High-precision calculation of the quark–gluon coupling from lattice QCD

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Based on:

- (ALPHA) M. Dalla Brida R. Höllwieser, F. Knechtli, T. Korzec A. Ramos, S. Sint, R. Sommer. *High-precision calculation of the quark–gluon coupling from lattice QCD*. Nature 652 (2026) 8109, 328-334. https://www.nature.com/articles/s41586-026-10339-4

CONSEJO SUPERIOR DE INVESTIGACIONES CIENTÍFICAS

VNIVERSITAT DE VALÈNCIA

INSTITUT DE FÍSICA CORPUSCULAR

EXCELENCIA SEVERO OCHOA

FUNDAMENTAL CONSTANTS IN NATURE

Electromagnetism

$$g_e - 2 : \alpha_{em} = 7.297\,352\,5698(24) \times 10^{-3}$$
$$\text{recoil} : \alpha_{em} = 7.297\,352\,585(48) \times 10^{-3}$$

Weak force

$$G_F = 1.166\,378\,7(6) \times 10^{5}$$

Gravity

$$G_N = 6.708\,61(31) \times 10^{-39}\,\text{GeV}^{-2}$$

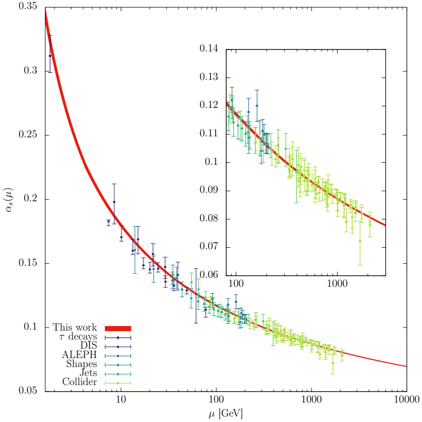
Strong force

$$\text{PDG Pheno average} : \alpha_s(M_Z) = 0.1175(10)$$
$$\text{This talk} : \alpha_s(M_Z) = 0.11876(58)$$

- This is an important parameter *in practice:*
  - Key input parameter to understand LHC physics!
  - Parametric uncertainty of α_s propagates to all new physics searches
  - Propagation of α_s uncertainty significant in Higgs theoretical cross sections
  - Vacuum estability, etc...
- Why is difficult?

The running coupling: $\alpha_s \equiv \bar{g}^2/(4\pi)$

$$S_{\rm QCD}[A_\mu,\psi,\overline{\psi}] = \int d^4x\, \frac{1}{4g^2} F^a_{\mu\nu}F^a_{\mu\nu} + \sum_f \overline{\psi_f}\gamma_\mu(\partial_\mu + m_q + \imath A_\mu)\psi_f$$



- Theory of quarks that interact by interchanging gluons
- Scale invariance broken at the quantum level
- Strong coupling depends on energy scale interactions

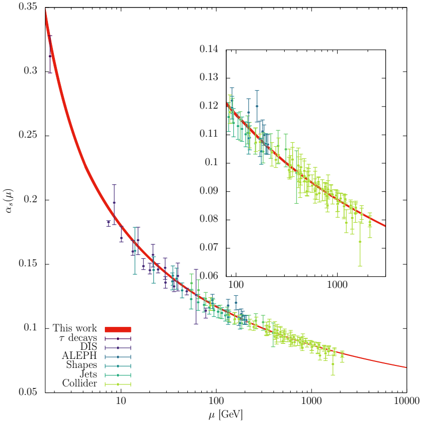
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu}\bar{g}(\mu) = \beta(\bar{g}) < 0$$

- Asymptotic freedom: $\bar{g}(\mu) \to 0$ as $\mu \to \infty$
- Perturbative behavior known (5-loops in $\overline{\rm MS}$)

$$\beta(\bar{g}) \overset{g\to 0}{\sim} -g^3\left(b_0 + b_1 g^2 + \ldots\right)$$

with first two perturbative coefficients (b_0, b_1) universal

Computing the strong coupling



Computing the strength of fundamental interactions

- Take some experimental observable $O(Q)$.
- Work hard to get ($\alpha_s \equiv \bar{g}_s^2/(4\pi)$)

$$O(Q) \overset{Q\to\infty}{\sim} \alpha_s(\mu) + \sum_{n=2}^{N} c_n(Q/\mu)\alpha_s^n(\mu) + \ldots$$

- Determine $\alpha_{\overline{\rm MS}}(\mu)$ by comparing experiment and theory computation

$$\begin{aligned} \tau : \alpha_s(M_Z) &= 0.1173(17) \\ e^+e^- : \alpha_s(M_Z) &= 0.1189(37) \end{aligned}$$

- Any observable works?
- Is the challenge really pushing computations to larger N?

