Quantum entanglement of partons in strongly coupled QFTs

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Based on:
2512.21228 &
in preparation

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EIC-Asia Workshop on QCD and Hadron Structure

Institute of Physics, Academia Sinica, 29 April - 1 May 2026

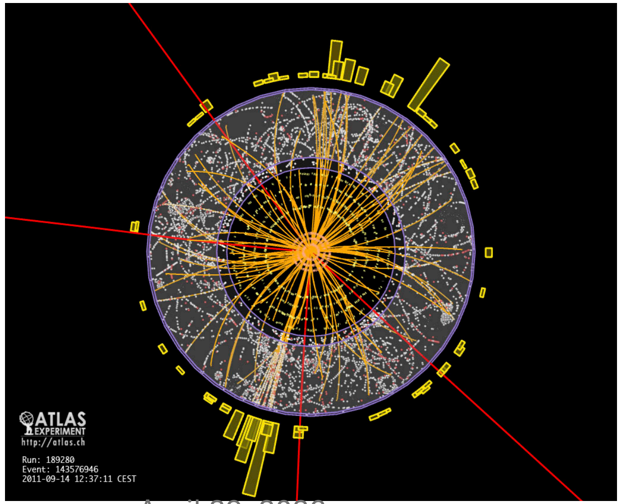
HEP in a nutshell

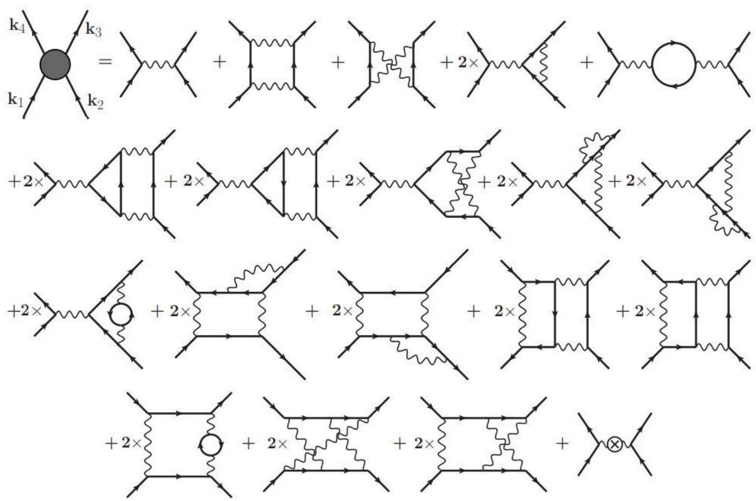
$$\sigma = \sum_{a,b} \int dx_a dx_b \hat{\sigma}_{ab\to X} f_{a/A}(x_a, \mu_F) f_{b/B}(x_b, \mu_F)$$

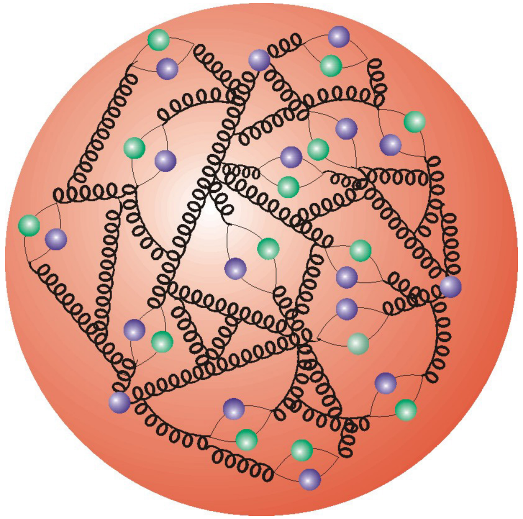
experimental measurements

perturbation theory

parton structure of the proton



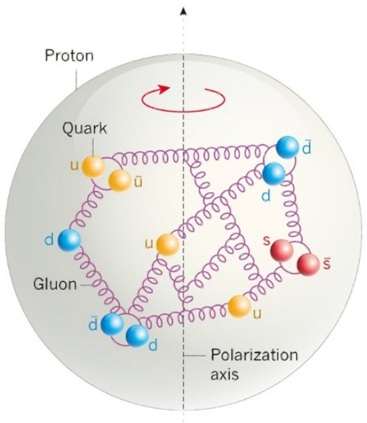




Parton distribution function (PDF)

- PDF $f(x)$ describes the probability density of finding a collinear parton of longitudinal momentum faction x
- Non-perturbative → obtained from global fits

quark model, Gell-Mann 1964

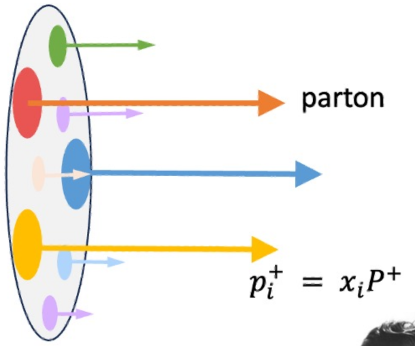


$T_{\text{int}} \sim \Lambda_{QCD}^{-1} = 10^{-23}\text{s}$

quarks are strongly coupled

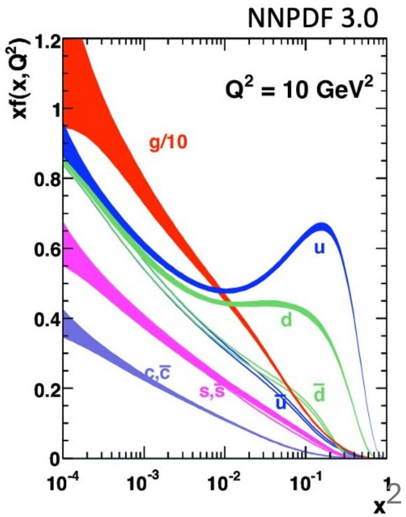
$P \to \infty$

parton model, Feynman 1969



$T_{\text{int}} \sim \gamma\Lambda_{QCD}^{-1} \gg 10^{-23}\text{s}$

partons are free!

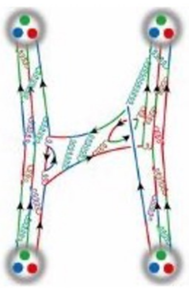
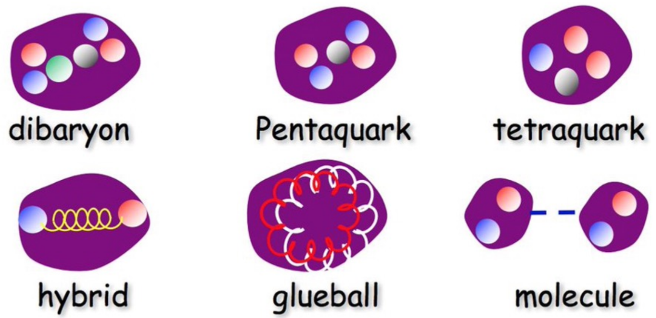


