From cold-atom polaron to nuclear physics:

---Alpha clusters in neutron matter---

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w/ Hiroyuki Tajima (U. Tokyo), Kei Iida(OUJ), Wataru Horiuchi (Osaka Metr. U.), and Hajime Moriya (Hokkaido U.)

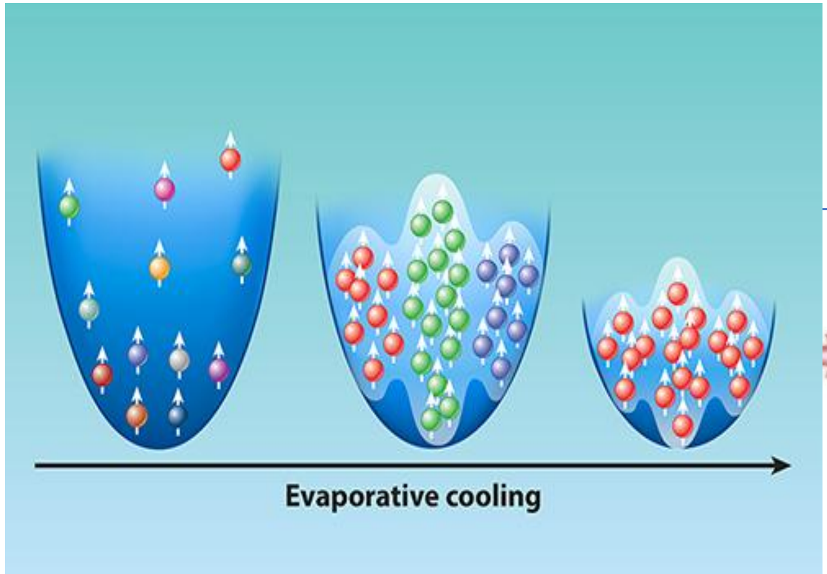
arXiv:2106.14469;arXiv:2207.13907;
arXiv:2304.00535; arXiv:2310.19422; arXiv:2408.15043

＠ **TNP2026, 宜蘭, on 21 Apr 2026**

contents：

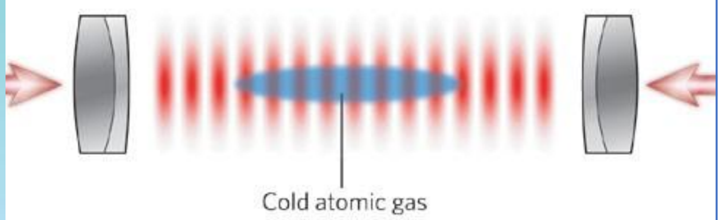
Quick view of cold-atom system (all most everything at will)

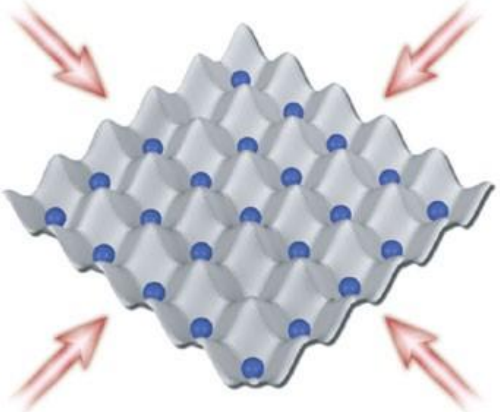
Cooling down to degenracy



APS viewpoint(2021), C. Mishra

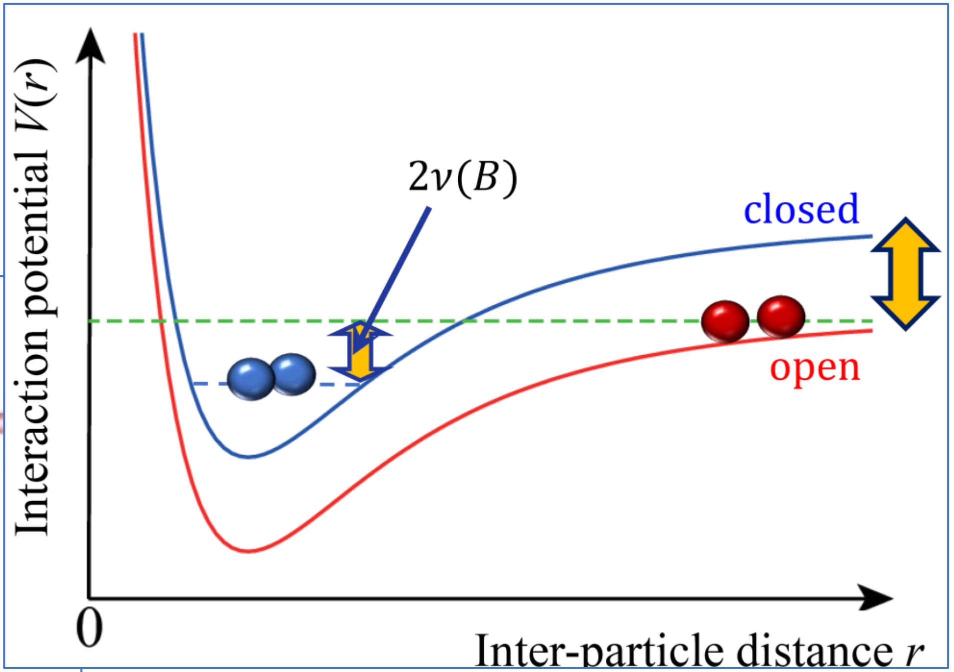
Trap geometry

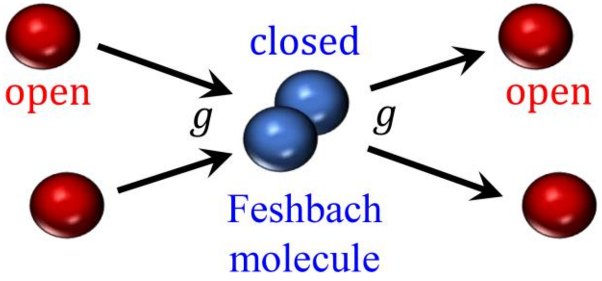




Nature(2008), I. Bloch

Atom-atom interaction

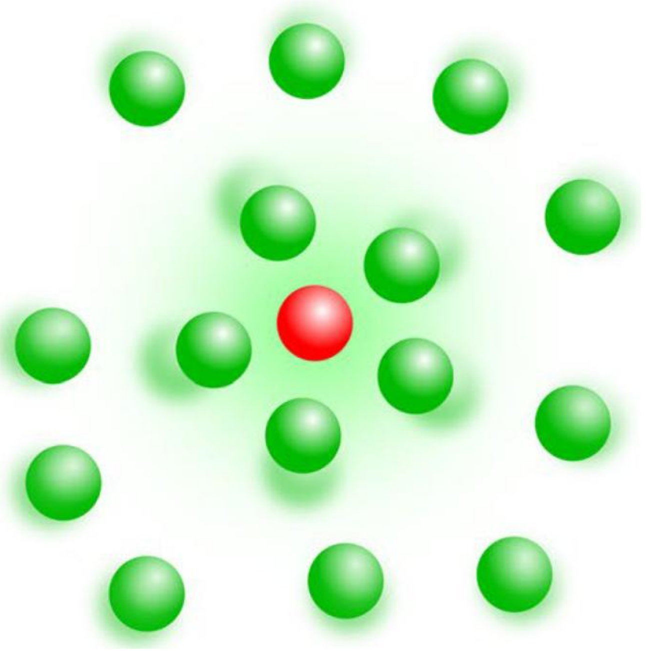




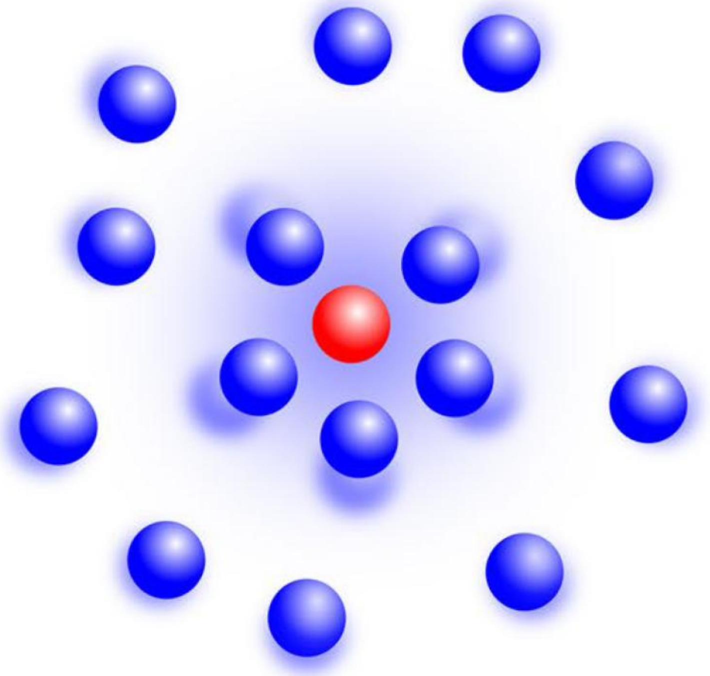
by H. Tajima

Cold-atom polarons (two categories):

Bose polaron (Impurity atom in BEC)

