

# Parton physics from a heavy-quark operator product expansion

Dynamical lattice QCD calculation of moments of the kaon  
light-cone distribution amplitudes

Speaker: Alex Chang (張聖彬)

National Yang Ming Chiao Tung University

2026/04/30

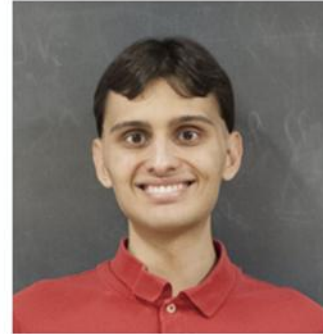
THE  
**H**  **PE**  
COLLABORATION



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(MIT)



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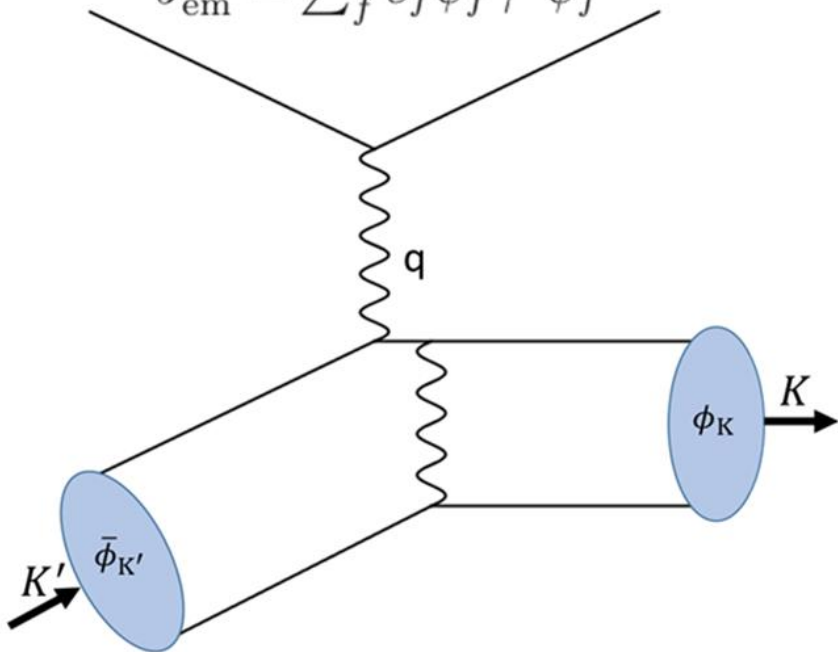
Matias I. Gutierrez E.  
(MIT)

# Kaon Electromagnetic Form Factors

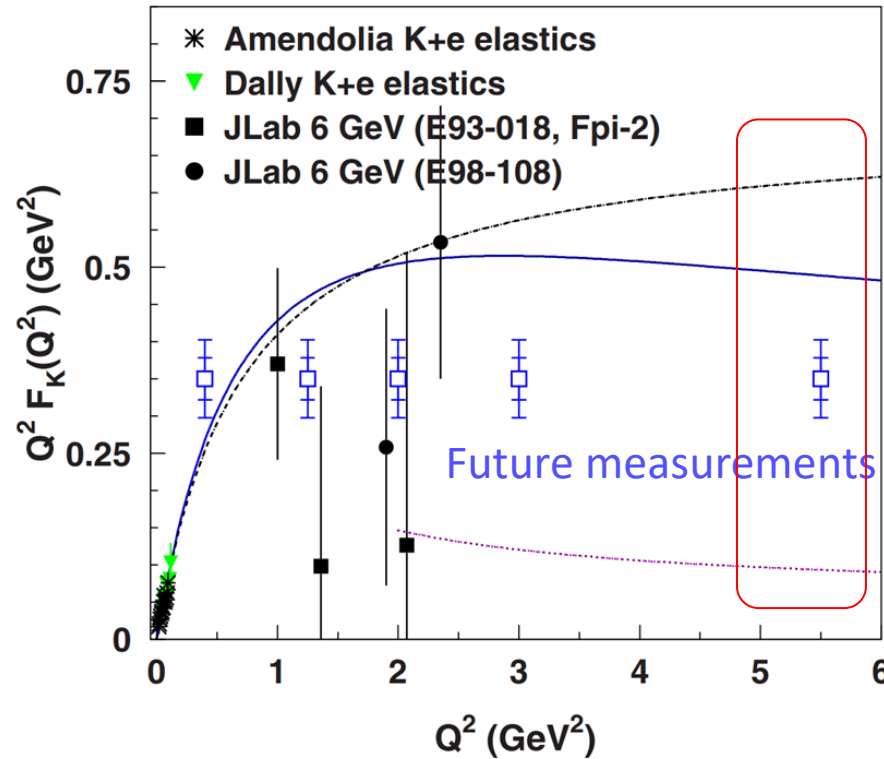
$$\langle K^+(P') | J_{em}^\mu | K^+(P) \rangle$$

$$= F_K(Q^2)(P^\mu + P'^\mu)$$

$$J_{em}^\mu = \sum_f e_f \bar{\psi}_f \gamma^\mu \psi_f$$



elastic electron-kaon scattering



Marco Carmignotto PhysRevC.97.025204

■ Model-dependent. We still do not fully understand this!!

■ High  $Q^2$  experimental data is very hard to get.

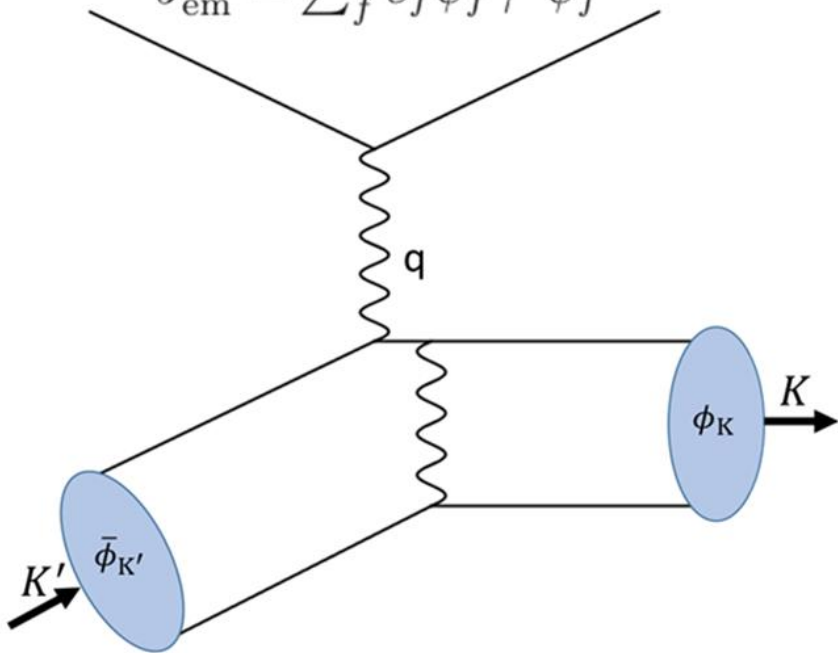
*so nonperturbative theory is important to know the Kaon Electromagnetic Form Factors*

# Kaon Electromagnetic Form Factors

$$\langle K^+(P') | J_{\text{em}}^\mu | K^+(P) \rangle$$

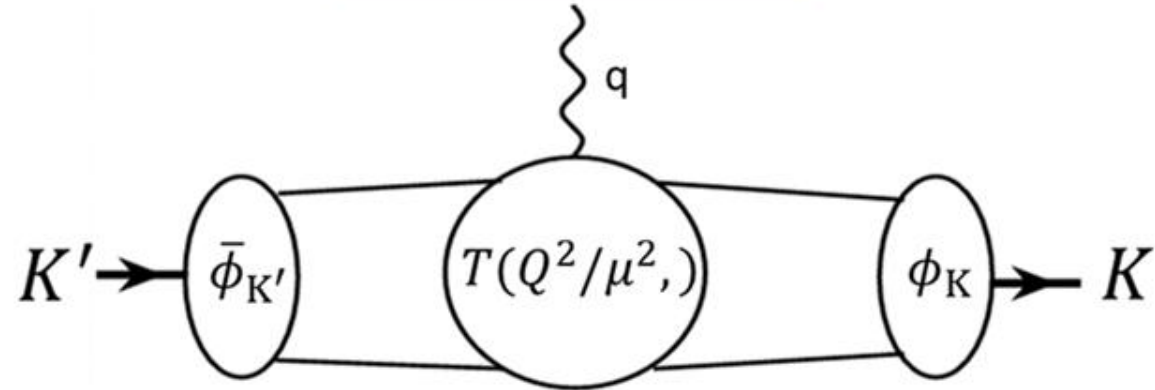
$$= F_K(Q^2)(P^\mu + P'^\mu)$$

$$J_{\text{em}}^\mu = \sum_f e_f \bar{\psi}_f \gamma^\mu \psi_f$$



elastic electron-kaon scattering

QCD Factorization



$$F_K(Q^2) = \int dx dy \bar{\phi}_K(x, Q^2) T(x, y, Q^2) \phi_K(y, Q^2)$$

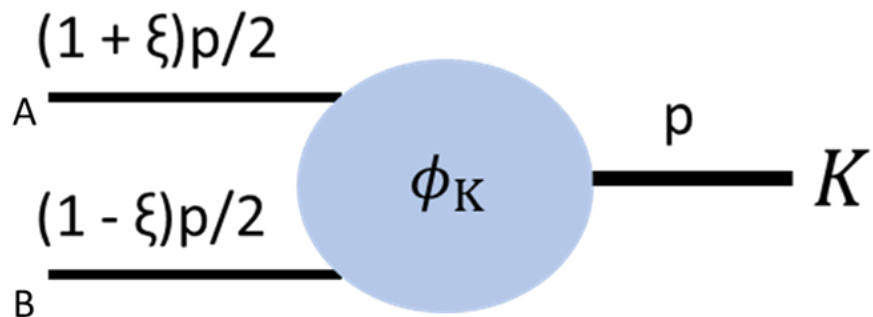
Hard scattering kernel  $T$  calculable in perturbative QCD.

LCDA  $\phi$  encoding the **non-perturbative** structure of the kaon.



