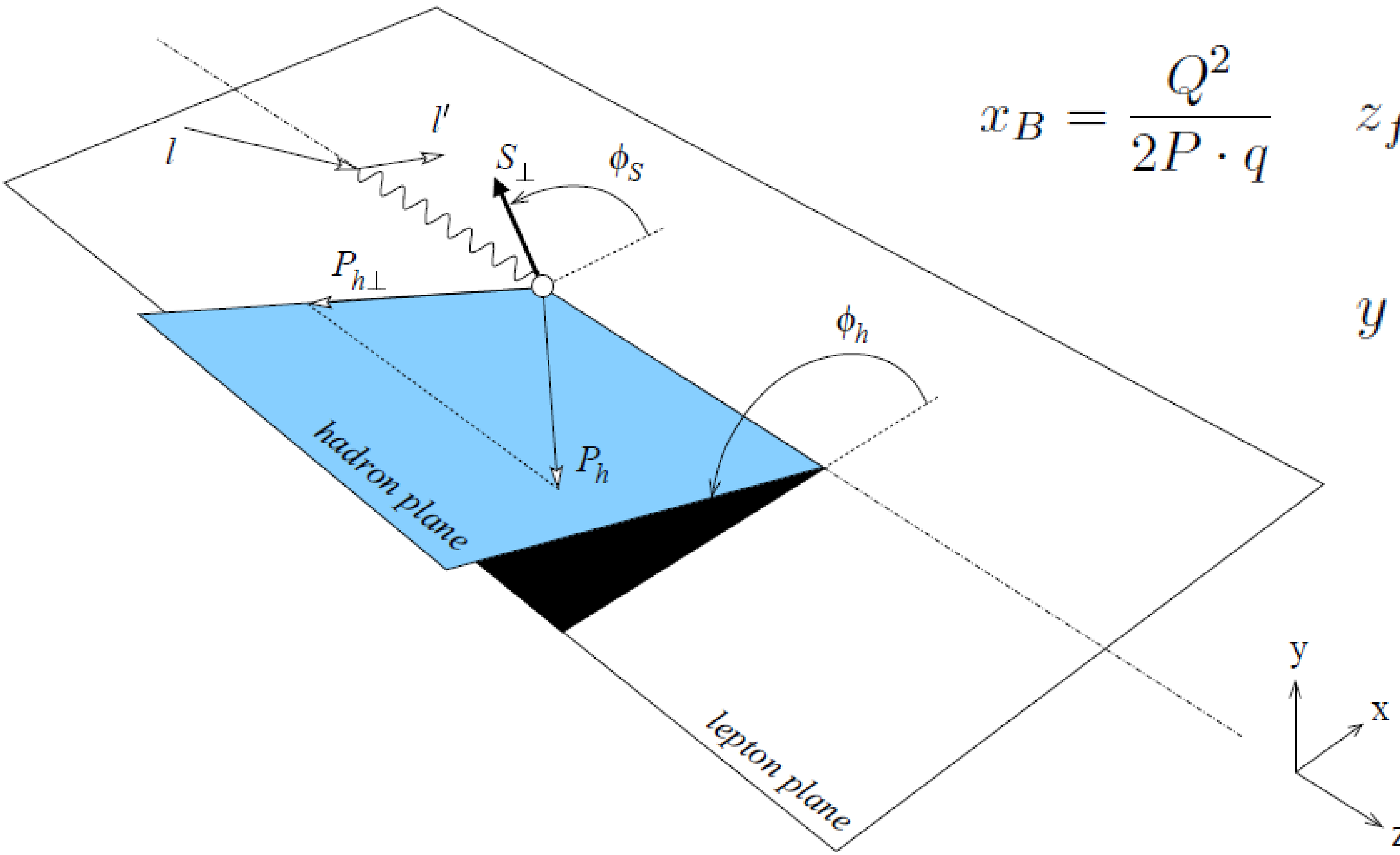
$$\frac{d\Delta\sigma}{dP_{h\perp}} \sim \mathcal{G}_{3T}(x) \otimes H_g \otimes D_1(z)$$

$$\sim \Delta G(x) \otimes H_g \otimes D_1(z)$$



Harmonics

$$A_{UT}^{\sin(\alpha\phi_h+\beta\phi_S)} = \frac{2\int_0^{2\pi} d\phi_h d\phi_S \sin(\alpha\phi_h+\beta\phi_S)\left[d\sigma(\phi_h,\phi_S) - d\sigma(\phi_h,\phi_S+\pi)\right]}{\int_0^{2\pi} d\phi_h d\phi_S \left[d\sigma(\phi_h,\phi_S) + d\sigma(\phi_h,\phi_S+\pi)\right]}$$



$$x_B = \frac{Q^2}{2P\cdot q} \qquad z_f = \frac{P\cdot P_h}{P\cdot q}$$

$$y = \frac{P\cdot q}{P\cdot l}$$

Kinematics

- COMPASS kinematics

$$0.003 \le x_B \le 0.7\,, \qquad 0.1 \le y \le 0.9\,, \qquad 0.2 \le z_f \le 1$$

$$Q^2 > 1\ \mathrm{GeV}^2, \qquad W^2 > 25\ \mathrm{GeV}^2 \qquad W^2 = (q+P)^2$$