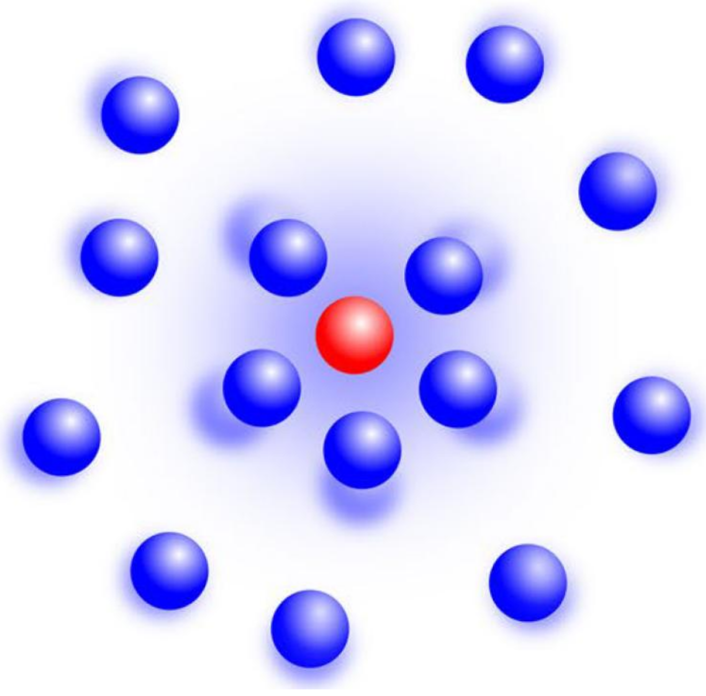


Fermi polaron (Impurity atom in Fermi sea)



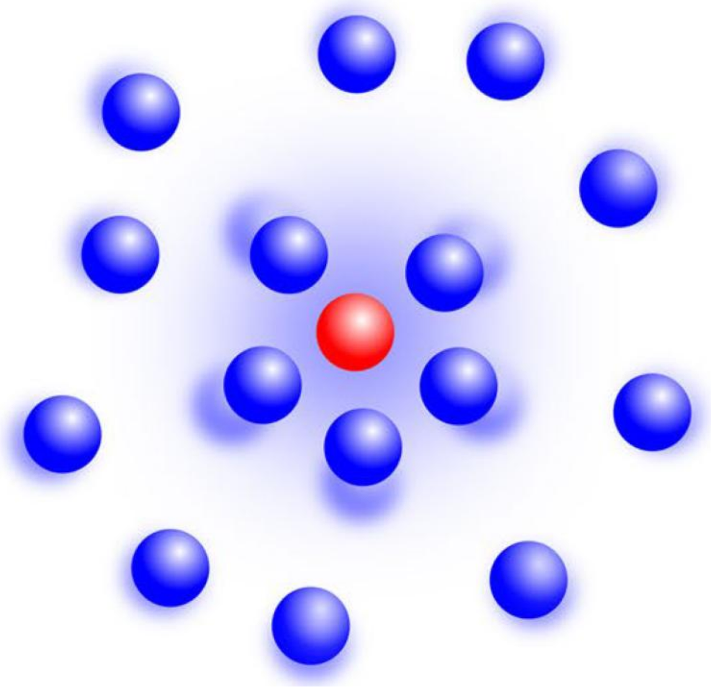
Cold-atom polarons = impurity atoms (very dilute gas) <u>dressed by *p*-*h* cloud (polarization) in degenerate medium (BEC or Fermi sea)</u>

Fermi polaron



Alpha particles in Neutron Fermi sea

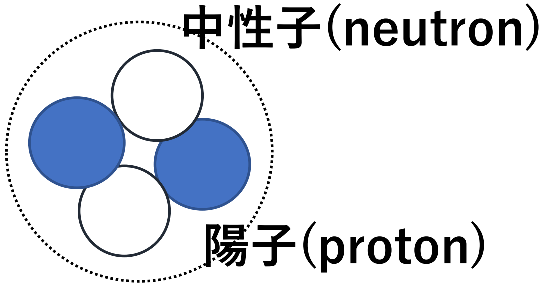
(=Polaronic Alpha particle)



Mapping

using EFT
developed
for cold-atom polarons

Alpha particle (^{4}He nuclei) : Why interesting ?



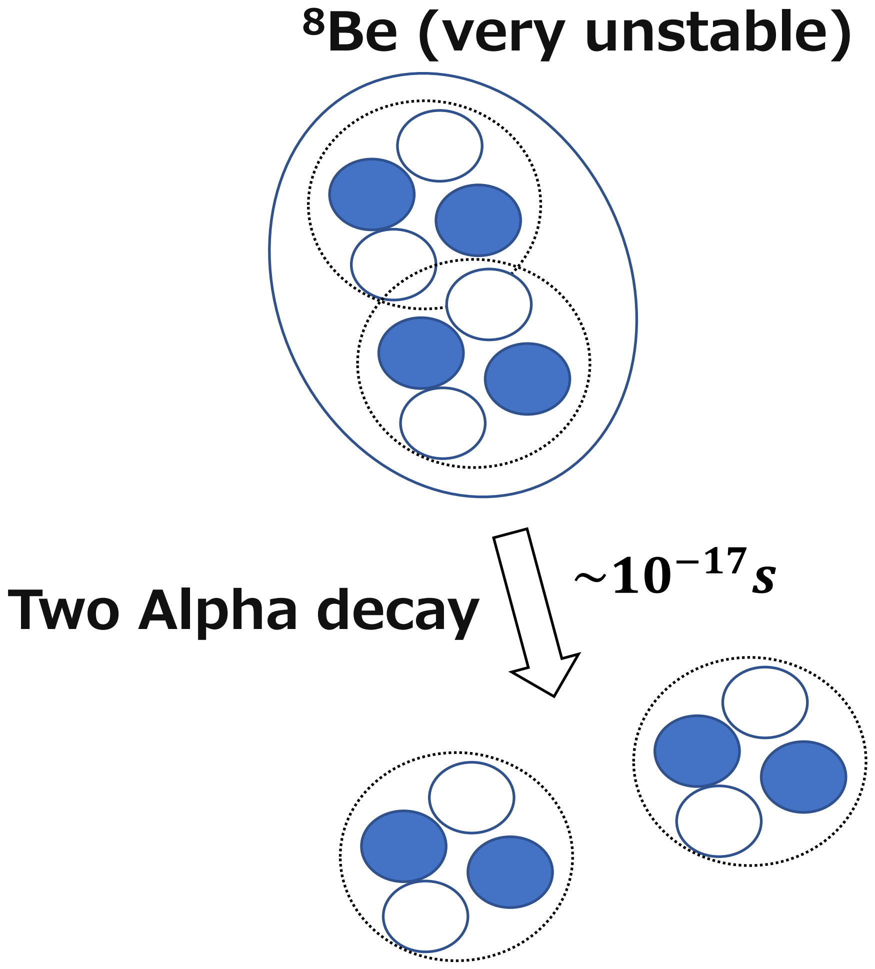
Binding energy ~ 7 MeV/nucleon ⇒ very stable nuclear <u>cluster</u>

(<u>effective DoF</u>)

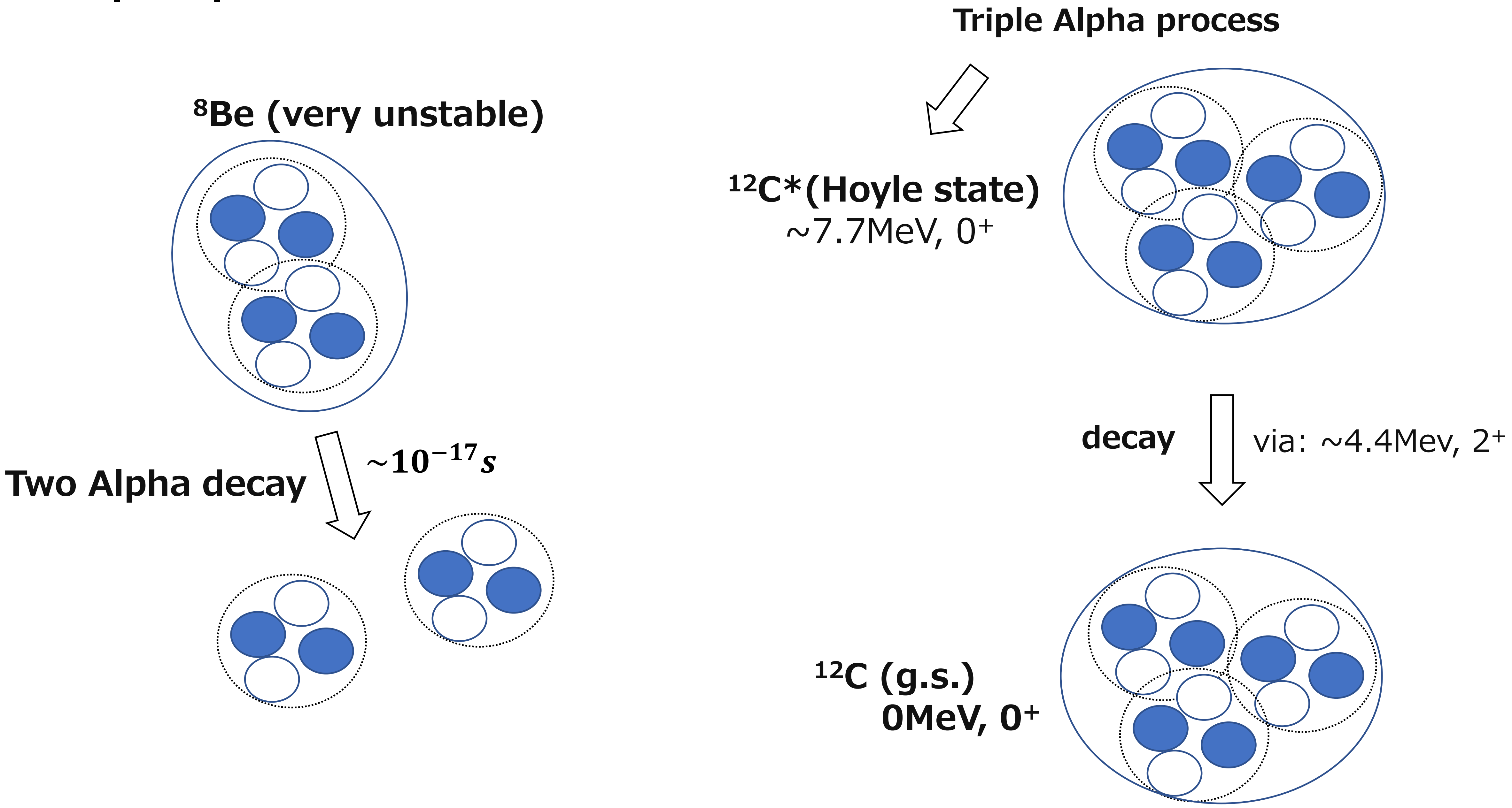
- Alpha decay (**in vacuum**)
- Surface in light neutron-rich nuclei (**in ‘medium’**)
- Inside neutron star/supernova (**in medium**)