Lattice QCD

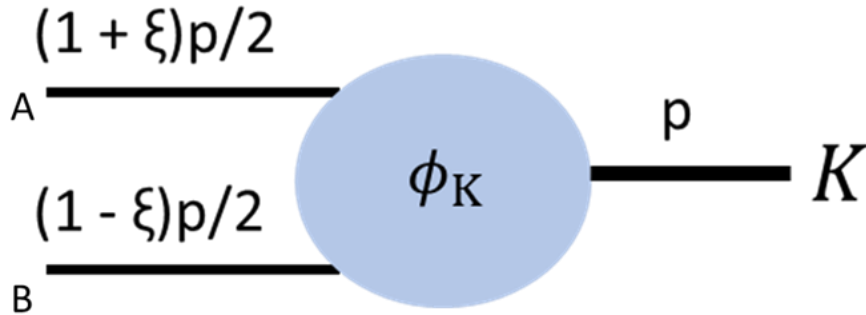
# Light-Cone Distribution Amplitudes (LCDAs)



$$\langle \Omega | \bar{\psi}_A(z) \gamma_\mu \gamma_5 W[z, -z] \psi_B(-z) | K^+(p) \rangle$$
$$= i f_K p_\mu \int_{-1}^1 d\xi e^{-i\xi p \cdot z} \phi_K(\xi, \mu^2)$$

- $f_K$  is the pseudoscalar kaon decay constant and  $W$  is light-like Wilson line

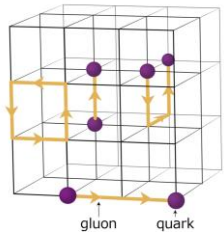
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Direct calculation of light-cone objects is impossible on a Euclidean lattice



Lattice QCD

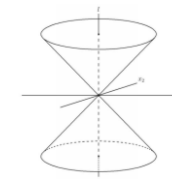


Directly

LCDAs

Wick rotation  $t \rightarrow -i\tau \Rightarrow$  Minkowski  $\rightarrow$  Euclidean

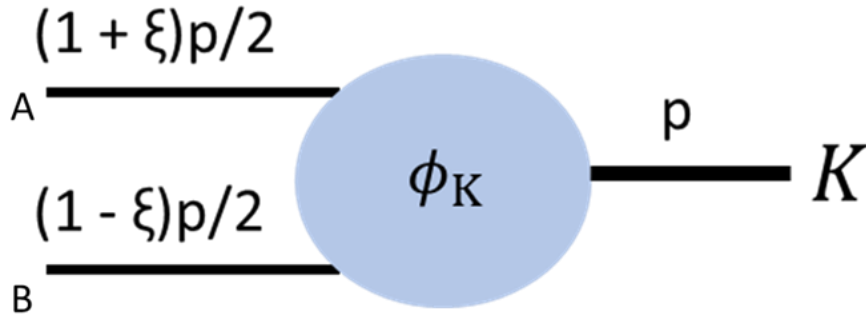
light cone,  
defined by  $z^2 = 0$



$0$

There is no notion of a light-cone in Euclidean space.

# Light-Cone Distribution Amplitudes (LCDAs)



$$\langle \Omega | \bar{\psi}_A(z) \gamma_\mu \gamma_5 W[z, -z] \psi_B(-z) | K^+(p) \rangle = i f_K p_\mu \int_{-1}^1 d\xi e^{-i\xi p \cdot z} \phi_K(\xi, \mu^2)$$

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## ◆ Light-Cone OPE

$$\bar{\psi}(z) \gamma^\mu \gamma_5 W[z, -z] \psi(-z) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} z_{\mu_1} \cdots z_{\mu_n} \bar{\psi}(0) \gamma^{\{\mu} \gamma_5 (i\overleftrightarrow{D}^{\mu_1}) \cdots (i\overleftrightarrow{D}^{\mu_n}) \psi(0) \Big|_{\text{traceless}}$$

$O^{\mu_0 \mu_1 \dots \mu_n}$

$$\langle 0 | [ \bar{\psi}_A \gamma^{\{\mu_0} \gamma_5 (i\overleftrightarrow{D}^{\mu_1}) \dots (i\overleftrightarrow{D}^{\mu_n}) \psi_B - \text{trace} ] | K^+(p) \rangle$$

local & twist-two  
twist = dim - spin

$$= f_K \langle \xi^n \rangle [ p^{\mu_0} p^{\mu_1} \dots p^{\mu_n} - \text{trace} ]$$

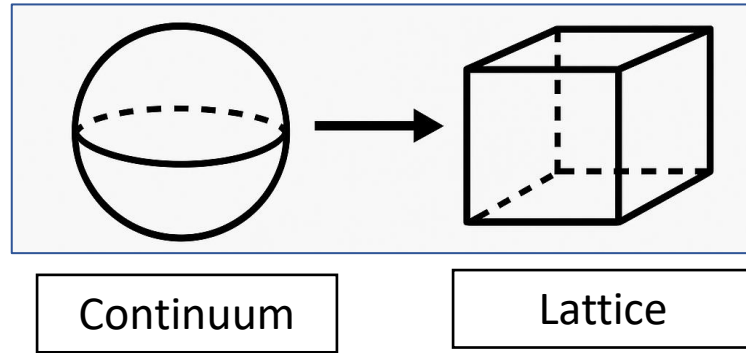
## ◆ Mellin moments

$$\langle \xi^n \rangle = \int_{-1}^1 d\xi \xi^n \phi_K(\xi, \mu^2)$$

# Compute Local matrix elements directly

## Limitations of Traditional OPE on Lattice:

The lattice regularization  $H(4)$  breaks  $SO(4)$



twist = dim - spin

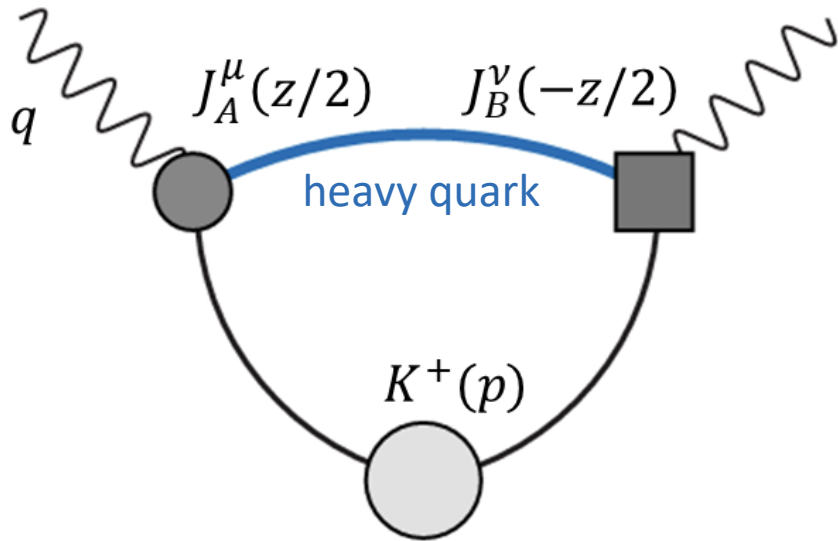
For large Mellin moments ( $\geq$ second moment)

mix with lower dimension operators and the mixing coefficients contain power divergences.

$$\mathcal{O}_i = \sum_j C_{ij}^{\text{latt}}(a) \mathcal{O}_j^{\text{latt}}$$

→ Traditional OPE infeasible beyond the first few Mellin moments

# HOPE Method



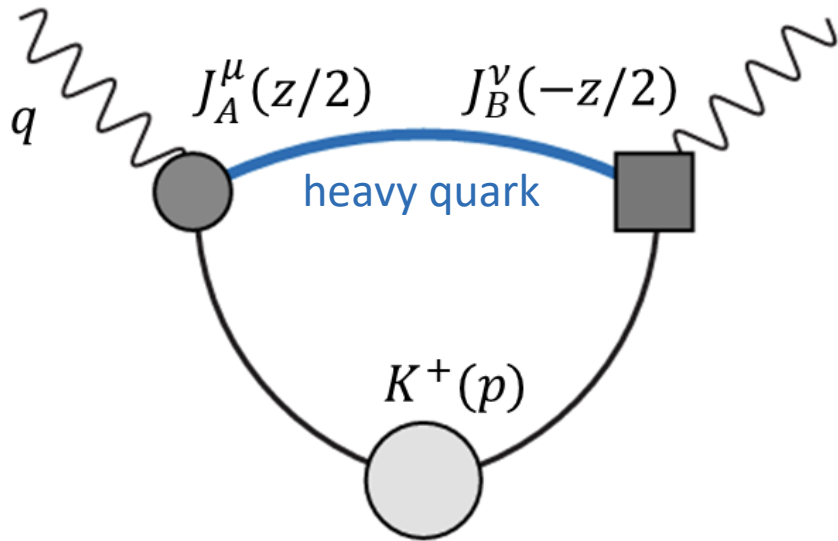
## ◆ Hadronic Tensor

$$V^{\mu\nu}(p, q) = \int d^4z e^{iq \cdot z} \langle \Omega | T \{ J_A^\mu(z/2) J_B^\nu(-z/2) \} | K^+(p) \rangle$$

$$J_A^\mu(z) = \bar{\Psi}(z) \gamma^\mu \gamma_5 \psi_A(z) + \bar{\psi}_A(z) \gamma^\mu \gamma_5 \Psi(z)$$

$\Psi$  is the fictitious valence heavy quark

# HOPE Method



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$$V^{[\mu\nu]}(q, p) = \frac{-2 i \epsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma}{\tilde{Q}^2} f_K \sum_{n=0}^{\infty} C_W^{(n)}(\tilde{Q}^2, \mu^2, m_\Psi) \langle \xi^n \rangle \left( \frac{\tilde{\omega}}{2} \right)^n$$

LQCD calculations:  
extract hadronic tensor

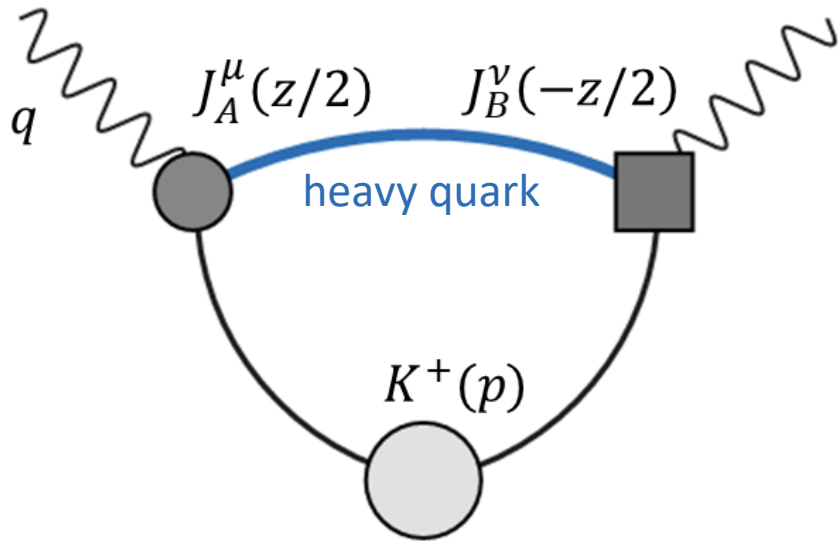
QCD perturbation theory:  
One-loop Wilson coefficients

$$\tilde{\omega} = (2 q \cdot p) / \tilde{Q}^2$$

$$\tilde{Q}^2 = -q^2 + m_\Psi^2$$

W. Detmold and C.-J. D. Lin, (2006), Phys. Rev. D 73, 014501

# HOPE Method



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LQCD calculations:  
extract hadronic tensor

