Lattice extraction of the Collins-Soper kernel via auxiliary fields

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with

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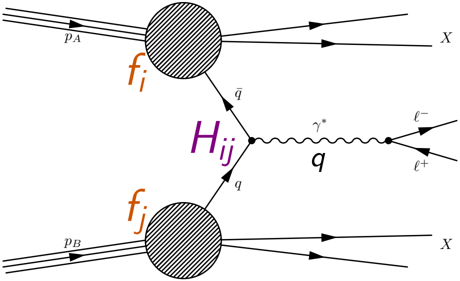
TMD factorization in Drell-Yan scattering



- $\Lambda_{\mathrm{QCD}} \lesssim |\vec{q}_\perp| \ll Q = \sqrt{q^2}$
- Collins-Soper scale, $\zeta_{a,b}$, dependence

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q\,\mathrm{d}Y\,\mathrm{d}^2\vec{q}_\perp} = \sum_{i,j} H_{ij}\left(Q,\mu\right) \int \mathrm{d}^2\vec{b}_\perp e^{i\vec{b}_\perp\cdot\vec{q}_\perp} f_i\left(x_a,\vec{b}_\perp,\mu,\zeta_a\right) f_j\left(x_b,\vec{b}_\perp,\mu,\zeta_b\right)$$

$$\times\left[1+\mathcal{O}\left(\frac{q_\perp^2}{Q^2},\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)\right] \qquad \zeta_{a,b} = 2(x_{a,b}P^+e^{-y_n})^2$$

TMDPDF, soft factor, and CS kernel

$$f_i\left(x_a, \vec{b}_\perp, \mu, \zeta_a\right) = \lim_{\substack{\epsilon\to 0\\ y_B\to-\infty}} Z_{\mathrm{UV}}\left(\mu, \zeta_a, \epsilon\right) \frac{f_i^{\mathrm{bare}}\left(x_a, \vec{b}_\perp, \epsilon, y_B, x_a P^+\right)}{\sqrt{S_i\left(b_\perp, \epsilon, 2y_n - 2y_B\right)}}$$

Evolution kernel:

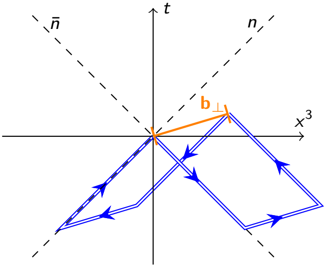
$$\gamma_\mu^q(\mu, \zeta) = \frac{\mathrm{d}\log f_q\left(x, \vec{b}_\perp, \mu, \zeta\right)}{\mathrm{d}\log \mu}$$

Collins-Soper (CS) kernel:

$$\gamma_q(b_\perp, \mu) = \frac{\mathrm{d}\log f_q\left(x, \vec{b}_\perp, \mu, \zeta\right)}{\mathrm{d}\log \sqrt{\zeta}}$$

$$S_q\left(b_\perp, y_A, y_B, \mu\right) = S_I\left(b_\perp, \mu\right) e^{\gamma_q(b_\perp, \mu)(y_A - y_B)}\left(1 + \mathcal{O}\left(e^{-2(y_A - y_B)}\right)\right)$$

Regulating the soft function



Regulate rapidity divergence with spacelike Wilson lines:

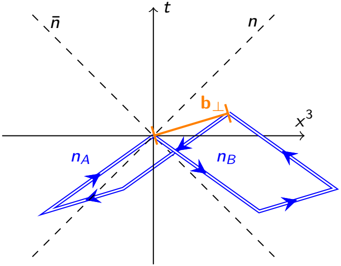
$$y_A, -y_B \to \infty,$$

$$n_A \equiv n - e^{-y_A}\bar{n},$$
$$n_B \equiv \bar{n} - e^{+y_B}n$$

Soft function defined on the lightcone:

$$n^2 = \bar{n}^2 = 0$$

$$n = (1, 0, 0, 1),$$
$$\bar{n} = (1, 0, 0, -1)$$



Relevance to lattice extraction of TMDPDFs

In the LaMET framework:

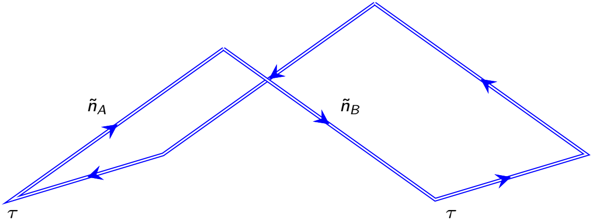
$$\sqrt{S_I(b_\perp,\mu)}\,\tilde{f}_\Gamma(x,b_\perp,P^z,\mu)$$
$$= f(x,b_\perp,\mu,\zeta)\,H_f(x,P^z,\mu)\exp\left[\frac{1}{2}\log\frac{(2xP^z)^2}{\zeta}\gamma_q(b_\perp,\mu)\right]$$
$$+\mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{xP^z},\frac{1}{b_\perp(xP^z)}\right)$$

[Ebert, *et. al.*, 2019], [Ji, Liu, Liu, 2020], [Ebert, *et. al*, 2022]

- Intrinsic soft function: S_I
  [Ji, Liu, Liu, 2020]
- quasi-TMD beam function: $\tilde{f}_\Gamma$
- TMDPDF: f
- Perturbative hard kernel: H_f
- CS kernel: γ_q

Euclidean space directional vectors with purely imaginary time components

$$\tilde{n}_A = \left(in_A^0, 0, 0, n_A^3\right), \quad \tilde{n}_B = \left(in_B^0, 0, 0, n_B^3\right)$$



$$r_a = \frac{n_A^3}{n_A^0} = \frac{1+e^{-2y_A}}{1-e^{-2y_A}}, \quad r_b = \frac{n_B^3}{n_B^0} = \frac{1+e^{2y_B}}{1-e^{2y_B}}$$

Auxiliary field definition of the Wilson line

Write Wilson line in terms of one dimensional 'fermions' that live along the path:

$$P \exp\left\{-ig\int_{s_i}^{s_f} \mathrm{d}s n^\mu A_\mu(y(s))\right\}$$
$$= Z_\psi^{-1}\int \mathcal{D}\psi\mathcal{D}\bar\psi\, \psi\bar\psi \exp\left\{ig\int_{s_i}^{s_f}\mathrm{d}s\bar\psi i\partial_s\psi - \bar\psi n\cdot A\psi\right\}$$

[Gervais, Nevau 1980], [Aref'eva 1980]

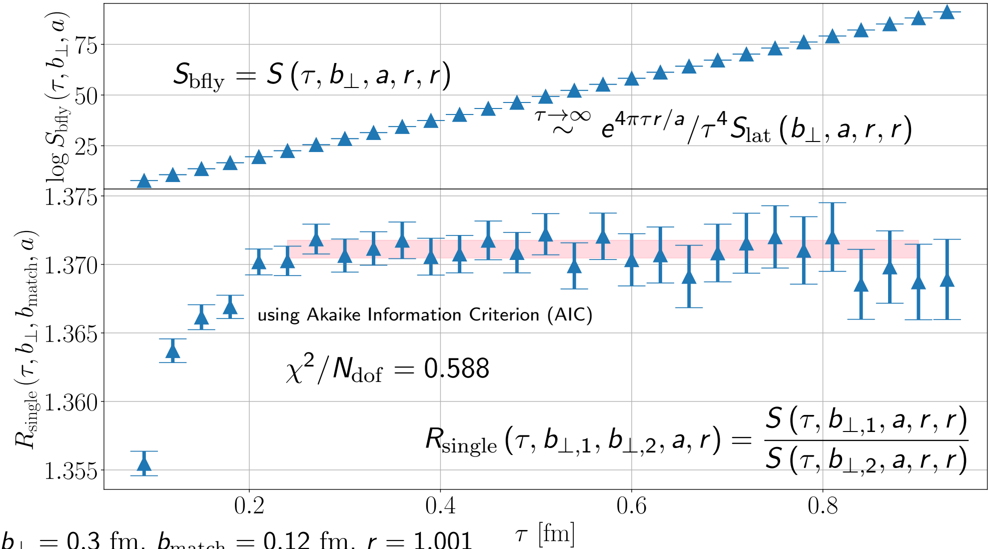
Auxiliary field propagator:

$$in\cdot DH_n(y) = \delta(y) \xrightarrow{\text{Euclidean space}} -i\tilde{n}\cdot D_E H_{\tilde{n}}(y) = \delta(y), \qquad \tilde{n} = (in_0, \vec{n})$$

Meaningful solution only obtained with a UV cutoff

[Aglietti, *et. al.* 1992], [Aglietti, 1994]

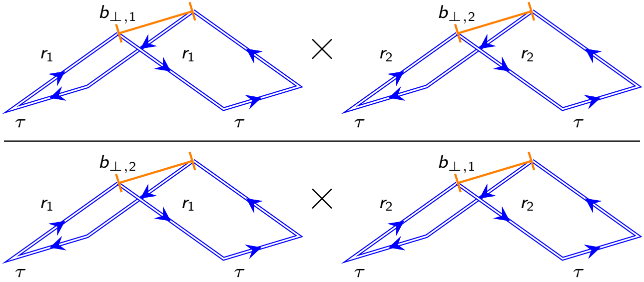
Single ratio, time dependence



$b_\perp = 0.3$ fm, $b_{\text{match}} = 0.12$ fm, $r = 1.001$

Quenched configurations from [Detmold, Endres, 2018]

$$R_{\text{double}}\left(b_{\perp,1}, b_{\perp,2}, a, r_1, r_2, \tau\right) = \frac{S\left(b_{\perp,1}, a, r_1, r_1, \tau\right)}{S\left(b_{\perp,2}, a, r_1, r_1, \tau\right)} \Bigg/ \frac{S\left(b_{\perp,1}, a, r_2, r_2, \tau\right)}{S\left(b_{\perp,2}, a, r_2, r_2, \tau\right)}$$
$$= \exp\left[\left(\gamma_q\left(b_{\perp,1}, a\right) - \gamma_q\left(b_{\perp,2}, a\right)\right) 2(y_1 - y_2)\right]$$



Double ratio

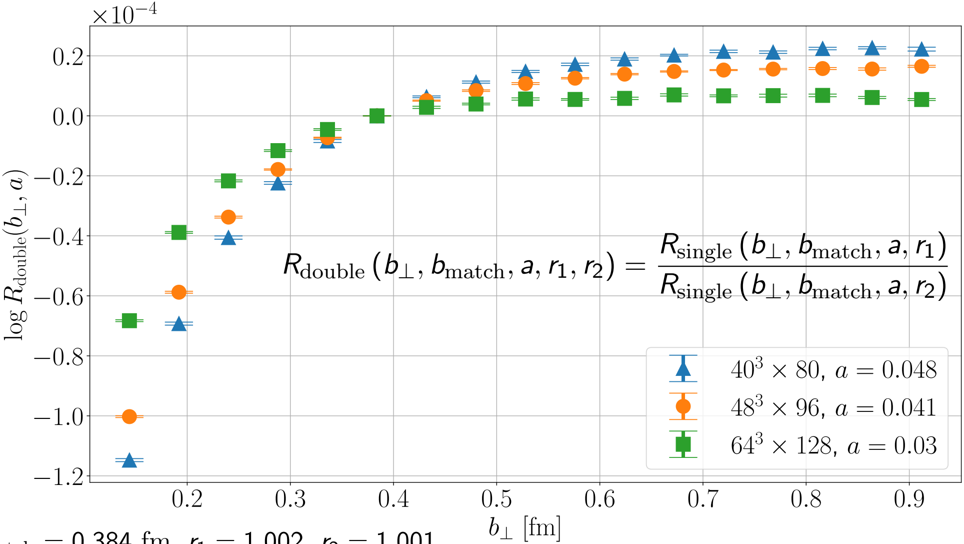
- Power counting for large lattice time:

$$R_{\text{double}}\left(b_{\perp,1}, b_{\perp,2}, a, r_1, r_2, \tau\right) = \frac{S_{\text{lat}}\left(b_{\perp,1}, a, r_1, r_1\right)}{S_{\text{lat}}\left(b_{\perp,2}, a, r_1, r_1\right)} \Bigg/ \frac{S_{\text{lat}}\left(b_{\perp,1}, a, r_2, r_2\right)}{S_{\text{lat}}\left(b_{\perp,2}, a, r_2, r_2\right)} + \mathcal{O}\left(\frac{b_1^2 - b_2^2}{\tau^2}\left(\frac{1}{r_1 - 1} - \frac{1}{r_2 - 1}\right), \frac{r_{1,2} - 1}{r_{1,2} + 1}, a^n\right)$$

- Match between lattice and continuum renormalization schemes

$$R_{\text{double}}\left(b_{\perp,1}, b_{\perp,2}, \mu, r_1, r_2\right) = C\left(r_1, r_2, \mu, a\right) \times R_{\text{double}}\left(b_{\perp,1}, b_{\perp,2}, a, r_1, r_2\right)$$

Interpolated double ratio, $b_\perp$ dependence

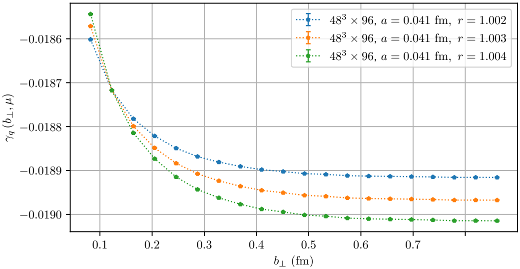


$b_{\rm match} = 0.384$ fm, $r_1 = 1.002$, $r_2 = 1.001$

$$\gamma_q\left(b_{\perp,1},\mu\right) = \gamma_q\left(b_{\perp,2},\mu\right) + \frac{\frac{1}{2}\log\left(R_{\mathrm{double}}\left(b_{\perp,1},b_{\perp,2},\mu,r_1,r_2\right)\right)}{\log\left(\frac{r_1+1}{r_1-1}\Big/\frac{r_2+1}{r_2-1}\right)}$$

- Implies that we need at least one perturbative value of γ_q.
- Another perturbative value of γ_q is needed, because $r_{1,2}$ is renormalized due to $O(4)$ symmetry breaking. [Aglietti, *et. al.* 1992], [Aglietti, 1994]

Bare vs renormalized rapidity factor



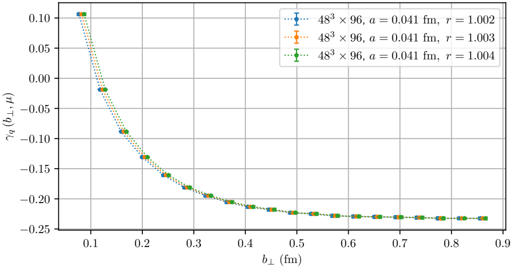
$$r_2 = 1.001, \quad r_1 = \{1.002, 1.003, 1.004\}$$

Bare rapidity:

$$2(y_1 - y_2) = \log\left(\frac{r_1+1}{r_1-1} \Big/ \frac{r_2+1}{r_2-1}\right)$$

With perturbative matching:

$$2(y_1^{\mathrm{ren}} - y_2^{\mathrm{ren}}) = \frac{R_{\mathrm{double}}\left(b_{\perp,1}^{\mathrm{pert}}, b_{\perp,2}^{\mathrm{pert}}, r_1, r_2, a\right)}{\gamma_q\left(b_{\perp,1}^{\mathrm{perp}}, \mu\right) - \gamma_q\left(b_{\perp,2}^{\mathrm{pert}}, \mu\right)}$$