w/other light clusters: Deuteron,Triton, *etc*

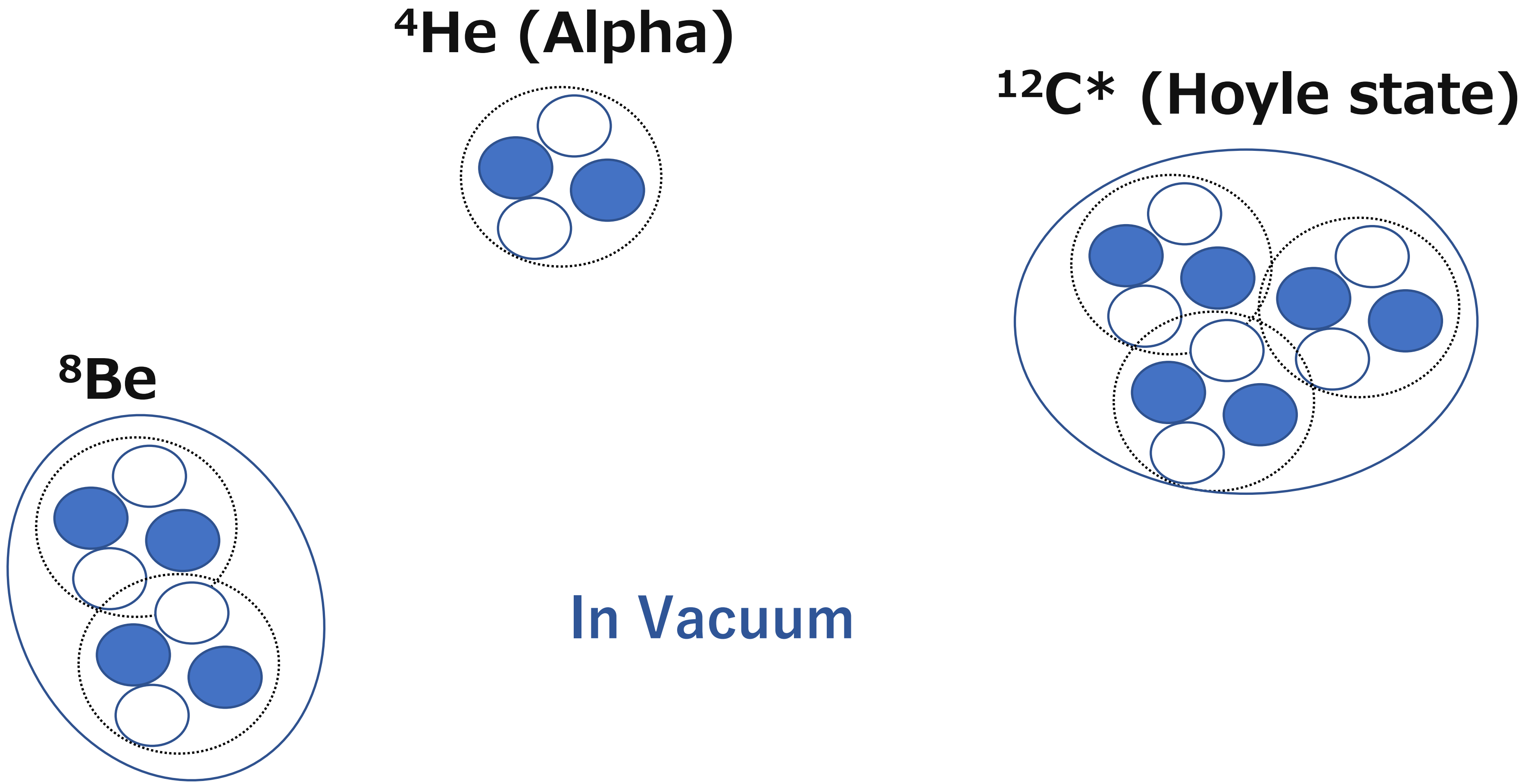
Alpha particles (clusters) in vacuum



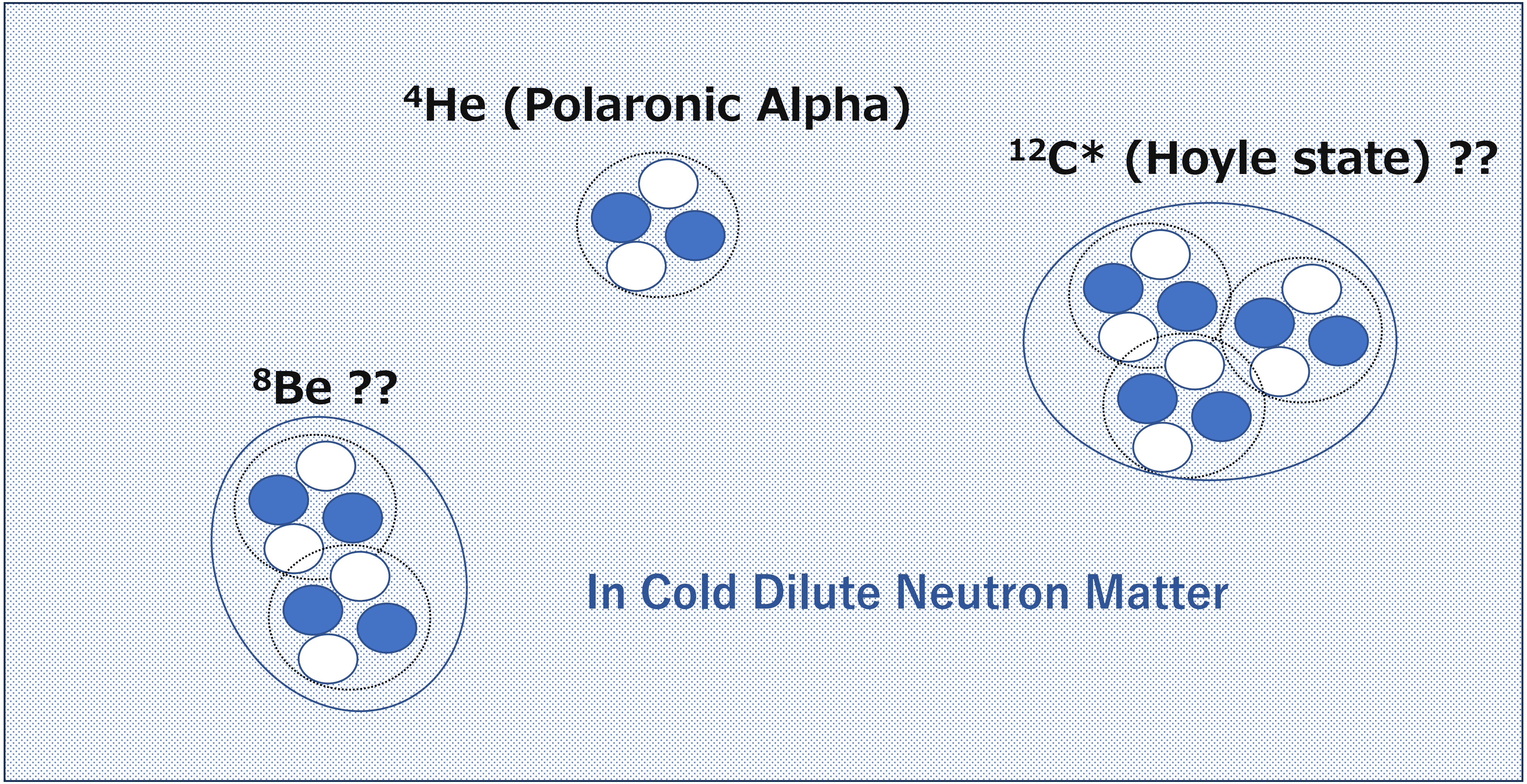
Alpha particles (clusters) in vacuum



What up if Alpha particles put in cold dilute neutron matter?



What up if Alpha particles put in cold dilute neutron matter?



Low-Energy Effective Hamiltonian for Alpha-Neutron system:

$$H(x) = \underbrace{\sum_{s,p} \frac{p^2}{2m} a^{\dagger}_{s,p} a_{s,p} + \frac{1}{2} \sum_{q,q',p,s,t,s',t'} a^{\dagger}_{t',q'-p} a^{\dagger}_{s',q+p} a_{s,q} a_{t,q'} \tilde{V}_{s't'st}(p)}_{\text{Neutron part}}$$

$$\underbrace{-\frac{\nabla_x^2}{2M}}_{\text{Alpha part}} + \underbrace{\sum_{p,q,s} g a^{\dagger}_{s,p} a_{s,q} e^{-i(p-q)x}}_{\text{Neutron-Alpha interaction}},$$

LS equation for S-wave scattering length :

$$g^{-1} = \frac{m_r}{2\pi\hbar^2 a} - \sum_p \frac{2m_r}{p^2} \qquad {m_r}^{-1} = m^{-1} + M^{-1}$$

Parameters determined from low-energy neutron-alpha scattering data

Neutron- Alpha scattering data analysis:

