



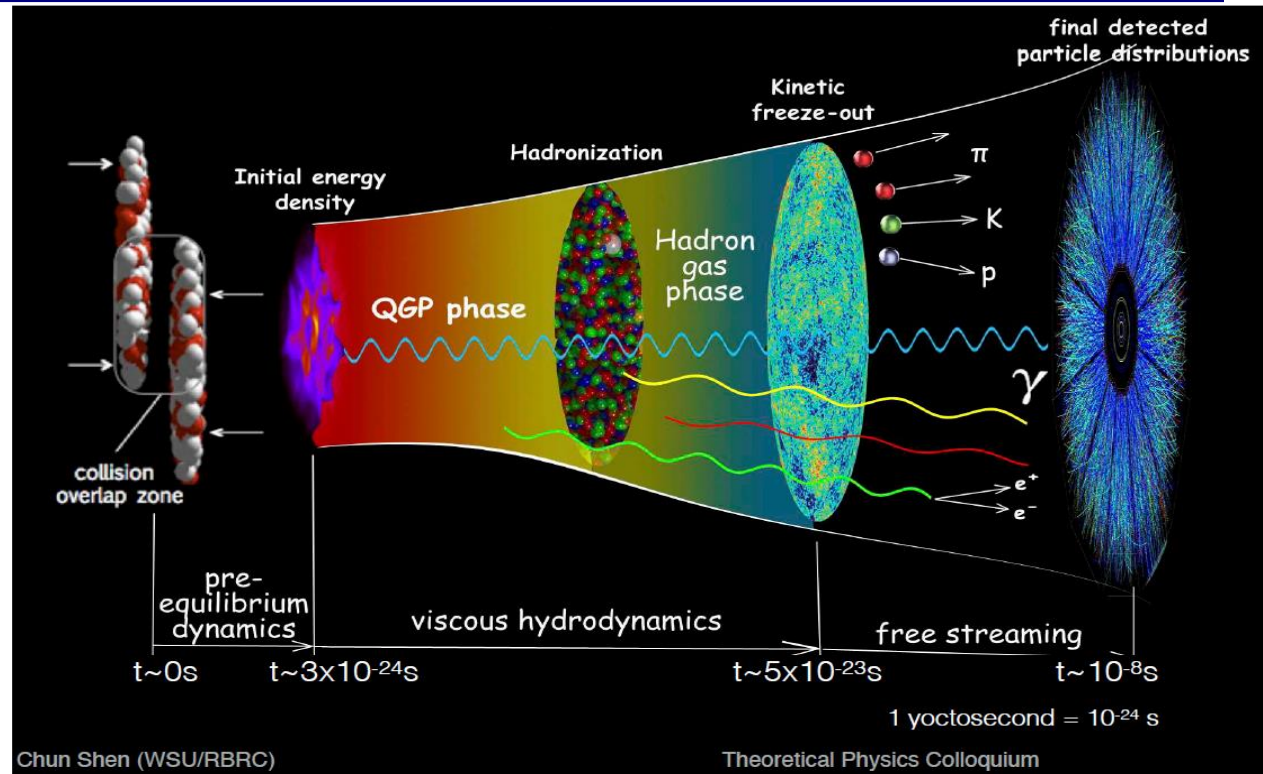
Exploration of spin transport in heavy ion collisions from quantum kinetic theory

Di-Lun Yang

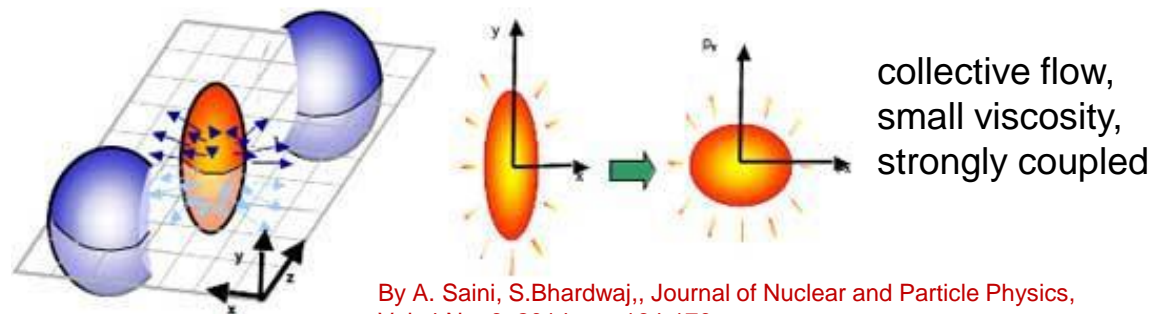
Institute of Physics, Academia Sinica
(2022 TQCD 1st meeting, Jan. 18th, 2022)

Relativistic heavy ion collisions

- Creating the quark gluon plasma (QGP) :

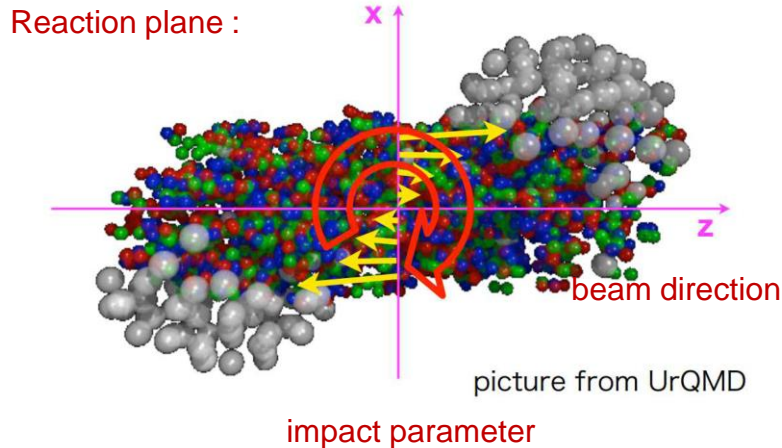


- Hydrodynamic evolution :

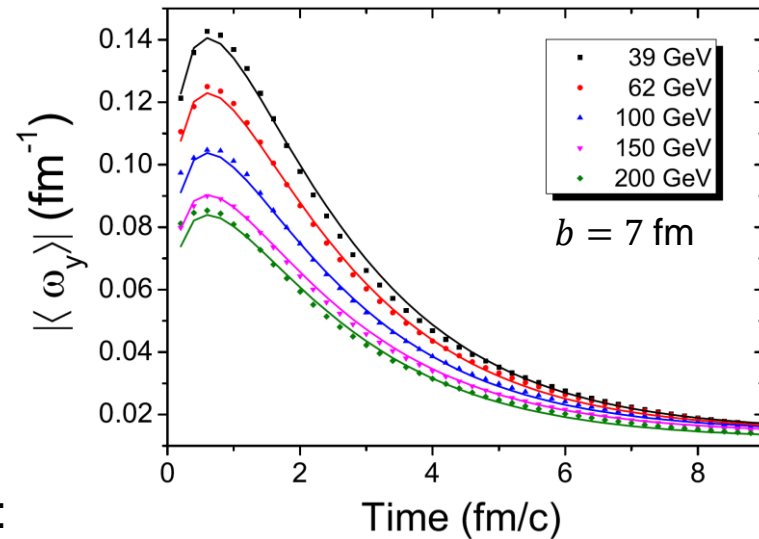


Subatomic swirls

Strong vortical fields in HIC :



Y. Jiang, Z.-W. Lin, J. Liao, PRC 94, 044910 (2016)
see also W.-T. Deng and X.-G. Huang, PRC 93, 064907 (2016)



❖ Angular momentum (AM) to vorticity :

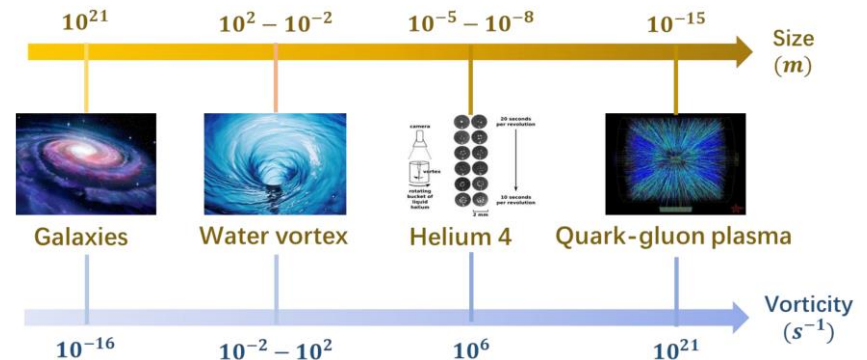
$$\mathbf{L} = \frac{1}{2} \int d^3 \mathbf{r} \epsilon |\mathbf{r}|^2 (1 - \hat{\omega} \cdot \hat{\mathbf{r}}) \boldsymbol{\omega},$$

$$\boldsymbol{\omega} = \nabla \times \mathbf{v} = \text{const.}$$

ϵ : energy density

❖ AM to spin polarization?