Model for the double ratio

- Model the double ratio as a piecewise function:

$$\log R^{\mathrm{fit}}_{\mathrm{double}} = R_a \left(\begin{cases} \gamma_q^{\overline{\mathrm{MS}}}(b_\perp, \mu) - \gamma_q^{\mathrm{match}}, & b_{\min} \le b_\perp \le b_{\mathrm{thresh}} \\ \{d_\ell\}, & b_\perp = \{b_\ell\}, b_\perp > b_{\mathrm{thresh}} \end{cases} + c_1 \left(\frac{a^2}{b_\perp^2} - \frac{a^2}{b_{\mathrm{match}}^2} \right) \right)$$

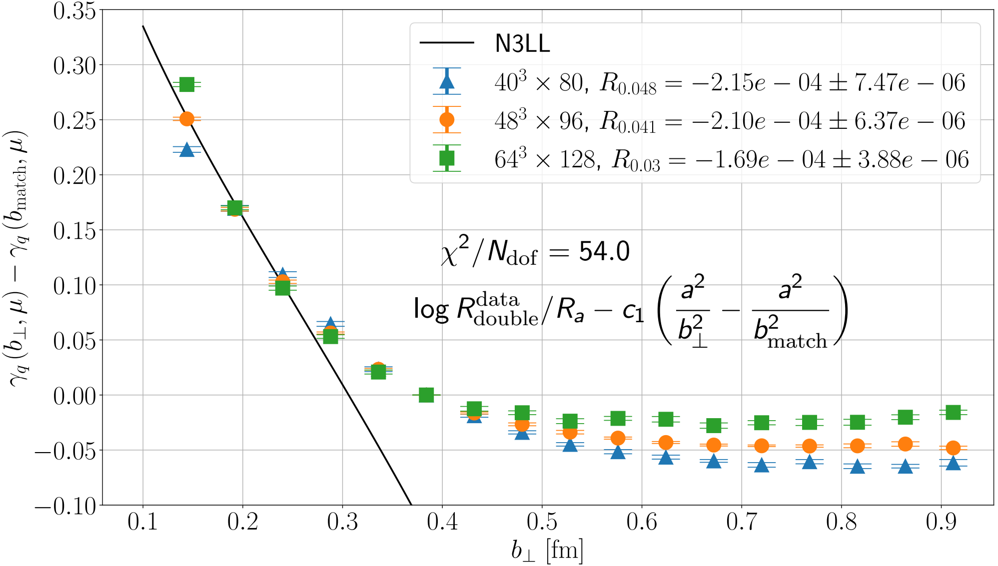
- Combined fit on three ensembles
- Model imposes a constraint on the large-$b_\perp$ region

$$b_{\min} = 0.144\ \mathrm{fm}, \quad b_{\mathrm{thresh}} = 0.24\ \mathrm{fm}, \quad b_{\mathrm{match}} = 0.384\ \mathrm{fm} > b_{\mathrm{thresh}}$$

