Hadron structure with lattice QCD for the EIC program

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AQTIVATE

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Extended Twisted Mass Collaboration

RESEARCH & INNOVATION FOUNDATION

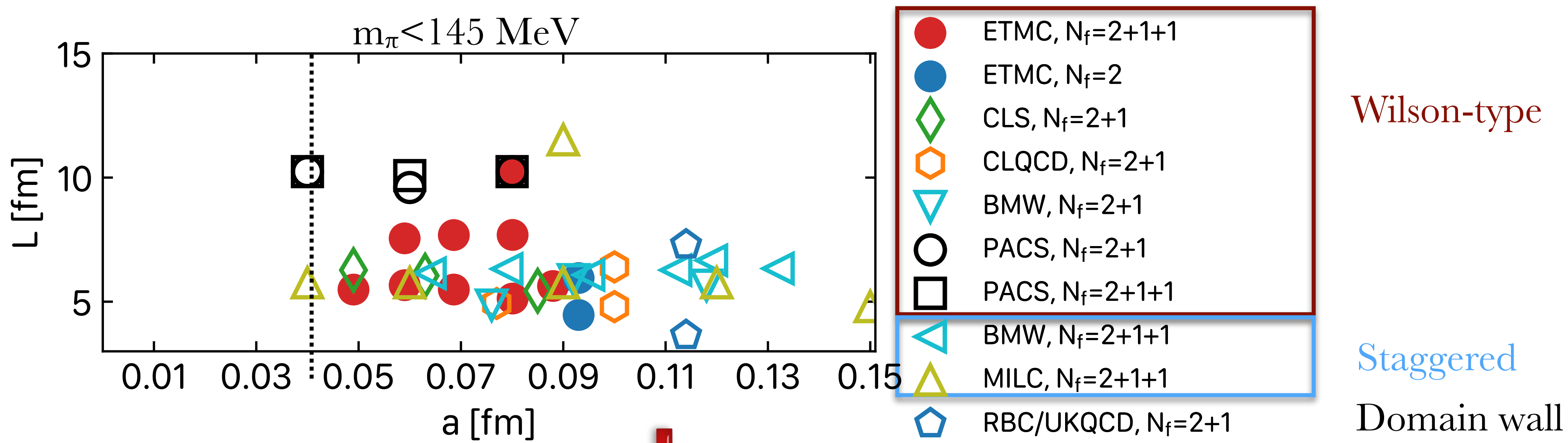
EXCELLENCE/0524/0459

Outline

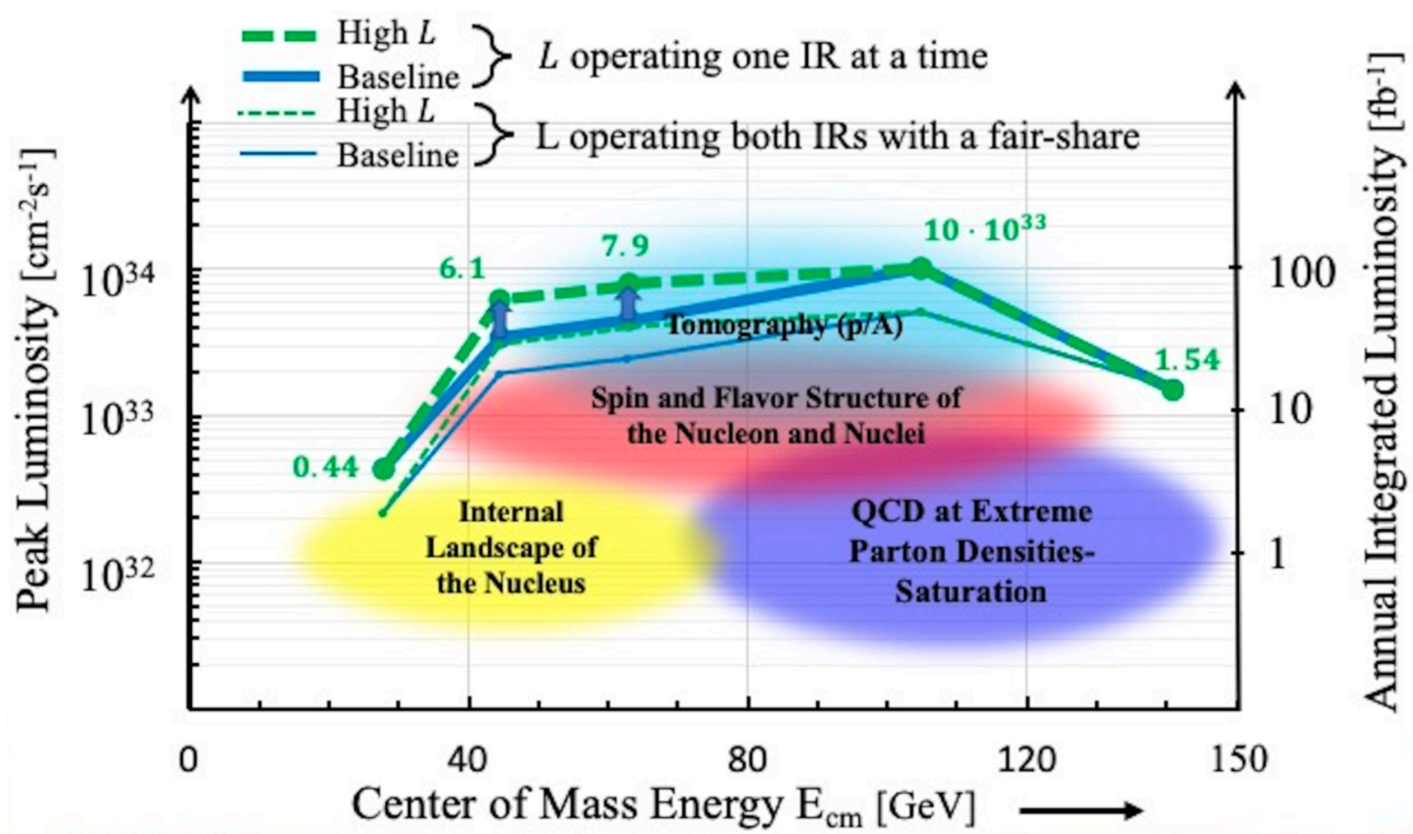
- Introduction
- 3D structure of the hadrons via Mellin moments (pion, kaon and nucleon)
  - first and second Mellin moments (charge, form factors, spin structure)
  - higher than second Mellin moments (reconstruction of PDFs and GPDs) - see also talks by A. Shindler and R. Perry
- Direct computation of hadron PDFs and GPDs (pion, kaon and nucleon)
- Conclusions

State-of-the-art gauge ensembles for hadron structure

$$U \in \frac{1}{Z}\int \mathcal{D}[U]\left(\prod_{f=u,d,s,c}\mathrm{Det}(D_f[U])\right)e^{-S_g[U]}$$



EIC physics



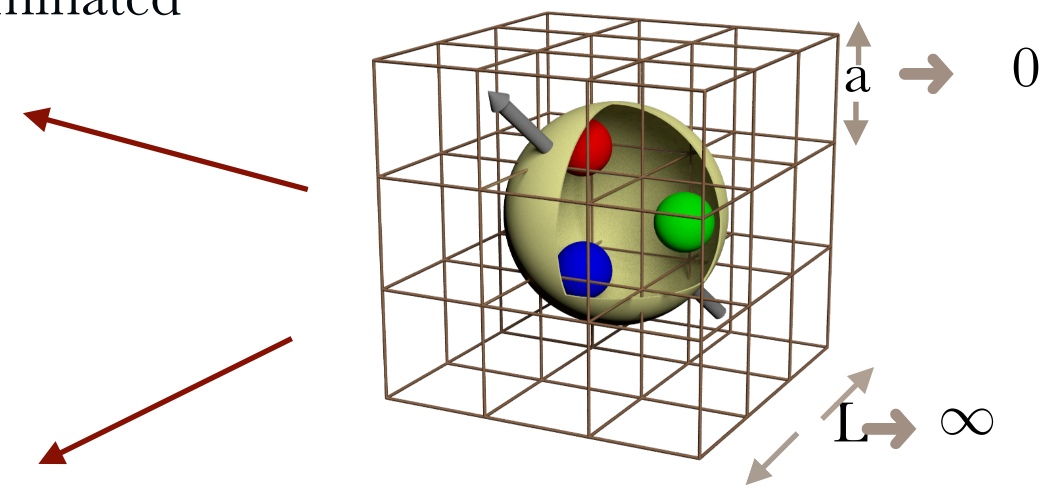
Lattice Systematics

- **Simulations directly at the physical point**

  Systematic effects from chiral extrapolation are eliminated

- **Discretisation effects:** Continuum limit

  —> need simulations for at least 3 lattice spacings



- **Finite volume effects:** Infinite volume limit

  —> need simulations for at least 3 volumes

- **Renormalisation**

  Non-perturbatively with improvements e.g using perturbative subtraction of lattice artefacts

- **Ground-state identification**

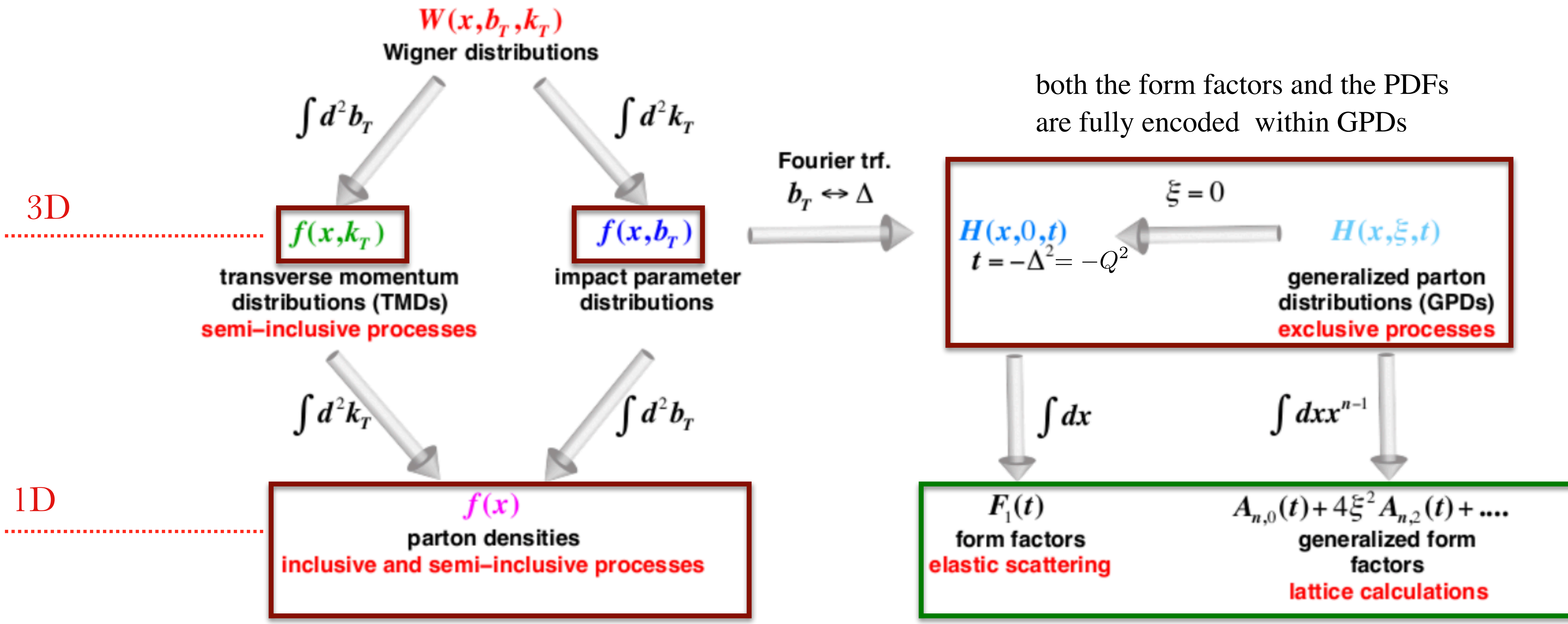
  Cross-check (one-, two- and three-state fits, summation)

  Two-particle state contributions complicate the identification of the ground state

- **Isospin corrections: QCD and QED**

  Very few lattice QCD results reach an accuracy to require isospin corrections but for precision physics they become relevant

Hadron tomography

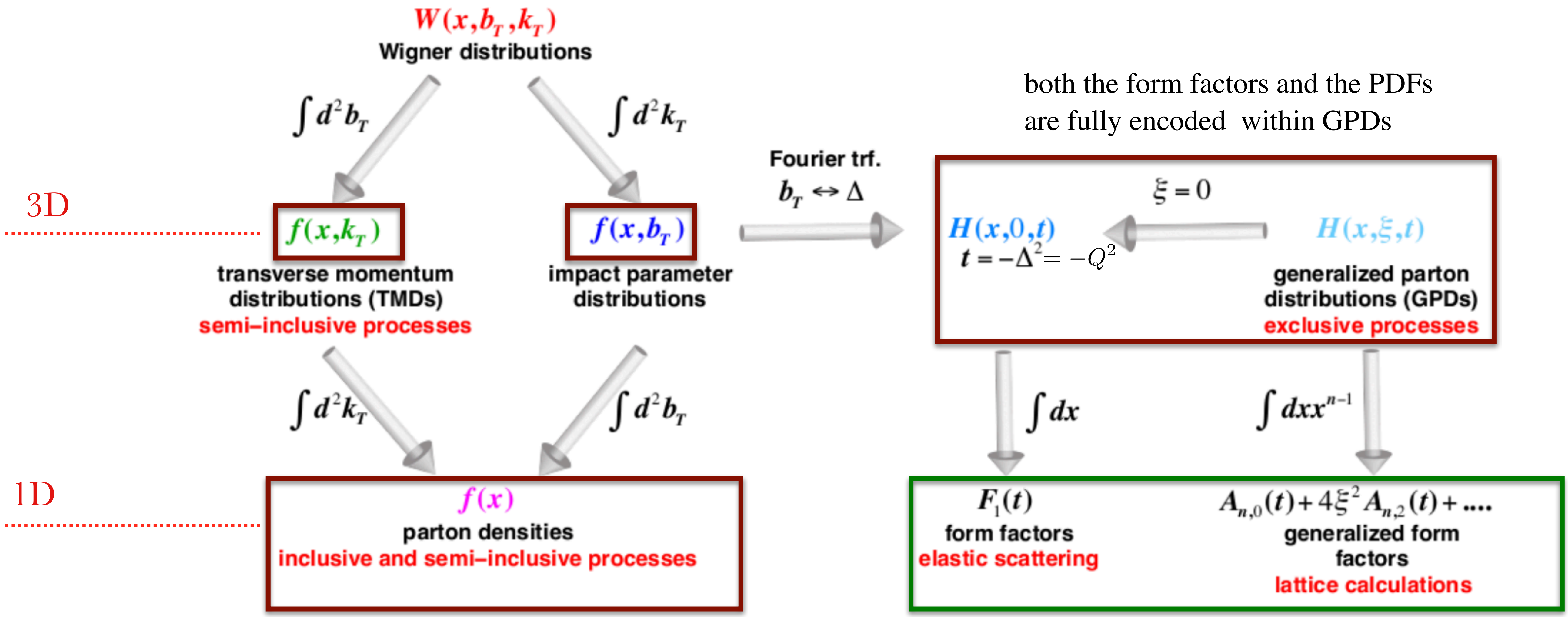


both the form factors and the PDFs are fully encoded within GPDs

Studies in lattice QCD since the 1990s

EIC report: A. Arcardi *et al.* arXiv:1212.1701

Hadron tomography



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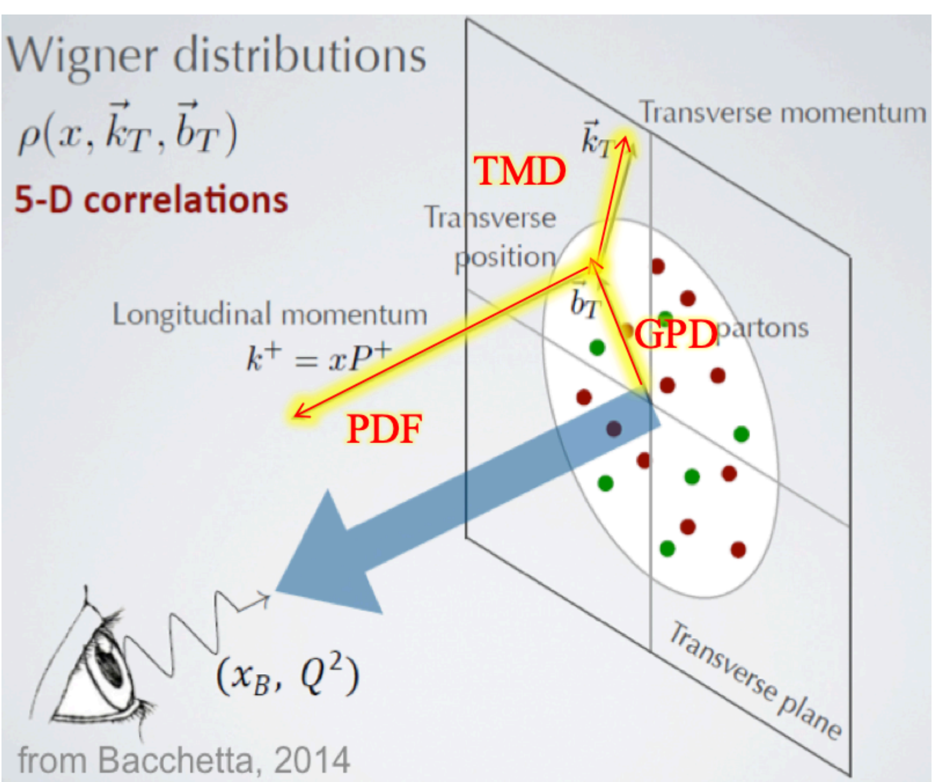
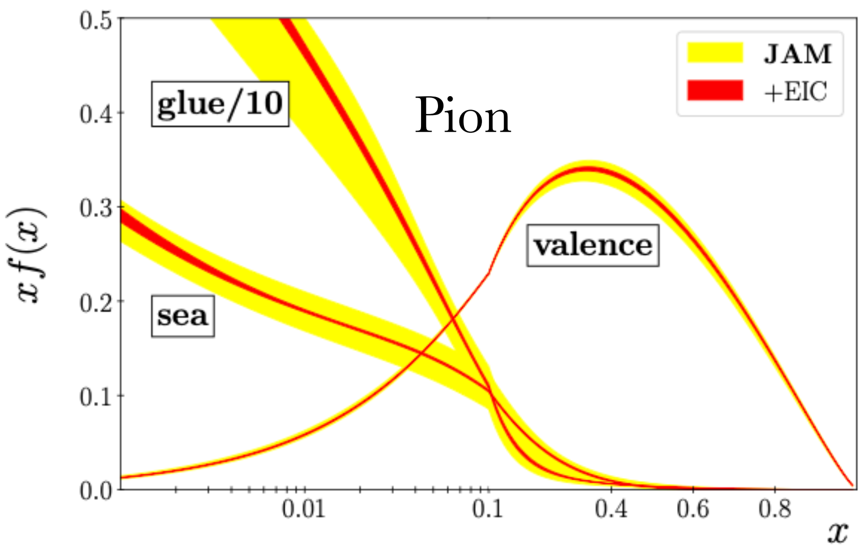


- PDFs —> momentum and spin distributions of quarks and gluons.
- GPDs —> correlation between the transverse position and longitudinal momentum of the partons.
- TMDs PDFs —> link the longitudinal and transverse momenta of the partons

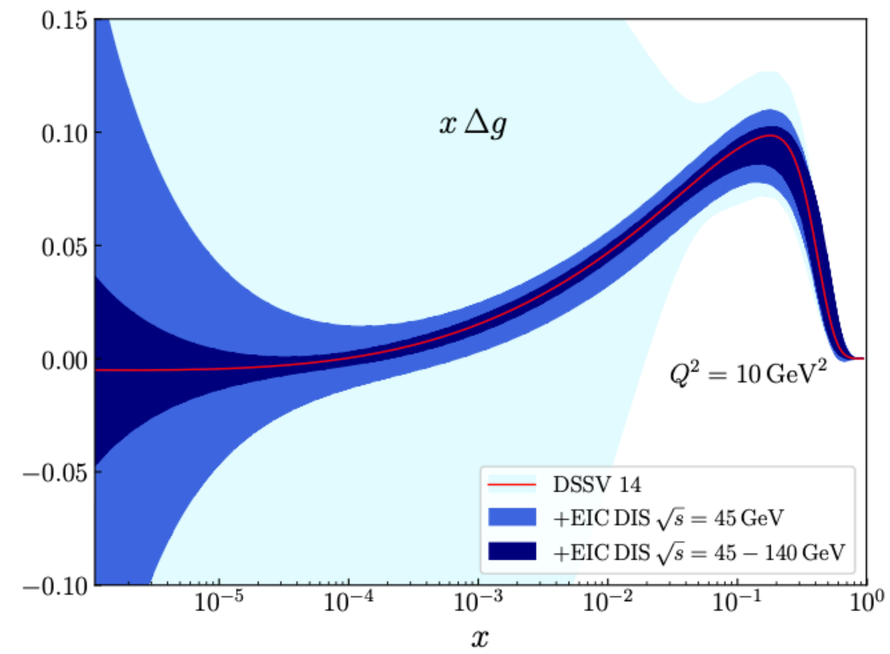
Talks by Yong Zhao and Wayne Morris

Examples of EIC hadron quantities (1D structure): PDFs

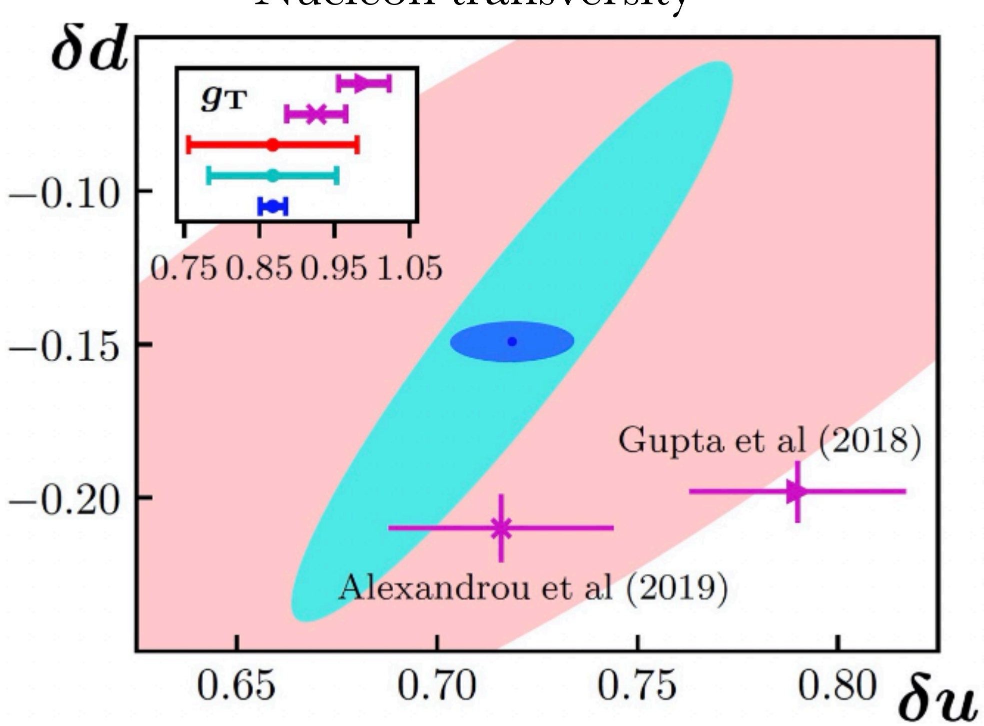
- Twist-2 PDFs with EIC accessing x<0.1

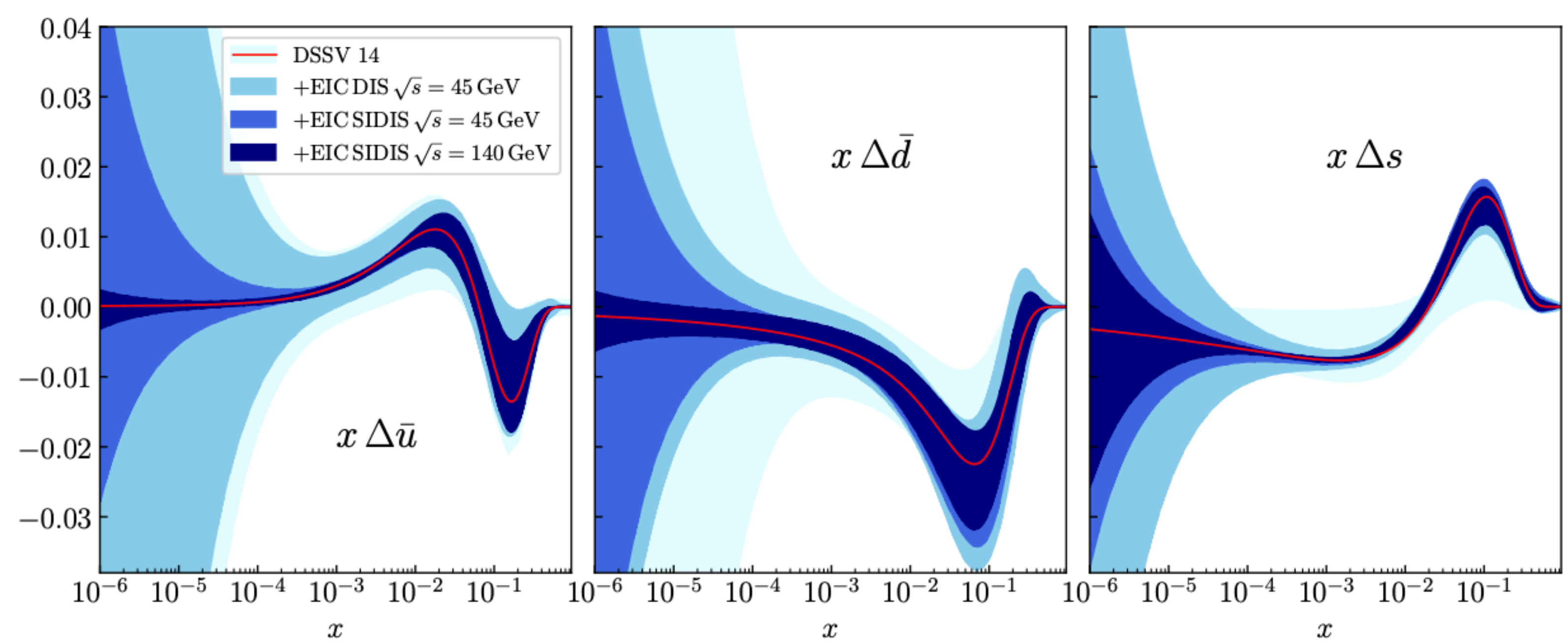


Nucleon helicity



Nucleon transversity





EIC Yellow report: arXiv:2103.05419
EIC White paper: arXiv:2211.15746

- Higher twist PDFs and moments probing multiparton correlations, e.g. twist-3 PDFs such as the scalar e(x), transversity $g_T(x)$ and the d_2 term