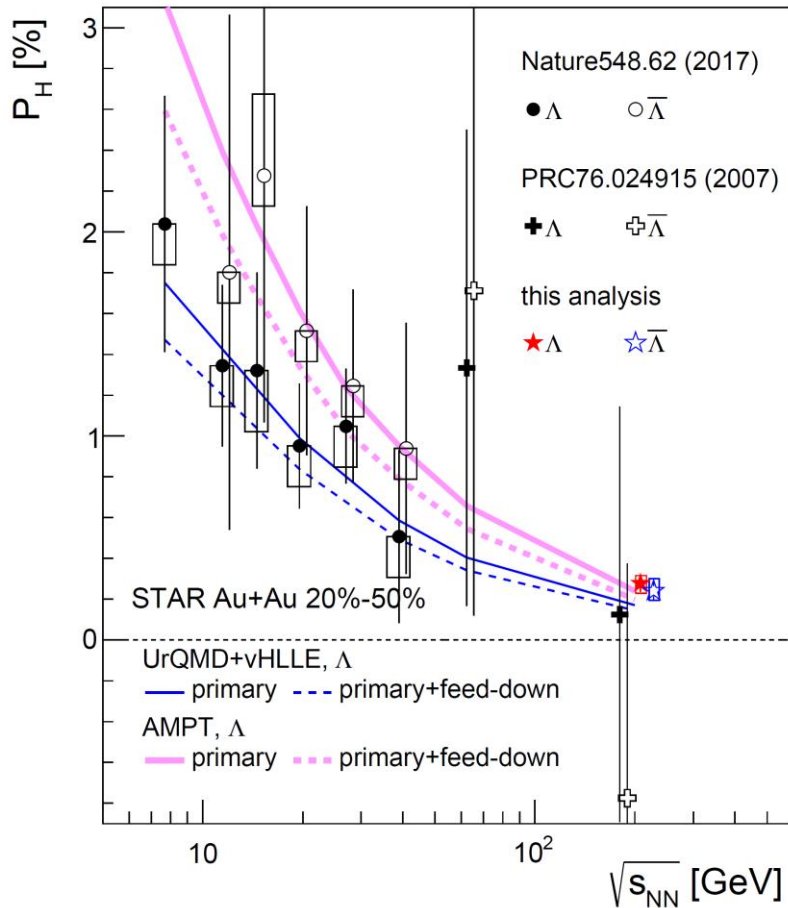
Z.-T. Liang and X.-N. Wang, PRL. 94, 102301 (2005)



X.-G. Huang, QM 19

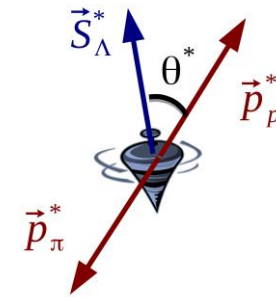
Λ polarization in HIC

Global polarization of Λ hyperons :



L. Adamczyk et al. (STAR), Nature 548, 62 (2017)

Self-analyzing via the weak decay :



T. Niida, QM18

Statistical model/Wigner-function approach (in global equilibrium):

F. Becattini, et al., Ann. Phys. 338, 32 (2013)

R. Fang, et al., PRC 94, 024904 (2016)

$$\mathcal{P}^\mu(q) \approx \frac{1}{8m} \epsilon^{\sigma\mu\nu\rho} q_\sigma \frac{\int d\Sigma \cdot q \omega_{\nu\rho} f_q^{(0)} (1 - f_q^{(0)})}{\int d\Sigma \cdot q f_q^{(0)}}$$

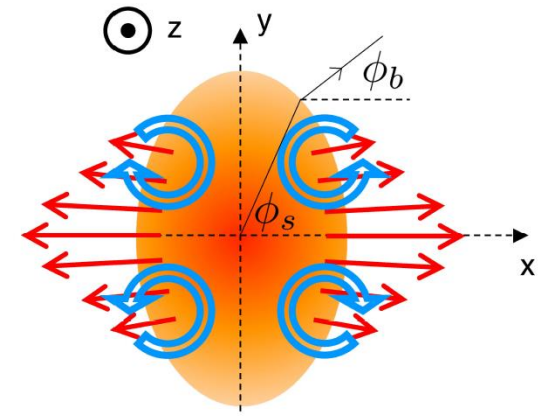
$$\omega_{\nu\rho} = \frac{1}{2} (\partial_\rho(u_\nu/T) - \partial_\nu(u_\rho/T)).$$

thermal vorticity

$$\xrightarrow{T=\text{const.}} \omega_{\alpha\beta} = -\epsilon_{\alpha\beta\mu\nu} \omega^\mu u^\nu$$

Local (longitudinal) polarization : a sign problem

- Local vorticity :
transverse expansion :
longitudinal vorticity & polarization



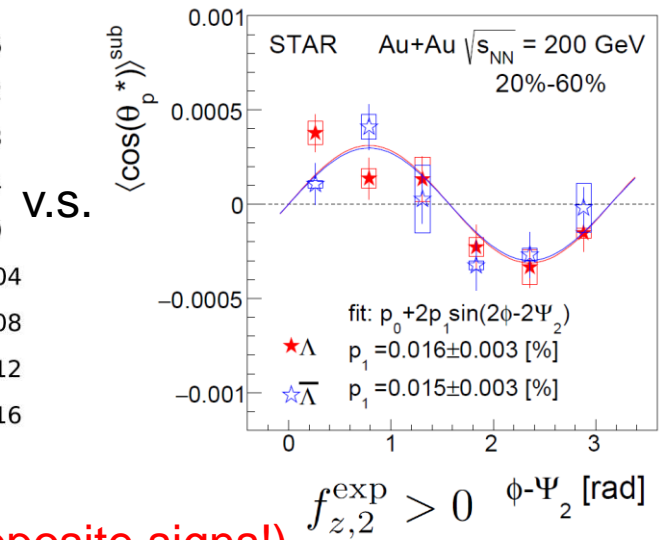
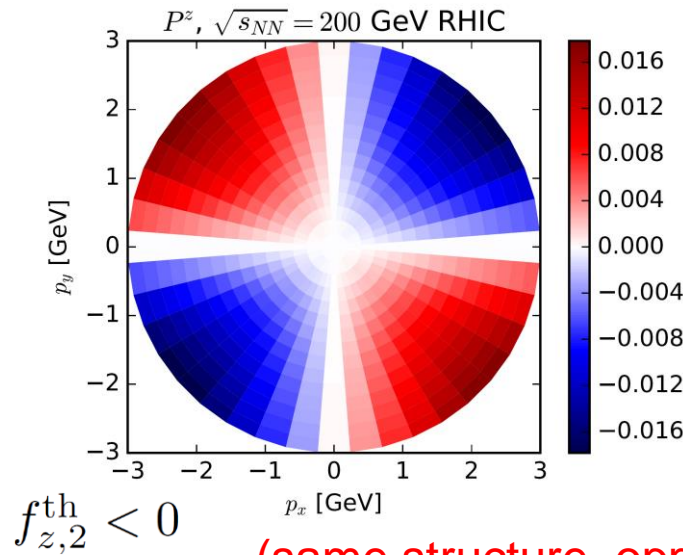
❖ A “sign problem” for longitudinal polarization

F. Becattini, I. Karpenko, PRL 120, 012302 (2018).

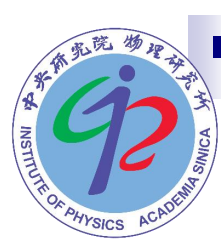
J. Adam et al. (STAR, PRL. 123, 132301 (2019))

Spin harmonics :

$$\frac{dP^z}{2\pi d\phi} = f_{z,0} + 2f_{z,2} \sin(2\phi)$$



(same structure, opposite signs!)



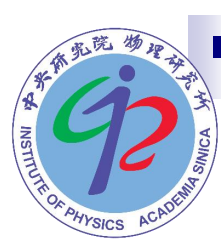
Go beyond global equilibrium

- The assumption for global equilibrium may be too naïve.
- To understand non-equilibrium effects and dynamical spin polarization :
 - Spin hydrodynamics (macroscopic)
 - Quantum kinetic theory (QKT) (microscopic)

How a (strange) quark traversing QGP becomes polarized?

$$\mathcal{P}^\mu(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^\mu(p, X)}{2m \int d\Sigma \cdot \mathcal{N}(p, X)}$$

- The key ingredient to generate dynamical spin polarization : quantum corrections in collisions (spin-orbit int.).
- Tracking the non-equilibrium evolution in phase space is challenging : solving multi-dimensional differential equations.
- Near equilibrium : QKT \Rightarrow spin hydro.