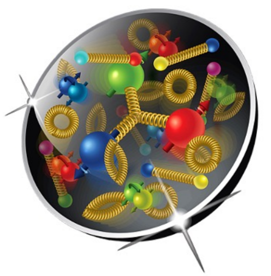
Parton entropy paradox

Kharzeev, 2021

$$S_{\text{Shn}}(f) = \ln \Delta x^{-1} - \int_0^1 dx\, f(x) \ln f(x) > 0$$

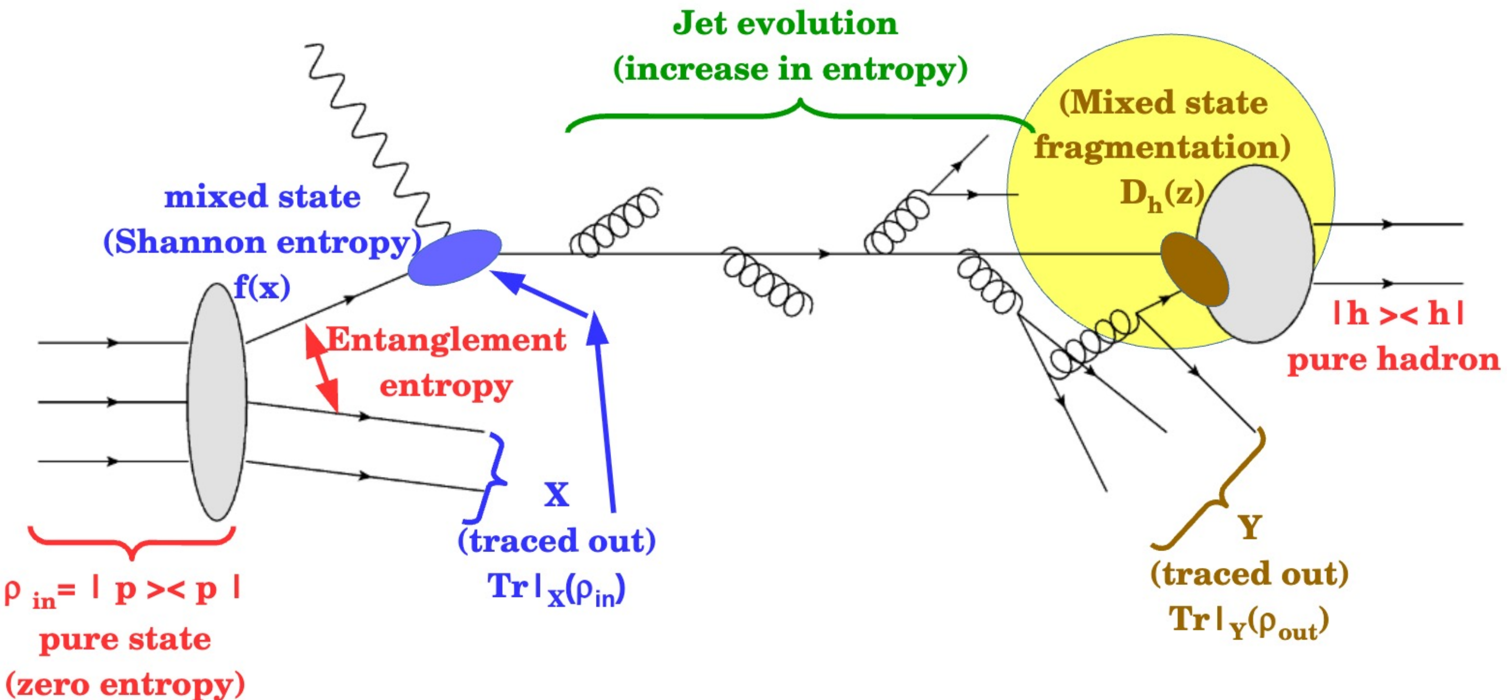
- But proton is a pure state $\rho = |\psi\rangle\langle\psi|$ with a vanishing entropy!
- Entropy = loss of information: some quantum information of the proton is lost in the parton picture
- Related to some of the big puzzles (e.g. confinement, origin of mass, origin of exotic hadrons) in the proton structure?



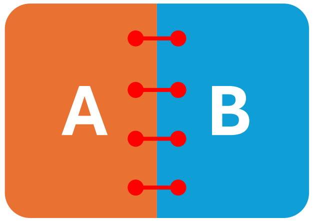
Parton entropy as quantum entanglement

- von Neumann entropy: $S_{\mathrm{vN}}(\rho_A) = -\mathrm{Tr}\rho_A \ln \rho_A$ measures the quantum entanglement between A and B, where $\rho_A = \mathrm{Tr}_B \rho$
- What is the relation between $S_{\mathrm{vN}}(\rho_A)$ and $S_{\mathrm{Shn}}(f)$?



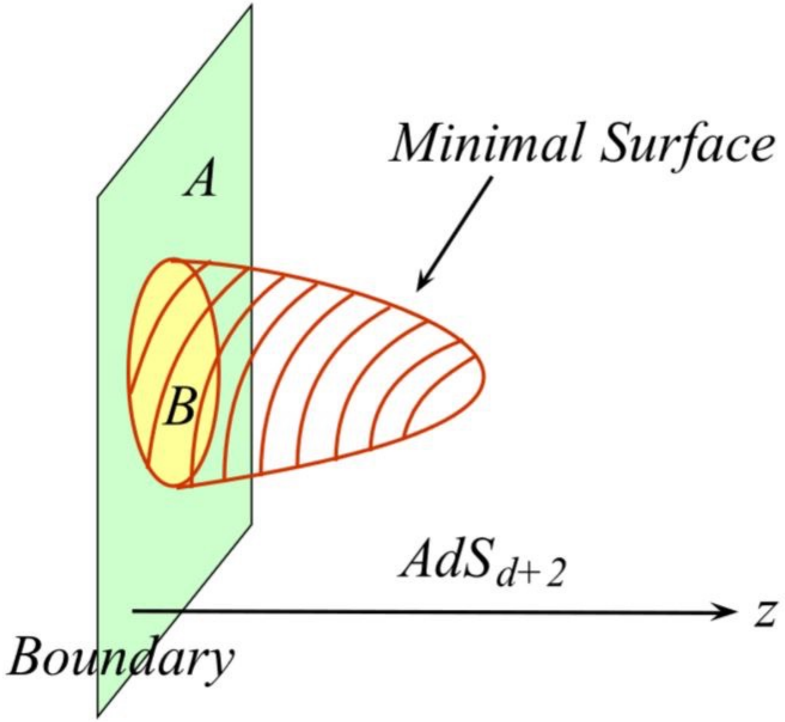
Shannon entropy:

$$S_{Shn}(\{p_i\}) = -\sum_i p_i \log p_i$$



$$S_A = S_B$$

Entanglement entropy in QFT



- Ryu-Takayanagi formula:

$$S_B = \min_{\partial V \sim \partial B} \frac{\mathcal{A}_V}{4G_N}$$

- Area law:

$$S_B = \frac{\mathcal{A}_{\partial B}}{\epsilon^{d-1}} + \text{finite corrections}$$

- Not directly measurable in high-energy physics experiments: particles vs geometry
- How to compute the entanglement between partons?

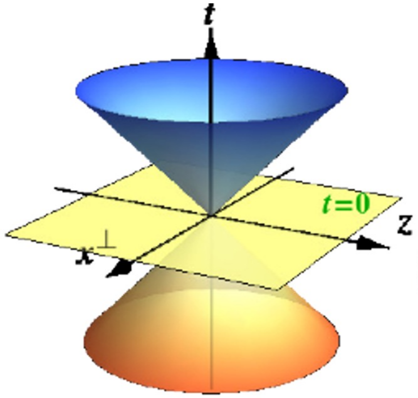
Light-front quantization

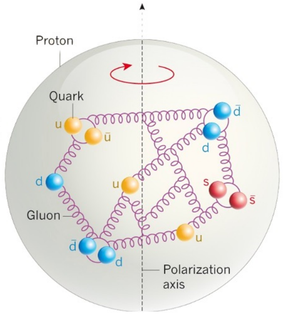
$$i\frac{\partial}{\partial x^\mu}|\Psi\rangle = \mathcal{P}_\mu|\Psi\rangle$$

- Choice of time: $x^0, x^+ \Rightarrow$ direction of dynamical evolution
- parton field theory = light-front quantization

Polyzou, 2021

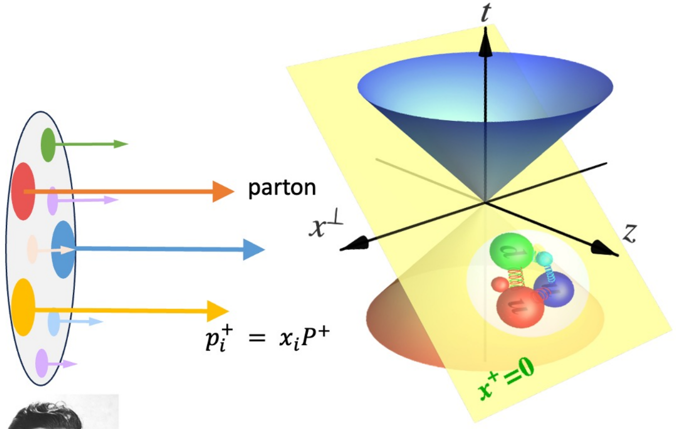
equal-time quantization
$t = 0$





infinite momentum frame $P_z \to \infty$

light-front quantization
$x^+ = t + z/c = 0$



$$i\frac{\partial}{\partial t}|\Psi\rangle = P^0|\Psi\rangle$$

$$i\frac{\partial}{\partial x^+}|\Psi\rangle = P_+|\Psi\rangle$$

Scalar Yukawa theory

$$\mathcal{L} = \partial_\mu N^\dagger \partial^\mu N - m^2 N^\dagger N + \frac{1}{2}\partial_\mu \pi \partial^\mu \pi - \frac{1}{2}\mu^2 \pi^2 + g N^\dagger N$$

- Nucleon-pion interaction – Yukawa potential

$$V(r) = -\frac{\alpha}{r} e^{-\mu r} \qquad \alpha = \frac{g^2}{16\pi m^2}$$

- Super-renormalizable: divergence only in 1-loop
- No gauge fields: no collinear divergence or gauge link
- Non-perturbative entanglement entropy in 3+1D based on the state vector

See, Fitzpatrick 2022 & 2025, for 1+1/2+1D QFTs in light-cone Hamiltonian approach