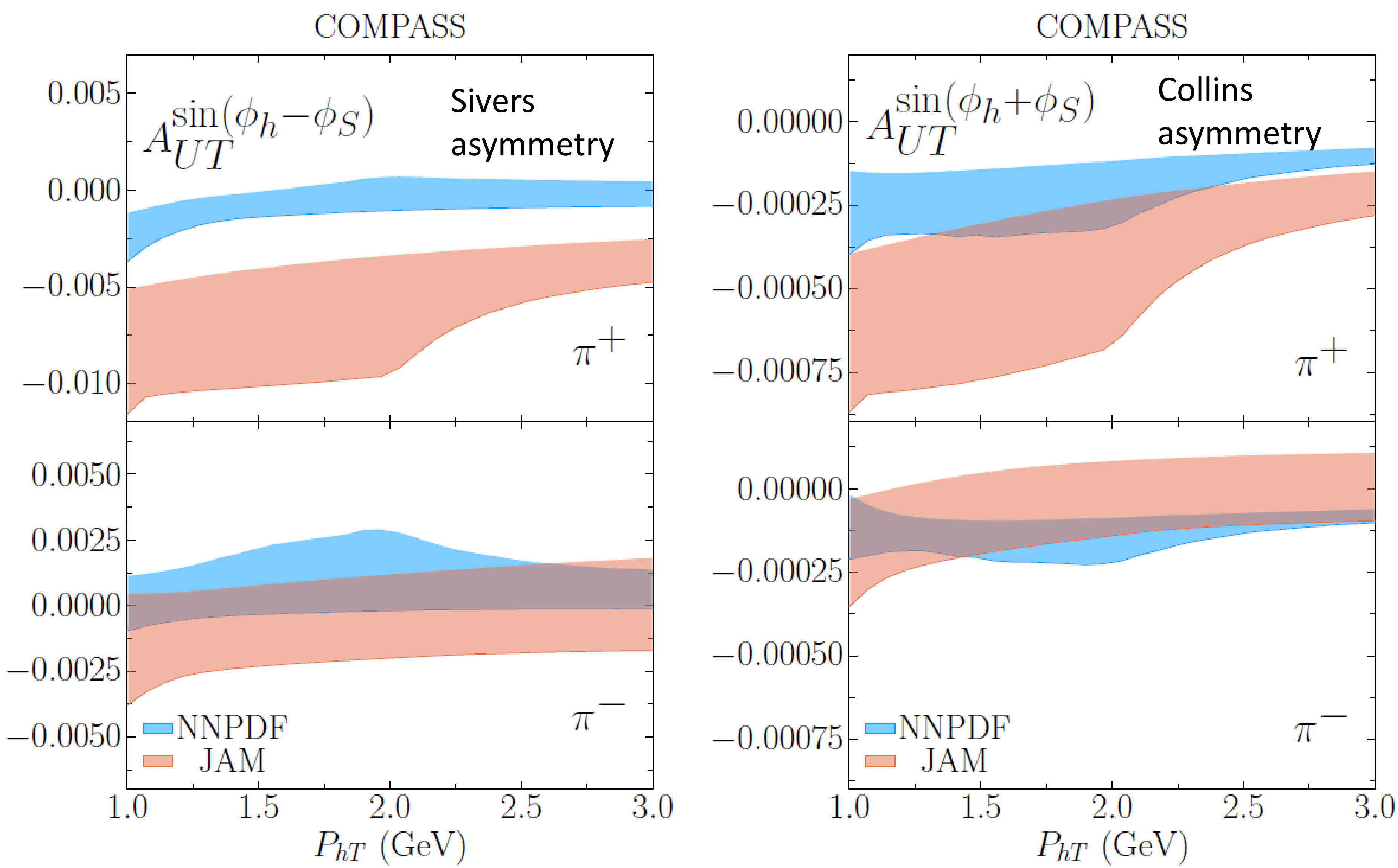
$$\sqrt{S_{\mu p}} \approx 17.4\ \mathrm{GeV}$$