← Fitting →

QCD perturbation theory:  
One-loop Wilson coefficients

### Fit parameters:

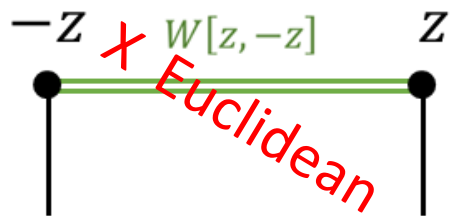
- $\langle \xi^n \rangle$  – Mellin moments
- $f_K$  – kaon decay constant
- $m_\Psi$  – heavy quark mass

$$\tilde{\omega} = (2 q \cdot p) / \tilde{Q}^2$$

$$\tilde{Q}^2 = -q^2 + m_\Psi^2$$

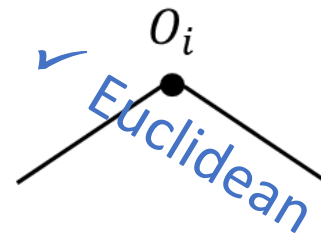
W. Detmold and C.-J. D. Lin, (2006), Phys. Rev. D 73, 014501

LCDA



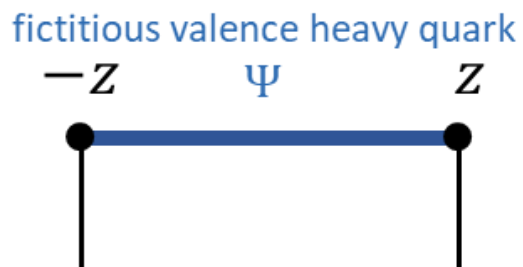
Light-Cone OPE

$$\sum_{i=0}^{\infty} C_W^{(i)}(z)$$



☹ operators mixing & power divergence

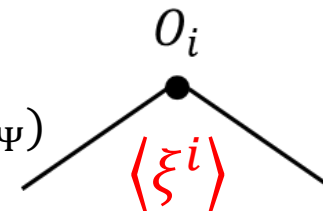
HOPE



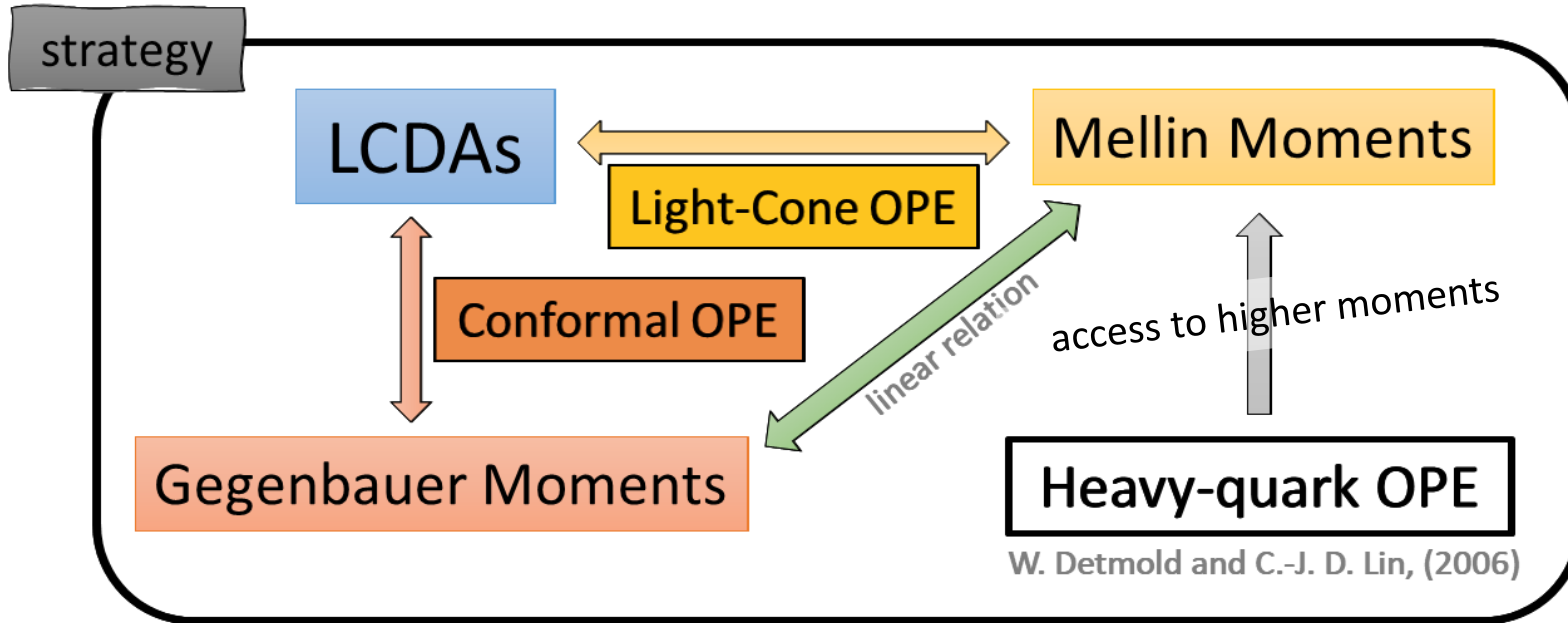
Short-distance & heavy scale

$\Lambda_{\text{QCD}} \ll m_\Psi$

$$\sum_{i=0}^{\infty} C_{\text{HOPE}}^{(i)}(z, m_\Psi)$$



☺ higher-twist effects are suppressed by  $1/\tilde{Q}^n$



LCDA(x)

**Quasi-DA / LaMET**

Xiangdong Ji,  
*Phys.Rev.Lett.* 110 (2013)

**Pseudo-DA**

Anatoly Radyushkin,  
*Phys.Lett.* B767 (2017)

Moments

**Braun–Müller approach**

Two currents separated by space-like distance  
V. M. Braun and D. Müller,  
*Eur. Phys. J. C* 55, 349 (2008).

- ◆ Constructing the full  $x$ -dependence is difficult near the end points.
- ◆ Moments provide observables that do not require constructing the full  $x$ -dependence.

# Effect of Moments on the Shape of the LCDA

## ◆ Gegenbauer OPE:

$$\phi_K(\xi, \mu^2) = \frac{3}{4} (1 - \xi^2) \sum_{n=0}^{\infty} \phi_n(\mu^2) C_n^{3/2}(\xi)$$

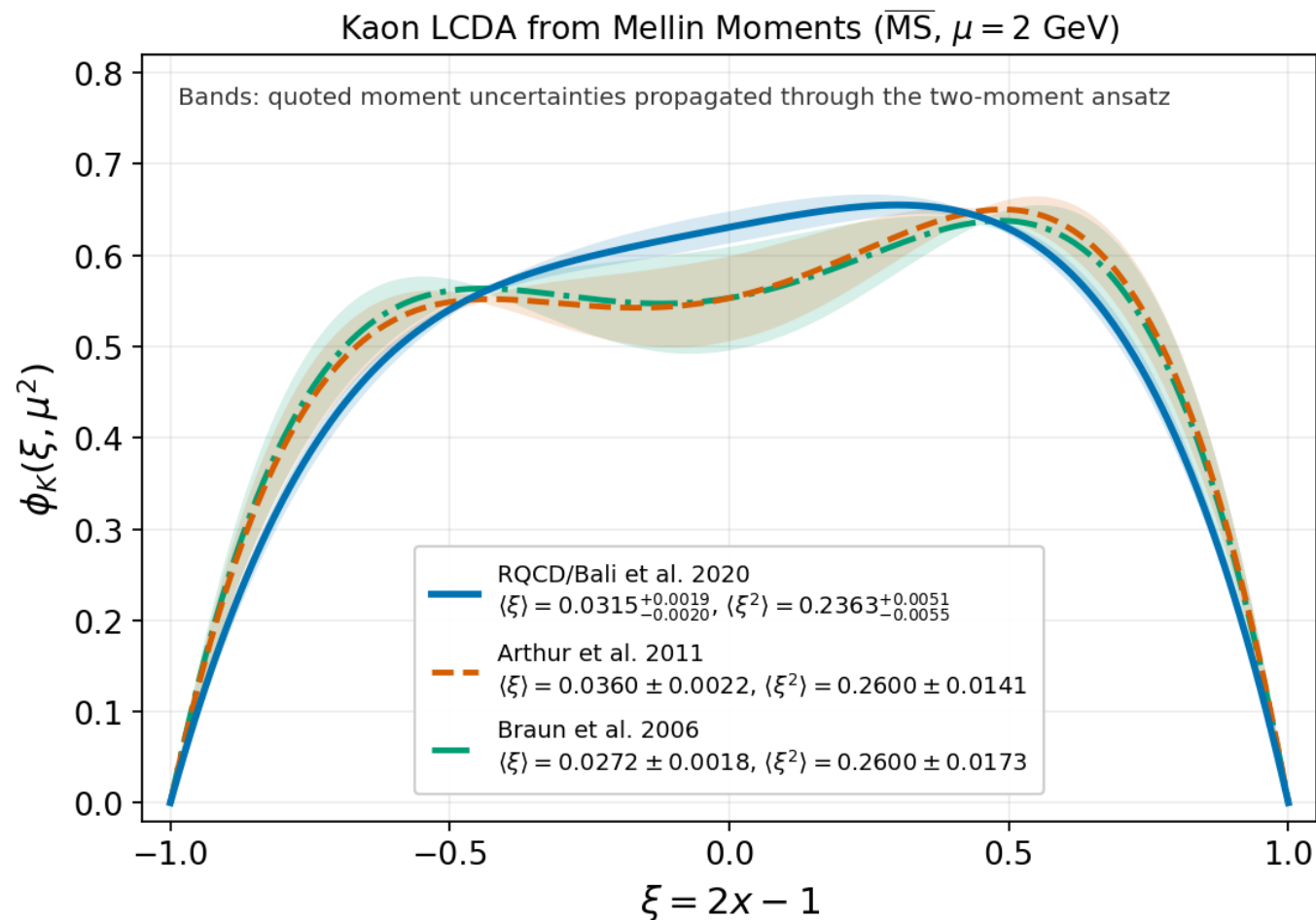
*Gegenbauer moments*   *Gegenbauer polynomials*

$$\phi_0 = \langle \xi^0 \rangle = 1, \quad \phi_1 = \frac{5}{3} \langle \xi \rangle, \quad \phi_2 = \frac{7}{12} (5 \langle \xi^2 \rangle - \langle \xi^0 \rangle)$$