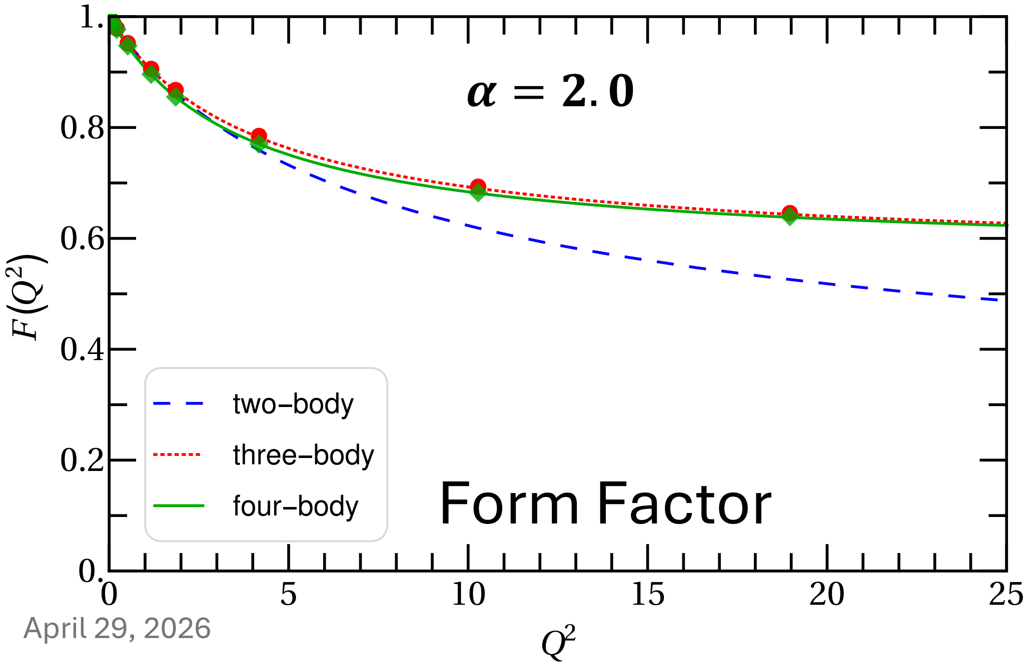
Light-cone Hamiltonian truncation

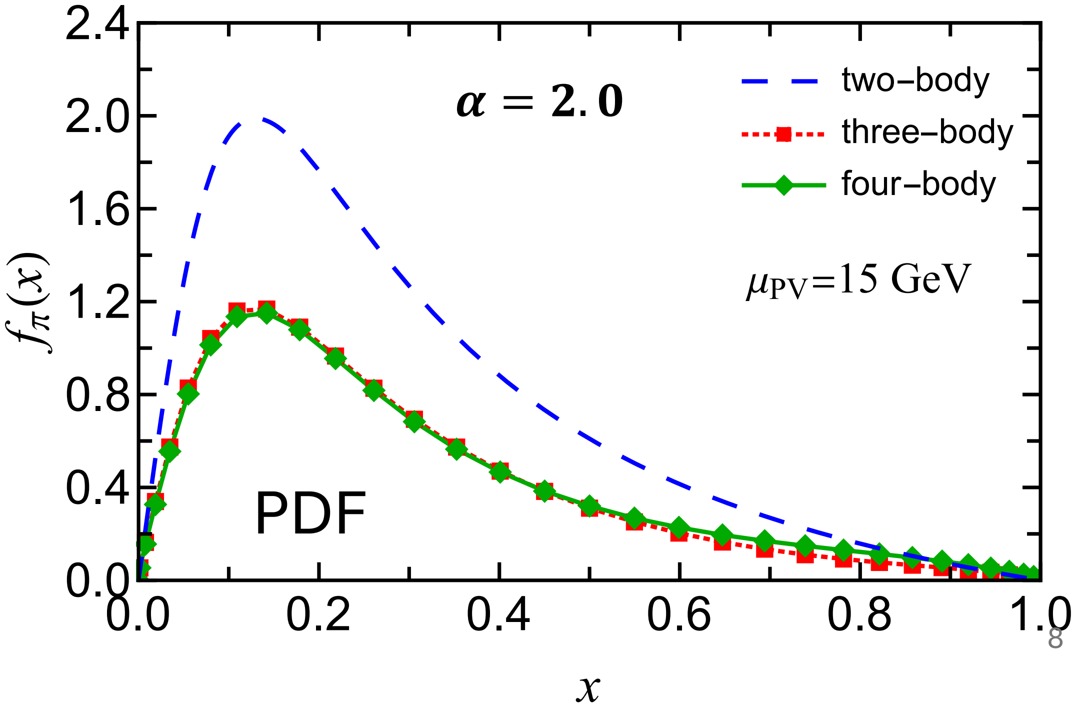
- Fock expansion:

$$|N\rangle_{\text{ph}} = |N\rangle + |\pi N\rangle + |\pi\pi N\rangle + |NN\bar{N}\rangle + |\pi\pi\pi N\rangle + \cdots$$

- Rapid convergence with Fock sector expansion

Li 2015, Karmanov 2016, Zhang 2025





Density matrix

- Reduced density matrix:

$$\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi| \qquad\qquad (\langle\Psi|\Psi\rangle = 1)$$

- *If ρ_A is diagonal*:

$$\rho_A = \sum_i p_i \, |i\rangle\langle i|$$

the von Neumann entropy is simply,

$$S_{\mathrm{vN}}(\rho_A) = -\mathrm{Tr}\rho_A \log \rho_A = -\sum_i p_i \ln p_i$$

- Is ρ_A diagonal ? Unfortunately, no.

Density matrix

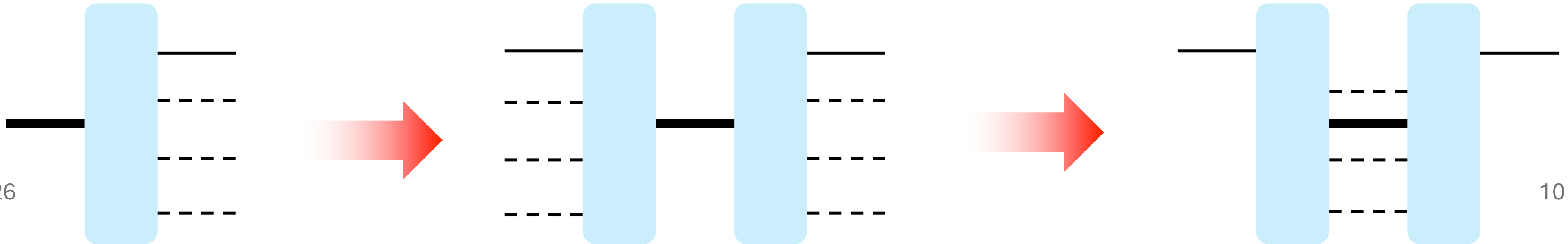
- Hadronic state vector:

$$|\Psi\rangle = \sum_F \int [d^3p_i]_F \, \psi_F(\{p_1, p_2, \cdots, p_n\}) |\{p_1, p_2, \cdots, p_n\}\rangle$$

- Density matrix

$$\rho_N = \mathrm{Tr}_\pi \sum_F \int [d^3p_i]_F \int [d^3p'_j]_F \, \Psi^*_F(\{p_i\})\Psi_F(\{p'_j\}) \underbrace{|p_1, p_2, \cdots,}_{\text{pion d.o.f.}} {\color{red}p_n}\rangle\langle \underbrace{p_1, p_2, \cdots,}_{\text{pion d.o.f.}} {\color{red}p'_n}|$$

- ρ_N is NOT diagonal in general



Density matrix in quenched approximation

$$|N\rangle_{\mathrm{ph}} = |N\rangle + |\pi N\rangle + |\pi\pi N\rangle + |NN\bar{N}\rangle + |\pi\pi\pi N\rangle + \cdots$$

- Quenched approximation: no sea nucleons – only one nucleon
- Factorization of the center-of-mass motion on the light front:

$$\Psi(p_1, p_2, \cdots, p_n) = \Psi(P)\psi\big(\{x_i, \vec{k}_i\}\big)$$

where $P = \sum_i p_i$, $x_i = p_i^+/P^+$, $\vec{k}_{i\perp} = \vec{p}_{i\perp} - x_i \vec{P}_\perp$

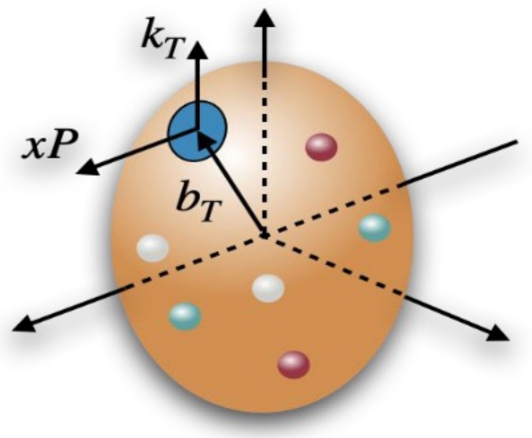
- In the narrow wavepacket limit (NWL), momentum conservation forces ρ_N to become diagonal, no need to do LF-time scrambling

$$\Psi(P) \to \delta^3(P - P_0)$$

- Not available in equal-time quantization!