- Electron-ion Collider (EIC) kinematics

$$0.1 < x_B < 0.9 \qquad 0.01 < y < 0.95 \qquad Q^2 > 1\ \mathrm{GeV}^2$$

- Adopt NNPDF, JAM inputs for PDFs, FFs

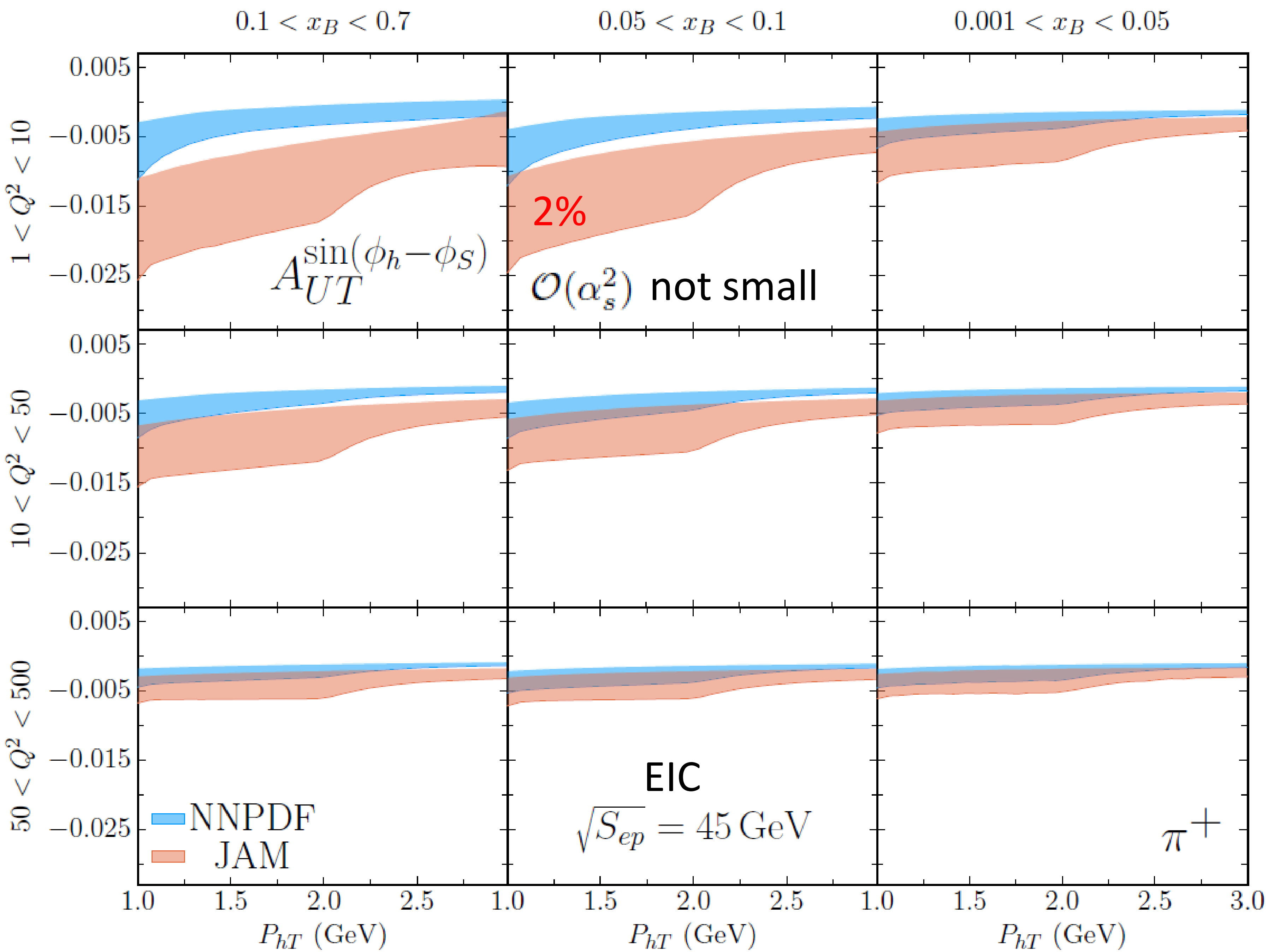
$$\mu = P_{hT}$$



uncertainty from PDFs, FFs and $0.5 < \mu/P_{hT} < 2.0$

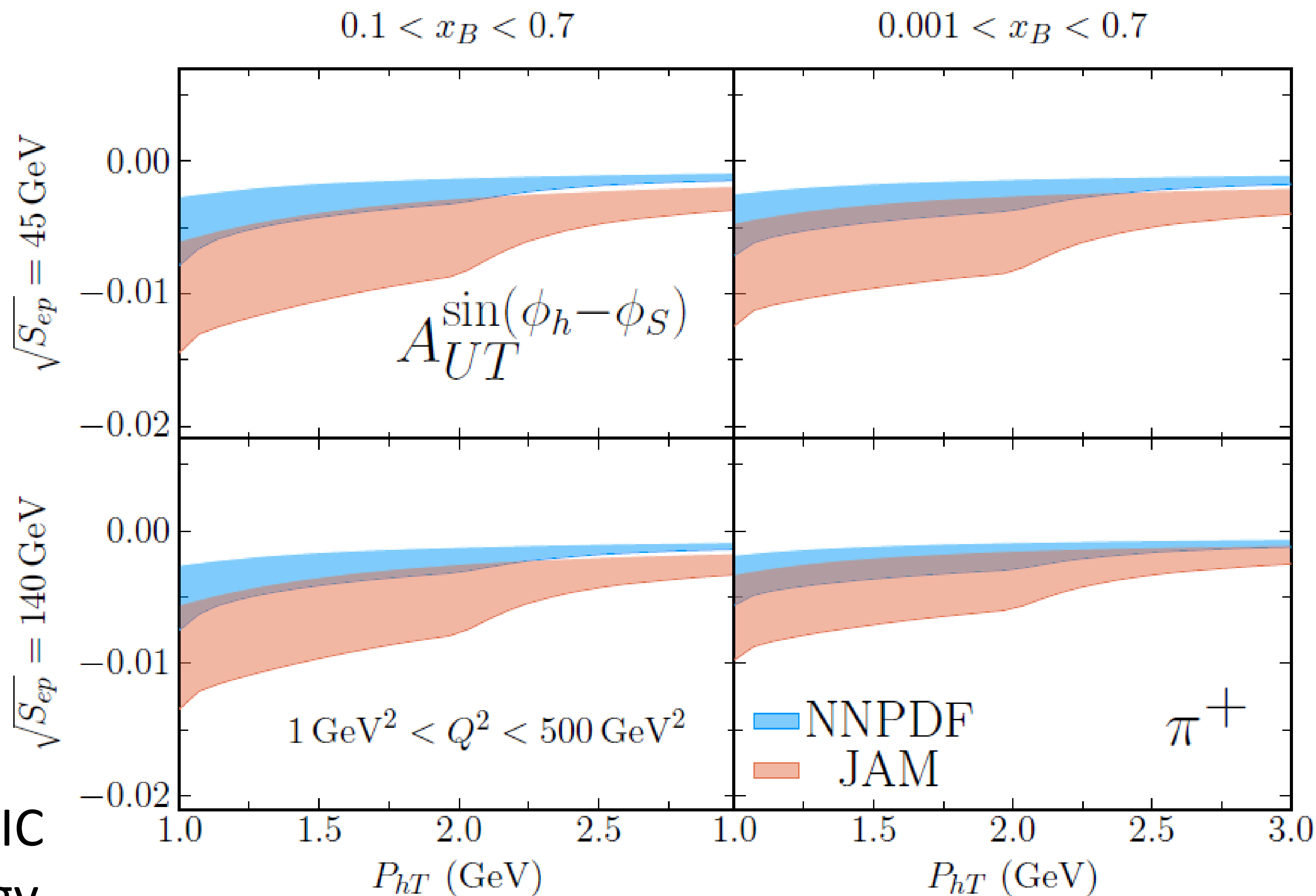
Discussions

- SSA from JAM, order of 1%, > NNPDF
- Our Sivers asymmetry has similar magnitude, but opposite sign compared to COMPASS data
- There cannot be single source, like Sivers function, expected for low pT processes
- More PDF inputs need to be included in global analysis on SSA
- Our Collins asymmetry is negligibly small