Extracting α_s from experimental data is hard

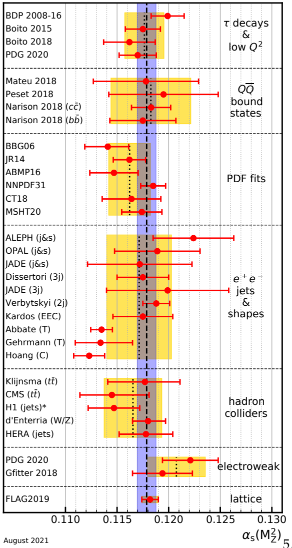
$$O(Q) \overset{Q\to\infty}{\sim} \alpha_s(Q) + \sum_{n=2}^{N} c_n \alpha_s^n(Q) + \mathcal{O}(\alpha_s^{N+1}(Q)) + \mathcal{O}\left(\frac{\Lambda^p}{Q^p}\right) + \dots$$

Difficulties in extracting α_s

- Difficult to compute (NP physics is difficult!)
- Difficult to estimate ($\alpha(Q)$ runs logarithmically)
- Asymptotic states are not quarks/gluons (hadronization, modeling, etc…)
- But most precise pheno extraction: τ decays ($m_\tau \approx 1.8$ GeV)
- Use large $Q \implies$ large uncertainties!

Real challenge: Determine α_s with precision and accuracy

- Clear meaning of error bars!



The scale of QCD: Λ-parameter

$$\mu \frac{d}{d\mu} g_s^2(\mu) = \beta_s(g_s) \Longrightarrow \Lambda_s = \mu \left[b_0 \bar{g}_s^2(\mu)\right]^{\frac{-b_1}{2b_0^2}} e^{-\frac{1}{2b_0 \bar{g}_s^2(\mu)}} \underbrace{\exp\left\{-\int_0^{\bar{g}_s(\mu)} dx \left[\frac{1}{\beta_s(x)} + \frac{1}{b_0 x^3} - \frac{b_1}{b_0 x}\right]\right\}}_{\mathcal{O}(\bar{g}_s{}^2(\mu))}$$

The intrinsic scale of QCD

- Λ_s same units as μ
- Λ_s is RGI: $d\Lambda_s/d\mu = 0$
- $\bar{g}_s^2(\mu)$ is a function of Λ_s/μ: Λ_s dictates what are "low" and "high" energies.
- (i.e. $\alpha_{\overline{\mathrm{MS}}}(M_Z)$ is trivial to compute if one knows $\Lambda_{\overline{\mathrm{MS}}}^{(5)}$)
- Scheme dependent, but

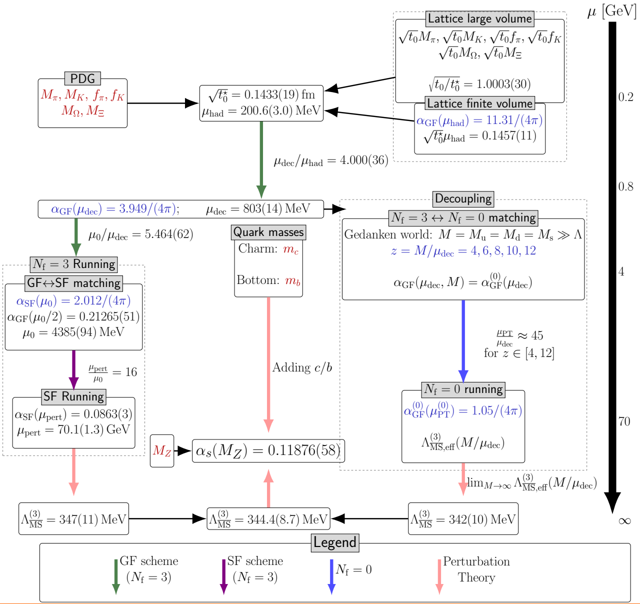
$$\bar{g}_{s'}^2(\mu) \overset{\bar{g}_s \to 0}{\sim} \bar{g}_s^2(\mu) + c_{ss'} \bar{g}_s^4(\mu) + \cdots \Longrightarrow \frac{\Lambda_{s'}}{\Lambda_s} = \exp\left(\frac{-c_{ss'}}{2b_0}\right).$$

- Defined non-perturbatively (even $\Lambda_{\overline{\mathrm{MS}}}$)
- Computing Λ_s requires PT (i.e. $\int_0 \ldots$). Take limit $\bar{g}_s^2(\mu) \to 0$ (i.e. extrapolation!)

The strategy

- Input: Hadron spectrum: $M_\pi, M_K, M_\Xi, f_k, f_\pi, M_\Omega, \ldots$
- Technical intermediate scale $\sqrt{t_0^\star}$
- "Solve" $N_f = 3$ QCD non perturbatively
  - From 200 MeV to EW scale
  - Match QCD with YM
- Use PT from EW scale on: $\Lambda^{(3)} = 344.4(8.7)$ MeV
- Use PT to cross c/b thresholds: $\alpha_s(M_Z) = 0.11876(58)$. [0.47%] precision

- Conservative error
- Statistical errors dominate



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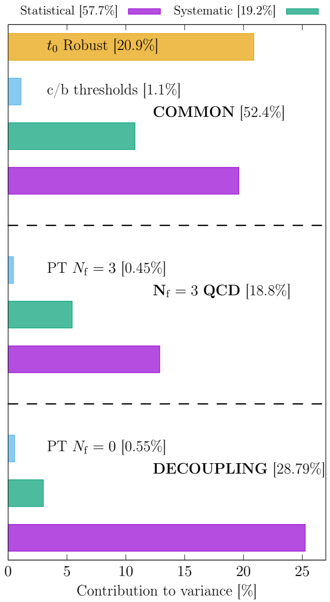
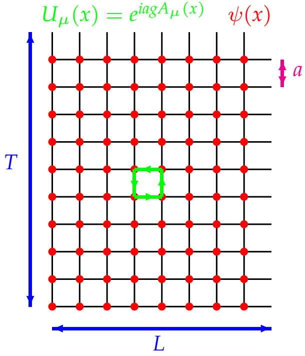


Computing path integrals: Lattice field theory

Lattice field theory $\longrightarrow$ Non Perturbative definition of QFT.