Progress of Theoretical Physics, Vol. 61, No. 5, May 1979

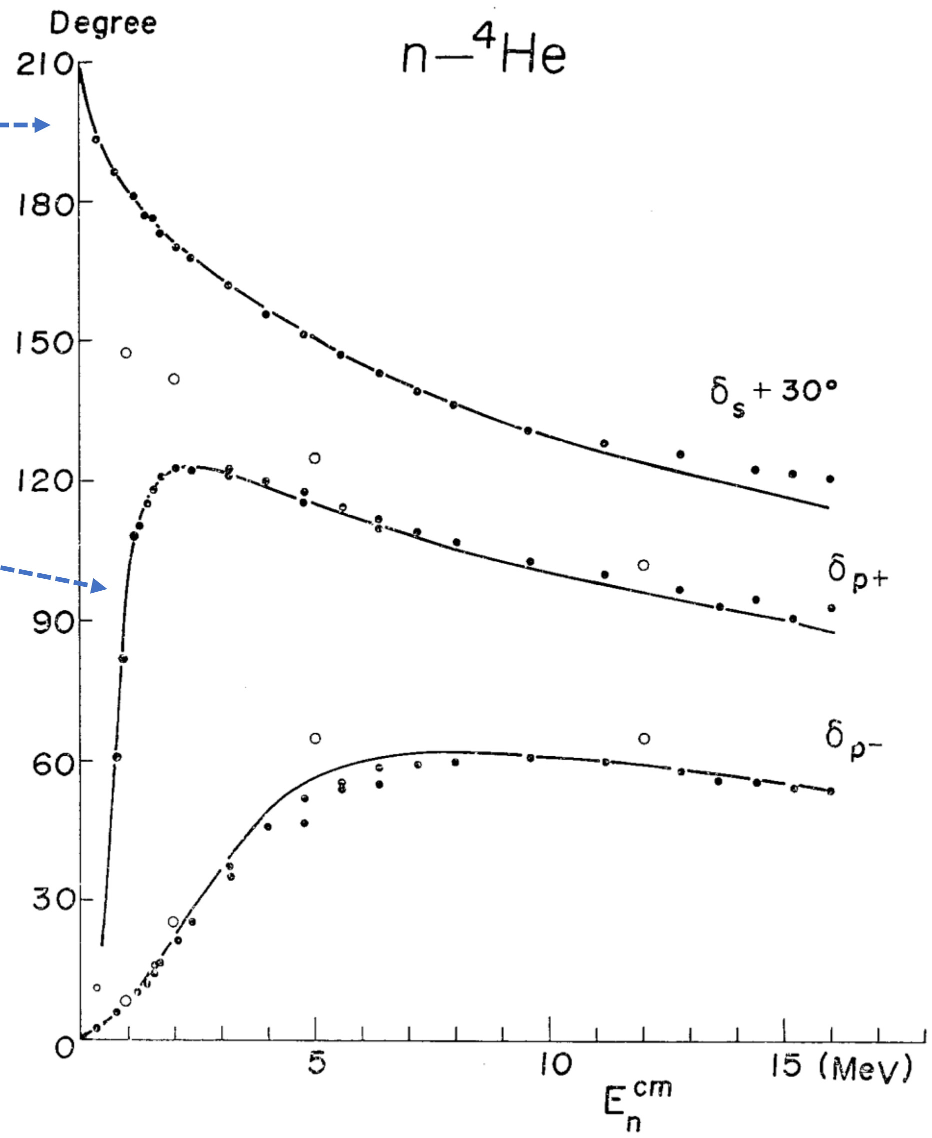
Microscopic Study of Nucleon-^{4}He Scattering and Effective Nuclear Potentials

Hiroyuki KANADA, Tsuneo KANEKO, Shinobu NAGATA* and Morikazu NOMOTO

phase shifts vs energy:

S-wave

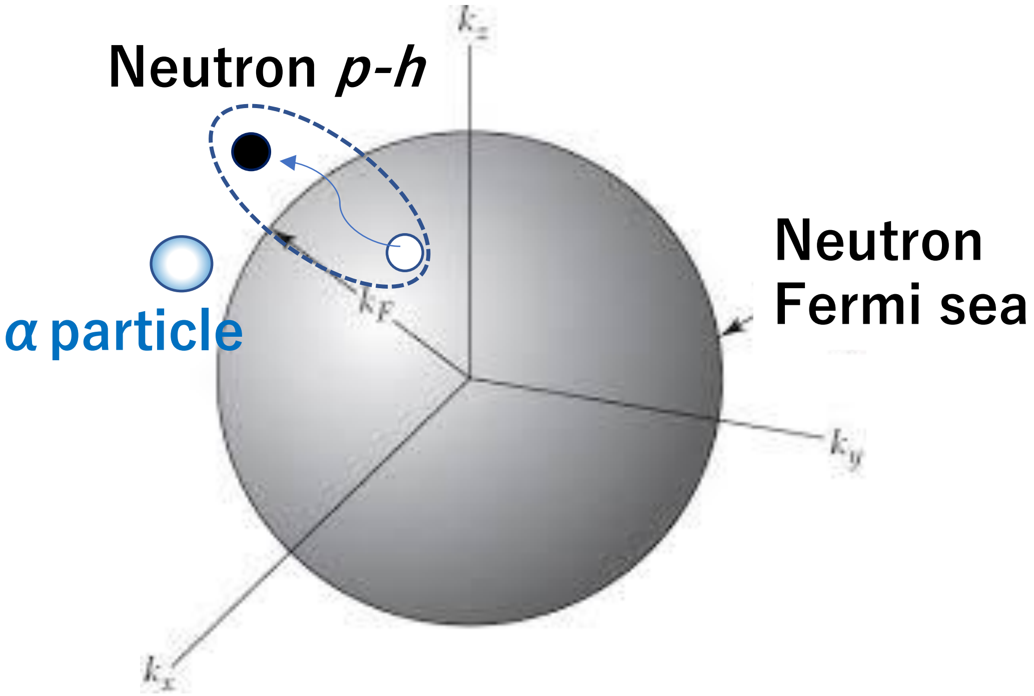
P-wave (P3/2) resonance



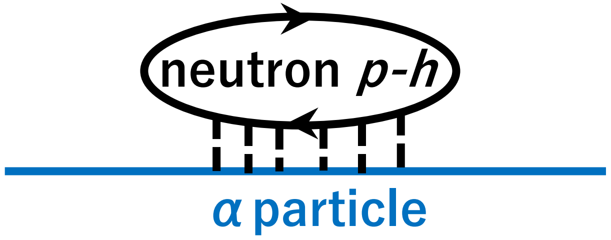
EFT with S-wave is limited
by Neutron Fermi energy < P-wave resonance:

$$k_F < 0.25\,\mathrm{fm}^{-1} \quad \rightarrow \quad \rho < 0.003\rho_0,$$

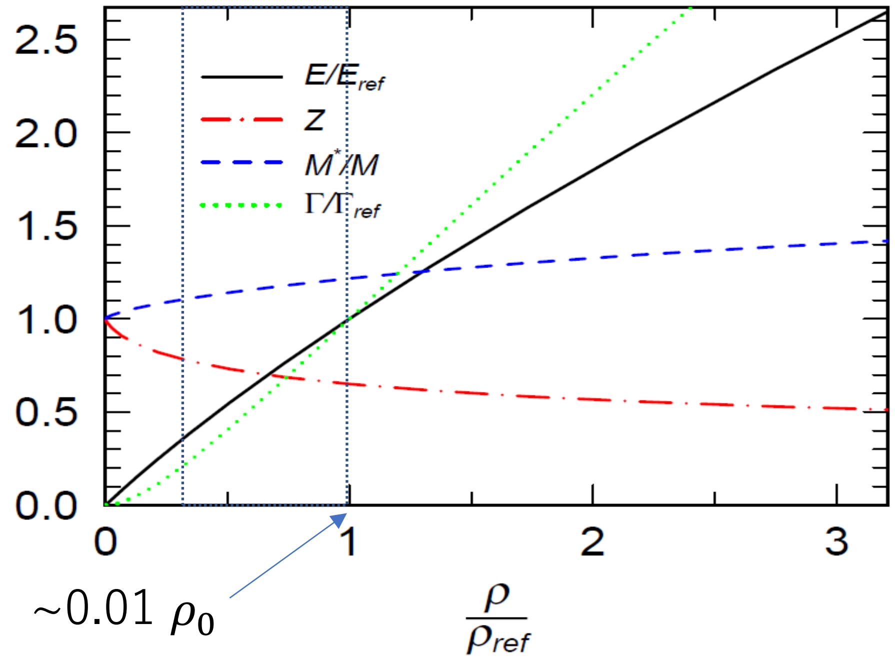
Single Polaronic Alpha particle:



Self-energy with medium effects



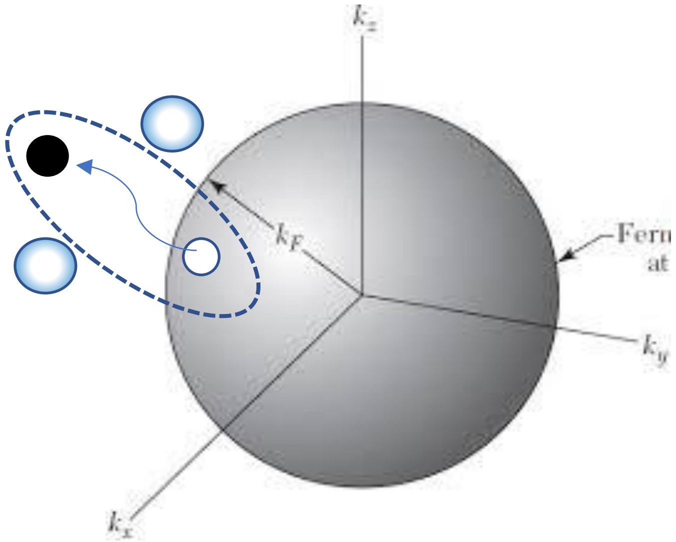
$$G^R(\omega, P) = \frac{1}{\omega + i0 - \frac{P^2}{2M} - \Sigma(\omega + i0, P)} \sim \frac{Z_P}{\omega - E_P + i\Gamma_P},$$



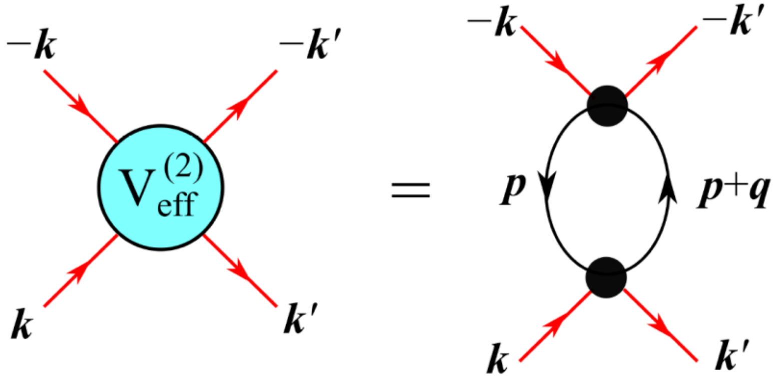
Quasi-particle properties:

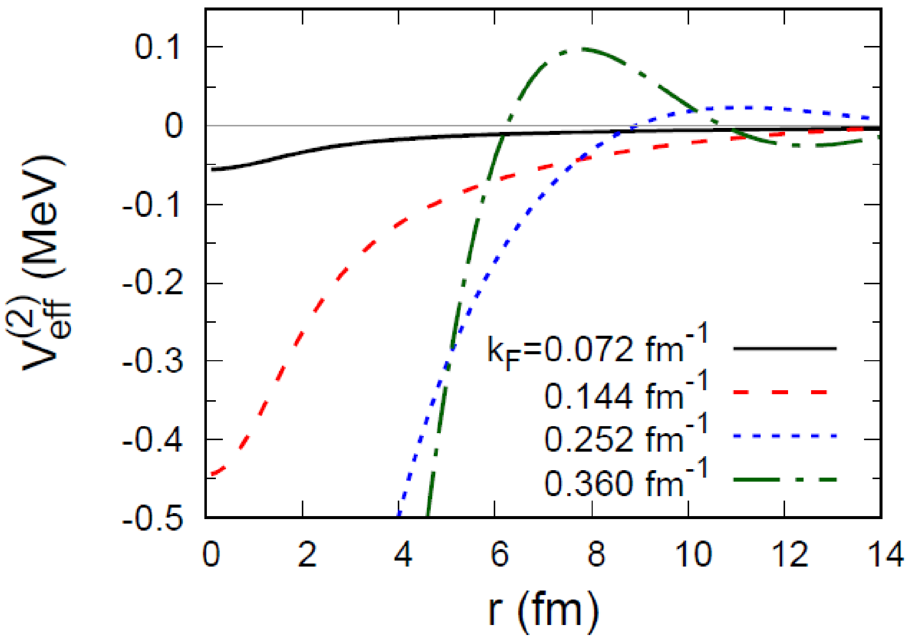
- Effective mass increase
- Small width ⇒good quasi-particle pic.

Two polaronic Alpha particles:



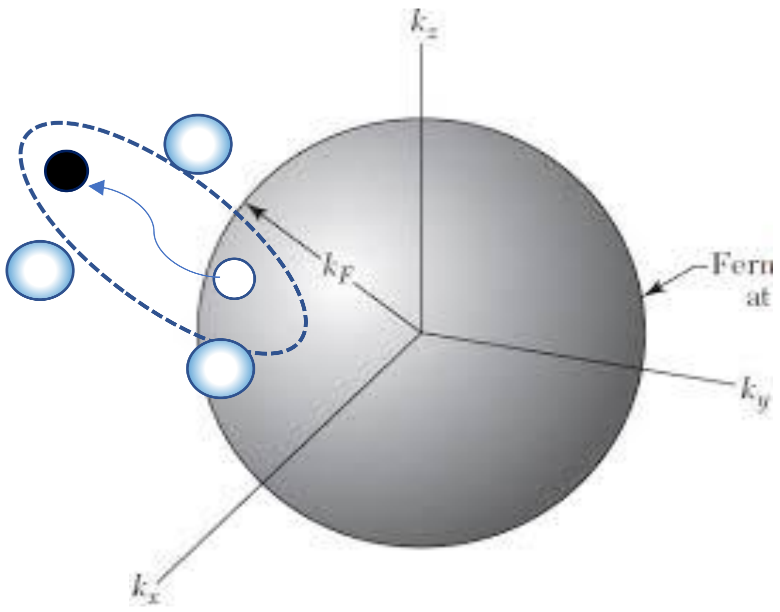
Medium-induced (RKKY type) two-body force (mostly attractive):



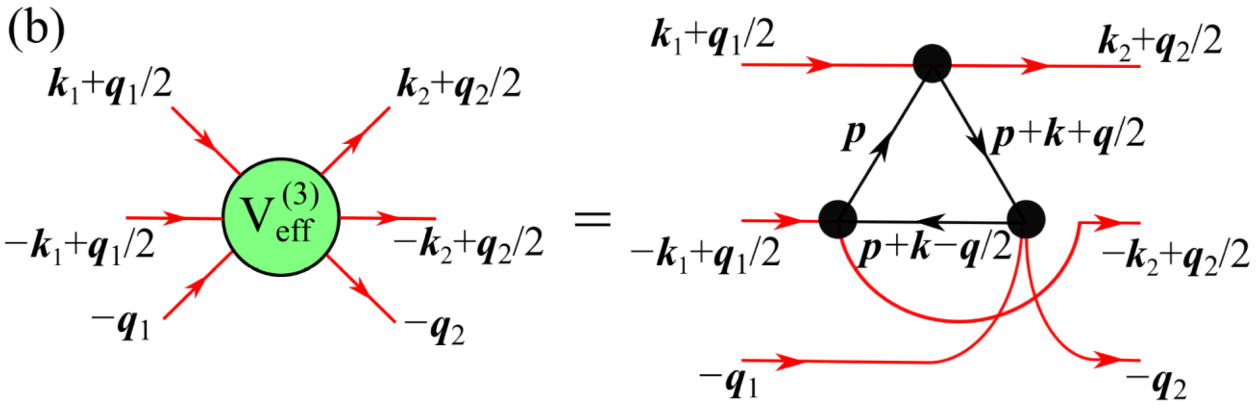


$$V_{\rm eff}^{(2)}(\boldsymbol{r}_1,\boldsymbol{r}_2)=\frac{m}{8\pi^3\hbar^2}\left(\frac{2\pi\hbar^2 a}{m_r}\right)^2 \times\frac{(2k_{\rm F}r)\cos(2k_{\rm F}r)-\sin(2k_{\rm F}r)}{r^4},$$

Three polaronic Alpha particles:



Medium-induced (3-particle irreducible)
α's three-body force (weakly repulsive)



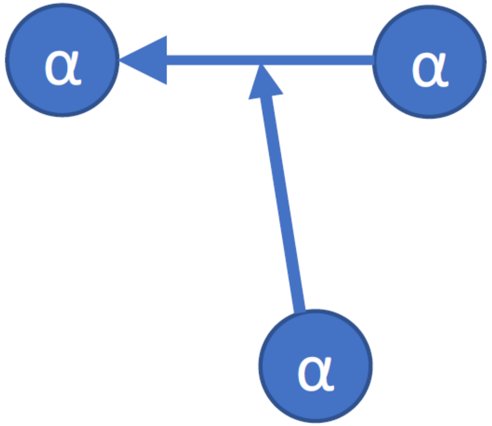
$$
V_{\rm eff}^{(3)}(\boldsymbol{k},\boldsymbol{q},i\nu_\ell,i\nu_u)=2\left(\frac{2\pi\hbar^2 a}{m_r}\right)^3
\times T\sum_{\sigma=\uparrow,\downarrow}\sum_{\boldsymbol{p},\omega_n}G_\sigma(\boldsymbol{p},i\omega_n)G_\sigma(\boldsymbol{p}+\boldsymbol{k}+\boldsymbol{q}/2,i\omega_n+i\nu_\ell)
\times G_\sigma(\boldsymbol{p}+\boldsymbol{k}-\boldsymbol{q}/2,i\omega_n+i\nu_\ell-i\nu_u),
$$

What called ??

Application of polaronic α particles to ^{8}Be and ^{12}C* (Hoyle state):

Orthogonality condition model
for two-body and three-body calculations: by Moriya and Horiuchi

$$H = \sum_i \frac{p_i^2}{2M^*} - T_{\mathrm{cm}} + \sum_{i>j}\left(V_{\alpha\alpha} + V_{\mathrm{Pauli}} + V_{\mathrm{eff}}^{(2)}\right) + V_{\alpha\alpha\alpha} + V_{\mathrm{eff}}^{(3)}$$



*C. Kurokawa, K. Kato, Phys. Rev. C **71**, 021301 (2005)

*C. Kurokawa, K. Kato, Nucl. Phys. A **792**, 87-101 (2007)

$$V_{\mathrm{Pauli}} = \gamma \sum_{nlm\in f} |\phi_{nlm}(ij)\rangle\langle\phi_{nlm}(ij)| \qquad f = \{0S, 1S, 0D\} \qquad \gamma = 10^5$$

*V. I. Kukulin, and V. N. Pomenertsev, Ann. Phys. (N. Y.) **111**, 330 (1978)

Two-body, Three-body Schroedinger equations for relative coordinates : solved with Gaussian wave basis

$$\Psi^{(k)} = \sum_{i=1}^{K} C_i^{(k)} \bar{G}(A_i, \boldsymbol{x}) = \sum_{i=1}^{K} C_i^{(k)} \mathcal{S} \exp\left(-\frac{1}{2}\tilde{\boldsymbol{x}} A_i \boldsymbol{x}\right)$$

$\boldsymbol{x}$: coordinates except for CoM

$$\sum_{j=1}^{K} H_{ij} C_j^{(k)} = E^{(k)} \sum_{j=1}^{K} B_{ij} C_j^{(k)}$$

$$H_{ij} = \langle \bar{G}(A_i, \boldsymbol{x}) | H | \bar{G}(A_j, \boldsymbol{x}) \rangle$$

$$B_{ij} = \langle \bar{G}(A_i, \boldsymbol{x}) | \bar{G}(A_j, \boldsymbol{x}) \rangle$$

