TMD Physics from Lattice QCD

EIC-Asia Workshop on QCD and Hadron Structure

Institute of Physics, Academia Sinica

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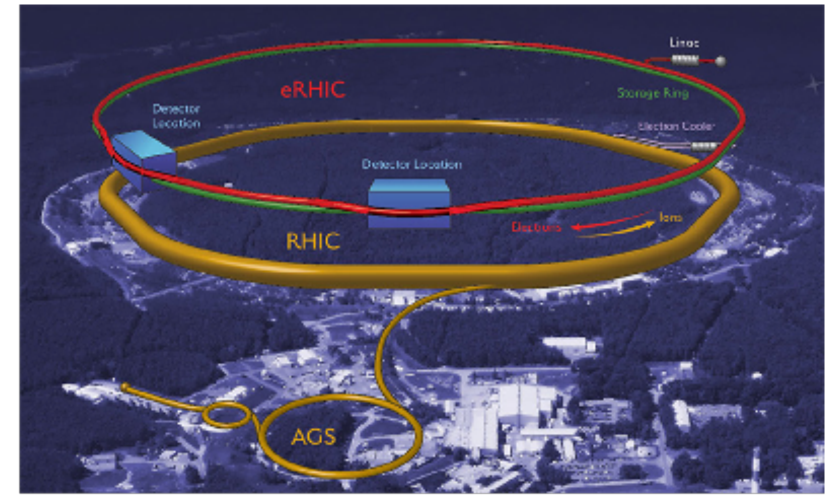
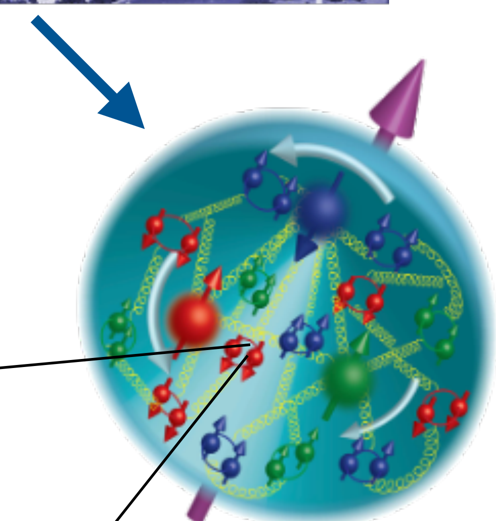
APRIL 29, 2026

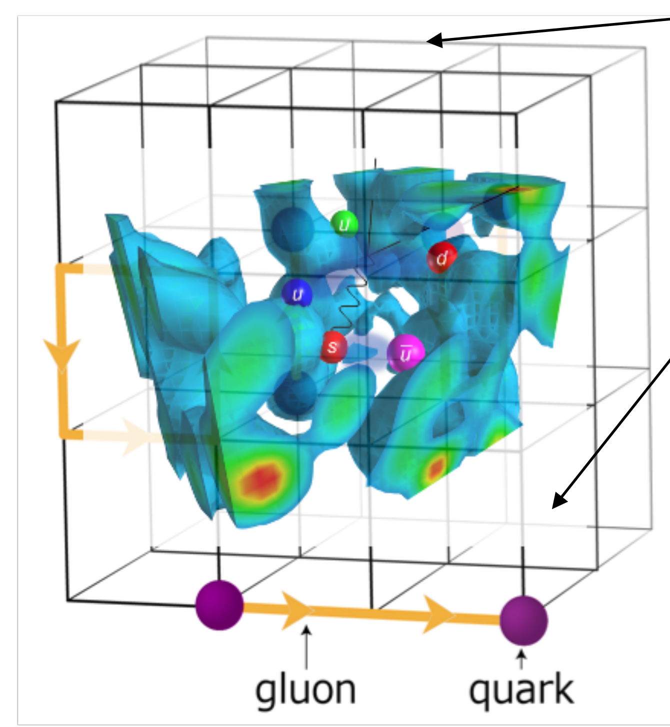
Argonne NATIONAL LABORATORY

Outline

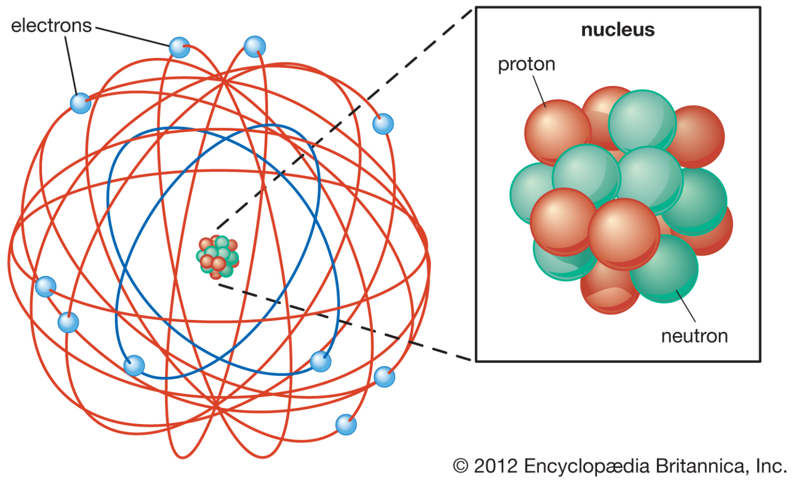
- **Introduction**
  - Overview of TMD physics
  - Large-Momentum Effective Theory
  - Status of lattice calculations
- **New approach without Wilson lines**
  - A quasi-TMD in the Coulomb gauge
  - Factorization
  - Applications
  - Other developments
- **Summary**

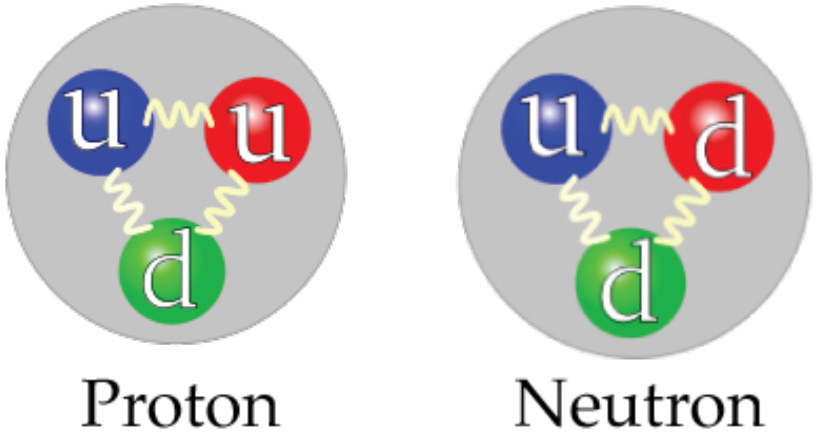






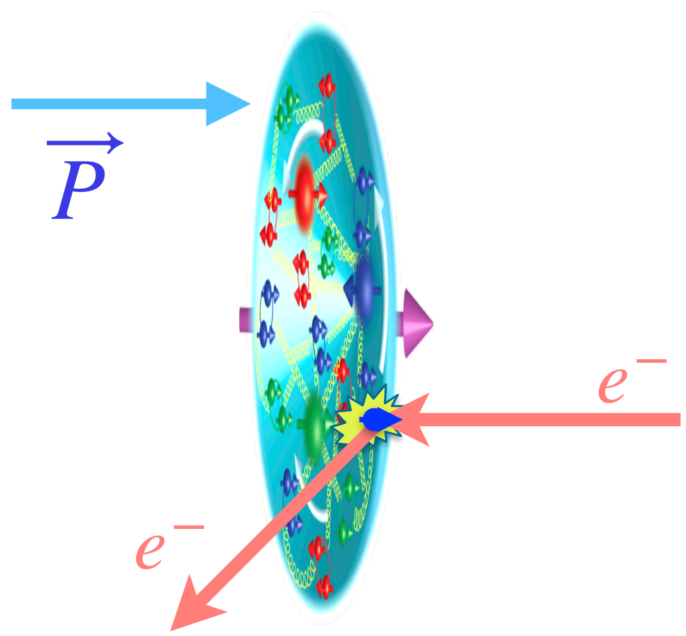
The visible universe built from protons and neutrons





Probing the quarks and gluons with high-energy scattering

The most important experiments are deep inelastic (e.g., *e*⁻+*p*) scatterings (DIS) and proton-proton collisions.



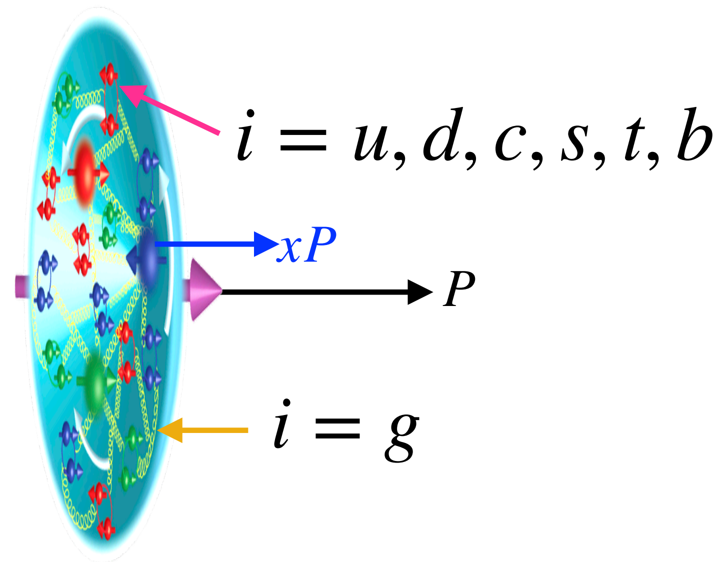
Richard P. Feynman

Feynman's parton model in the infinite momentum frame (1969):

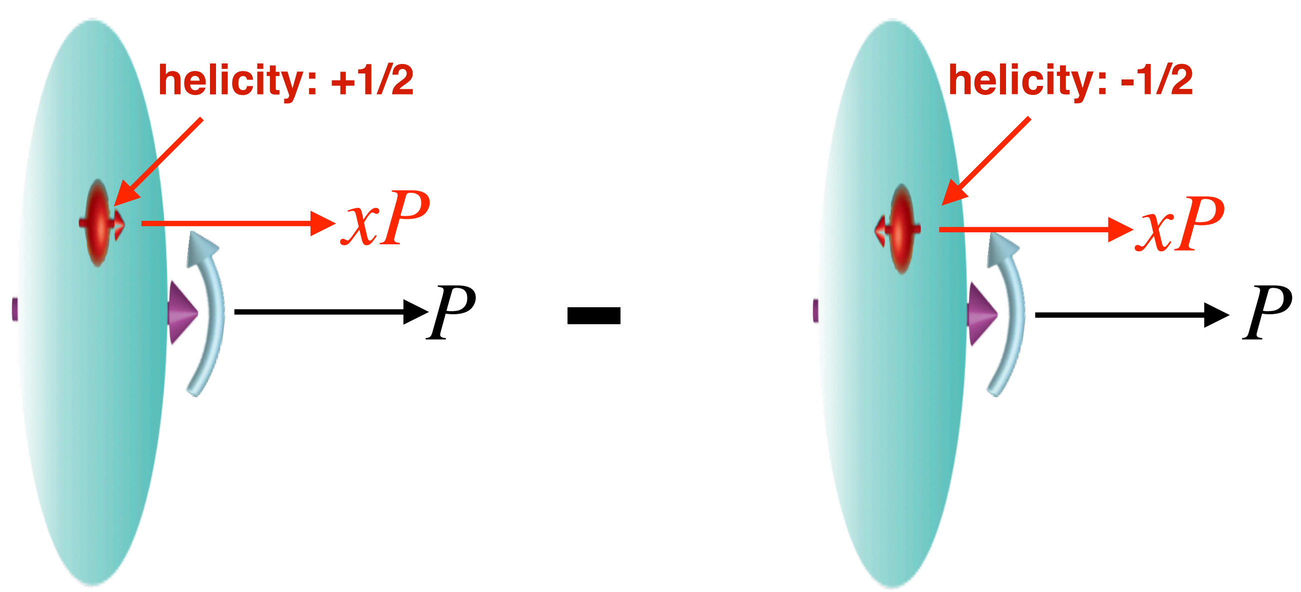
- Quarks and gluons are "frozen" in the transverse plane;
- During a hard collision, the struck quark/gluon (parton) appears like free particles, i.e., partons.

Parton distribution functions (PDFs)

- Unpolarized PDF $f_i(x)$:

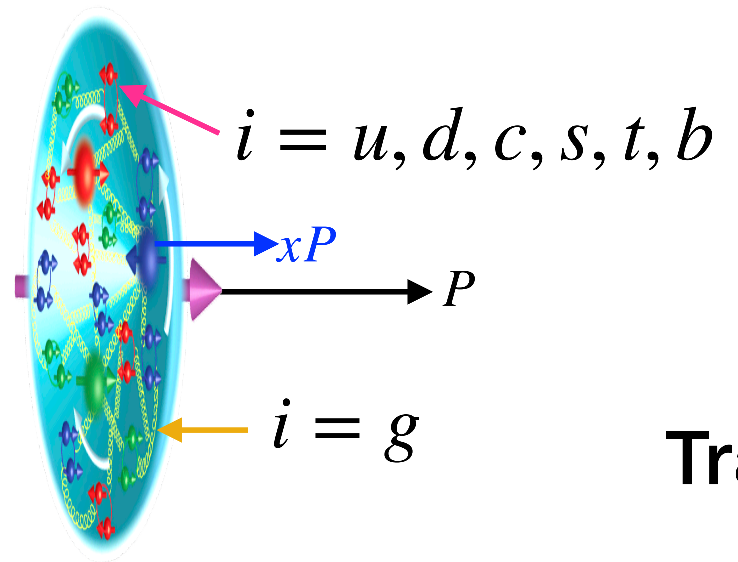


- Helicity PDF $\Delta f_i(x)$:

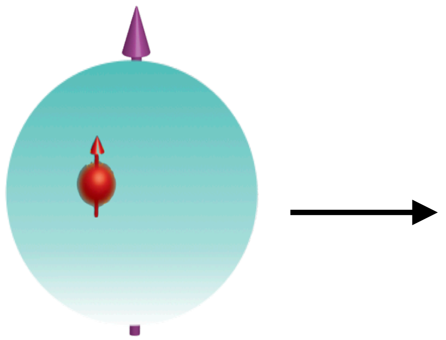


Parton distribution functions (PDFs)

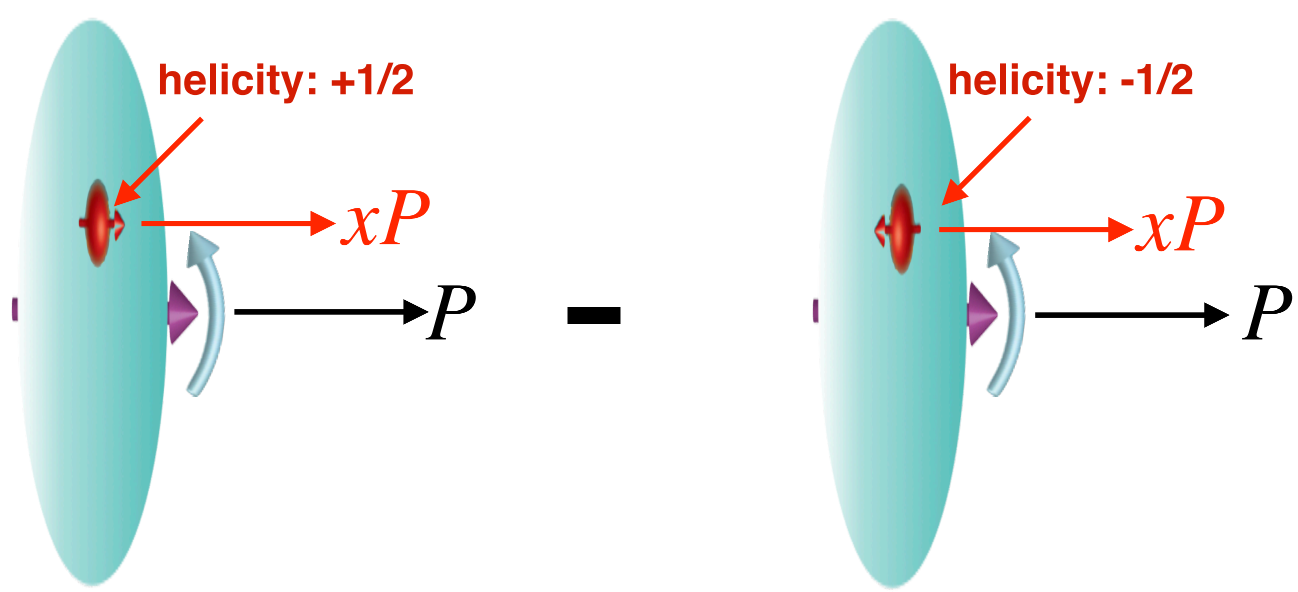
- Unpolarized PDF $f_i(x)$:



Transversity PDF $h_{1i}(x)$

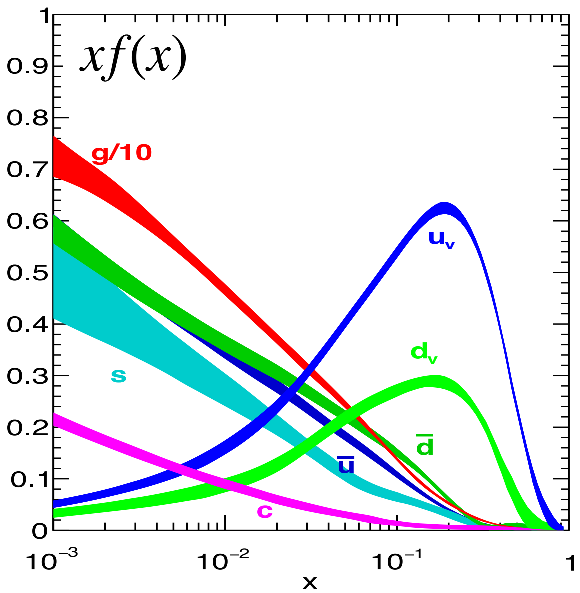


- Helicity PDF $\Delta f_i(x)$:



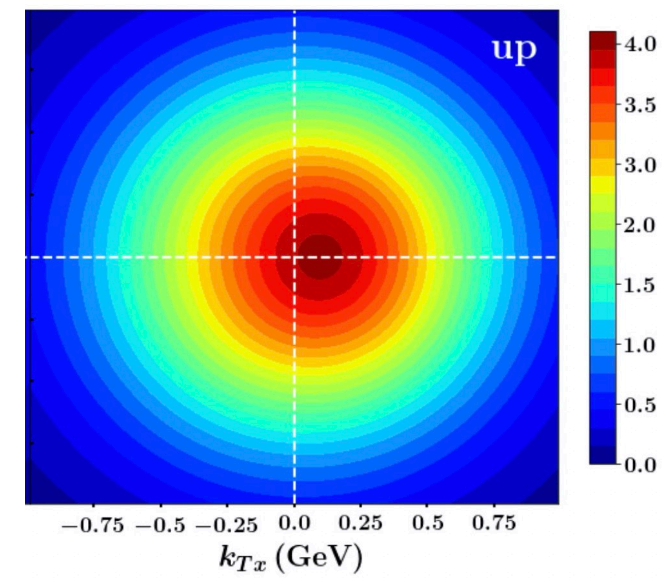
3D imaging of the proton → nuclei

PDFs



NNPDF, EPJ C77 (2017)

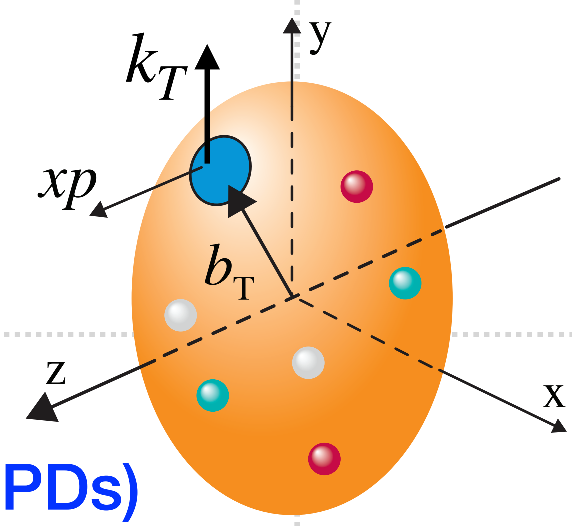
Transvers momentum distributions (TMDs)



Cammarota, et al. (JAM), PRD **102** (2020).

