April 22, 2026

TNP
Retreat
2026

Anisotropic transport in strongly magnetized relativistic matter

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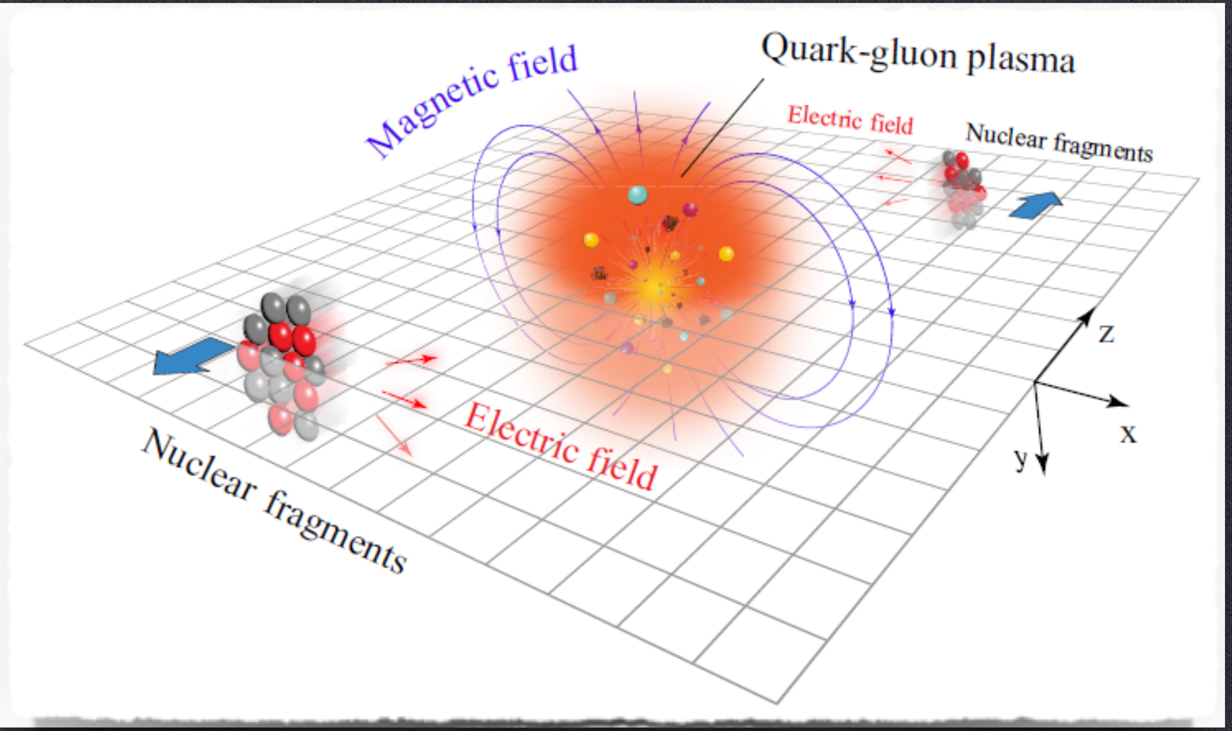


Based on:

- **The fermion self-energy and damping rate in a hot magnetized plasma,**
  Authors: *R Ghosh*, I. A. Shovkovy, Phys.Rev.D 109 (2024) 9, 096018
- **Anisotropic charge transport in strongly magnetized relativistic matter,**
  Authors: *R Ghosh*, I. A. Shovkovy, Eur. Phys. J. C 84, 1179 (2024)
- **Electrical conductivity of hot relativistic plasma in a strong magnetic field,**
  Authors: *R Ghosh*, I. A. Shovkovy, Phys.Rev.D 110 (2024) 09, 096009
- **Review of heat and charge transport in strongly magnetized relativistic plasmas**
  Authors: I. A. Shovkovy, *R Ghosh*, AAPPS Bulletin, 35(1), arXiv: 2506.14956

OUTLINE

- Background and motivation
- Fermion in magnetic field

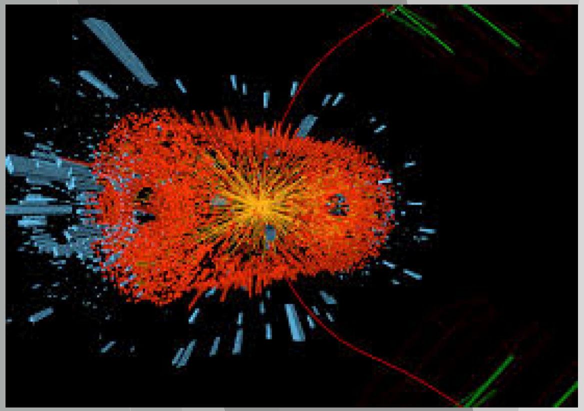
- damping rate
  - from self-energy
  - from the poles of the propagator

- Electrical conductivity in extreme condition
  - -QED conductivity
  - -QCD conductivity