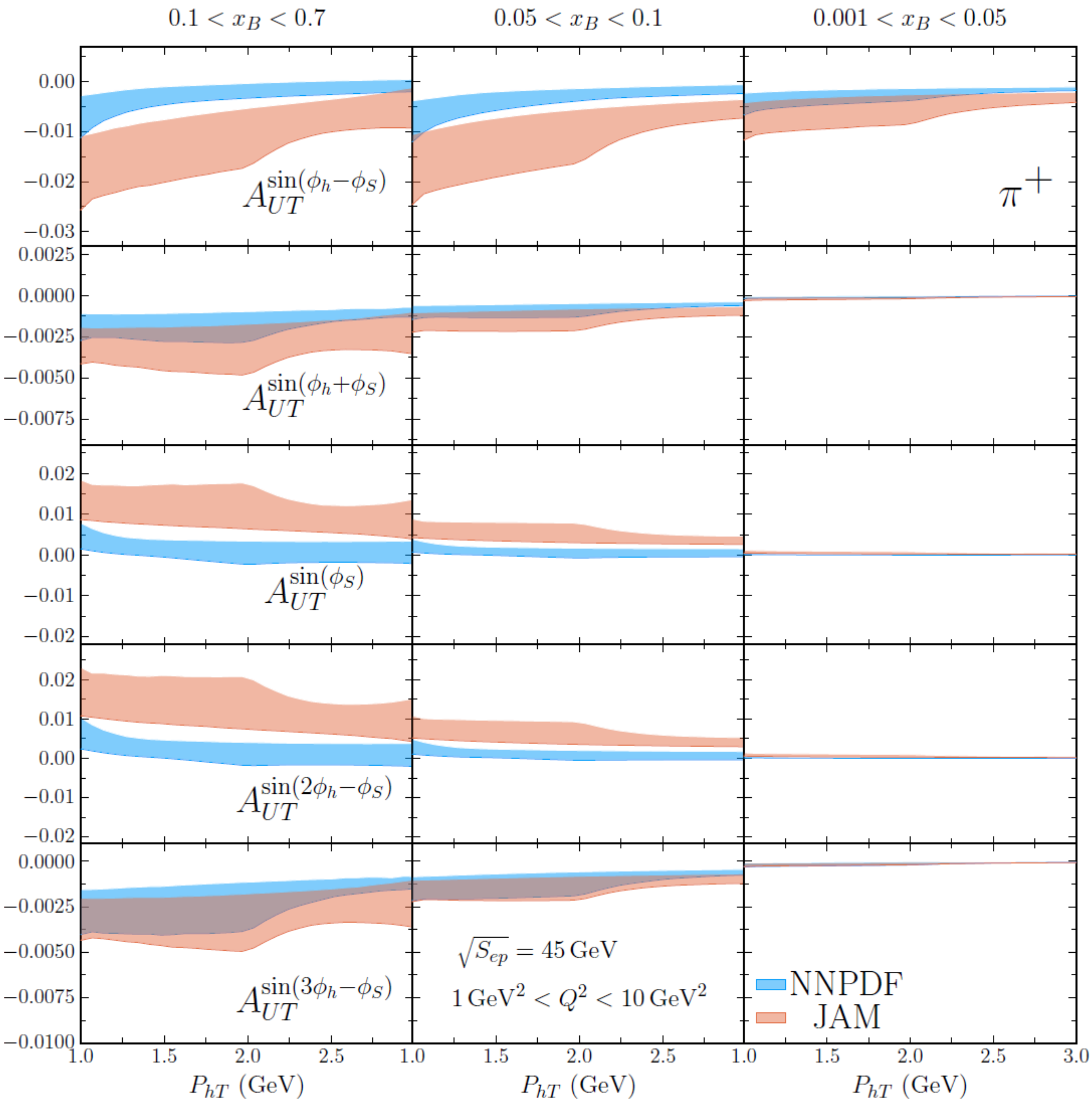
$0.1 < x_B < 0.7$
$0.05 < x_B < 0.1$
$0.001 < x_B < 0.05$
$1 < Q^2 < 10$
$10 < Q^2 < 50$
$50 < Q^2 < 500$
$A_{UT}^{\sin(\phi_h-\phi_S)}$
2%
$\mathcal{O}(\alpha_s^2)$ not small
EIC
$\sqrt{S_{ep}} = 45\,\mathrm{GeV}$
π^+
NNPDF
JAM
P_{hT} (GeV)

SSA insensitive to COM energy



top EIC energy

energy dependence cancels between numerator and denominator



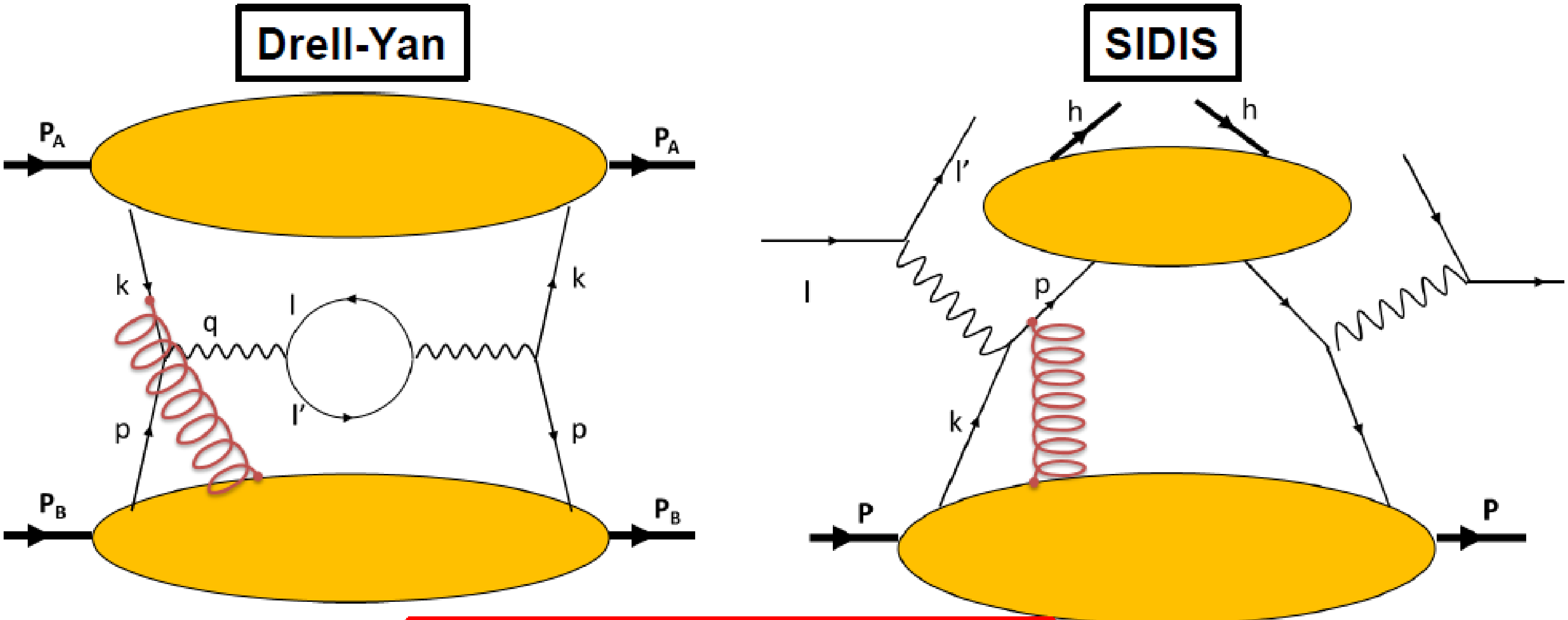
predictions for various harmonics, some reach 2%

