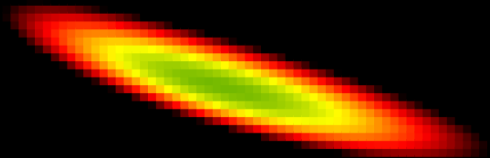
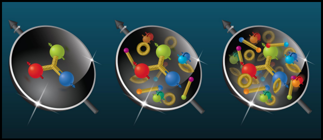
Pion TMDPDFs from phenomenology

Valentin Moos with Wen-Chen Chang, Chia-Yu Hsieh, Chung-Wen Kao, C.-J. David Lin and Wayne Morris

EIC-Asia Workshop on QCD and Hadron Structure 2026, Academia Sinica, Taipei





What is this talk about

- **WHAT** do we study: TMDPDFs:
  (naively) an extension of (collinear) **P**arton**D**istribution**F**unction that captures the total momentum distribution of the hadron among its constituents (partons).
  TMDs are **T**ransverse **M**omentum **D**ependent PDFs.

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- **WHAT** do we study: TMDPDFs:
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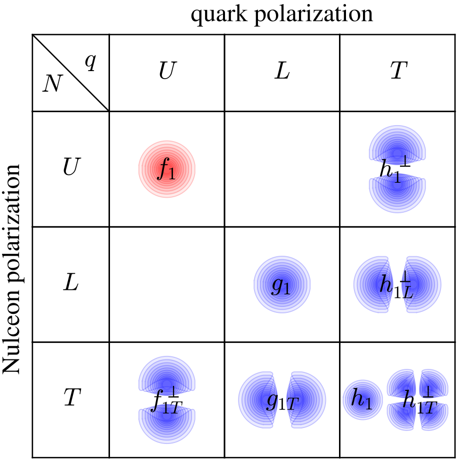
  TMDs are **T**ransverse **M**omentum **D**ependent PDFs.

- **HOW** do we access them:
  TMDs can describe the cross section of hadronic scattering events
  available data are from Drell-Yan measurements
  $\longrightarrow$ Method = parametrise and fit.

8 TMD distributions



Parametrised forms of TMDs include 8 functions.

Polarisation of quark
~ **internal** spinor operator (Γ)

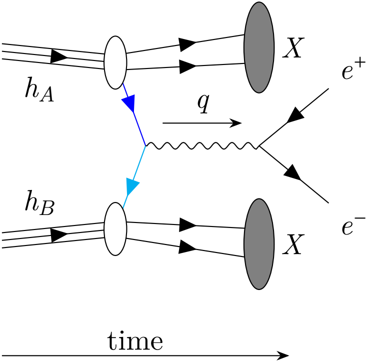
Posalisation of hadron
~ **exterior** state of nucleon

Our microscope: The Drell Yan process

A *clean* process:

$$h_1 + h_2 \longrightarrow l + \bar{l} + X$$

Two colliding hadrons → two parton distribution functions necessary.



- momentum of lepton pair q
  = momentum of virtual boson
  = momentum of colliding quarks
- only PDFs involved
- small transverse momentum q_T of photon ~ notion of small transverse motion of quarks

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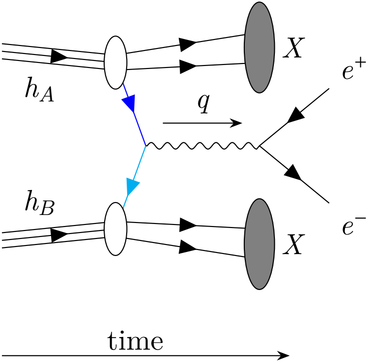


Kinematics:

$$q = k + k' \qquad Q^2 = q^2$$

$$x_{1,2} = \frac{q^\pm}{p^\pm_{1,2}} \qquad 0 < q_T = |q \perp \langle p_1, p_2 \rangle|$$

$$x_F = x_1 - x_2 \qquad y = \frac{1}{2} \ln\left(\frac{x_1}{x_2}\right)$$

TMD cross section

...on the example of γ^* production in DY

$$\frac{d\sigma}{dQ^2\,dy\,dq_T^2} = \frac{2\pi\alpha_{\rm em}^2(Q)}{3N_c s Q^2}\left(1+\frac{q_T^2}{2Q^2}\right)\sum_f C_{\rm DY}(Q)$$
$$\times \int_0^\infty db\, b\, J_0(bq_T)\left(\frac{Q^2}{\zeta_Q(b)}\right)^{-2\mathcal{D}(b,Q)} f_{1,f}(x_1,b) f_{1,\bar{f}}(x_2,b)$$