Non zero odd moment:

the strange and light quarks share longitudinal momentum unequally

flavor asymmetry in the kaon LCDA



# Effect of Moments on the Shape of the LCDA

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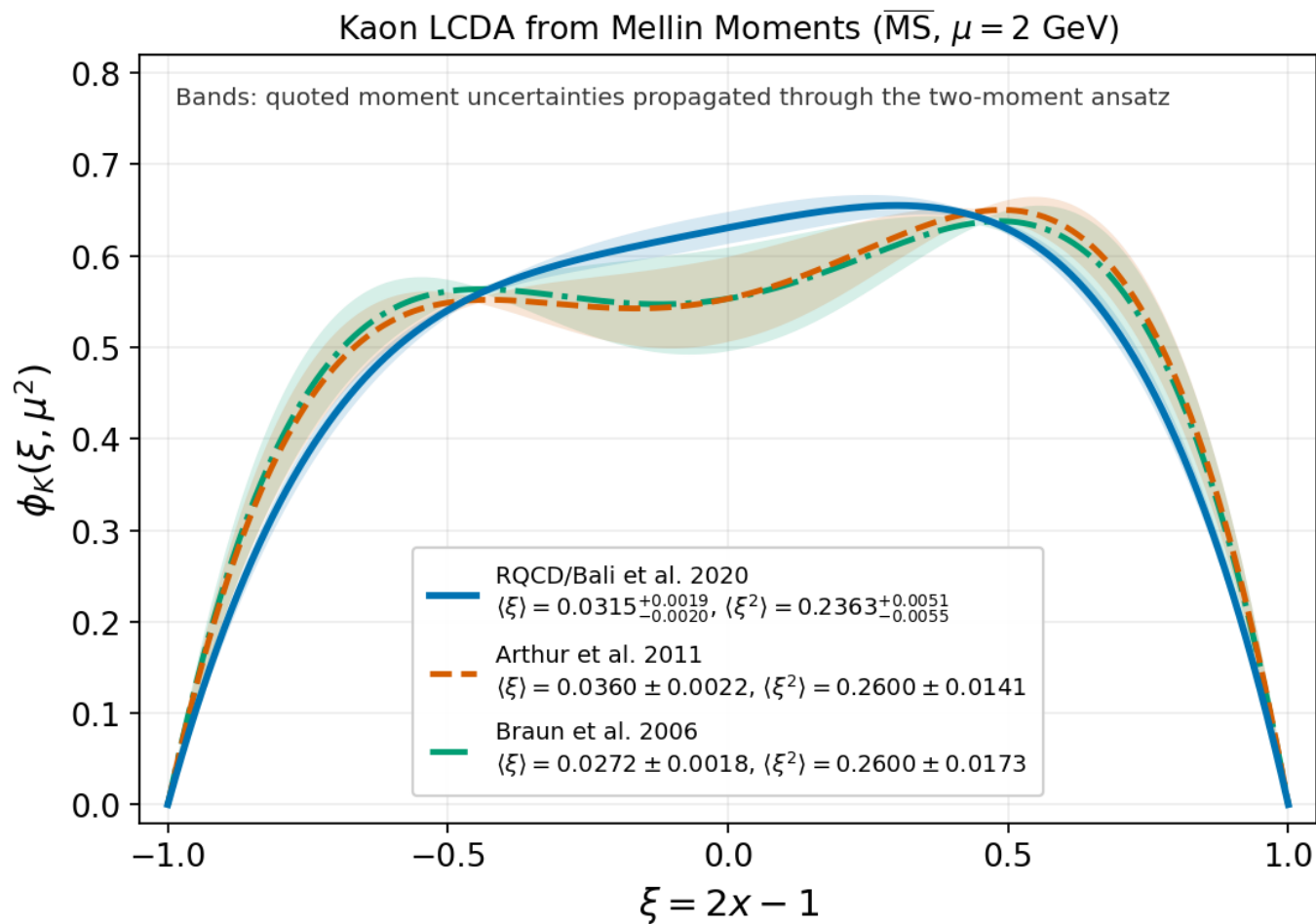
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Non zero odd moment:

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flavor asymmetry in the kaon LCDA

**What about the third moment?**



# HOPE Method for Kaon LCDA

$$V^{[\mu\nu]}(q, p) = \frac{-2 i \epsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma}{\tilde{Q}^2} f_K \sum_{n=0}^{\infty} C_W^{(n)}(\tilde{Q}^2, \mu^2, m_\Psi) \langle \xi^n \rangle \left(\frac{\tilde{\omega}}{2}\right)^n$$

$$\begin{aligned} \tilde{\omega} &= (2 q \cdot p) / \tilde{Q}^2 \\ \tilde{Q}^2 &= -q^2 + m_\Psi^2 \end{aligned}$$

## ◆ Separated by **even** and **odd** Mellin moments

- Antisymmetric ( $q \rightarrow -q$ )

$$V_{even}^{[\mu\nu]}(q, p) = \frac{1}{2} \left( V^{[\mu\nu]}(q, p) - V^{[\mu\nu]}(-q, p) \right)$$

– Even Mellin Moments

- Symmetric ( $q \rightarrow -q$ )

$$V_{odd}^{[\mu\nu]}(q, p) = \frac{1}{2} \left( V^{[\mu\nu]}(q, p) + V^{[\mu\nu]}(-q, p) \right)$$

– Odd Mellin Moments

# HOPE Method for Kaon LCDA

$$V^{[\mu\nu]}(\mathbf{q}, \mathbf{p}) = \frac{-2i \epsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma}{\tilde{Q}^2} f_K \sum_{n=0}^{\infty} C_W^{(n)}(\tilde{Q}^2, \mu^2, m_\Psi) \langle \xi^n \rangle \left(\frac{\tilde{\omega}}{2}\right)^n$$

Complex

$$\tilde{\omega} = (2 \mathbf{q} \cdot \mathbf{p}) / \tilde{Q}^2$$

$$\tilde{Q}^2 = -q^2 + m_\Psi^2$$

$$\Lambda_{\text{QCD}} \ll m_\Psi \sim \sqrt{q^2} \ll \frac{1}{\hat{a}}$$

Special kinematics chosen

$$i \epsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma = i(q_0 p_3 - p_0 q_3) = -q_4 p_3 - iE_k q_3$$

$$\text{chose } p_e(0,0,1), p_m(1/2,0,-1) \rightarrow p_3 = 0$$

$$i \epsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma = -iE_k q_3 \quad \text{pure imaginary}$$

Fitting parameters in each channel:

	Imag part	Real part
$V_{\text{even}}^{[\mu\nu]}(\mathbf{q}, \mathbf{p})$	$f_K \cdot m_\Psi$	$\langle \xi^2 \rangle$
$V_{\text{odd}}^{[\mu\nu]}(\mathbf{q}, \mathbf{p})$	$\langle \xi^1 \rangle \cdot \langle \xi^3 \rangle$	$\langle \xi^1 \rangle \cdot \langle \xi^3 \rangle$

$$|\tilde{\omega}| < 1$$

large  $p \rightarrow$  large  $|\tilde{\omega}| \rightarrow$  sensitivity to higher moments

# Fourier transform of HOPE

$$V^{[\mu\nu]}(p, q) = \int_{-\infty}^{\infty} d\tau e^{i\tau q_4} R^{[\mu\nu]}(\tau; p, q)$$

HOPE formula

Construct ratio from correlators

Inverse Fourier transform of HOPE formula

$$R^{[\mu\nu]}(\tau, p, q) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dq_4 e^{-i\tau q_4} \frac{-2i \epsilon^{\mu\nu\rho\sigma} q_\rho p_\sigma}{\tilde{Q}^2} f_K \sum_{n=0}^{\infty} C_W^{(n)}(\tilde{Q}^2, \mu^2, m_\Psi) \langle \xi^n \rangle \left(\frac{\tilde{\omega}}{2}\right)^n$$

numerical Fourier transform

# Resource

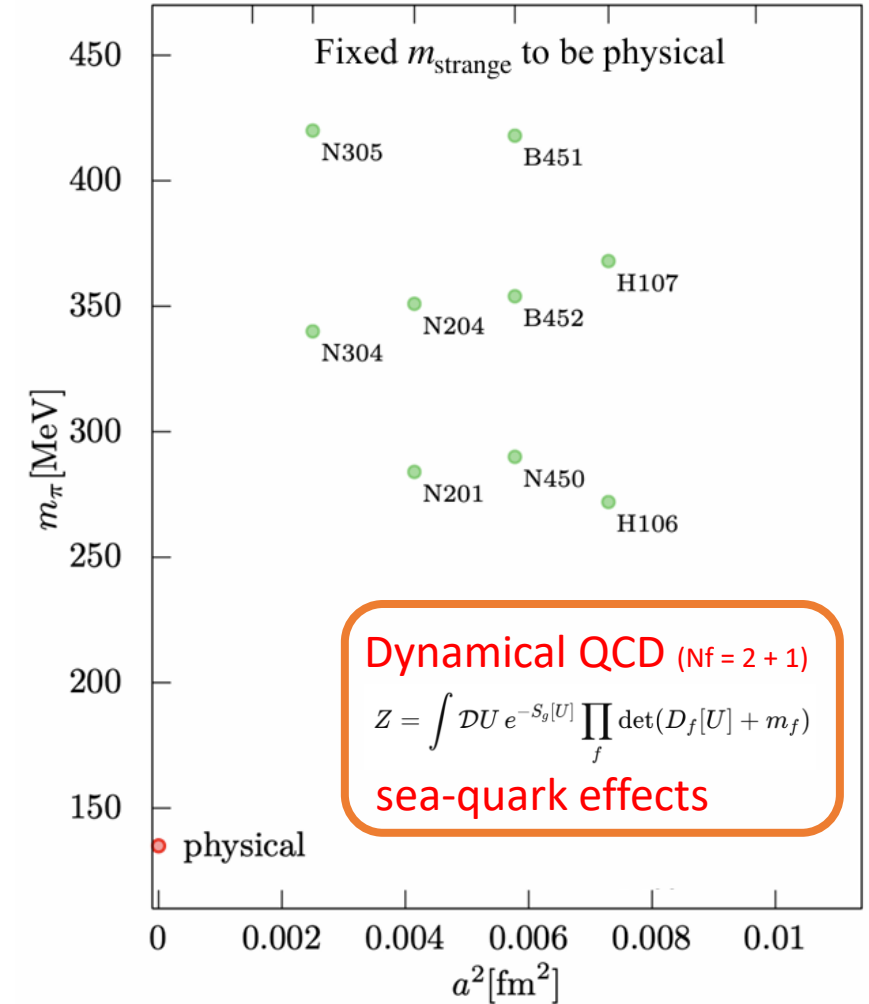
the Fugaku project id: hp220312, hp230466.