Matter in extreme conditions

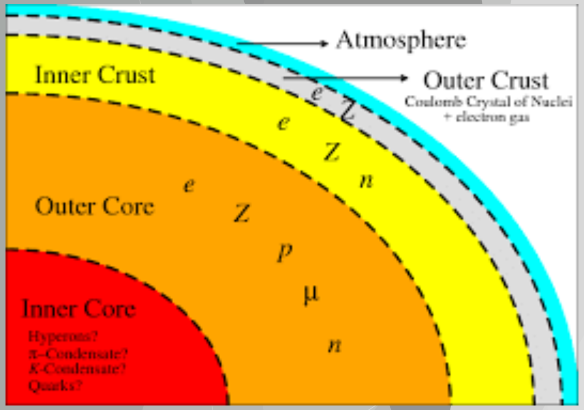
- High temperature $\sim 10^{12}$ K

Early universe, Quark gluon plasma in heavy ion collisions



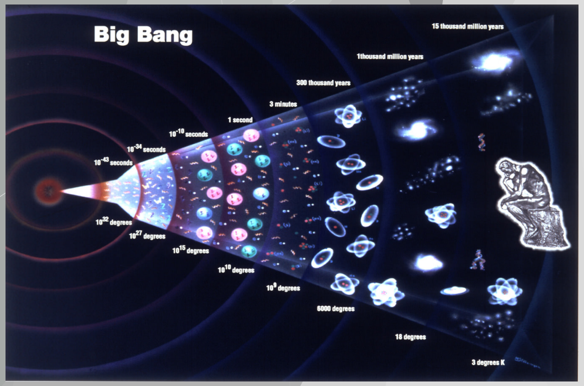
- High densities $\sim 5n_0$, where $(n_0 \sim 3 \times 10^{17})$ kg/m^3

Compact stars core, experimental lab (FAIR...)



- High magnetic field $\sim 10^{14} - 10^{18}$ G

Inside compact star, non-central heavy-ion collisions



- Compact stars

  - equation of state, mass-radius relation, gravitational collapse/merger, neutrino emission from star

    [Duncan et al. *Astrophys.J.Lett.* 392 (1992) L9]
    [Ferrar et al. arXiv: 1009.3521]
    [Anderson et al. *Phys.Rev.Lett.* 100 (2008) 191101]
    [Ghosh & Shovkovy, arXiv: 2501.03318 , *JHEP* 04 (2025) 110]
    [Ghosh & Shovkovy, arXiv: 2504.20138 , *JHEP* 04 (2025) 110]

- In the early universe

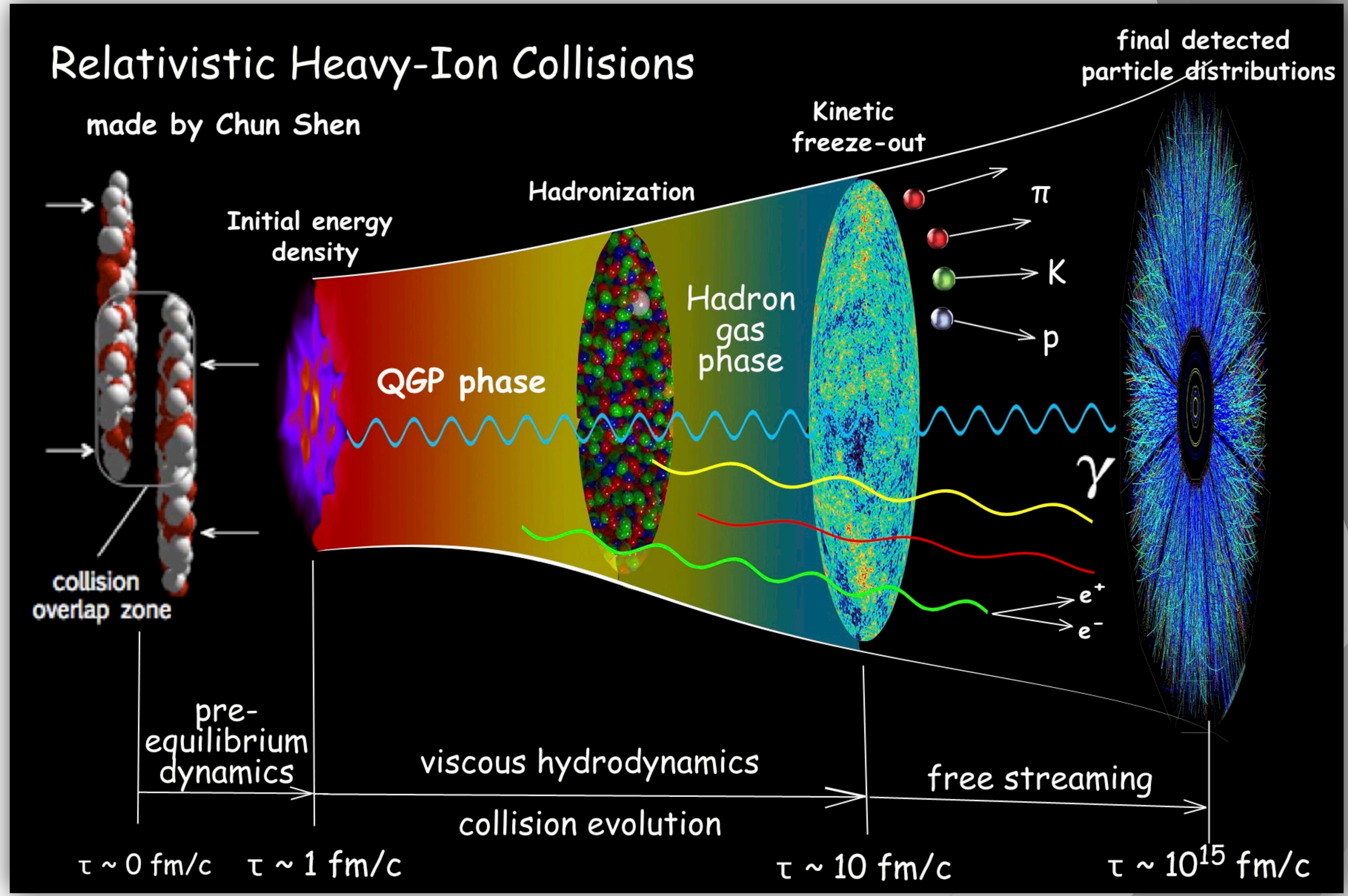
  - Magnetic fields from cosmological phase transitions

    [Vachaspati et al. *Phys.Lett.B* 265 (1991) 258-261]
    [Enqvist, Olesen, *Phys.Lett.B* 319 (1993) 178-185]