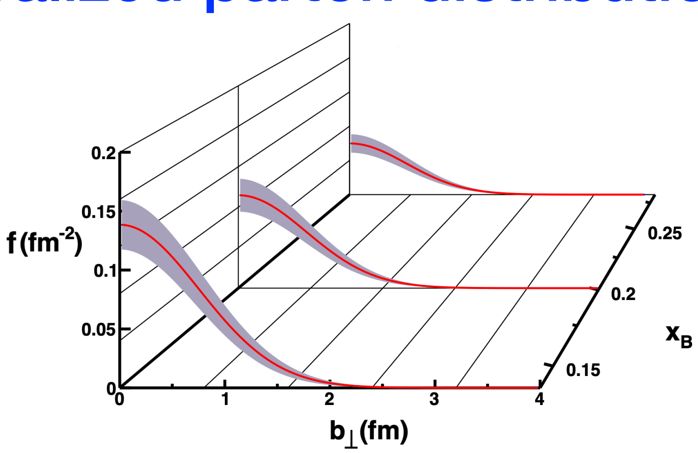
$\int d^2\vec{k}_T$

$\int d^2\vec{b}_T$



$\int d^2\vec{b}_T$

Generalized parton distributions (GPDs)



W. Armstrong et al., arXiv: 1708.00888.

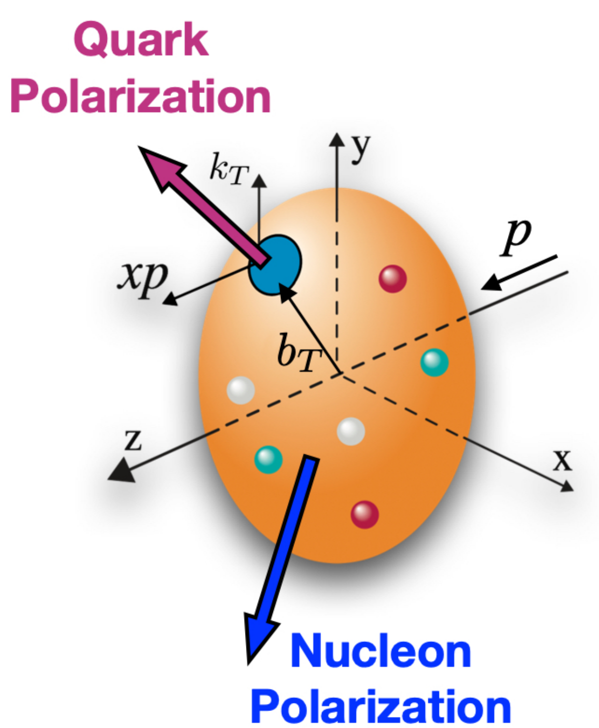
$\int d^2\vec{k}_T$

Wigner distributions/Generalized TMDs

$$W(x, \vec{k}_T, \vec{b}_T)$$

Transverse Momentum Distributions (TMDs)

TMDs are essential for 3D proton imaging in momentum space:



Leading Quark TMDPDFs

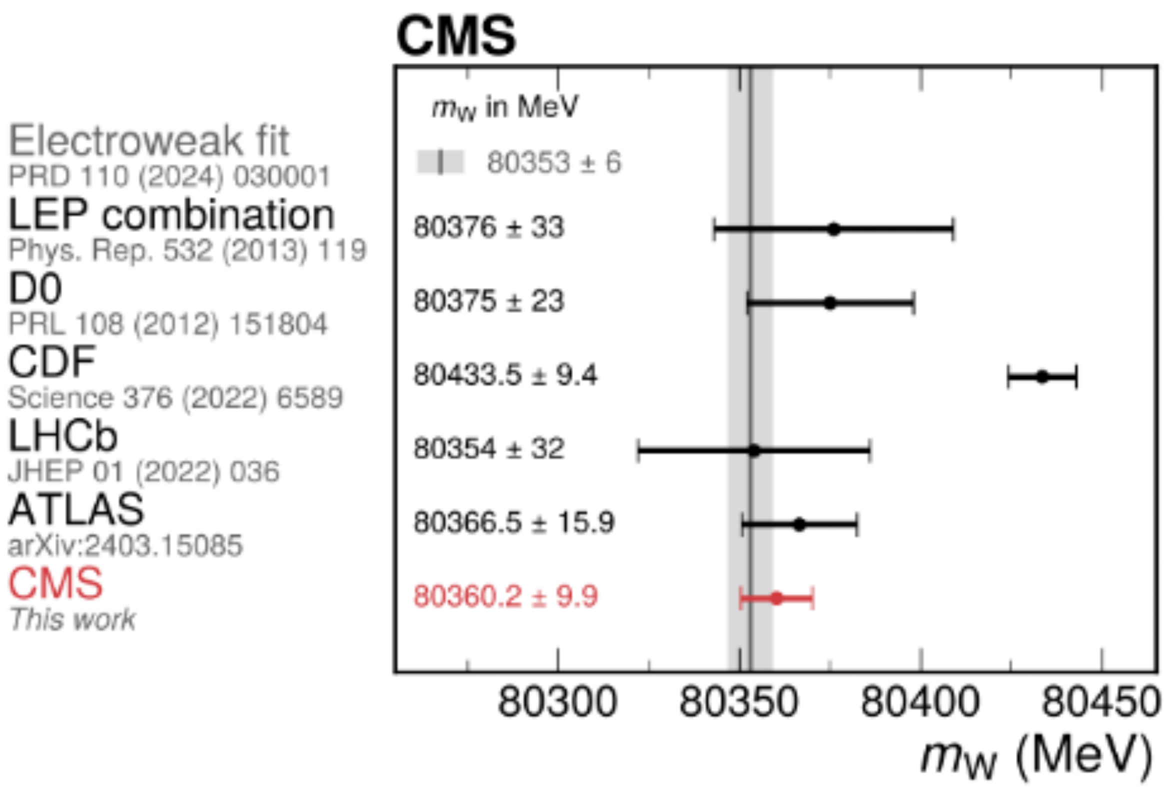
Nucleon Spin   Quark Spin

		Quark Polarization		
		Un-Polarized (U)	**Longitudinally Polarized (L)**	**Transversely Polarized (T)**
Nucleon Polarization	**U**	f_1 = Unpolarized		$h_1^\perp$ = − Boer-Mulders
	L		g_1 = − Helicity	$h_{1L}^\perp$ = − Worm-gear
	T	$f_{1T}^\perp$ = − Sivers	$g_{1T}^\perp$ = − Worm-gear	h_1 = − Transversity $h_{1T}^\perp$ = − Pretzelosity

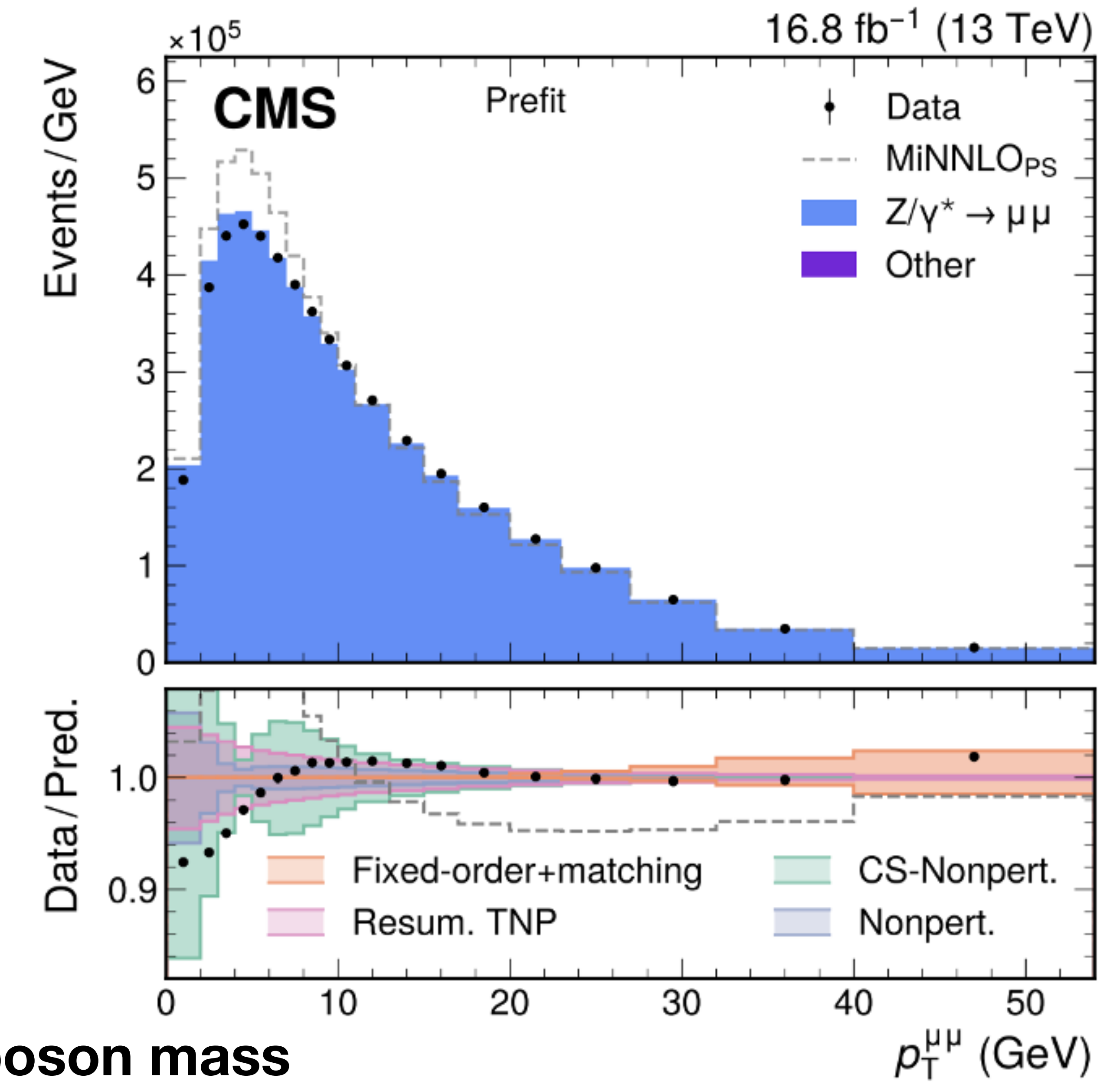
TMD Handbook, TMD Topical Collaboration, arXiv: 2304.03302.

W & Z mass measurements

High-precision measurement of the W boson mass with the CMS experiment at the LHC



CMS Collaboration, Nature, 652 (2026).



Reanalysis of CDF measurement of *W*-boson mass with different non-perturbative inputs:

A Avkhadiev (ANL), Y Fu (MIT), J Isaacson (MSU), M Wagman (FNAL), in prepration

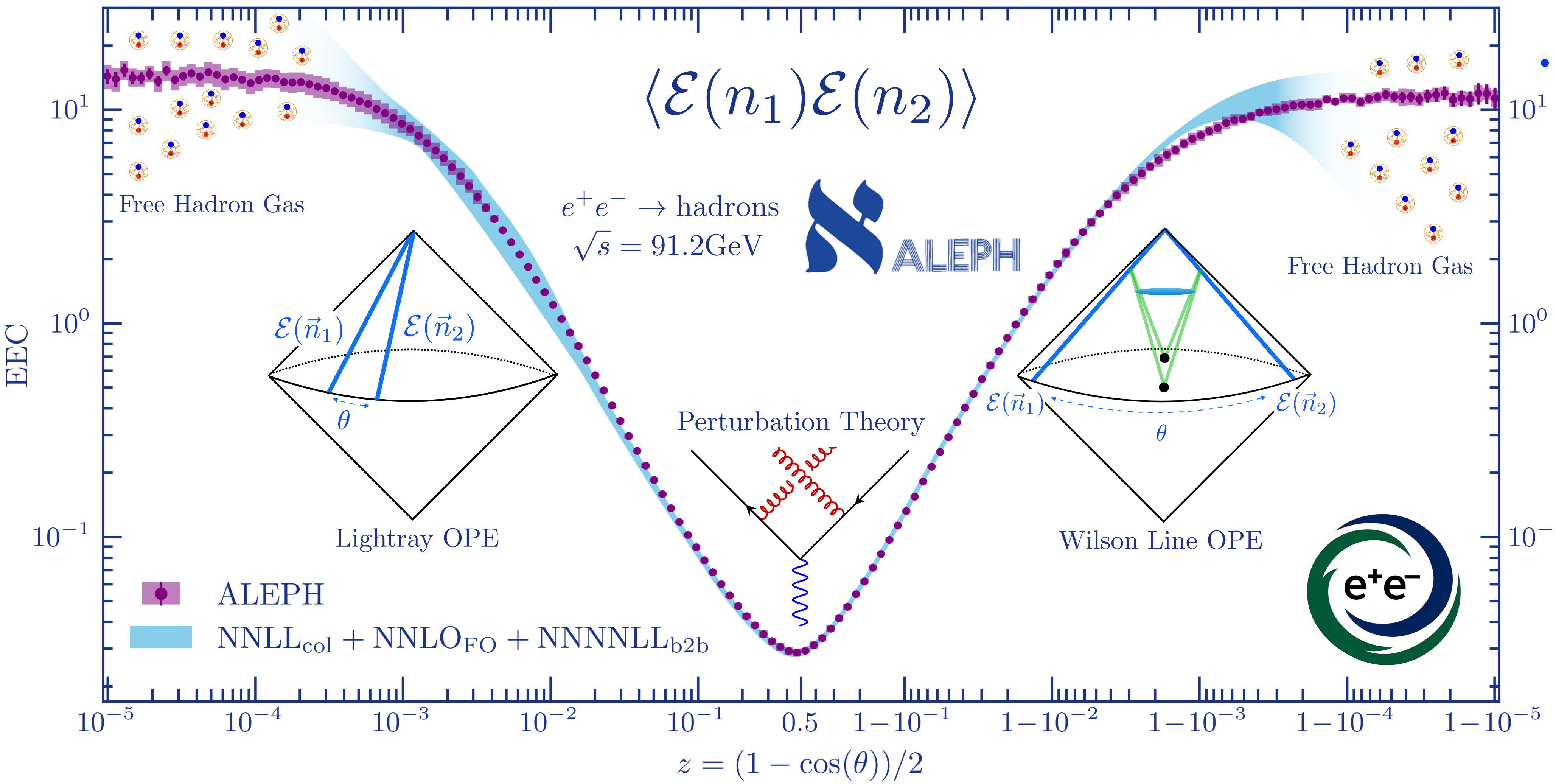
Slide provided by A. Avkhadiev

(Track-based) energy-energy correlators

Comparison to archival ALEPH data

See Z. Kang's talk.

- Over **order-of-magnitude** gain in angular resolution with tracks:



- **Excellent agreement between theory and data for a wide range of angles**

Lee et al. (theory: Jaarsma, Li, Moult, Waalewijn, and Zhu.

Slide provided by W. Waalewijn

(Track-based) energy-energy correlators

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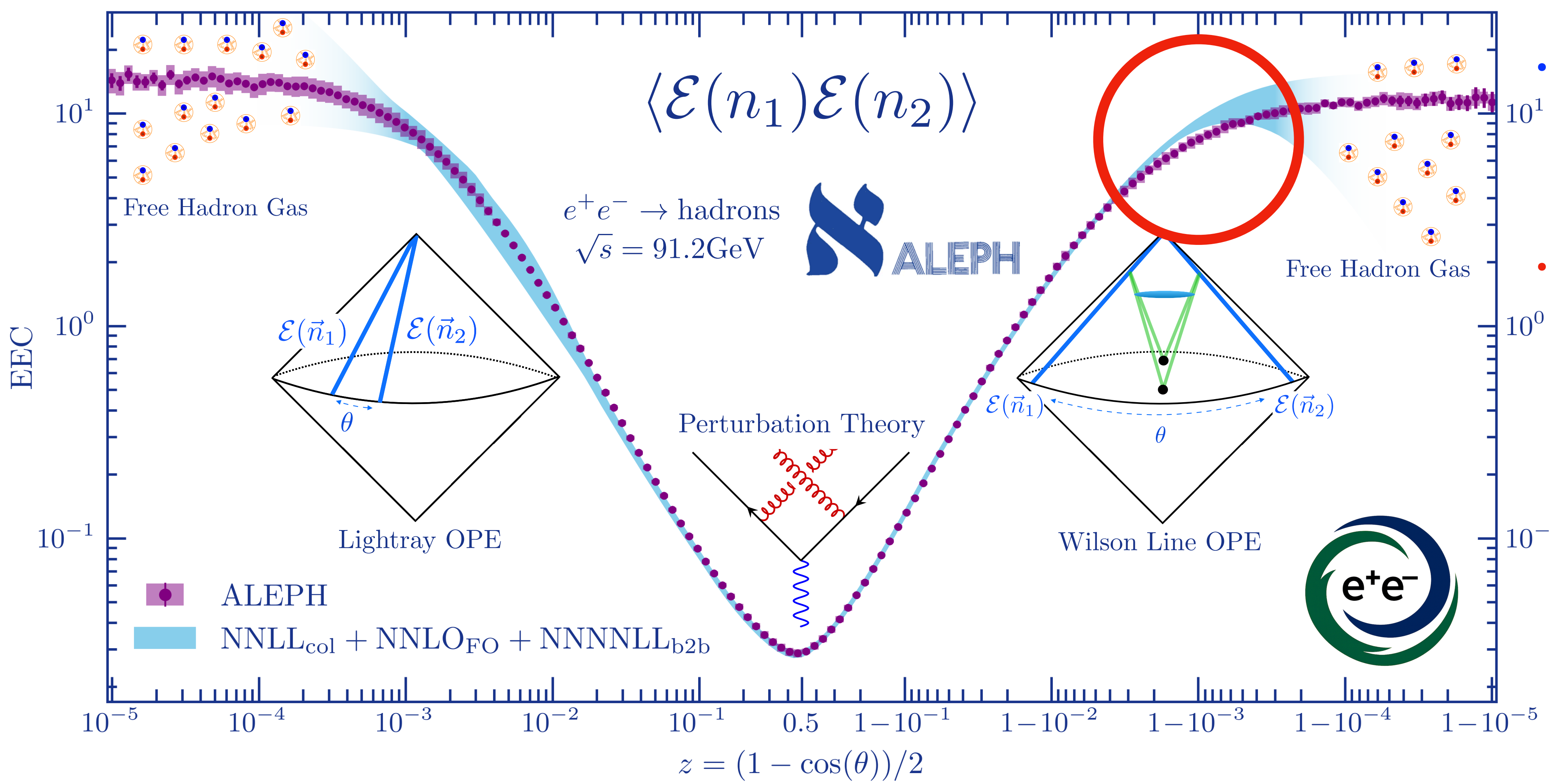


- **Excellent agreement between theory and data for a wide range of angles**
- **Deviation in the back-to-back region is due to the non-perturbative TMD parameters.**

Lee et al. (theory: Jaarsma, Li, Moult, Waalewijn, and Zhu.