HPCI High Performance  
Computing Infrastructure

RIKEN  
R-CCS  
Center for  
Computational Science



Supercomputer  
Fugaku

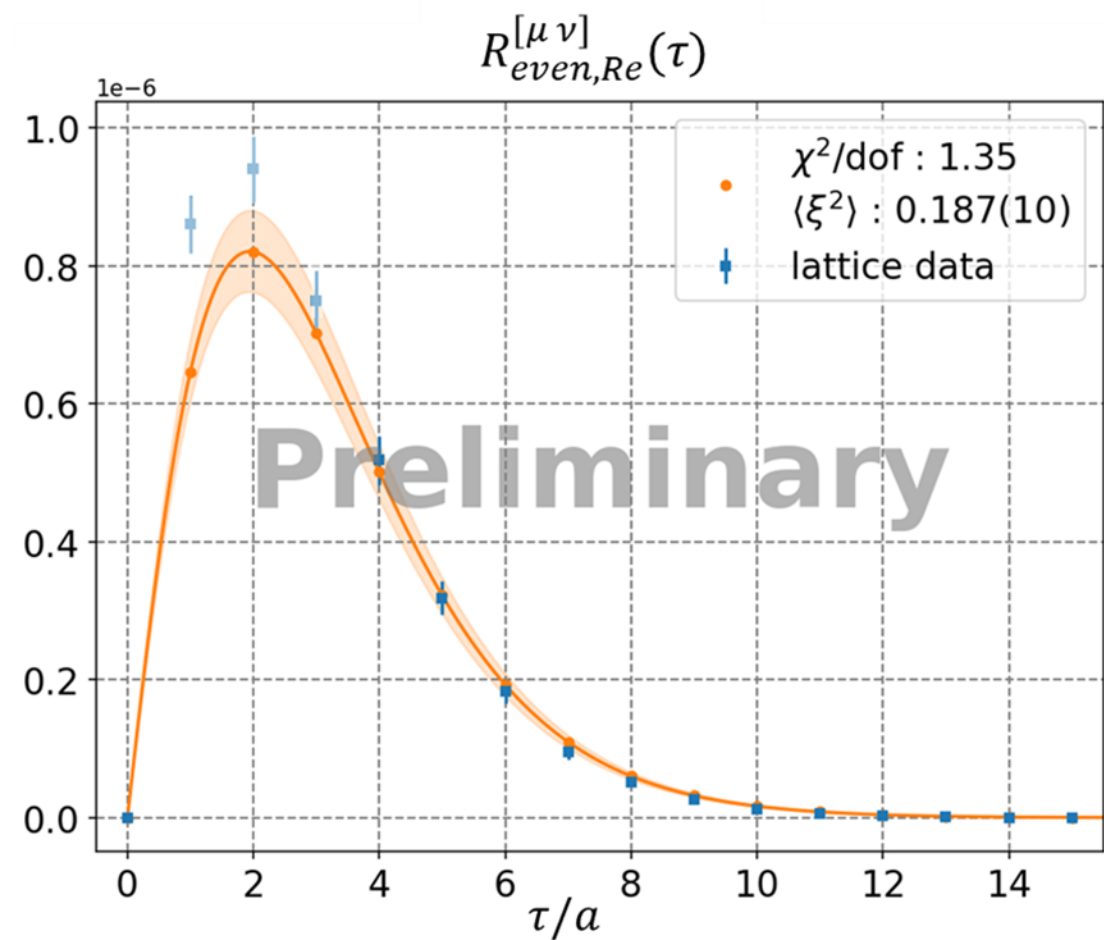
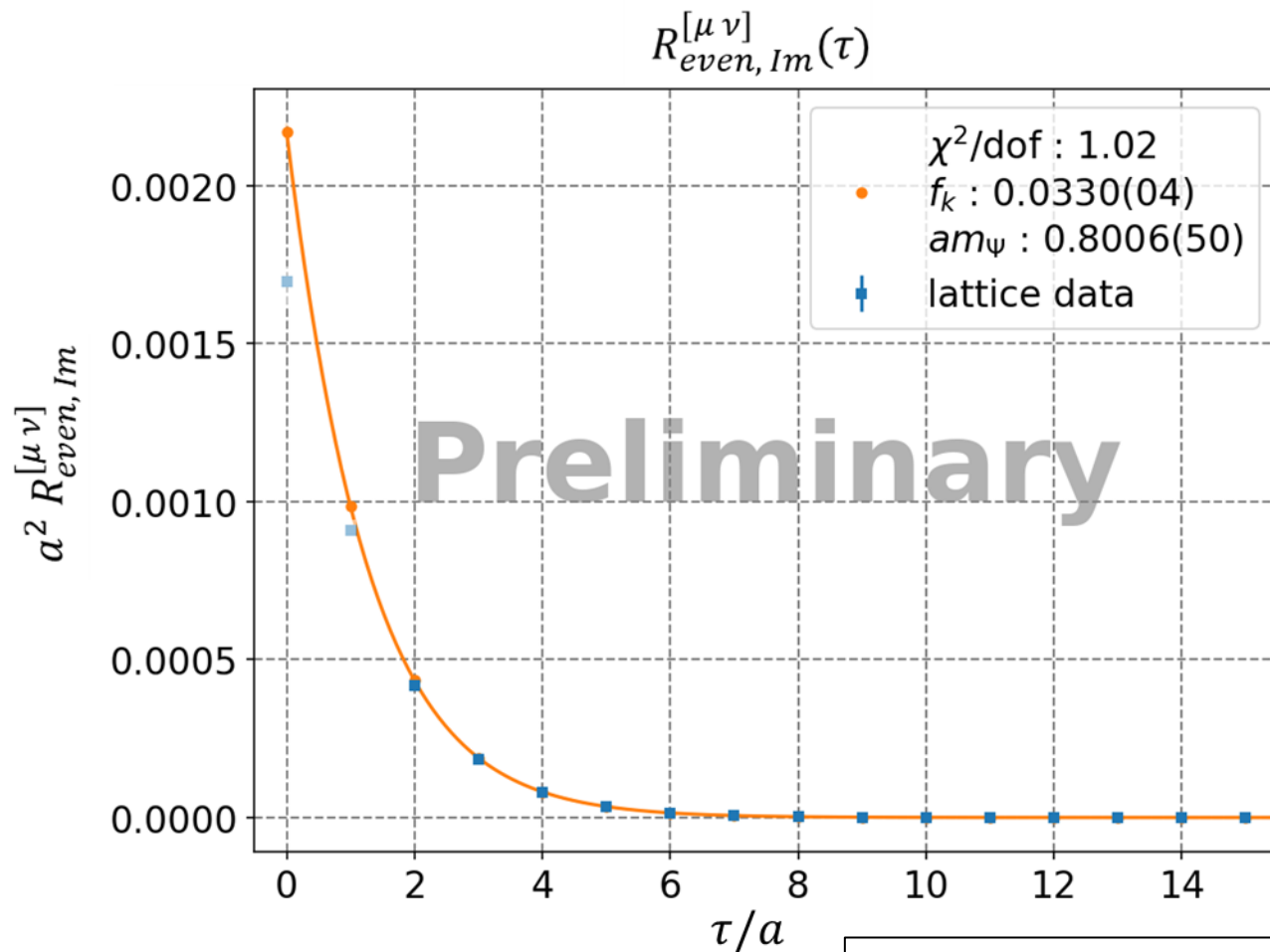


Planned our calculations on these CLS ensembles

Thanks to CLS and R-CCS

# Result

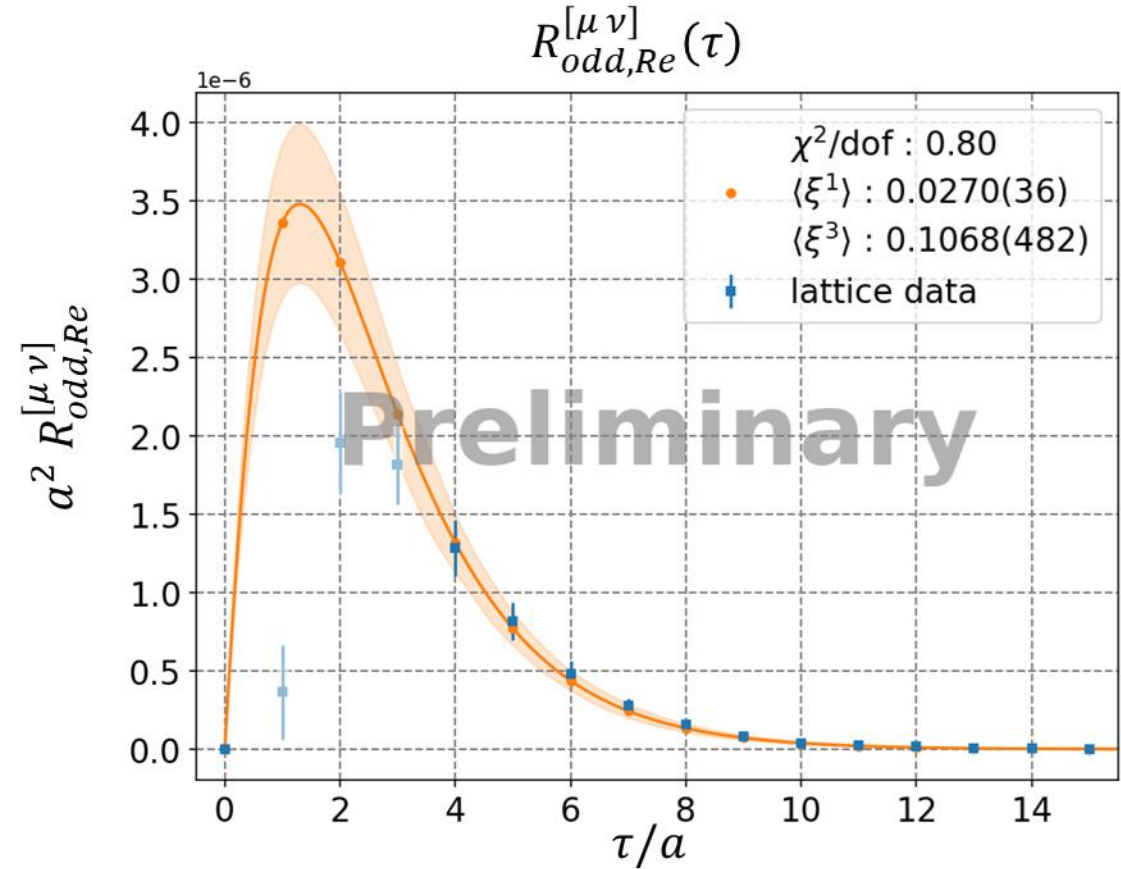
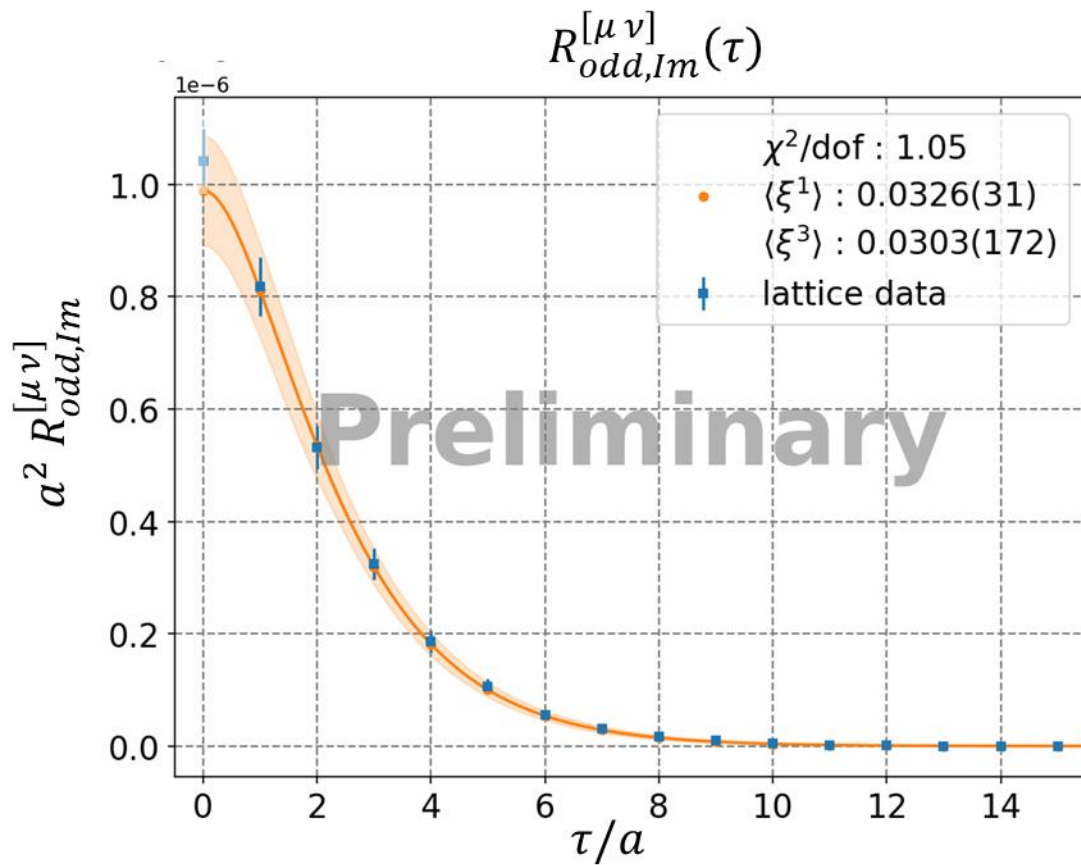
$(L/a)^3 * T/a$	$(a)$ (fm)	Configurations Used	Sources/Config	$M_\pi$ (MeV)	$M_k$ (MeV)
$32^3 * 64$	0.0750	400	16	355.8(1.2)	551.1(0.9)