Entanglement entropy in quenched theory

$$\rho_N = \frac{1}{P^+V}\int \frac{dx}{2x}\int \frac{d^2k_\perp}{(2\pi)^3} f_N(x,k_\perp)|p\rangle\langle p|$$



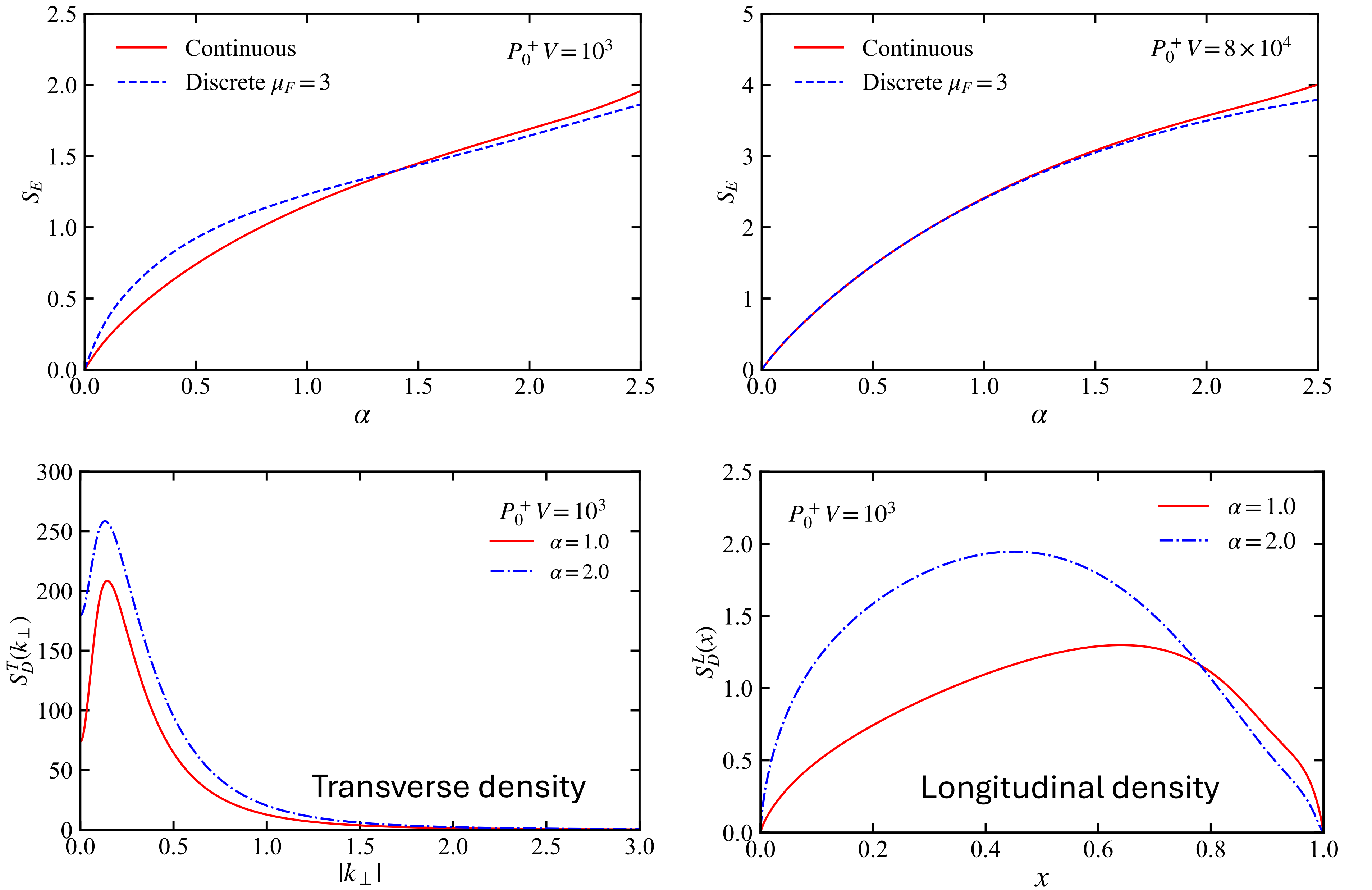
here, $f_N(x,k_\perp)$ is the nucleon transverse momentum dependent parton distribution (TMD)

- Entanglement entropy:

$$S_{\mathrm{vN}}(\rho_N) = \ln P^+V - \int dx \int \frac{d^2k_\perp}{(2\pi)^3} f_N(x,k_\perp)\ln f_N(x,k_\perp)$$

$$= S_{\mathrm{Shn}}(f_N)$$

- Entanglement entropy = Shannon entropy



Continuous
Discrete $\mu_F = 3$
$P_0^+ V = 10^3$
S_E
α
Continuous
Discrete $\mu_F = 3$
$P_0^+ V = 8 \times 10^4$
S_E
α
$P_0^+ V = 10^3$
$\alpha = 1.0$
$\alpha = 2.0$
$S_D^T(k_\perp)$
Transverse density
$|k_\perp|$
$P_0^+ V = 10^3$
$\alpha = 1.0$
$\alpha = 2.0$
$S_D^L(x)$
Longitudinal density
x

Entanglement entropy in unquenched theory

$$|N\rangle_{\rm ph} = |N\rangle + |\pi N\rangle + |\pi\pi N\rangle + |NN\bar{N}\rangle + |\pi\pi\pi N\rangle + \cdots$$

- Unquenched theory: add back the sea
- Problem: the reduced density matrix is no longer diagonal in the narrow wavepacket limit (NWL),

$$\rho_N = |N\rangle\langle N| + \mathrm{Tr}_\pi |\pi N\rangle\langle \pi N| + \mathrm{Tr}_\pi |\pi\pi N\rangle\langle \pi\pi N| + \mathrm{Tr}_{\bar{N}} |NN\bar{N}\rangle\langle NN\bar{N}|$$

  - Cross terms, such as $\mathrm{Tr}_\pi |\pi\pi N\rangle\langle \pi N|$, vanish
  - The sea contribution $\mathrm{Tr}_{\bar{N}} |NN\bar{N}\rangle\langle NN\bar{N}|$ is non-diagonal

Non-diagonal part

- Numerical diagonalization: is analytical expression still possible?
- Note that

$$\rho_N = \rho_N^{(1)} \oplus \rho_N^{(2)}$$

  - $\rho_N^{(1)} = |N\rangle\langle N| + \mathrm{Tr}_\pi|\pi N\rangle\langle\pi N| + \mathrm{Tr}_\pi|\pi\pi N\rangle\langle\pi\pi N|$ is shown to be diagonal in NWL
  - $\rho_N^{(2)} = \mathrm{Tr}_{\bar{N}}|NN\bar{N}\rangle\langle NN\bar{N}|$ is non-diagonal in NWL
  - Can diagonalize $\rho_N^{(1)}, \rho_N^{(2)}$ separately:

$$\rho_N = \begin{pmatrix} \rho_N^{(1)} & \\ & \rho_N^{(2)} \end{pmatrix}$$

$$S_{\mathrm{vN}}(\rho_N) = S_{\mathrm{vN}}\left(\rho_N^{(1)}\right) + S_{\mathrm{vN}}\left(\rho_N^{(2)}\right)$$

Diagonal — Non-diagonal

Non-diagonal part

$$\varrho \equiv |NN\bar{N}\rangle\langle NN\bar{N}|$$
$$\varrho_{\bar{N}} = \mathrm{Tr}_N|NN\bar{N}\rangle\langle NN\bar{N}|$$
$$\varrho_N = \mathrm{Tr}_{\bar{N}}|NN\bar{N}\rangle\langle NN\bar{N}|$$