Spin polarization in local equilibrium

- Spin polarization for massless fermions in local equilibrium can be obtained from the CKT with Coulomb scattering. [Y. Hidaka, S. Pu, DY, PRD 97, 016004 \(2018\)](#)

$$\mathcal{J}_5^\mu = \mathcal{J}_{\text{thermal}}^\mu + \mathcal{J}_{\text{shear}}^\mu + \mathcal{J}_{\text{accT}}^\mu + \mathcal{J}_{\text{chemical}}^\mu + \mathcal{J}_{\text{EB}}^\mu, \quad (+ \text{dissipative terms})$$

$$\mathcal{J}_{\text{thermal}}^\mu = a \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} p_\nu \partial_\alpha \frac{u_\beta}{T}, \quad \Rightarrow \quad J_5^\mu = \sigma_{5\omega} \omega^\mu \quad (\text{chiral vortical effect})$$

[A. Vilenkin, PRD 20, 1807 \(1979\)](#)

$$\mathcal{J}_{\text{shear}}^\mu = -a \frac{1}{(u \cdot p)T} \epsilon^{\mu\nu\alpha\beta} p_\alpha u_\beta p^\sigma \partial_{\langle\sigma} u_{\nu\rangle},$$

$$\mathcal{J}_{\text{accT}}^\mu = -a \frac{1}{2T} \epsilon^{\mu\nu\alpha\beta} p_\nu u_\alpha \left(Du_\beta - \frac{1}{T} \partial_\beta T \right),$$

$$a = 4\pi \hbar \text{sign}(u \cdot p) \delta(p^2) f_V^{(0)} (1 - f_V^{(0)}).$$

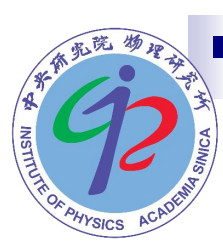
[C. Yi, S. Pu, DY, PRC 104, 064901\(2021\)](#)

(“naïve” generalization to massive fermions)

- Generalization to the massive case was also derived from the linear response theory and statistical field theory. [S. Y. F. Liu and Y. Yin, PRD 104, 054043 \(2021\)](#)

[S. Y. F. Liu, Y. Yin, JHEP 07, 188 \(2021\)](#)

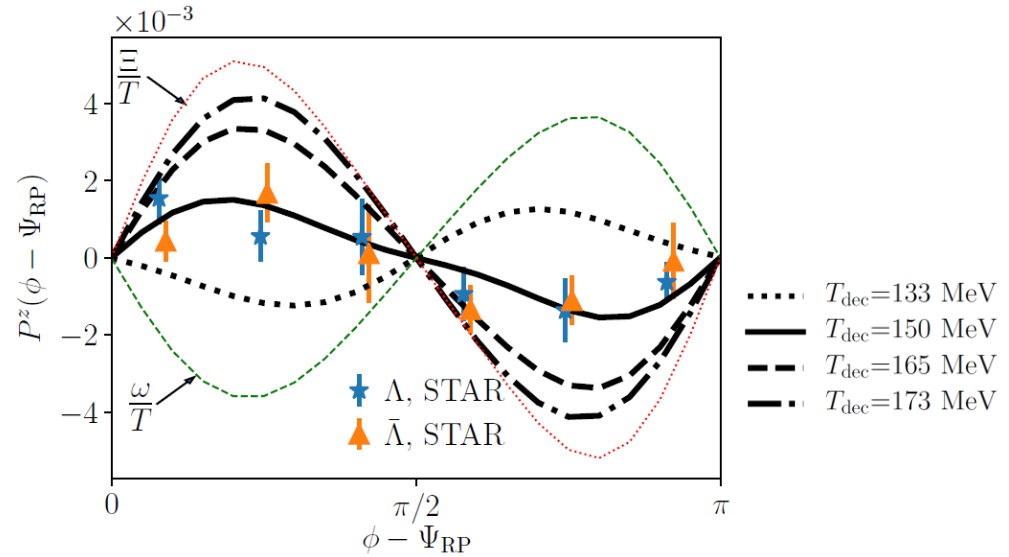
[F. Becattini, M. Buzzegoli, A. Palermo, PLB 820,136519 \(2021\)](#)



Shear corrections on longitudinal polarization

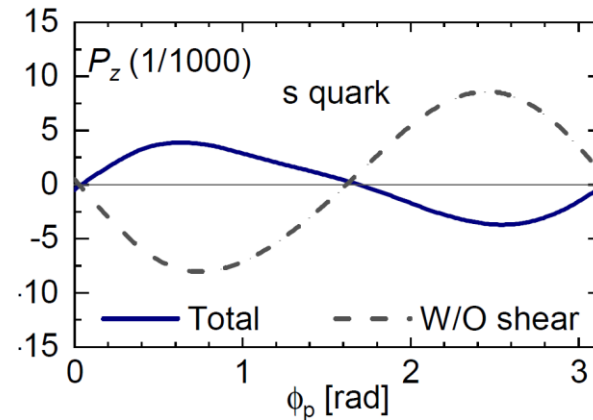
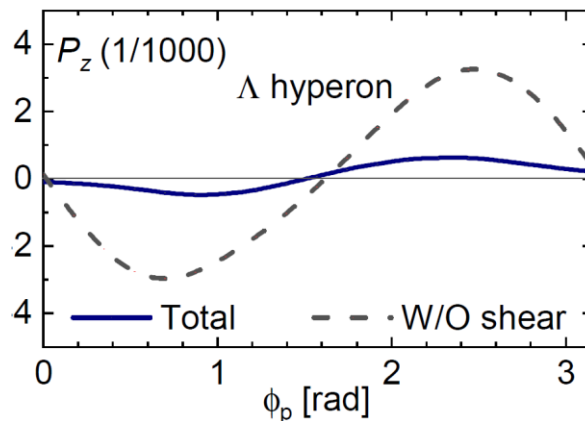
■ Isothermal approximation :

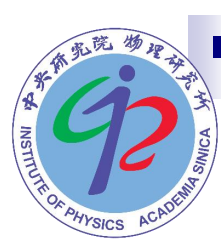
F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, and I. Karpenko, PRL 127 (2021) 27, 272302



■ Λ and s equilibrium scenarios:

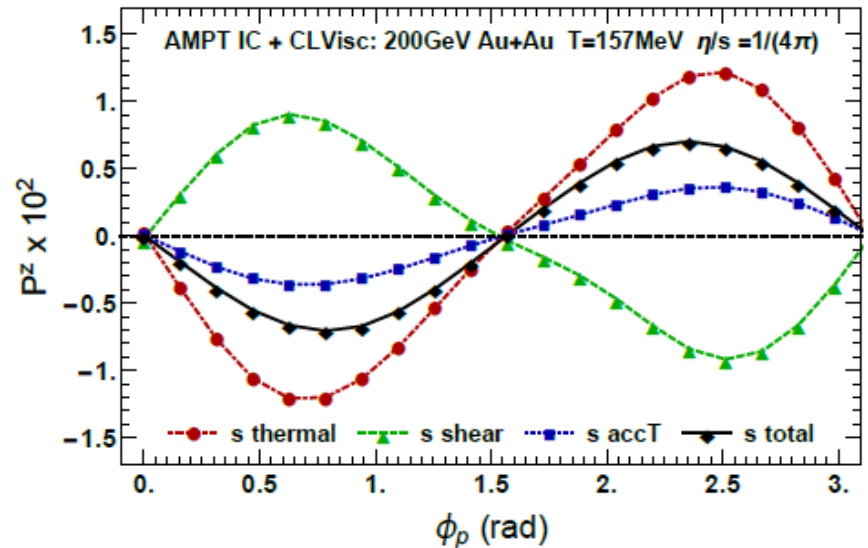
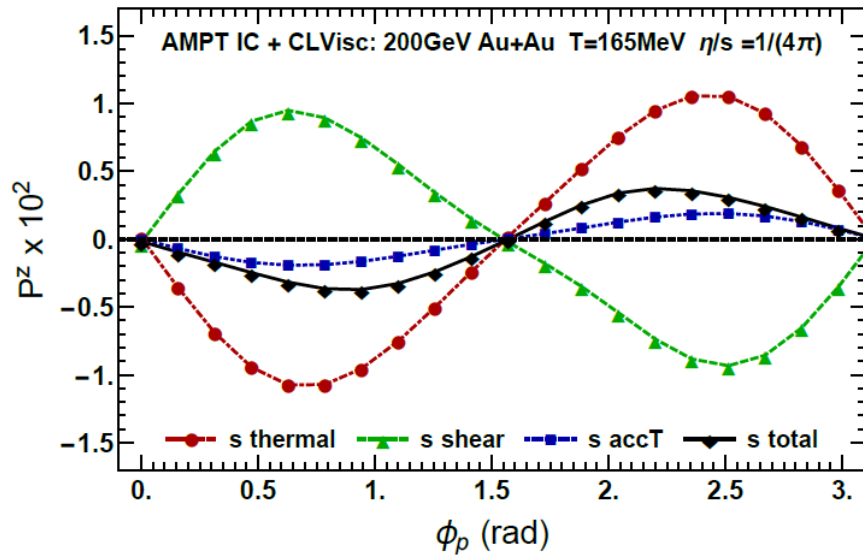
B. Fu, S. Y. F. Liu, L. Pang, H. Song, and Y. Yin, PRL 127, 142301 (2021)



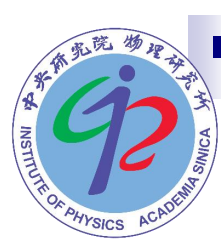


Is the spin sign problem solved?

- Sensitive to the equation of state and freeze-out T : out-of-equilibrium corrections depending on interaction should be still considered.