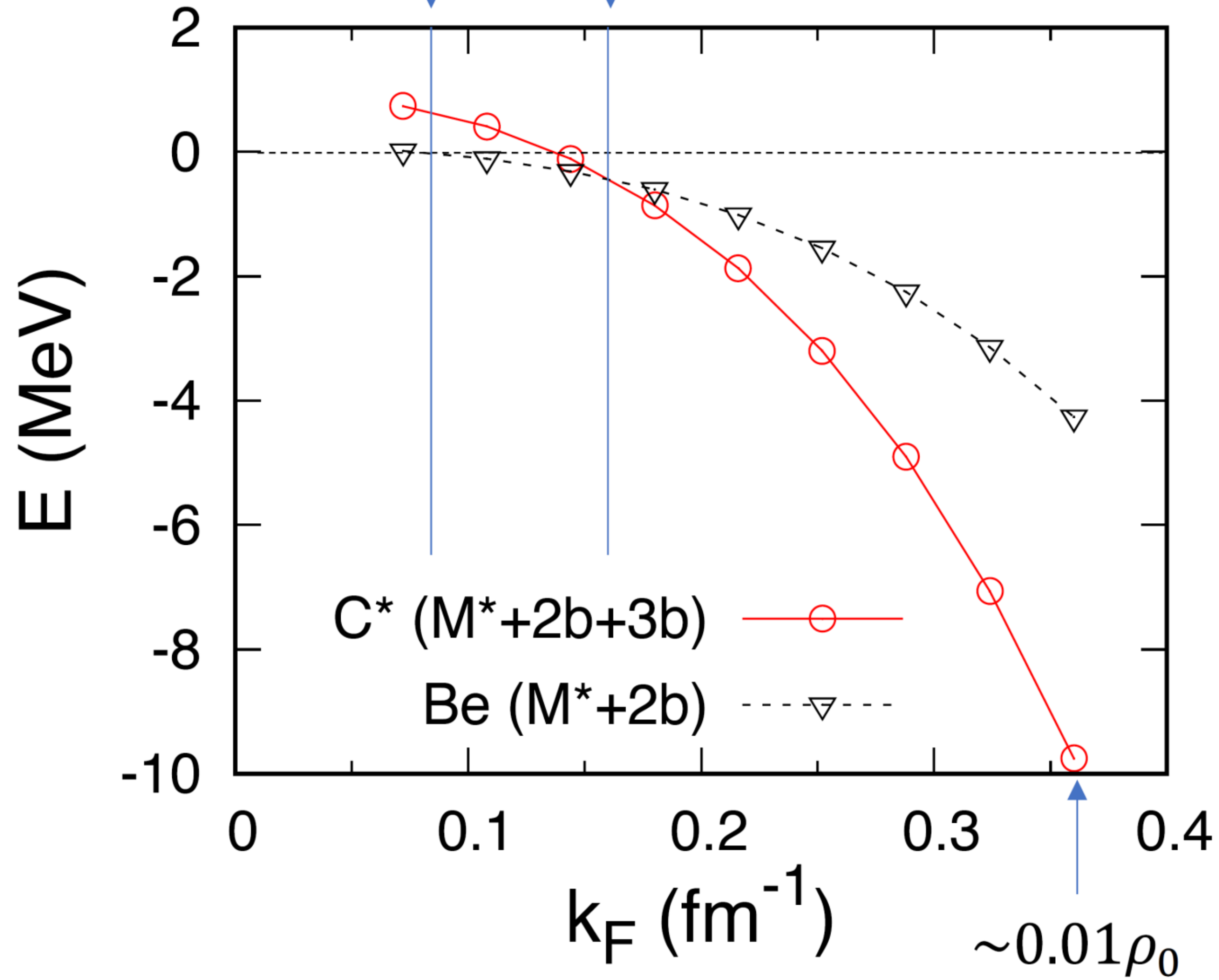
① variation method for ground state

② orthogonalization ⇒ excited states

Numerical results:
In-medium ^{8}Be and ^{12}C (Hoyle state) energy vs Neutron Fermi momentum

$k_F = 0.08$ fm^{-1}
$\rho \sim 10^{-4} \rho_0$

$k_F = 0.16$ fm^{-1}
$\rho \sim 10^{-3} \rho_0$



$\sim 0.01 \rho_0$

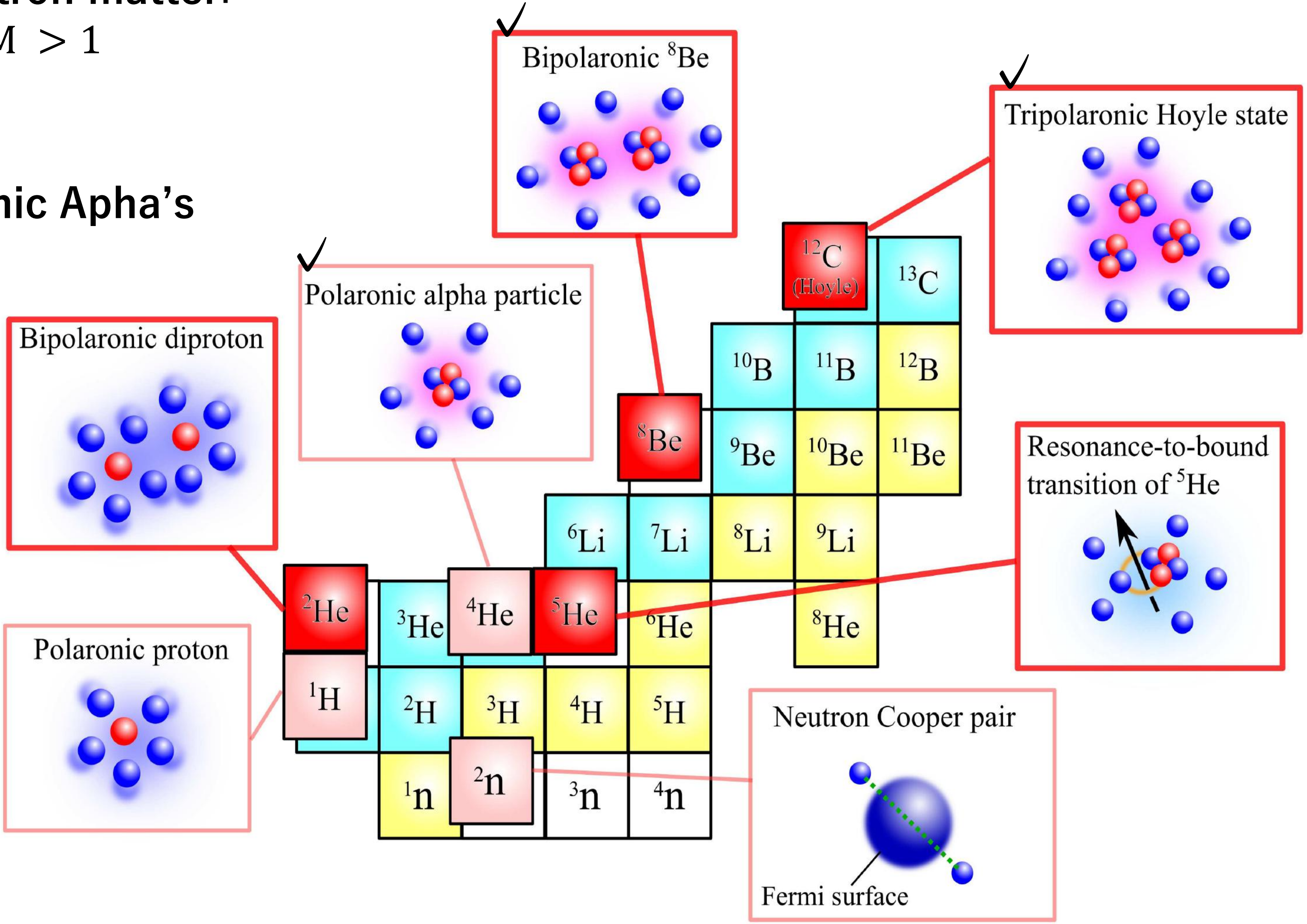
Stabilized due to
Enhanced mass and
Two-body Attraction

Summary:

- **Polaronic Alpha in dilute neutron matter**:
  good quasi-particle with $M^*/M > 1$

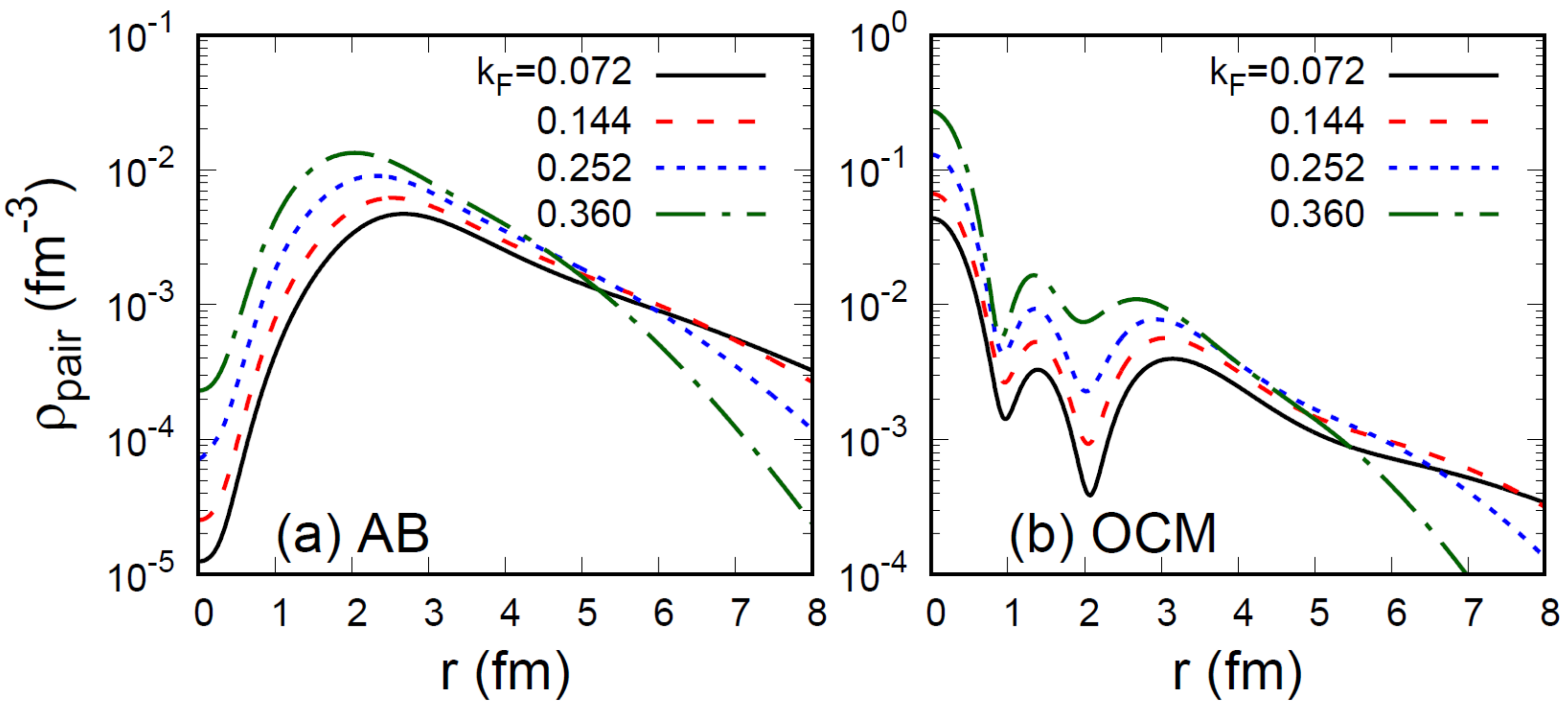
- **Two-** and **Three-body Polaronic Apha's**
  ^{8}Be and **^{12}C***(Hoyle state)
  become **bound in medium**
  due to $V_{\rm eff}^{(2)}$, $V_{\rm eff}^{(3)}$,and M^*