- Discretize space-time in an hyper-cubic lattice (spacing a)
- Path integral $\longrightarrow$ multiple integral (one variable for each field at each point)
- Compute the integral numerically $\rightarrow$ Monte Carlo sampling.

$$\langle O \rangle = \frac{1}{N_{\rm conf}} \sum_{i=1}^{N_{\rm conf}} O(U_i) + \mathcal{O}(1/\sqrt{N_{\rm conf}})$$

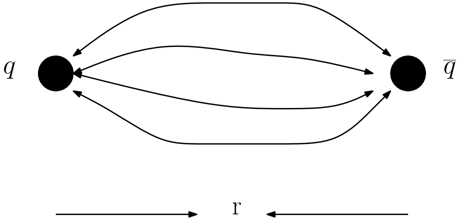
Observable computed averaging over samples

- This works both in the perturbative and non-perturbative regimes!

$$S_G[U] = \frac{\beta}{6} \sum_{p \in \text{Plaquettes}} Tr(1 - U_p - U_p^+) \xrightarrow[a\to 0]{} -\frac{1}{2}\int d^4x\, \mathrm{Tr}(F_{\mu\nu}F_{\mu\nu})$$

Our particle accelerators: HLRN CRAY supercomputer

The strength of YM: "physical" coupling definitions



- Take $O(Q) = \frac{3r^2}{4} F(r)\Big|_{Q=1/r}$
- This defines the "potential scheme". Non-perturbative coupling definition.

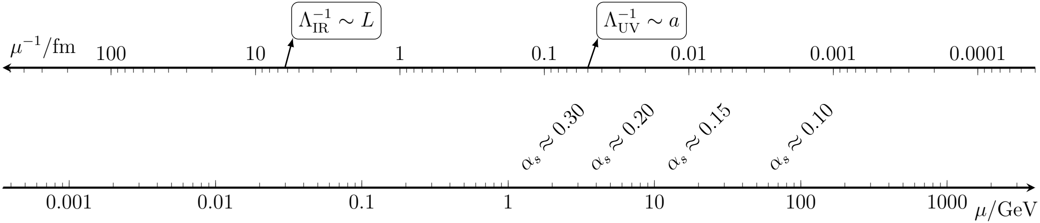
$$\alpha_{qq}(Q) = \frac{3r^2}{4} F(r)\Big|_{Q=1/r}$$

- ****If**** $\alpha(Q)$ is small (small r), perturbation theory tells

$$\alpha_{qq}(Q) \overset{Q\to\infty}{\sim} \alpha_{\overline{\mathrm{MS}}}(Q) + c_1 \alpha^2_{\overline{\mathrm{MS}}}(Q) + \ldots$$

- "Any" observable can be used for a non perturbative definition of the strong coupling, but...
- We need to evaluate $O(Q)$ non-peturbatively $\Longrightarrow$ Lattice

Lattice QCD typical scales

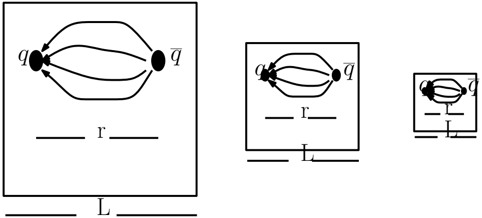


CLS ensembles ($N_f = 3$ QCD) **[Bruno et al. '15]**

Lattice sp. a	UV cutoff a^{-1}	L^{-1}
0.086 fm	2.3 GeV	35 – 70 MeV
0.064 fm	3.1 GeV	50 – 64 MeV
0.05 fm	3.9 GeV	60 MeV
0.04 fm	4.97 GeV	75 MeV

- Difficult to have **two relevant scales** (L, Q)
- Pushing $1/a \longrightarrow 100\,\mathrm{GeV} \Longrightarrow \times 6400000$ in CPU cost (scaling $\propto (L/a)^6$).
- Reducing α **exponentially** difficult problem!
- No way to perform a solid extrapolation!

The solution: Finite volume renormalization schemes [Lüscher, Weisz, Wolff '91]



Fix QL = constant

- Coupling $\alpha(Q)$ depends on no other scale but L
- Small $L \Longrightarrow$ small $\alpha(L)$
- $aQ \ll 1$ easy: $L/a \sim 10 - 40$
- $1/L$ is a IR cutoff $\Rightarrow$ simulate directly $m_q = 0$

Step scaling function

- How much changes $\alpha(Q)$ if $Q \to Q/2$?

$$\sigma(u) = \alpha(Q/2)\Big|_{\alpha(Q)=u}$$

- Simply change $L/a \to 2L/a$!