C. Yi, S. Pu, DY, PRC 104 (2021) 6, 064901



Helicity polarization

- A better observable to probe the strength of local vorticity?
- Helicity polarization : $S^h = \hat{\mathbf{p}} \cdot \mathbf{S}(\mathbf{p}) = \hat{p}^x S^x + \hat{p}^y S^y + \hat{p}^z S^z$,

F. Becattini et al., Phys.Lett.B 822 (2021) 136706

J.-H. Gao, Phys. Rev. D 104, 076016

C. Yi, S. Pu, DY, (2021) 2112.15531

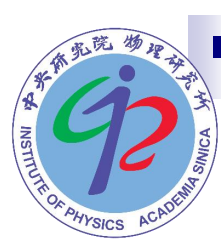
$$S_{\text{thermal}}^h(\mathbf{p}) = \int d\Sigma^\sigma F_\sigma p_0 \epsilon^{0ijk} \hat{p}_i \nabla_j \left(\frac{u_k}{T} \right),$$

$$S_{\text{shear}}^h(\mathbf{p}) = - \int d\Sigma^\sigma F_\sigma \frac{\epsilon^{0ijk} \hat{p}^i p_0}{(u \cdot p) T} \{ p^\sigma (\partial_\sigma u_j + \partial_j u_\sigma - u_\sigma D u_j) u_k \},$$

$$S_{\text{accT}}^h(\mathbf{p}) = \int d\Sigma^\sigma F_\sigma \frac{1}{T} \epsilon^{0ijk} \hat{p}^i p_0 u_j (D u_k - \frac{1}{T} \partial_k T),$$

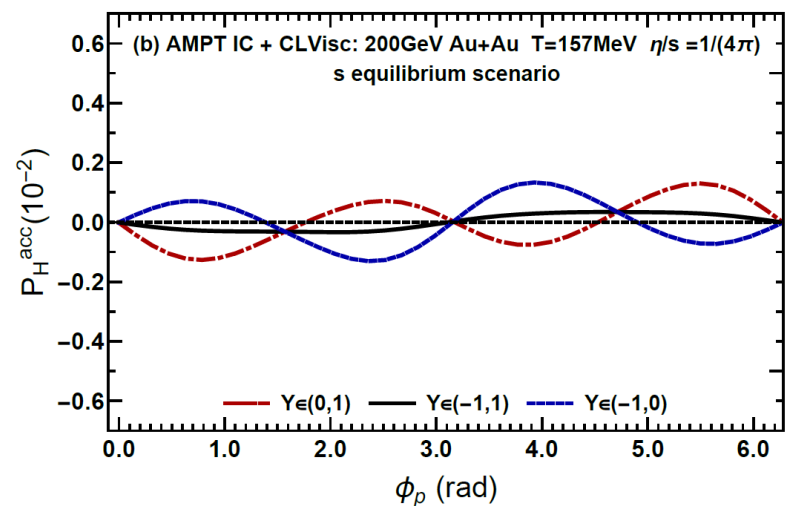
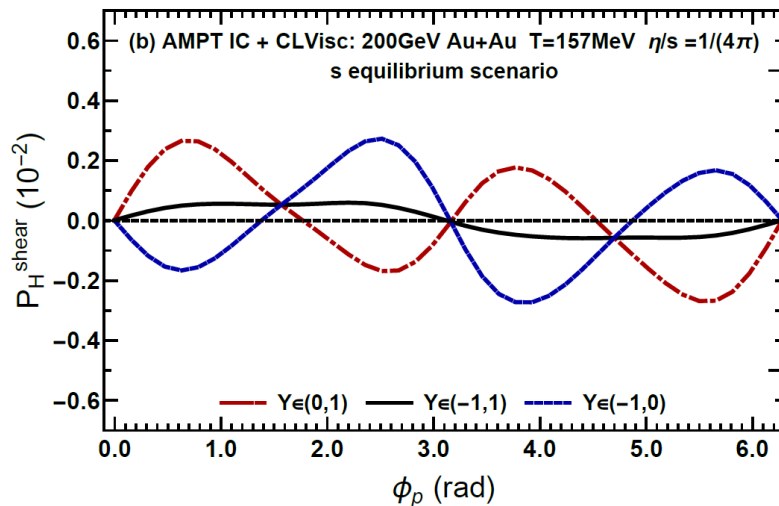
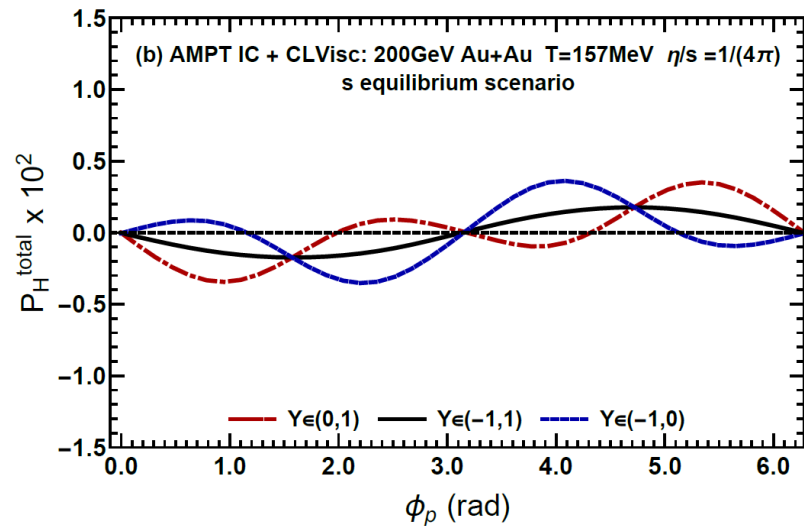
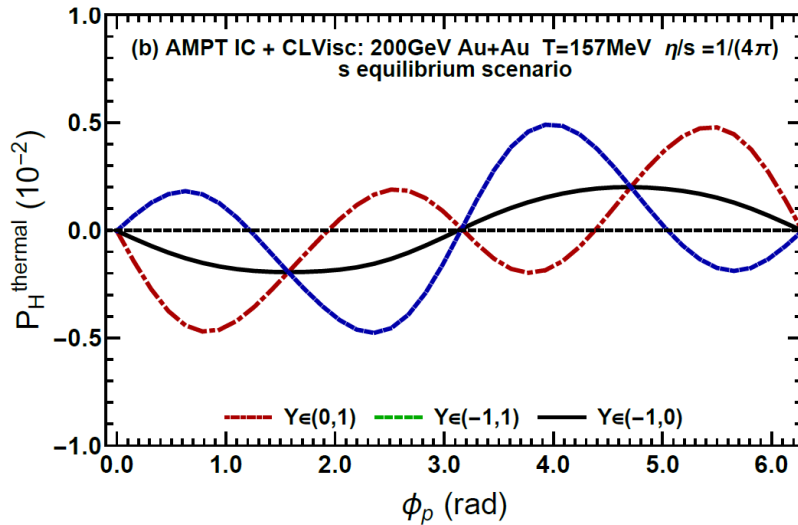
$$F^\mu = \frac{\hbar}{8m_\Lambda N} p^\mu f_V^{(0)} (1 - f_V^{(0)})$$

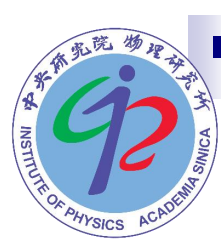
- When the fluid velocity is small, (fluid) vorticity contribution becomes dominant.



Hydrodynamic helicity polarization

- The dominant contribution from thermal vorticity :

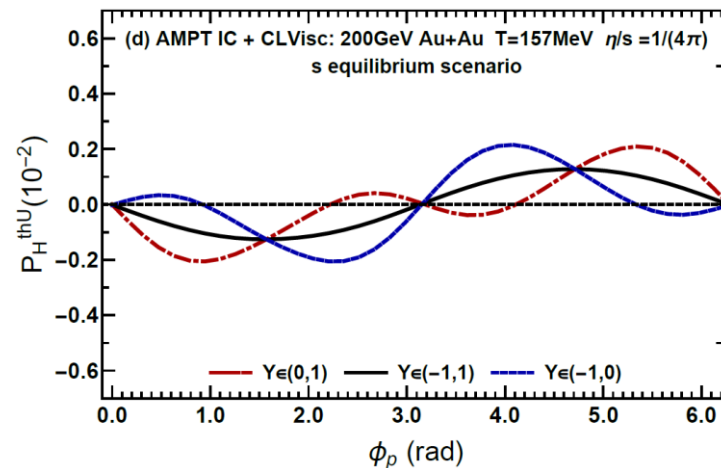
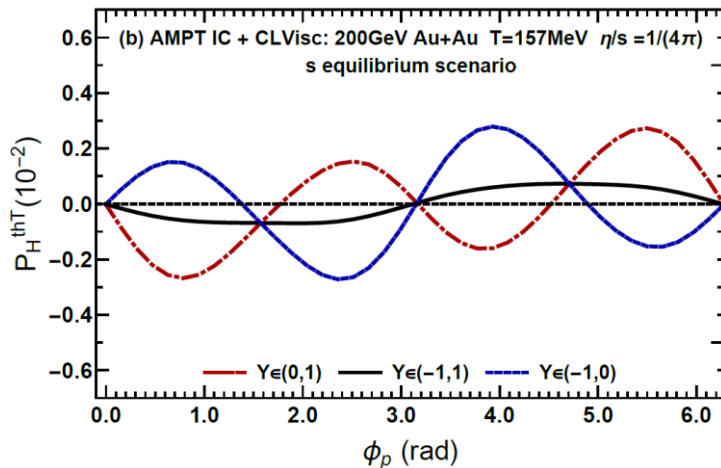