3D structure of hadrons via Mellin moments - precision era of lattice QCD

Computation of lower Mellin moments of GPDs

- Moments are readily accessible in lattice QCD from matrix elements of local operators with computations in early 90s
- To leading order we have the twist-2 operators

Spin-1/2

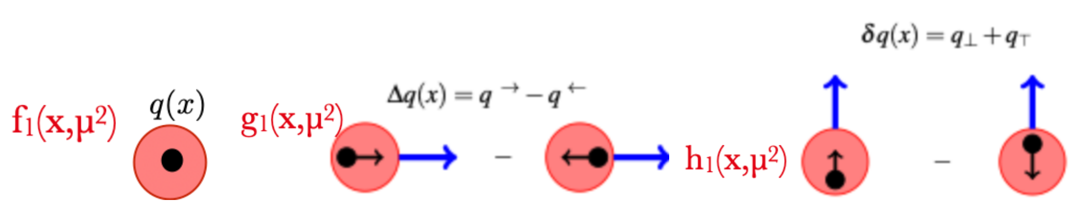
$$\mathcal{O}_V^{\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma^{\{\mu_1} i\overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi \quad \overset{unpolarized}{\rightarrow} \quad \langle x^{n-1}\rangle_q = \int_0^1 dx\, x^{n-1}\left[q(x) - (-1)^{n-1}\bar{q}(x)\right]$$

$$\mathcal{O}_A^{\mu_1\mu_2\ldots\mu_n} = \bar{\psi}\gamma_5\gamma^{\{\mu_1} i\overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi \quad \overset{helicity}{\rightarrow} \quad \langle x^{n-1}\rangle_{\Delta q} = \int_0^1 dx\, x^{n-1}\left[\Delta q(x) + (-1)^{n-1}\Delta\bar{q}(x)\right]$$

$$\mathcal{O}_T^{\rho\mu_1\mu_2\ldots\mu_n} = \bar{\psi}i\sigma^{\rho\{\mu_1} i\overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi \quad \overset{transversity}{\rightarrow} \quad \langle x^{n-1}\rangle_{\delta q} = \int_0^1 dx\, x^{n-1}\left[\delta q(x) - (-1)^{n-1}\delta\bar{q}(x)\right]$$

$$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow, \quad \delta q = q_\top + q_\perp$$

Made traceless and {} denotes symmetrization



Moments:

Pol. \ Γ	γ^+	$\gamma^+\gamma_5$	$i\sigma^{+j}\gamma_5$
U	H		E_T
L		$\tilde{H}$	$\tilde{E}_T$
T	E	$\tilde{E}$	H_T, $\tilde{H}_T$

- n=1 —> vector, axial and tensor charges, g_V, g_A g_T and form factors, intrinsic spin, etc
- n=2 —> unpolarized, helicity and tensor generalized form factors factors (GFFs), spin structure, transverse densities, etc

Computation of lower Mellin moments of GPDs

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$$\mathcal{O}_T^{\rho\mu_1\mu_2\ldots\mu_n} = \bar{\psi}i\sigma^{\rho\{\mu_1} i\overleftrightarrow{D}^{\mu_2}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi \quad \xrightarrow{transversity} \quad \langle x^{n-1}\rangle_{\delta q} = \int_0^1 dx\, x^{n-1}\left[\delta q(x) - (-1)^{n-1}\delta\bar{q}(x)\right]$$

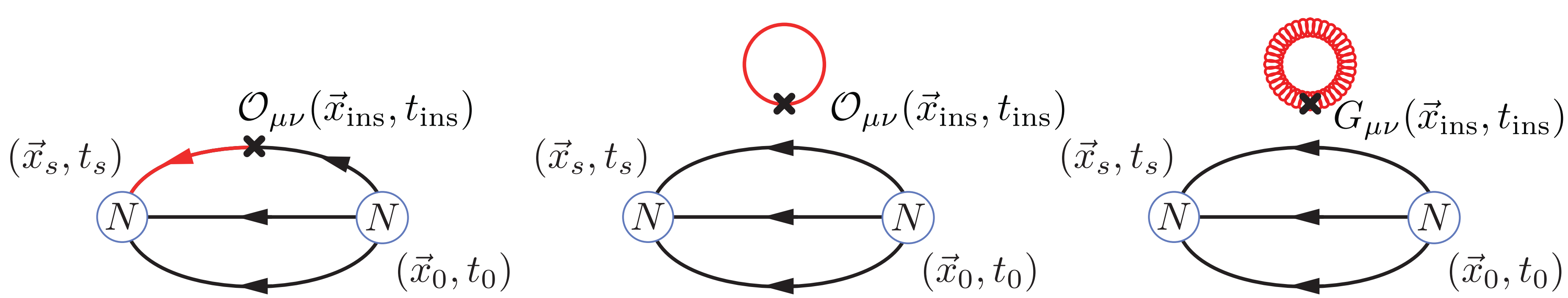
$$q = q_\downarrow + q_\uparrow, \quad \Delta q = q_\downarrow - q_\uparrow, \quad \delta q = q_\top + q_\perp$$

Moments: $\langle x\rangle_q = A_{20}(0),\ \langle x\rangle_{\Delta q} = \tilde{A}_{20}(0),\ \langle x\rangle_{\delta q} = A_{20}^T(0)$

Angular momentum: $J_{q,g} = \frac{1}{2}[A_{20}^{q,g}(0) + B_{20}^{q,g}(0)]$

Sum rules: $\sum_q\left[\frac{1}{2}\Delta\Sigma_q + L_q\right] + J_g = \frac{1}{2},\ \sum_q\langle x\rangle_q + \langle x\rangle_g = 1$

- n=1 —> vector, axial and tensor charges, g_V, g_A g_T and form factors, intrinsic spin, etc
- n=2 —> unpolarized, helicity and tensor generalized form factors factors (GFFs), spin structure, transverse densities, etc



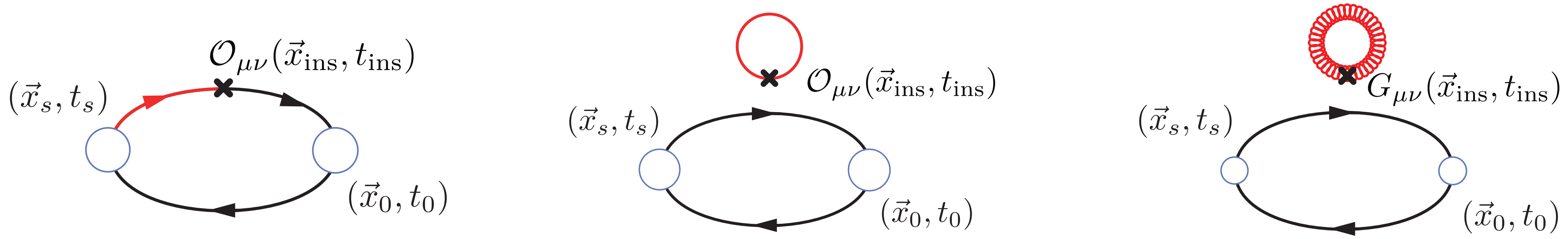
Spin-0 particles: pion, kaon

Spin-0

- n=1 —> vector and tensor charges, g_V, g_T and form factors: FF: $F_V(Q^2)$, $F_T(Q^2)$
- n=2 —> unpolarized and tensor generalized form factors factors (GFFs): $A_{20}(Q^2)$, $C_{20}(Q^2)$, $B_{T20}(Q^2)$

- Most studies are for the pion EM form factor

- Surprising few results for pion tensor and scalar and all kaon form factors, e.g. ETMC for one ensemble



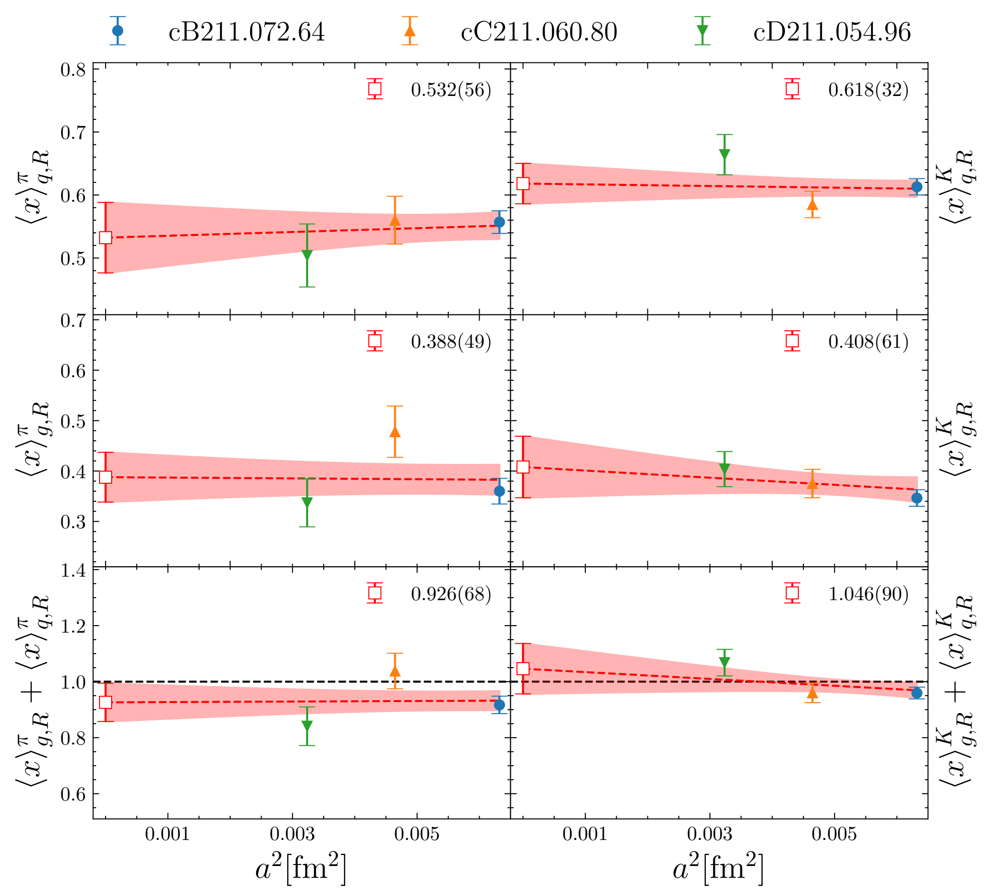
Pion and Kaon momentum fractions

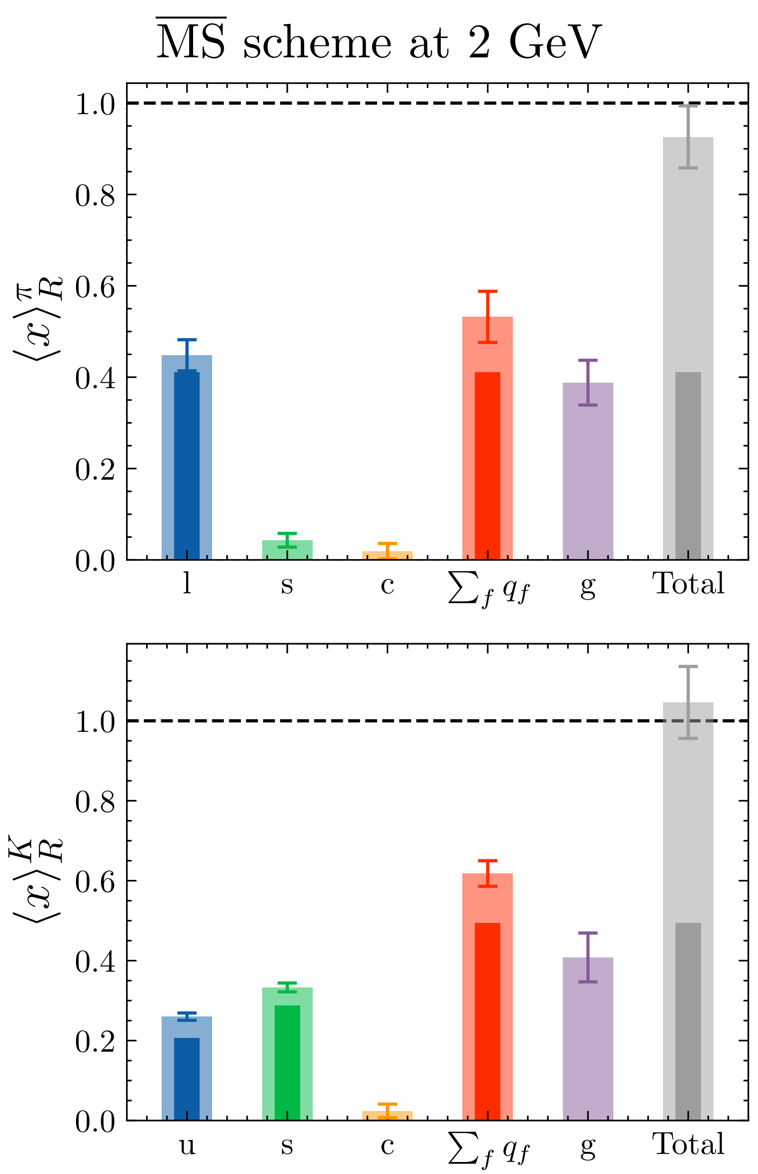
❋ ETMC: continuum limit taken directly at physical pion mass

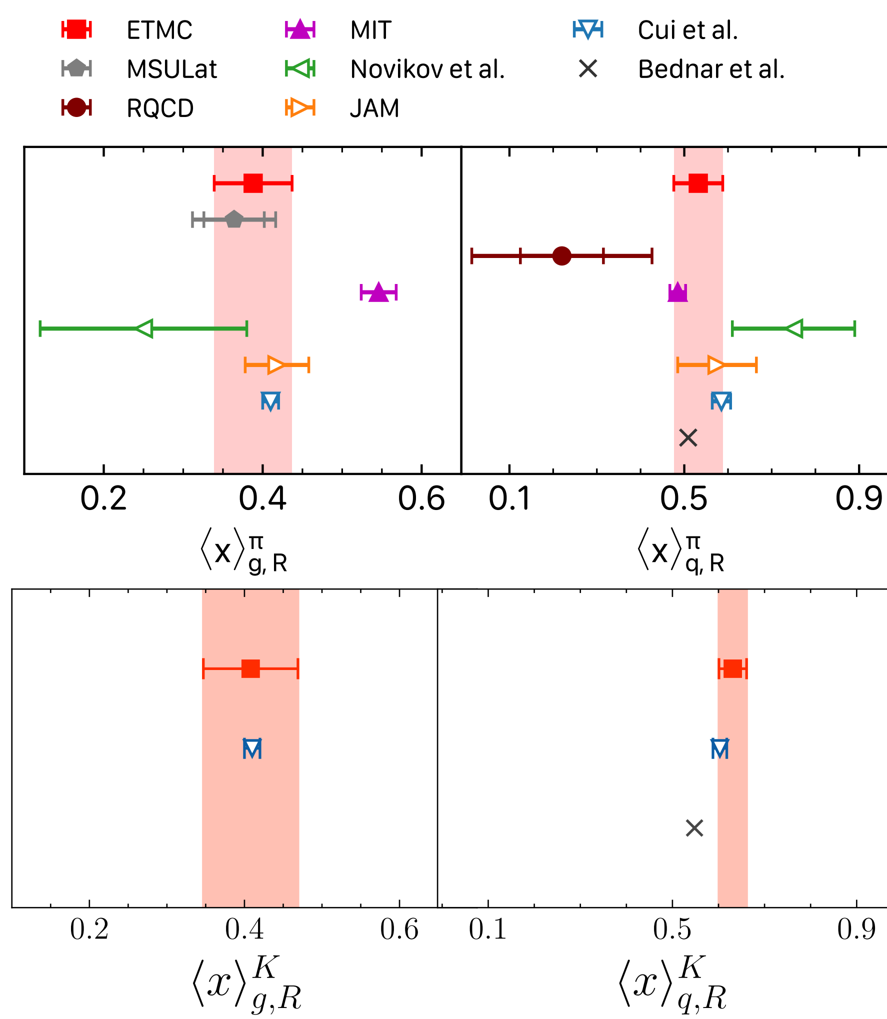
$N_f = 2+1+1$, Twisted – mass fermions : $64^3 \times 128, a = 0.080$ fm, $m_\pi = 140$ MeV B64

$80^3 \times 160, a = 0.069$ fm, $m_\pi = 137$ MeV C80

$96^3 \times 192, a = 0.057$ fm, $m_\pi = 141$ MeV D96







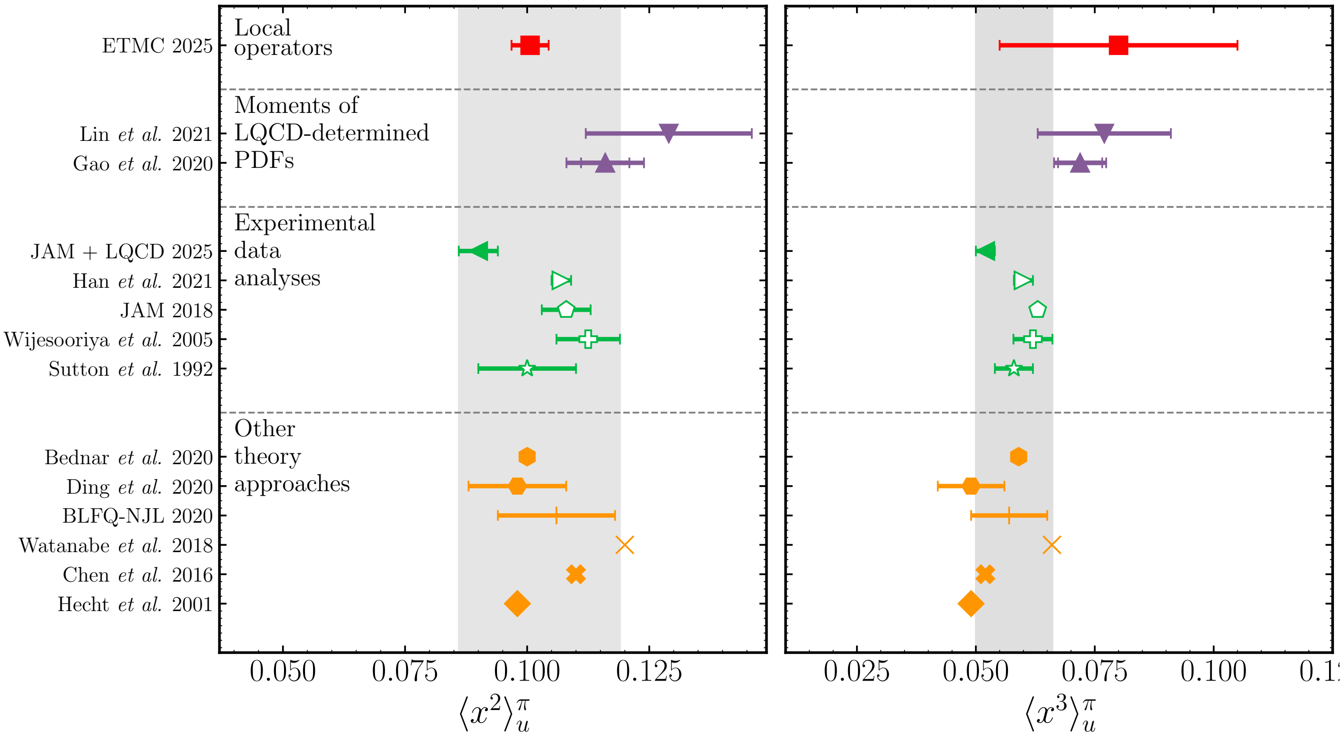
C. A. *et al.* (ETMC), Phys. Rev. Lett. 127 (2021) 252001
C. A. *et al.* (ETMC), Phys. Rev. Lett 134 (2025) 131902

✦ About 40% comes from the gluon contribution and about 10% from sea quarks

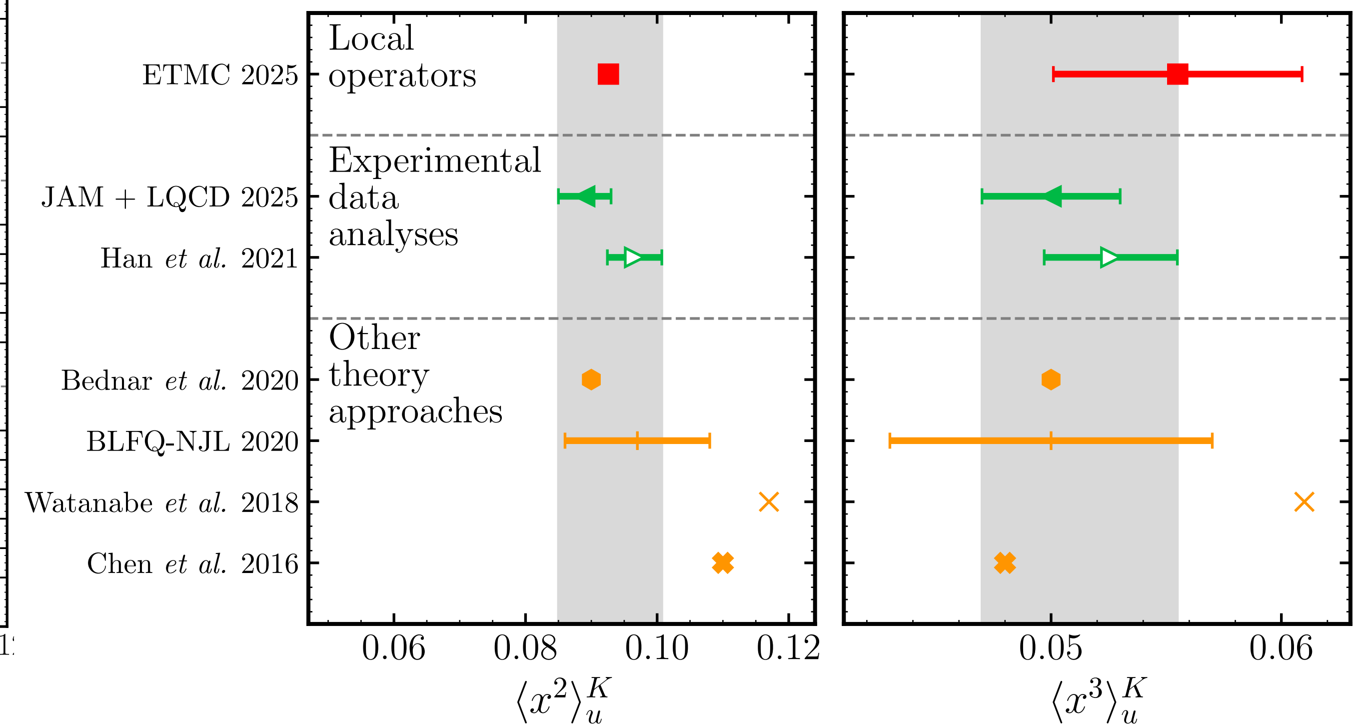
✦ Momentum sum satisfied when all components are added