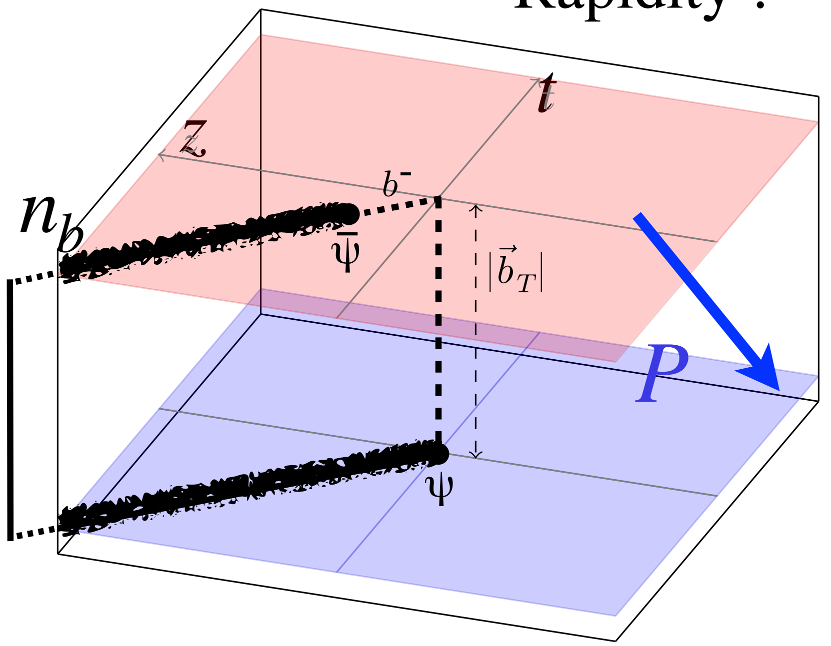
Slide provided by W. Waalewijn

TMD definition in QCD

- Beam function:

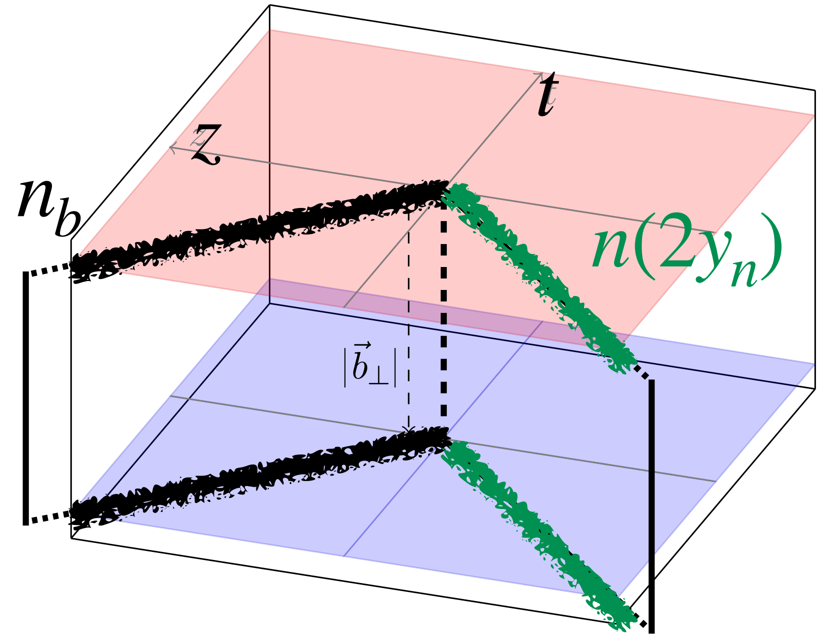
$$n_b^2 = 0$$

$$\text{Rapidity}: \quad y_B = \frac{1}{2}\ln\left|\frac{n_b^+}{n_b^-}\right| = -\infty$$



Hadronic matrix element

- Soft function :



Vacuum matrix element

$$f_i(x, \mathbf{b}_T, \mu, \zeta) = \lim_{\epsilon\to 0} Z_{\mathrm{UV}} \lim_{\tau\to 0} \frac{B_i}{\sqrt{S^q}}$$

Collins-Soper scale: $\zeta = 2(xP^+e^{-y_n})^2$

Rapidity divergence regulator

TMD definition in QCD

- Beam function:

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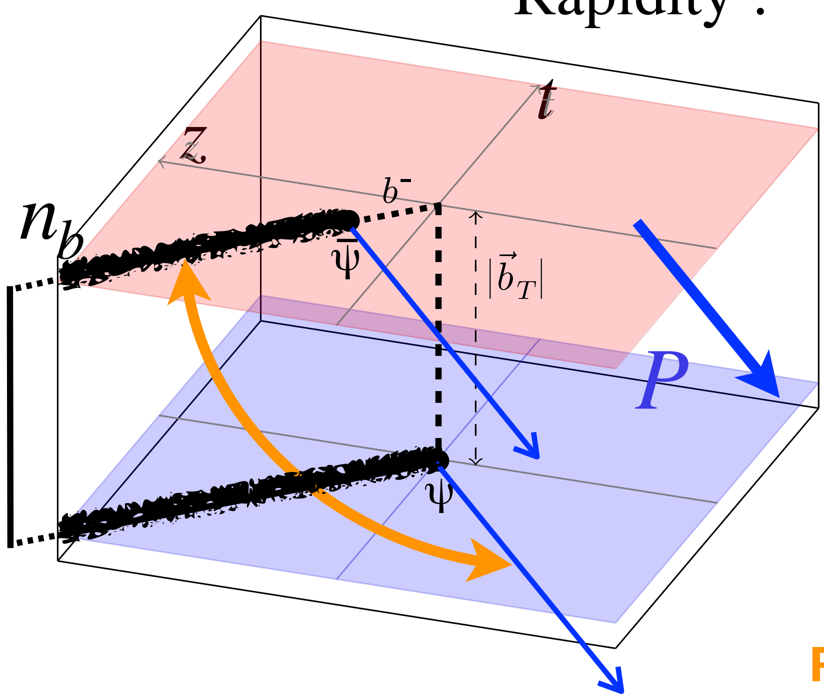
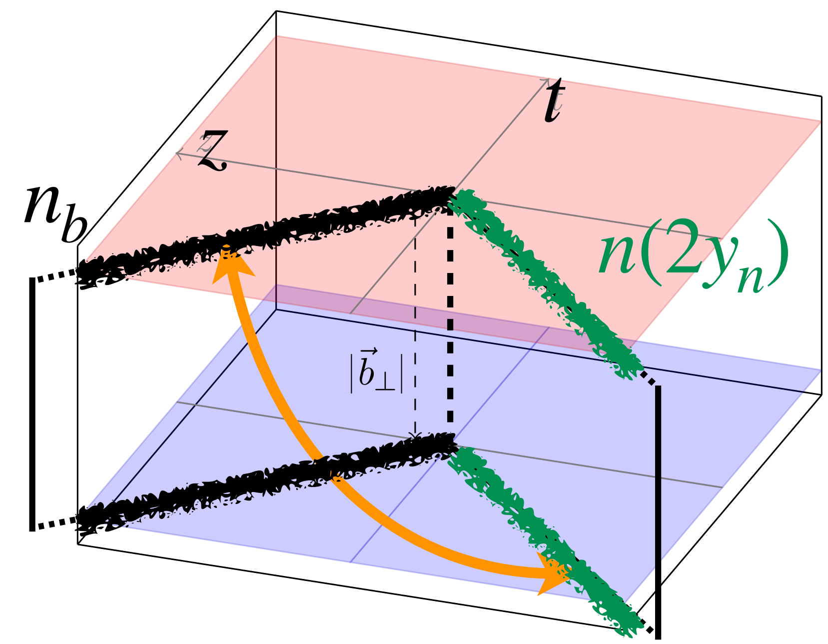


Hadronic matrix element

Rapidity divergences

- Soft function :



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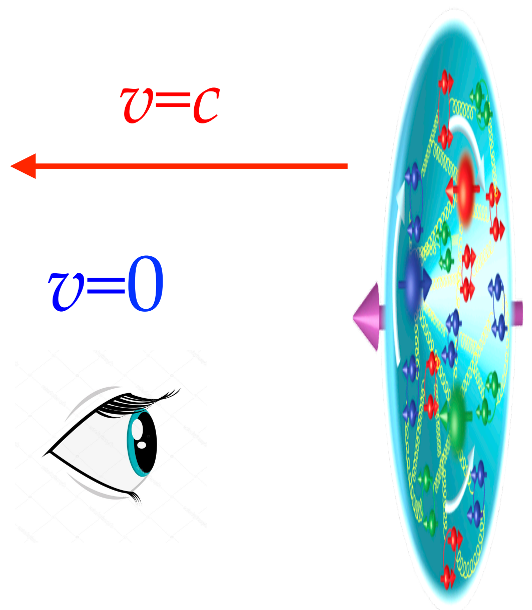
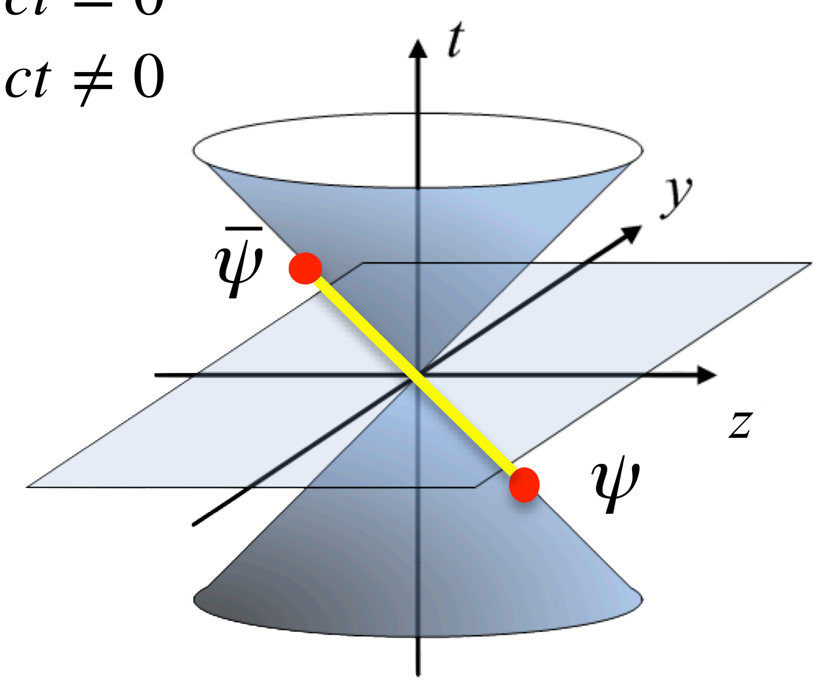
Rapidity divergence regulator

Simulating partons on the Euclidean lattice?

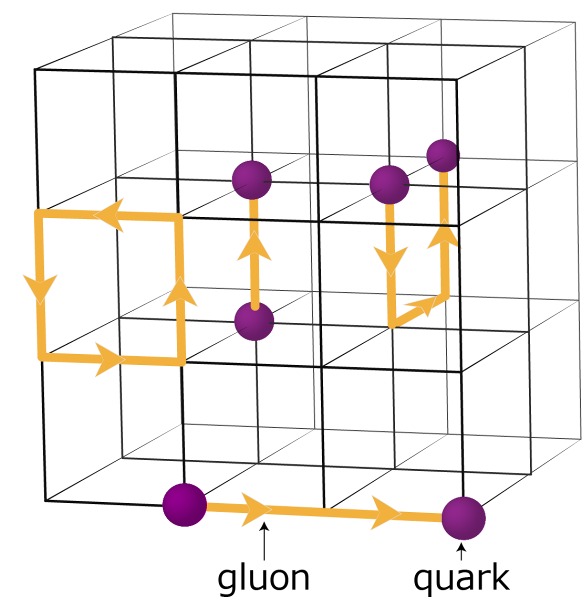


$z + ct = 0$

$z - ct \neq 0$



$t \to i\tau$

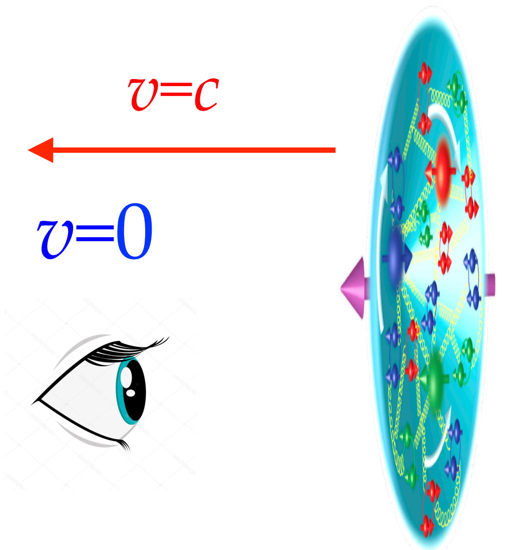


$O(i\tau) \overset{?}{\to} O(t)$

Time-dependence makes it impossible to calculate the lightlike (or close-to-lightlike) correlations directly on the Euclidean lattice. 🙁

Large-Momentum Effective Theory (LaMET)

Revisit the infinite momentum frame picture by Feynman:



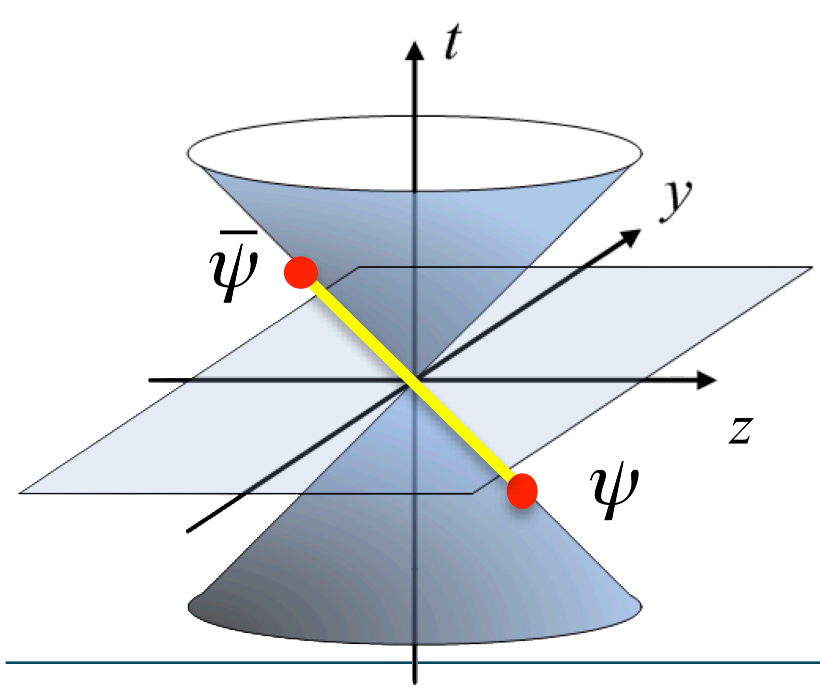
Simulating $\langle P=\infty\,|\,\tilde{O}(t=0)\,|\,P=\infty\rangle$?

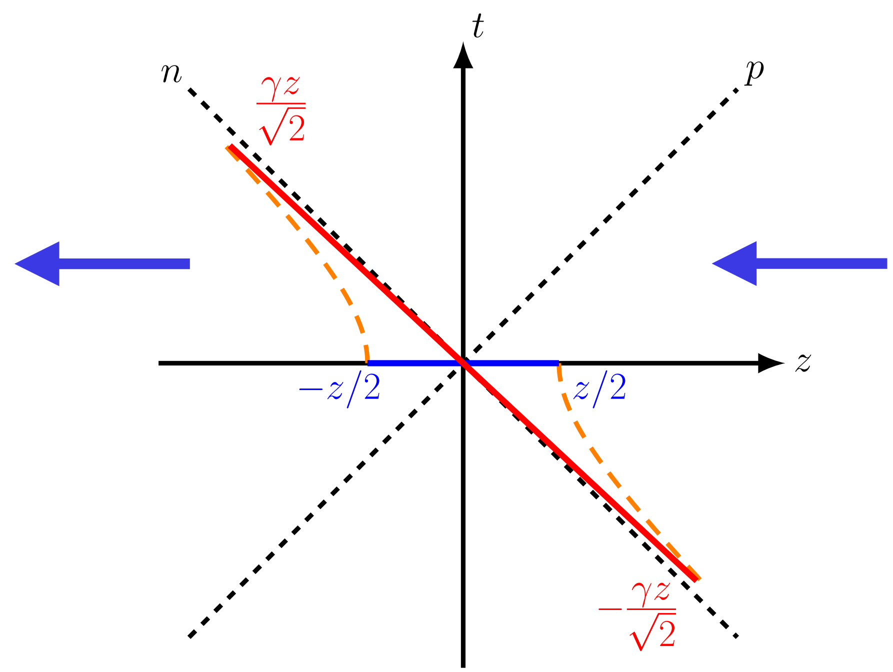
$$P \ll \frac{2\pi}{a}!$$

Nevertheless, it is possible to simulate a proton at large *P*:

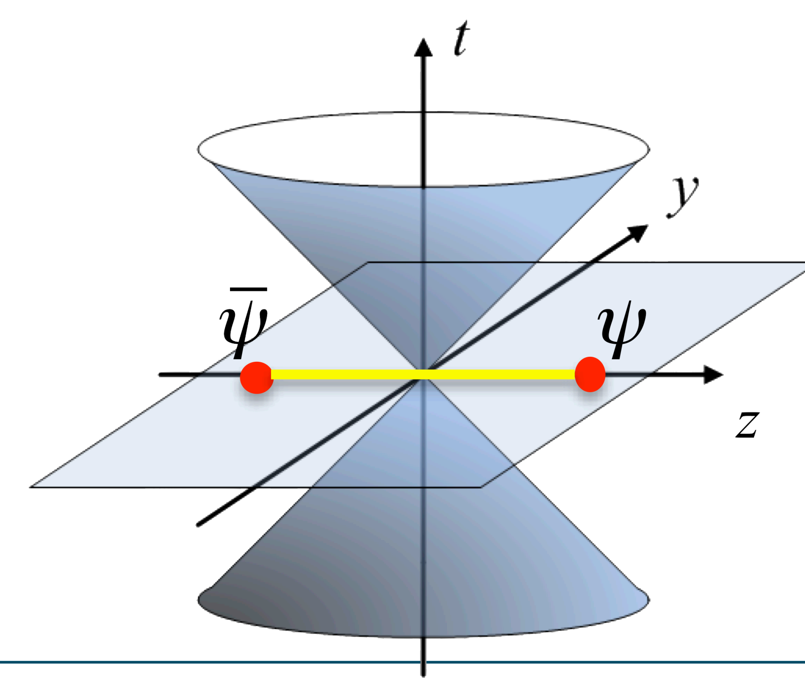
- X. Ji, PRL **110** (2013); SCPMA **57** (2014).
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

$z+ct=0,\quad z-ct\neq 0$





$t=0,\quad z\neq 0$



Large-Momentum Effective Theory (LaMET)

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{2xP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \ + \ \mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(xP^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((1-x)P^z)^2}\right)$$

- Power expansion in parton momentum
  - X. Ji, PRL **110** (2013); SCPMA **57** (2014).
  - X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).
- Valid for a moderate range of *x* at finite P^z
- No parameterization from global analyses
- Matching kernel available at N3LO now

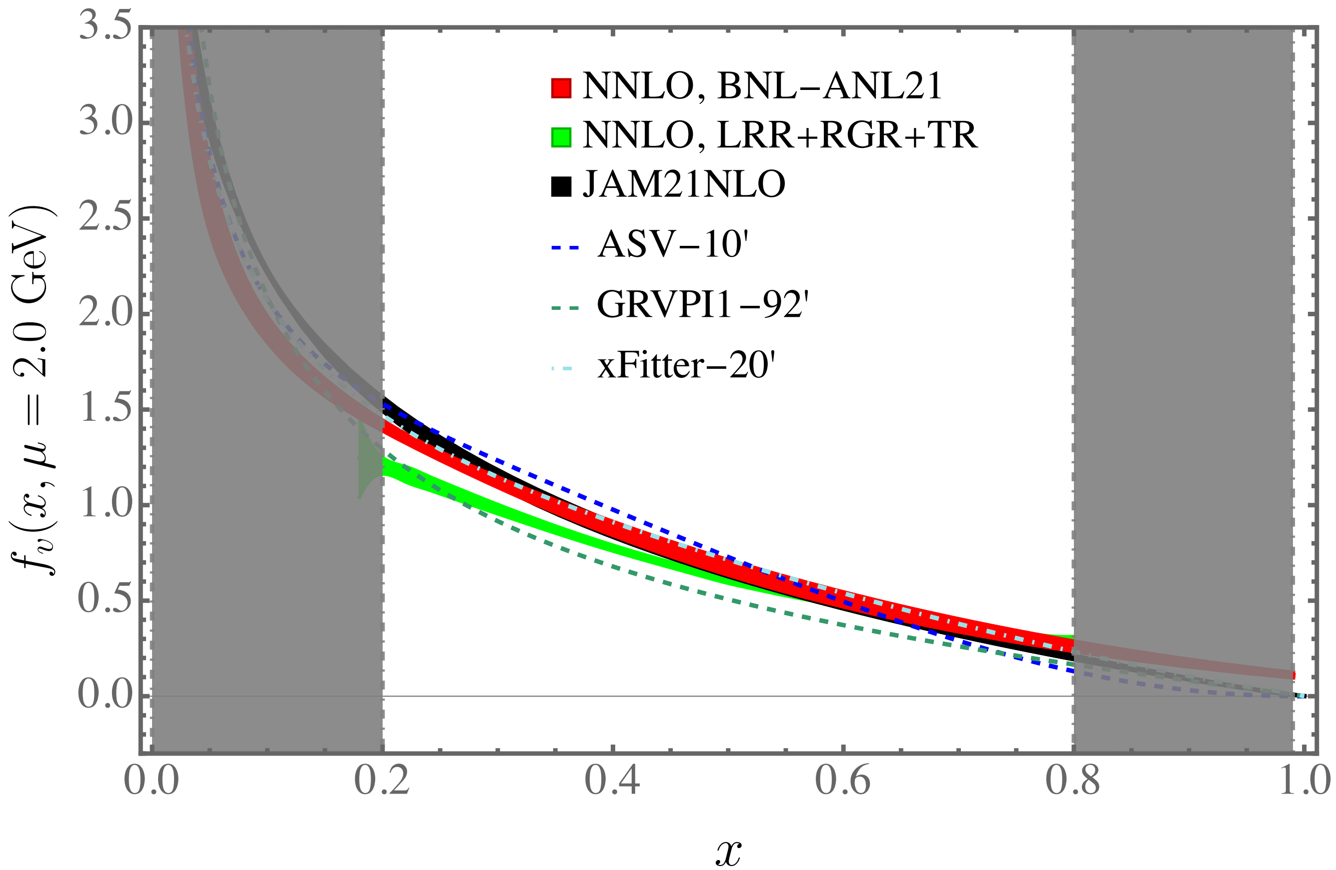
NNLO
- Chen, Zhu and Wang, PRL. **126** (2021);
- Li, Ma and Qiu, PRL **126** (2021);

N3LO
- Cheng, Huang, Li, Li and Ma, PRL 134 (2025).