Summary

- There are many sources of SSA at subleading level in collinear or TMD factorizations
- gT PDF contributes with 2-loop hard kernel
- Both quark- and gluon-initiated channels computed, latter being negligible
- Our Sivers asymmetry reaches 2% at pT=1 GeV, but opposite in sign compared to COMPASS data
- There cannot be single source at low pT
- More sources need to be included in global analysis on SSA
- Some other harmonics also reach 2%

Back-up slides

Sign change of Sivers function



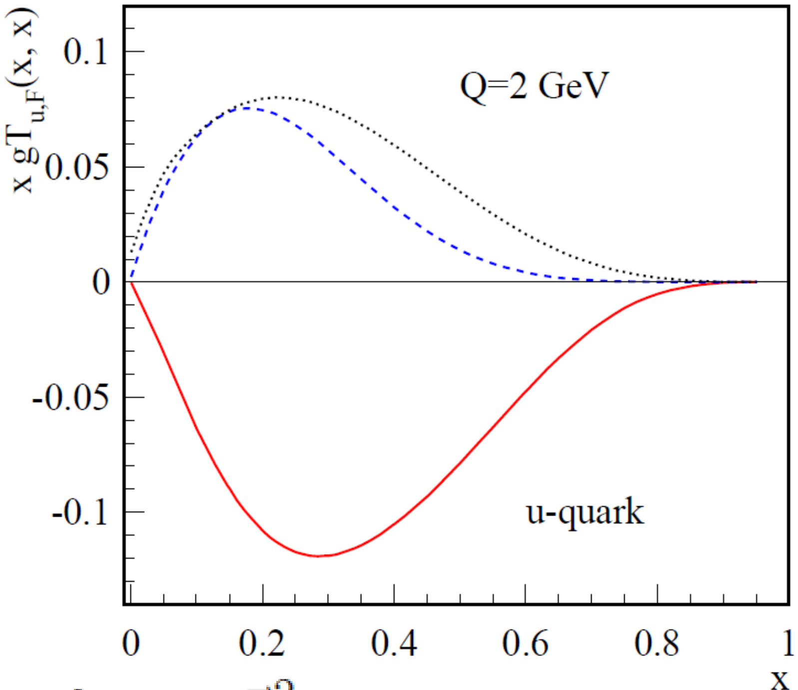
$$\text{Sivers}\,|_{DY} = -1 * \text{Sivers}\,|_{SIDIS}$$

Sign-mismatch problem

- No sign flip seen in $p^{\uparrow}p \rightarrow \pi + X$

Kang, Qiu, Vogelsang, Yuan 2011

correlation function for polarized proton, assumed to dominate



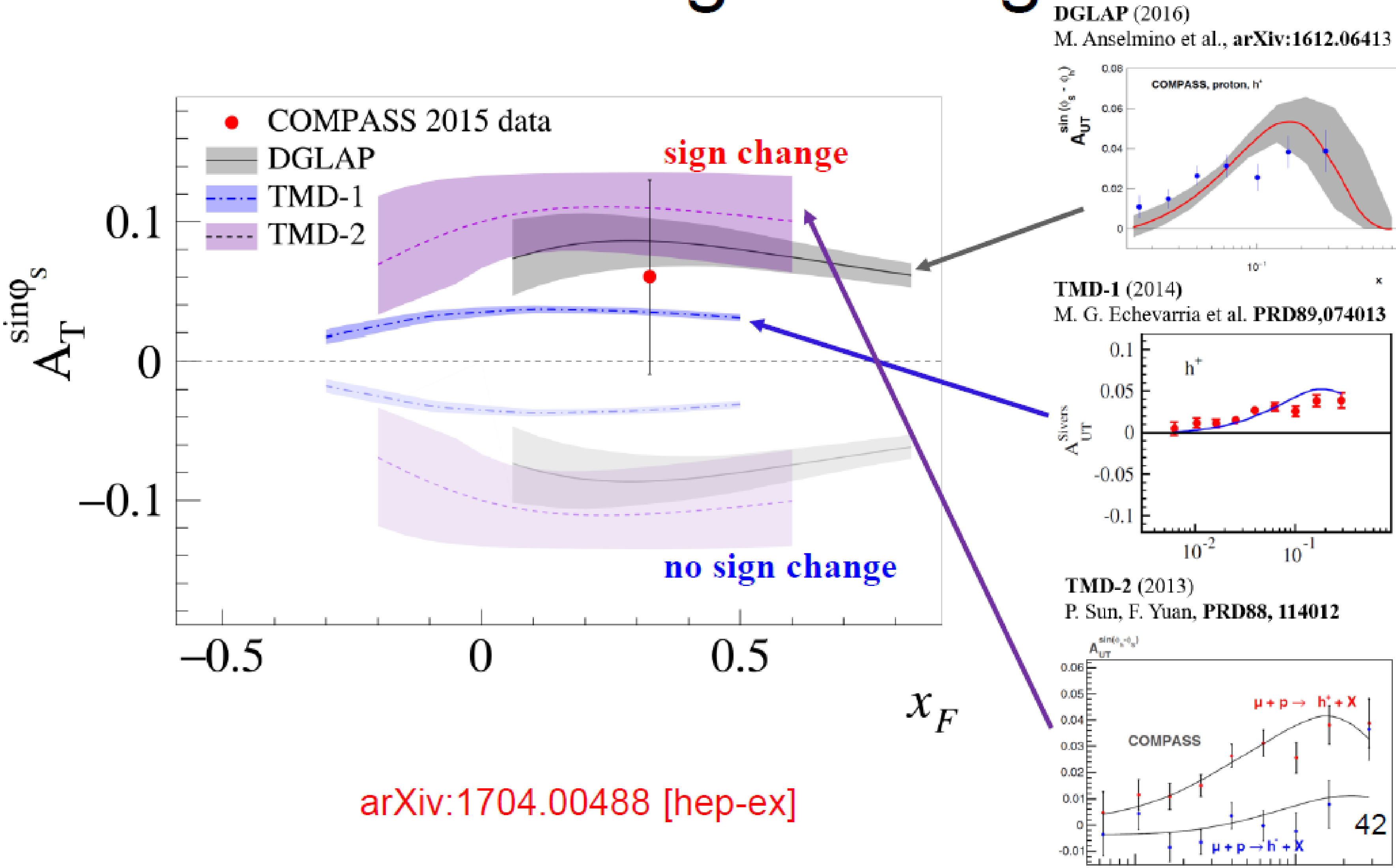
expectation from SIDIS data (HERMES, COMPASS) under sign flip