⇒**Nuclear chart modified**
in cold dilute neutron matter



Pair correlation in three Alpha system：

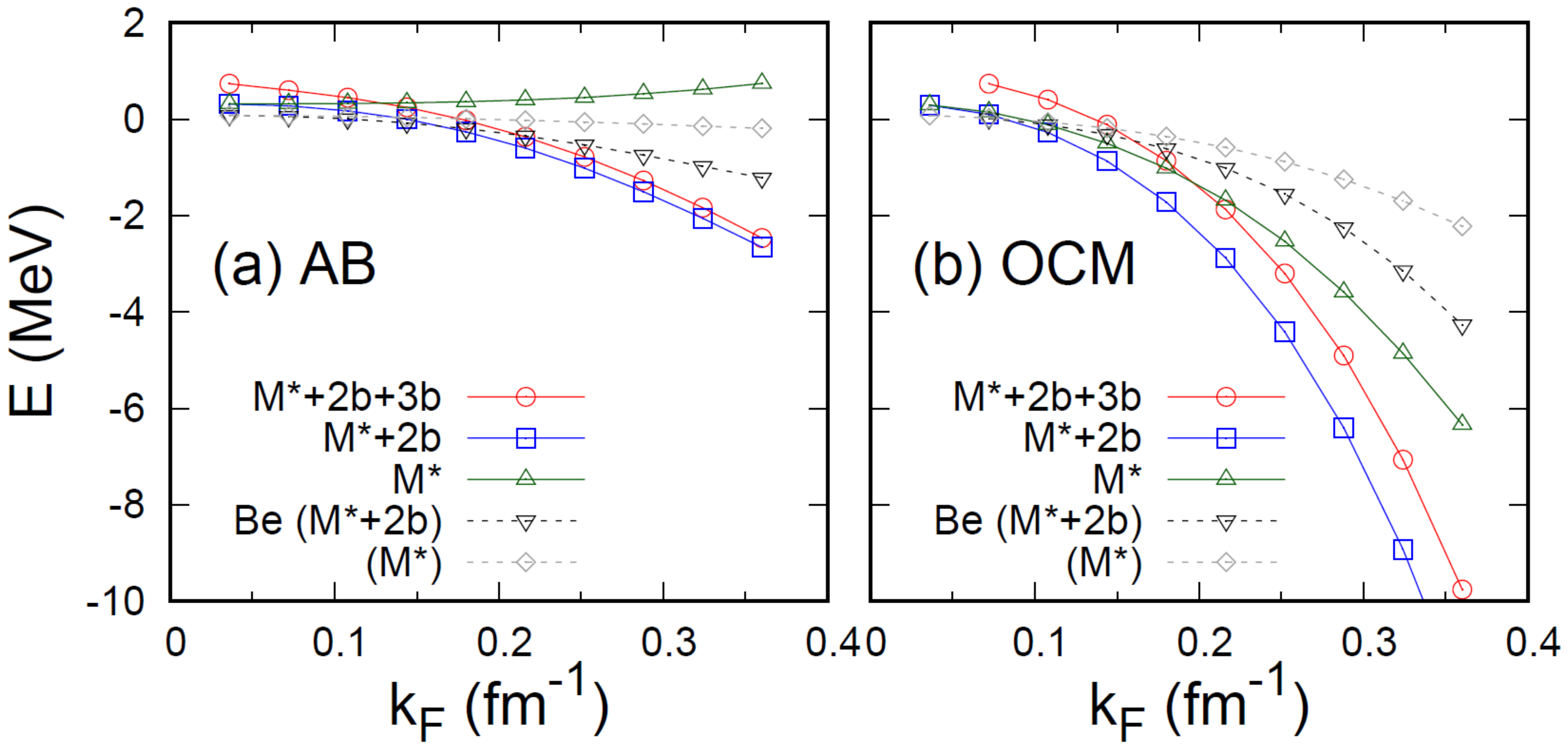
$$\rho_{\mathrm{pair}}(r) = \left\langle \frac{\delta(|\boldsymbol{r}_1 - \boldsymbol{r}_2| - r)}{4\pi r^2} \right\rangle$$



OCM more concentrated in short range region

⇒**Shrinkage of Two-body wave functions**

結果：２体、２体アルファのエネルギー vs 中性子フェルミ運動量



考察：M^*はk_Fと10％～20％増加 ⇒ ゆらぎ抑制局在化 ⇒ 浅いABより近距離引力のOCMで束縛しやすくなる傾向。媒質効果をフルに入れると、2モデルともk_F~0.1 fm^{-1}くらいで束縛。

数値計算：入力パラメタ

・Scattering length from alpha-neutron Low energy scattering

H. Kanada, T. Kaneko, S. Nagata, and M. Nomoto, Prog. Theor. Phys. **61**, 1327 (1979).

Reproduce the experimental phase shift by 堀内

散乱長：$a_{\rm ref} = 2.64$ fm　　有効レンジ：$r_0 = 1.43$ fm

(repulsive branch)

・中性子物質の密度(~ supernova core, < neutron skin of heavy nuclei)

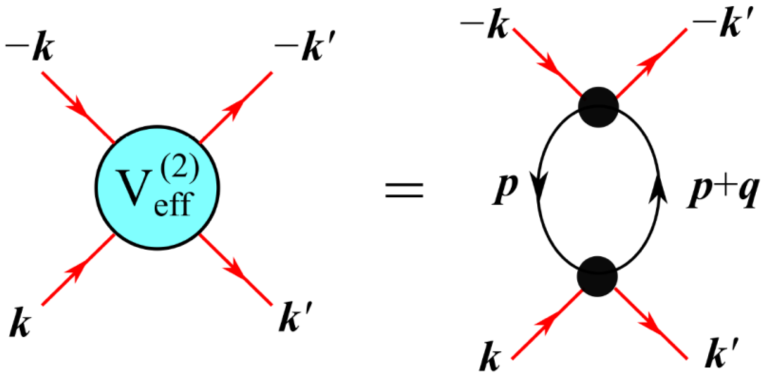
$\rho_{\rm ref} = 0.01\,\rho_0$　　$\rho_0 = 0.16$ fm^{-3} is the normal nuclear density.

$a_{\rm ref} k_{F{\rm ref}} = 0.95 \simeq 1,$　　$r_0 k_{F{\rm ref}} = 0.51$

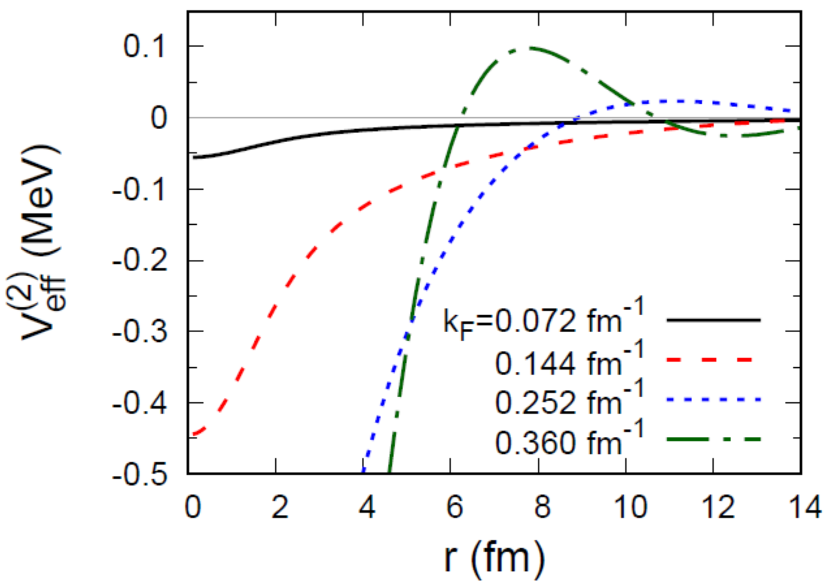
強結合領域への入り口

⑥　中性子媒質に誘起されるアルファ粒子間有効相互作用

中性子１ループ媒介の２アルファ粒子間有効相互作用



$$V^{(2)}_{\rm eff}(\boldsymbol{q}, i\nu_\ell) = -\left(\frac{2\pi\hbar^2 a}{m_r}\right)^2 \times T \sum_{\sigma=\uparrow,\downarrow} \sum_{\boldsymbol{p},\omega_n} G_\sigma(\boldsymbol{p}+\boldsymbol{q}, i\omega_n + i\nu_\ell) G_\sigma(\boldsymbol{p}, i\omega_n),$$