Schematic:

$$\sigma = H_{\rm pert.} \otimes F_1^{\rm NP} \otimes F_2^{\rm NP}$$

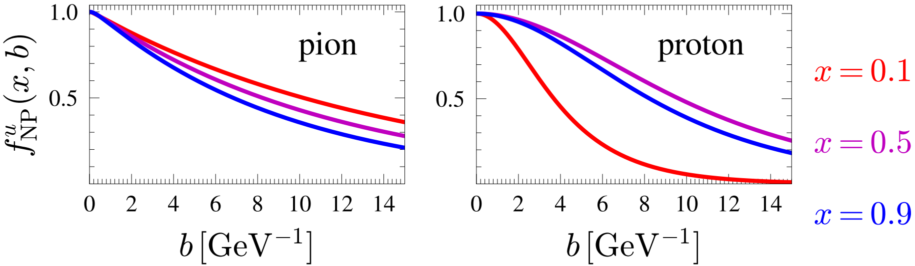
Scale dependence of TMDs – both pert. and NP.

How do we "construct" the TMDPDFs:

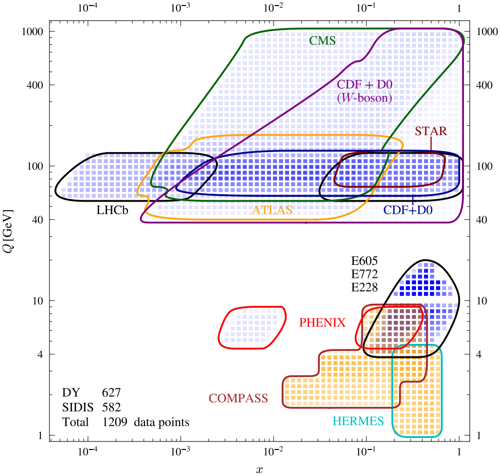
$$f_{1,f}(x,b) = \left(C_{f\to f'} \otimes q_{f'}\right)(x,b) \cdot f_{\rm NP}^{f}(x,b)$$

⟶ use perturbative expanision in collinear PDFs
multiplied by a NP parametrisation for large distance decay

$$f_{\rm NP}^{f,\pi}(x,b) = \exp\left\{-\left(\lambda_1(1-x)+\lambda_2 x\right)\left(\sqrt{1+\frac{b^2}{x\lambda_3}}-1\right)\right\}$$



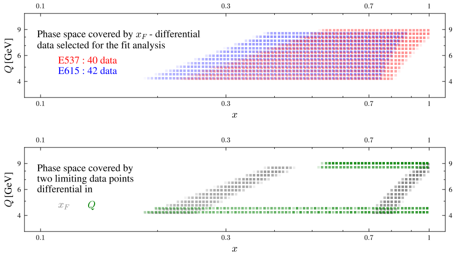
Kinematic range of included data that constrains the nucleon...



Features:

- large range of resolution scale: 1 GeV → 1 TeV
- *W* production in DY
- SIDIS (flavour sensitive!) and DY data
- $\frac{q_T}{Q} < 0.25$ (TMD region!)

… and the pion



Features:

- small range of resolution scale: 4 GeV → 8 GeV
- small range of momentum fraction $0.18 < x_\pi < 1.0$
- basically only valence quark: $\overline{u} + u$ contribution dominates $\pi^- + p/n$
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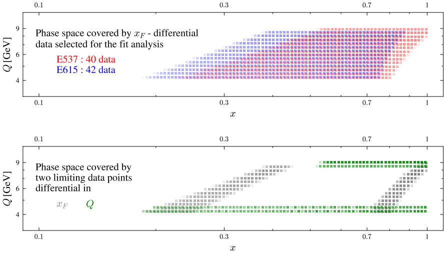

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- **LARGE normalisation uncertainty**

TMD cross section

...on the example of γ^* production in DY

$$\frac{d\sigma}{dQ^2\,dy\,dq_T^2} = \frac{2\pi\alpha_{\rm em}^2(Q)}{3N_c\,s\,Q^2}\left(1+\frac{q_T^2}{2Q^2}\right)\sum_f C_{\rm DY}(Q)$$

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Schematic:

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Scale dependence of TMDs – both pert. and NP.

definition of the test function – what to minimise

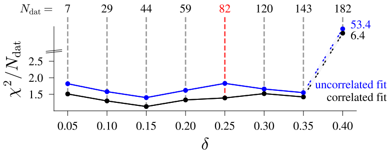
χ^2-testfunction:

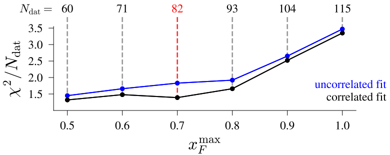
$$\chi^2 = (t - d)^T C^{-1} (t - d)$$

Covariance matrix

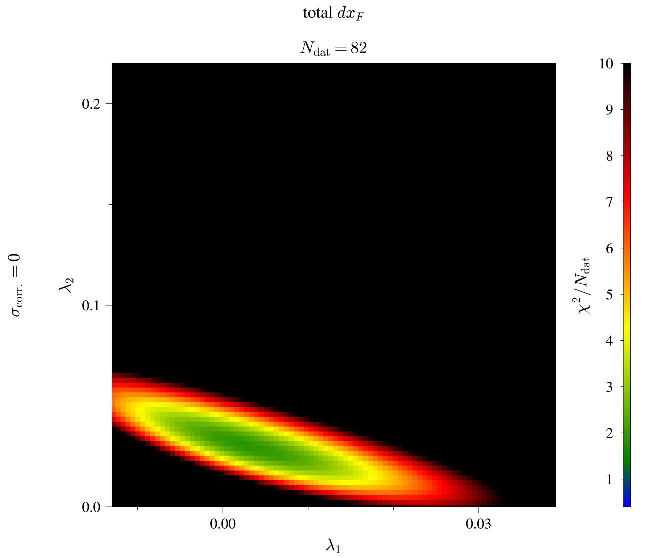
$$C_{ij} = \delta_{ij}\sigma^2_{\mathrm{uncorr},i} + \sigma_{\mathrm{corr},i}\sigma_{\mathrm{corr},j}$$

Our kinematic cuts on the data ($\delta = q_T/Q$)

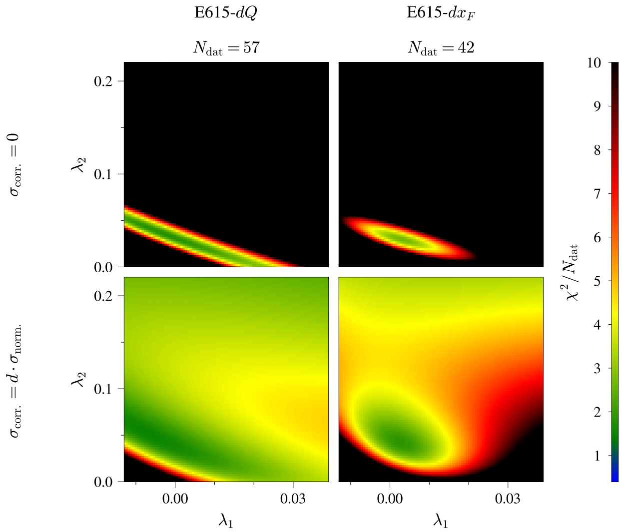




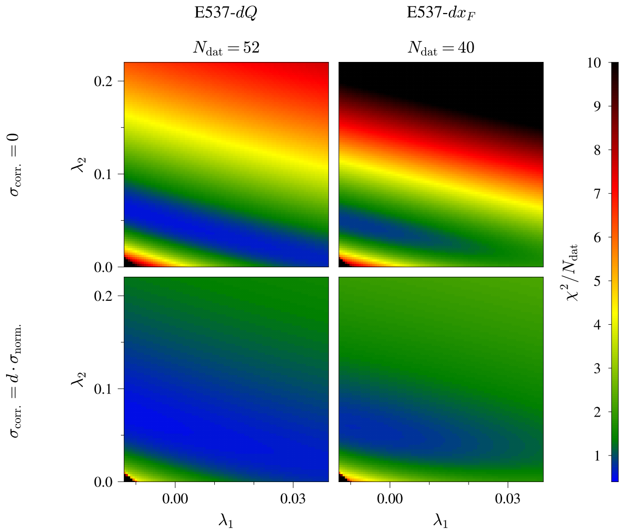
Example for parameter space χ^2 scan



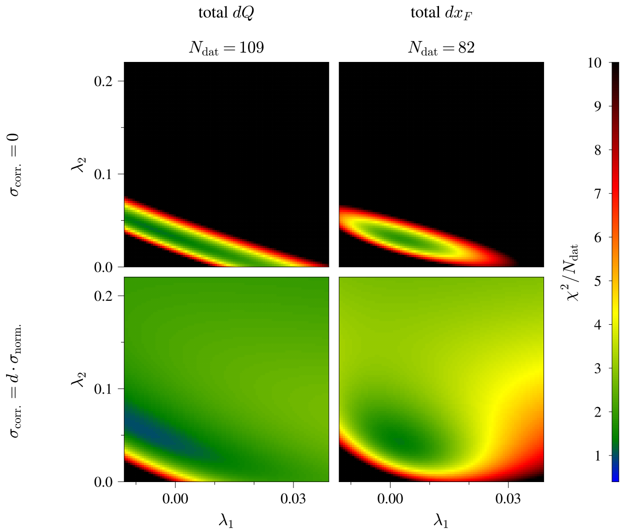
E615: small σ_{stat} & $\sigma_{\text{norm}} = 16\%$