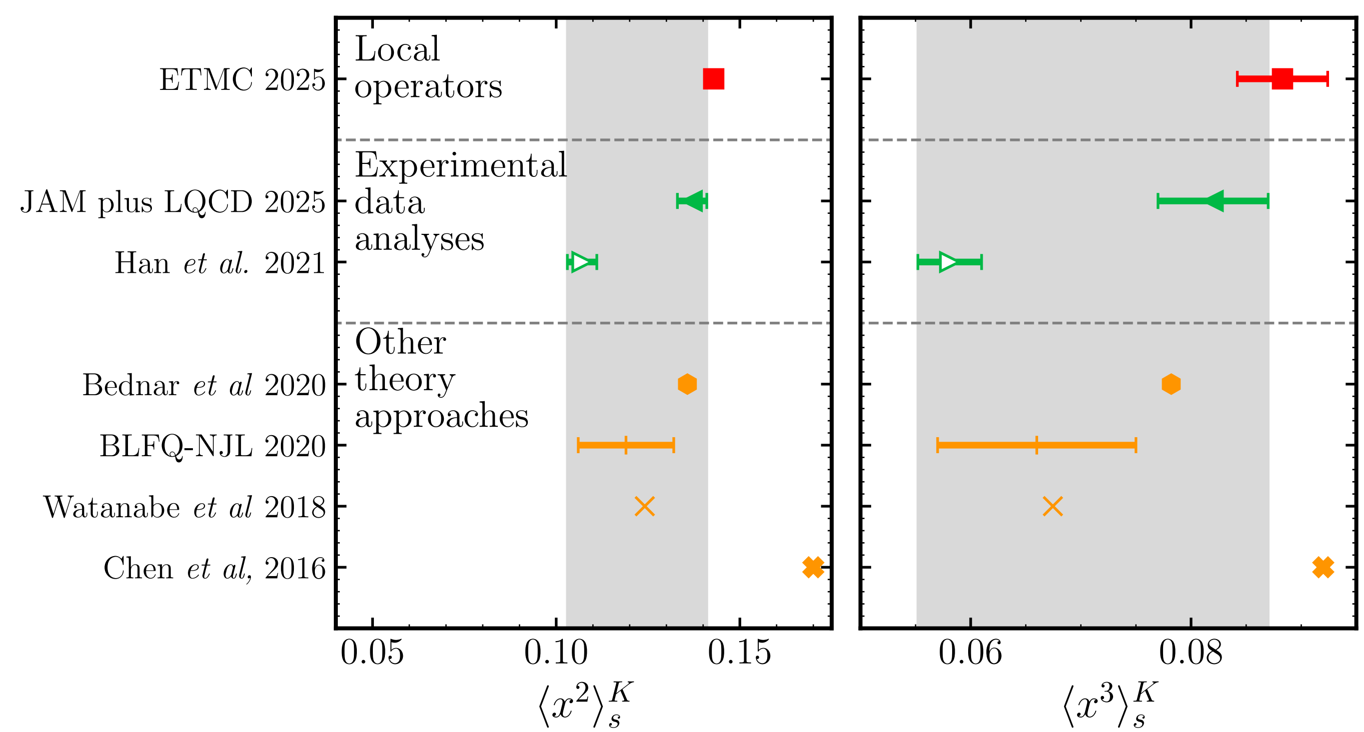
Higher Mellin moments for pion and kaon

Pion



Kaon





❋ Third and fourth Mellin moment

$$N_f = 2+1+1, \text{ Twisted mass fermions}$$

$$64^3 \times 128, a = 0.08 \text{ fm}, m_\pi = 140 \text{ MeV}$$

$$\frac{\langle x\rangle_u^K}{\langle x\rangle_s^K} = 0.715(5), \quad \frac{\langle x^2\rangle_u^K}{\langle x^2\rangle_s^K} = 0.647(8), \quad \frac{\langle x^3\rangle_u^K}{\langle x^3\rangle_s^K} = 0.632(67)$$

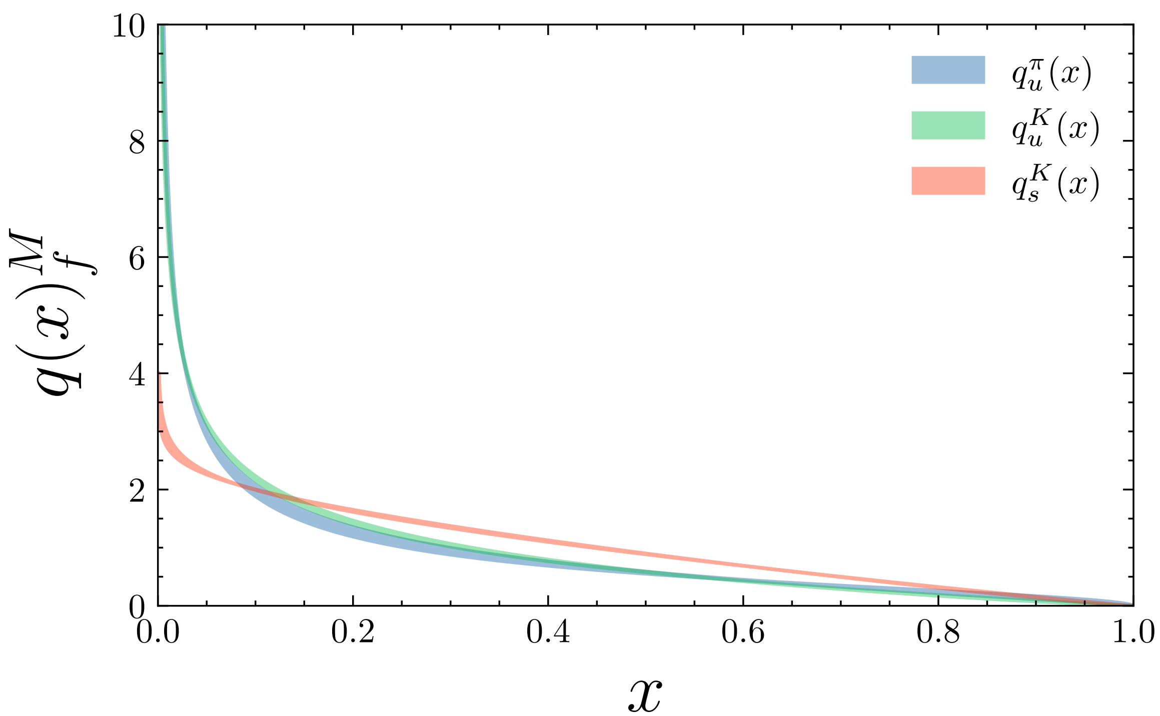
- Results point to the strange quark PDF having its support at larger x than the up quark PDF in the kaon
- SU(3) symmetry breaking is more pronounced for higher moments

Reconstruction of PDFs using moments

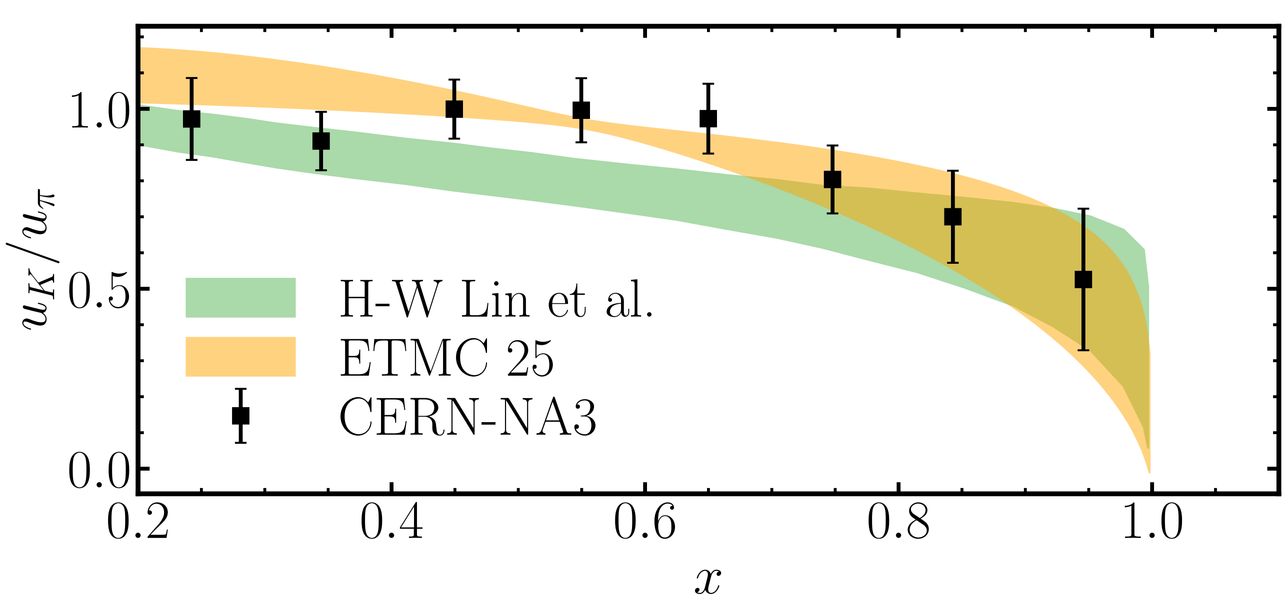
- Use moments up to <x^3>

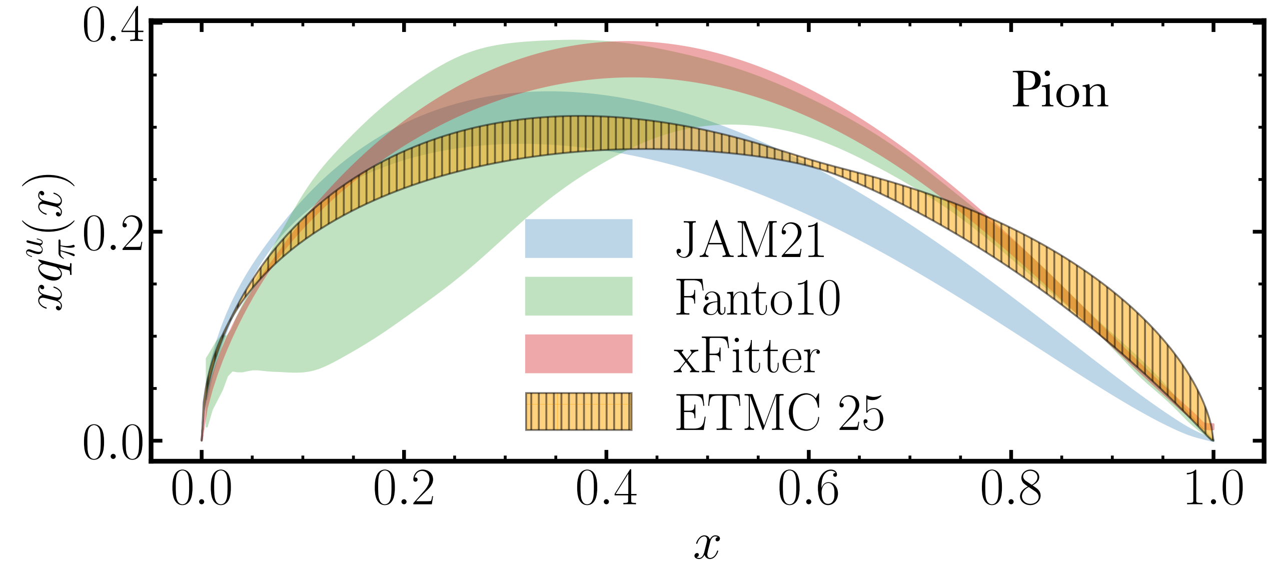
	$\langle x \rangle$	$\langle x^2 \rangle$	$\langle x^3 \rangle$
u^π	0.2194(22)	0.1021(34)	0.079(25)
u^K	0.2151(12)	0.0925(11)	0.0557(54)
s^K	0.30081(67)	0.14312(86)	0.0881(41)

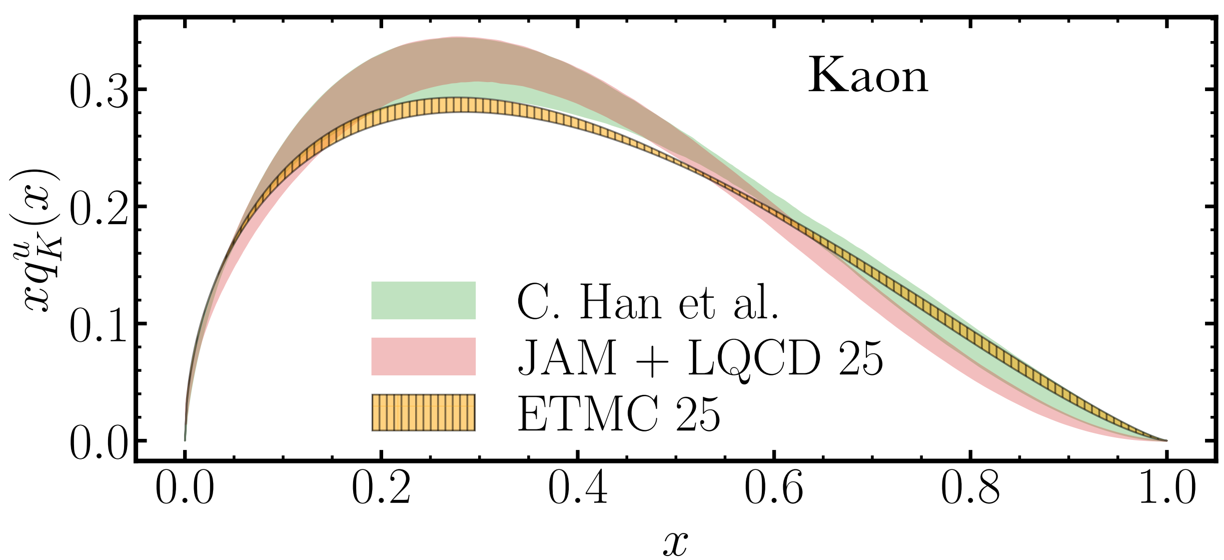
$$q^f_{\pi,K}(x) = N x^{\alpha} (1-x)^{\beta} (1 + \gamma \sqrt{x}) \qquad \langle 1 \rangle_M = \int_0^1 q_M(x) = 1$$

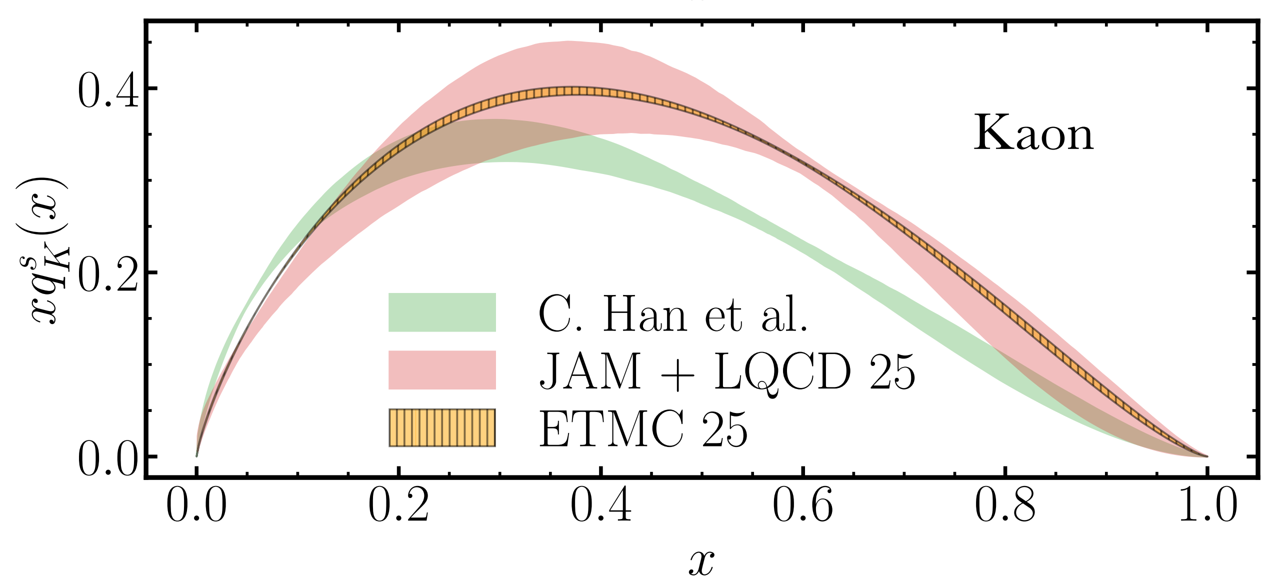


Pion\kaon



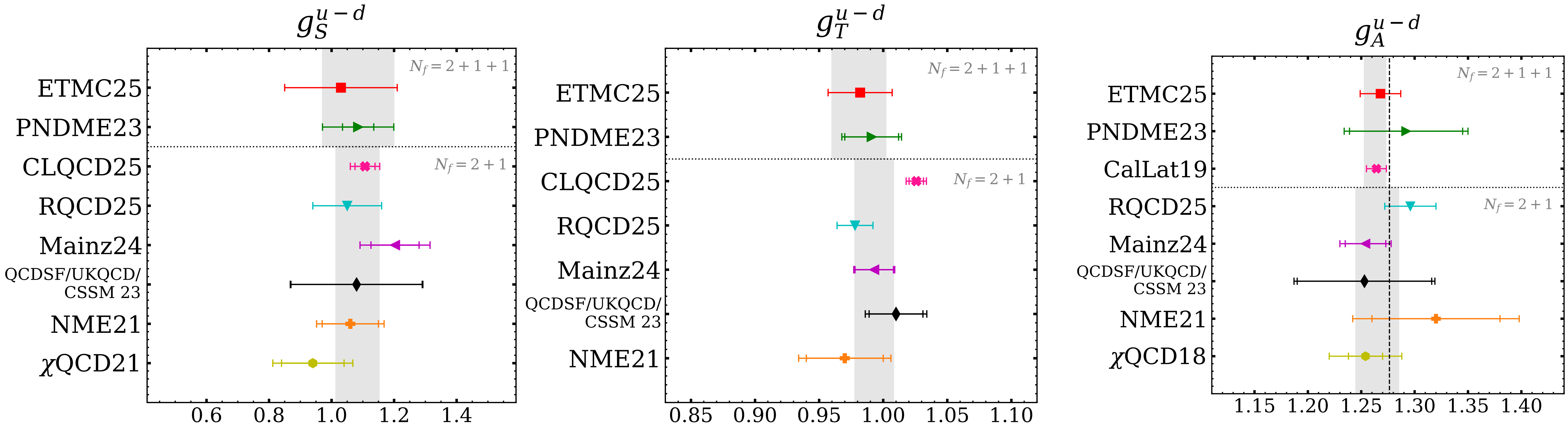






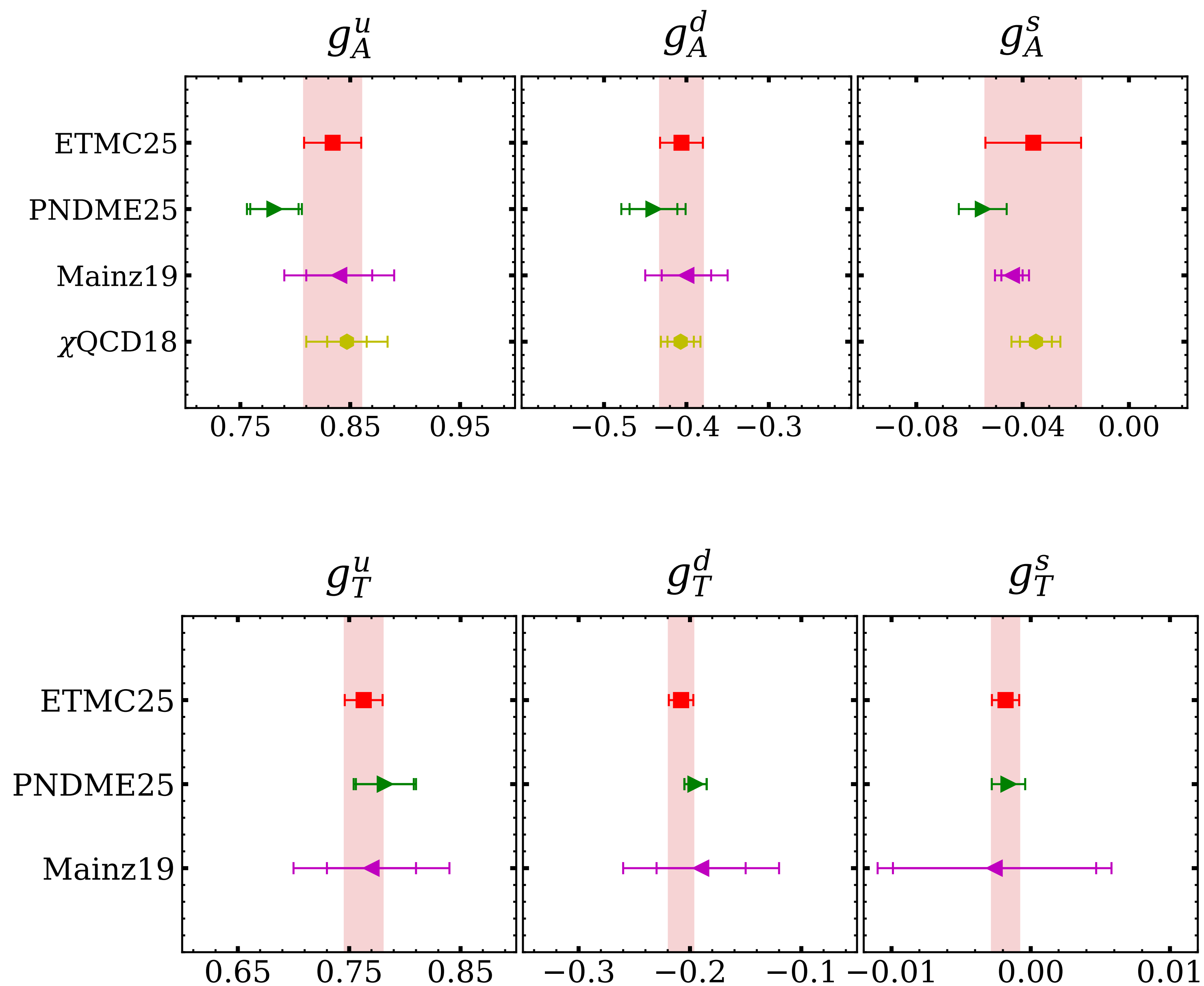
Nucleon isovector charges

- Nucleon isovector charges are well-studied by many collaborations

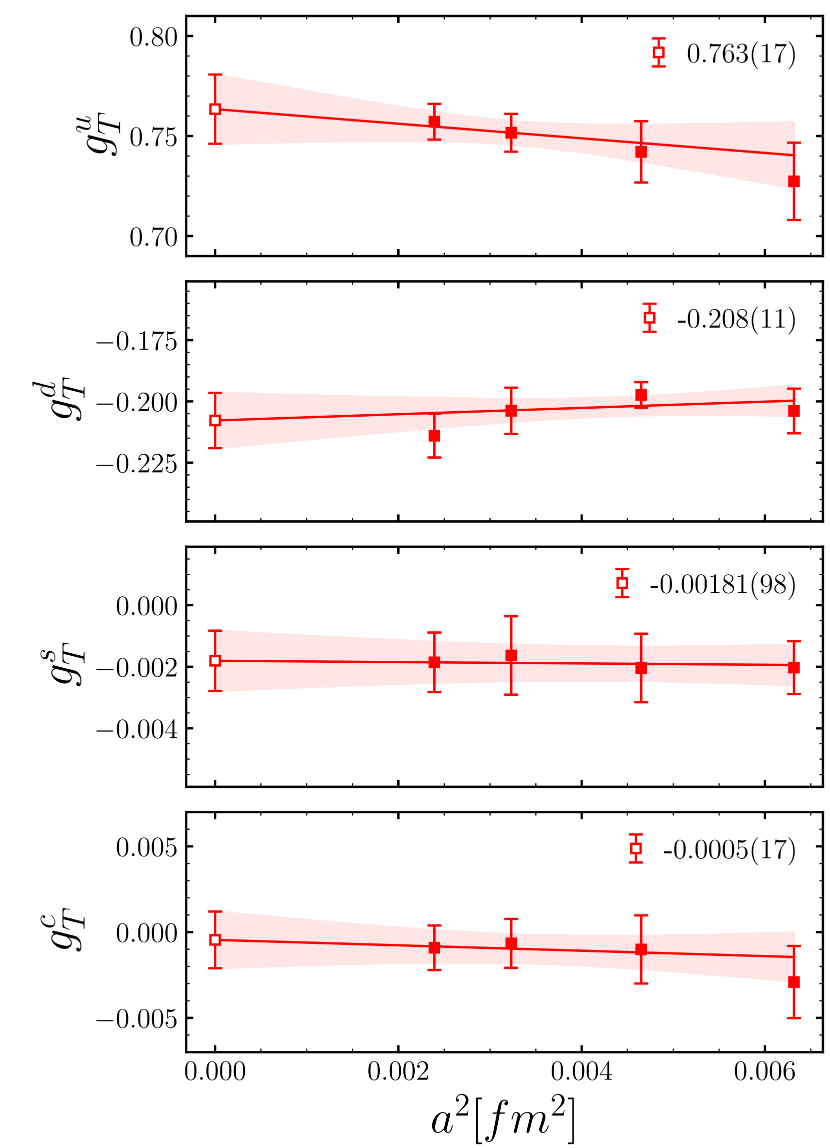


Nucleon charges

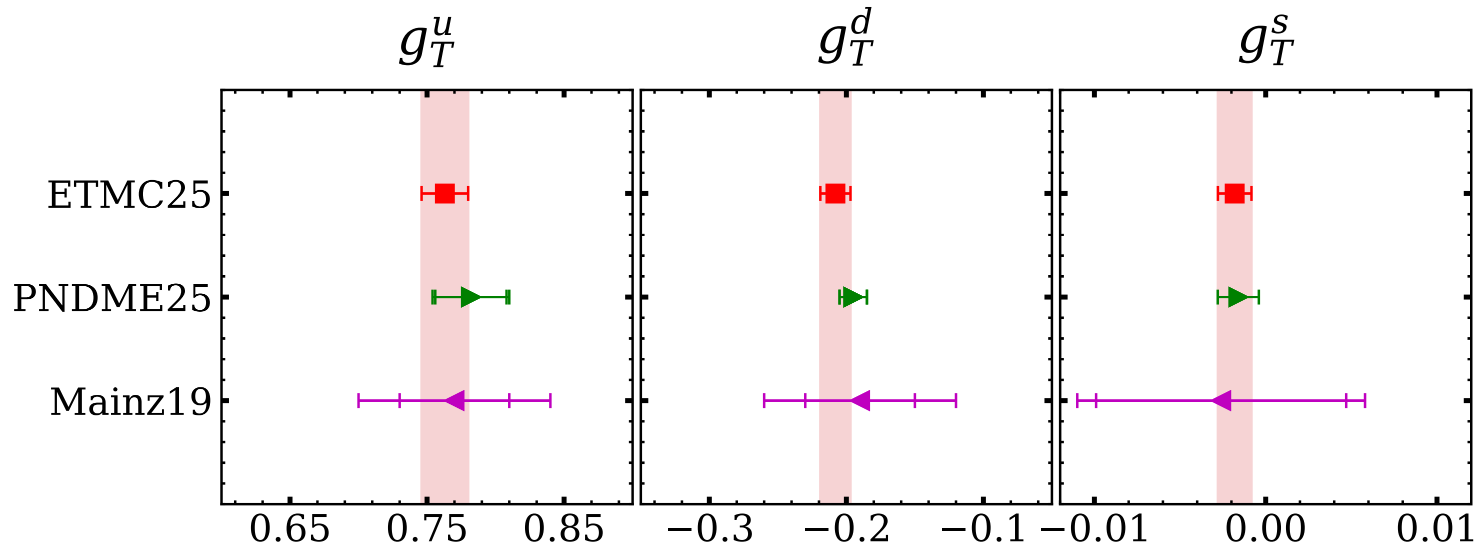
$N_f = 2+1+1$, Twisted − mass fermions : $64^3 \times 128, a = 0.080$ fm, $m_\pi = 140$ MeV
$80^3 \times 160, a = 0.069$ fm, $m_\pi = 137$ MeV
$96^3 \times 192, a = 0.057$ fm, $m_\pi = 141$ MeV
$112^2 \times 224, a = 0.049$ fm, $m_\pi = 136$ MeV