We need dedicated simulations of the **femto-universe**

Example: massless running in $N_f = 3$ QCD [ALPHA '17]

Gradient flow scheme [ALPHA; Phys.Rev.D 95 (2017)]

- Determine lattice version of SSF

$$\Sigma(u, L/a) = \alpha(Q/2)\Big|_{\alpha(Q)=u,\text{fixed}L/a}$$

Use $8 \to 16, 12 \to 24, 16 \to 32, (20 \to 40, 24 \to 48, 32 \to 64)$ at fixed (g_0, am_0)

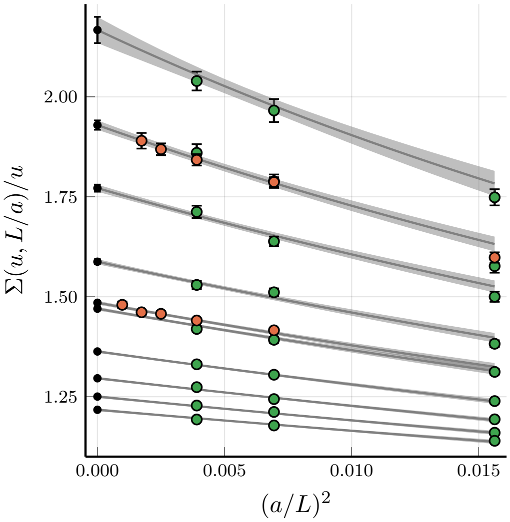
- Continuum limit

$$\sigma(u) = \lim_{a/L\to 0} \Sigma(u, L/a)\,.$$

- Use to determine β-function

$$\log(2) = \int_u^{\sigma(u)} \frac{\mathrm{d}x}{\beta(x)}\,.$$

- **Continuum limit under control**
- Cover energy range 200 MeV $\to$ 4 GeV.



Example: massless running in $N_f = 3$ QCD [ALPHA '17]

Gradient flow scheme [ALPHA; Phys.Rev.D 95 (2017)]

- Define

$$\alpha_{GF}(\mu_{had}) = 11.31/(4\pi)$$

with

$$\mu_{had} \times \sqrt{t_0^\star} = 0.1457(11)_{stat}(1)_{sys}(11)_{tot}$$

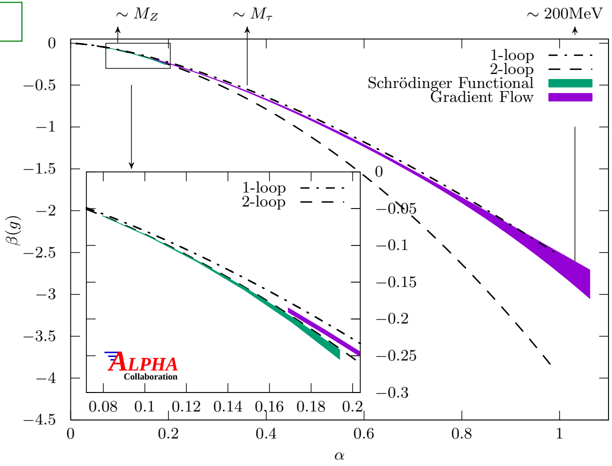
- Using $\alpha_{GF}(\mu_0/2) = 0.21265(51)$

$$\frac{\mu_0}{\mu_{had}} = 21.85(30)_{stat}(17)_{sys}(34)_{tot}$$

and

$$\mu_0 = 4385(71)_{stat}(36)_{sys}(51)_{robust}(94)_{tot}\ \mathrm{MeV}\,.$$

- Reached scale of cutoff of typical lattice simulations with full control over continuum extrapolation!

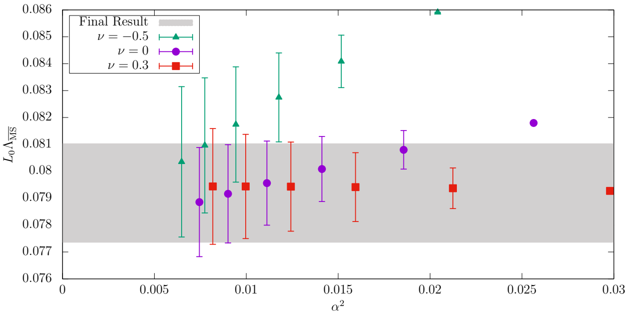


Determination of α_s

- One can use PT directly at $\mu_0 \approx 4$ GeV

$$\Lambda^{(3)}_{\overline{\rm MS}} = 347.6(6.3)\,\text{MeV}$$

- [0.34%] error in α_s...
- ...But what about PT
  - Missing orders
  - Power corrections
- Continue to high energies **[ALPHA' 2019]**
  - Improved $\mathcal{O}(a)$