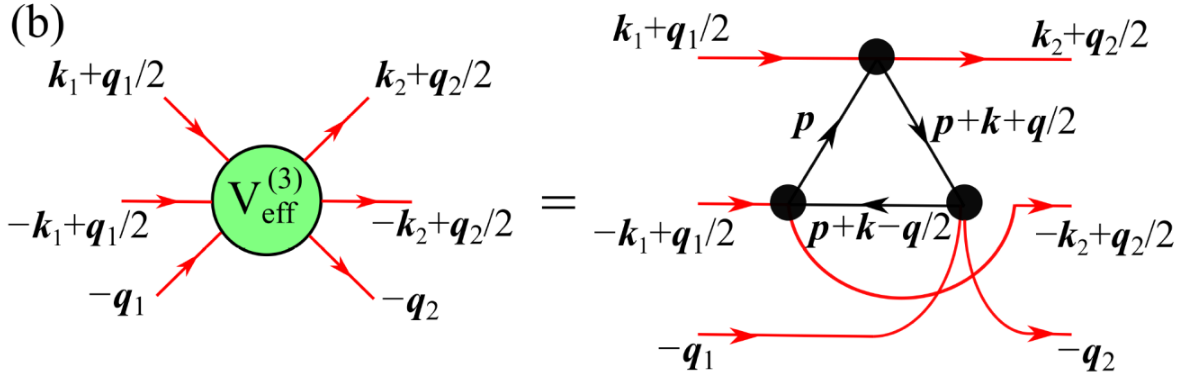
RKKY相互作用と同じ形

フーリエ逆変換

$$V^{(2)}_{\rm eff}(\boldsymbol{q}, 0) = -\frac{m k_{\rm F}}{2\pi^2\hbar^2}\left(\frac{2\pi\hbar^2 a}{m_r}\right)^2 \times \left[1 + \frac{k_{\rm F}}{q}\left(1 - \frac{q^2}{4k_{\rm F}^2}\right)\ln\left|\frac{q+2k_{\rm F}}{q-2k_{\rm F}}\right|\right].$$

$$V^{(2)}_{\rm eff}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{m}{8\pi^3\hbar^2}\left(\frac{2\pi\hbar^2 a}{m_r}\right)^2 \times \frac{(2k_{\rm F}r)\cos(2k_{\rm F}r) - \sin(2k_{\rm F}r)}{r^4},$$

中性子１ループ媒介の３アルファ粒子間有効相互作用



$$V_{\rm eff}^{(3)}(\boldsymbol{k},\boldsymbol{q},i\nu_\ell,i\nu_u)=2\left(\frac{2\pi\hbar^2 a}{m_r}\right)^3$$
$$\times\, T\sum_{\sigma=\uparrow,\downarrow}\sum_{\boldsymbol{p},\omega_n}G_\sigma(\boldsymbol{p},i\omega_n)G_\sigma(\boldsymbol{p}+\boldsymbol{k}+\boldsymbol{q}/2,i\omega_n+i\nu_\ell)$$
$$\times\, G_\sigma(\boldsymbol{p}+\boldsymbol{k}-\boldsymbol{q}/2,i\omega_n+i\nu_\ell-i\nu_u), \qquad ($$

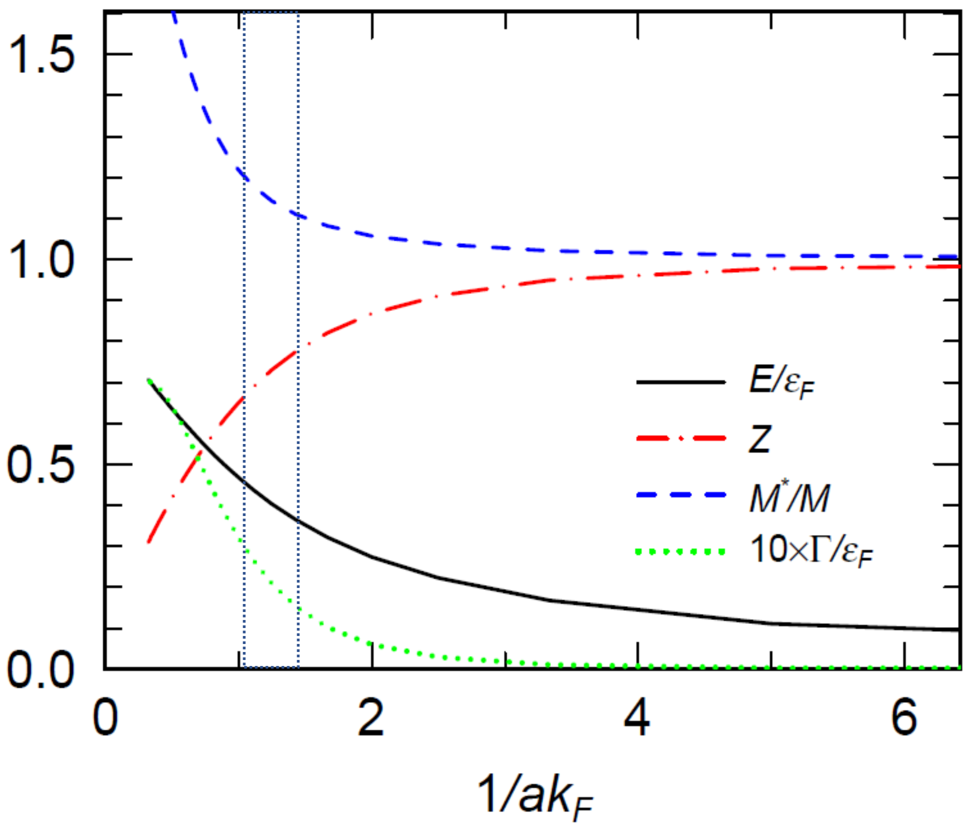
$$V_{\rm eff}^{(3)}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3)=\sum_{\boldsymbol{k},\boldsymbol{q}}V_{\rm eff}^{(3)}(\boldsymbol{k},\boldsymbol{q},0,0)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}_1+i\boldsymbol{q}\cdot\boldsymbol{x}_2}$$

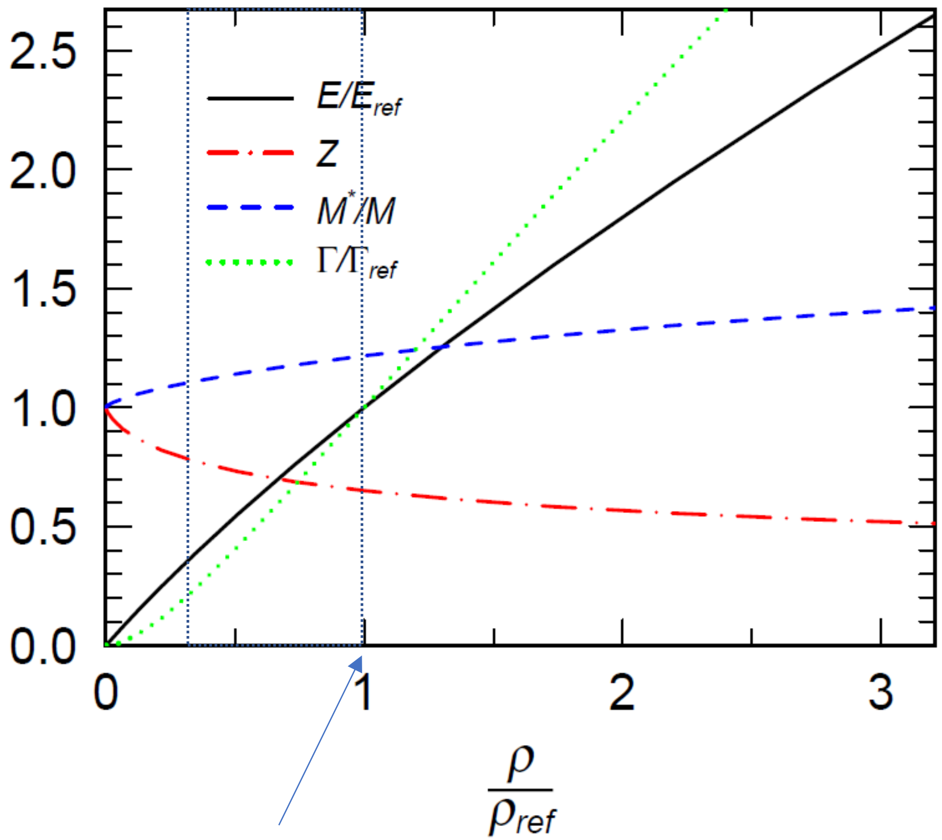
今回は、簡単のため３体相互作用の空間構造を無視して、$V^{(3)}(0,0,0,0)$として、
熱力学的ポテンシャルの３体パラメタで見積もる：$-P=V^{(3)}(0,0,0,0)n_\alpha^3$

$$V_{\rm eff}^{(3)}(\boldsymbol{r}_1,\boldsymbol{r}_2,\boldsymbol{r}_3)=\frac{m^2}{\pi^2\hbar^4 k_{\rm F}}\left(\frac{2\pi\hbar^2 a}{m_r}\right)^3\delta(\boldsymbol{x}_1)\delta(\boldsymbol{x}_2)$$

where $\boldsymbol{x}_1=\boldsymbol{r}_1-\boldsymbol{r}_2$ and $\boldsymbol{x}_2=\boldsymbol{r}_3-(\boldsymbol{r}_1+\boldsymbol{r}_2)/2$

数値計算２　無次元散乱パラメタと中性子密度への依存性





~0.01 ρ_0

中性子密度： ρ=0.003~0.01 ρ_0 （**縦破線範囲**）

通常核密度：$\rho_0 = 0.16\ \mathrm{fm}^{-3}$

E ： 静止エネルギー　< 1.26 MeV
Z ： 留数　　> 0.65
M^*/M ： 有効質量比　1.1 ~ 1.2
Γ ： 崩壊幅　< 0.0086 MeV

⇨ **α粒子の準粒子描像、悪くない。**