Helicity polarization from fluid vorticity

- Decomposition of the polarization from thermal vorticity:

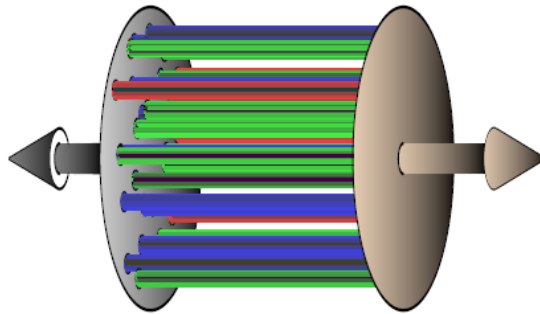
$$S_{\text{thT}}^h(\mathbf{p}) = \int d\Sigma^\sigma F_\sigma \frac{p_0}{T^2} \hat{\mathbf{p}} \cdot (\mathbf{u} \times \nabla T), \quad S_{\text{thU}}^h(\mathbf{p}) = \int d\Sigma^\sigma F_\sigma \frac{p_0}{T} \hat{\mathbf{p}} \cdot \boldsymbol{\omega}$$



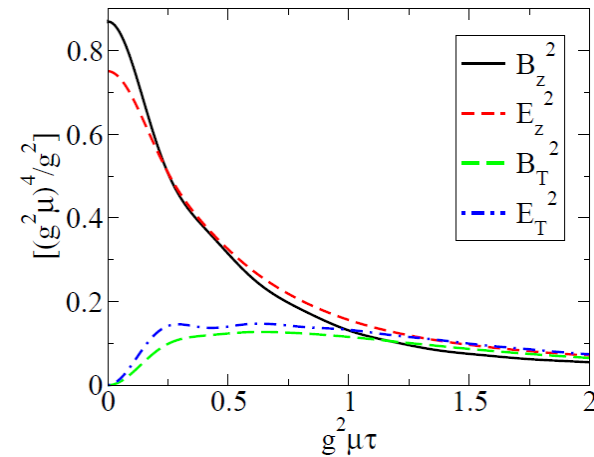
- For low-energy collisions, fluid vorticity increases and the velocity decreases.
 - ➡ Helicity polarization from fluid-vorticity becomes more dominant
- Probing strongest local fluid vorticity from helicity polarization with the beam energy scan?

Chromo-electromagnetic fields in HIC

- Color flux tubes in the glasma phase : longitudinal chromo-EM fields in early times.



review: F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, *Ann.Rev.Nucl.Part.Sci.*60:463-489,2010



- Plasma instability could enhance the color fields.

T. Lappi, *Phys.Lett.B* 643 (2006) 11-16

- No local parity violation :

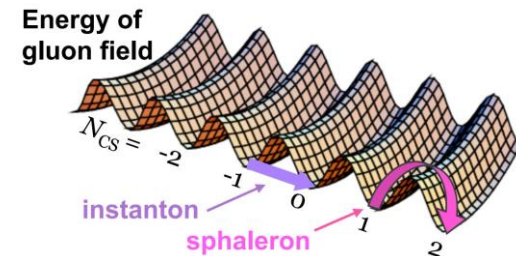
S. Mrowczynski, *PLB* 214, 587 (1988), *PLB*314,118 (1993)

P. Romatschke and M. Strickland, *PRD* 68, 036004 (2003)

$$\langle B_\mu^a(X) B_\nu^a(X') \rangle \neq 0, \quad \langle E_\mu^a(X) E_\nu^a(X') \rangle \neq 0, \quad \langle B_\mu^a(X) E_\nu^a(X') \rangle = 0.$$

- Local-parity violation : $\langle B_\mu^a(X) E_\nu^a(X') \rangle \neq 0$

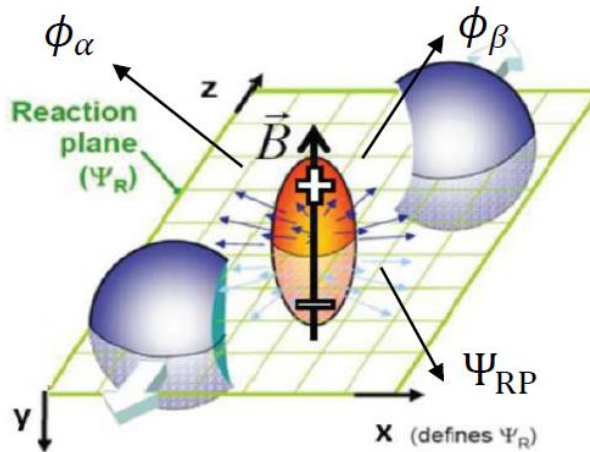
e.g. N. Tanji, N. Mueller, and J. Berges, *PRD* 93, 074507 (2016)



(Correlators in the QGP phase are unknown)

Probing local parity violation in QCD matter

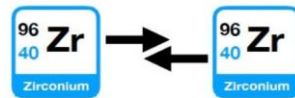
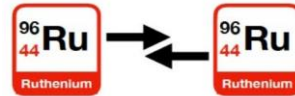
- Probing the local parity violation via the chiral magnetic effect (CME) :



$$\mathbf{J}_V = \frac{\mu_5}{2\pi^2} \mathbf{B}$$

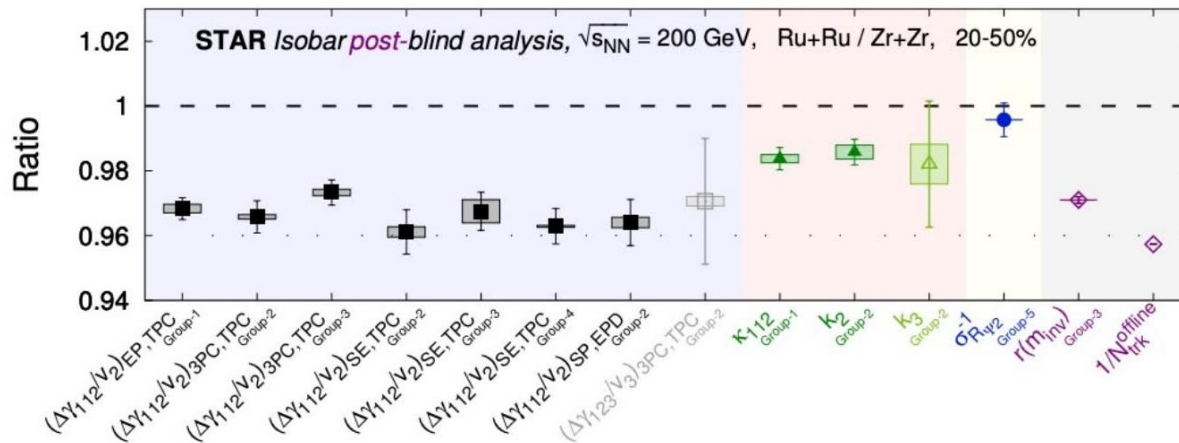
D. E. Kharzeev et al.,
Prog. Part. Nucl. Phys. 88, 1 (2016)

Isobar collisions : (same shape=background?)

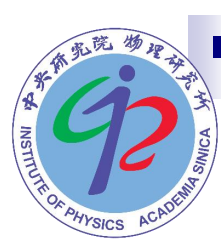


(diff. B fields=signal)

STAR, M. Abdallah et al., (2021), 2109.00131



<1
no
CME?



Spin alignments of vector mesons

- Decay daughter : $\frac{dN}{d\cos\theta^*} \propto [1 - \rho_{00} + \cos^2\theta^*(3\rho_{00} - 1)]$

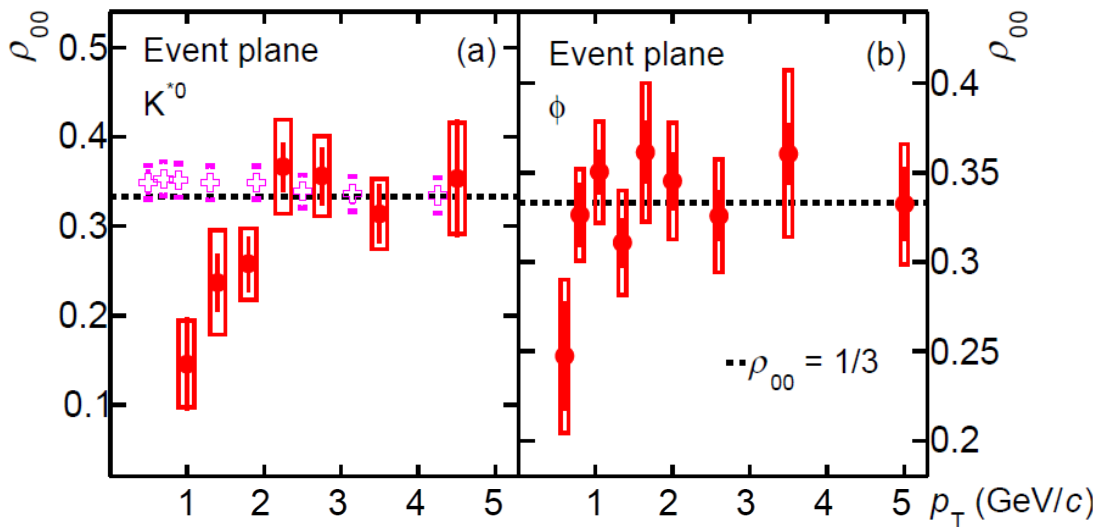
$$\rho_{00} = \frac{1 - \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i}{3 + \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i}$$

Z.-T. Liang and X.-N. Wang, PLB 629, 20 (2005)

$\rho_{00} \neq 1/3$: spin polarization

S. Acharya et al. (ALICE), PRL.125, 012301 (2020)

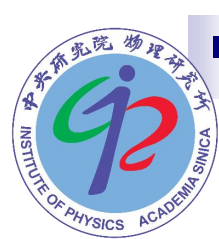
What is the source for the large deviations ??