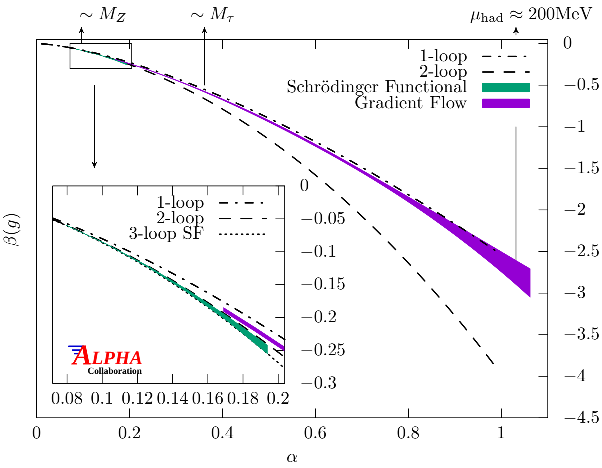
ALPHA approach

Use PT at genuinely high energy: 70 GeV

$$\Lambda^{(3)}_{\overline{\rm MS}} = 347(11)\,\text{MeV}$$

Check with a one-parameter family of observables! ν

Example: massless running in $N_f = 3$ QCD [ALPHA '17]



- NP determination of the β-function
- Large range of energy scales:

$$Q \approx 0.2\,\text{GeV to } 140\,\text{GeV}\,.$$

- Using PT for $Q \in [100\,\text{GeV}, \infty]$, and new data we get

$$\Lambda^{(3)}_{\overline{\text{MS}}} = 347(11), \text{MeV}\,.$$

- Most of the uncertainty:

NP running from 4 − 100 GeV

Checkpoint

- Input: Hadron spectrum: $M_\pi, M_K, M_\Xi, f_k, f_\pi, M_\Omega, \ldots$
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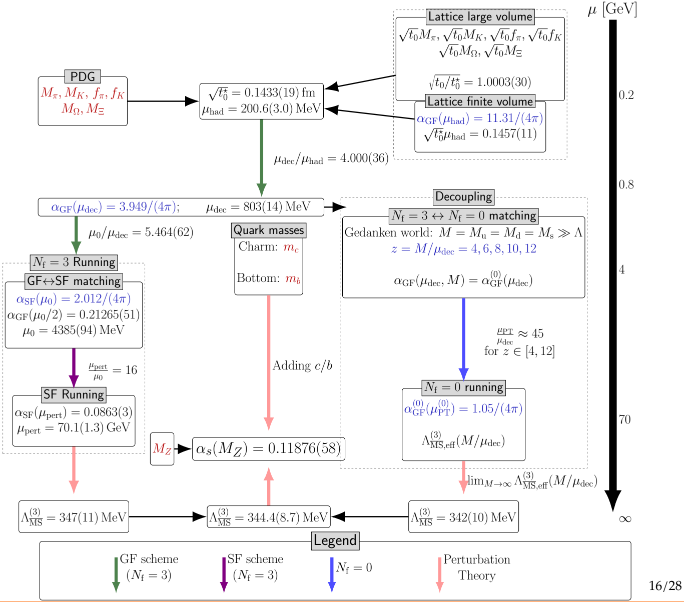


Massless renormalization schemes

Computation of observables

$$O(Q) \overset{\alpha \to 0}{\sim} \sum_n c_n(Q/\mu)\alpha^n_{\overline{\rm MS}}(\mu)$$

- Coupling $\alpha_{\overline{\rm MS}}(\mu)$ defined by: counterterms only include the divergences
- Divergences do not depend on values of quark masses
- $\overline{\rm MS}$ is an example of a massless scheme. High order computations.
- In LQCD we also prefer massless schemes: Conditions imposed at zero mass
  - Schrödinger Functional (SF)
  - (S)RI/MOM
  - …

Decoupling of heavy quarks in massless schemes

Matching between theories

- For $E < Q$ forget about all quarks with $m > Q$
- "Nice" perturtbative expressions if you only use **active** quarks
- Matching between effective theory (with **active quarks**) and fundamental theory (with **active** and heavy quarks)

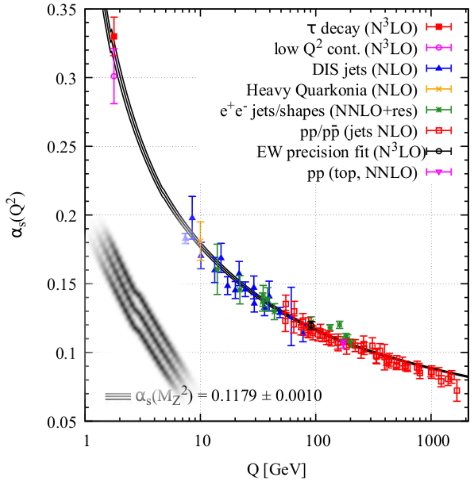
$$\alpha^{(N_f-1)}_{\overline{\rm MS}}(\mu) = \alpha^{(N_f)}_{\overline{\rm MS}}(\mu)\times\left\{1 + a_1(m_h/\mu)\alpha^{(N_f)}_{\overline{\rm MS}}(\mu) + \dots\right\}$$

- Also matching between Λ parameters

$$\frac{\Lambda^{(N_f)}}{\Lambda^{(N_f-1)}} = P_{N_f,N_f-1}(\Lambda/M)$$

Abuse of language: A single $\alpha_{\overline{\rm MS}}(\mu)$ that "jumps" at quark thresholds

- $\alpha_{\overline{\rm MS}}(4\,{\rm GeV})$: This is the four flavor coupling
- $\alpha_{\overline{\rm MS}}(M_Z)$: This is the five flavor coupling