- Non-central heavy-ion collisions

  - chiral magnetic effect, anisotropies, elliptic flow

    [Kharzeev et al., arXiv:0711.0950]
    [Fukushima et al. arXiv:1209.5064]



Relativistic Heavy-Ion Collisions
made by Chun Shen
final detected
particle distributions
Kinetic
freeze-out
Hadronization
Initial energy
density
Hadron
gas
phase
QGP phase
π
K
p
γ
e+
e−
collision
overlap zone
pre-
equilibrium
dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c

Chun Shen

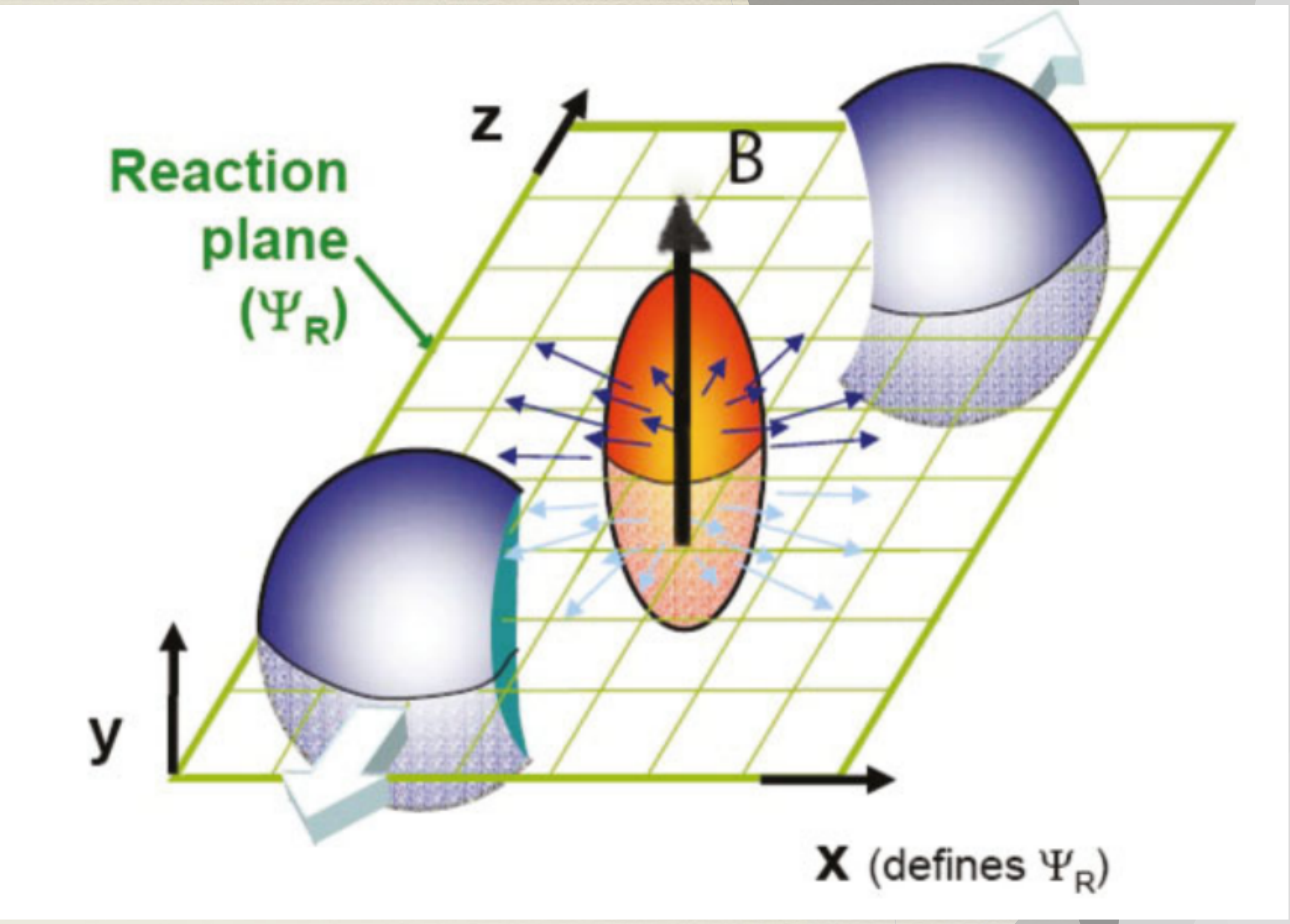
Magnetic field @RHICs

Off-center heavy ion collision

Magnetic field strength $|eB| \sim m_\pi^2$

$m_\pi^2 \sim 10^{18} G$

Short lived 10^{-24} s



[Ferrer, Incera, arXiv: 1603.08226]