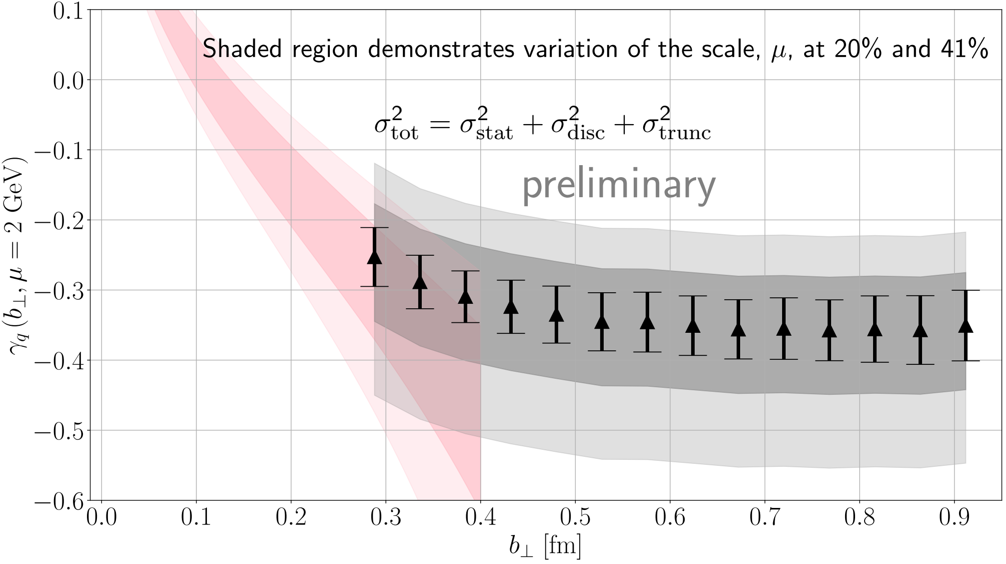
$$d_\ell = \gamma_q(b_\ell, \mu) - \gamma_q(b_{\mathrm{match}}, \mu), \qquad \gamma_q^{\mathrm{ext}}(b_\ell, \mu) = \gamma_q^{\mathrm{match}} + d_\ell$$

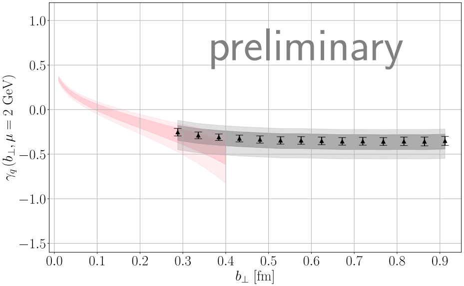
- Model does not adequately estimate the lattice spacing dependence

Fit results

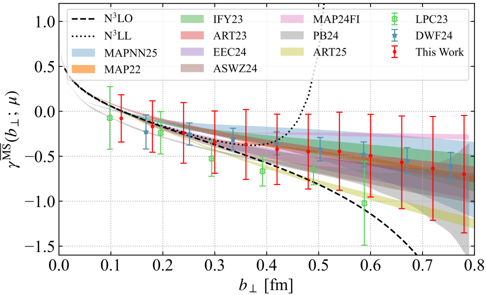

N3LL
$40^3 \times 80,\ R_{0.048} = -2.15e-04 \pm 7.47e-06$
$48^3 \times 96,\ R_{0.041} = -2.10e-04 \pm 6.37e-06$
$64^3 \times 128,\ R_{0.03} = -1.69e-04 \pm 3.88e-06$
$\chi^2/N_{\mathrm{dof}} = 54.0$
$\log R^{\mathrm{data}}_{\mathrm{double}}/R_a - c_1\left(\frac{a^2}{b_\perp^2} - \frac{a^2}{b^2_{\mathrm{match}}}\right)$
$\gamma_q(b_\perp,\mu) - \gamma_q(b_{\mathrm{match}},\mu)$
$b_\perp$ [fm]
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9

CS kernel with systematic error





Our result



Other recent results presented in [Bollweg, *et. al.*, 2025]

- We obtain results competitive with recent lattice extractions of the Collins-Soper kernel.
- Our result is pure gauge. Comparison plots have $n_f = 3, 4$.

- Euclidean space soft function maps directly to Minkowski space result
- Lattice data for 'butterfly loop' at high statistical precision
- Double ratio method gives: $\gamma_q(b_\perp, \mu) - \gamma_q(b'_\perp, \mu)$
- Error primarily dominated by unkown discretization effects, scale uncertainty
- Even with uncertainty, our results are competitive with existing lattice results

Open questions and future direction:

- Theoretical understanding of lattice spacing dependence after renormalization
- Improve scale uncertainty, perform perturbative matching at smaller distances

Thank you!