- Leading renormalon subtraction which improves power accuracy
  - Holligan, Ji, Lin, Su and Zhang, NPB **993** (2023);
  - Zhang, Ji, Holligan and Su, PLB **844** (2023).
- DGLAP ($x \to 0$) and threshold ($x \to 1$) resummations
  - X. Gao, K. Lee, S. Mukherjee, C. Shugert and YZ, **PRD** 103 (2021);
  - Y. Su, X. Ji, and J. Holligan et al., **NPB** 991 (2023);
  - X. Ji, Y. Liu and Y. Su, **JHEP** 08 (2023) 037;
  - Y. Liu and Y. Su, **JHEP** 2024 (2024) 204;
  - X. Ji, Y. Liu, Y. Su and R. Zhang, **JHEP** 03 (2025).
  - J. Holligan, H.-W. Lin, R. Zhang and YZ, **JHEP** 207 (2025).

The *x*-dependence of PDFs

Pion valence PDF



YZ, Research 9 (2026)

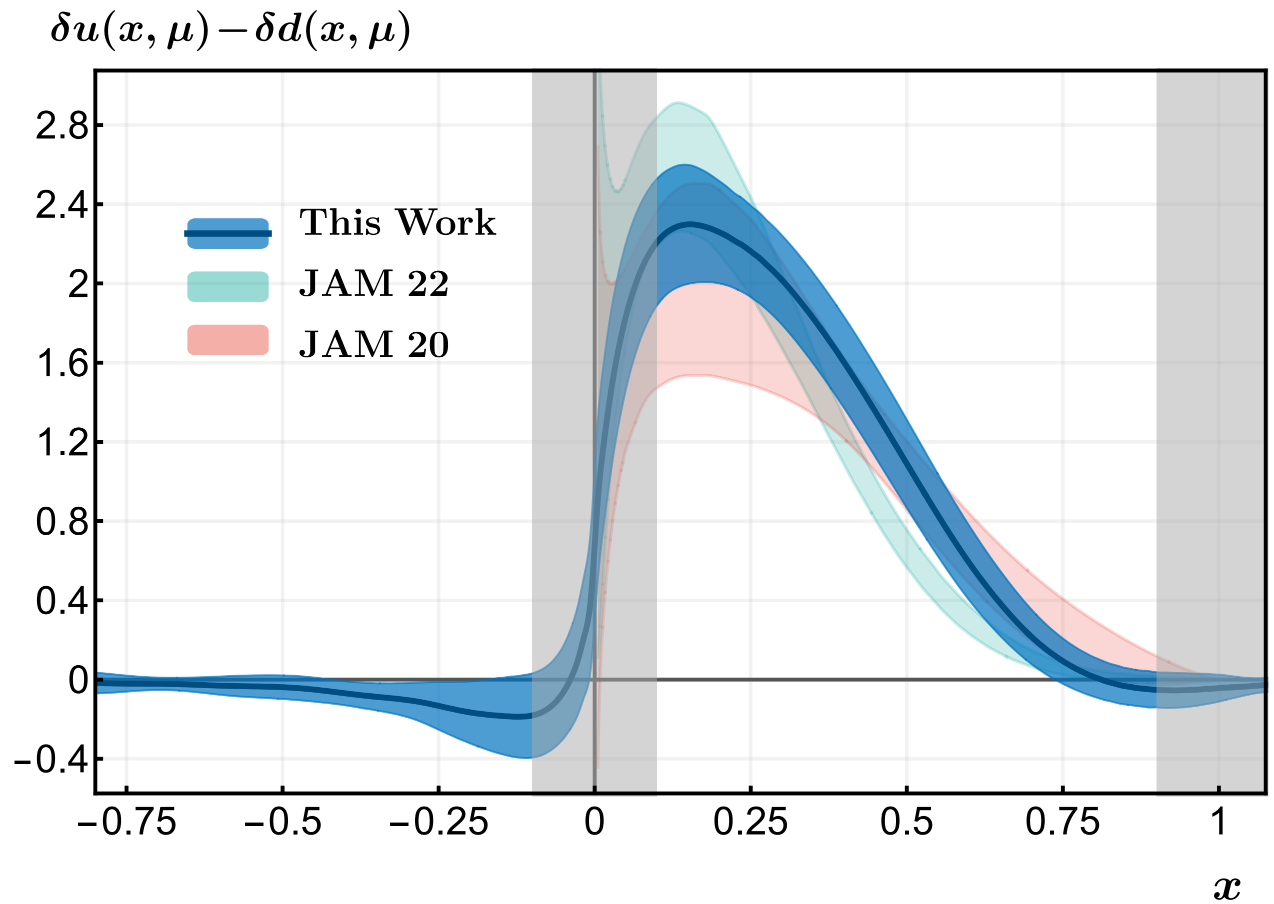
NNLO, BNL-ANL21

X. Gao, A. Hanlon and YZ, et al., PRL **128** (2022)

NNLO, LRR+RGR+TR

X. Ji, Y. Liu, Y. Su and R. Zhang, JHEP 03 (2025)

Nucleon isovector transversity PDF

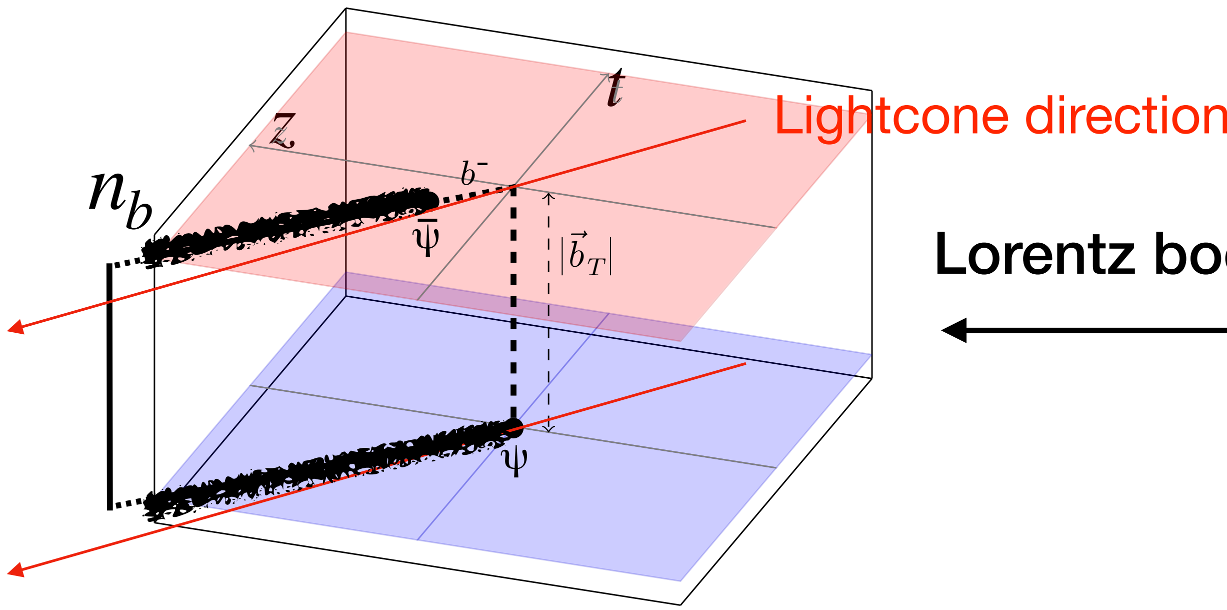


F. Yao et al. (Lattice Parton Collaboration), PRL **131** (2023)

See C. Alexandrou's talk for a broader review.

TMDs from LaMET

- Beam function (Collins's scheme):

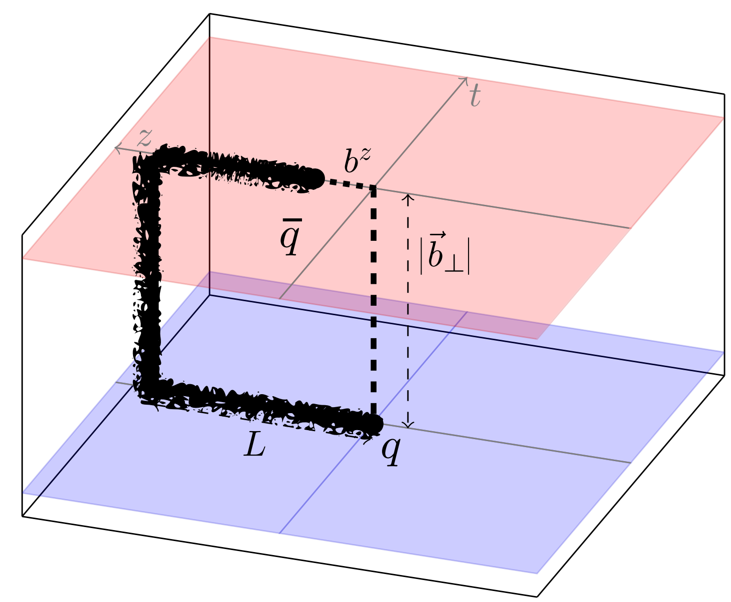


$$n_b^\mu(y_B) = (n_b^+, n_b^-, \vec{0}_\perp) = (-e^{2y_B}, 1, \vec{0}_\perp)$$

Spacelike but close-to-light-cone ($y_B \to -\infty$) Wilson lines, not calculable on the lattice ☹️

- Quasi beam function :

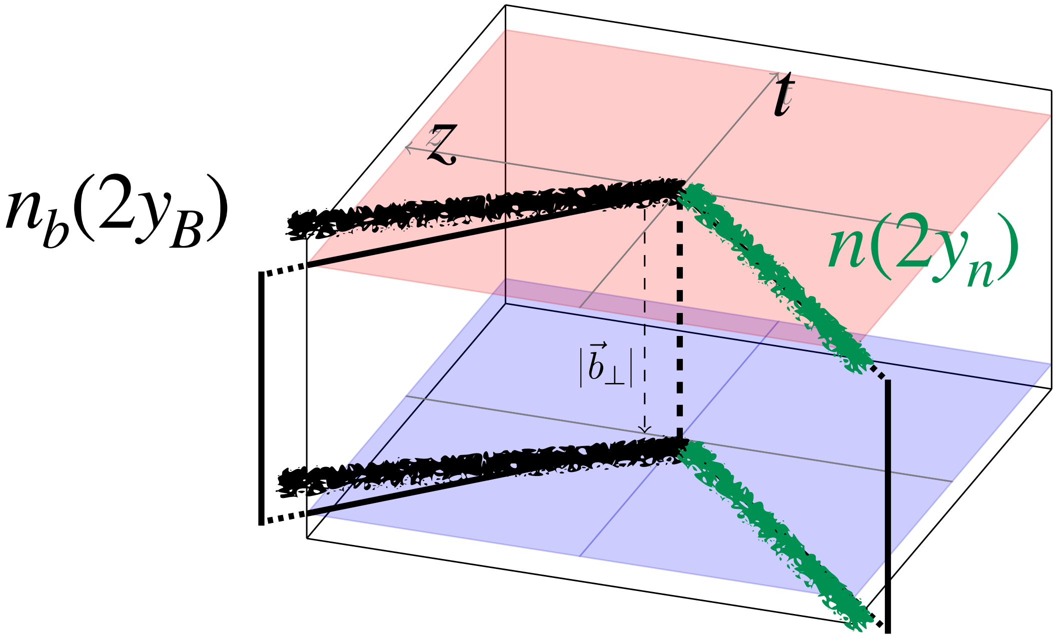
Lorentz boost and $L \to \infty$



Equal-time Wilson lines, directly calculable on the lattice🙂

Ebert, Schindler, Stewart and YZ, JHEP **04** (2022).

Soft factor



$$\xrightarrow{y_n - y_B \to \infty} S_r(b_T, \mu)\; e^{-2(y_n - y_B)\gamma_\zeta(b_T,\mu)}$$

Collins-Soper kernel

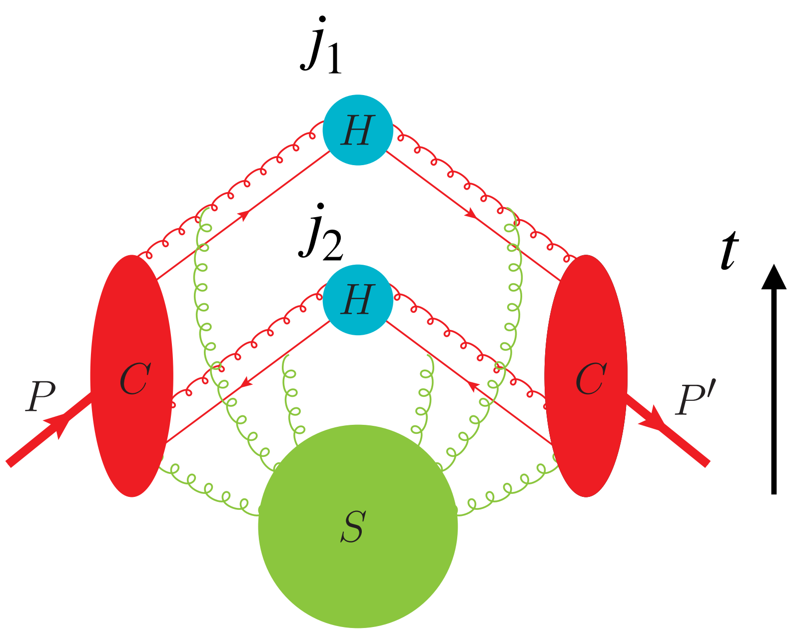
Reduced soft factor

Light-meson form factor:

$$F(b_T, P^z) = \langle \pi(-P) | j_1(b_T) j_2(0) | \pi(P) \rangle$$

$$\stackrel{P^z \gg m_N}{=} S_r(b_T, \mu) \int dx dx' \; H(x, x', \mu)$$

$$\times \Phi^\dagger(x, b_T, P^z, \mu) \Phi(x', b_T, P^z, \mu)$$



$\Phi(x, b_T, P^z, \mu)$: **quasi-TMD wave function**

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ji and Liu, PRD **105** (2022);
- Deng, Wang and Zeng, JHEP **09** (2022).

Factorization formula for the quasi-TMDs

$$\frac{\tilde{f}_{i/p}^{\mathrm{naive}[s]}(x, \mathbf{b}_T, \mu, \tilde{P}^z)}{\sqrt{S_r(b_T, \mu)}} = C\left(\mu, x\tilde{P}^z\right) \exp\left[\frac{1}{2}\gamma_\zeta(\mu, b_T)\ln\frac{(2x\tilde{P}^z)^2}{\zeta}\right]$$
$$\times f_{i/p}^{[s]}(x, \mathbf{b}_T, \mu, \zeta)\left\{1 + \mathcal{O}\left[\frac{1}{(x\tilde{P}^z b_T)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(x\tilde{P}^z)^2}\right]\right\}$$

- Ji, Sun, Xiong and Yuan, PRD91 (2015);
- Ji, Jin, Yuan, Zhang and YZ, PRD99 (2019);
- Ebert, Stewart, YZ, PRD99 (2019), JHEP09 (2019) 037;
- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- Ebert, Schindler, Stewart and YZ, JHEP 09 (2020);
- Vladimirov and Schäfer, PRD 101 (2020);
- Ji, Liu, Schäfer and Yuan, PRD 103 (2021);
- Ebert, Schindler, Stewart and YZ, JHEP 04, 178 (2022).

✳ Collins-Soper kernel $\gamma_\zeta(\mu, b_T)$;

✳ Flavor separation; $\dfrac{f_{i/p}^{[s]}(x, \mathbf{b}_T)}{f_{j/p}^{[s']}(x, \mathbf{b}_T)} = \dfrac{\tilde{f}_{i/p}^{\mathrm{naive}[s]}(x, \mathbf{b}_T)}{\tilde{f}_{j/p}^{\mathrm{naive}[s']}(x, \mathbf{b}_T)}$

✳ Spin-dependence;

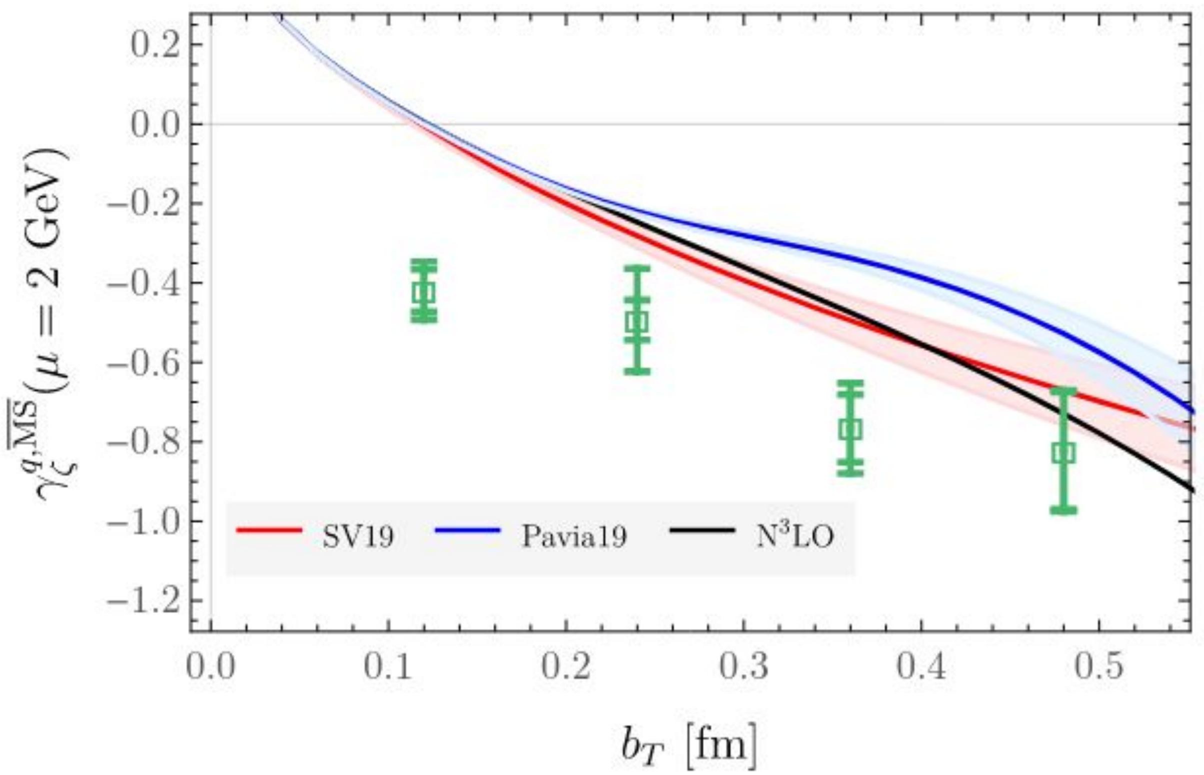
✳ Full TMD kinematic dependence.