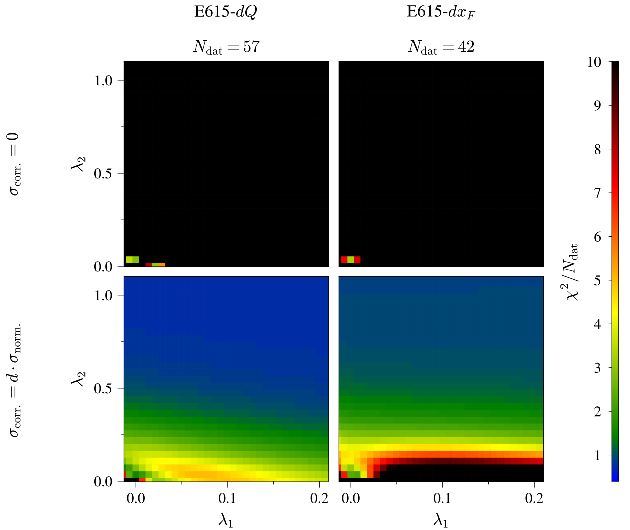
E537: large(!) σ_{stat} & $\sigma_{\text{norm}} = 8\%$



E537 and E615 combined



E615: small σ_{stat} & $\sigma_{\text{norm}} = 16\%$ – large parameter range



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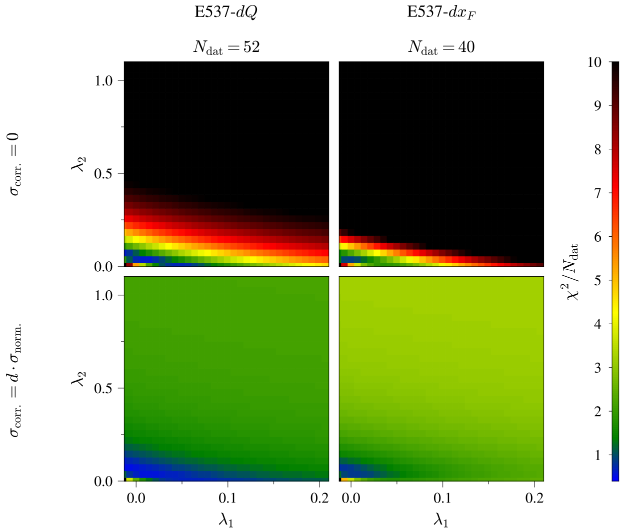

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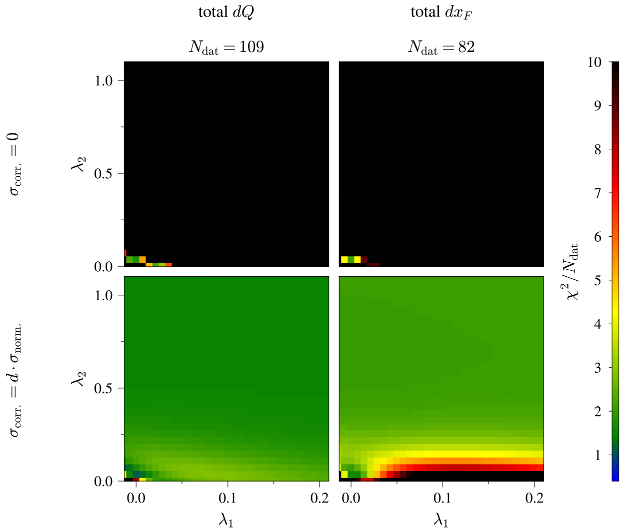
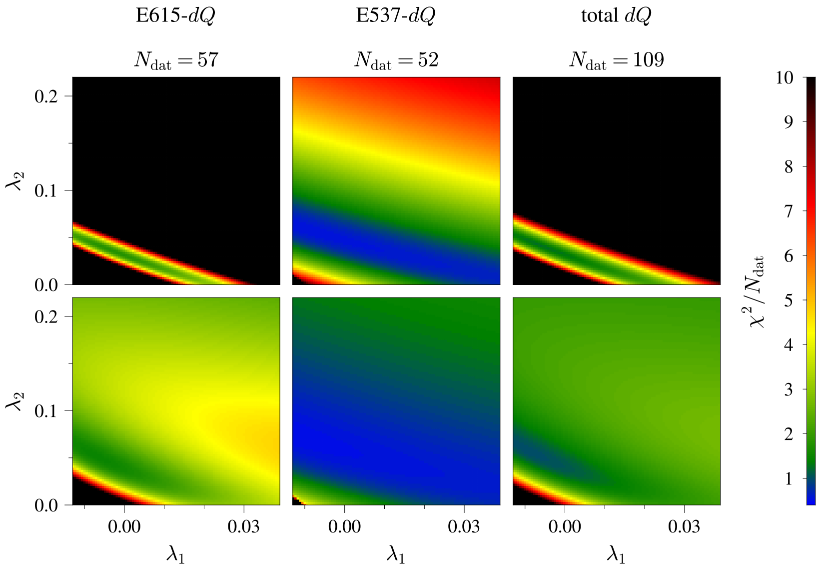
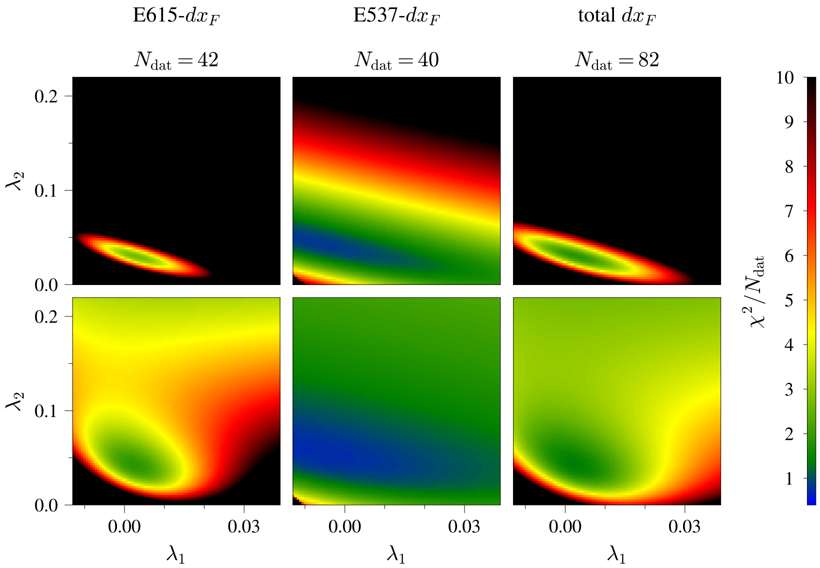