result extracted from data (E704, STAR, PHENIX, BRAHMS)

$$T_F^q(x,x) = -\int d^2\vec{p}_\perp \frac{\vec{p}_\perp^{\,2}}{M} f_{1T}^{\perp q}(x,\vec{p}_\perp^{\,2})\Big|_{\mathrm{SIDIS}}$$

- Now there are other twist-3 contributions…

Sivers Asymmetry in Drell-Yan: Hint of Sign Change!

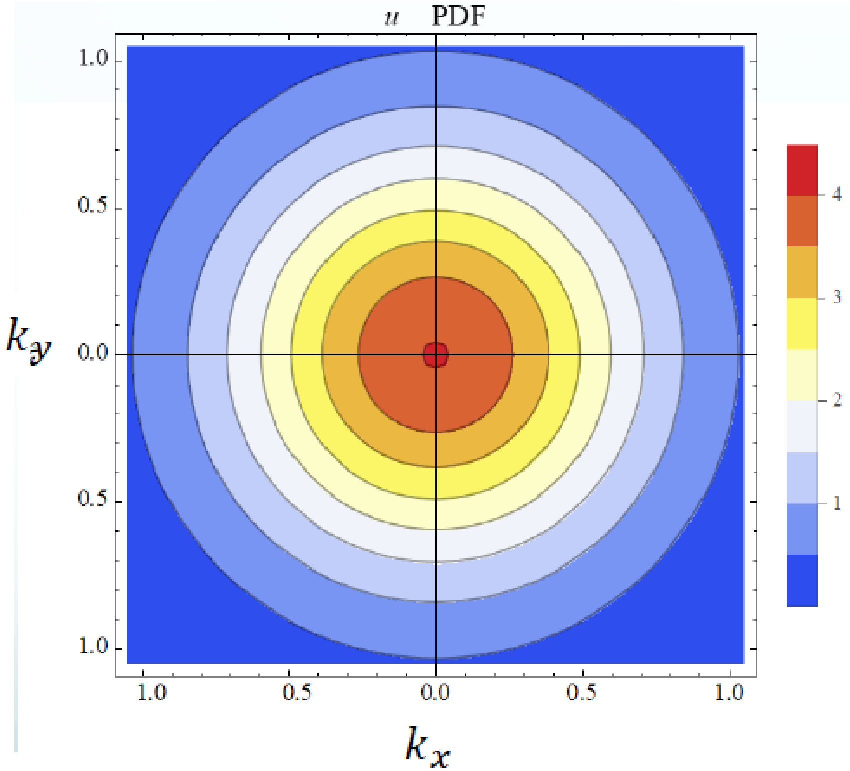


arXiv:1704.00488 [hep-ex]

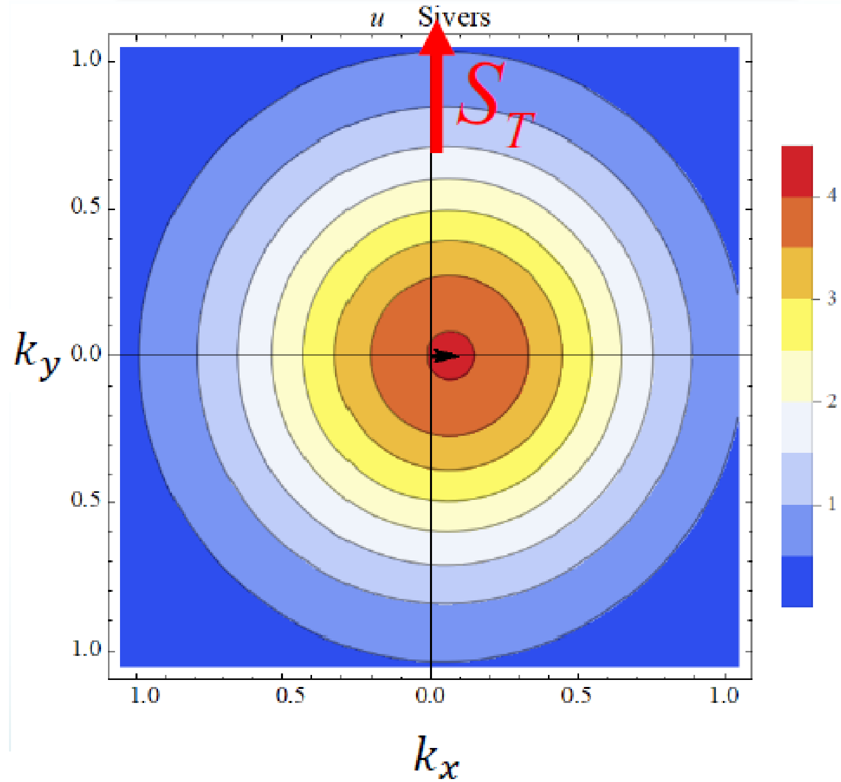
Spin-transverse-momentum correlation

$$f_{q/p\uparrow}(x, k_T, \vec{S}_T) = f_{q/p}(x, k_T) - \frac{1}{M} f_{1T}^{\perp q}(x, k_T) \vec{S}_T \cdot (\hat{p}_h \times k_T)$$