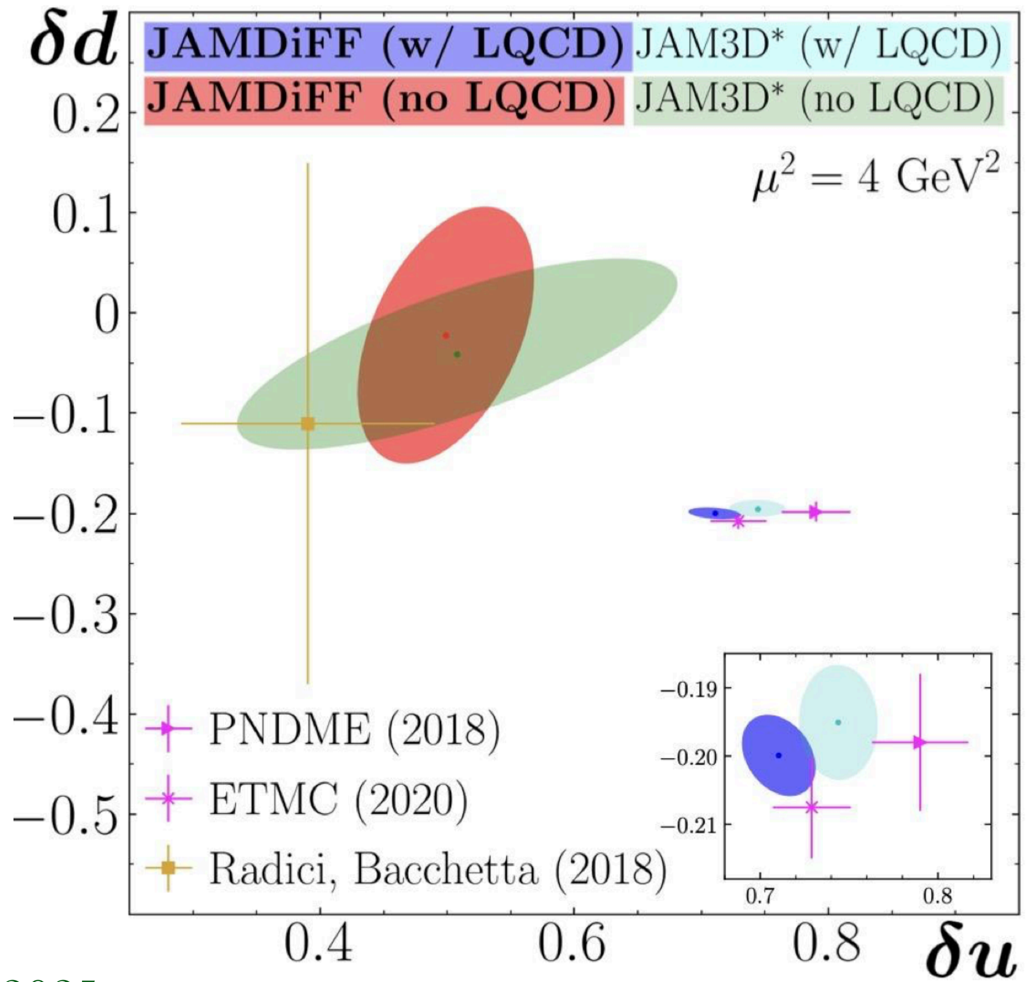
ETMC: continuum limit taken directly at physical pion mass



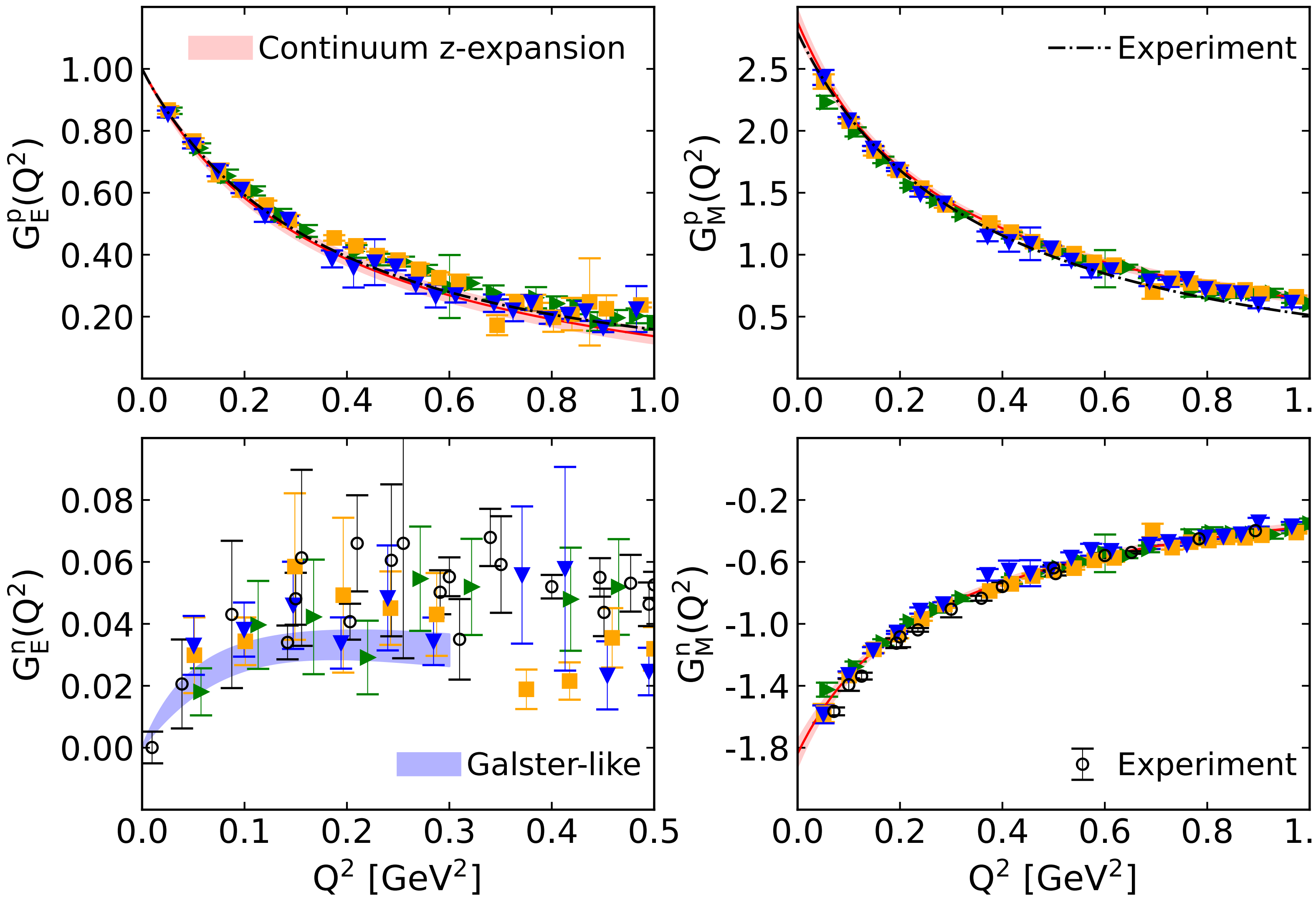
Nucleon tensor charge input



- Tensor charge provides input for phenomenology



Nucleon EM and axial form factors by ETMC



Two-fit forms for Q^2 dependence:

- Model independent z-expansion

$$G(Q^2) = \sum_{k=0}^{k_{\max}} c_k z^k(Q^2)$$

$$z(Q^2) = \frac{\sqrt{t_{\rm cut} + Q^2} - \sqrt{t_{\rm cut} + t_0}}{\sqrt{t_{\rm cut} + Q^2} + \sqrt{t_{\rm cut} + t_0}}$$

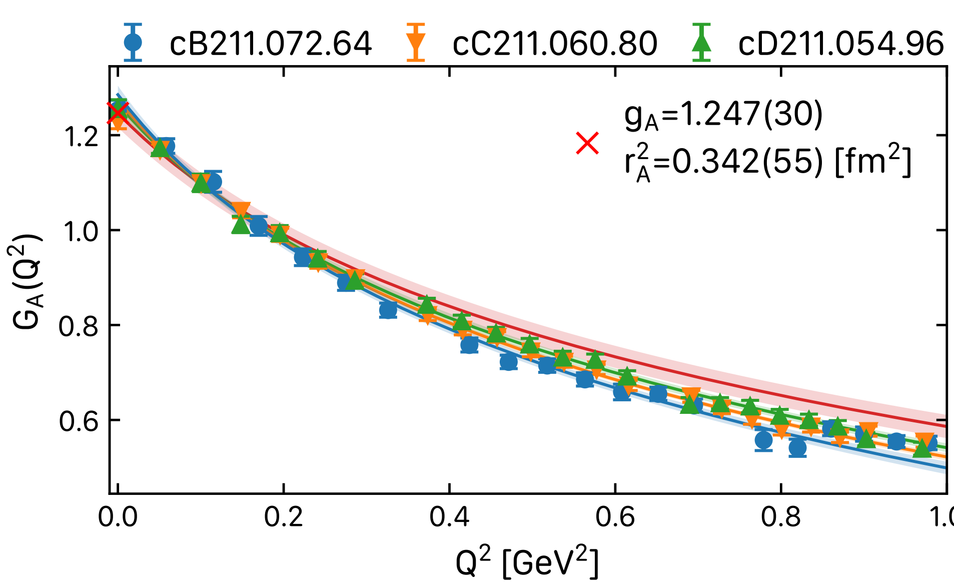
t_{cut}= particle production threshold and t_0=0

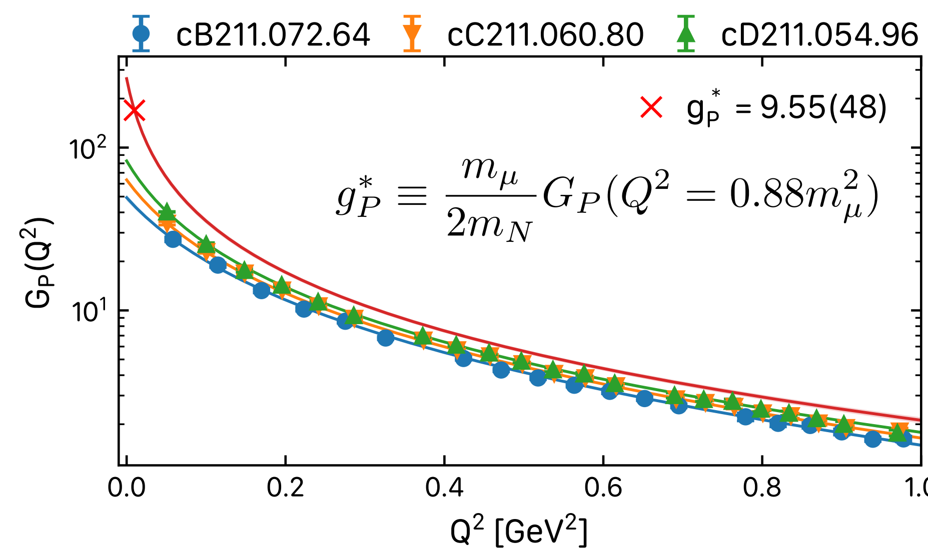
- Dipole or Galster form

$$G(Q^2) = \frac{g}{\left(1 + \frac{Q^2}{M^2}\right)^2}$$

$$G_E^n(Q^2) = \frac{Q^2 A}{4m_N^2 + Q^2 B} \frac{1}{\left(1 + \frac{Q^2}{0.71\text{GeV}^2}\right)^2}$$

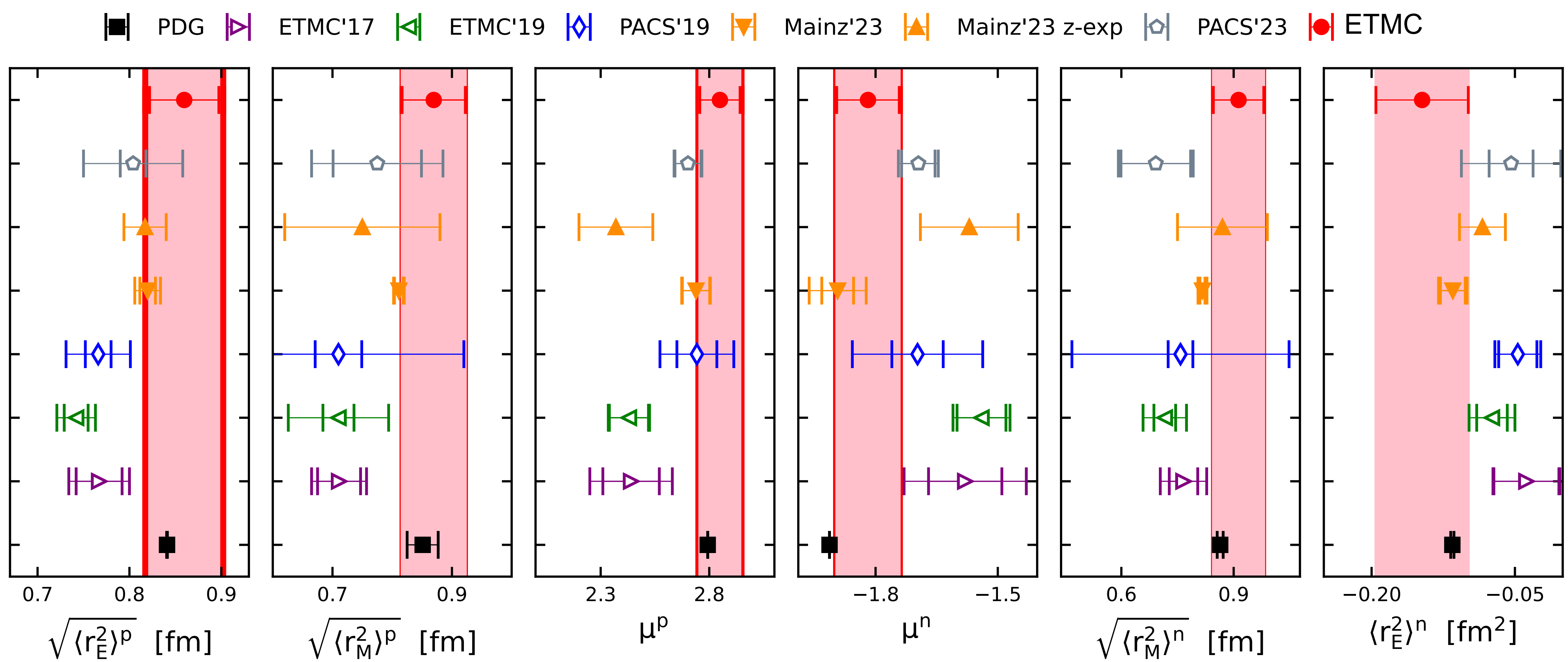
ETMC: C.A., S. Bacchio, G. Koutsou, B. Prassad, G. Spanoudes, arXiv:2507.20910

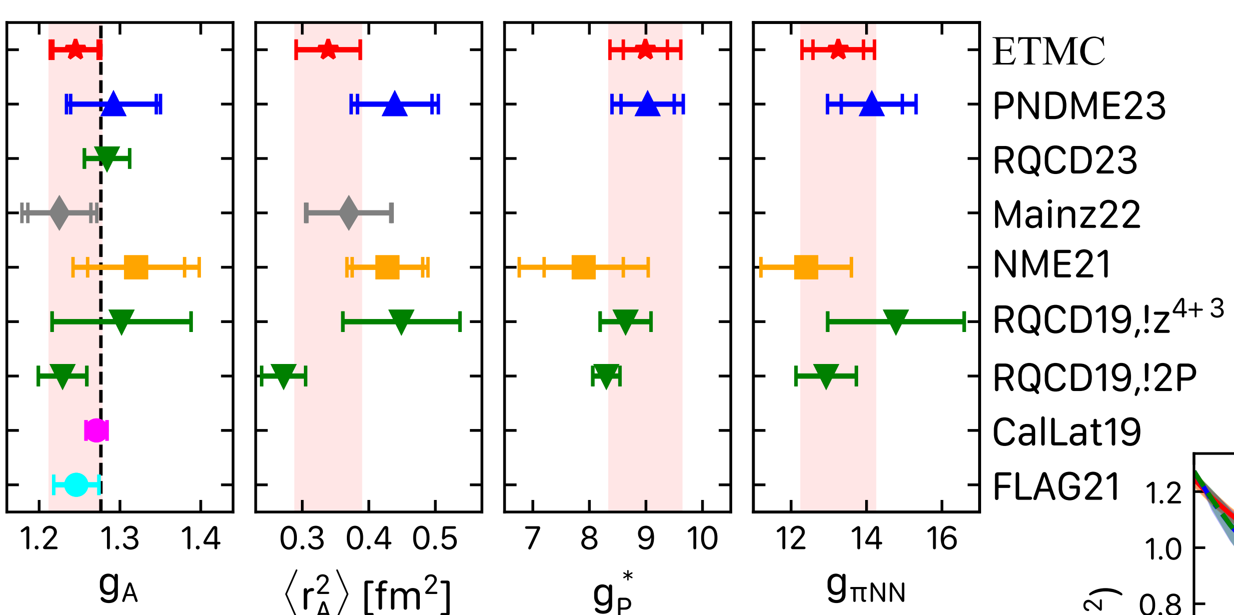


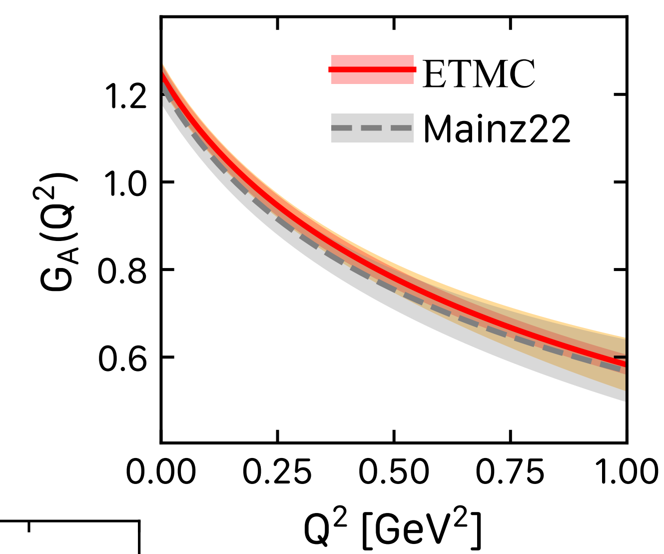


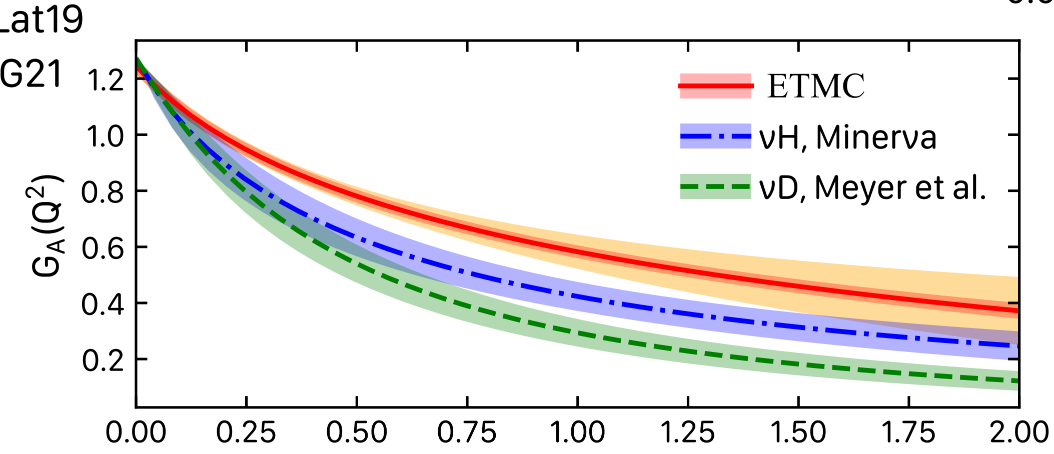
ETMC: C.A. *et al.*, Phys. Rev. D 109(2024)3, 034503