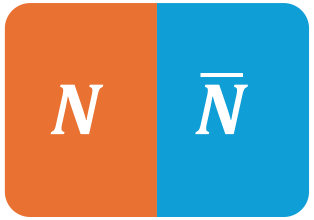
- How to diagonalize $\rho_N^{(2)} = \mathrm{Tr}_{\bar{N}}|NN\bar{N}\rangle\langle NN\bar{N}| \equiv \varrho_N$?
- Note that $\varrho_{\bar{N}} \equiv \mathrm{Tr}_N|NN\bar{N}\rangle\langle NN\bar{N}|$ is diagonal in the NWL

$$\varrho_{\bar{N}} = \frac{1}{P^+V}\int\frac{dx}{2x}\int\frac{d^2k_\perp}{(2\pi)^3}\, f_{\bar{N}}(x,k_\perp)|p\rangle\langle p|$$

$$\Rightarrow S_{\mathrm{vN}}(\varrho_{\bar{N}}) = Z_{NN\bar{N}}\log P^+V - \int dx\int\frac{d^2k_\perp}{(2\pi)^3} f_{\bar{N}}(x,k_\perp)\log f_{\bar{N}}(x,k_\perp)$$

- On the other hand, since the subspace is bipartite,

$$S_{\mathrm{vN}}(\varrho_N) = S_{\mathrm{vN}}(\varrho_{\bar{N}})$$



$S_N = S_{\bar{N}}$

Total entanglement entropy

$$S_{\mathrm{vN}}(\rho_N) = (1 - Z)\log P^+V \ - Z\log Z$$
$$- \int dx \int \frac{d^2k_\perp}{(2\pi)^3} f_N^{(\pi^n N)}(x,k_\perp)\log f_N^{(\pi^n N)}(x,k_\perp)$$
$$- \int dx \int \frac{d^2k_\perp}{(2\pi)^3} f_{\bar{N}}(x,k_\perp)\log f_{\bar{N}}(x,k_\perp)$$

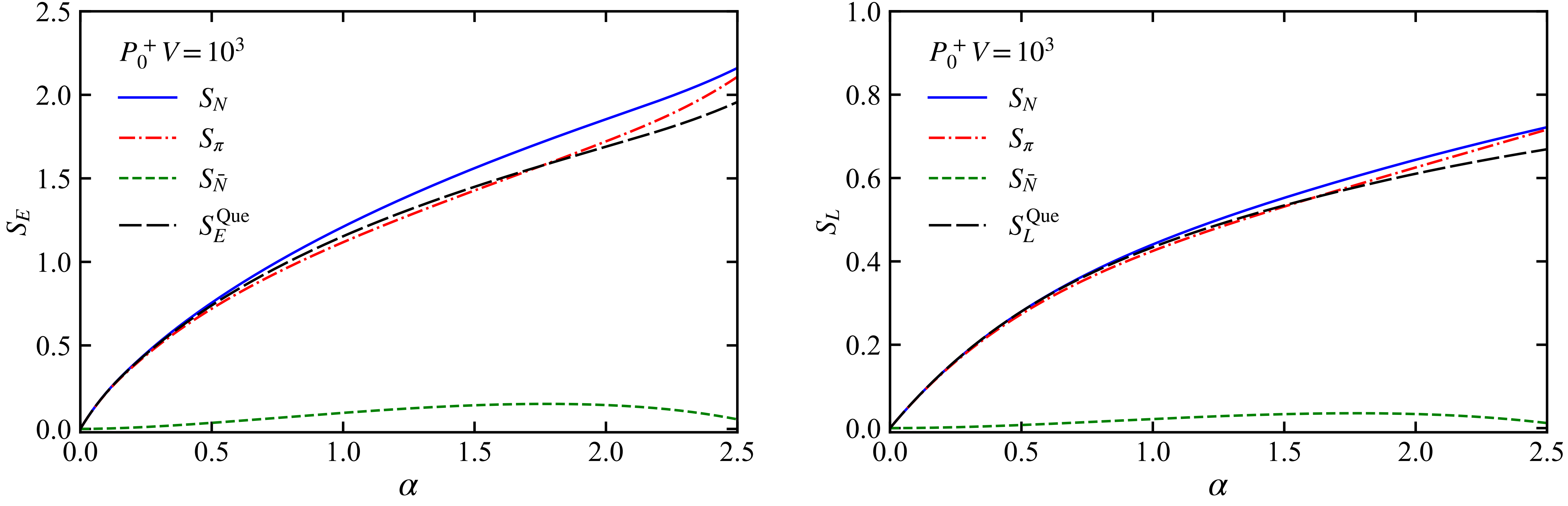
where, Z is the field renormalization constant, and

$$f_N^{(\pi^n N)} \equiv f_N^{(\pi N)} + f_N^{(\pi\pi N)} + \cdots$$
$$\neq f_N \equiv f_N^{(\pi N)} + f_N^{(\pi\pi N)} + f_N^{(NN\bar{N})} + \cdots$$

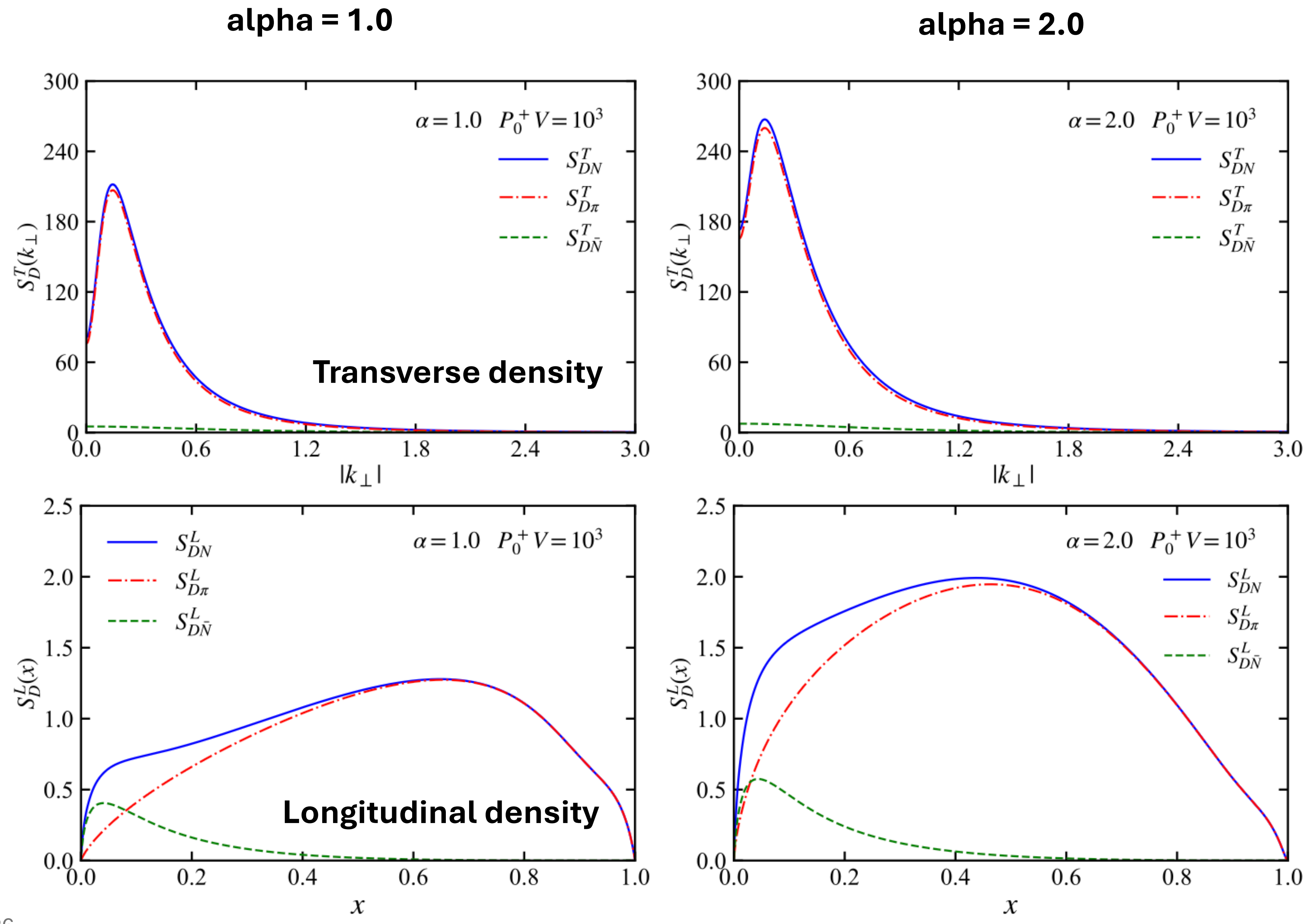
Therefore, $S_{\mathrm{vN}}(\rho_N) \neq S_{\mathrm{Shn}}(f_N)$

Entanglement entropy $S_{\mathrm{vN}}(\rho) = -\mathrm{Tr}\,\rho \log \rho$

Linear entropy $S_{\mathrm{Ln}}(\rho) = -\mathrm{Tr}\,\rho^2$



$S_{\mathrm{vN}}(\rho) = \frac{1}{n-1} \lim_{n\to 1} \mathrm{Tr}\,\rho^n$ (Renyi entropy)