data show significant lattice artifacts

# Result

$(L/a)^3 * T/a$	(a) (fm)	Configurations Used	Sources/Config	$M_\pi$ (MeV)	$M_k$ (MeV)
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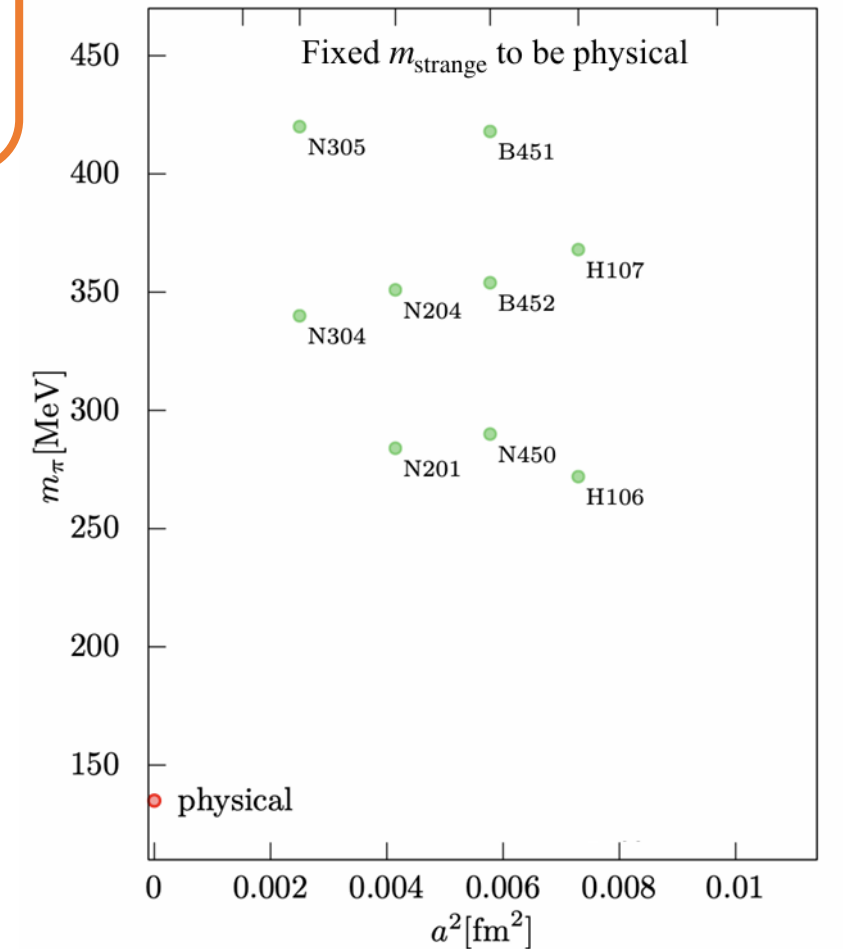
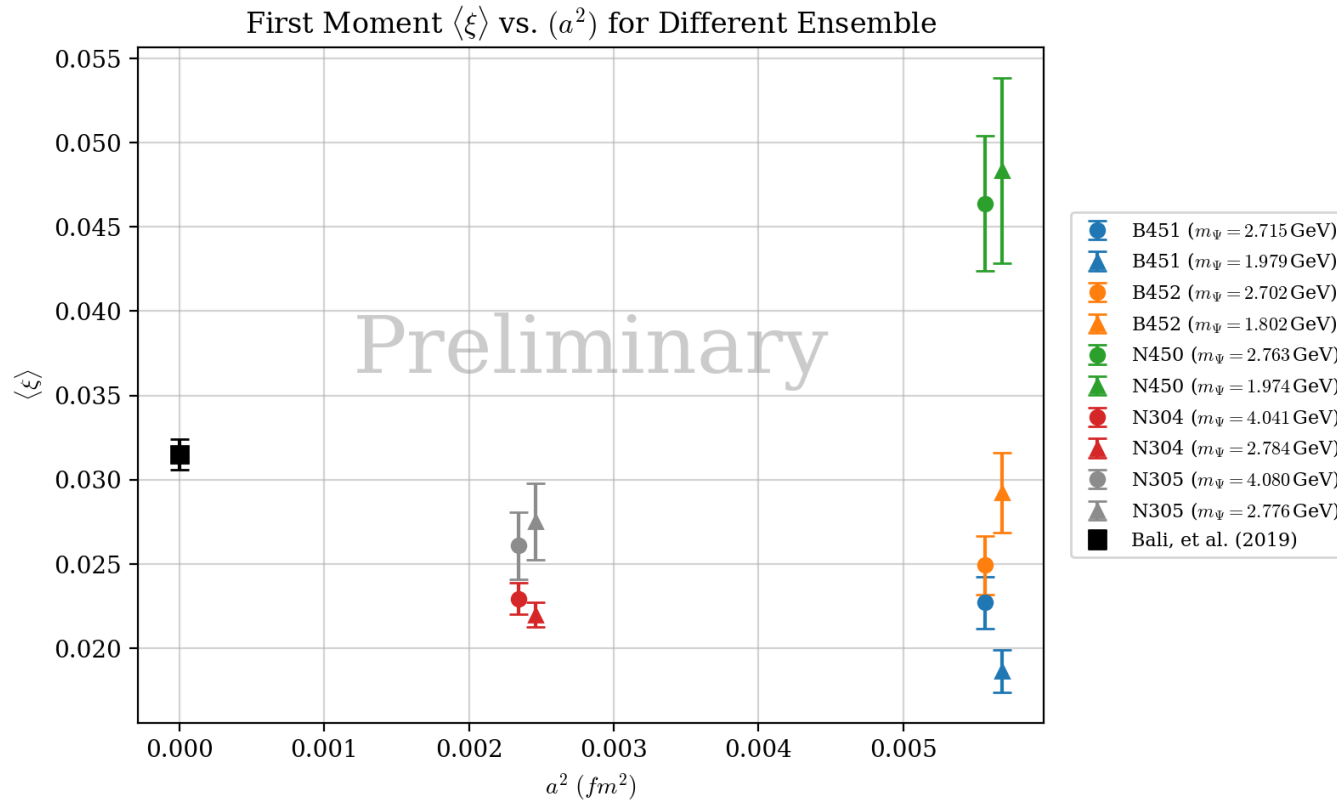


◆ data show significant lattice artifacts

# Continuum and Twist-2 Extrapolation

$$\langle \xi^n \rangle(a, m_\Psi) = \langle \xi^n \rangle + \frac{A_n}{m_\Psi} + B_n a^2 + C_n a^2 m_\Psi + D_n a^2 m_\Psi^2 + E_n (m_K^2 - m_\pi^2).$$

where  $\langle \xi^n \rangle$ ,  $A_n$ ,  $B_n$ ,  $C_n$ ,  $D_n$ , and  $E_n$  are fit parameters.