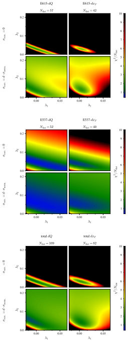
Predictive power of data: Q binning



Predictive power of data: x_F binning



Summary: problematic data



- parameters anticorrelated (expected)
- $\overline{u}$ contribution dominates $\pi^- + p/n$ (no flav. dep.)
- E537 very large stat. uncertainties (small χ^2 and weak predictive power)
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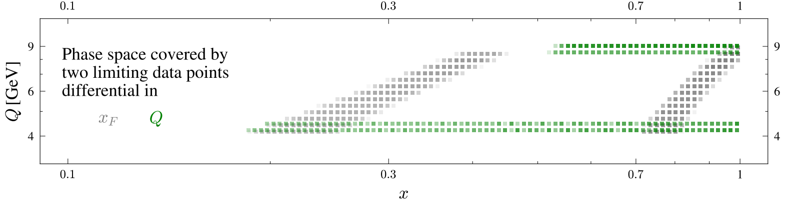

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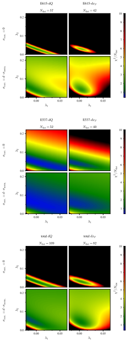

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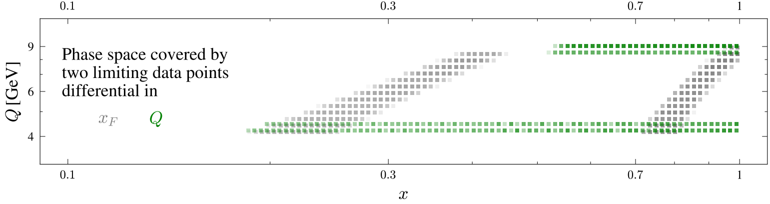

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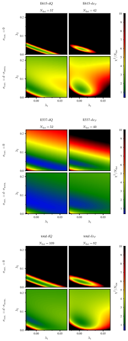

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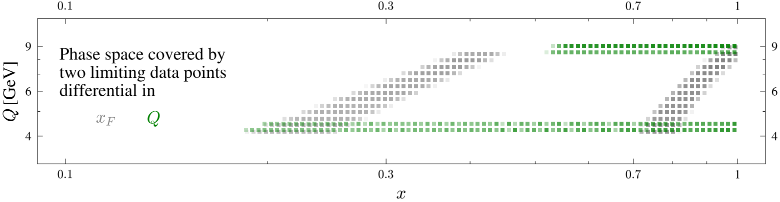
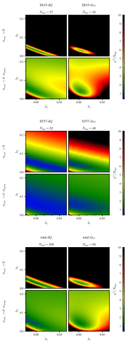
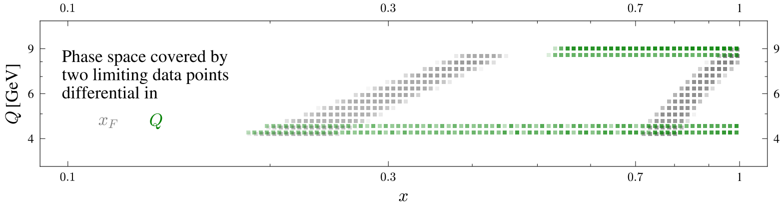
- x_F binning: local minimum (Good!)