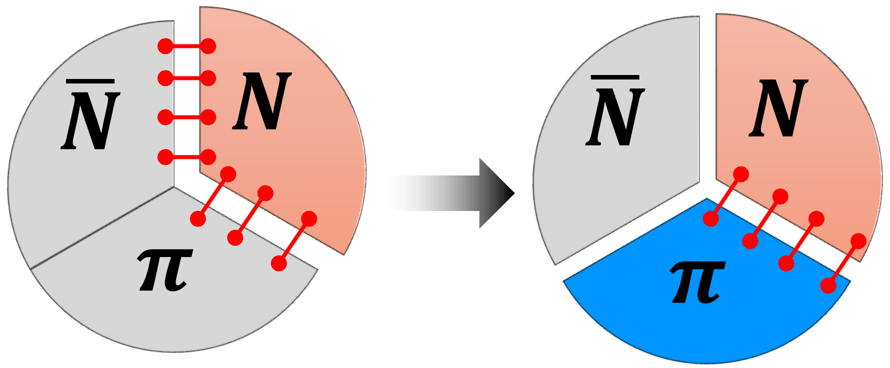

alpha = 1.0
alpha = 2.0
$\alpha = 1.0$ $P_0^+ V = 10^3$
$\alpha = 2.0$ $P_0^+ V = 10^3$
S^T_{DN}
$S^T_{D\pi}$
$S^T_{D\bar{N}}$
$S^T_D(k_\perp)$
$|k_\perp|$
Transverse density
S^L_{DN}
$S^L_{D\pi}$
$S^L_{D\bar{N}}$
$S^L_D(x)$
x
Longitudinal density

Entanglement witnesses

- Entanglement entropy contains classical info, good for pure state
- Separable or unentangled state:

$$\rho_{AB} = \sum_k p_k \, \rho_k^A \otimes \rho_k^B \;\; \Rightarrow \;\; |\psi\rangle = |\psi\rangle_A \otimes |\psi\rangle_B$$

- An entanglement witness is an observable which distinguishes a specific entangled state from separable ones
  - Renyi entropy (e.g. linear entropy)
  - Mutual information
  - Quantum negativity
  - Entanglement of formation, …

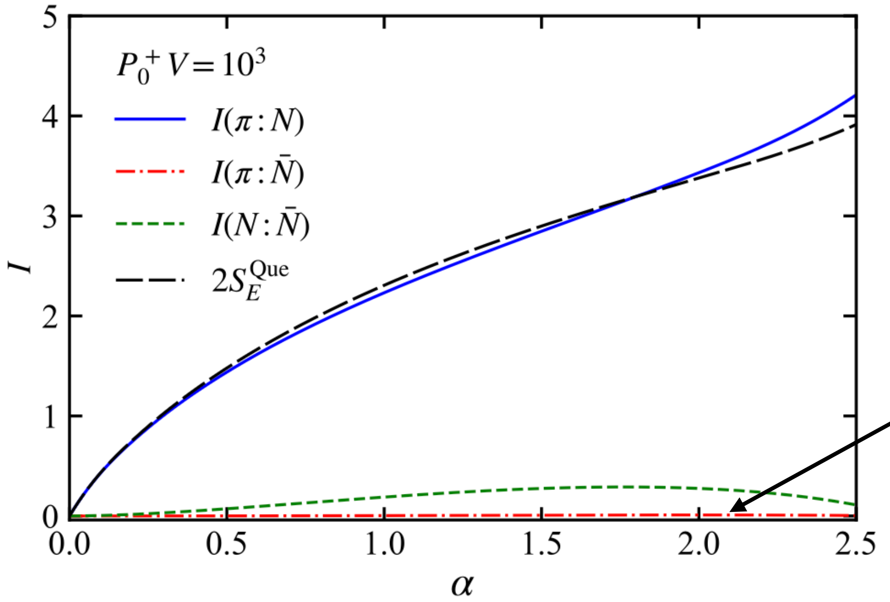


$\rho_N = \mathrm{Tr}_{\bar{N},\pi}\rho$ $\qquad$ $\rho_{N\pi} = \mathrm{Tr}_{\bar{N}}\rho$

Mutual information

- Mutual information $I(A\!:B)$ characterizes the correlation between subsystems A & B
- For example:

$$I(\pi\!:N) = S_{vN}(\rho_\pi) + S_{vN}(\rho_N) - S_{vN}(\rho_{\bar{N}}) \xrightarrow{\text{quenched apprx.}} 2S_{vN}(\rho_N)$$



Lowest Fock sector with both $\bar{N}$ and π: $|NN\bar{N}\pi\rangle$

Positive partial transpose (PPT)

- Detect quantum entanglement between two subsystems, e.g. N, π
  - Entanglement entropy: not applicable to mixed states
  - Mutual information: contains classical correlation
- PPT criterion (Peres-Horodecki criterion): If ρ_{AB} is separable, ρ_{AB}^T must be positive semi-definite

$$\rho_{AB} = \sum_{ijkl} C_{ijkl} |i\rangle_A |j\rangle_B \langle k|_A \langle l|_B \ \Rightarrow \ \rho_{AB}^T = \sum_{ijkl} C_{ijkl} |i\rangle_A |l\rangle_B \langle k|_A \langle j|_B$$

  - Existing negative eigenvalues of ρ_{AB}^T $\Rightarrow$ A、B are entangled
  - Necessary condition for the separability of a mixed state ρ_{AB}

Quantum negativity

$$\mathcal{N}(\rho_{AB}) = -\sum_{\lambda_i<0} \lambda_i \equiv \frac{1}{2}\left(Tr\sqrt{\rho_{AB}^{T\dagger}\rho_{AB}^{T}} - 1\right)$$

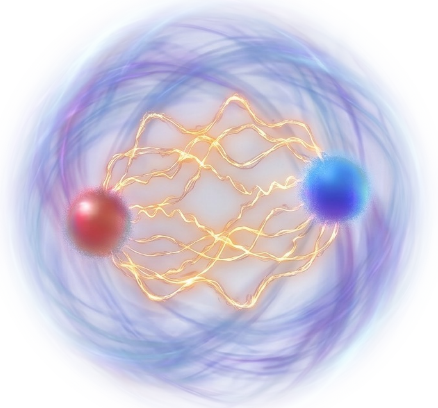
where, λ_i are eigenvalues of ρ_{AB}^{T}

- Characterizes the quantumness of a resource
- Measures the quantum correlation beyond local interactions
- Two-body truncation:

$$\mathcal{N}(\rho_{\pi N}) = \sqrt{P^{+}VZ}f_{\pi N} + P^{+}Vf_{\pi N}^{2}$$

where, $f_{\pi N}$ is the wave function at the origin, i.e. the decay constant

Entanglement within quarkonium



- Reduced density matrix in the quenched ansatz:

$$\rho_q^\Lambda = \sum_{s,s'} \int \frac{d^3p}{(2\pi)^3 2p^+} \rho_{ss'}^\Lambda(s, \vec{k}_\perp) \frac{1}{N_c} \sum_i |p, s', i\rangle\langle p, s, i|$$