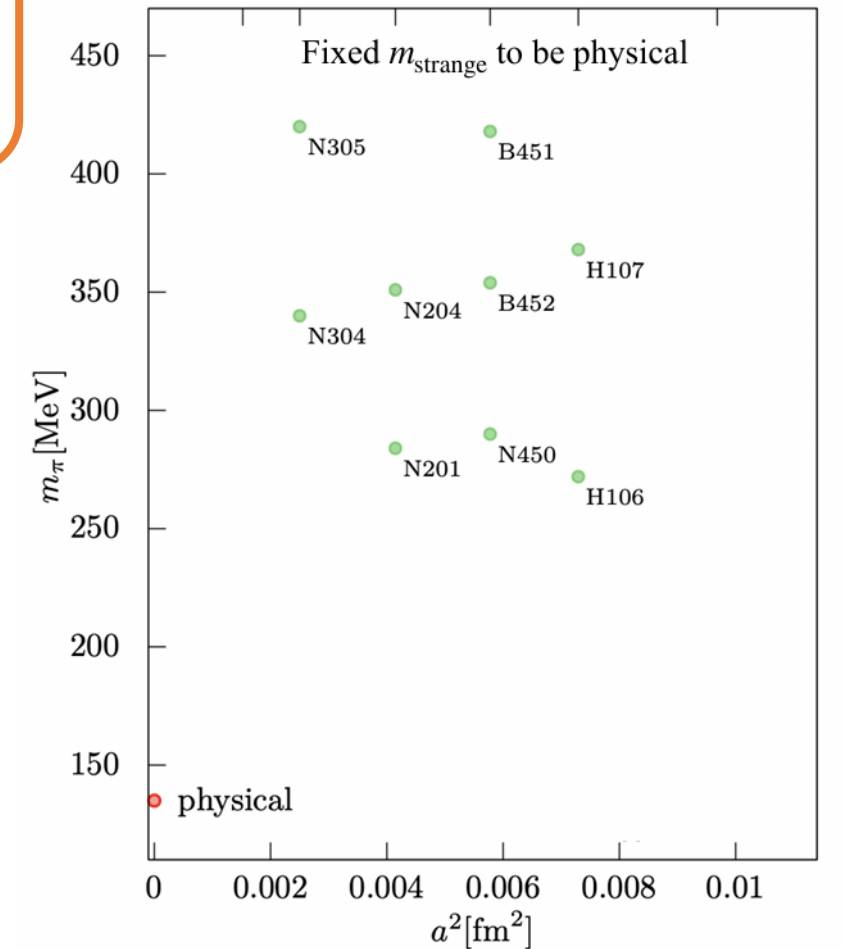
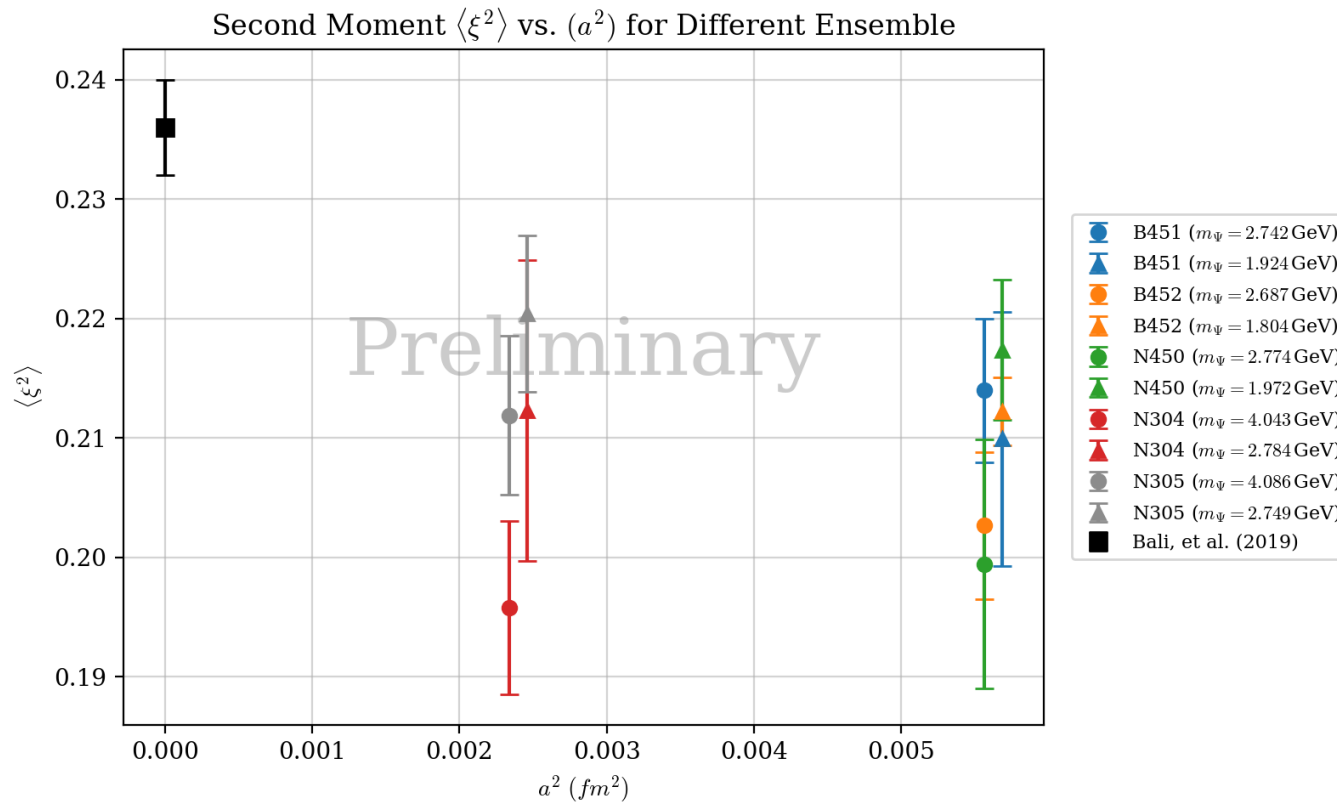


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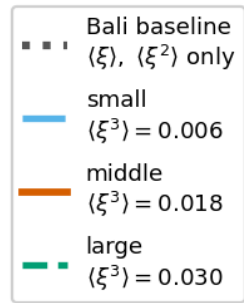
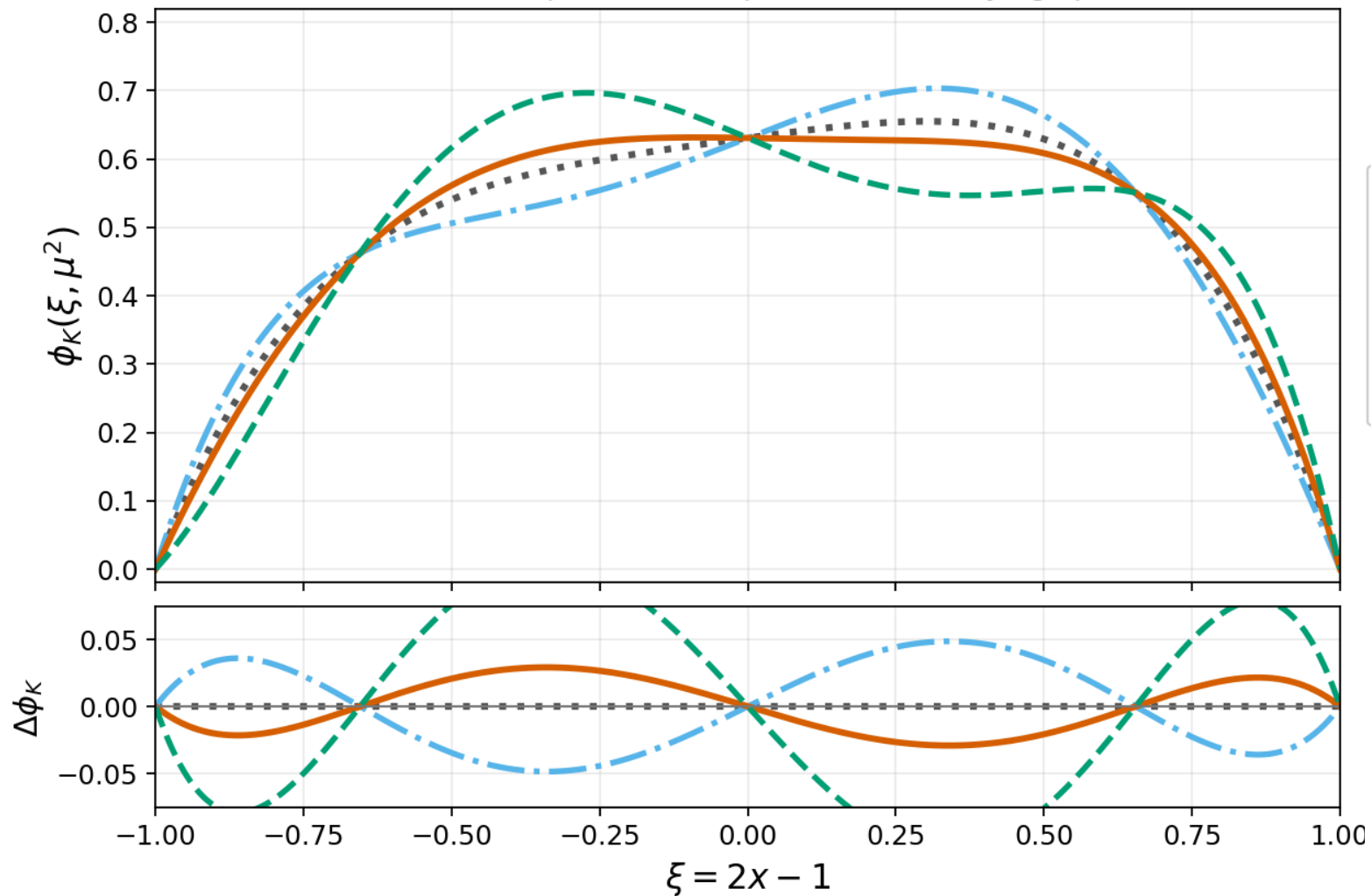
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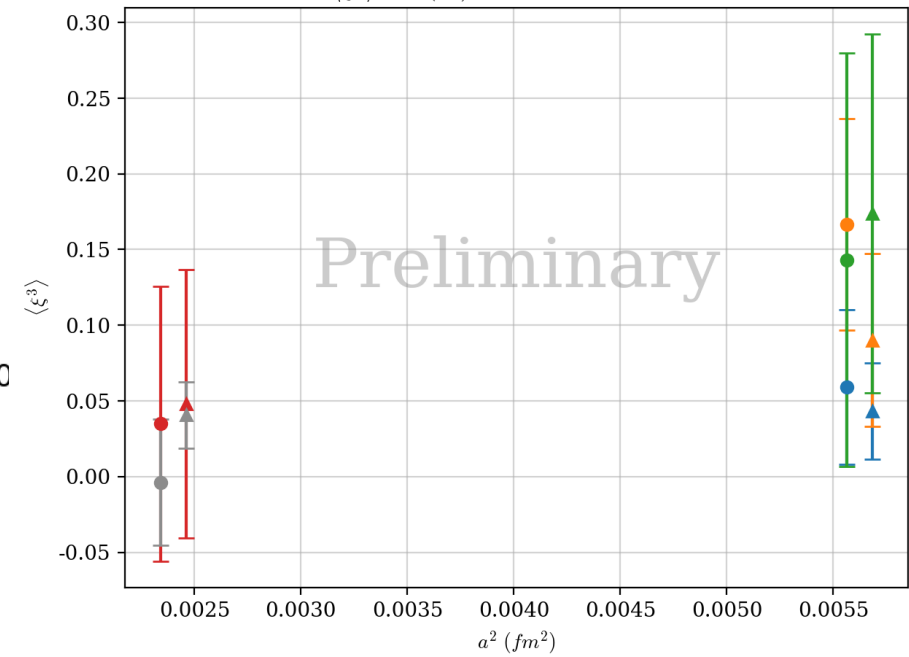


Bali fixed  $\langle \xi \rangle = 0.0315$ ,  $\langle \xi^2 \rangle = 0.2363$ ; varying  $\langle \xi^3 \rangle$



The continuum limit value of  $\langle \xi^3 \rangle$  is not yet available, so we vary it within the current lattice range to estimate its impact on the LCDA shape.

Third Moment  $\langle \xi^3 \rangle$  vs.  $(a^2)$  for Different Ensemble (odd real)



# Summary

## Summary

- We use the **Heavy-Quark Operator Product Expansion (HOPE)** to access the **moments of kaon LCDAs** from **Lattice QCD** calculations.

✓ Second and Fourth Moments of the Pion (**quenched**)

First, Second and Third Moments of the Kaon (**dynamical**)

## Future Work

- Incorporate **more lattice data (different heavy quark mass)** to improve **Continuum and Twist-2 Extrapolation** and **Chiral extrapolation**.