Consequences for our analysis:



- use x_F dependent data as better constraint of x dependence in TMDPDFs from both experiments



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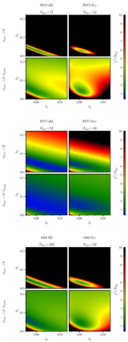

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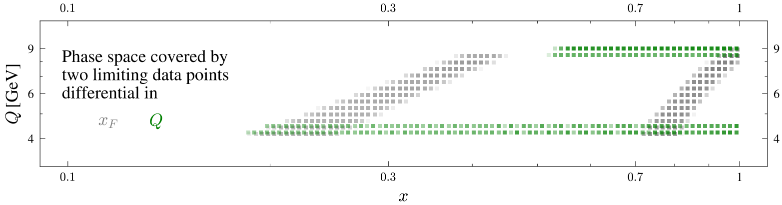

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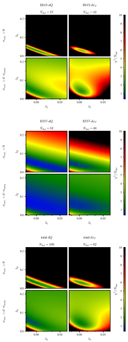
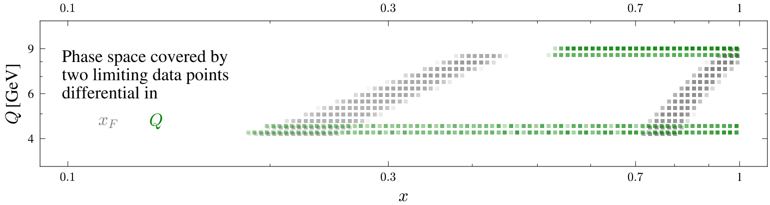
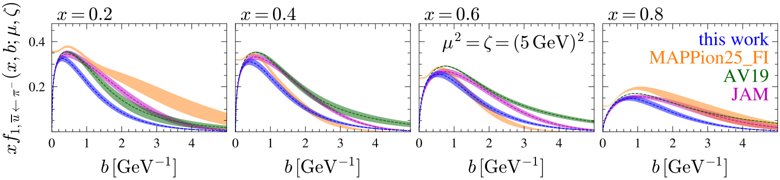


- use x_F dependent data as better constraint of x dependence in TMDPDFs from both experiments
- fit 1: disregard correlated uncertainties, then fit 2: based on fit 1, find **local** correlated χ^2 minimum
- sensible & good χ^2 results

data	N_{dat}	χ^2_D/N_{dat}	$\chi^2_\lambda/N_{\text{dat}}$	χ^2/N_{dat}	sys. shift
E537-dQ	57	0.572	0.001	0.573	1.0%
E615-dQ	52	1.789	0.015	1.804	14.0%
E537-dx_F	40	0.764	0.029	0.793	8.6%
E615-dx_F	42	1.920	0.012	1.932	11.3%



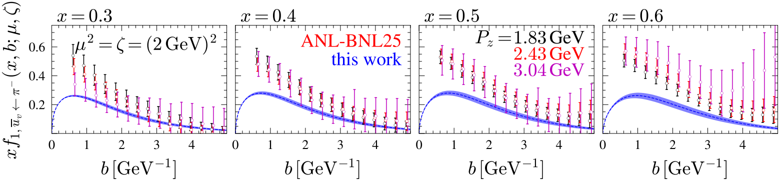
pion TMDPDFs
valence flavour ($\overline{u}$) compared with other extractions



differences in

- proton TMDPDFs, CS-kernel (rapidity scale evolution)
- kinematic binning of data (Q or x_F)
- fit strategy and (resulting) normalisation

pion TMDPDFs
valence combination $(\overline{u} - u)$ compared with LaMET



- Large Momentum Effective Theory needs to extrapolate to large hadron momenta P_z
- Promising complementary method to phenomenology (this talk) to determine pion distributions!

- large b trend ~ similar...
- small b difference can be due to collinear PDF (normalisation)
- x behaviour different (normalisation/other pheno)

Summary on a technical level

data input

- $\frac{q_T}{Q} = \delta < 0.25$ (TMD regime: $\delta \ll 1$)
- $x_F = x_\pi - x_{\text{tar.}} < 0.7$ $\longrightarrow$ 82 data

non-perturbative input:

- `ART25` (TMDs:target (PDFs:`MSHTPDF20`) **and** CS-kernel)
- `xfitter` (NLO) (PDFs:pion)

perturbative input

Γ_{cusp}	γ_V	$\mathcal{D}_{\text{pert}}$	$C_{f\to f'}$	C_{h}
N^{4}LO	N^{3}LO	N^{3}LO	N^{3}LO	N^{4}LO
α_s^5	α_s^4	α_s^4	α_s^3	α_s^4

human footprint (!)