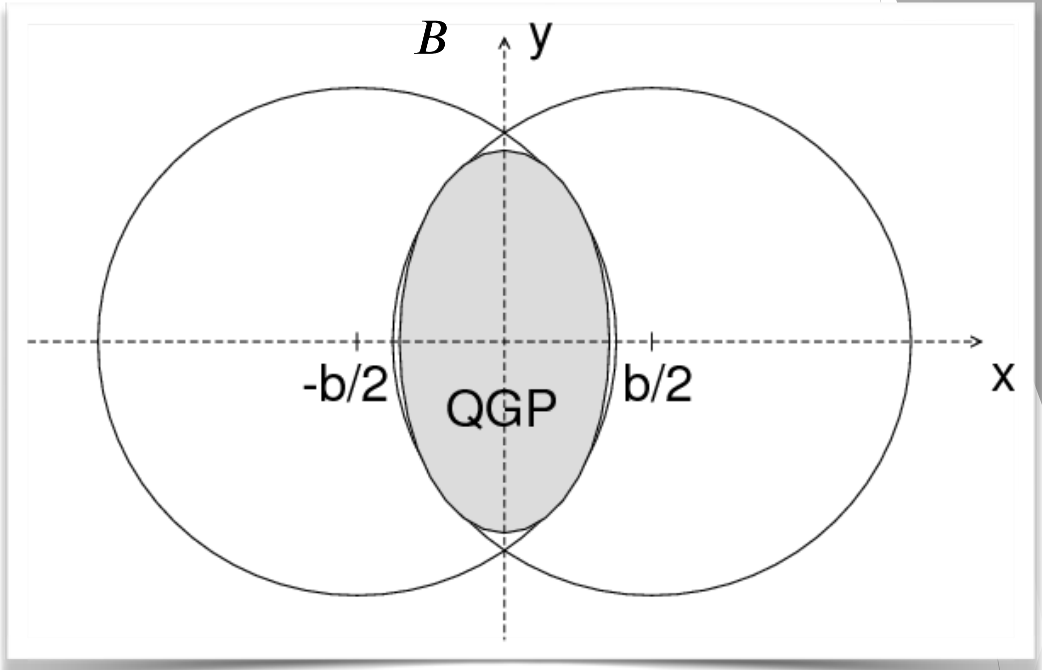
Earth magnetic field~ 10^{-1} G
refrigerator magnets ~ 100 G
Neutron star~ $10^{14} - 10^{17}$ G

[Skokov et. al, 0907.1396 [nucl-th]]
[Zhong et. al, 1408.5694 [hep-ph]]

Lienard-Wiechert potential:

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n - R_n\mathbf{v}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{(R_n - \mathbf{R}_n\cdot\mathbf{v}_n)^3}(1-v_n^2)$$



$\mathbf{R_n} = \mathbf{x} - \mathbf{x_n}$

Non-relativistic limit, $v_n \ll 1$

Coulomb's law:

$$e\mathbf{E}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{R}_n}{R_n^3},$$

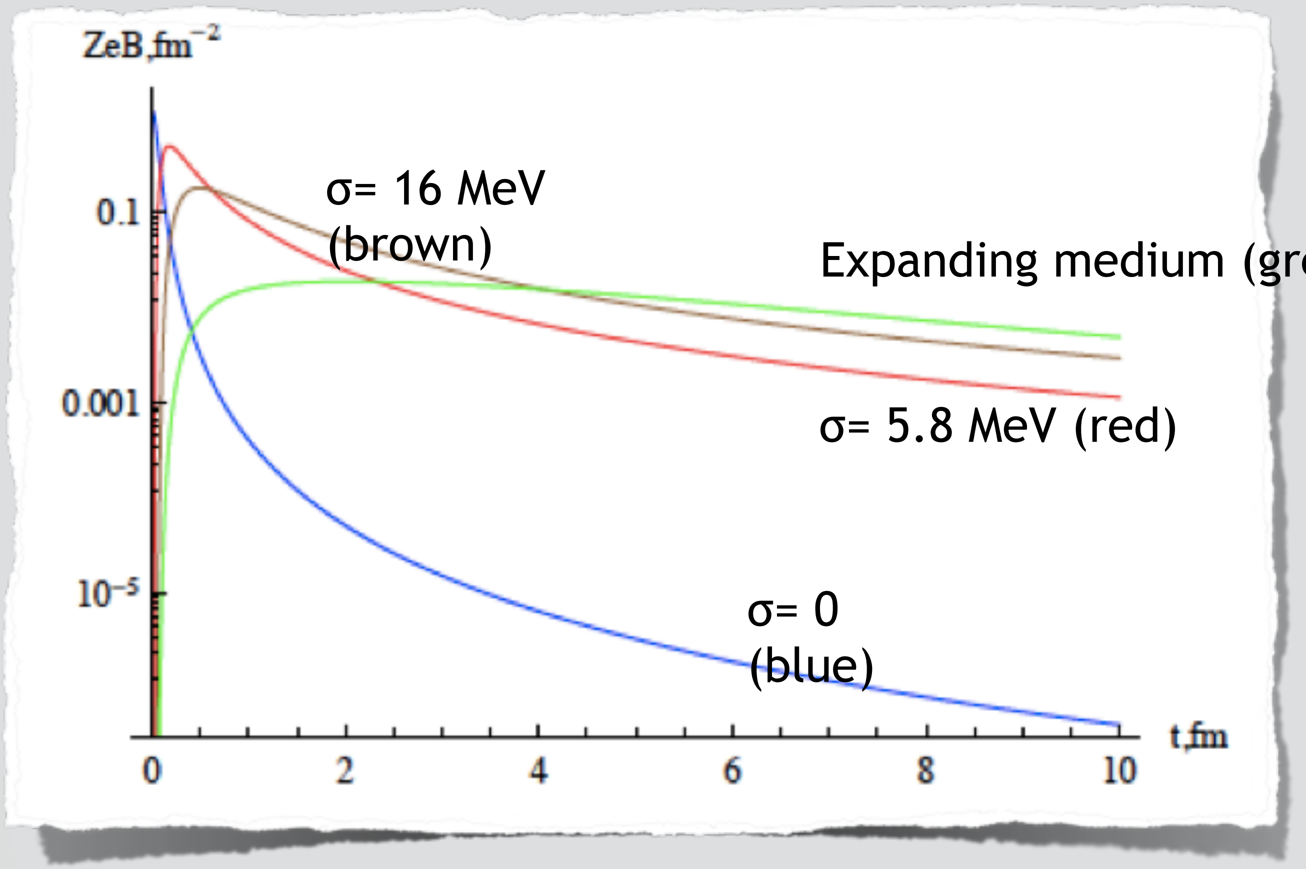
Biot-Savart law:

$$e\mathbf{B}(t,\mathbf{r}) = \frac{e^2}{4\pi}\sum_n Z_n \frac{\mathbf{v}_n \times \mathbf{R}_n}{R_n^3}.$$

[Rafelski & Müller, PRL, 36, 517 (1976)] [Kharzeev et al., arXiv: 0711.0950]
[Skokov et al., arXiv:0907.1396]
[Voronyuk et al., arXiv: 1103.4239]

Conductivity?