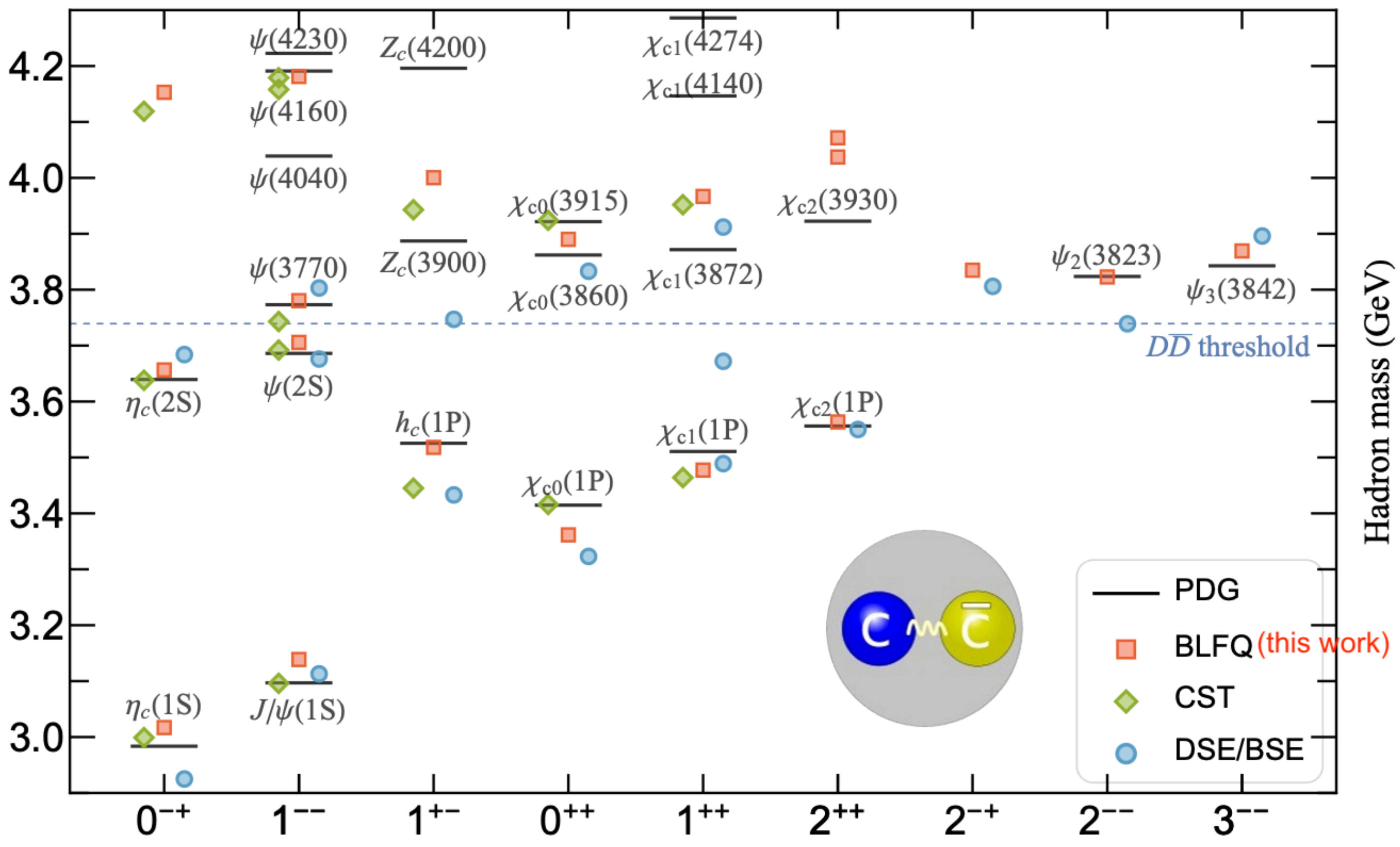
where,

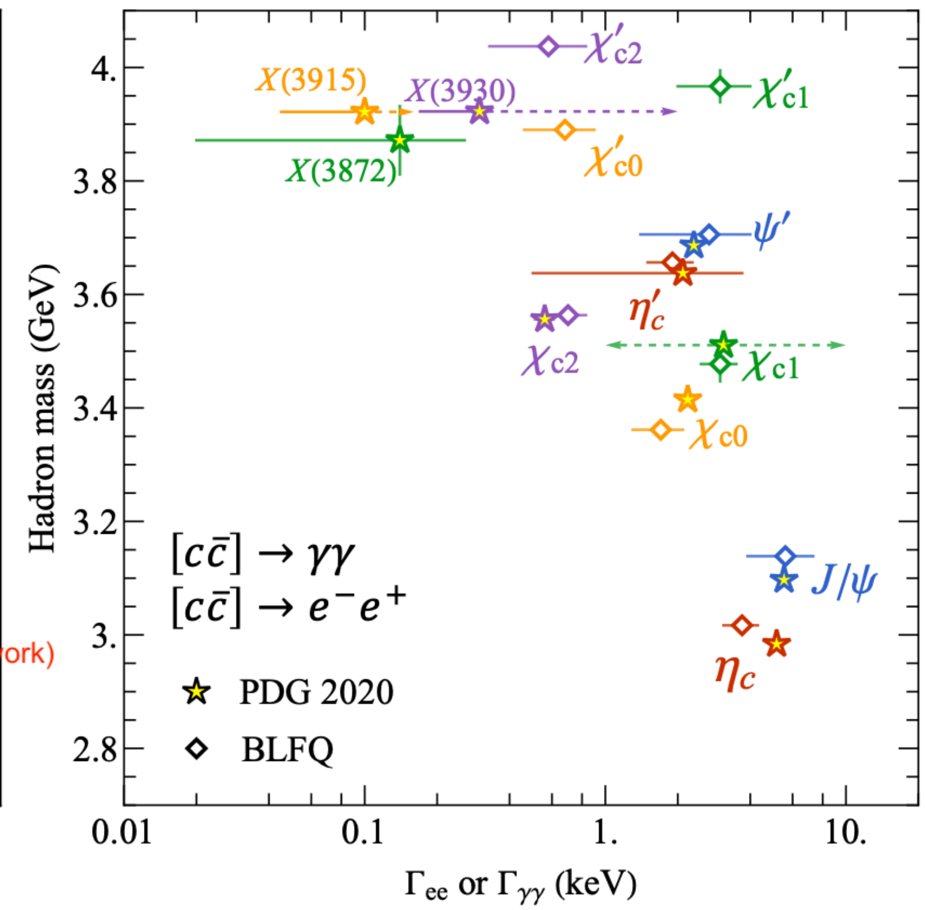
$$\rho^\Lambda = \frac{1}{2}\begin{pmatrix} \Phi_\Lambda^{[\gamma^+]} + \Phi_\Lambda^{[\gamma^+\gamma_5]} & \Phi_\Lambda^{[i\sigma^{1+}\gamma_5]} + i\Phi_\Lambda^{[i\sigma^{2+}\gamma_5]} \\ \Phi_\Lambda^{[i\sigma^{1+}\gamma_5]} - i\Phi_\Lambda^{[i\sigma^{2+}\gamma_5]} & \Phi_\Lambda^{[\gamma^+]} - \Phi_\Lambda^{[\gamma^+\gamma_5]} \end{pmatrix}$$

- Not diagonal due to spin part even in the quenched approximation
- Related to quark TMDs $\Phi_\Lambda^{[\Gamma]}$

Charmonium: hydrogen atom of QCD

Li, 2015 & 2017





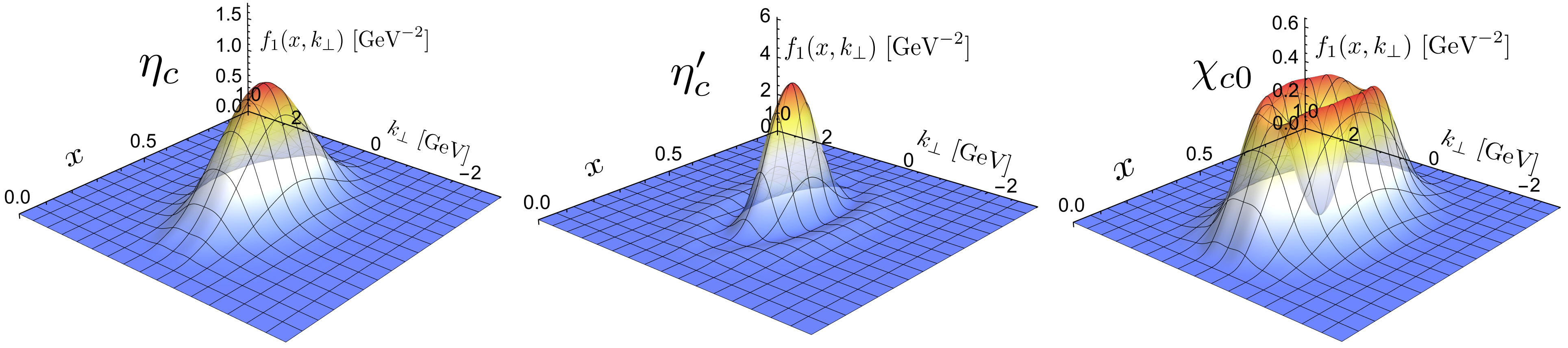
- Two free parameters (m_c, κ), rms deviation: 30 MeV
- Good agreement with the PDG data for both the masses and the widths

[Gross:2022hyw]

[LFWFs published on Mendeley Data, doi: 10.17632/cjs4ykv8cv.2]