Nucleon EM and axial form factors - comparison









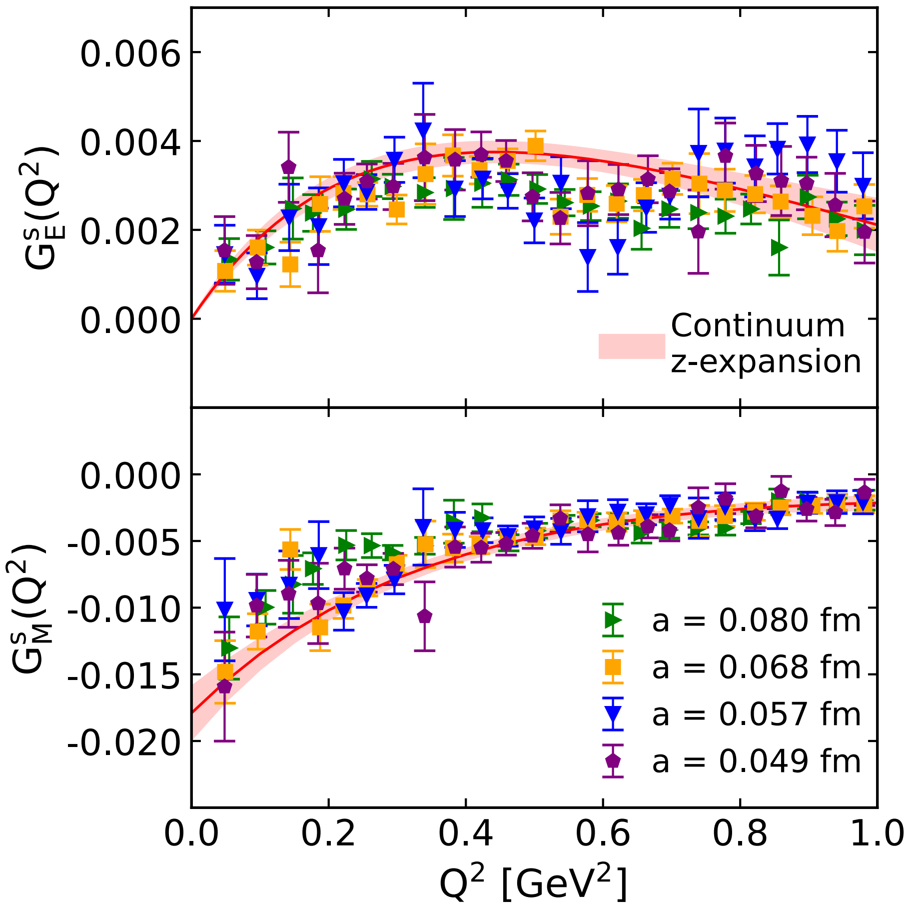
Strangeness in the nucleon from ETMC

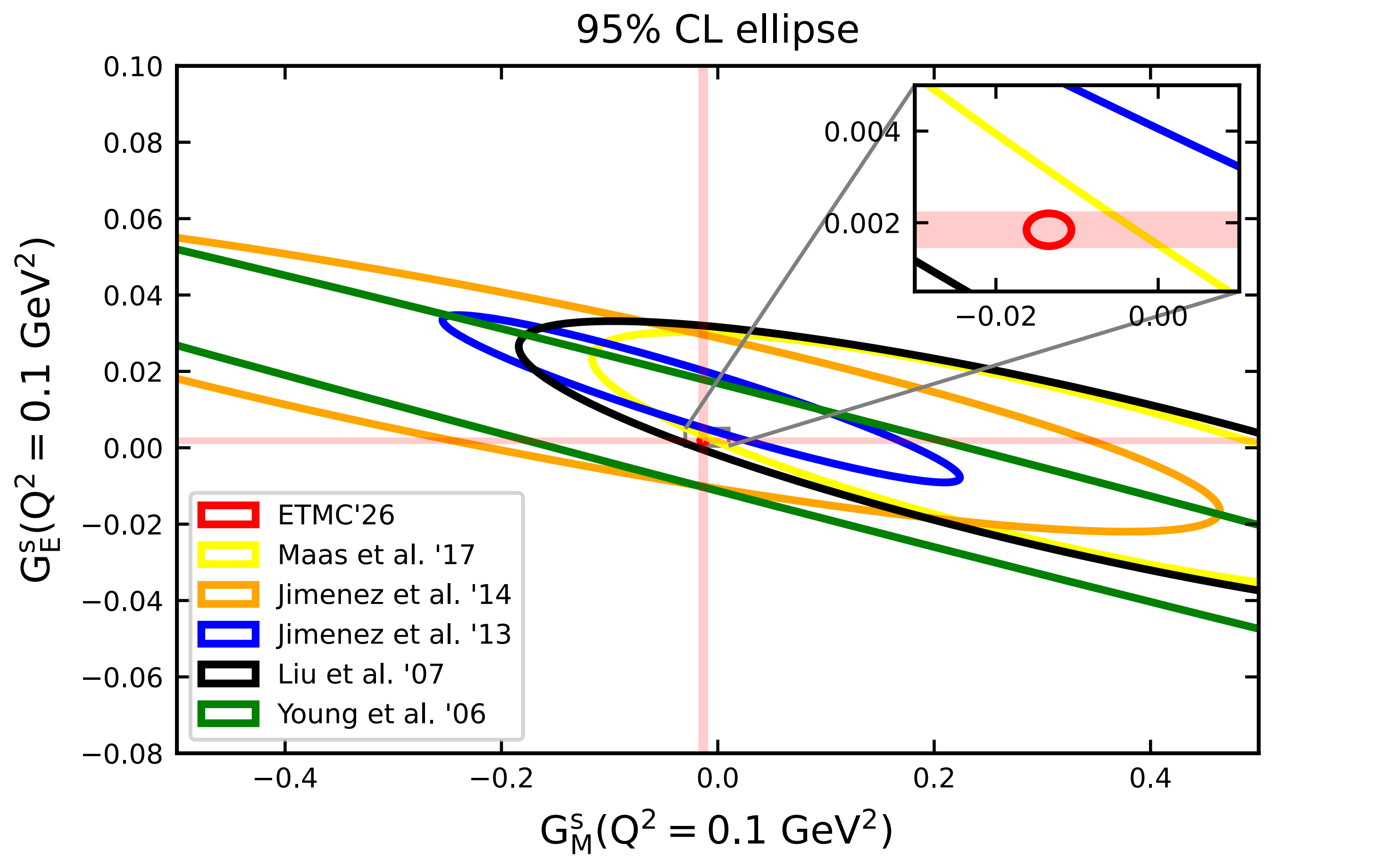
$N_f = 2+1+1$, Twisted − mass fermions : $64^3 \times 128, a = 0.080$ fm, $m_\pi = 140$ MeV B64

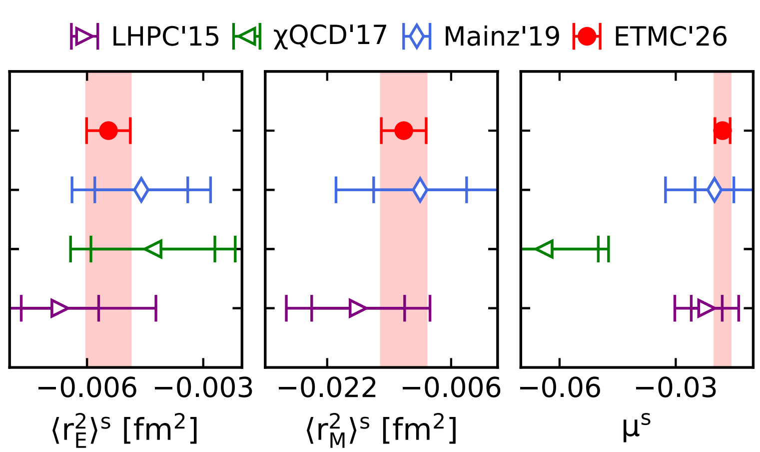
$80^3 \times 160, a = 0.069$ fm, $m_\pi = 137$ MeV C80

$96^3 \times 192, a = 0.057$ fm, $m_\pi = 141$ MeV D96

$112^2 \times 224, a = 0.049$ fm, $m_\pi = 136$ MeV E112







Significant input to PV experiments e.g. for Q-weak, G0 & HAPPEX @JLab

ETMC: C.A. *et al.*, arXiv:2603.26600; arXiv:2603.2691

- First computation directly at physical pion mass with continuum extrapolation - clearly shows the accuracy that can be reached by lattice QCD on sea quark effects

Nucleon second moments

❋Nucleon momentum and spin sums

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$80^3 \times 160, a = 0.069$ fm, $m_\pi = 137$ MeV C80

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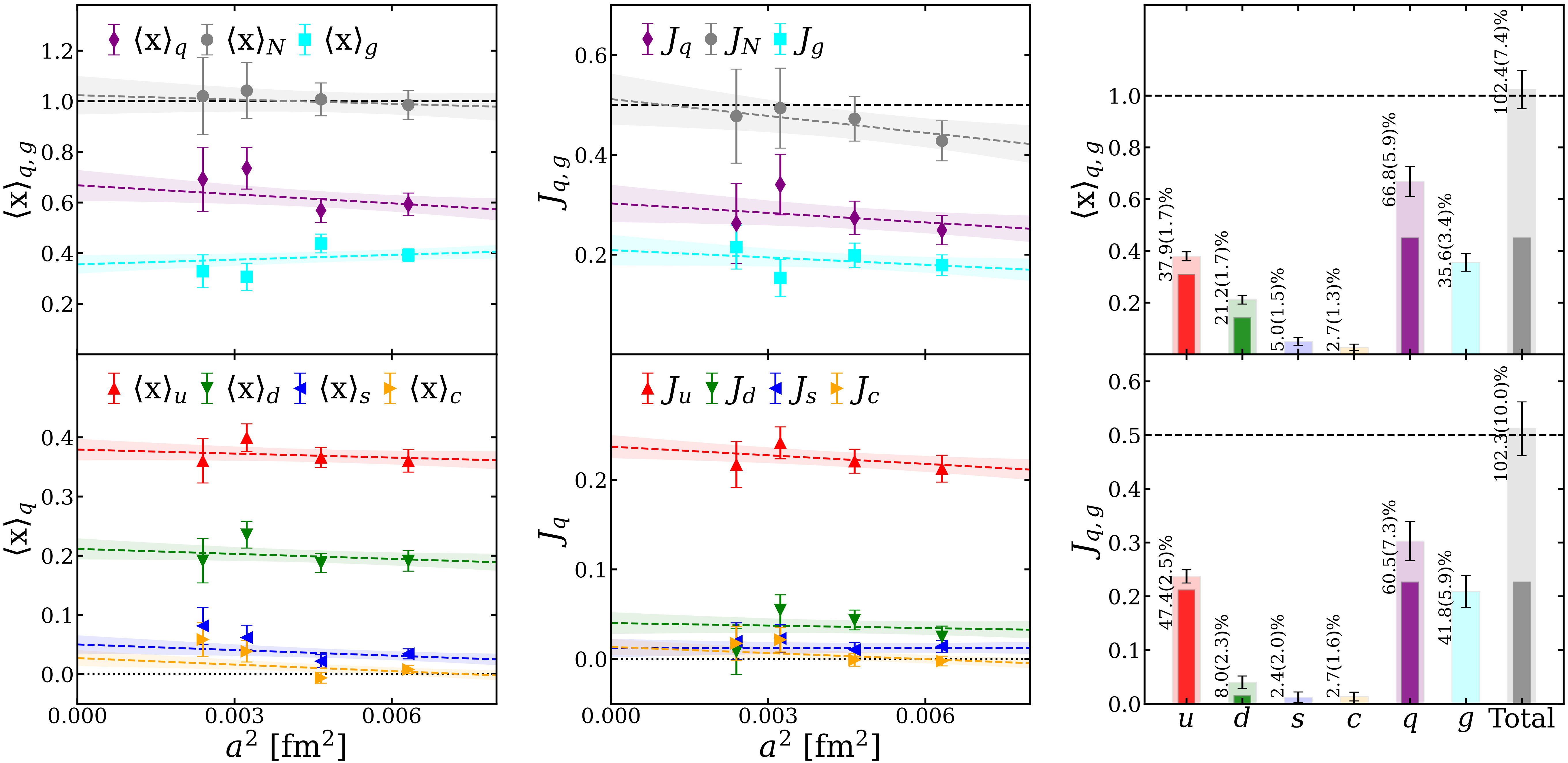

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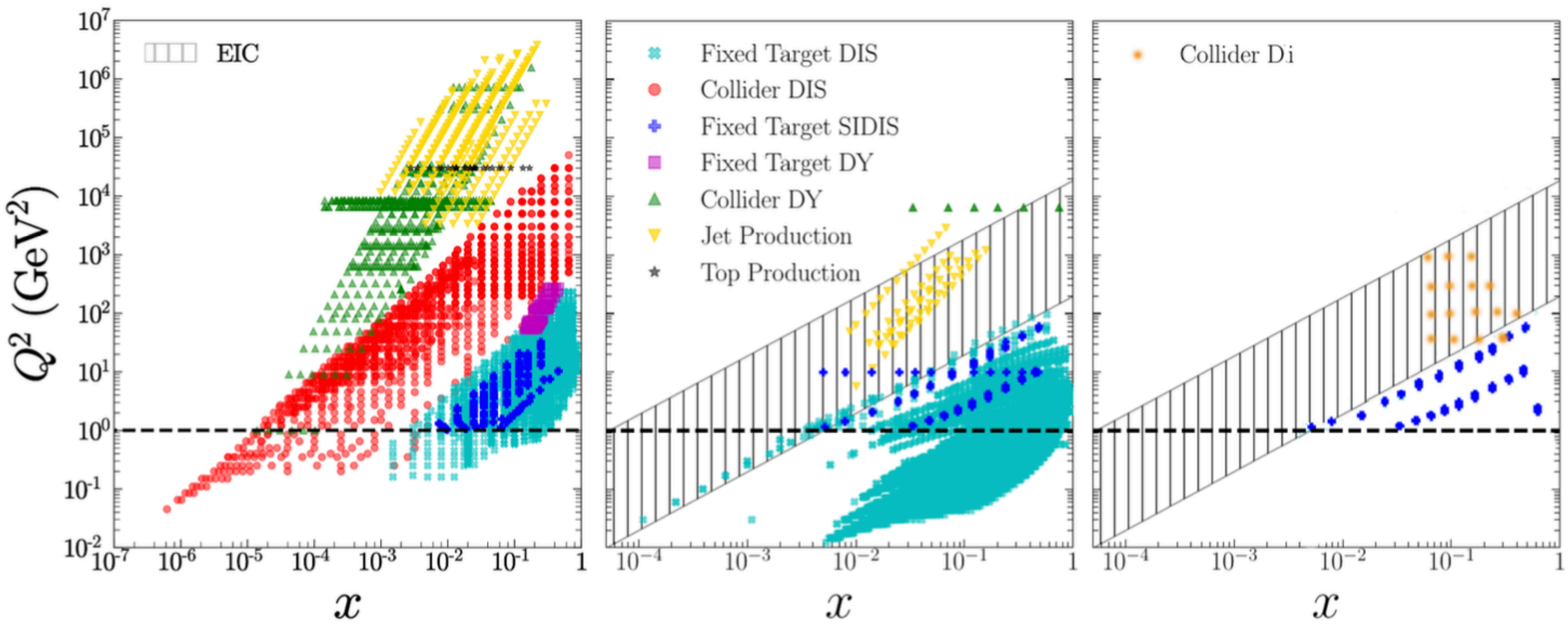
✦Momentum sum satisfied when all components are added

Direct computation of GPDs



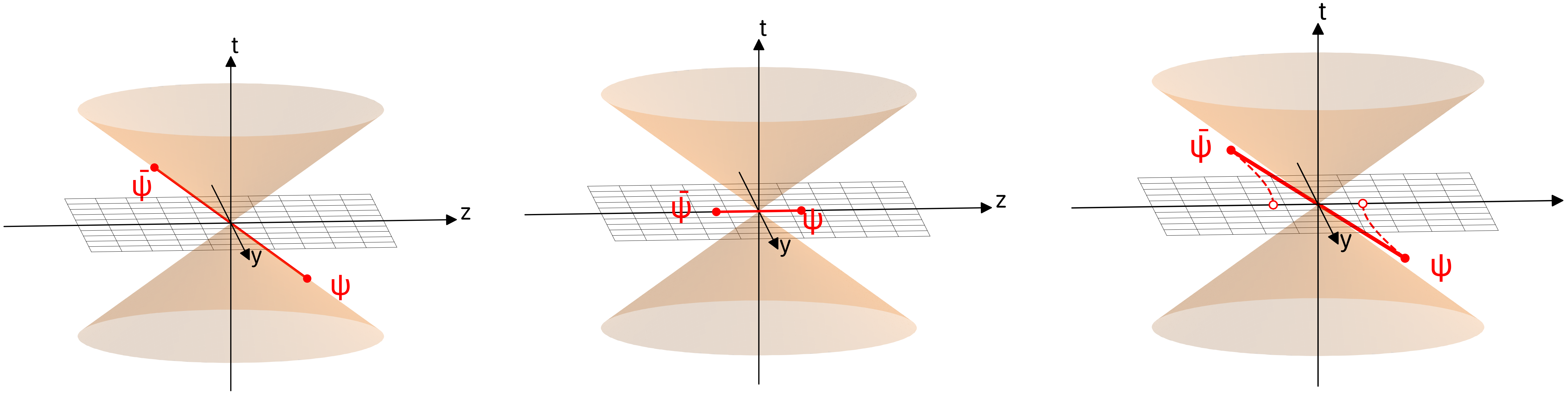
EIC will probe the region of low x —> non trivial flavor and quark-anti-quark differences

Direct computation of GPDs

- Quasi-approach - X. Ji 2013
- Pseudo-approach - A. V. Radyushkin 2017
- Hadronic tensor - K.-F. Liu 2016
- Forward Compton amplitude - A. Chambers *et al.* 2017
- Auxiliary heavy quark - W. Detmold and C. D. Lin 2006
- Good lattice cross sections - Y.-Q. Ma and J.-W. Qiu 2018
- …

❋ Large momentum effective theory (LaMET) X. Ji, Phys. Rev. Lett. 110 (2013) 262002

- Define spatial correlators and boost nucleon state to large momentum —> quasi PDFs (have same IR behavior)
- Match to the infinite momentum frame using the matching kernel computed in perturbation theory (possible due to asymptotic freedom of QCD)
- Allow momentum transfer —> generalised parton distributions



❋Short distance factorization (SDF) —>Pseudo- distributions V. Radyushkin, Phys. Rev. D 96 (2017) 034025

❋Quasi- and Pseudo- distributions are extracted from the same spatial corrector

Computation of quasi- and pseudo-PDFs

❋ Compute space-like matrix elements for boosted states $M_\Gamma(z,P_3) = \langle P_3|\,\overline{\psi}(0)\,\Gamma W(0,z)\,\psi(z)|\,P_3\rangle$

- **Quasi-distributions:** Take Fourier transform and the large boost limit

$$\tilde{F}_\Gamma(x,P_3,\mu) = 2P_3\int_{-\infty}^{\infty}\frac{dz}{4\pi}e^{-ixP_3 z}\,M_\Gamma(z,P_3)|_\mu$$

Renormalise non-perturbatively, $Z(z,\mu)$
Need to eliminate both UV and exponential divergences

Match using LaMET

Perturbative kernel — Higher twist

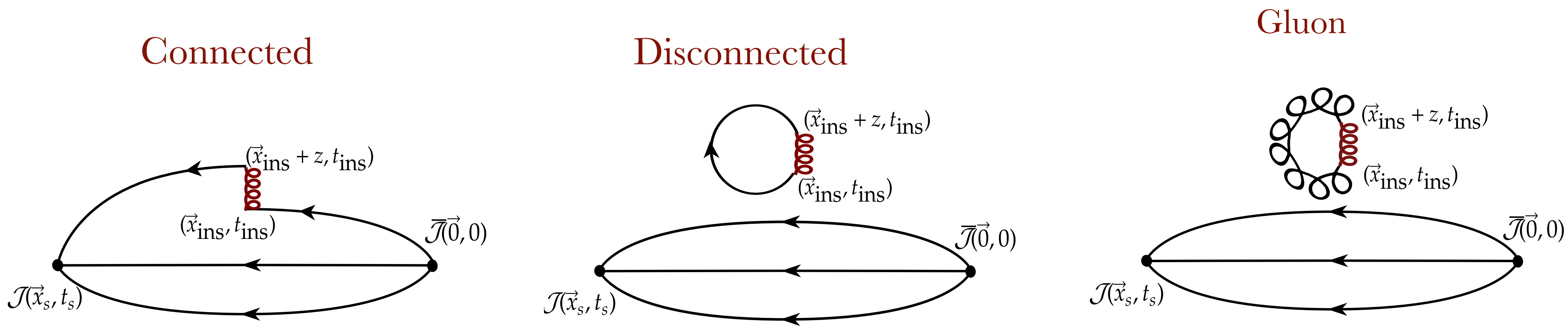
$$\tilde{F}_\Gamma(x,P_3,\mu) = \int_{-1}^{1}\frac{dy}{|y|}\,C\left(\frac{x}{y},\frac{\mu}{yP_3}\right)F_\Gamma(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{x^2P_3^2},\frac{\Lambda^2_{QCD}}{(1-x)^2P_3^2}\right)$$

Validity range:

$$x \in (x_{\min} - x_{\max}) \approx (0.2 - 0.8)$$

$\Gamma =$	γ_0	unpolarised
	$\gamma_5\gamma_3$	helicity
	$\sigma_{3i}, i=1,2$	transversity

- **Nucleon**



Computation of quasi- and pseudo-PDFs

❋ Compute space-like matrix elements for boosted states $M_\Gamma(z, P_3) = \langle P_3 | \overline{\psi}(0)\, \Gamma W(0,z)\, \psi(z) | P_3 \rangle$

- **Quasi-distributions:** Take Fourier transform and the large boost limit

$$\tilde{F}_\Gamma(x, P_3, \mu) = 2P_3 \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-ixP_3 z}\, M(z, P_3)|_\mu$$

Renormalise non-perturbatively, $Z(z,\mu)$
Need to eliminate both UV and exponential divergences

Match using LaMET

Perturbative kernel — Higher twist — Validity range:

$$\tilde{F}_\Gamma(x, P_3, \mu) = \int_{-1}^{1} \frac{dy}{|y|}\, C\left(\frac{x}{y}, \frac{\mu}{yP_3}\right) F_\Gamma(y,\mu) + \mathcal{O}\left(\frac{\Lambda^2_{QCD}}{x^2 P_3^2}, \frac{\Lambda^2_{QCD}}{(1-x)^2 P_3^2}\right) \qquad x \in (x_{\min} - x_{\max}) \approx (0.2 - 0.8)$$

- **Pseudo-distributions:** use same matrix elements as quasi-distributions and write in terms of $\nu = z.P$ Ioffe time

Renormalize matrix element e.g. $\tilde{\mathcal{M}}_\Gamma(\nu, z^2) = \dfrac{\bar{M}_\Gamma(\nu, z^2)}{\bar{M}_\Gamma(0, z^2)}$

Match in coordinate space via short distance factorization (SDF)

$$\tilde{\mathcal{M}}_\Gamma(z^2, \nu) = \int_{-1}^{1} dy\, C(y) \mathcal{M}_\Gamma(y\nu, \mu) + \mathcal{O}(z^2 \Lambda^2_{\mathrm{QCD}})$$

Ioffe time distribution — Higher twist

limits max. ν_{max} value —> max. moments $x^{n_{\nu_{\max}}}$

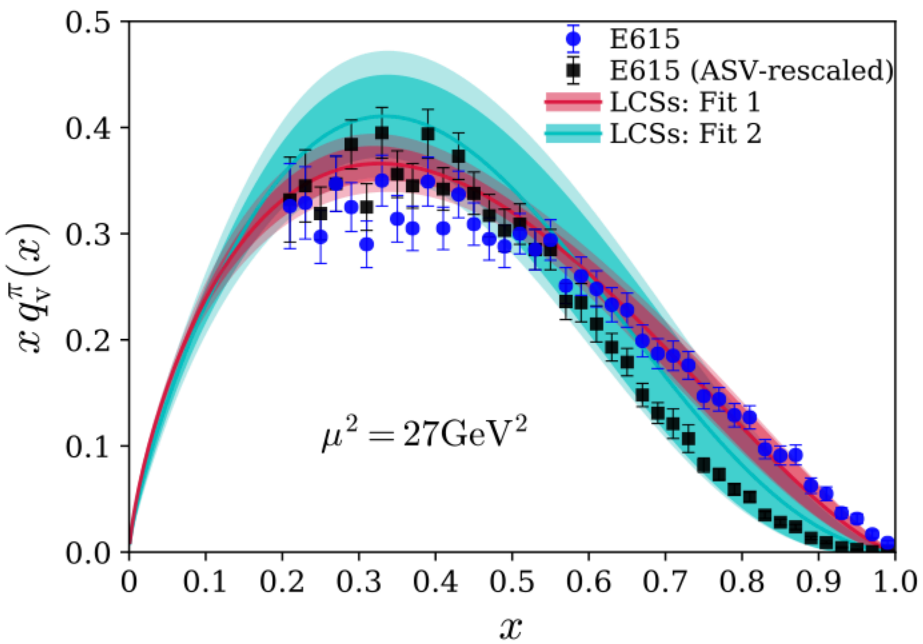
$$\mathcal{M}_\Gamma(\nu, \mu) = \int_{-1}^{1} d\nu e^{ix\nu} F_\Gamma(x, \mu)$$

❋ Quasi- and pseudo- are complementary X. Ji, Research 8 (2025) 0695

Pion and kaon unpolarized PDF

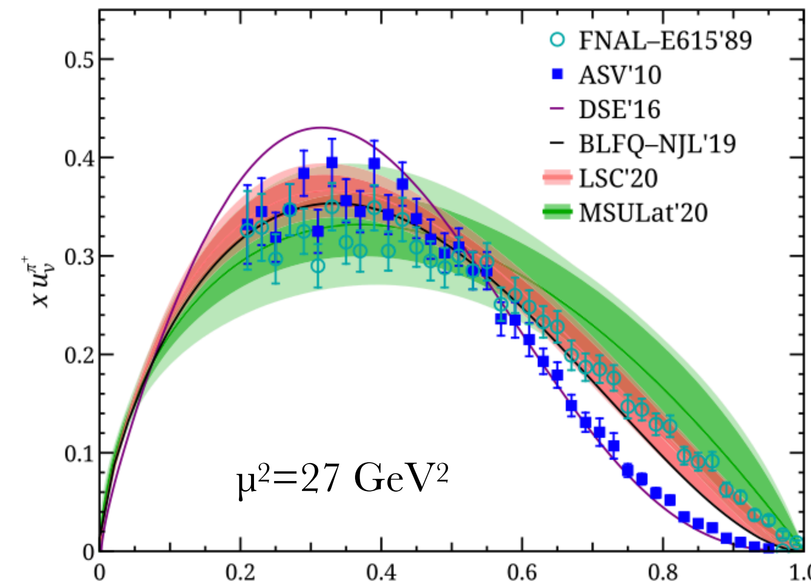
❋ Pion

R. S. Sufian *et al.*, Phys. Rev. D 102 (2020) 054508



"Good lattice cross-sections" approach with N_f=2+1 clover, m_π~410-280 MeV and a=0.127 and 0.094