- From the QKT with background color fields : B. Müller, DY, (2021), 2110.15630
DY, (2021), 2112.14392

$$\mathcal{P}^\mu(\mathbf{p}) \approx \frac{-\pi^{3/2} \tau_c \beta \int d\Sigma \cdot p (\langle E^a \cdot B^a \rangle u^\mu + \langle B^{a\mu} E^{a\nu} \rangle p_\nu \beta (1 - 2f_{\text{eq}}(p \cdot u))) f_{\text{eq}}(p \cdot u) (1 - f_{\text{eq}}(p \cdot u))}{4mp \cdot u \int d\Sigma \cdot p f_{\text{eq}}(p \cdot u)}$$

- Could spin alignments complement the CME study to probe the local parity violation?



Conclusions

- ✓ QKT plays an essential role to understand dynamical spin polarization in HIC.
- ✓ Although local-equilibrium corrections (in particular the shear) yield substantial contributions to local spin polarization, non-equilibrium effects should be involved.
- ✓ Helicity polarization may be a better probe for local vorticity.
- ✓ Not only the collisions, but also dynamically generated color fields may affect the spin polarization of quarks in QGP.
- ✓ Anomalous spin polarization could be triggered by parity-odd correlators of color fields, which may possibly explain the spin alignments in high-energy nuclear collisions.

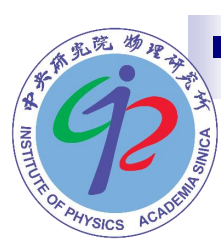
Refs: [1] Cong Yi, Shi Pu, Di-Lun Yang, PRC 104, 064901(2021).

[2] Cong Yi, Shi Pu, Di-Lun Yang, (2021), 2112.15531.

[3] Berndt Müller and Di-Lun Yang, (2021), 2110.15630.

[4] Di-Lun Yang, (2021), 2112.14392.

[5] Yoshimasa Hidaka, Shi Pu, Di-Lun Yang, Qun Wang, "Foundations and Applications of Quantum Kinetic Theory" (review), submitted to prog.part.nucl.phys.

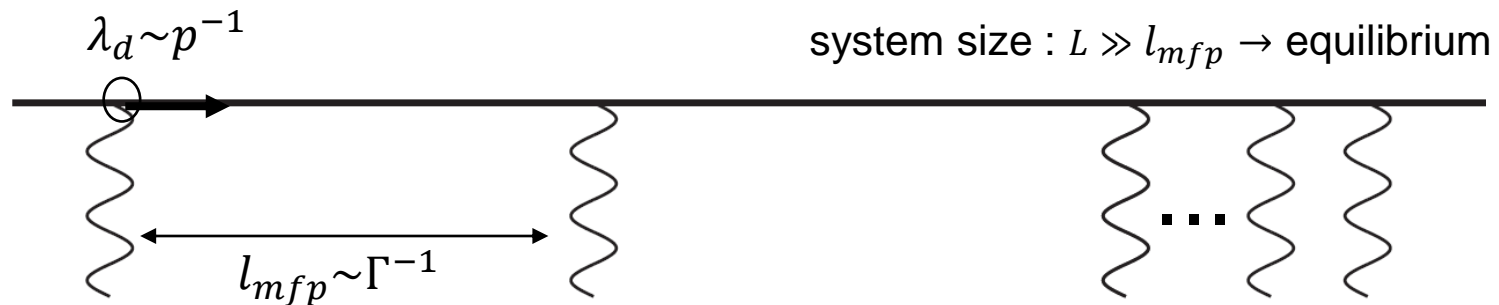


Thank you!

Some properties of kinetic theory

- Kinetic theory : microscopic theory for quasi-particles in phase space

- ❖ Boltzmann (Vlasov) Eq. : $q^\mu \Delta_\mu f(q, X) = q^\mu C_\mu[f]$, $\Delta_\mu = \partial_\mu + F_{\nu\mu} \frac{\partial}{\partial q^\nu}$.
- ❖ Physical quantities : $J^\mu(X) = \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{q^\mu}{E_q} f(q, X)$, $T^{\mu\nu}(X) = \int \frac{d^3 \mathbf{q}}{(2\pi)^3} \frac{q^\mu q^\nu}{E_q} f(q, X)$.
- ❖ Valid for weak coupling : mean free path \gg de Broglie wavelength



- ❖ Near equilibrium : kinetic theory \Rightarrow hydrodynamics
- Classical kinetic theory $\Rightarrow \partial_\mu J^\mu = 0$, $\partial_\mu T^{\mu\nu} = F^{\nu\rho} J_\rho$.
- CKT: $(q \cdot \Delta + \hbar \tilde{\Delta}) f_R = q \cdot C[f_R] + \hbar \tilde{C}[f_R] \Rightarrow \partial_\mu J_R^\mu = -\frac{\hbar}{4\pi^2} E \cdot B$
(for right-handed fermions) (chiral anomaly)

CKT with collisions

- WF up to $\mathcal{O}(\hbar)$: (for right-handed fermions)

quantum corrections

$$\dot{S}^<(q, X) = \bar{\sigma}_\mu 2\pi \bar{\epsilon}(q \cdot n) \left(q^\mu \delta(q^2) f_q^{(n)} + \hbar \delta(q^2) S_{(n)}^{\mu\nu} \mathcal{D}_\nu f_q^{(n)} + \hbar \epsilon^{\mu\nu\alpha\beta} q_\nu F_{\alpha\beta} \frac{\partial \delta(q^2)}{2\partial q^2} f_q^{(n)} \right)$$

$$\mathcal{D}_\beta f_q^{(n)} = \Delta_\beta f_q^{(n)} - \mathcal{C}_\beta$$

- CKT with collisions ($\partial_\rho n^\mu = 0$) :

Y. Hidaka, S. Pu, DY,
PRD 95, 091901 (2017),
PRD 97, 016004 (2018)

$$\delta \left(q^2 - \hbar \frac{B \cdot q}{q \cdot n} \right) \left\{ \left[q \cdot \Delta + \hbar \frac{S_{(n)}^{\mu\nu} E_\mu}{(q \cdot n)} \Delta_\nu + \hbar S_{(n)}^{\mu\nu} (\partial_\mu F_{\rho\nu}) \partial_q^\rho \right] f_q^{(n)} - \tilde{\mathcal{C}} \right\} = 0,$$

magnetic-moment
coupling

spin tensor : $S_{(n)}^{\mu\nu} = \frac{\epsilon^{\mu\nu\alpha\beta}}{2(q \cdot n)} q_\alpha n_\beta$

($F^{\mu\nu} = 0$: the quantum corrections only appear in collisions)

- Quantum corrections on the collision term :

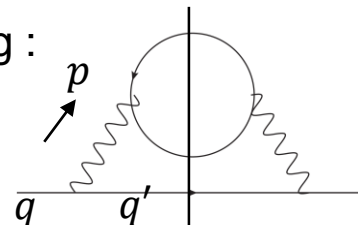
$$\tilde{\mathcal{C}} = q \cdot \mathcal{C} + \hbar \frac{S_{(n)}^{\mu\nu} E_\mu}{(q \cdot n)} \mathcal{C}_\nu + \hbar S_{(n)}^{\alpha\beta} \left((1 - f_q^{(n)}) \Delta_\alpha \Sigma_\beta^< - f_q^{(n)} \Delta_\alpha \Sigma_\beta^> \right),$$

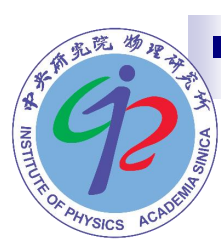
induced by inhomogeneity of the medium

$$\mathcal{C}_\beta = \Sigma_\beta^< (1 - f_q^{(n)}) - \Sigma_\beta^> f_q^{(n)}.$$

also include \hbar corrections

2-2 scattering :





Axial kinetic theory

- QKT for massive fermions :