Unpolarized proton



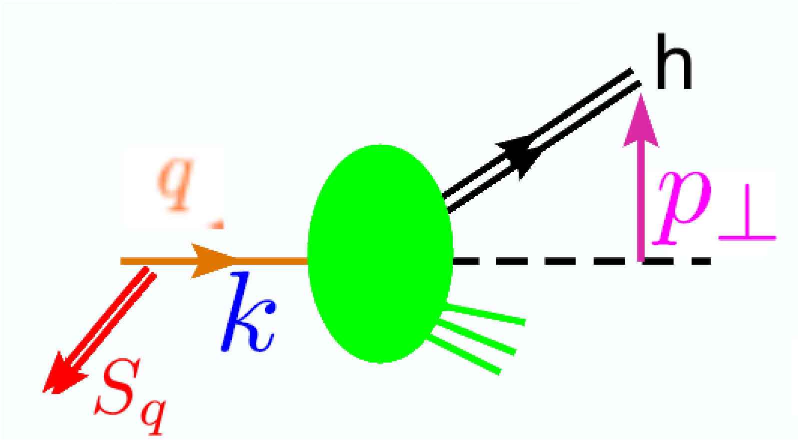
Transversely-polarized proton



produced hadron tends to move to right

Mechanism

- Transversity distribution for polarized proton determines preferred direction of quark spin
- This polarized quark scattered into final state
- Collins function then determines direction of polarized quark (produced hadron) preferred by correlation
- Without preferred direction of quark spin from initial state, Collins function cannot work



Factorization of transverse gluon

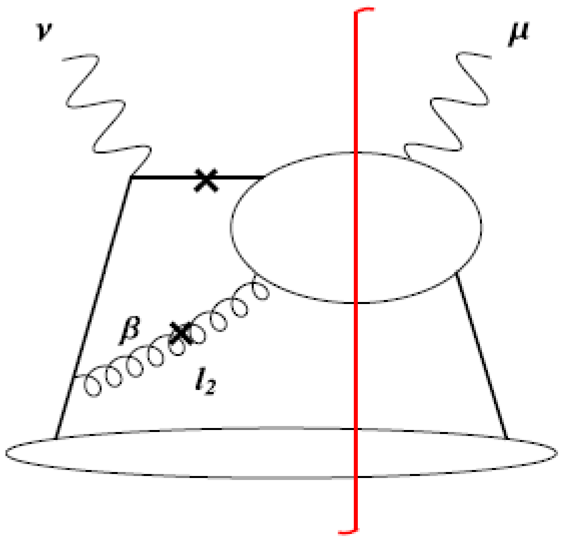
- As soft gluon carries transverse polarization, outgoing quark line cannot be eikonalized
- Collinear divergence in (+,+) combination goes into three-parton TMD, whose collinear version is Efremov-Teryaev-Qiu-Sterman (ETQS) function   Efremov, Teryaev 1982; Qiu, Sterman, 1991
- Similar construction for three-parton fragmentation functions

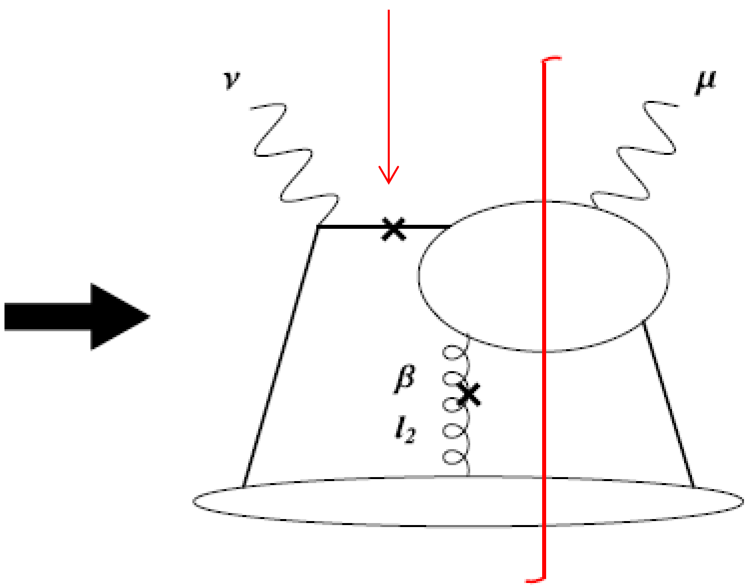
Kang, Yuan, Zhou, 2010

ETQS function

- Twist-3 ETQS function is PDF, not TMD, regarded as leading source of SSA in collinear factorization applicable to large pT