✳ Twist-3 PDFs from small b_T expansion of TMDs. Ji, Liu, Schäfer and Yuan, PRD 103 (2021).

✳ Higher-twist TMDs. Rodini and Vladimirov, JHEP 08 (2022).

Collins-Soper (CS) kernel

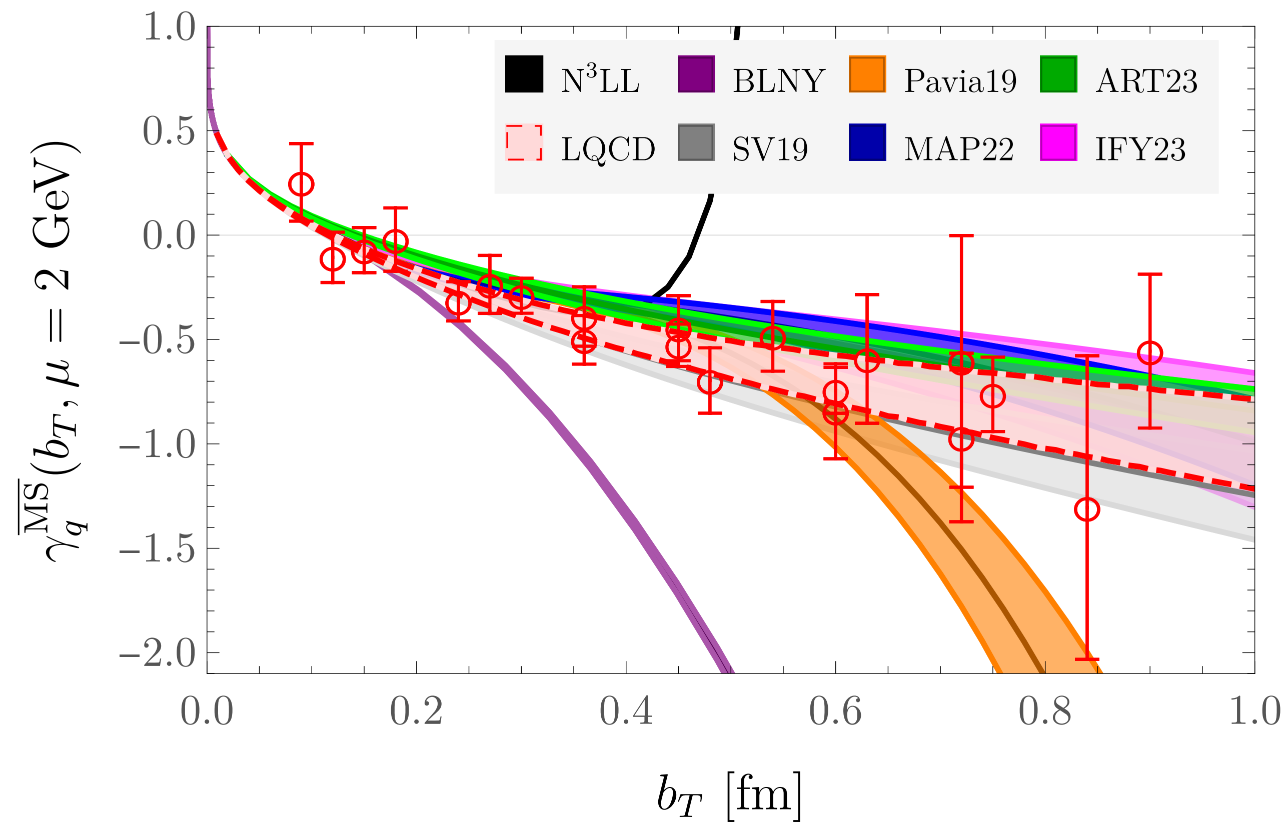
2021



Shanahan, Wagman and YZ, PRD **104** (2021).

- Pion mass 580 MeV
- Largest momentum P^z=1.5 GeV
- a=0.12 fm
- Next-to-leading order (NLO) order matching

2024



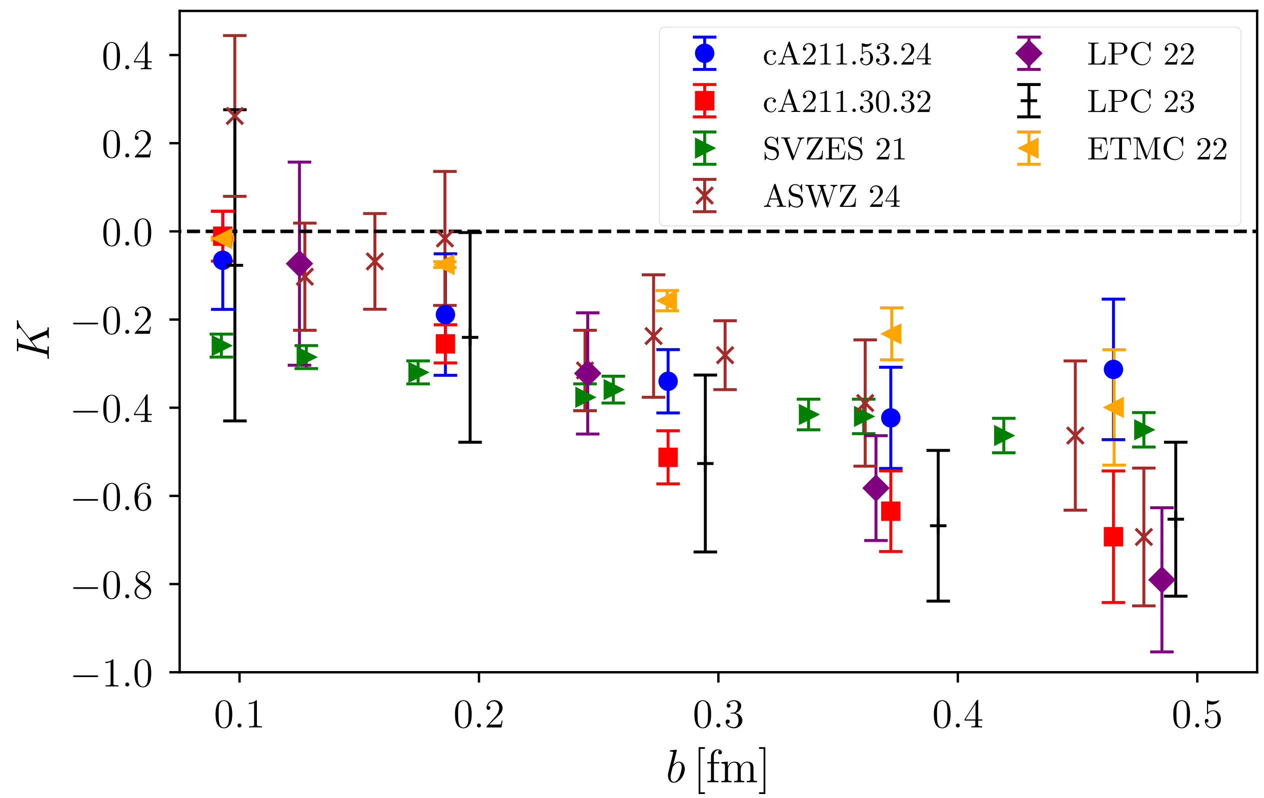
Avkhadiev, Shanahan, Wagman and YZ, PRD **108** (2023), **PRL 132** (2024).

- Physical quark masses, large Lorentz boosts
- Continuum extrapolation with a=0.15, 0.12, 0.09 fm
- Controlled Fourier transform
- Lattice renormalization and operator mixing subtraction
- Next-to-next-to-leading logarithmic (NNLL) order matching

First gluon CS-kernel to appear soon!

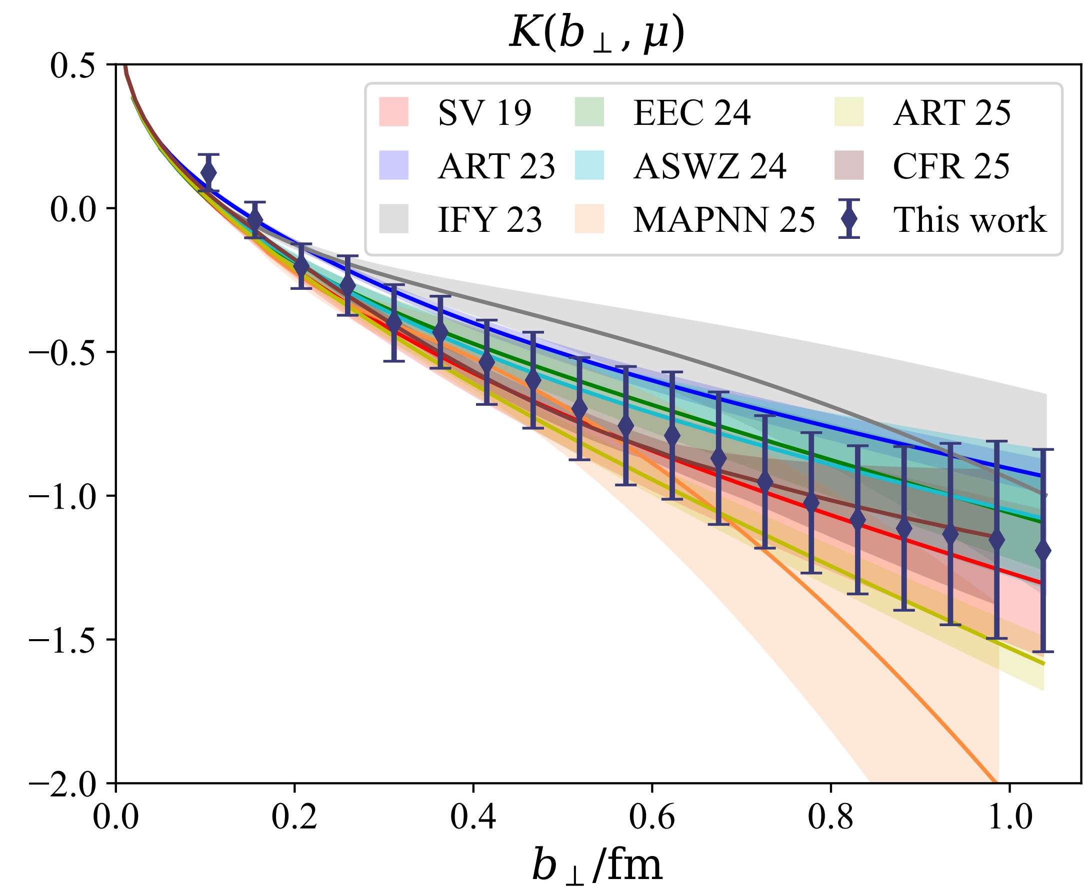
Collins-Soper (CS) kernel

More recent results:



C. Alexandrou, S. Bacchio, K. Cichy et al., ETM Collaboration, PRD **113** (2026).

- Pion mass 640 and 830 MeV
- Largest momentum P^z=3.3 GeV
- a=0.093 fm
- Next-to-leading order (NLO) order matching

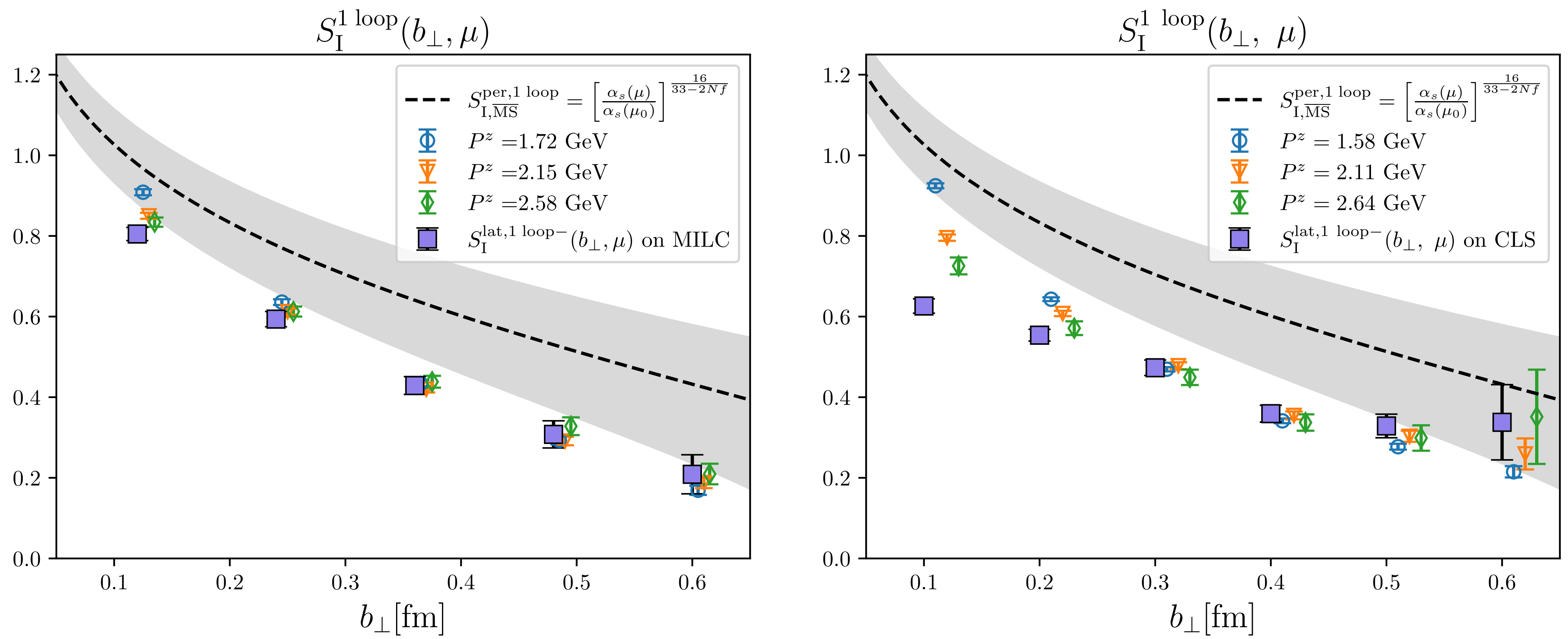


J.-X. Tan, Z.-C. Gong, J. Hua et al., LPC, PRD **113** (2026).

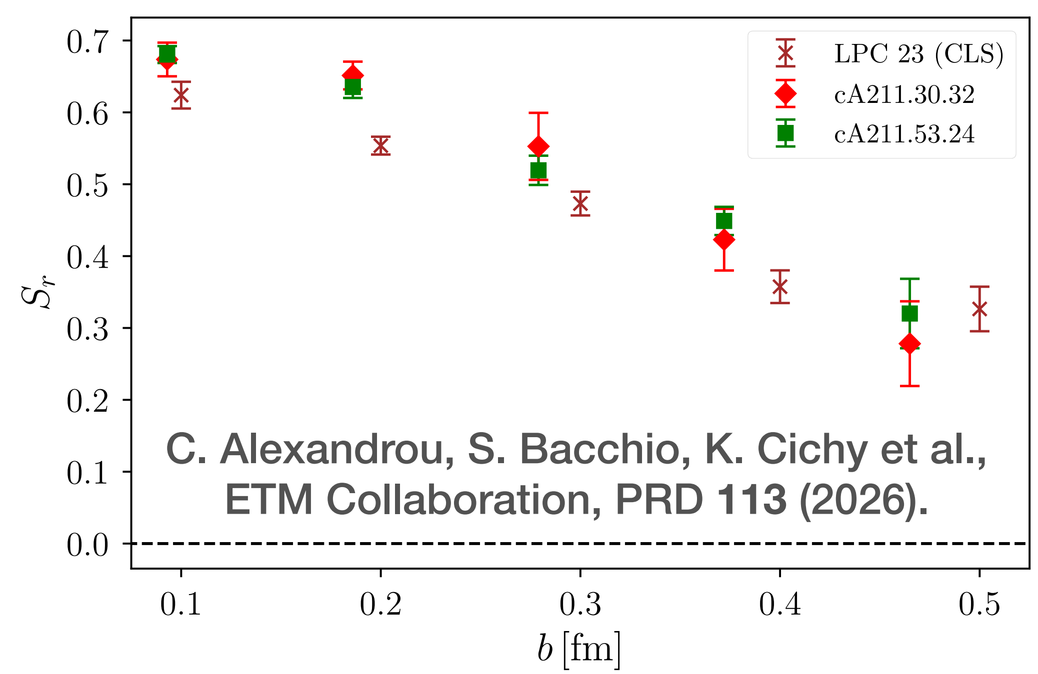
- Pion mass 140, 230 and 300 MeV
- Largest momentum P^z=3.0 GeV
- a=0.05, 0.08, 0.1 fm
- NNLL matching

An autonomous AI "Physics Master" analysis, J.-X. Tan et al., 2603.22471

(Reduced) soft function



M.-H. Chu, et al. (LPC), JHEP 08 (2023).