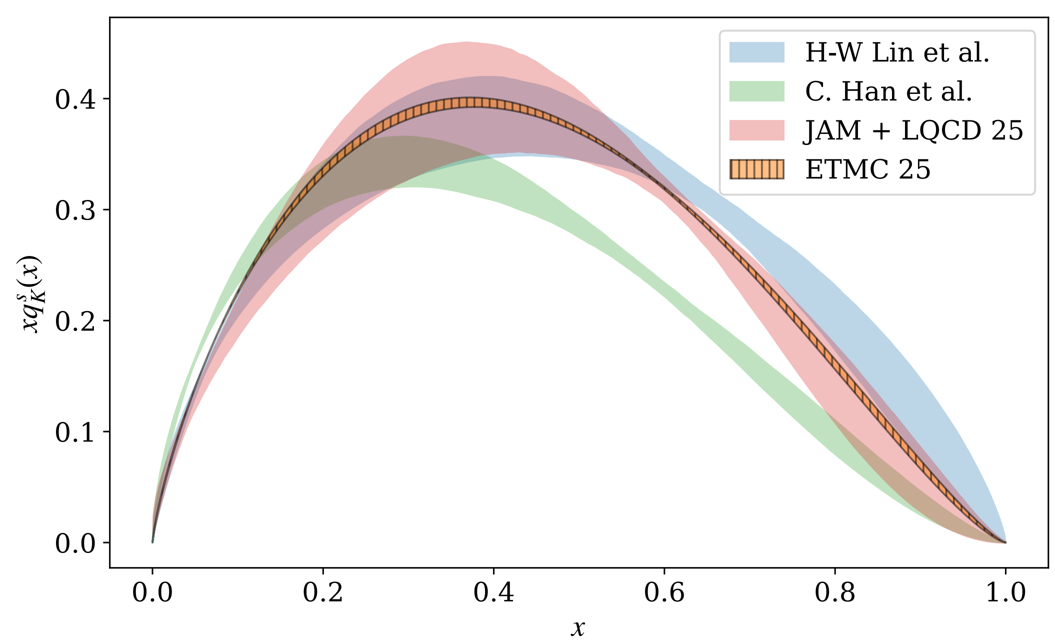
H.-W. Lin *et al.*, Phys.Rev.D 103 (2021) 014516

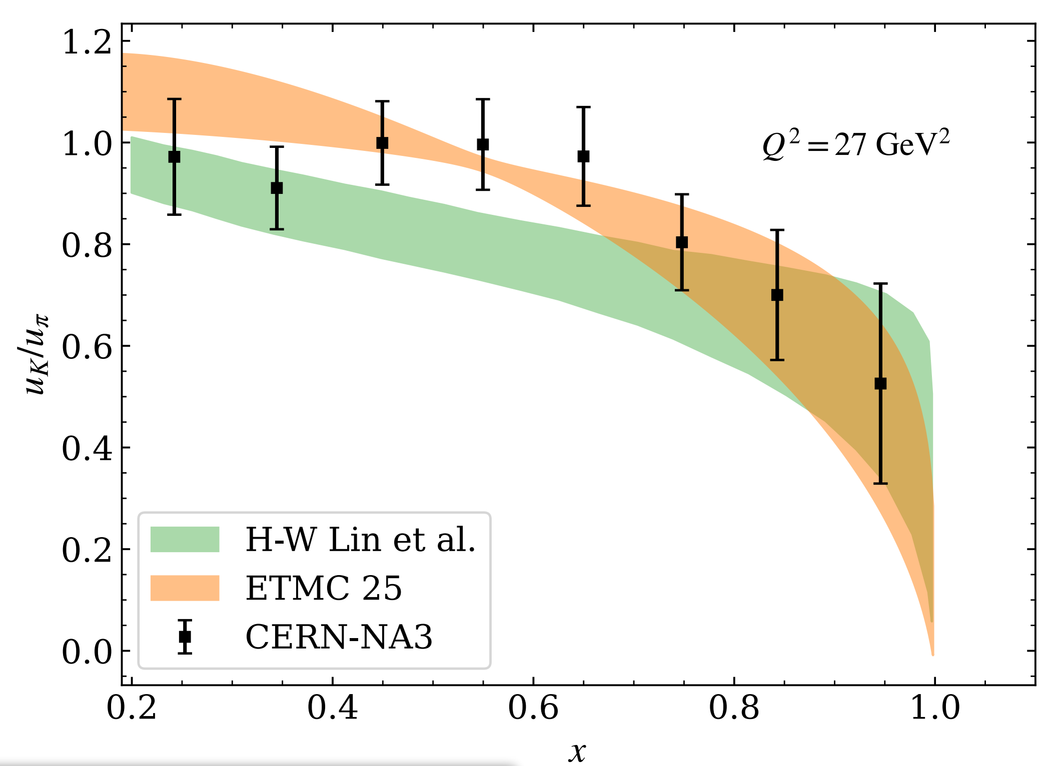


Quasi-PDF approach with clover valence on staggered sea with m_π=220, 310, 690 MeV and a=0.12, 0.06 fm with extrapolation to the continuum and physical m_π via $c_0 + c_1 m_\pi^2 + c_3 a^2$

❋ Kaon

H.-W. Lin *et al.*, Phys.Rev.D 103 (2021) 014516





To be measured by the AMBER experiment at CERN

Nucleon helicity and strange quark PDFs

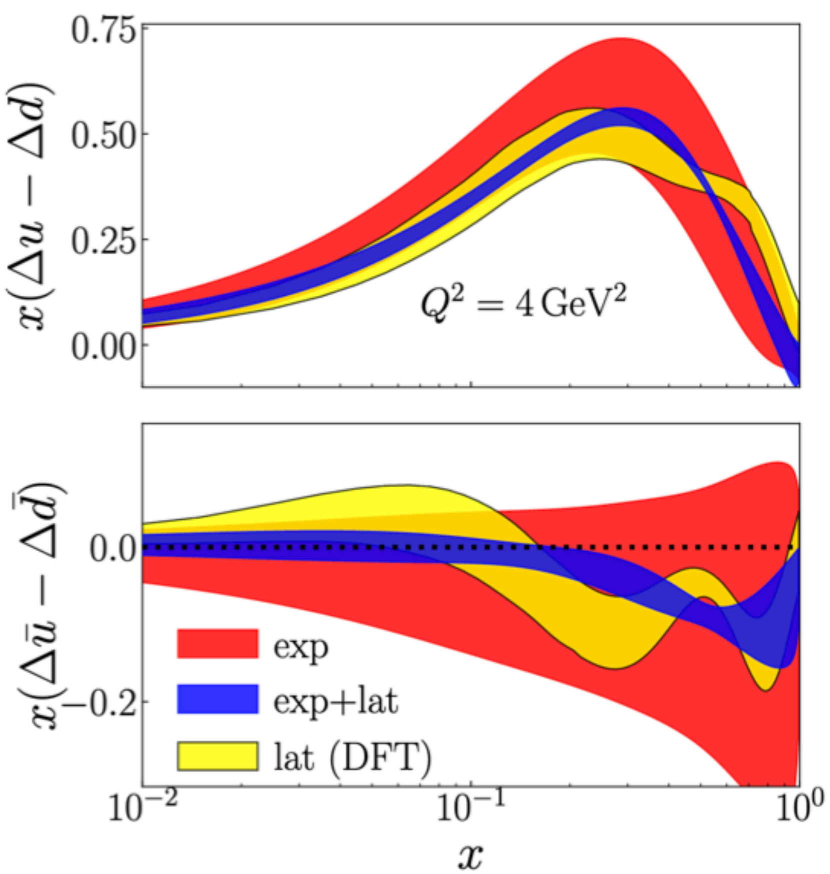
- Studies in the quasi-PDF approach helicity have had an impact on phenomenology

- Isovector at physical pion mass

- LP³: N_f=2+1+1, m_π~135 MeV, a=0.09 fm
  H.-W. Lin *et al.* Phys. Rev. Lett. **121**, 242003 (2018)

- ETMC: N_f=2, m_π=135 MeV, a=0.094 fm

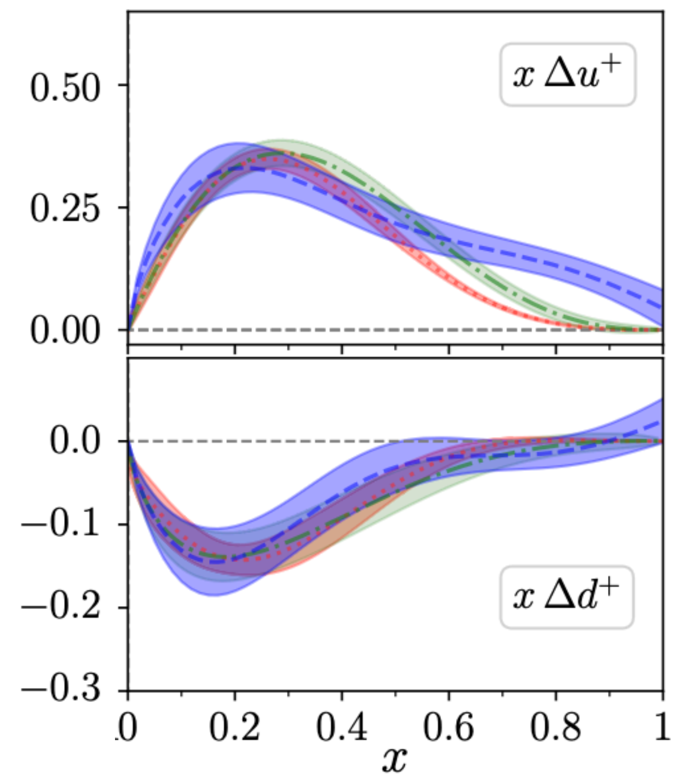
C.A. *et al.* (ETMC) Phys. Rev. Lett. 121 (2018) 112001

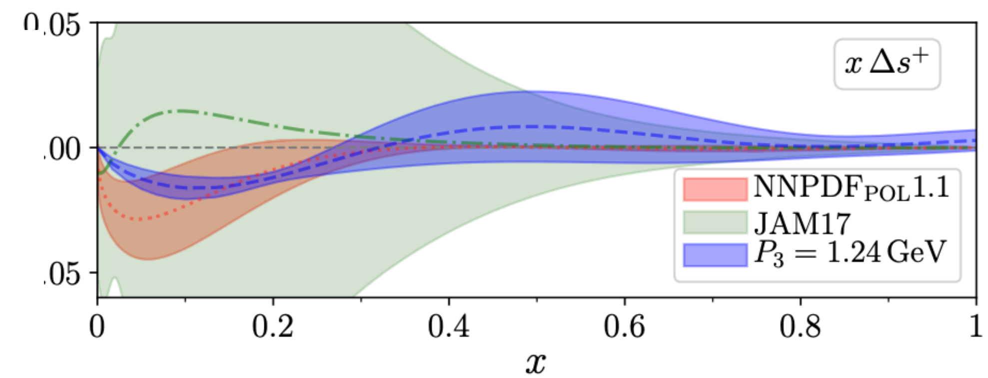


J. Bringewatt *et al.* (JAM) Phys.Rev.D 103 (2021) 016003

- Disconnected at heavier than physical pion mass

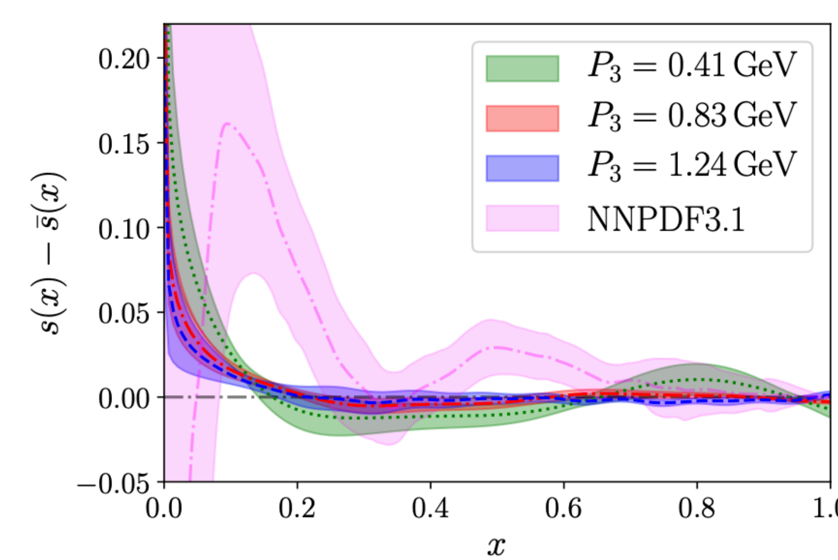
- ETMC: N_f=2+1+1, m_π=260 MeV, a=0.094 fm





C. A., M. Constantinou, K. Hadjiyiannakou, K. Jansen, F. Manigrasso, Phys. Rev. Lett. 126 (2021) 102003

C.A. *et al.*, Phys.Rev.D 104 (2021) 5, 054503



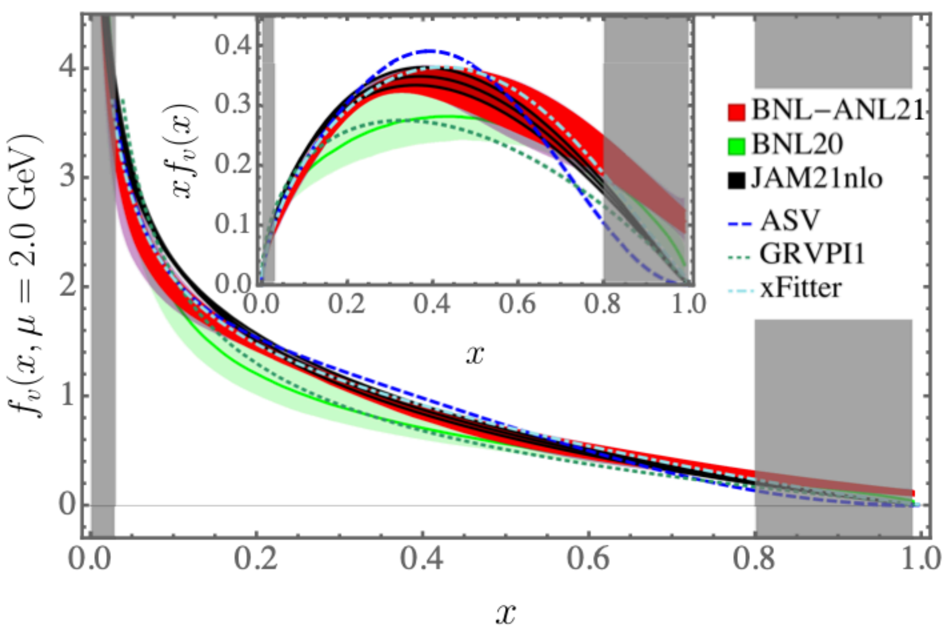
Pion and isovector nucleon unpolarized PDFs

❋ Improvements in renormalization and matching by e.g. LPC, BNL, ETMC, MSULAT, …

- PDFs using NNLO matching and the hybrid renormalization scheme by BNL

Pion

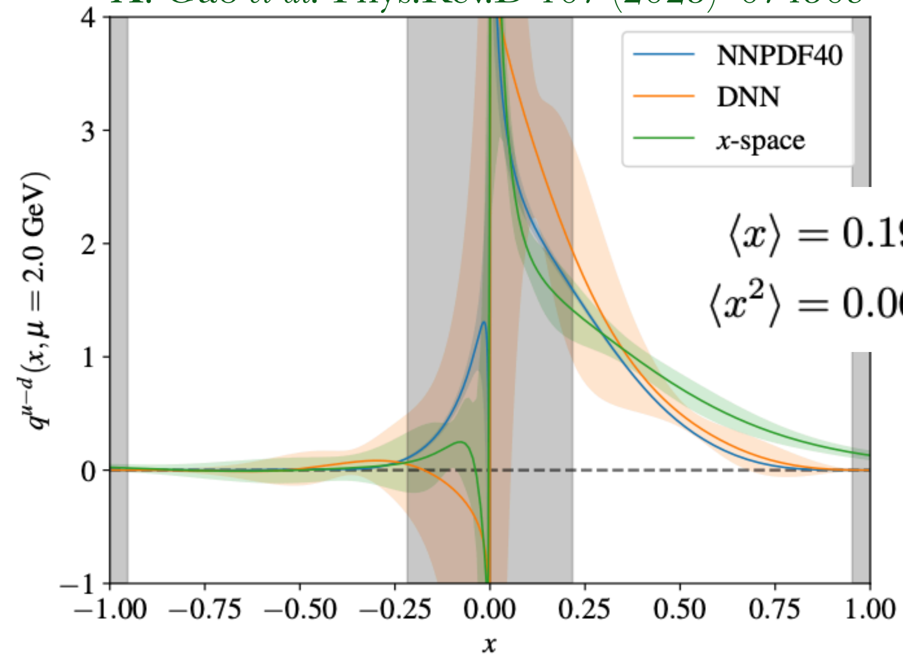
X. Gao *et al.* Phys. Rev. Lett. 128 (2022) 142003



Quasi-approach with NNLO matching using hybrid action: clover on $N_f=2+1$ staggered $m_\pi=300$ MeV, a=0.04 fm

Nucleon isovector

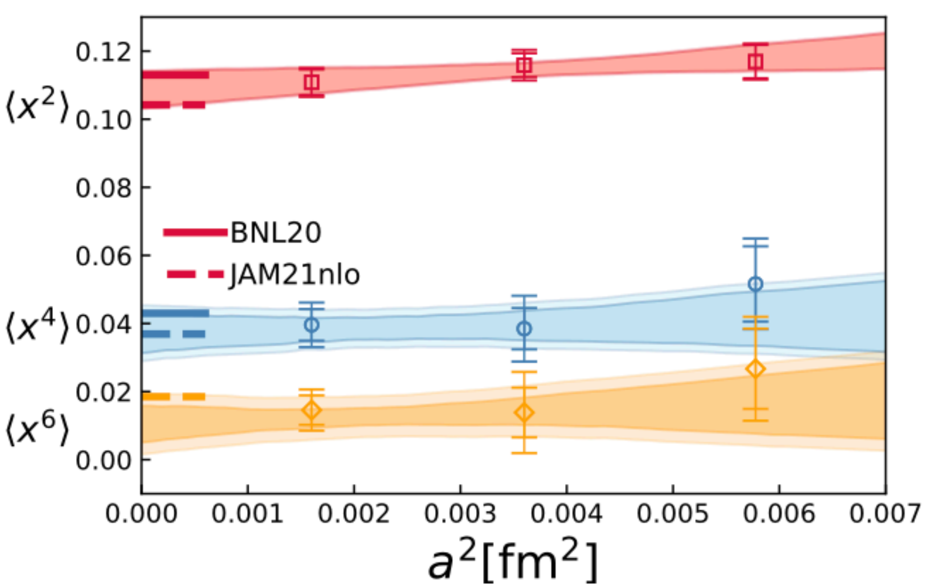
X. Gao *et al.* Phys.Rev.D 107 (2023) 074509

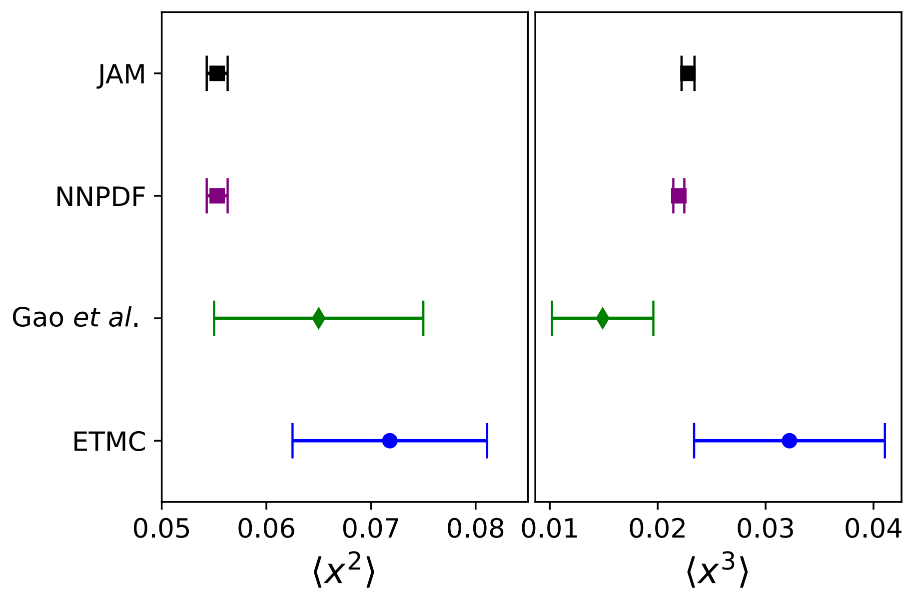


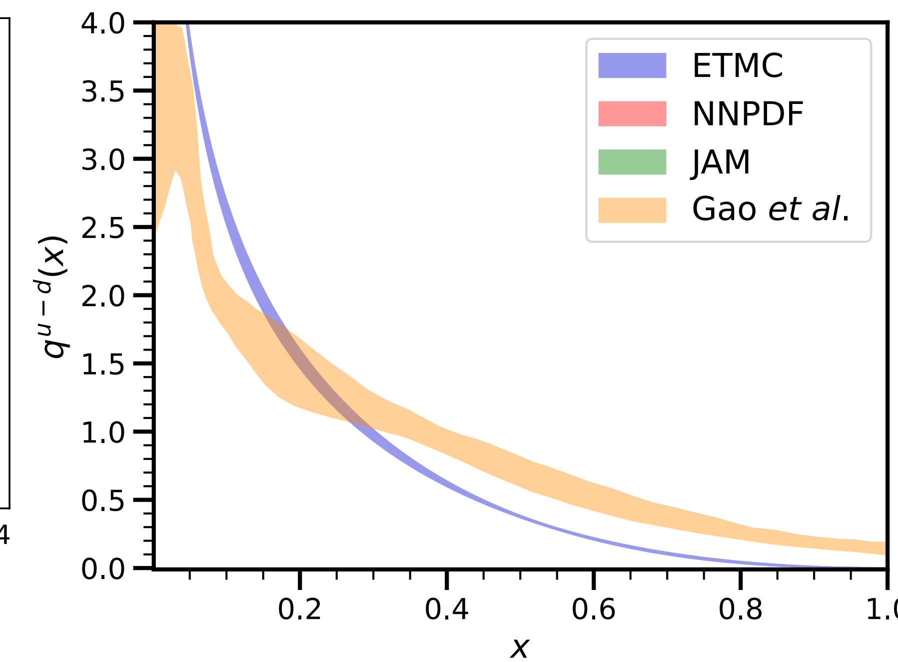
$$\langle x\rangle = 0.191(14),\ \langle x^3\rangle = 0.0149(47),$$
$$\langle x^2\rangle = 0.065(10),\ \langle x^4\rangle = 0.0083(43),$$

$m_\pi=140$ MeV, 64^4, a=0.076 fm in same setup as pion

X. Gao *et al.* Phys.Rev.D 106 (2022) 11, 114510







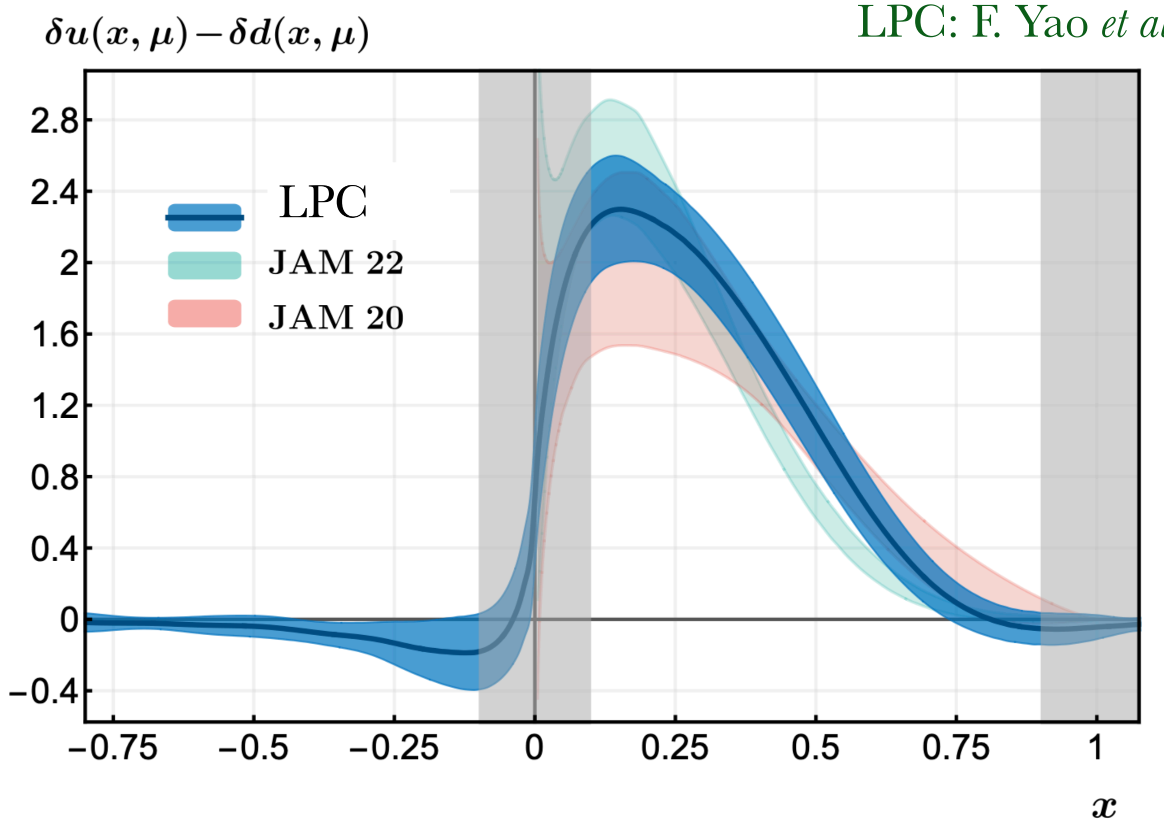
Pseudo-approach to extract moments with $m_\pi=140$ MeV, a=0.076 fm and $m_\pi=300$ MeV, a=0.06, 0.04 fm