3M: A universe with three heavy degenerate quarks $(M \gg \Lambda)$

Alice uses fundamental theory

$$S_{\text{fund}}[A_\mu, \psi, \bar{\psi}] = \int \mathrm{d}^4x \left\{ -\frac{1}{2g^2} \mathrm{Tr}\,(F_{\mu\nu}F_{\mu\nu}) + \sum_{i=1}^{3} \bar{\psi}_i(\gamma_\mu D_\mu + M)\psi_i \right\}$$

Bob uses effective theory

$$S_{\text{eff}}[A_\mu] = -\frac{1}{2g_{\text{eff}}^2} \int \mathrm{d}^4x \,\{\mathrm{Tr}\,(F_{\mu\nu}F_{\mu\nu})\} + \frac{1}{M^2} \sum_k \omega_k \int \mathrm{d}^4x \,\mathcal{L}_k^{(6)} + \dots$$

3M: A universe with three heavy degenerate quarks $(M \gg \Lambda)$

Alice uses fundamental theory

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Decoupling

- Dimensionless "low energy quantities" $\sqrt{t_0}/r_0, w_0/\sqrt{8t_0}, r_0/w_0, \ldots$ from effective theory

$$\frac{\mu_1^{\text{fund}}(M)}{\mu_2^{\text{fund}}(M)} = \frac{\mu_1^{\text{eff}}}{\mu_2^{\text{eff}}} + \mathcal{O}\left(\frac{\mu^2}{M^2}\right)$$

Bob and Alice determination of the strong coupling

- Decoupling also allows to compute the strong coupling (Λ). An implicit equation for $\Lambda^{(3)}_{\overline{\rm MS}}/\mu_{\rm dec}$ (**note:** $\Lambda^{(3)}_{\overline{\rm MS}}/M = \Lambda^{(3)}_{\overline{\rm MS}}/\mu_{\rm dec} \times \mu_{\rm dec}/M$)

$$\frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\mu_{\rm dec}} = \frac{\Lambda^{(0)}_{\overline{\rm MS}}}{\mu_{\rm dec}(M)} \times \frac{1}{P\left(\Lambda^{(3)}_{\overline{\rm MS}}/M\right)} + \mathcal{O}(\alpha^4(m^\star)) + \mathcal{O}\left(\frac{\mu^2_{\rm dec}}{M^2}\right)$$

- Exact relation:

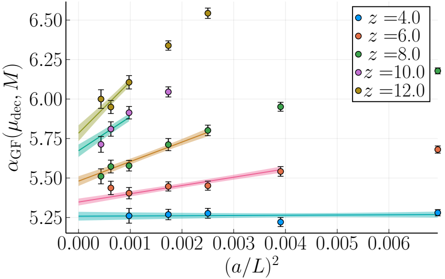
$$\frac{\Lambda^{(3)}_{\overline{\rm MS}}}{\mu_{\rm dec}} = \lim_{M\to\infty} \frac{\Lambda^{(0)}_{\overline{\rm MS}}}{\mu_{\rm dec}(M)} \times \frac{1}{P\left(\Lambda^{(3)}_{\overline{\rm MS}}/M\right)}$$

- We need:
  - Running in pure gauge: $\Lambda^{(0)}/\mu$
  - Matching between a coupling $\alpha(\mu_{\rm dec}, M)$ in a world with degenerate massive quarks: $\alpha(\mu_{\rm dec}(M), M) = \alpha^{(0)}(\mu_{\rm dec}) \Longrightarrow \mu_{\rm dec}(M) \approx \mu_{\rm dec}$

We do not live in $3M$, but we can simulate it!

$$\mu_{\rm dec}(M) = \mu^{\rm phys}_{\rm dec} \times \lim_{a\to 0} \frac{a\mu_{\rm dec}(M)}{a\mu^{\rm phys}_{\rm dec}}$$

The continuum extrapolation of massive couplings



Coupling changes with M [ALPHA '23]

- Precise extrapolations

Effective field theory description of cutoff effects+mass dependence

$$\bar{g}^2(z_i, a) = C_i + p_1[\alpha_(a^{-1})]^{\hat{\Gamma}}(a\mu_{\mathrm{dec}})^2 + p_2[\alpha_(a^{-1})]^{\hat{\Gamma}'}(aM_i)^2\,.$$

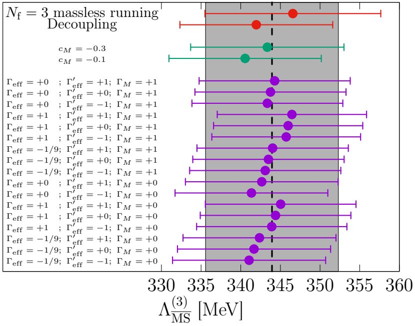
- Continuum values (our target quantity)
- Mass independent cutoff effects
- Mass dependent cutoff effects
- Loop corrections in effective theory: $-1 \leq \hat{\Gamma} \leq 1$ and $-1/9 \leq \Gamma' \leq 1$