- High electrical conductivity can prolong the lifetime of magnetic fields



- induced currents help sustain the magnetic field through Lenz's Law, reducing its decay rate.

$1\, fm/c = 3.3 \times 10^{-24} sec$ $\mathrm{fm}^{-2} \approx 2 m_\pi^2$

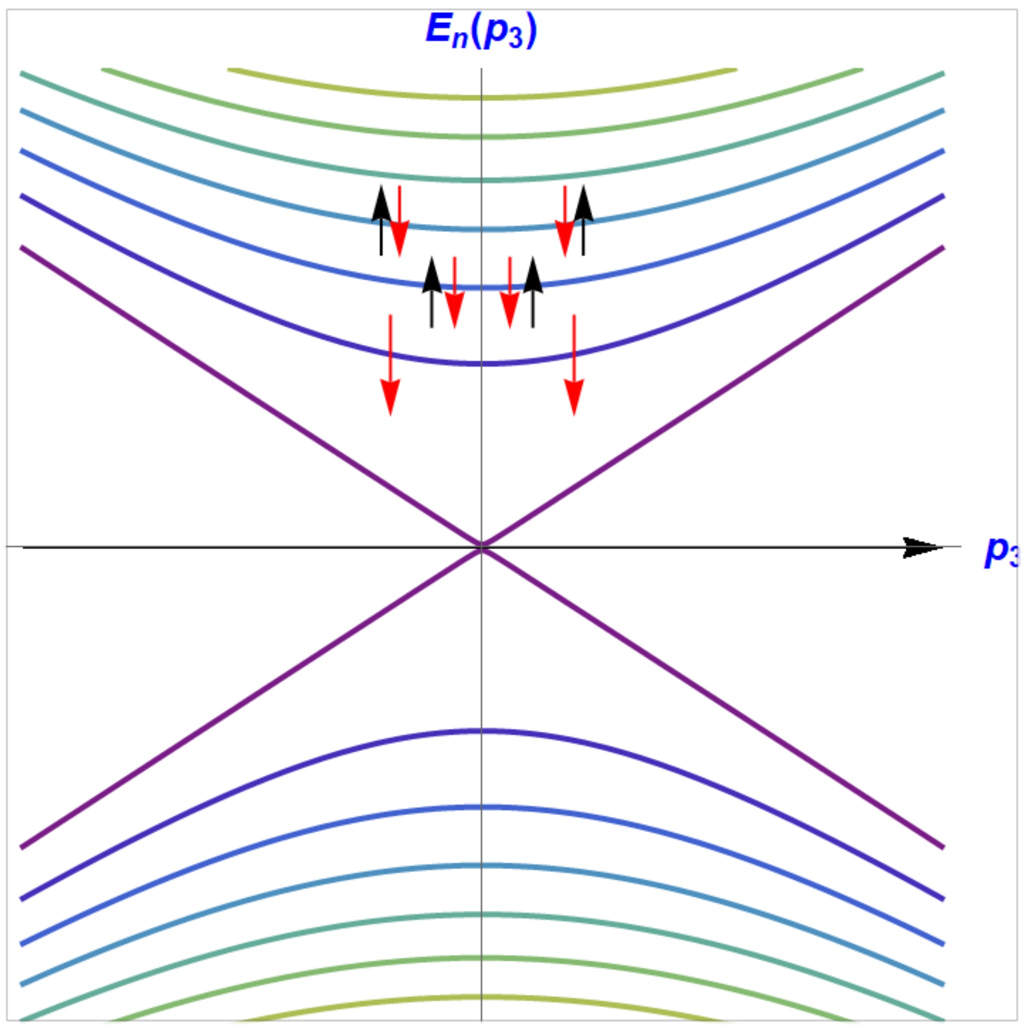
[Tuchin, Adv.High Energy Phys. 2013 (2013) 490495]

Landau Levels

Relativistic:

$E_{kin} \geq E_{rest}$

$v \sim c$



Dirac Equation

$$\left\{ i\gamma^0 \partial_0 - i\vec{\gamma} \cdot (\nabla - iq\vec{A}) \right\} \Psi = 0$$

Energy spectrum:

$$E_n(p_3) = \pm\sqrt{p_3^2 + 2n|eB| + m^2}$$

$$n = s + k + \frac{1}{2}$$

Spin quantum number: s=± 1/2

Orbital: k=0,1,2,...

[Miranski & Shovkovy, Phys.Rept. 576 (2015) 1-209]

Fermion propagator @B≠0

Field theoretical approach

The fermion propagator in coordinate space:

$$G(p_\parallel, u_\perp, u'_\perp) = e^{i\Phi(u_\perp, u'_\perp)} \bar{G}(p_\parallel, u_\perp - u'_\perp)$$