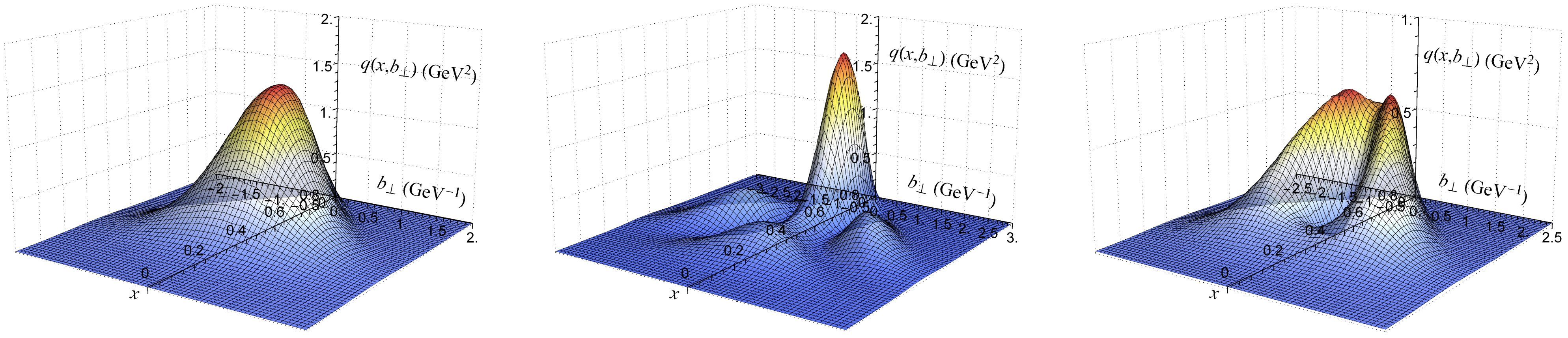
Charmonium structures

TMDs



GPDs

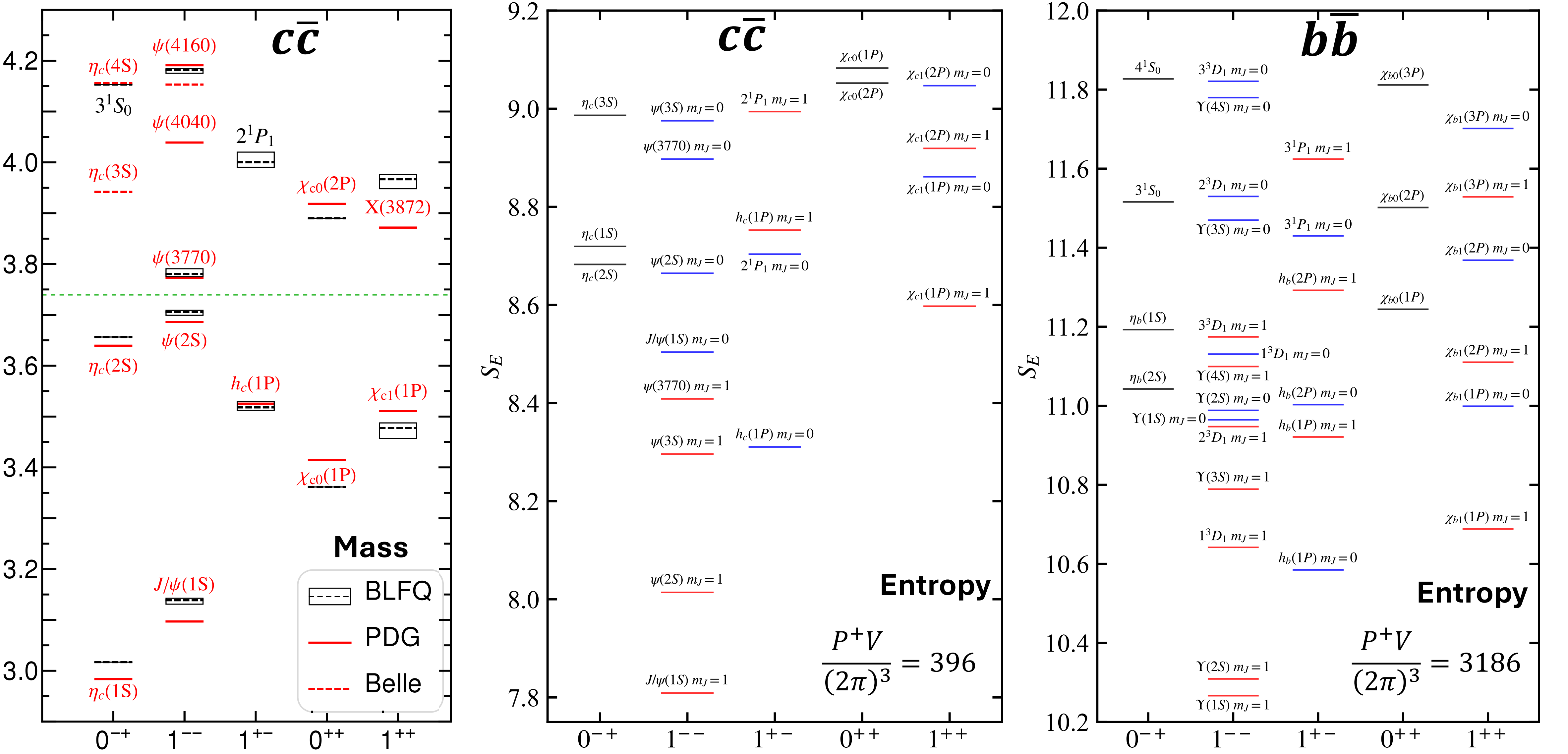
(impact-parameter space)



η_c η'_c χ_c

Entropy spectrum

$$\frac{P^+V}{(2\pi)^3} = 6L_{max}N_{max}\kappa^{-2}$$



Spin entanglement within quarkonium

- In quark model, quarkonium wavefunction factorizes, e.g.

$$\psi_{\eta_c}(\vec{r}) \approx \phi(r)\,\frac{\uparrow\downarrow - \downarrow\uparrow}{\sqrt{2}}$$

- Entanglement of quarks ≈ entanglement of spins
- Spin density matrix obtained by tracing over the momenta,

$$\rho^\Lambda = \sum_{s,\bar{s},s',\bar{s}'} I^\Lambda_{s,\bar{s};s',\bar{s}'} \, |s\bar{s}\rangle\langle s'\bar{s}'|$$

N.B. ρ^Λ is a mix-state DM, and entanglement entropy may not be a good gauge

- Need additional entanglement witnesses

Spin entanglement within quarkonium

- Quantum negativity $\mathcal{N}(\rho)$: for qubit-qubit and qutrit- qutrit systems, the PPT criterion is sufficient and necessary
- Concurrence $\mathcal{C}(\rho)$: for qubit-qubit system, $\mathcal{C}(\rho) = \frac{1}{2}\mathcal{N}(\rho)$
- Separable: $\mathcal{C}(\rho) = 0$; entangled: $\mathcal{C}(\rho) > 0$; maximally entangled: $\mathcal{C}(\rho) = 1$

	J^{PC}	m_J	L	$S^{\Lambda}_{\text{spin}}$	$\mathcal{C}[\rho^{\Lambda}_{\text{spin}}]$	$\mathcal{C}_{\text{NRQM}}$
$\eta_c(1S)$	0^{-+}	0	0	0.31602	0.85147	1
$J/\psi(1S)$	1^{--}	0	0	0.00504	0.99890	1
$J/\psi(1S)$	1^{--}	1	0	0.13949	0.01068	0
$\chi_{c0}(1P)$	0^{++}	0	1	1.08799	0.00000	–
$\chi_{c1}(1P)$	1^{++}	0	1	0.92394	0.00000	–
$\chi_{c1}(1P)$	1^{++}	1	1	0.67860	0.64530	–
$h_c(1P)$	1^{+-}	0	1	0.13387	0.95017	1
$h_c(1P)$	1^{+-}	1	1	0.20803	0.91686	1
$\eta_c(2S)$	0^{-+}	0	0	0.18471	0.92561	1

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	J^{PC}	m_J	L	$S^{\Lambda}_{\text{spin}}$	$\mathcal{C}[\rho^{\Lambda}_{\text{spin}}]$	$\mathcal{C}_{\text{NRQM}}$
$\psi(2S)$	1^{--}	0	0	0.01546	0.99610	1
$\psi(2S)$	1^{--}	1	0	0.09368	0.00000	1
$\psi(3770)$	1^{--}	0	2	1.07793	0.00000	0
$\psi(3770)$	1^{--}	1	2	0.52833	0.00000	–
$\chi_{c0}(2P)$	0^{++}	0	1	1.03304	0.00000	–
$\chi_{c1}(2P)$	1^{++}	0	1	0.88668	0.00000	–
$\chi_{c1}(2P)$	1^{++}	1	1	0.47852	0.81399	1
2^1P_1	1^{+-}	0	1	0.08394	0.97177	1
2^1P_1	1^{+-}	1	1	0.13827	0.95252	1

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Summary

- Using light-cone Hamiltonian formalism, we find

$$S_{\mathrm{vN}}(\rho_i) = S_{\mathrm{Shn}}(f_i) + \text{quantum corrections}$$

- Other entanglement witnesses: mutual info., quantum negativity etc reveal various quantum aspects of hadrons
- Application to QCD: quarkonium, spin entanglement
- Challenges: UV & IR divergences, collinear div. & factorization, gauge invariance, ...

Thank you!