Transversity PDF

- Lattice Parton Collaboration (LPC) analyzed CLS ensembles with 4 lattice spacings (0.098-0.049 fm), pion masses ranging from 220 to 350 MeV and momentum boosts from 1.6 GeV to 2.8 GeV
  - Continuum and chiral extrapolations are performed as well as the large momentum limit
  - Renormalization done using a hybrid scheme separating the short and long distances

$$\tilde{h}_R(z,P^z) = \frac{1}{\tilde{h}(0,P^z,1/a)}\left[\frac{\tilde{h}(z,P^z,1/a)}{\tilde{h}(z,0,1/a)}\theta(z_s-|z|) + \frac{Z_R(z_s,1/a)}{Z_R(z,1/a)}\frac{\tilde{h}(z,P^z,1/a)}{\tilde{h}(z_s,0,1/a)}\theta(|z|-z_s)\right]$$

LPC: F. Yao *et al.*, Phys. Rev. Lett. 131 (2023) 26, 261901



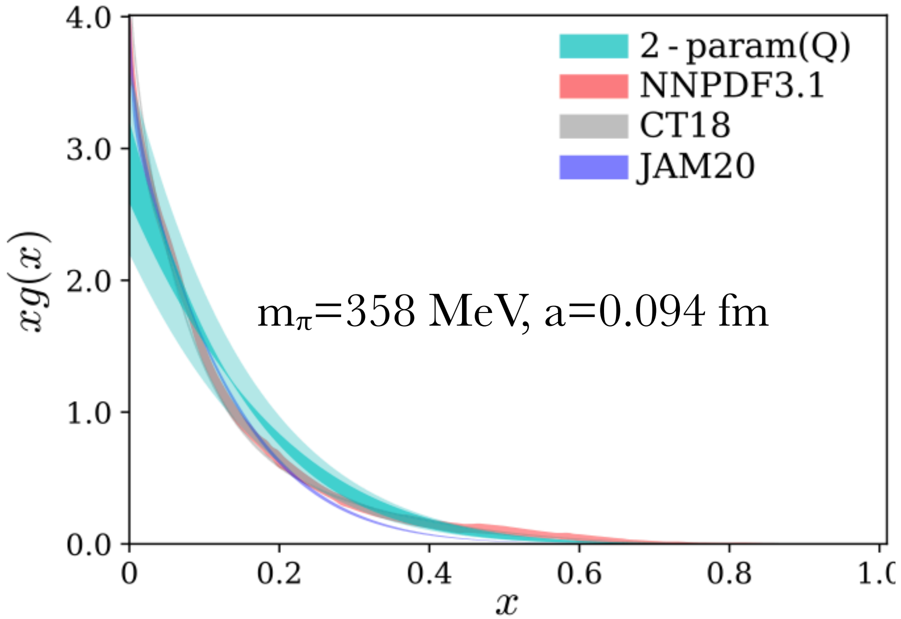
- BNL also computed the transversity isovector and isoscalar PDFs using clover on staggered and one ensemble at physical pion mass for both the quasi- and pseudo-approaches

X. Gao *et al.*, Phys. Rev. D 109 (2025) 054506

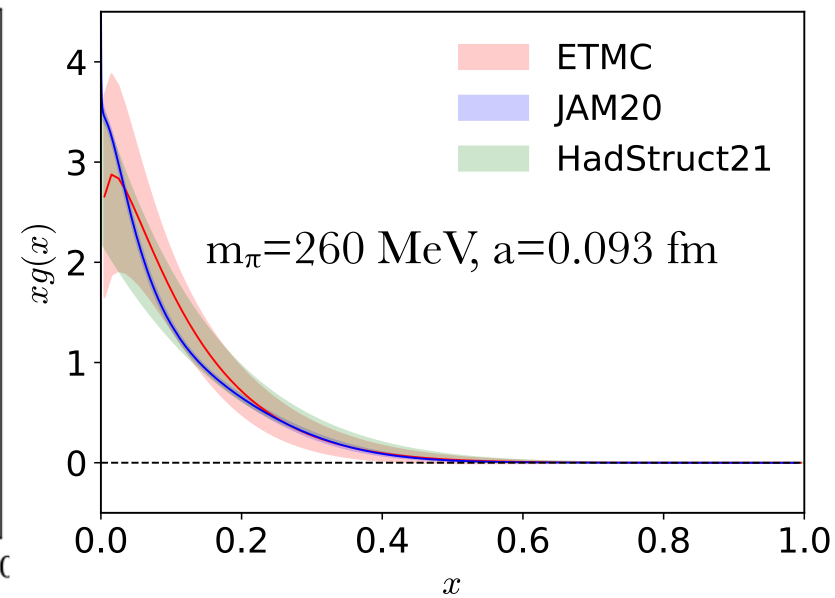
Gluon PDFs

❋ Most studies in the pseudo-pdf approach

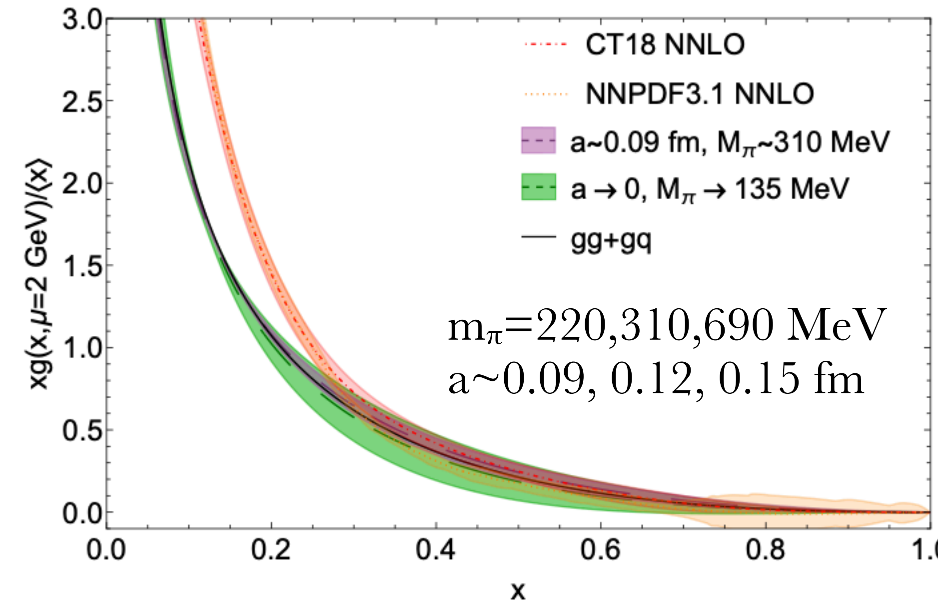
- Nucleon unpolarized PDF



HadStruc Collaboration: T. Khan *et al.*, Phys. Rev. D 104, (2021) 094516

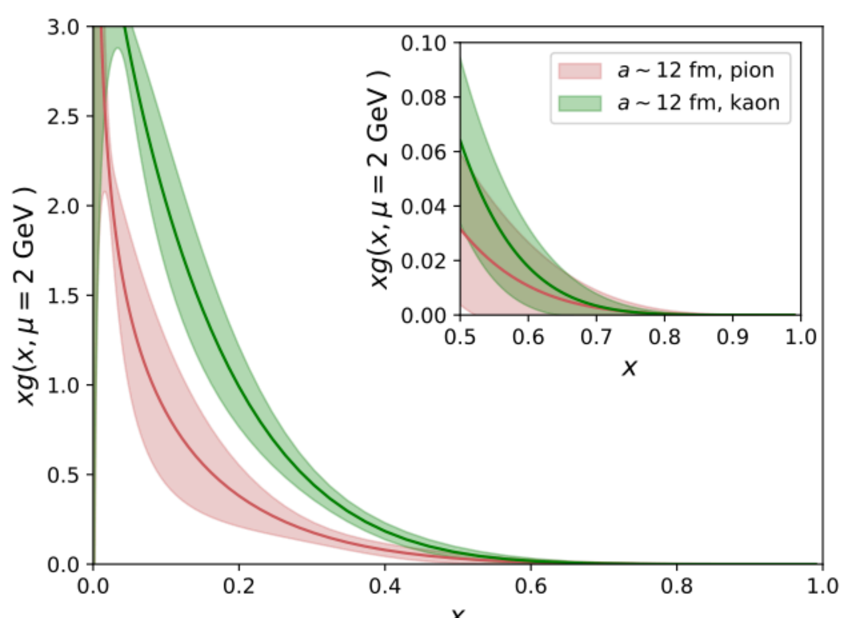


ETMC: J. Delmar *et al.*, Phys. Rev. D 108 (2023) 094515



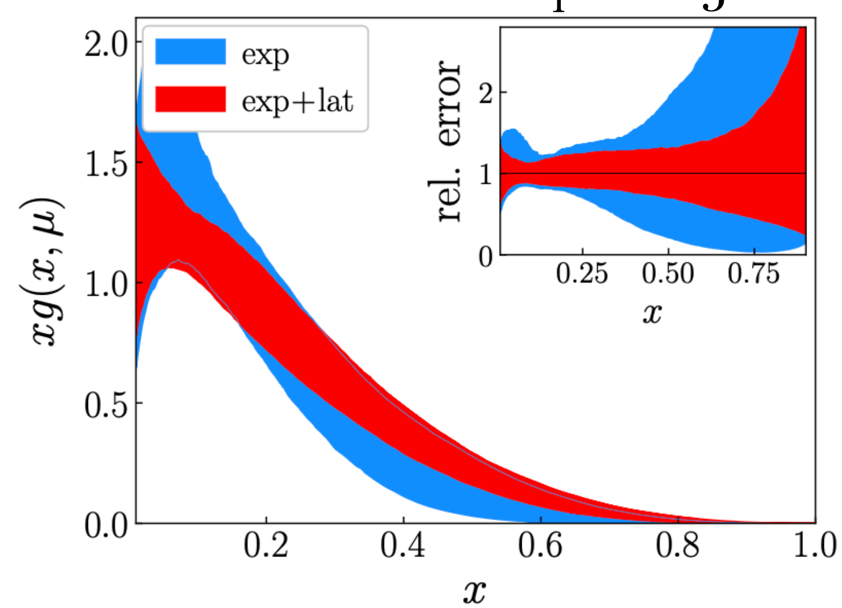
MSULAT: Z. Fan, W. Good, H.-W. Lin, Phys. Rev. D 108 (2023) 014508

- Pion and kaon gluon PDFs by MSULAT



A. NieMiera, W. Good, H.-W. Lin, Phys.Rev.D 112 (2025) 074504

MSULAT data on pion + JAM

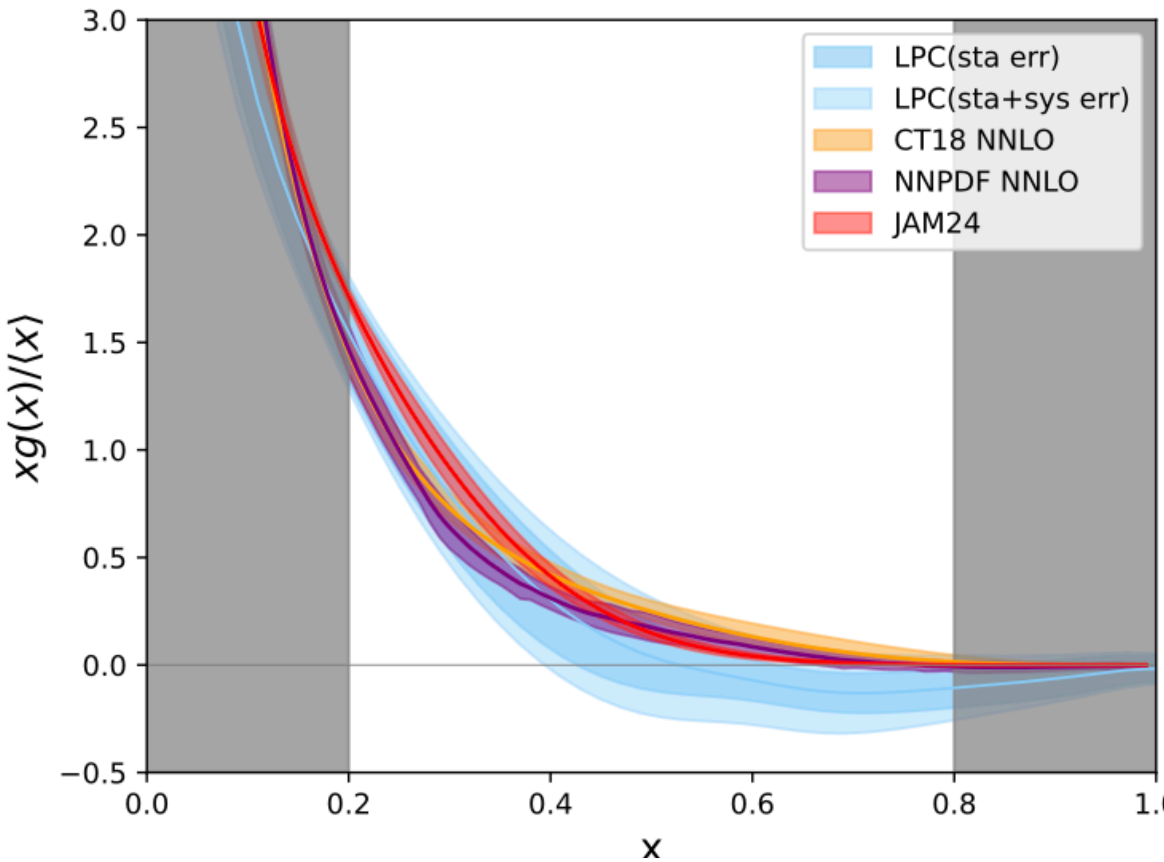


W. Good *et al.* arXiv: 2507.22730

Unpolarized gluon PDF

- LPC and MSULAT used the quasi-distribution approach at 3 lattice spacings and $m_\pi \sim 300$ MeV
- Continuum momentum limit is performed
- Renormalization done using a hybrid scheme separating the short and long distances

$$\tilde{h}_R(z, P^z) = \frac{1}{\tilde{h}(0, P^z, 1/a)} \left[\frac{\tilde{h}(z, P^z, 1/a)}{\tilde{h}(z, 0, 1/a)} \theta(z_s - |z|) + \frac{Z_R(z_s, 1/a)}{Z_R(z, 1/a)} \frac{\tilde{h}(z, P^z, 1/a)}{\tilde{h}(z_s, 0, 1/a)} \theta(|z| - z_s) \right]$$



LPC: Ch. Chen *et al.*, arXiv:2510.26425

- Similar results by MSULAT A. NieMiera , W. Good, H.-W. Lin, F. Yao, arXiv:2510.17758

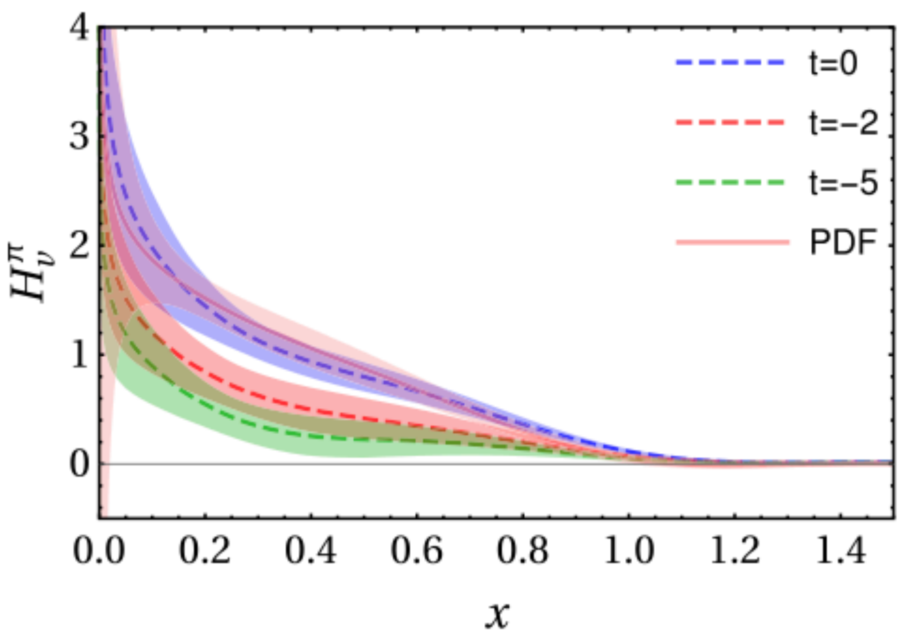
Pion and nucleon GPDs

❋Quasi-approach: Compute space-like matrix element with different initial and final nucleon boosts e.g. in the Breit frame

$$h_\Gamma(z,\tilde{\xi},Q^2,P_3) = \langle N(P_3\hat{e}_z+\vec{Q}/2)|\,\overline{\psi}(z)\,\Gamma W(z,0)\,\psi(0)|\,N(P_3\hat{e}_z-\vec{Q}/2)\rangle$$

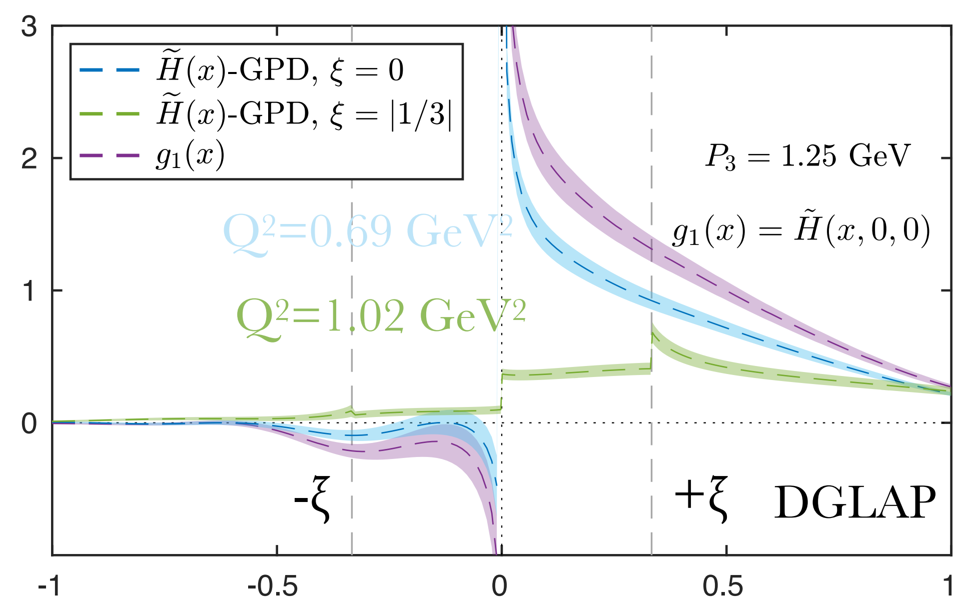
$$\tilde{\xi} = -\frac{Q_3}{2P_3}\ :\ \text{quasi}-\text{skewness} \quad \tilde{\xi} = \xi + \mathcal{O}(\frac{1}{P_3^2})$$

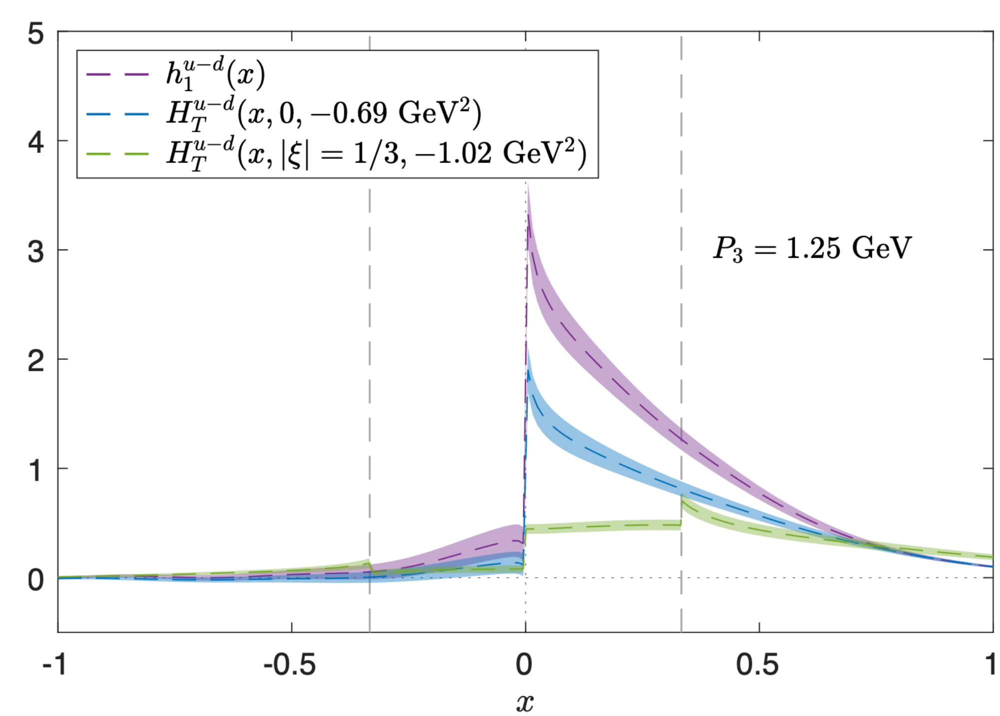
- Pion GPD at ξ=0 and m_π=300 MeV hybrid action



J.W. Chen, H.W. Lin, J.H. Zhang, Nucl. Phys. B 952, 114940 (2020), 1904.12376

- Isovector nucleon GPDs, m_π=260 MeV





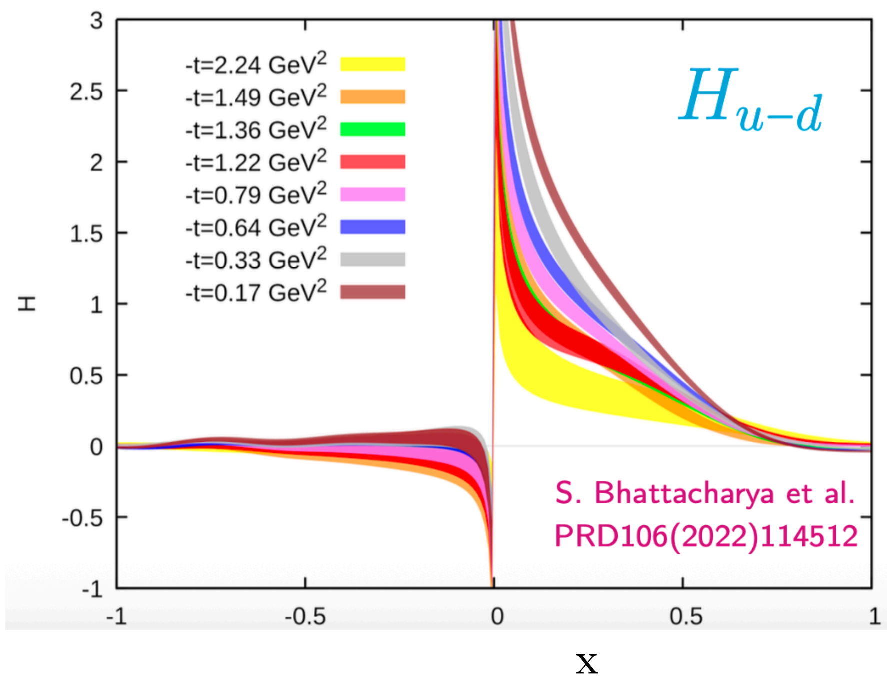
—> GPDs flatten as Q^2 increases

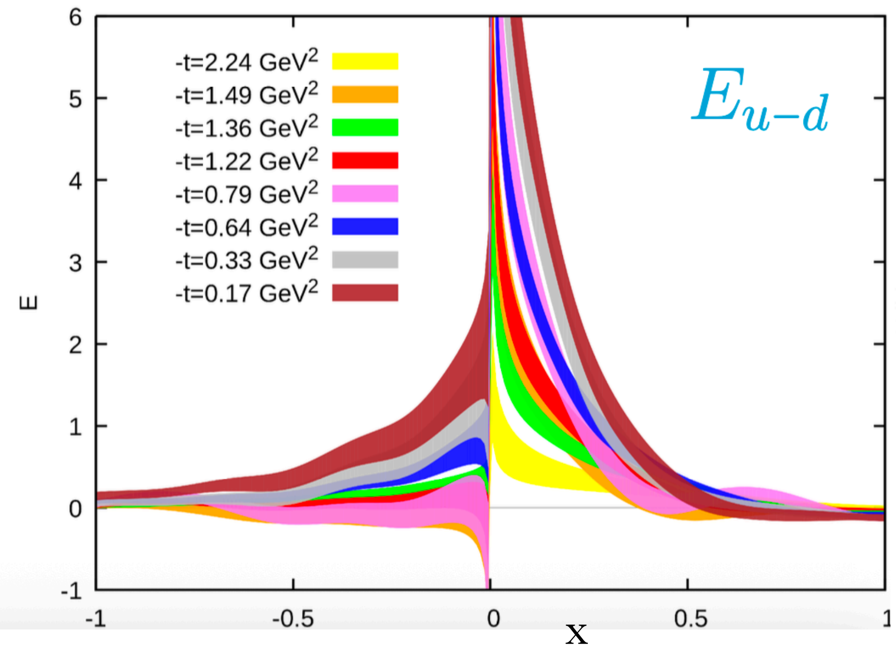
C. A. *et al.* (ETMC), Phys. Rev. Lett. 125 (2020) 262001, 2008.10573
C.A. *et al.*(ETMC), Phys. Rev. D 105 (2022) 3, 034501, 2108.10789