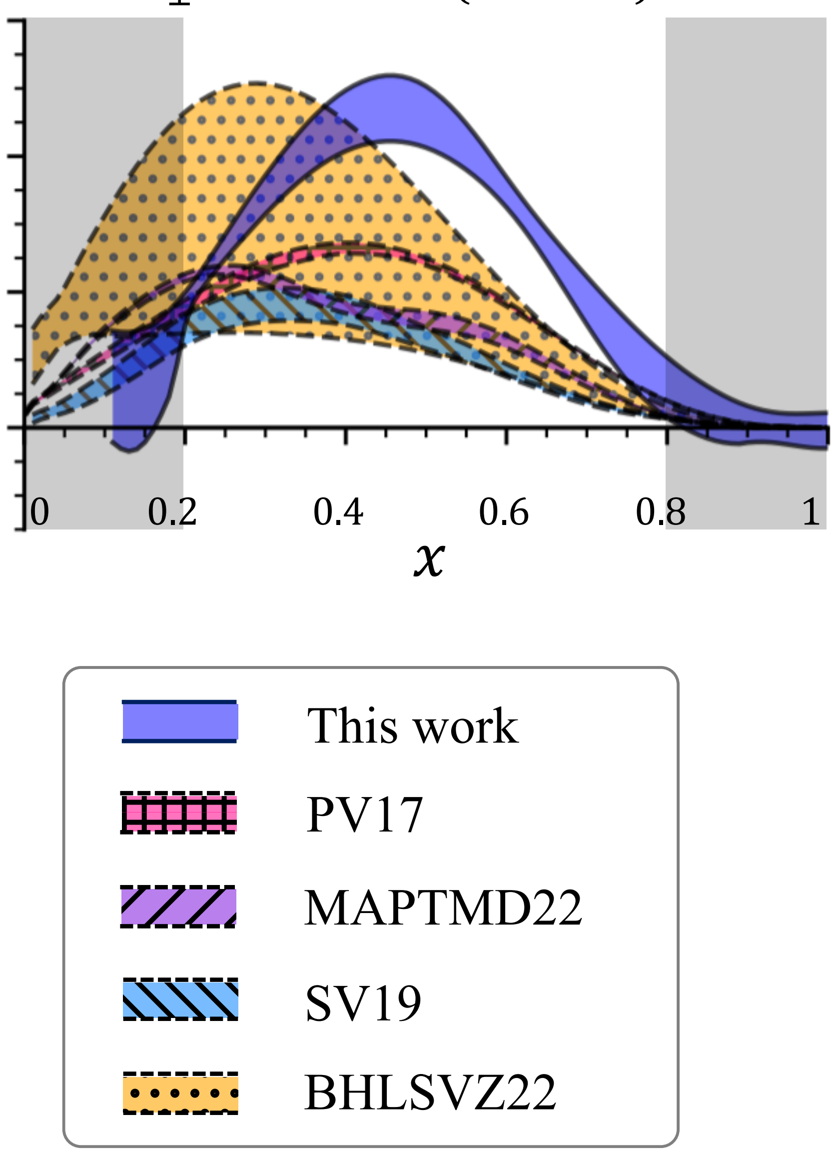


C. Alexandrou, S. Bacchio, K. Cichy et al., ETM Collaboration, PRD 113 (2026).

Nucleon isovector TMDs

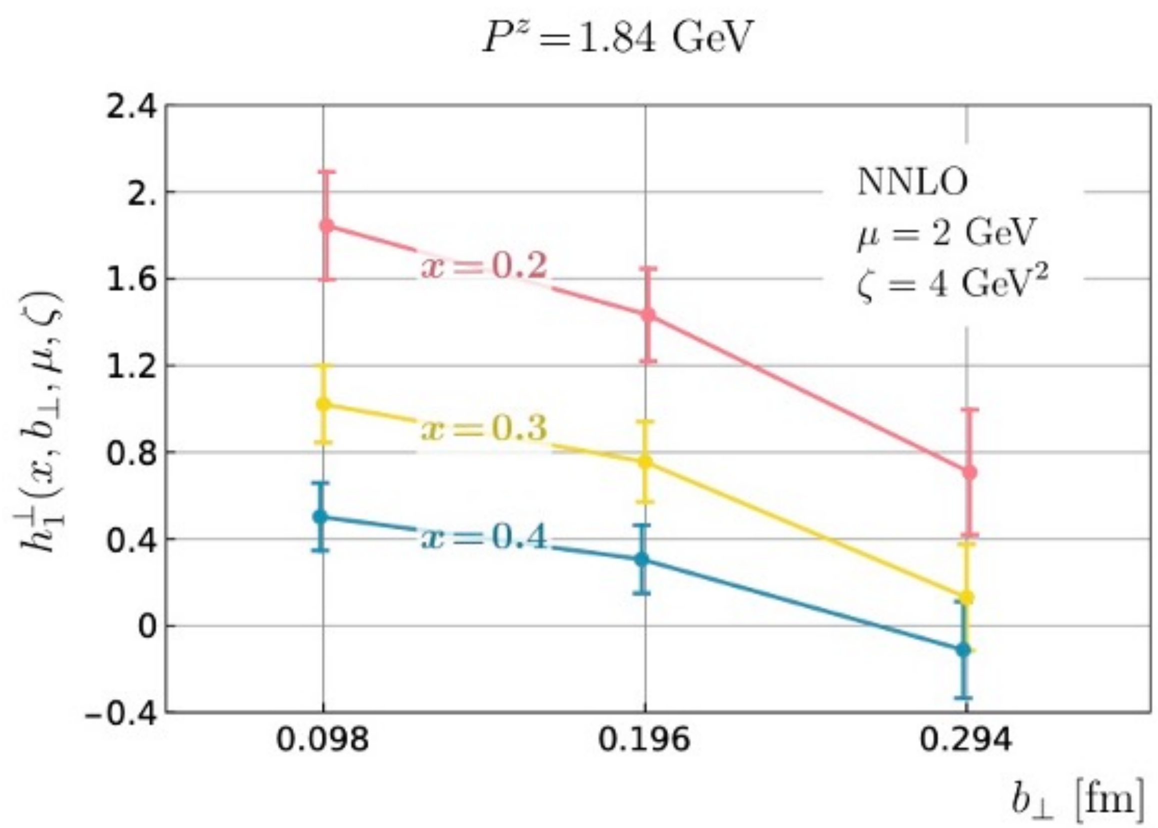
Unpolarized TMD

$b_\perp = 0.36\text{fm} = (0.55\text{GeV})^{-1}$



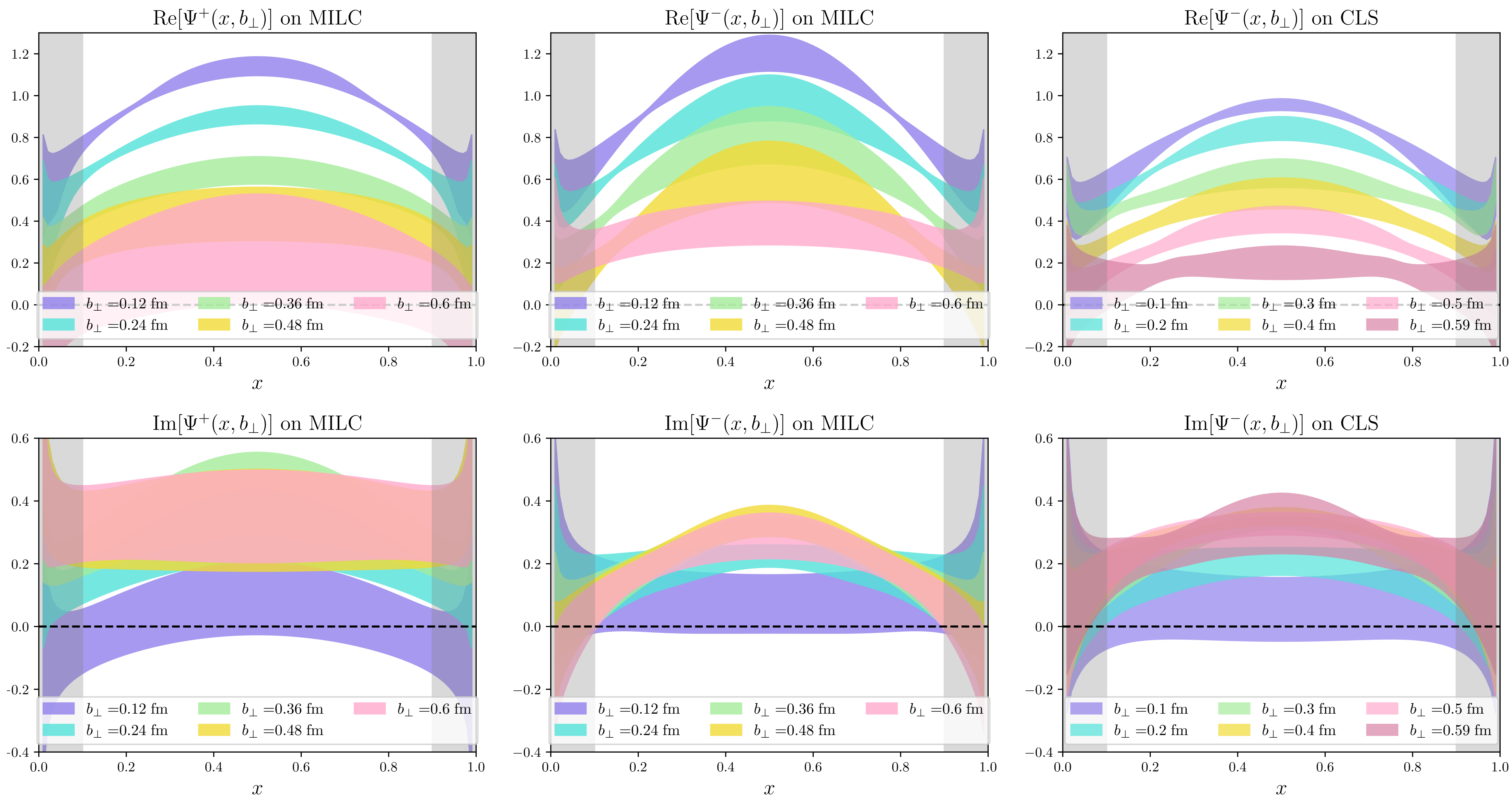
J.-C. He et al. (Lattice Parton Collaboration), PRD **109** (2024).

Boer-Mulders Function



L. Ma et al. (Lattice Parton Collaboration), JHEP **08** (2025).

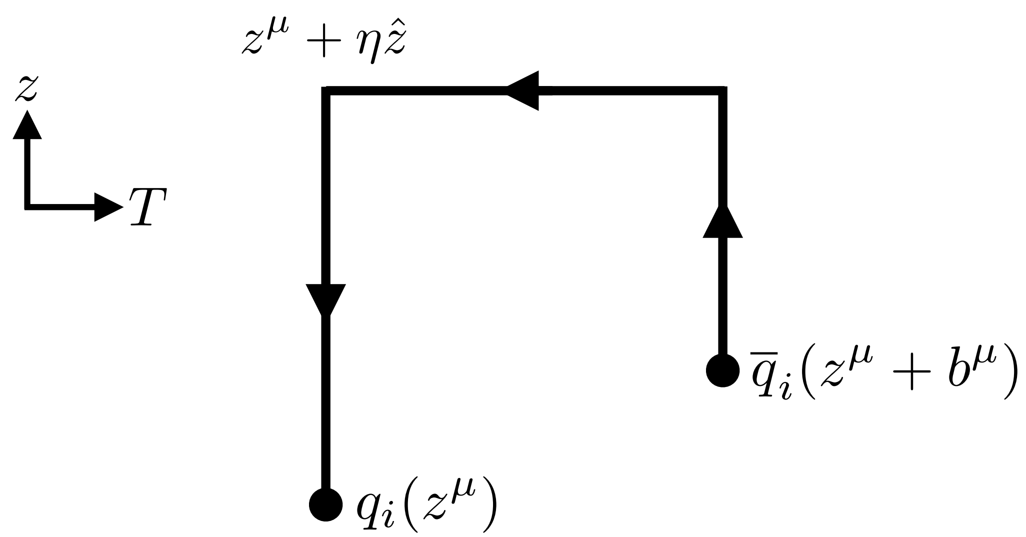
TMD wave function of the pion



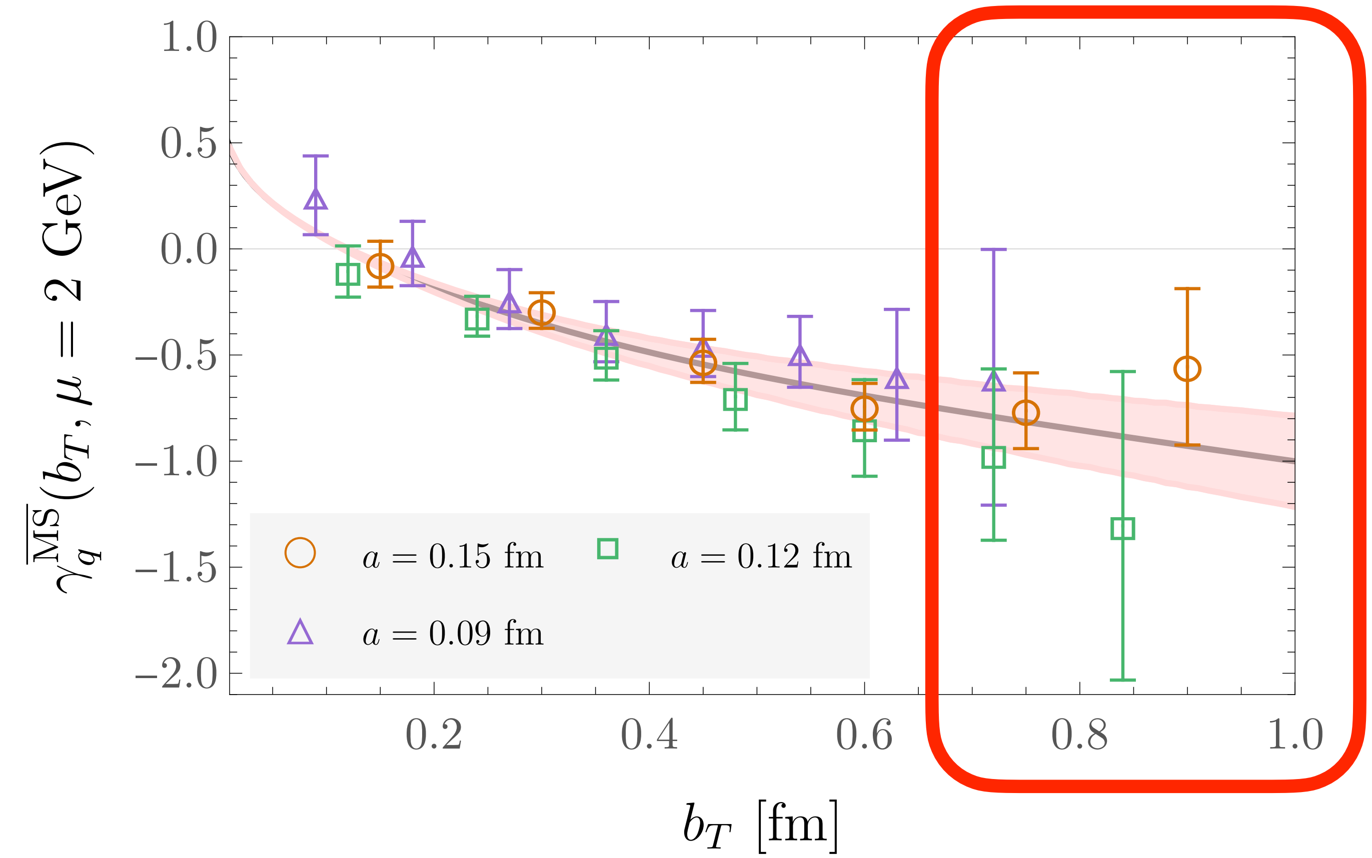
M.-H. Chu et al. (Lattice Parton Collaboration), PRD **109** (2024).

Some key systematics in lattice calculation

Staple-shaped Wilson line



$\eta \gg \{b^z, b_T\}$, $xP^z \gg 1/b_T$



Can we go further non-perturbative?

- Gauge link induces statistical noise, while signal decays exponentially at large b_T due to the linear divergence $e^{-\delta m(a)[2\eta + b_T]}$ in the Wilson line self-energy;
- Complex operator mixings due to the breaking of symmetries by the staple;
- Additional systematics due to multiple scales $\{b^z, b_T, \eta\}$ involved.

Universality class in LaMET

Parton distributions probe the correlation of energetic quarks and gluons, which can be formulated by dressing them in a physical gauge.

"Universality Class"

- Y. Hatta, X. Ji, and YZ, PRD **89** (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, RMP **93** (2021).

$$G(A) = 0, \quad G(A) = \underbrace{A^0, A^z, A^+}_{\text{Axial gauges}}, \underbrace{\nabla \cdot \mathbf{A}}_{\text{Coulomb gauge}}$$

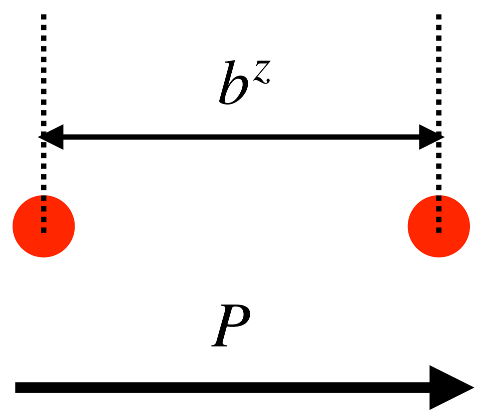
Coulomb gauge

Axial gauges: equivalent to the Wilson-line operators up to a boundary condition.

Quasi-TMD in the Coulomb gauge

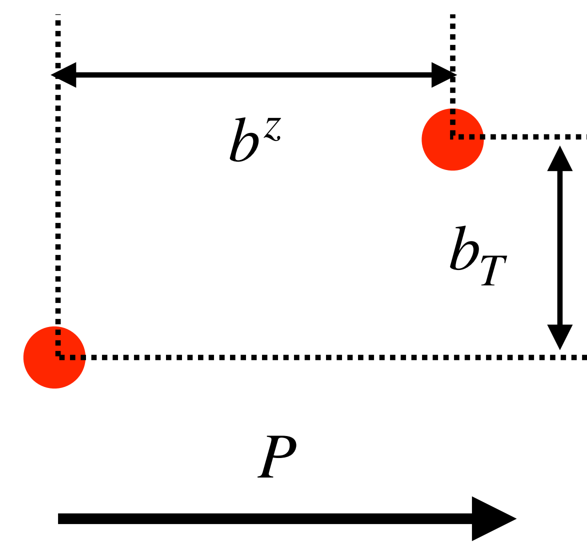
$$\tilde{f}(x, b_T, P^z, \mu) = \frac{P^z}{P^t}\int_{-\infty}^{\infty}\frac{db^z}{4\pi}\, e^{ixP^z b^z}\langle P\,|\,\bar{\psi}(\vec{b})\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0}\,|\,P\rangle$$

Quasi-PDF



X. Gao, W.-Y. Liu and YZ, PRD 109 (2024).

Quasi-TMD

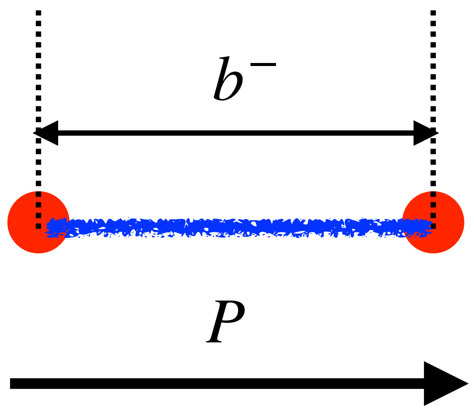


YZ, PRL 133 (2024).

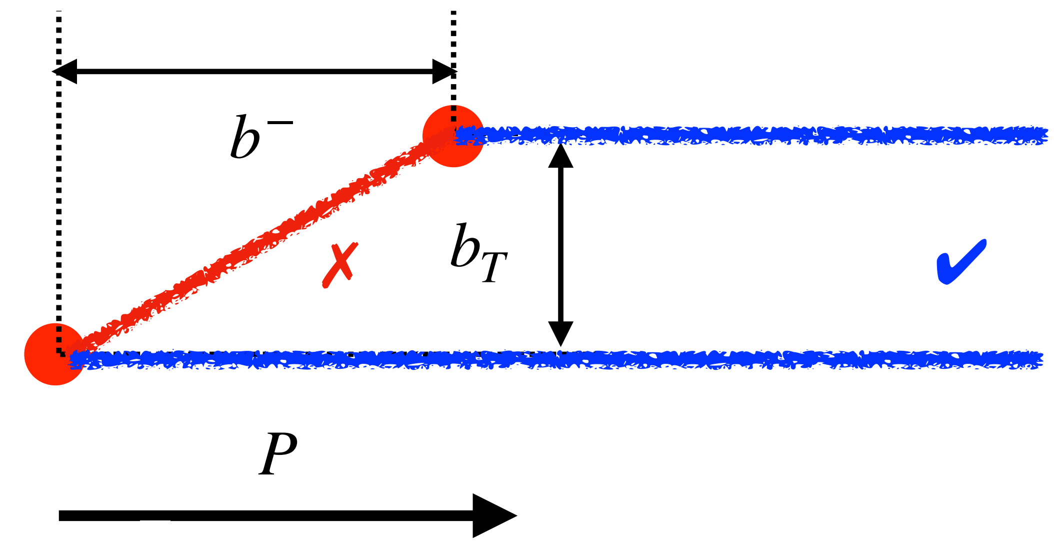
Quasi-TMD under the infinite boost

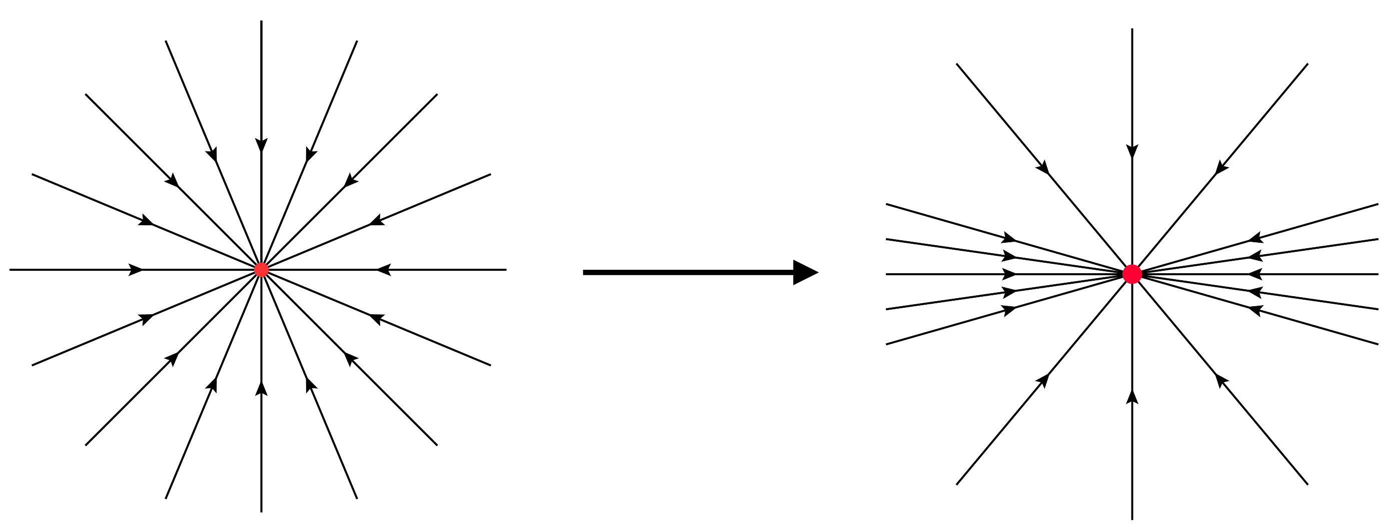
$$\tilde{f}(x, b_T, P^z, \mu) = \frac{P^z}{P^t}\int_{-\infty}^{\infty}\frac{db^z}{4\pi}\, e^{ixP^zb^z}\langle P\,|\,\bar{\psi}(\vec{b})\gamma^t\psi(0)\Big|_{\nabla\cdot\mathbf{A}=0}\,|\,P\rangle$$

PDF



TMD





Factorization at large momentum

YZ, PRL 133 (2024).

$$\frac{\tilde{B}_C(x, b_\perp, \mu, P^z)}{\tilde{S}_C(b_\perp, \mu, 0)} = |C(xP^+/\mu)|^2 \exp\left[\frac{1}{2}\gamma_\zeta(b_\perp, \mu)\ln\frac{2(xP^+)^2}{\zeta}\right] \times f(x, b_\perp, \mu, \zeta) + O\left(\frac{\Lambda^n_{\rm QCD}}{(xP^z)^n}, \frac{1}{(xP^z b_\perp)^n}\right)$$

Quasi soft factor:

$$\frac{d}{dy_n}\ln\tilde{S}_C(b_\perp, \mu, y_n) = \gamma_\zeta(b_\perp, \mu)$$

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- YZ, PRL 133 (2024).