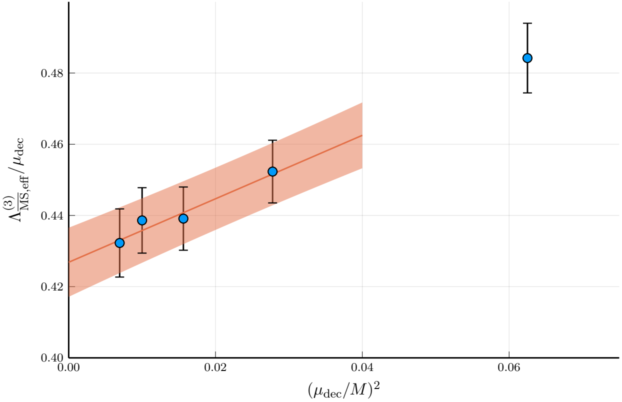
Strong coupling from decoupling

- Precise result for α_s

$$\Lambda_{\overline{\mathrm{MS}}}^{(3)} = 342(10)\ \mathrm{MeV}\,.$$

- Different models to extrapolate to continuum
- Different modesl to extrapolate $M \to \infty$
- Use of **already existing** pure gauge results





Final results

Final result for α_s

- Two strategies, consistent results

$$\text{Massless running:} \quad \Lambda^{(3)}_{\overline{\rm MS}} = 347(11)\,\text{MeV}\,,$$
$$\text{Decoupling:} \quad \Lambda^{(3)}_{\overline{\rm MS}} = 342(10)\,\text{MeV}\,.$$

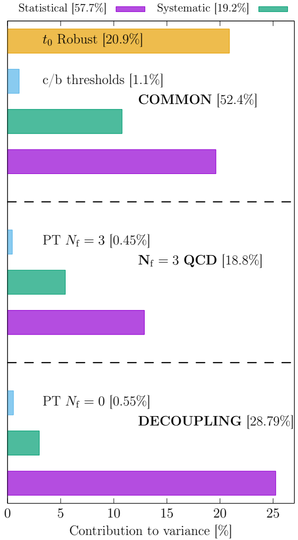
- Average

$$\Lambda^{(3)}_{\overline{\rm MS}} = 344.4(8.7)\,\text{MeV}\,.$$

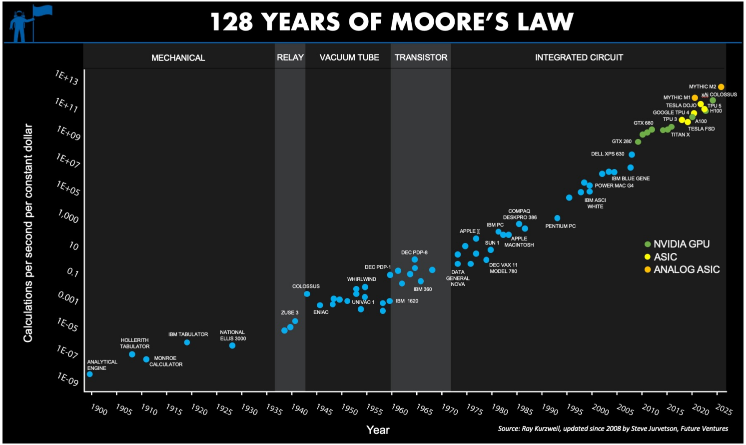
- "Crossing" c/b thresholds

$$\alpha_s(m_Z) = 0.11876(58) \quad [0.46\%]\,.$$

- Two times more precise than all pheno determinations combined!
- PT errors negligible
- Error dominated by statistics



Precision LQCD = New theoretical approaches + Efficient machines/algorithms



- Step Scaling
- Preconditioning
- Large Volume simulations
- SF in QCD
- High order PT computations
- Domain decomposition
- Mass preconditioning
- ...
- GF couplings
- Decoupling

Conclusions

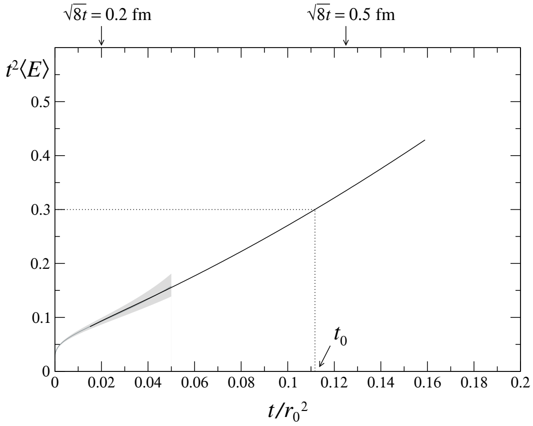
- Dedicated approaches allow to solve QCD non-perturbatively from hadronic scales to EW scale
- Finite volume renormalization schemes: key role!
- New decoupling idea: connect QCD with Yang-Mills
  - Running in $N_\mathrm{f} = 0$ numerically easier
- Precise result

$$\alpha_s(m_Z) = 0.11876(58) \quad [0.46\%]\,.$$

  - 2x more precise than combination of pheno determinations
  - Negligible PT uncertainties
  - Uncertainty limited by statistical precision of our simulations
  - Experimental input: Low energy QCD spectrum (no NP!)
  - No correlation with LHC data (use for PDFs)
- Number should be **used as input** in LHC analysis
- Combined with EIC: unprecedented check of QCD running coupling!

MANY THANKS!