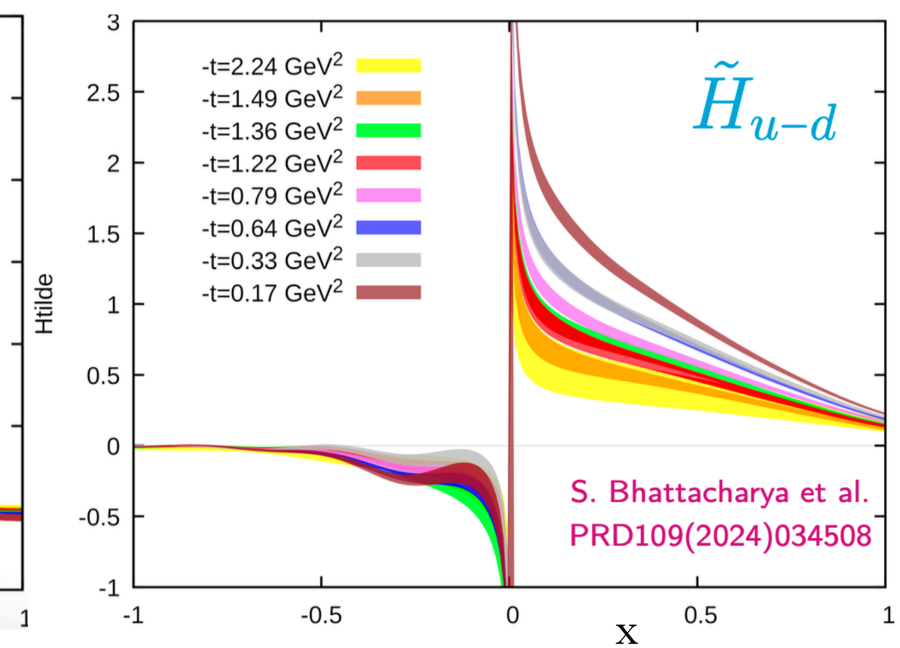
Asymmetric frame

- Express GPDs in terms of Lorentz invariant amplitudes that can be computed in any frame allowing easier access to a range of Q^2 similar to FFs

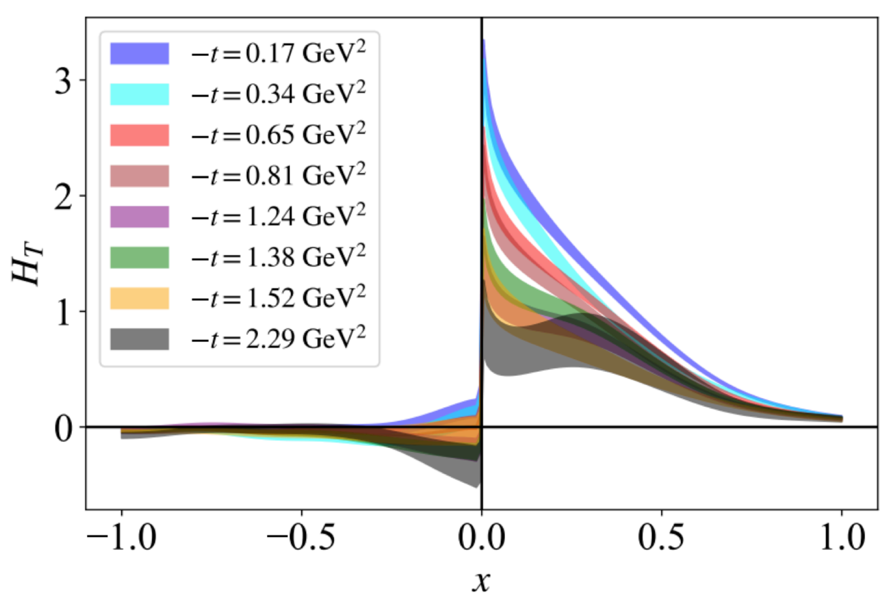
S. Bhattacharya *et al.* Phys. Rev. D 106 (2022) 114512; Phys. Rev. D 109 (2024) 034508; Phys. Rev. D 112 (2025) 11, 114504

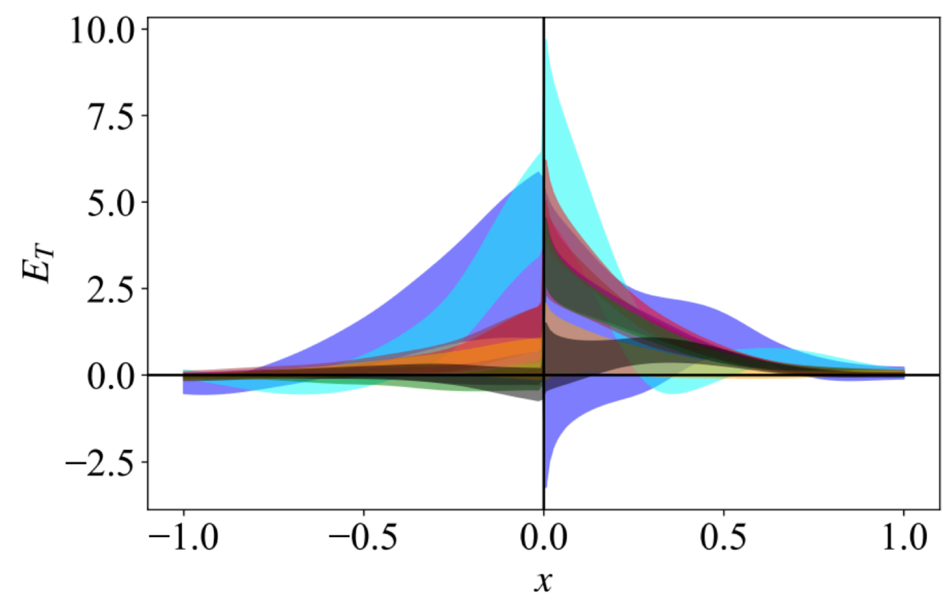






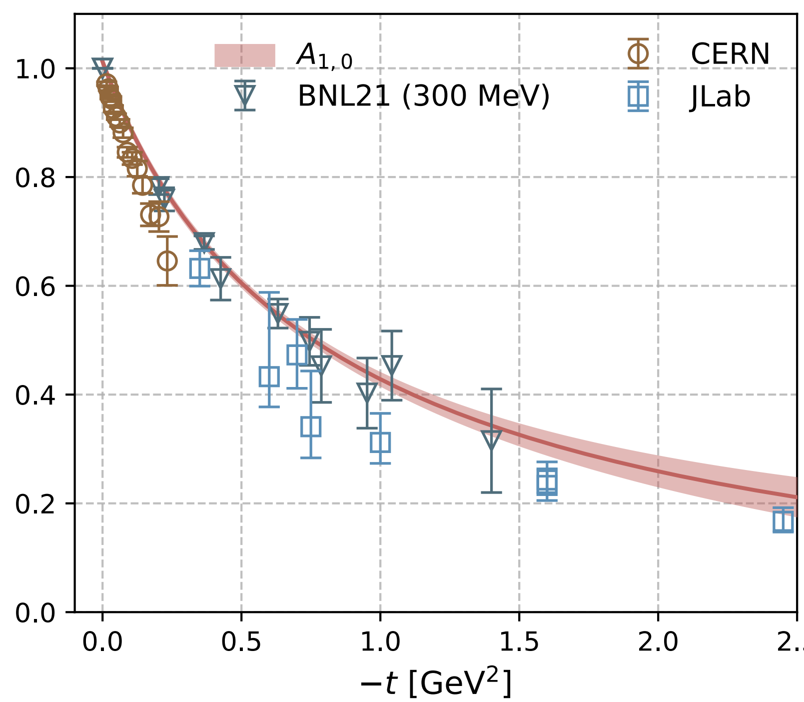
K. Cichy, EINN 2025

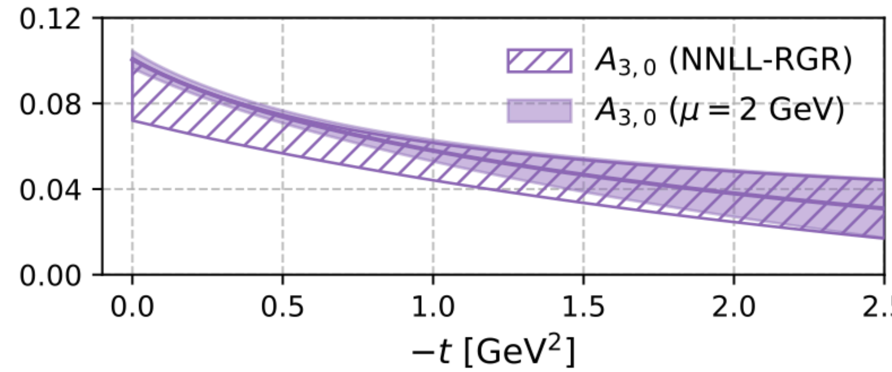


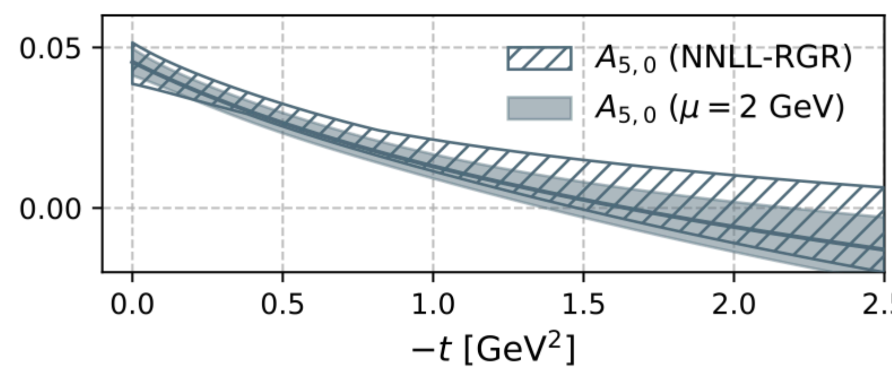


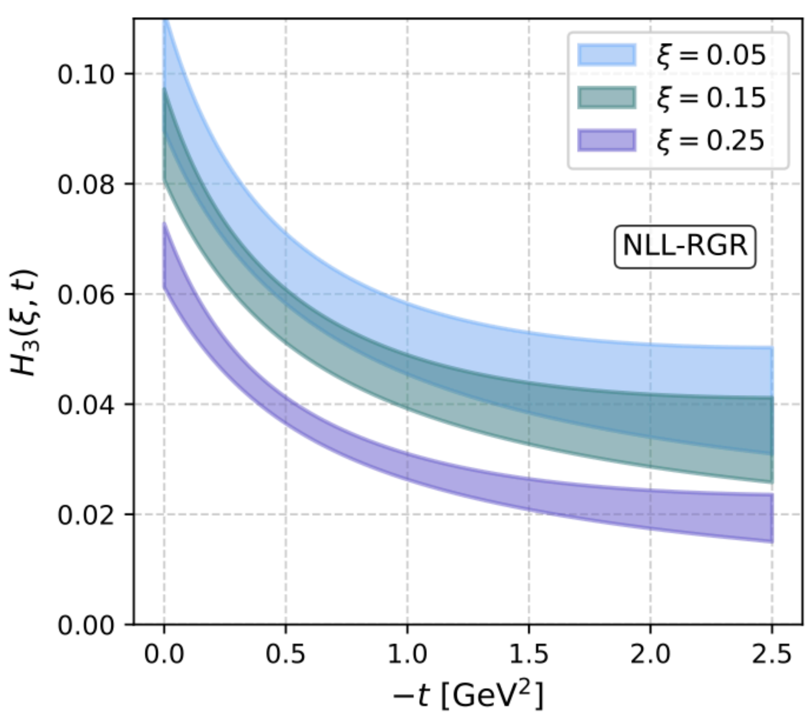
Pion unpolarized GFFs

- An N_f=2+1 ensemble of staggered fermions at a=0.04 fm 64^4 and clover valence at m_π=300 MeV
- Computation is done using the gauge invariant decomposition and ratio method to renormalize
- Renormalization with ratio and short distance expansion to compute moments for zero and non-zero skewness









n^{th} moment

$$H_n(\xi,t)=\sum_{k=0,2,..}^{n-1} A_{n,k}(t)(2\xi)^k \pm \mathrm{mod}(n-1,2)(2\xi)^2 C_n(t)$$

GPD Mellin moments decrease as ξ increases

X. Gao, S. Mukherjee, Qi Shi, Fei Yao and Yong Zhao, arXiv:2511.01818

Conclusions

❋ **EIC will bring a wealth of data on pion, kaon (also AMBER@CERN) and proton structure:**

1. Measurement of form factors to large momentum transfer

2. Study the low x-region of PDFs and GPDs improving our knowledge of sea quark and gluon distributions
   —> spin structure of proton, mass generation, mechanical properties

   —> 3D imaging of pion, kaon and proton

3. Measure helicity and transversity TMD PDFs such as the Sivers structure and the Boer-Mulders functions and higher twist PDFs probing multi-particle correlations

❋ **Lattice QCD can greatly impact the scientific program of EIC using complementary approaches:**

1. Precise computations of charges, form factors and Mellin moments of PDFs and GPDs taking into account all major lattice systematics

   As 1% is reached in some of these quantities one needs to include isospin breaking

   —>New era of QCD+QED

2. Direct computations of PDFs, GPDs and TMDs including twist-3 with better matching and renormalisation procedures

Thanks to ETMC members for their crucial contributions

Extended Twisted Mass Collaboration

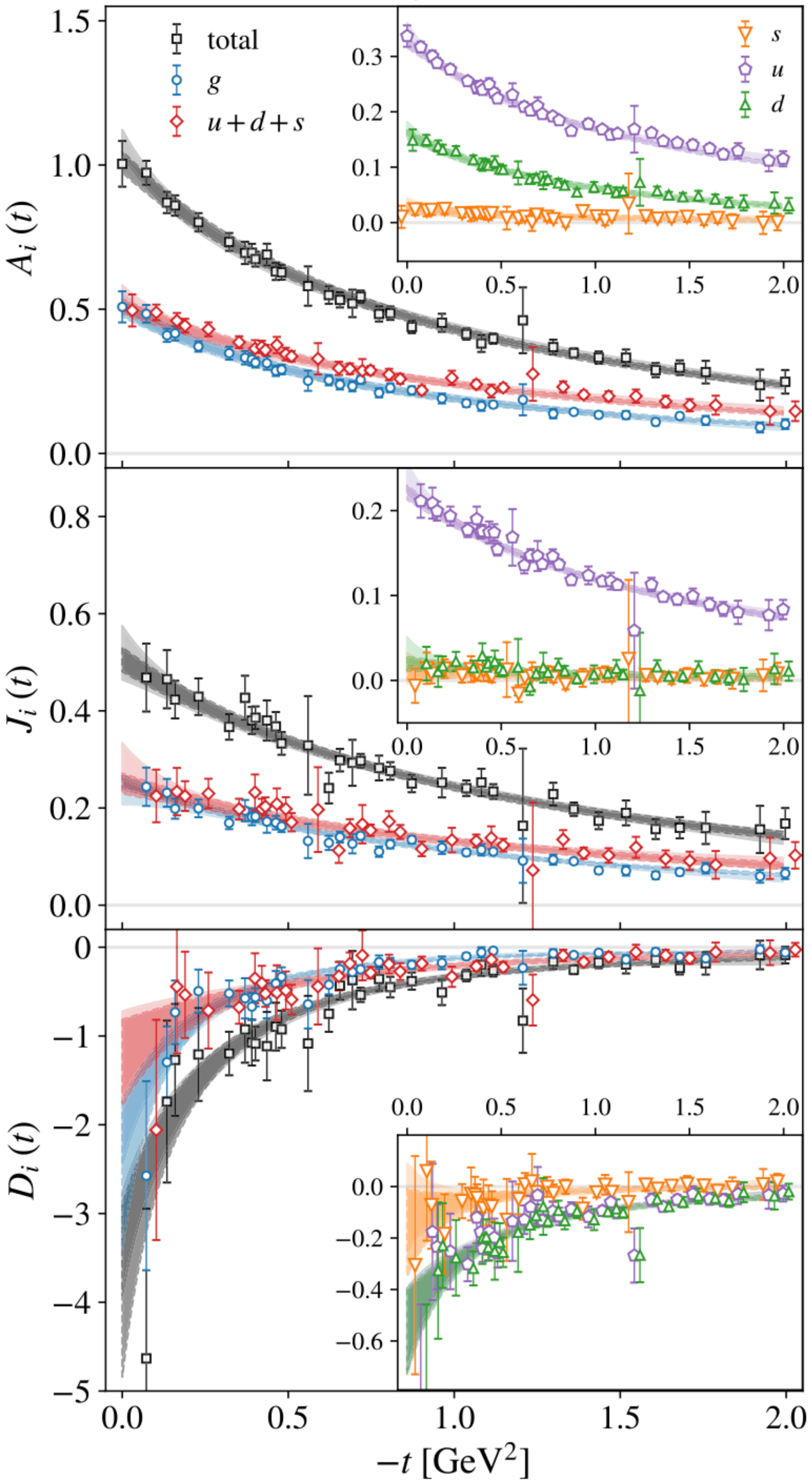
Backup slides

Generalised form factors by MIT group

$N_f = 2+1$, Clover fermions : $48^3 \times 96, a = 0.091(1)$fm

Nucleon at m_π~170 MeV

($\overline{\mathrm{MS}}$, $\mu = 2$ GeV)



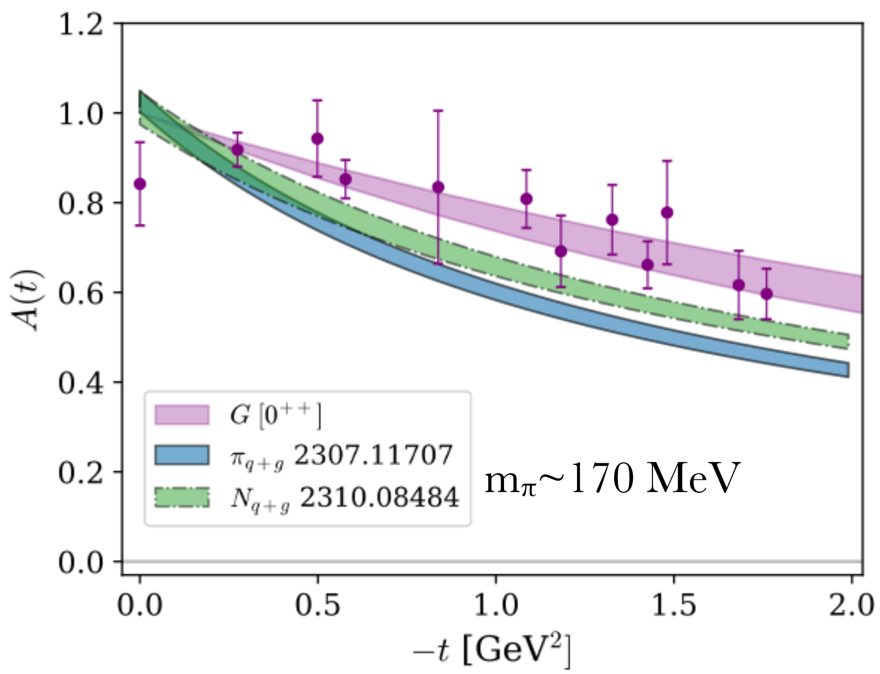
Momentum + spin sums fulfilled

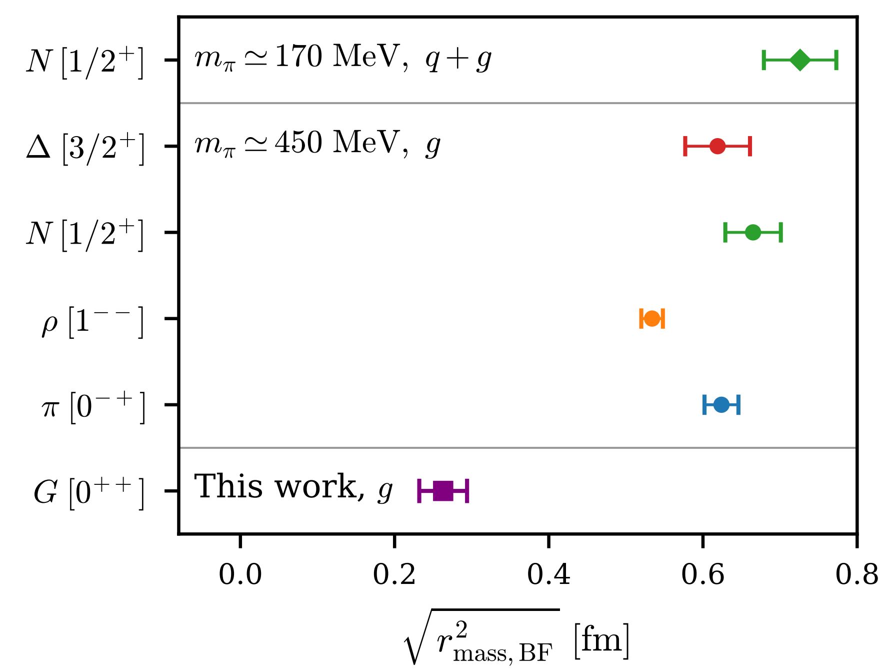
D. C. Hackett, D. A. Pefkou and P. E. Shanahan, Phys. Rev. Lett 132 (2024) 251904

Scalar glueball

Compute root mean square radius of the energy density of the glueball in the Breit frame

$$r^2_{\mathrm{mass,BF}} = \frac{1}{A(0)}\left[6\frac{dA(t)}{dt}|_{t=0} - \frac{3}{4m^2}(A(0)+2D(0))\right]$$





Glueball has a very small mass radius

R. Abbot *et al.*, arXiv:2508.21821

Nucleon higher Mellin moments

- Third Mellin moments of the twist-2 and twist-3 helicity structure functions $g_1(x)$ and $g_2(x)$
- The twist-3 d_2 is connected to the Sivers function for $b_\perp \to 0$ and probes quark-gluon correlations
- First computed by QCDSF QCDSF: M.Goeckeler *et al.*, Phys. Rev. D 72, (2005) 054507

- Recent calculation by RQCD using clover fermions with six different spacings from 0.039 fm to 0.098 fm and 220 < m_π <420 MeV to perform chiral and continuum extrapolation

RQCD: M.Buerger *et al.*, Phys.Rev.D 105 (2022) 054504

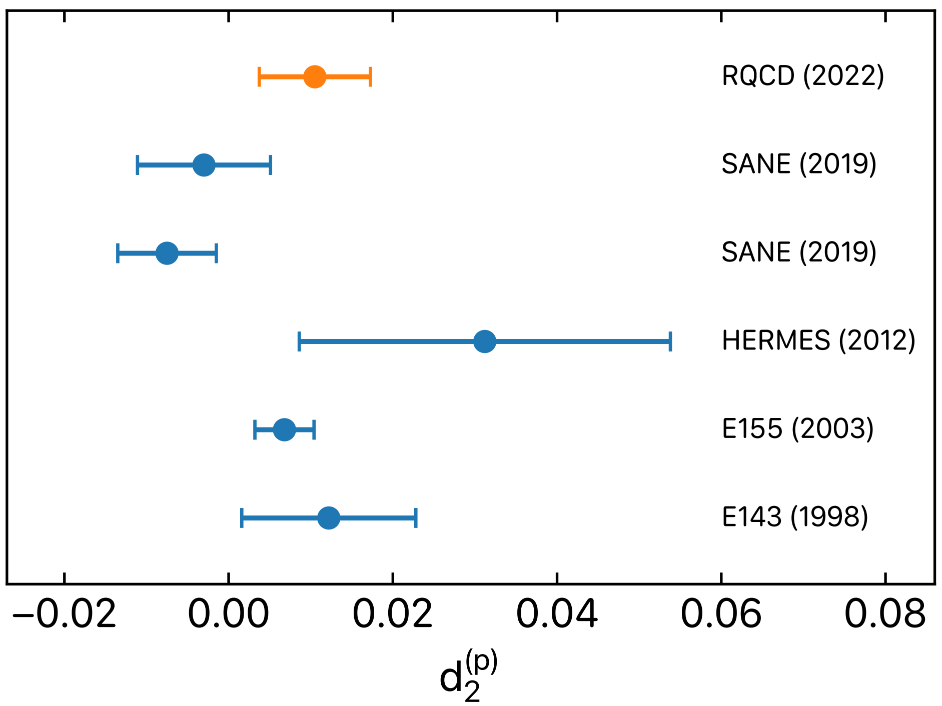
$$d_2(\mu) = 4\int_0^1 dx x^2 \left[g_1(x,Q^2) + \frac{3}{2} g_2(x,Q^)\right]$$

$\mu^2 = Q^2 = 4$ GeV^2

$$d_2^p = 0.0105(19)(65),\ d_2^n = -0.0009(14)(69)$$

Helicity third moment in $\overline{\text{MS}}$

$$< x^2 >_{\Delta p} = 0.035(3)(8),\ < x^2 >_{\Delta n} = 0.0034(17)(41)$$



- Third Mellin moments of unpolarized, helicity and transversity