THANKS

# HOPE Method

## 3pt-function

$$C_3^{\mu\nu}(\tau_e, \tau_m; p_e, p_m) = \int d^3x_e d^3x_m e^{ip_e \cdot x_e} e^{ip_m \cdot x_m} \langle 0 | \mathcal{T} [J_A^\mu(\tau_e, x_e) J_B^\nu(\tau_m, x_m) \mathcal{O}_K^\dagger(0)] | 0 \rangle$$

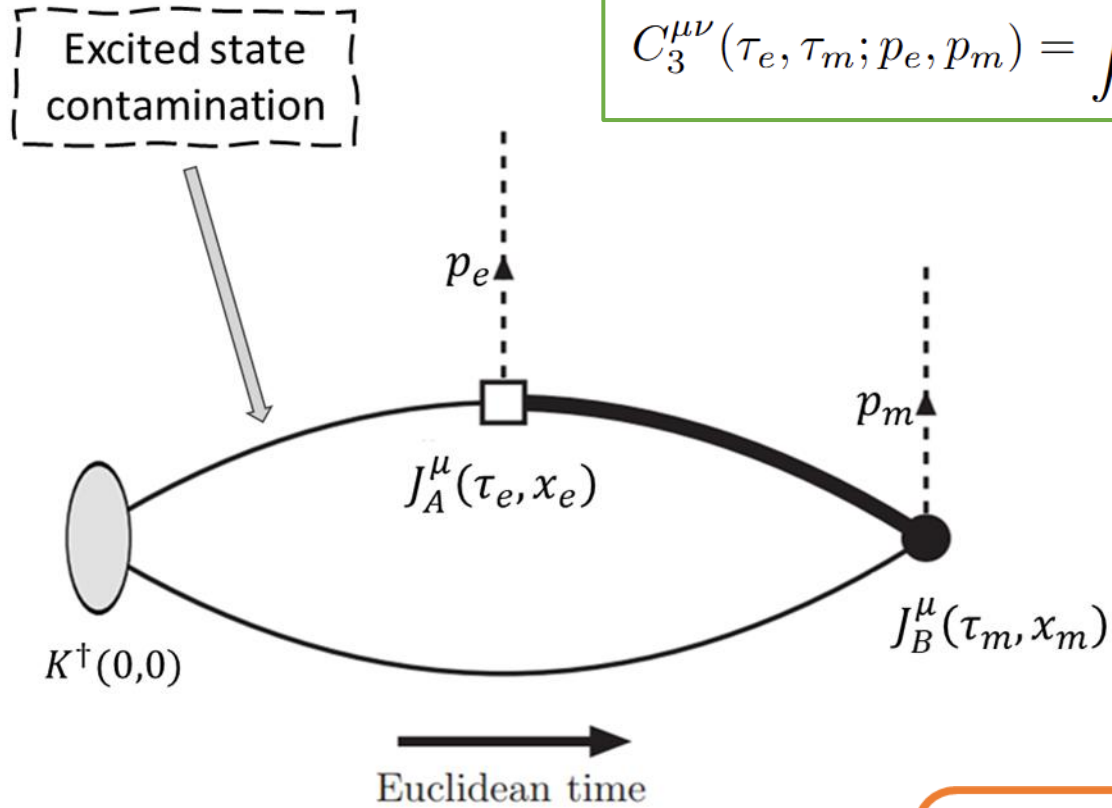
$$\langle 0 | \mathcal{T} \{ J_A^\mu(\tau_e, p_e) J_B^\nu(\tau_m, p_m) K^+(0, p) \} | 0 \rangle$$

Fourier transform of Hadronic Tensor

$$R^{\mu\nu}(\tau, p, q) = \langle 0 | \mathcal{T} \{ J_A^\mu(\tau_e, p_e) J_B^\nu(\tau_m, p_m) \} | K^+(p) \rangle$$

## 2pt-function

$$C_2(\tau, p) = \int d^3x e^{ip \cdot x} \langle 0 | \mathcal{O}_K(\tau, x) \mathcal{O}_K^\dagger(0, 0) | 0 \rangle$$



$$\begin{aligned} p &= p_e + p_m \\ q &= (q_e - q_m)/2 \\ \tau &= \tau_e - \tau_m \end{aligned}$$

$$R^{\mu\nu}(\tau; p, q) \sim C_3^{\mu\nu}(\tau_e, \tau_m; p_e, p_m) \frac{2 E_K(p)}{Z_K(p)} e^{-E_K(p)(\tau_e + \tau_m)/2}$$

$$G_3^{\mu\nu}(x, y) = \langle \Omega | \mathcal{T} \{ J_A^\mu(x) J_A^\nu(y) \mathcal{O}_\pi^\dagger(0) \} | \Omega \rangle$$

$$J_A^\mu = Z_A^{(0)} (1 + \tilde{b}_A a \tilde{m}_{ij}) \left[ \bar{\psi} \gamma_\mu \gamma_5 \Psi + a c_A \partial_\mu \bar{\psi} \gamma_5 \Psi - a \frac{c'_A}{4} \left( \bar{\psi} \gamma_\mu \gamma_5 (\vec{D} + m_\Psi) \Psi - \bar{\psi} (\overleftarrow{D} + m_\psi) \gamma_\mu \gamma_5 \Psi \right) + (\psi \leftrightarrow \Psi) \right]$$

$$\begin{aligned} G_3^{\mu\nu}(x, y) &= Z^2(a) Z_\pi \langle \Omega | T \left\{ \left( \bar{\Psi}^{(0)}(x) \gamma^\mu \gamma^5 \psi^{(0)}(x) + \bar{\psi}^{(0)}(x) \gamma^\mu \gamma^5 \Psi^{(0)}(x) \right. \right. \\ &\quad \left. \left. + a c_A \partial^\mu \left\{ \bar{\Psi}^{(0)}(x) \gamma^5 \psi^{(0)}(x) \right\} + a c_A \partial^\mu \left\{ \bar{\psi}^{(0)}(x) \gamma^5 \Psi^{(0)}(x) \right\} \right) \right. \\ &\quad \times \left( \bar{\Psi}^{(0)}(y) \gamma^\nu \gamma^5 \psi^{(0)}(y) + \bar{\psi}^{(0)}(y) \gamma^\nu \gamma^5 \Psi^{(0)}(y) \right. \\ &\quad \left. \left. + a c_A \partial^\nu \left\{ \bar{\Psi}^{(0)}(y) \gamma^5 \psi^{(0)}(y) \right\} + a c_A \partial^\nu \left\{ \bar{\psi}^{(0)}(y) \gamma^5 \Psi^{(0)}(y) \right\} \right) \\ &\quad \left. \times [\bar{\psi}^{(0)}(0) \gamma^5 \psi^{(0)}(0)]^\dagger \right\} | \Omega \rangle + \mathcal{O}(a^2), \end{aligned}$$

$$a c_A i p_2^\nu \int d^4 x d^4 y e^{-i p_1 \cdot x} e^{-i p_2 \cdot y} \langle \Omega | T \{ (\bar{\Psi} \gamma^\mu \gamma^5 \psi)(x) (\bar{\psi} \gamma^5 \Psi)(y) [\bar{\psi} \gamma^5 \psi(0)]^\dagger \} | \Omega \rangle.$$

$$\begin{aligned} \frac{a c'_A}{4} \sum_{\mathbf{x}_e, \mathbf{x}_m, x} & [D_u^{-1}(0|x_m)_{da}^{\eta\alpha}] [D_d^{-1}(x_e|0)_{cd}^{\varepsilon\zeta}] [D_\Psi^{-1}(x|x_e)_{bc}^{\gamma\delta}] [D_\Psi(x_m|x)_{ab}^{\beta\gamma}] \\ & (\gamma^\nu \gamma^5)^{\alpha\beta} (\gamma^\mu \gamma^5)^{\delta\varepsilon} (\gamma^5)^{\zeta\eta} e^{i\mathbf{p}_e \cdot \mathbf{x}_e} e^{i\mathbf{p}_m \cdot \mathbf{x}_m}. \end{aligned}$$

$$\sum_x D_\Psi(x_m|x)_{ab}^{\beta\gamma} D_\Psi^{-1}(x|x_e)_{bc}^{\gamma\delta} = \delta^{(4)}(x_m - x_e) \delta^{\beta\delta} \delta_{ac}$$

$$|\pi(\mathbf{p})\rangle \rightarrow |u(\frac{1}{2}(1+x_0)p, \uparrow)\bar{d}(\frac{1}{2}(1-x_0)p, \downarrow)\rangle$$

$$V_q^{\mu\nu(0)}(q, p) = \bar{v}(\frac{1}{2}(1-x_0)p, \downarrow) \left[ \gamma^\mu \gamma_5 \frac{i}{\not{q} + \frac{x_0\not{p}}{2} - m_\Psi} \gamma^\nu \gamma_5 + \gamma^\nu \gamma_5 \frac{i}{-\not{q} + \frac{x_0\not{p}}{2} - m_\Psi} \gamma^\mu \gamma_5 \right] u(\frac{1}{2}(1+x_0)p, \uparrow)$$

$$\begin{aligned} V_q^{[\mu\nu](0)}(q, p, m_\Psi, x_0) &= \epsilon^{\mu\nu\rho\sigma} q_\rho \bar{v} \gamma_\sigma \gamma_5 u \left[ \frac{1}{q^2 + x_0 p \cdot q - m_\Psi^2} + \frac{1}{q^2 - x_0 p \cdot q - m_\Psi^2} \right] \\ &= -\frac{2\epsilon^{\mu\nu\rho\sigma} q_\rho}{\tilde{Q}^2} \bar{v} \gamma_\sigma \gamma_5 u \sum_{\substack{n=0, \\ \text{even}}}^{\infty} \mathcal{F}_n^{(0)}(\tilde{\omega}) \phi_n^{(0)}(x_0), \end{aligned}$$

$$\mathcal{F}_n^{(0)}(\tilde{\omega}) = \frac{3}{2} \frac{\sqrt{\pi}(n+1)(n+2)n!}{2^{n+2}\Gamma(n+\frac{5}{2})} \left(\frac{\tilde{\omega}}{2}\right)^n {}_2F_1\left(\frac{n+1}{2}, \frac{n+2}{2}; n+\frac{5}{2}; \frac{\tilde{\omega}^2}{4}\right).$$

$$\mathcal{F}_n(\tilde{Q}^2, \mu^2, \tilde{\omega}, \tau) = \mathcal{F}_n^{(0)}(\tilde{\omega}) \left[ 1 + \frac{\alpha_s C_F}{4\pi} \gamma_n^{(0)} \ln \frac{\mu^2}{\tilde{Q}^2} \right] + \frac{\alpha_s C_F}{4\pi} \mathcal{R}_n^{(1)}(\tilde{Q}^2, \mu^2, \tau, \tilde{\omega}) + \mathcal{O}(\alpha_s^2) \quad \phi_n(x_0, \epsilon) = \phi_n^{(0)}(x_0) \left[ 1 + \frac{\alpha_s C_F}{4\pi} \frac{\gamma_n^{(0)}}{\epsilon'} + \mathcal{O}(\alpha_s^2) \right]$$

Beyond leading logarithm, the conformal symmetry is broken, so the Gegenbauer moments start to mix.

### AIC reweighted fit: B452, even\_im, 1-loop, mu=RQ, R=1.00