$u \equiv (t, x, y, z)$ translation invariant part

Schwinger phase:

$$\Phi(u_\perp, u'_\perp) = \frac{qB}{2}(x + x')(y - y')$$

$p^\mu_\perp = (p_x, p_y)$

$p^\mu_\parallel = (p_0, p_z)$

Fourier transform of the translation invariant part (free propagator)

$$\bar{G}^f(p_\parallel, \boldsymbol{p}_\perp) = ie^{-p_\perp^2 \ell^2} \sum_{n=0}^{\infty} (-1)^n \frac{D_n^{(0)}(p_\parallel, \boldsymbol{p}_\perp)}{p_\parallel^2 - \bar{m}_0^2 - 2n|qB|}$$

spin projectors:

$$\mathcal{P}_\pm = \left(1 \pm s_\perp i\gamma^1\gamma^2\right)/2$$

$$s_\perp = \text{sign}(qB)$$

$$\ell = 1/\sqrt{|qB|}$$

$$D_n^{(0)}(p_\parallel, \boldsymbol{p}_\perp) = 2\left[(p_\parallel \cdot \gamma_\parallel) + \bar{m}_0\right]\left[\mathcal{P}_+ L_n\left(2p_\perp^2\ell^2\right) - \mathcal{P}_- L_{n-1}\left(2p_\perp^2\ell^2\right)\right] + 4(\boldsymbol{p}_\perp \cdot \boldsymbol{\gamma}_\perp) L_{n-1}^1\left(2p_\perp^2\ell^2\right).$$

generalized Laguerre polynomials

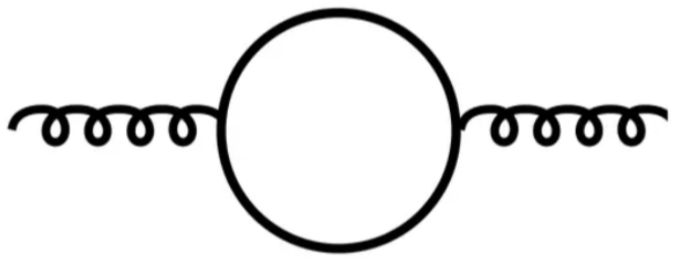
Fermion propagator in the spectral form:

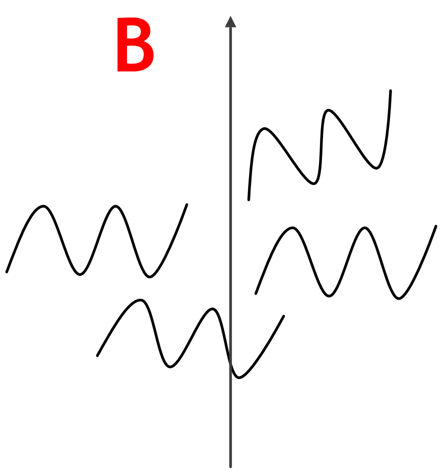
$$\bar{G}^f(i\omega_k, \boldsymbol{k}) = \int_{-\infty}^{\infty} \frac{dk_0 A_{\boldsymbol{k}}^f(k_0)}{i\omega_k - k_0 + \mu_f}$$

The spectral function carries information about the quasiparticle properties in plasma

Gluons or photons in strong field

No ‘electric charge’
-does not feel B at zeroth order





$$\Pi^{\mu\nu}(i\Omega_m;\mathbf{k}) = 4\pi N_c \sum_{f=u,d} \alpha_f T \sum_{k=-\infty}^{\infty} \int \frac{dp_z}{2\pi} \int d^2\mathbf{r}_\perp e^{-i\mathbf{r}_\perp\cdot\mathbf{k}_\perp} \mathrm{tr}[\gamma^\mu \bar{G}_f(i\omega_k, p_z;\mathbf{r}_\perp)\gamma^\nu \bar{G}_f(i\omega_k - i\Omega_m, p_z - k_z; -\mathbf{r}_\perp)]$$

Effective mass through coupling with fermions $M^2 \propto \alpha|eB|$

[Mustafa et. al Prog.Part.Nucl.Phys. 148 (2026) 104234]

[Miranski & Shovkovy, Phys. Rev. D 66, 045006 (2002)]

Electrical Conductivity

$$j^i = \sigma^{ij} E^j$$

$B = 0$

$$\sigma_{ij} = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}$$

$$\vec{J} = \sigma_0 \vec{E}$$

$\vec{B} = B\hat{z}$

The conductivity tensor becomes anisotropic

$$\sigma_{ij} = \begin{bmatrix} \sigma_\perp & -\sigma_H & 0 \\ \sigma_H & \sigma_\perp & 0 \\ 0 & 0 & \sigma_\parallel \end{bmatrix}$$

σ_H

Hall conductivity

nondissipative transport : non-zero even in free theory

[Shovkovy et. al, PRB **89**, 085126 (2014)]

Conductivity @ B≠0

Phenomenological models:

[JHEP 08, 083 (2013)]
[Phys. Rev. D 105, 054016 (2022)]
[Phys. Rev. D 96, 114026 (2017)]
[Phys. Rev. D 101, 034027 (2020)]
[Phys. Rev. D 104, 056030 (2021)]
[EPJC 83, 489 (2023)]

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\frac{\partial f}{\partial \boldsymbol{r}} - e(\boldsymbol{E} + [\boldsymbol{v} \times \boldsymbol{H}])\frac{\partial f}{\partial \boldsymbol{p}} = I[f]$$

limitations of kinetic theory @B≠0!

Attempts within a gauge theory (LLL approximation or "longitudinal" kinetic theory):

[PRD 94, 114032 (2016)]
[Phys. Rev. D 95, 076008 (2017)]
[Phys. Rev. Lett. 120, 162301 (2018)]
[JHEP 04, 162 (2020)]

Limitations:

- Restricted to the **lowest Landau level (LLL)**
- **No access to higher Landau level transitions**
- **Perpendicular (transverse) transport not described**

We compute the complete conduction picture in magnetized matter, including higher Landau levels and transverse transport, and reveal rich underlying physics.