- parametrisation
- fit strategy

output

unpolarised π^- TMDPDFs

$$\chi^2/N_{\text{pt}} = 1.38$$

I showed you

WHAT physical function we extract (unpol. pion TMDPDFs)

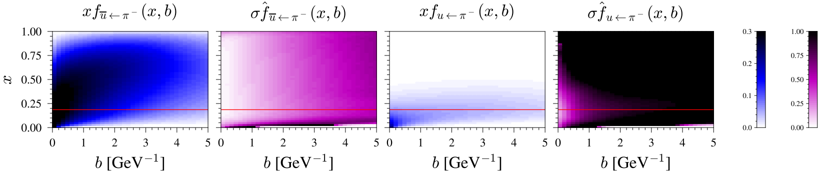
and **HOW** we do it (DY data + parametrisation (intuition) + fit)

Problems are a small phase space(total), large phase space(per data point), normalisation uncertainty,
large x_F region, consistency between PDFs and TMDPDFs (σ_{norm}),...

A paper should appear soon on the arXive.

☺

pion TMDPDFs in collinear momentum - transverse distance - space



- $0.18 < x < 1.0$ accessed by data (red line)
- flavour dependent to illustrate the uncertainty of sea quarks

Scale dependence of TMDs:

PDF $f(x;\mu) \longrightarrow$ TMDPDF $f(x,b;\mu,\zeta)$

Evolution equations

$$\mu^2 \frac{d}{d\mu^2} f(x,b;\mu,\zeta) = \frac{\gamma_F(\mu,\zeta)}{2} f(x,b;\mu,\zeta)$$

$$\zeta \frac{d}{d\zeta} f(x,b;\mu,\zeta) = -\mathcal{D}(b,\mu) f(x,b;\mu,\zeta)$$

μ evolution is perturbative ✓

ζ evolution is not:

Parametrisation of TMD Evolution: (Collins-Soper kernel $\mathcal{D}$)

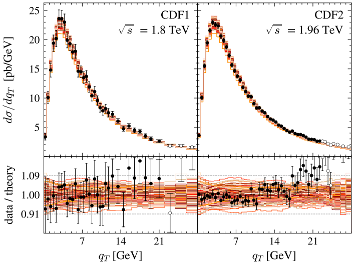
$$\begin{aligned}\mathcal{D}(b,\mu) &= \mathcal{D}_{\text{small-b}}(b^*,\mu) && +\mathcal{D}_{\text{NP}}(b) \\ &= \mathcal{D}_{\text{small-b}}(b^*,\mu^*) + \int_{\mu^*}^{\mu} \frac{d\mu'}{\mu'} \Gamma_{\text{cusp}}(\mu') && +\mathcal{D}_{\text{NP}}(b)\end{aligned}$$

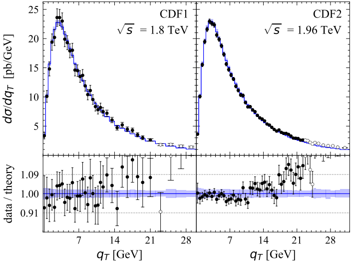
Uncertainty processing in fit (ART23 figures)

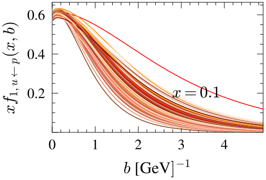
replica of data + replica of PDF $\xrightarrow{\text{fit}}$ TMDPDF replica