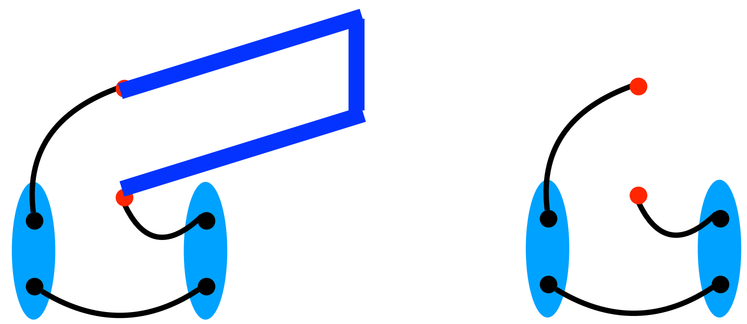
Calculable from the same meson form factor:

$$F(b_T, P^z) = \langle \pi(-P)\,|\,j_1(b_T)j_2(0)\,|\,\pi(P)\rangle$$

$$\stackrel{P^z \gg m_N}{=} \tilde{S}_r(b_T, \mu, 0)\int dx dx'\; H(x, x', \mu) \times \Phi^\dagger_C(x, b_T, P^z, \mu)\Phi_C(x', b_T, P^z, \mu)$$

Why the Coulomb gauge?

- Significantly improved statistical precision, access to larger b_T;



- Absence of linear power divergence and multiplicative renormalization;

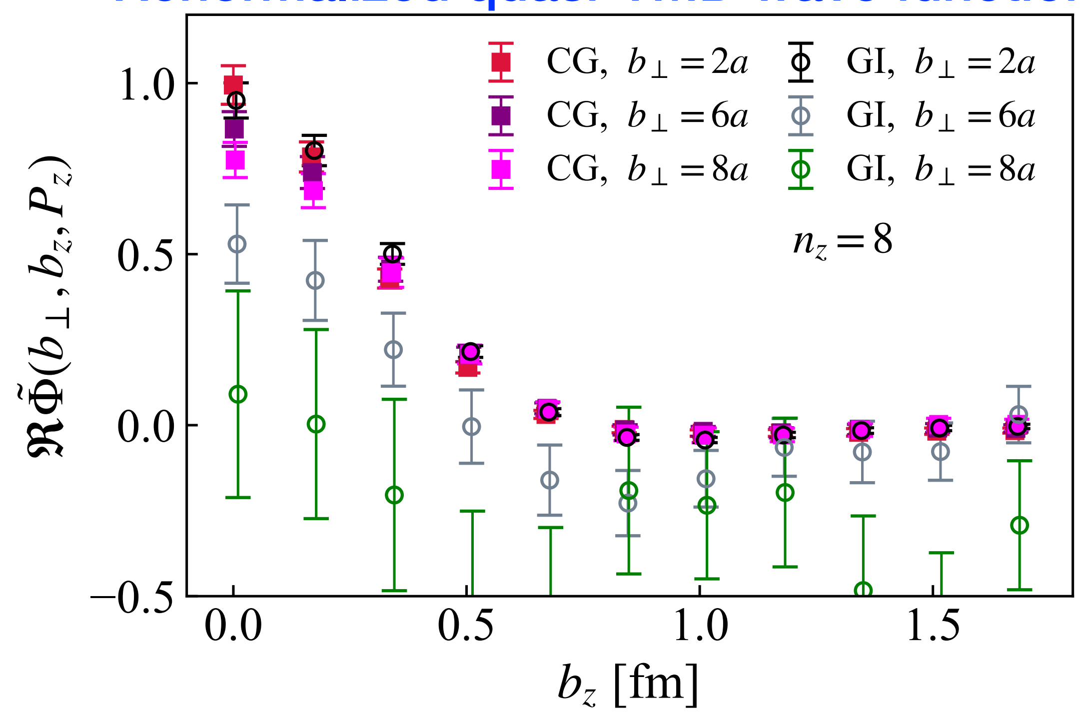
$$\bar{\psi}_B(b)\Gamma\psi_B = Z_\psi(a)\left[\bar{\psi}(b)\Gamma\psi(0)\right]_R$$

X. Gao, W.-Y. Liu and YZ, PRD 109 (2024)

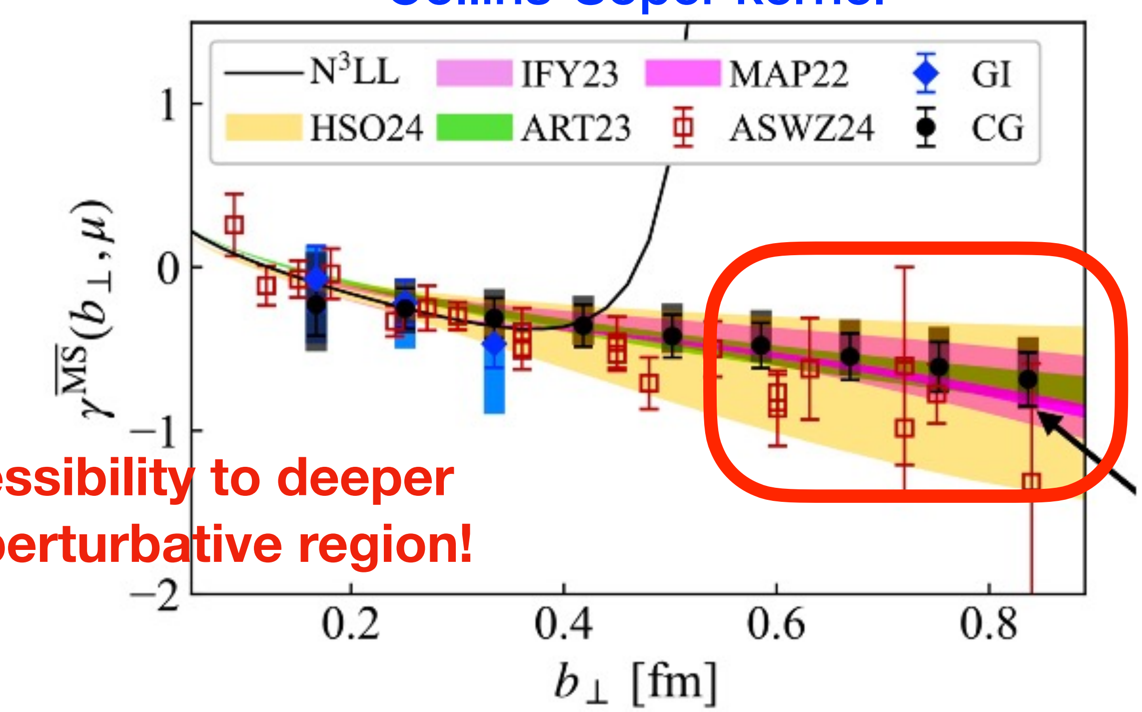
- Access to larger off-axis momenta.

$$\vec{P} = (0, P^z, P^z), \qquad \vec{b} = (b_\perp, b^z, b^z)$$

Renormalized quasi-TMD wave function



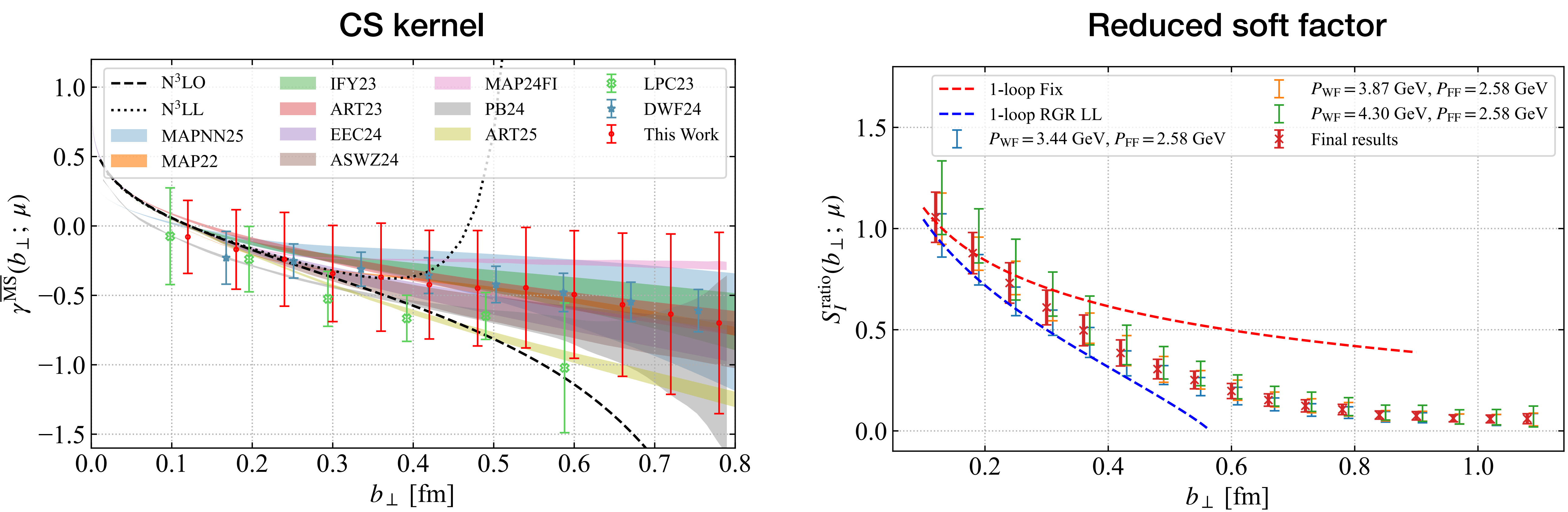
Collins-Soper kernel



Accessibility to deeper non-perturbative region!

D. Bollweg, X. Gao, S. Mukherjee and YZ, PLB **852** (2024)

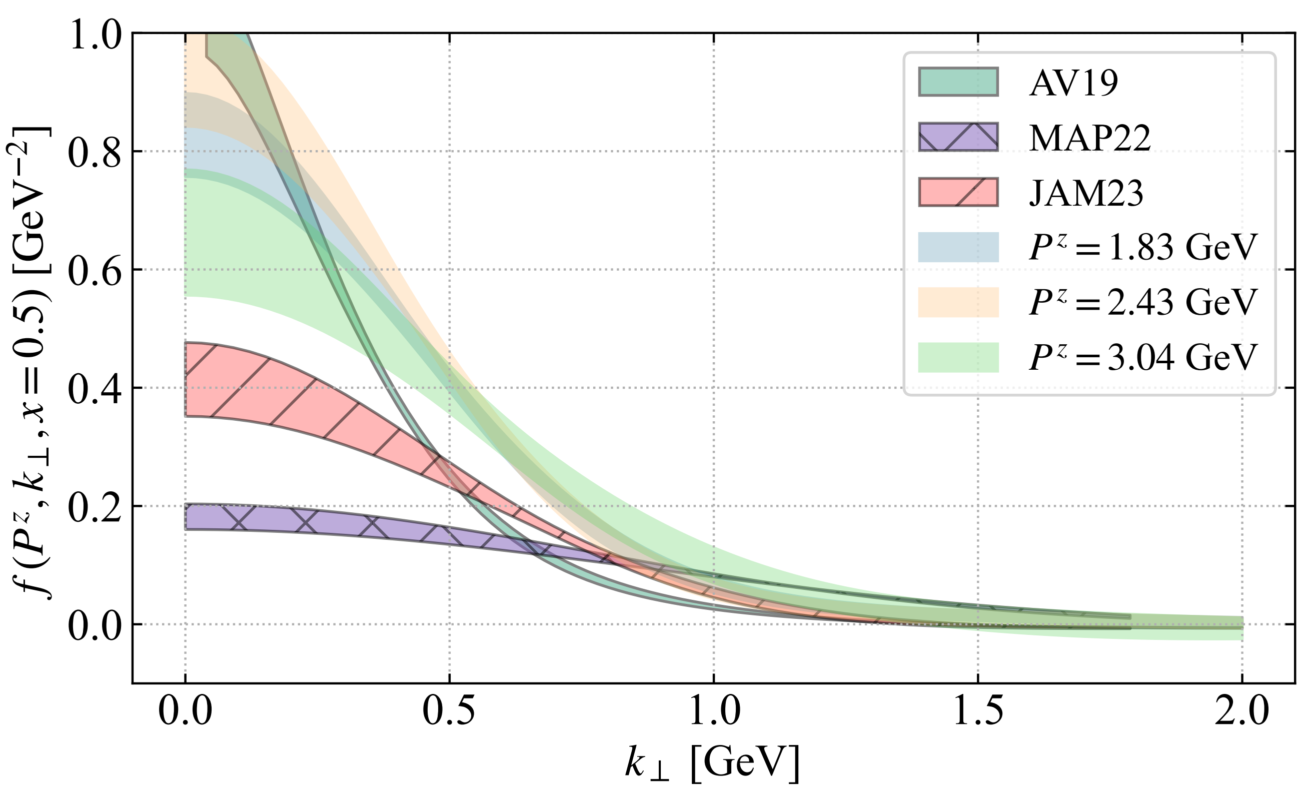
CS kernel and soft factor



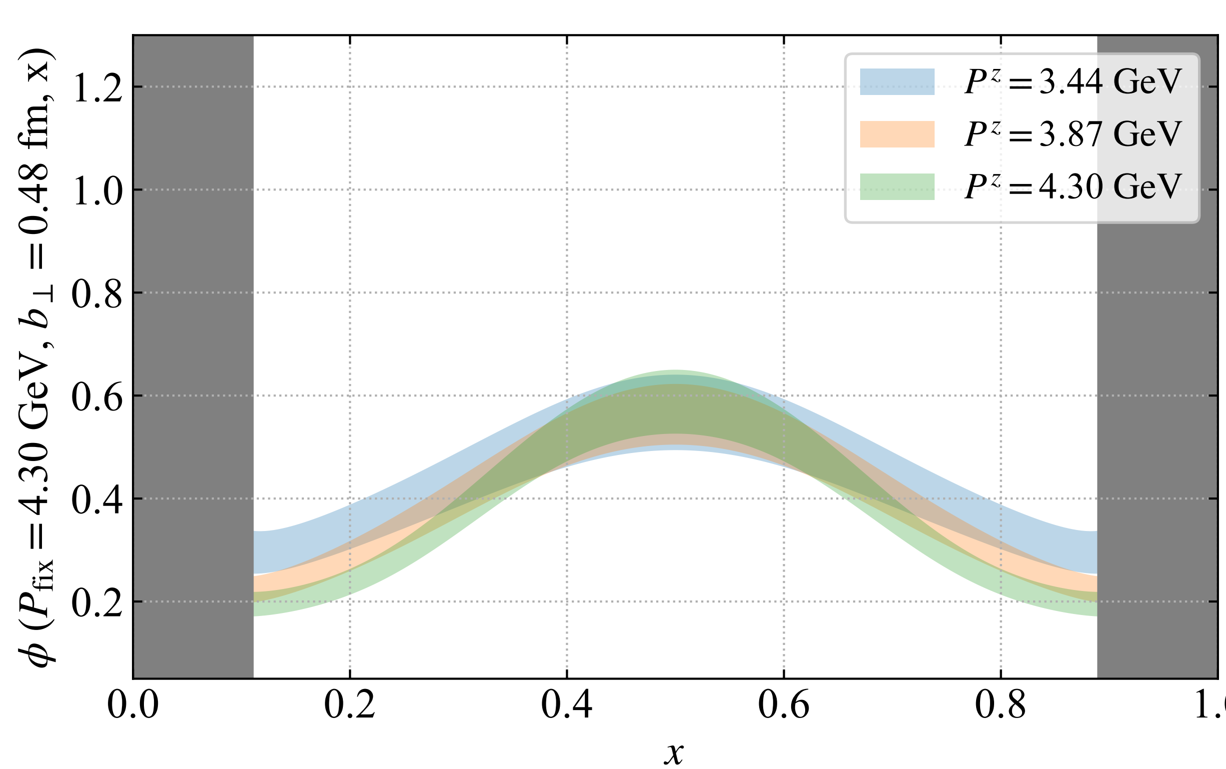
X. Gao, J. He, YZ et al, PRD **112** (2025).