- Wigner functions : $S^<(p, X) = \int d^4Y e^{ip \cdot Y/\hbar} \langle \bar{\psi}(y) U(y, x) \psi(x) \rangle$

vector/axial-vector
components :

$$\mathcal{V}^\mu(p, X) = \frac{1}{4} \text{tr} (\gamma^\mu S^<(p, X)), \quad \mathcal{A}^\mu(p, X) = \frac{1}{4} \text{tr} (\gamma^\mu \gamma^5 S^<(p, X))$$

- Dynamical variables in $\mathcal{V}^\mu / \mathcal{A}^\mu$: $f_{V/A}(q, X)$ & $a^\mu(q, X)$ **spin 4-vector**
($\tilde{a}^\mu = a^\mu f_A$)

K. Hattori, Y. Hidaka, D.-L. Y, PRD100,
096011 (2019)

$$\xrightarrow{m=0} a^\mu = q^\mu, \quad f_V = (f_R + f_L)/2, \quad f_A = f_R - f_L.$$

- Axial kinetic theory : scalar/axial-vector kinetic eqs. (SKE/AKE)

with quantum corrections

- Collisions for AKE : $\square^{(n)} \mathcal{A}^\mu = \hat{\mathcal{C}}_{\text{cl}}^\mu + \hbar \hat{\mathcal{C}}_{\text{Q}}^{(n)\mu}$ D.-L. Y, K. Hattori, Y. Hidaka,
JHEP 20, 070 (2020)

$\propto L^{\mu\nu} \tilde{a}_\nu$
spin diffusion
 $\propto H^{\mu\nu} \partial_\nu f_V$
spin polarization coupled to vector charge

- ❖ E.g. diffusion for massive quarks in weakly coupled QGP : S. Li, H.-U. Yee, PRD100, 056022 (2019)

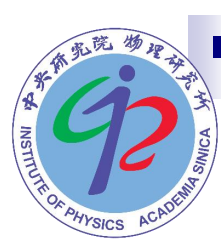
- ❖ The quantum correction is only studied for purely fermionic interactions.

Z. Wang, X. Guo, P. Zhuang, Eur. Phys. J. C 81, 799 (2021)
N. Weickgenannt, et al., PRL 127, 052301 (2021)



global-equilibrium sol. is reproduced from detailed
balance (local equilibrium?)

- ❖ The role of gluons and color dof. for spin polarization is unknown.



WFs and AKE with source terms

- Incorporation of background color fields into WFs and kinetic theory.

- Color decomposition : $O = \boxed{O^s} I + O^a t^a$ U. W. Heinz, Phys. Rev. Lett. 51, 351 (1983)
-> physical observable e.g. $J_5^\mu = 4 \int \frac{d^4 p}{(2\pi)^4} \text{Tr}_c \mathcal{A}^\mu(p, X)$
 $O^s : \mathcal{O}(g^0), O^a : \mathcal{O}(g).$

- SKE, AKE, WFs are decomposed into color-singlet & octet components.

- Perturbatively, we may rewrite $f_V^a, \tilde{a}^{a\mu}$ in terms of $f_V^s, \tilde{a}^{s\mu}$.

- Modified Cooper-Frye formula :

$$\mathcal{P}^\mu(\mathbf{p}) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^\mu(p, X)}{2m \int d\Sigma \cdot \mathcal{N}(p, X)} \quad \mathcal{N}^\mu(p, X) = 4N_c(p^\mu f_V^s),$$

$$\mathcal{J}_5^\mu(p, X) = 4N_c(\tilde{a}^{s\mu} + \hbar \bar{C}_2 \mathcal{A}_Q^\mu),$$

- **Source term in WFs :** $\mathcal{A}_Q^\mu = \frac{\partial_{p\kappa}}{2} \int_{k, X'}^p p^\beta \langle \tilde{F}^{a\mu\kappa}(X) F_{\alpha\beta}^a(X') \rangle \partial_p^\alpha f_V^s(p, X')$

B. Müller and D.-L. Y, (2021), 2110.15630

M. Asakawa, S. A. Bass, B. Muller, PRL. 96, 252301 (2006)

- SKE & AKE : $0 = p \cdot \partial f_V^s(p, X) - \boxed{\partial_p^\kappa \mathcal{D}_\kappa[f_V^s]} \sim g^2 > \text{collisions} \sim g^4 : \text{anomalous viscosity}$
 $0 = p \cdot \partial \tilde{a}^{s\mu}(p, X) - \boxed{\partial_p^\kappa \mathcal{D}_\kappa[\tilde{a}^{s\mu}]} + \boxed{\hbar \partial_p^\kappa (\mathcal{A}_\kappa^\mu[f_V^s])}$
diffusion source

Spin polarization from color fields

- The real color- field correlators can only be obtained from real-time simulations.

- Physical assumptions : $\langle F_{\kappa\lambda}^a(X) F_{\alpha\rho}^a(X') \rangle = \langle F_{\kappa\lambda}^a F_{\alpha\rho}^a \rangle e^{-(t-t')^2/\tau_c^2}$

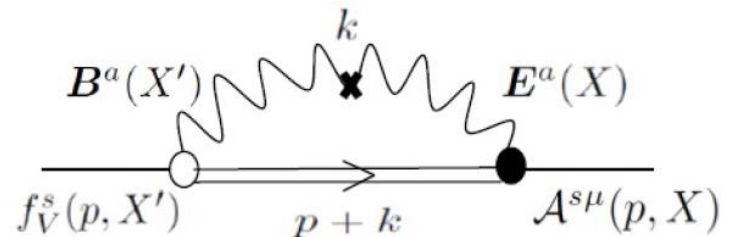
$$|\langle B_\mu^a B_\nu^a \rangle| \gg |\langle E_\mu^a B_\nu^a \rangle| \gg |\langle E_\mu^a E_\nu^a \rangle|$$

- f_V reaches thermal equilibrium : $\mathcal{J}_5^\mu(p, X) = 4N_c(\tilde{a}^{s\mu} + \hbar\bar{C}_2\mathcal{A}_Q^\mu)$,

$$\tilde{a}^\mu(t, p) = -\frac{\hbar\bar{C}_2(t-t_0)}{2p_0^2}(\partial_{p_0} f_{\text{eq}}(p_0))(\langle B^{a\mu} E^{a\nu} \rangle p_\nu - \langle B^a \cdot E^a \rangle p_\perp^\mu)$$

$$(\mathcal{A}_Q^\mu)_{\text{eq}} = \frac{\pi^{3/2}\tau_c}{2p_0}\delta(p^2 - m^2)(\langle E^a \cdot B^a \rangle u^\mu - \langle B^{a\mu} E^{a\beta} \rangle p_\beta \partial_{p_0})\partial_{p_0} f_{\text{eq}}(p_0)$$

- Origin of the source terms:
correlation of the Lorentz force &
anomalous force from quantum
corrections



The axial charge currents and Ward identity

- A constant axial charge current (finite τ_c):

$$J_5^\mu = 4N_c \int \frac{d^4p}{(2\pi)^4} \text{sign}(p_0) \mathcal{A}^{s\mu}(p, X) = -\frac{\hbar u^\mu}{8\pi^2} \sqrt{\pi} \tau_c \langle E^a \cdot B^a \rangle \mathcal{I} \quad \mathcal{I} = 1 \text{ for } m = 0$$

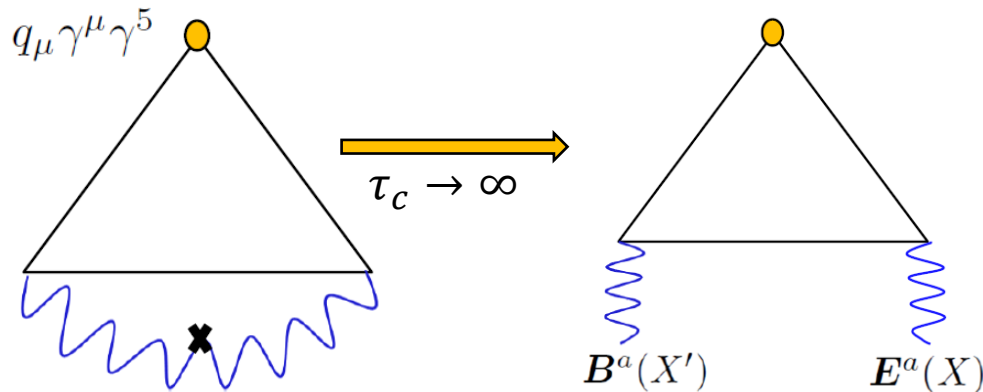
(similar to the steady state in Weyl semimetals $n_5 \sim \tau_R E \cdot B$)

- Vanishing axial-Ward identity : $\partial \cdot J_5 = 0$

- Constant-field limit ($\tau_c \rightarrow \infty$) : $\partial_\mu J_5^\mu(X) = -\hbar \frac{\langle B^a \cdot E^a \rangle}{4\pi^2} + 2m \langle \bar{\psi} i \gamma_5 \psi \rangle$,

D.-L. Y, in preparation

$$\langle \bar{\psi} i \gamma_5 \psi \rangle = -\frac{\hbar \langle B^a \cdot E^a \rangle}{8m\pi^2} \int_0^\infty d|\mathbf{p}| \left(1 - \frac{|\mathbf{p}|}{\epsilon_p}\right) \frac{d}{d|\mathbf{p}|} [f_{\text{eq}}(\epsilon_p) - f_{\text{eq}}(-\epsilon_p)]$$