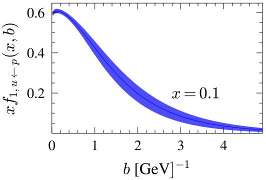
ensemble of replicas

average value and 68% CI

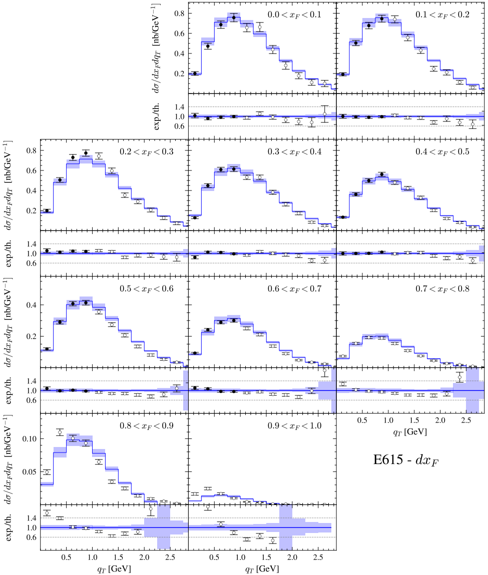








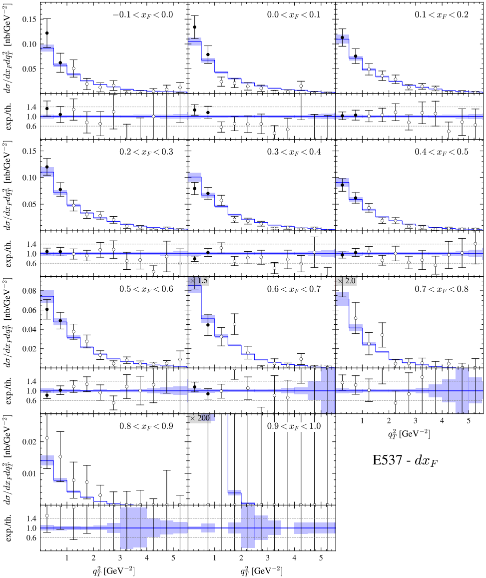
data - theory agreement after fit I



$\pi^- + W \to$ DY q_T spectrum in bins of x_F

- $\bullet$ data in fit
- $\circ$ data not in fit

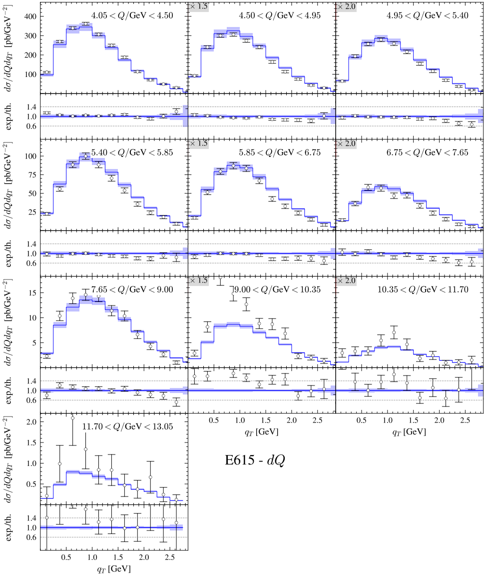
data - theory agreement after fit II



$\pi^- + W \to$ DY q_T spectrum in bins of x_F

- • data considered in fit
- ∘ data are not

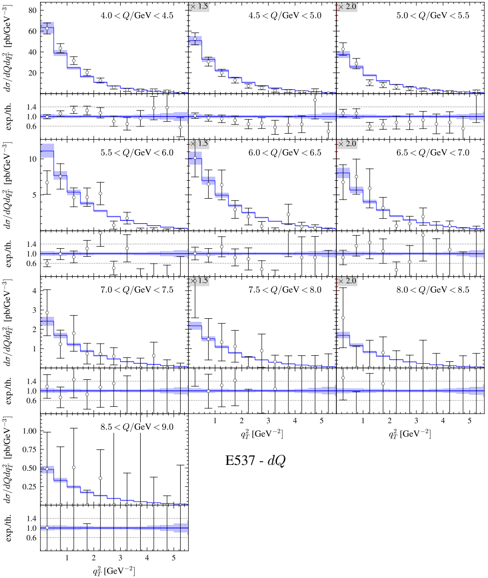
data - theory agreement after fit III



$\pi^- + W \to \mathrm{DY}\ q_T$ spectrum in bins of Q

- • data considered in fit
- ∘ data are not

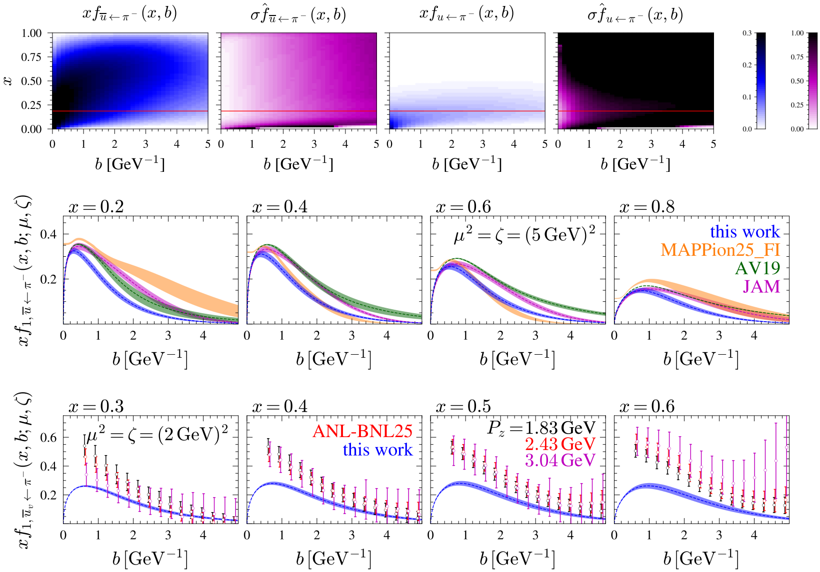
data - theory agreement after fit IV



$\pi^- + W \to$ DY q_T spectrum in bins of Q

- • data considered in fit
- ◦ data are not

Let me flash a few results... the extracted distributions



χ^2 heatmap with alternative model