Pion and TMDPDF and TMD wave function

Pion unpolarized TMDPDF



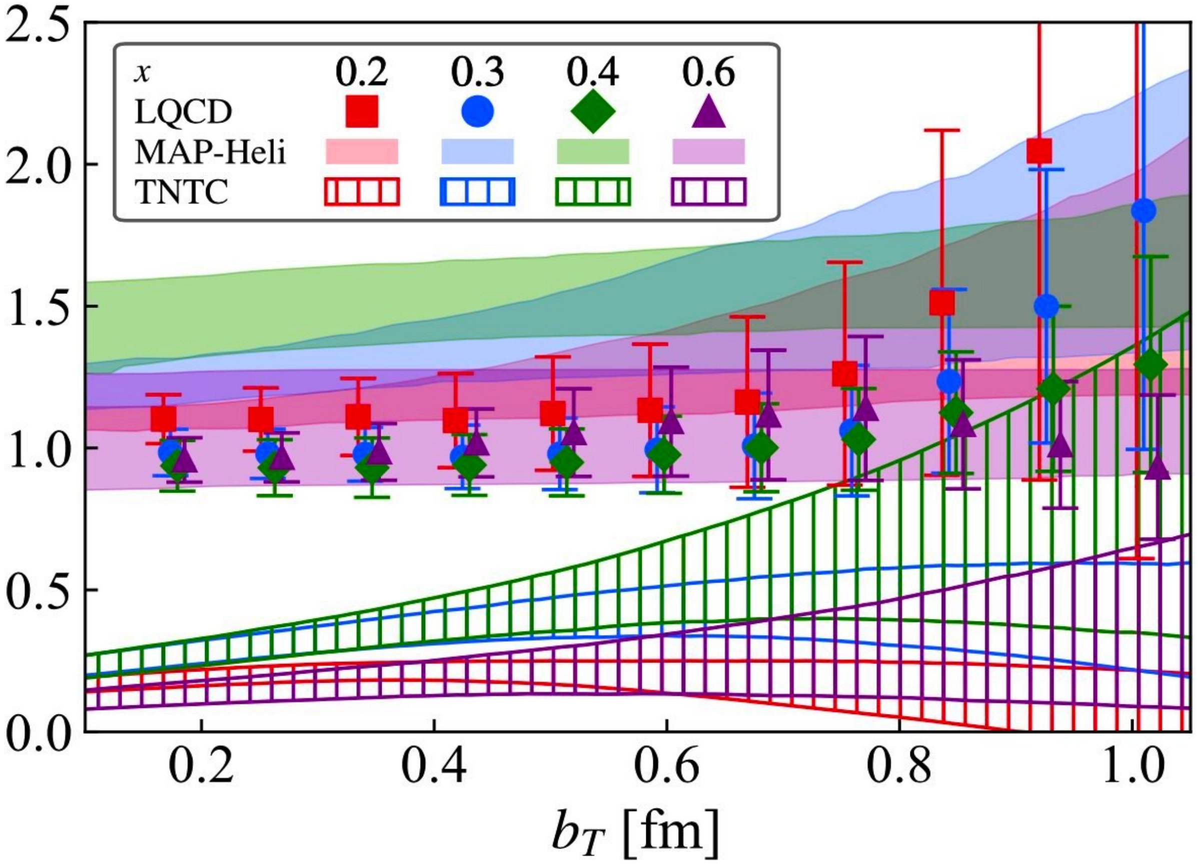
Pion TMD wave function



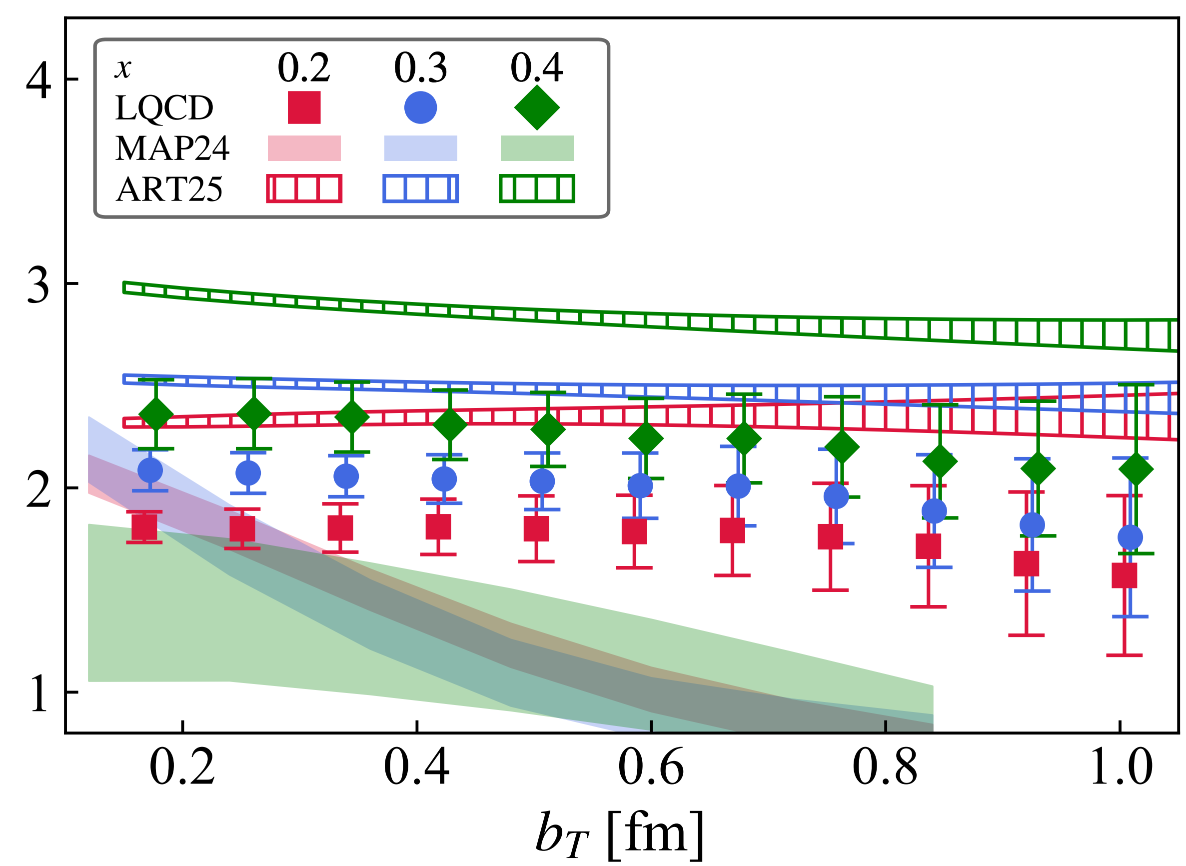
X. Gao, J. He, YZ et al, PRD **112** (2025).

Nucleon (spin-dependent) TMDPDFs

Ratios of proton helicity to unpolarized TMDPDFs



Ratios of proton unpolarized up to down quark valence TMDPDFs



D. Bollweg, X. Gao, S. Mukherjee and YZ, PRL **135** (2025).

MAP-Heli: PRL **134** (2025).
TNTC: PRL **134** (2025).

MAP24: JHEP **08** (2024).
ART25: JHEP **11** (2025).

New: inverse-problem-free parton distributions

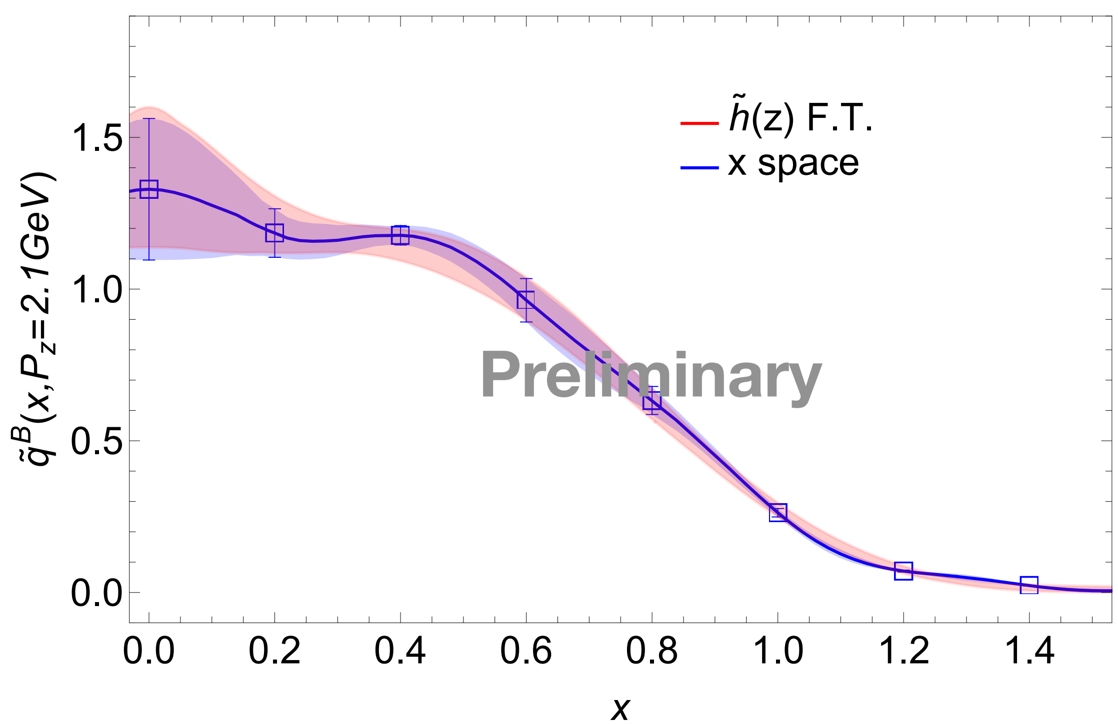
A momentum-density operator directly calculable on the lattice:

$$\tilde{f}(x, \vec{k}_T) \propto \int \frac{d^3\vec{b}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{b}} \langle P | \bar{\psi}(\vec{b})\gamma^t\psi(0) \Big|_{\nabla\cdot\mathbf{A}=0} | P \rangle$$

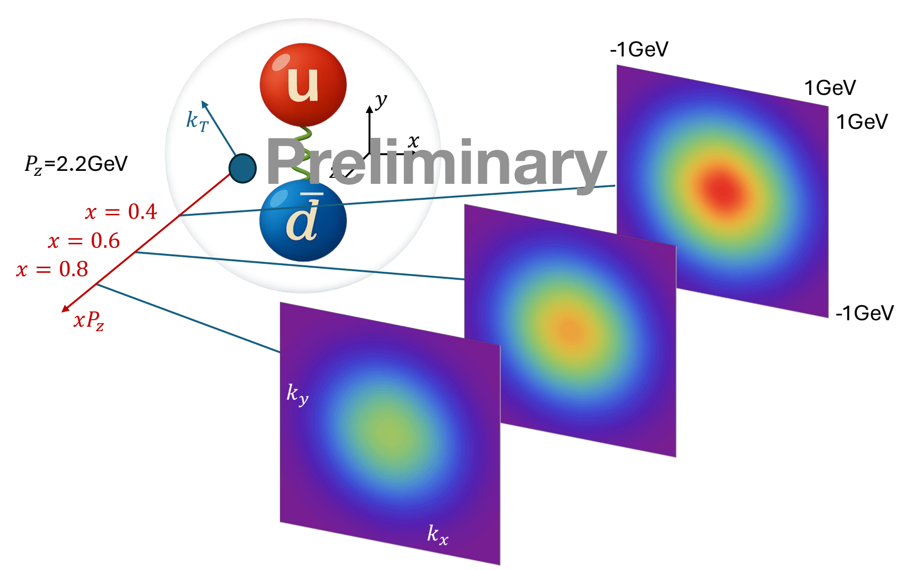
Enabled by the commutativity between renormalization and Fourier transform

$$= \langle P | \int \frac{d^3\vec{b}}{(2\pi)^3} e^{i\vec{k}\cdot\vec{b}} \bar{\psi}(\vec{b})\gamma^t\psi(0) \Big|_{\nabla\cdot\mathbf{A}=0} | P \rangle \propto \langle P | \psi^\dagger(\vec{k})\psi(\vec{k}) | P \rangle$$

Pion valence quasi-PDF



Pion valence quasi-TMD: a 3D image



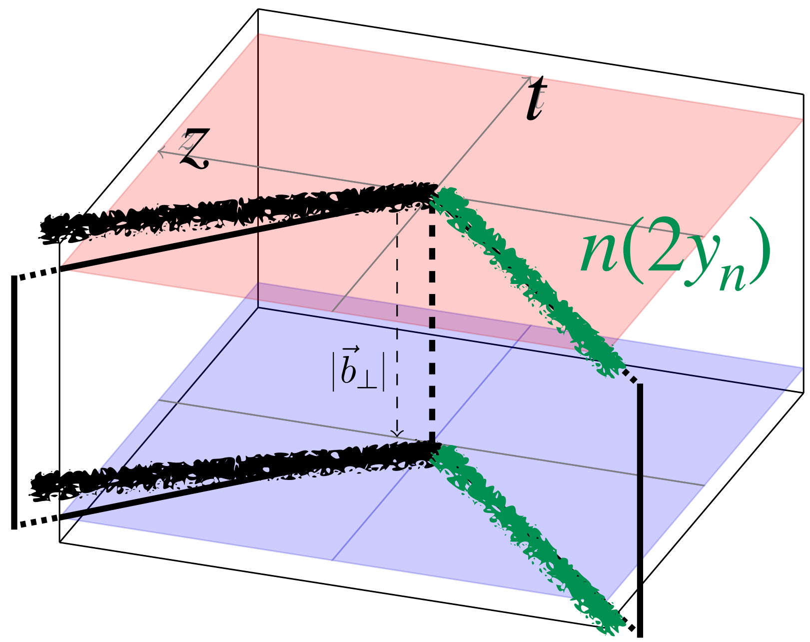
R. Zhang, A. Grebe, D. Hackett, M. Wagman and YZ, in preparation.

New: CS kernel from auxiliary fields on the lattice

$$P\exp\left\{-ig\int_{s_i}^{s_f}\mathrm{d}s n^\mu A_\mu(y(s))\right\}$$
$$= Z_\psi^{-1}\int \mathcal{D}\psi\mathcal{D}\bar\psi\,\psi\bar\psi\exp\left\{ig\int_{s_i}^{s_f}\mathrm{d}s\bar\psi i\partial_s\psi - \bar\psi n\cdot A\psi\right\}$$

[Gervais, Nevau 1980], [Aref'eva 1980]

$n_b(2y_B)$

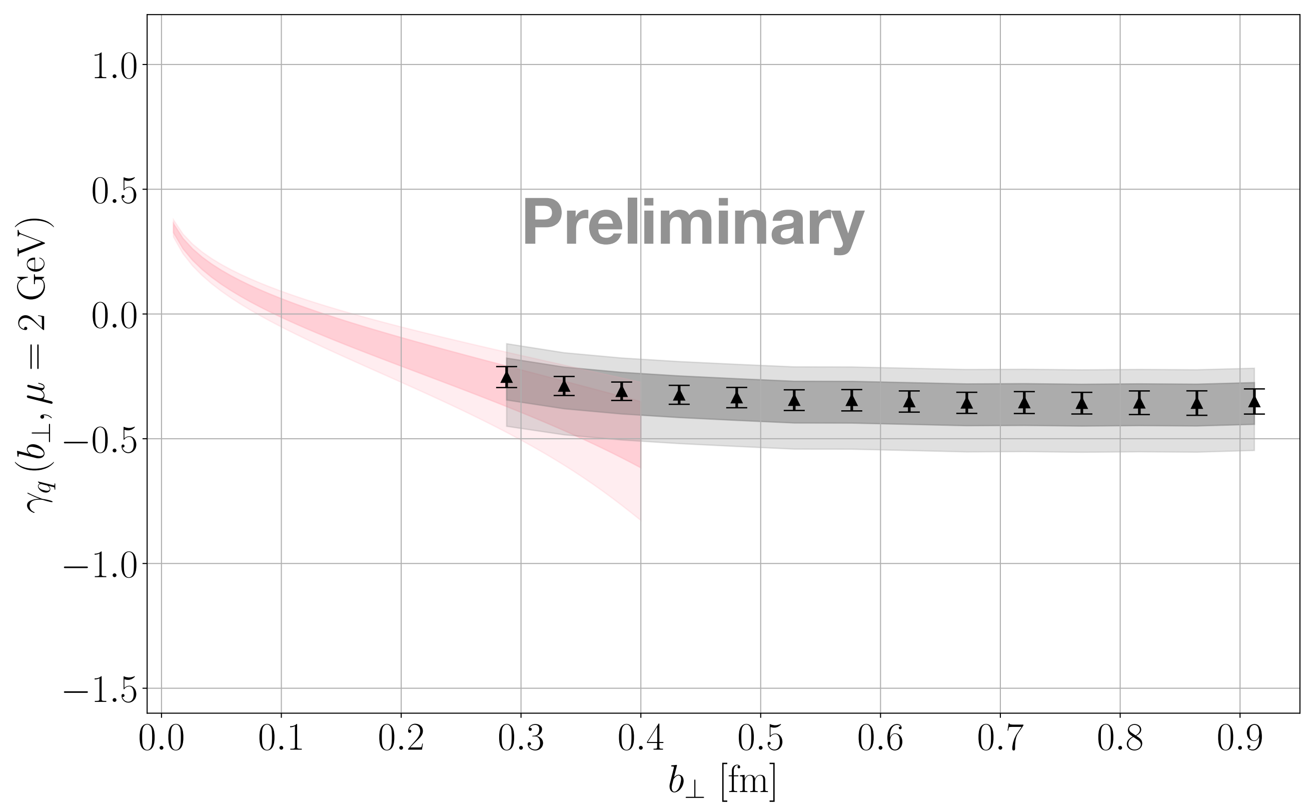


$$\xrightarrow{y_n - y_B\to\infty} S_r(b_T,\mu)\ e^{-2(y_n-y_B)\gamma_\zeta(b_T,\mu)}$$

See W. Morris' talk.

L= 40^3x80, a=0.048 fm
L= 48^3x96, a=0.041 fm
L= 64^3x128, a=0.03 fm

Exploratory calculation on quenched ensembles



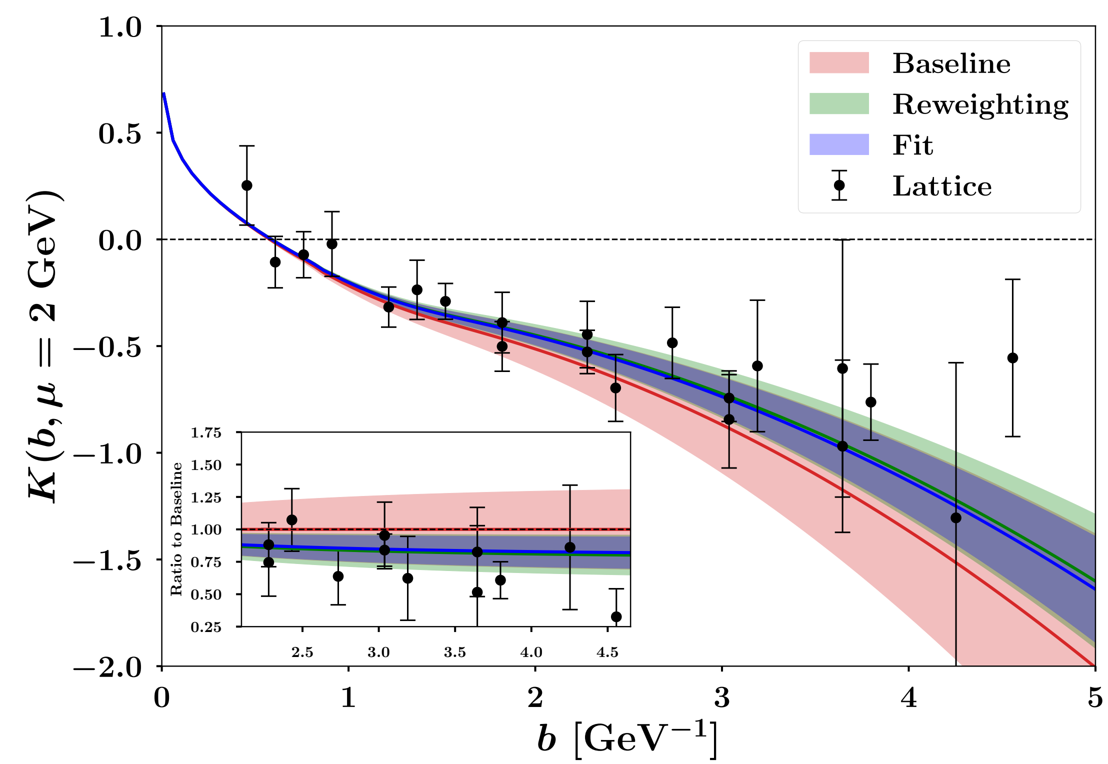
A. Francis, I. Kanamori, D. C-J Lin, W. Morris, and YZ, in preparation

Impact of lattice input on phenomenology

Joint global fit of the CS kernel:

- Neural-network parametrization of TMDs
- Bayesian reweighting of existing fits of TMDs with lattice data
- Joint TMD fit to lattice and experimental data
- Central value of the CS kernel shifted by ~10% from existing fit
- Uncertainties reduced by 40-50%

$$\hat{\gamma}_\zeta(b_T, g_2) = -g_2 b_T^2$$



A. Avkhadiev, V. Bertone, and YZ et al., accepted by PRL.

Summary and Outlook

- LaMET provides a general framework for calculating 3D partonic structure of the nucleon from lattice QCD;
- Significant progress has been made towards the systematic calculations of PDFs, GPDs and TMDs;
- New methods and techniques are being developed to improve the precision of lattice calculations;
- There are still many exciting areas to explore in LaMET;
- Lattice QCD is going to play a crucial role in complementing the experiments at the current and future colliders.