$t_0^\star$ as an intermediate reference scale



Gradient Flow [M. Lüscher '10].

- Evolve gauge fields along the gradient flow

$$\frac{dB_\mu(x,t)}{dt} = D_\nu G_{\nu\mu}(x,t); \quad \left(\sim -g_0^2 \frac{\delta S_{\mathrm{YM}}[B]}{\delta B_\mu}\right)$$

flow time dimensions of $[\text{length}]^2$

- Finite (renormalized) observables(for $t > 0$)

$$E(x,t) = \frac{1}{4}\langle G_{\mu\nu}(x,t)G_{\mu\nu}(x,t)\rangle$$

- $t^2\langle E(t)\rangle$ is dimensionless and depends on scale t
- It is a renormalized coupling!!

$$g^2_{\mathrm{GF},\infty}(\mu) \propto t^2\langle E(t)\rangle\Big|_{\mu=1/\sqrt{8t}}$$

- Ideal candidate for scale setting: t_0
- $t_0^{\star 2}\langle E(t_0^\star)\rangle = 0.3$ at $m_\pi = m_K = 420\,\mathrm{MeV}$

Scale setting: The case with significant systematics

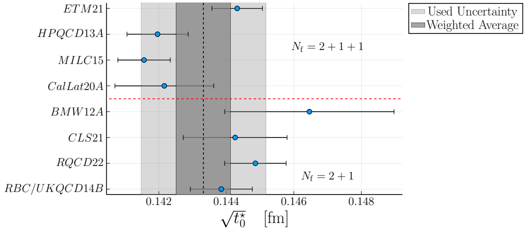
Technical scale $t_0^\star$

- Different collaborations determine dimensionless:

$$\sqrt{t_0^\star} \times (M_\pi, M_K, M_\Xi, M_\Omega, f_\pi, f_K, \dots)$$

- Use all results entering FLAG average
- $\chi^2/\mathrm{dof} = 2.8$
- **Robust** error: all "precise" results covered

$$\sqrt{t_0^\star} = 0.1433(7)_{\mathrm{stat}}(4)_{\mathrm{sys}}(17)_{\mathbf{robust}}(19)_{\mathrm{tot}}\ \mathrm{fm}\,.$$



- Only case of significant systematic in our work
- **Robust** error band covers all (precise) central values
- Effect propagated in all quantities.
- Our error 2.5× larger than "standard" (i.e. FLAG/PDG) error inflation
- Small effect in final error of α_s: $58 \times 10^{-4} \to 51 \times 10^{-4}$

From $\Lambda^{(3)}_{\overline{\text{MS}}}$ to $\Lambda^{(5)}_{\overline{\text{MS}}}$

$$\frac{\Lambda^{(5)}_{\overline{\text{MS}}}}{\Lambda^{(3)}_{\overline{\text{MS}}}} = P_{3,5}$$

Use RunDec to explore PT corrections in crossing c/b thresholds

loop-orders	$P^{\text{ref}}_{3,4}$	$P^{\text{ref}}_{4,5}$	$P^{\text{ref}}_{3,5}$	$\alpha_s^{\text{ref}}(m_Z)$
5/4	0.87548	0.72143	0.63160	0.11872
loop-orders	$100 \times \delta P_{3,4}$	$100 \times \delta P_{4,5}$	$100 \times \delta P_{3,5}$	$10^5 \times \Delta\alpha_s(m_Z)$
4/3	−0.2536	0.2056	−0.3313	−5.992
3/2	−0.8503	0.5758	−1.2237	−22.20
2/1	−3.8555	1.2228	−6.8235	−126.3
SI,$m^\star$	0.0	0.0		
SI,$2m^\star$	−0.4364	−0.0702		
SI,$m^\star/2$		−0.0117		
$\overline{\text{MS}}$,$\mu = \mu_h$	−0.0299	−0.0014		
$\overline{\text{MS}}$,$\mu = 2\mu_h$	−0.1036	−0.0105		
$\overline{\text{MS}}$,$\mu = \mu_h/2$	0.0016	0.0119		

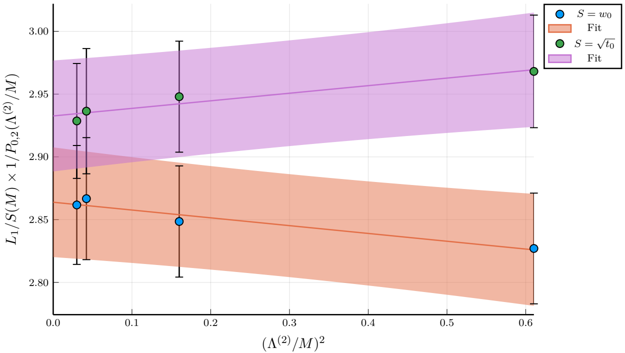
Small corrections

- PT corrections 0.3%
- NP corrections 0.1%

From $\Lambda^{(3)}_{\overline{\rm MS}}$ to $\Lambda^{(5)}_{\overline{\rm MS}}$

$$\frac{\Lambda^{(5)}_{\overline{\rm MS}}}{\Lambda^{(3)}_{\overline{\rm MS}}} = P_{3,5}$$

Use RunDec to explore PT corrections in crossing c/b thresholds



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