so-called hard pole contribution to SSA

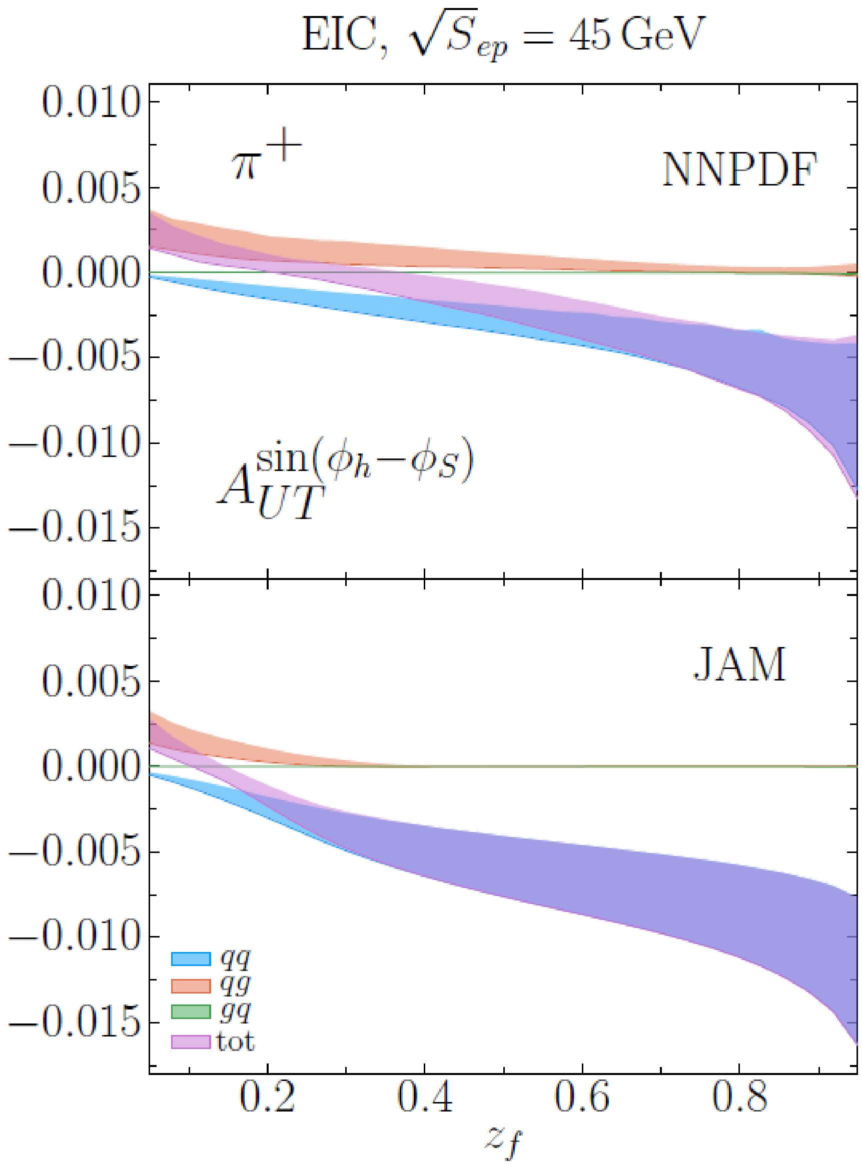




Lesson learned

- Sivers, ETQS, Collins functions all have same origin, resulting from different factorization
- Siver, Collins contributions start from LO hard kernel; ETQS starts from NLO
- If allowed to go to higher-order hard kernels, other projectors can be used
- data with Q ~ few GeV (such as COMPASS) and pT ~ 1GeV, hard kernel effect may be sizable
- Hard kernel is process-dependent
- Rich phenomenology!

JAM > NNPDF

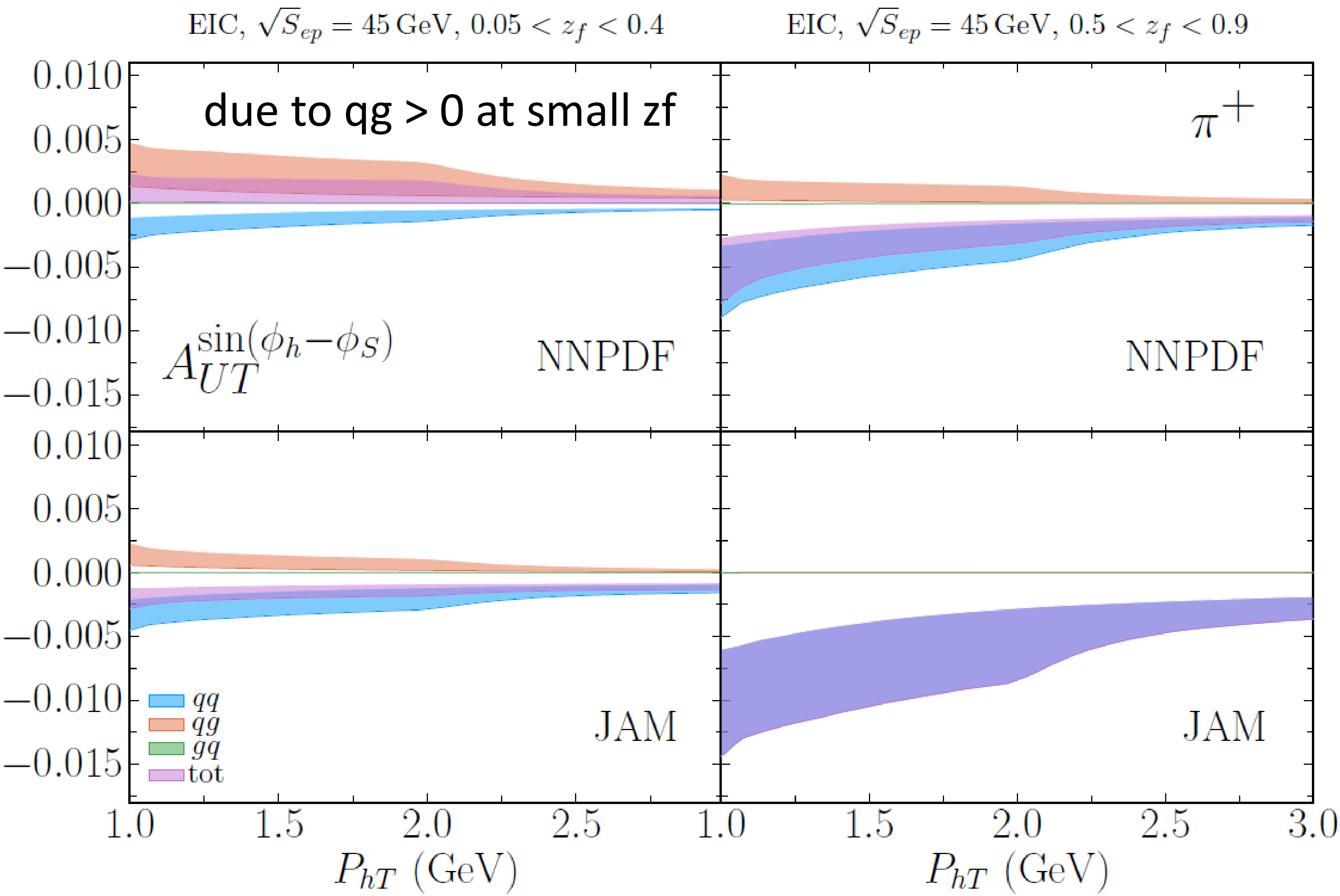


q->q dominates at large zf
qg dominates at small zf

qq, qg opposite in sign,
because q,g fly back-to-back

gg negligible

qq from JAM larger than NNPDF
qg from JAM decreases faster than NNPDF



Sivers asymmetry from NNPDF can flip sign between small and large zf bins