Probing local parity violation in QCD matter

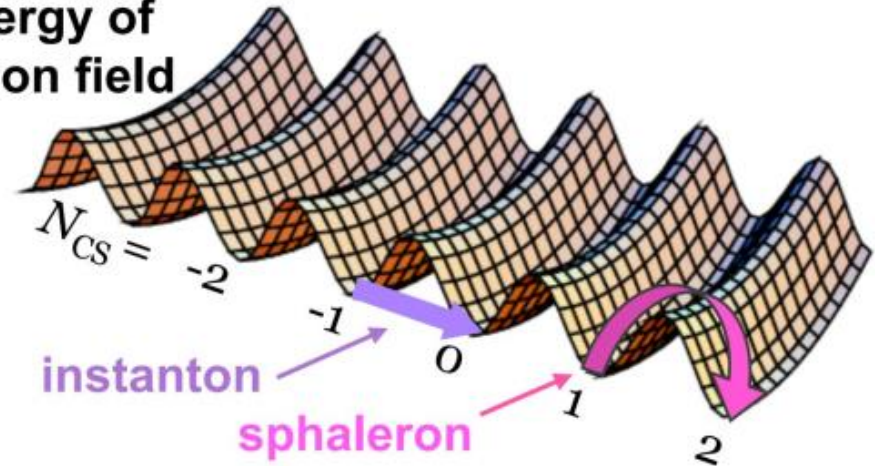
- Transition between topological sectors in QCD vacuum yields parity violation and chirality production.

$$\partial_t(N_L - N_R) = 2g^2 \partial_t N_{CS},$$

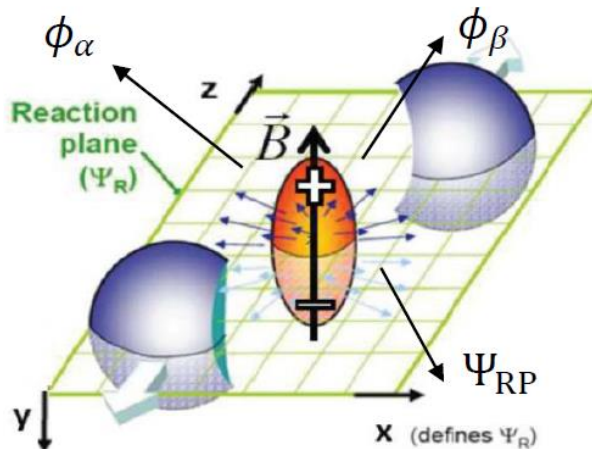
$$N_{CS} \equiv \int d^3x K_0,$$

$$K^\mu = \frac{\epsilon^{\mu\nu\rho\sigma}}{16\pi^2} \left(A_\nu^a \partial_\rho A_\sigma^a + \frac{1}{3} f^{abc} A_\nu^a A_\rho^b A_\sigma^c \right).$$

Energy of gluon field



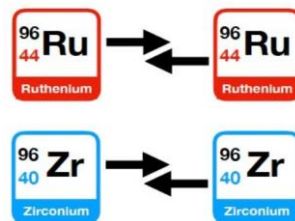
- Probing the local parity violation via the chiral magnetic effect (CME) :



$$\mathbf{J}_V = \frac{\mu_5}{2\pi^2} \mathbf{B}$$

D. E. Kharzeev et al.,
Prog. Part. Nucl. Phys. 88, 1 (2016)

Isobar collisions : so far, negative



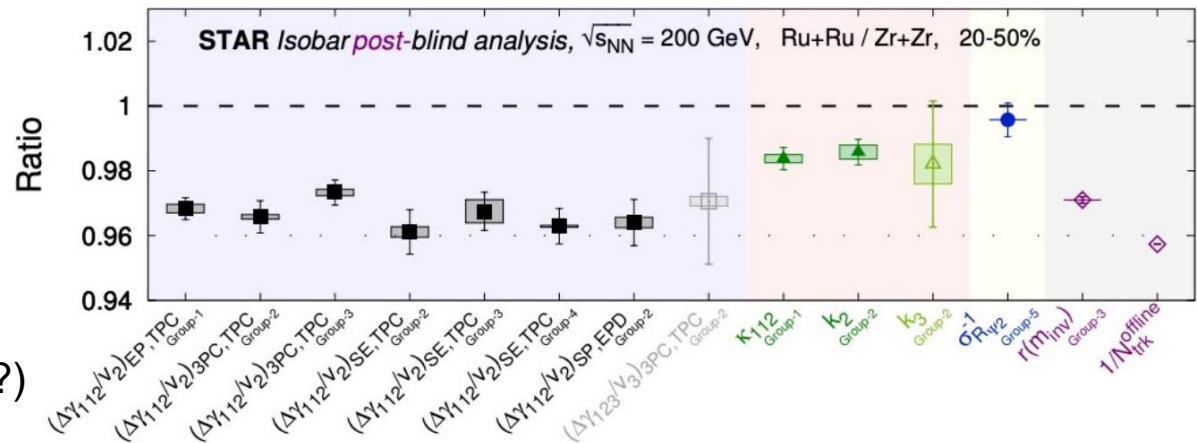
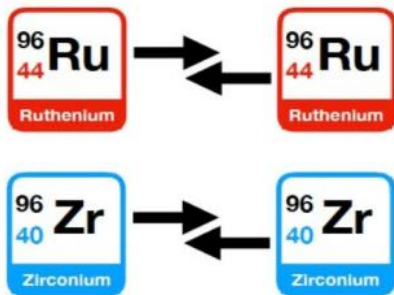
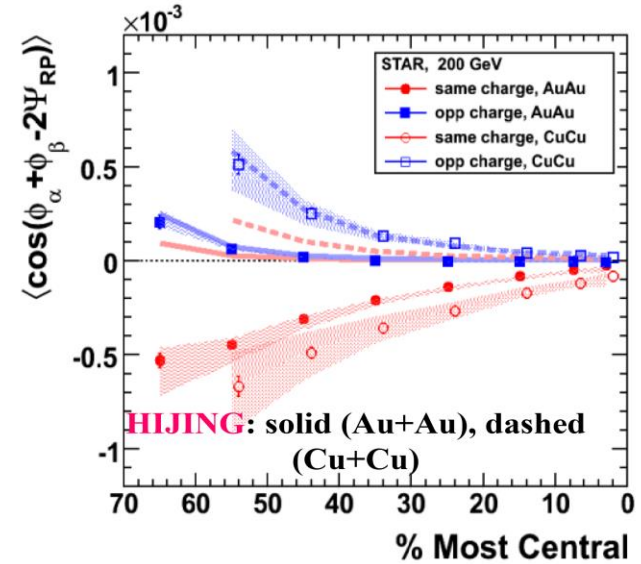
STAR, M. Abdallah et al., (2021), 2109.00131

(observed in Weyl semimetals)

Qiang Li, et.al., Nature Phys. 12 (2016) 550-554

Searching for CME in heavy ion collisions

- CME : correlations of same & opposite charge particles
- Background \gg signal
- Separating the signal from background : isobar collisions



<1
no
CME?

(same shape=background?)

(diff. B fields=signal)

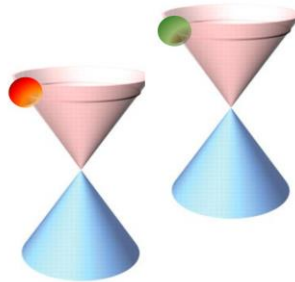
CME in Weyl Semimetals

- Chiral matter in the condensed matter system.

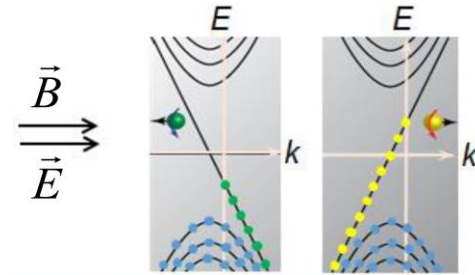
Weyl semimetals :

$$\partial_\mu J_5^\mu = \frac{\mathbf{E} \cdot \mathbf{B}}{2\pi^2} - \frac{n_5}{\tau_R}$$

relaxation time



TaAs
NbAs
NbP
TaP



charge pumping via parallel E & B : generate $\mu_5 \sim n_5 \sim E \cdot B$

steady-state approximation :

$$n_5 = \tau_R \frac{\mathbf{E} \cdot \mathbf{B}}{2\pi^2}$$

thermal equilibrium

$$= \frac{4\mu_5^3}{3\pi^2 v_f^3} + \frac{\mu_5}{3v_f^3} \left(T^2 + \frac{4\mu_V^2}{\pi^2} \right)$$

$$T \sim \mu_V \gg \mu_5$$

$$\mathbf{J}_V = \frac{\mu_5}{2\pi^2} \mathbf{B} = \frac{3v_f^3 \tau_R |\mathbf{B}|^2}{2(\pi^2 T^2 + 4\mu_V^2)} \mathbf{E}$$

$$\rho = \frac{1}{\sigma} \sim \frac{1}{B^2} \quad \text{"negative magnetoresistance" (the signal of CME)}$$