ELECTRICAL CONDUCTIVITY

Kubo formula:

$$\sigma_{ij} = \lim_{\Omega \to 0} \frac{\mathrm{Im}\left[\Pi_{ij}(\Omega + i0; \mathbf{0})\right]}{\Omega}$$

Matsubara sum

with N_f fermion species having electric charges e_f

$$\Pi_{ij}(i\Omega_l; \mathbf{0}) = 4\pi\alpha \sum_{f=1}^{N_f} q_f^2 T \sum_{k=-\infty}^{\infty} \int \frac{d^3\boldsymbol{k}}{(2\pi)^3} \times \mathrm{tr}\left[\gamma^i \bar{G}^f(i\omega_k, \boldsymbol{k}) \gamma^j \bar{G}^f(i\omega_k - i\Omega_l, \boldsymbol{k})\right]$$

$$\bar{G}^f(i\omega_k, \boldsymbol{k}) = \int_{-\infty}^{\infty} \frac{dk_0 A_{\boldsymbol{k}}^f(k_0)}{i\omega_k - k_0 + \mu_f}$$

spectral function

$$A_{\boldsymbol{k}}^f(k_0) = \frac{ie^{-k_\perp^2 \ell_f^2}}{\pi} \sum_{\lambda=\pm} \sum_{n=0}^{\infty} \frac{(-1)^n}{E_{f,n}} \Big\{ \left[E_{f,n}\gamma^0 - \lambda k_z \gamma^3 + \lambda m_f\right] \times \left[\mathcal{P}_+ L_n\left(2k_\perp^2 \ell_f^2\right) - \mathcal{P}_- L_{n-1}\left(2k_\perp^2 \ell_f^2\right)\right] + 2\lambda(\boldsymbol{k}_\perp \cdot \boldsymbol{\gamma}_\perp) L_{n-1}^1\left(2k_\perp^2 \ell_f^2\right) \Big\} \frac{\Gamma_{f,n}}{(k_0 - \lambda E_{f,n})^2 + \Gamma_{f,n}^2}$$

$$\frac{1}{\omega - \omega' - \Omega - i0} = \mathcal{P}\frac{1}{\omega - \omega' - \Omega} + i\pi\delta(\omega - \omega' - \Omega)$$

ELECTRICAL CONDUCTIVITY

[Ghosh, Shovkovy, Phys.Rev.D 110 (2024) 09, 096009] &
[Ghosh, Shovkovy, Eur. Phys.J.C 84, 1179 (2024)]

Kubo formula:

$$\sigma_{ij} = \lim_{\Omega\to 0} \frac{\mathrm{Im}\left[\Pi_{ij}(\Omega+i0;\mathbf{0})\right]}{\Omega} = -\frac{\alpha}{8\pi T}\sum_{f=1}^{N_f} q_f^2 \int \frac{dk_0 d^3\boldsymbol{k}}{\cosh^2\frac{k_0-\mu_f}{2T}} \mathrm{tr}\left[\gamma^i A_{\boldsymbol{k}}^f(k_0)\gamma^j A_{\boldsymbol{k}}^f(k_0)\right].$$

Fermion spectral function

Spectral function ∝ fermion damping rate (Γ_n)

$$\sigma_{\parallel} \propto \alpha \sum_{n=0}^{\infty} \int dp_z \frac{F_{\parallel}(E_n, p_z)}{\Gamma_n(p_z)}$$

$$\sigma_{\perp} \propto \alpha \sum_{n=0}^{\infty} \int dp_z \Gamma_n(p_z) F_{\perp}(E_n, E_{n+1}, p_z)$$

$\Gamma_n \to 0$ ⇒ $\sigma_{\parallel} \to \infty$, $\sigma_{\perp} \to 0$

When interactions included:

$\sigma_{\parallel}$ Finite because of scattering

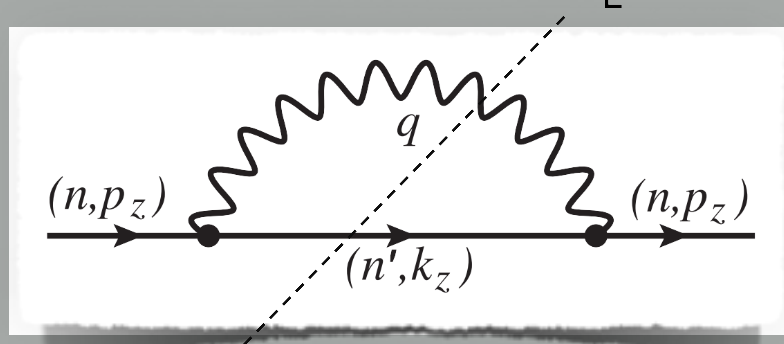
$\sigma_{\perp} \neq 0$ Because of Landau level hopping

[Ghosh, Shovkovy, Phys. Rev. D 109, 096018 (2024)]

Fermion damping rate from self-energy: Γ_n

$$B \neq 0$$

$$\Gamma_n^{\rm ave}(p_z) = \frac{1}{2p_0}\int d^4u' \int d^4u \,{\rm Tr}\left[\frac{2\pi\ell^2}{V_\perp}\int dp \sum_s \bar{\Psi}_{n,p,s}(u')\,{\rm Im}\Sigma(u',u)\Psi_{n,p,s}(u)\right]$$



$$= {\rm Im}\Sigma_n(p_z)$$

Γ_n Order of $\alpha|eB|/T$

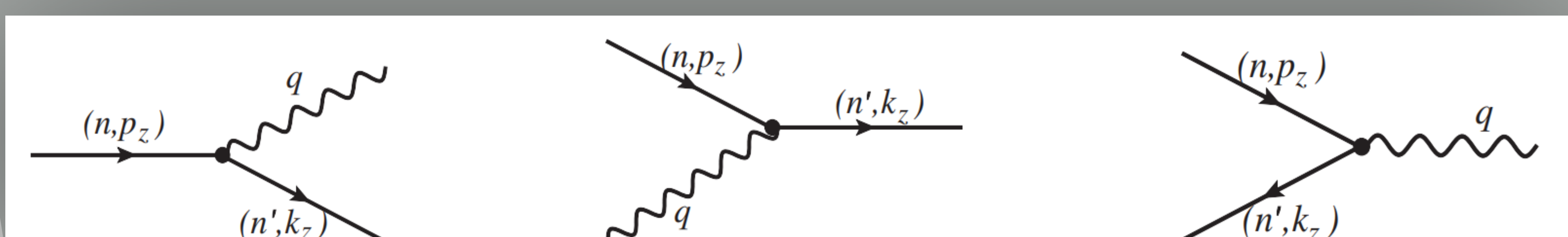
- Underlying processes:

$eB \gg \alpha T^2$

$p_0 = s_1 E_{n',k_z} + s_2 E_q$

$s_1, s_2 = \pm 1$

$n > n'$ $\qquad$ $n' > n$



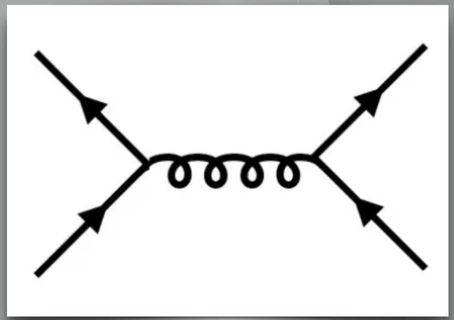
Sub-leading $\alpha^2 T$

kinematically forbidden In the absence of a magnetic field

Leading order processes:

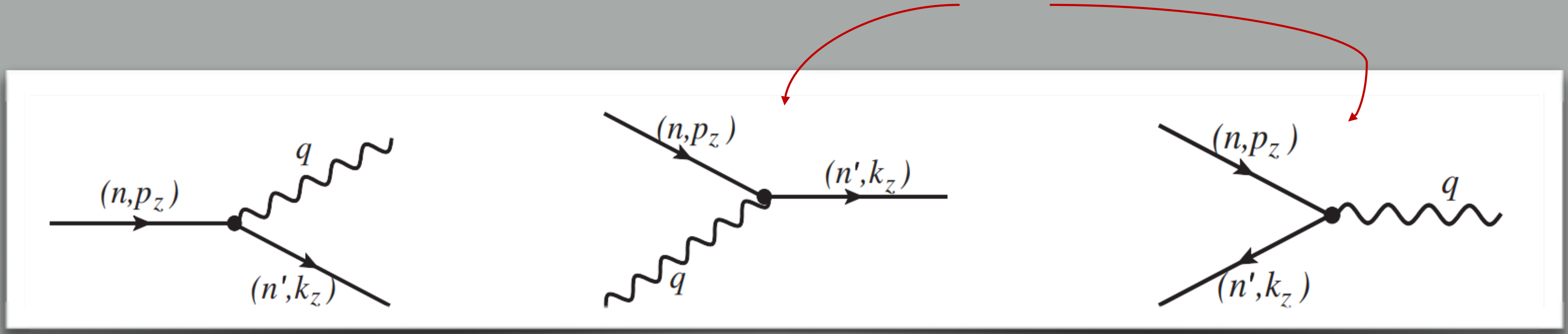
$B = 0$: $2 \to 2$ are leading processes.

$1 \leftrightarrow 2$ Kinematically forbidden



$B \neq 0$: $1 \leftrightarrow 2$ are leading processes.

$n = 0$



$n > n'$ $n' > n$

[Ghosh, Shovkovy, Phys. Rev. D 109, 096018 (2024)]

Analytic expression:

$$\Gamma_n^{\text{ave}}(k_z) = \frac{\alpha}{2\ell^2\beta_n E_n}\sum_{n'=0}^{\infty}\sum_{\{s\}}\int \frac{d\xi\left[1-n_F(E_{f,s'})+n_B(E_{\gamma,s'})\right]\mathscr{M}_{n,n'}(\xi)}{s_1 s_2\sqrt{(\xi-\xi_{n,n'}^{-})(\xi-\xi_{n,n'}^{+})}}$$

where,

$$\mathcal{M}_{n,n'}(\xi) = -\left(n+n'+\bar{m}_0^2\ell^2\right)\left[\mathcal{I}_0^{n,n'}(\xi)+\mathcal{I}_0^{n-1,n'-1}(\xi)\right]+(n+n')\left[\mathcal{I}_0^{n,n'-1}(\xi)+\mathcal{I}_0^{n-1,n'}(\xi)\right]$$ $\propto$ Matrix amplitude squared

- positive definite quantity

$$\mathcal{I}_0^{n,n'}(\xi) = \frac{(n')!}{n!}e^{-\xi}\xi^{n-n'}\left(L_{n'}^{n-n'}(\xi)\right)^2$$

$$\xi^{\pm} = \tfrac{1}{2}\left[\sqrt{2n'+(\bar{m}_0/qB)^2}\pm\sqrt{2n+(\bar{m}_0/qB)^2}\right]^2$$

$$\beta_n = 2-\delta_{n,0}$$

Energy conservation:

$$p_0 = s_1 E_{n',k_z} + s_2 E_q$$

$$\psi_n \to \psi_{n'} + \gamma \quad (s_1>0,\ s_2>0): \qquad 0<\xi<\xi^{-},$$

$$\psi_n + \gamma \to \psi_{n'} \quad (s_1>0,\ s_2<0): \qquad 0<\xi<\xi^{-},$$

$$\psi_n + \bar{\psi}_{n'} \to \gamma \quad (s_1<0,\ s_2>0): \qquad \xi^{+}<\xi<\infty.$$

Conductivity of QED plasma

[Ghosh, Shovkovy, Phys.Rev.D 110 (2024) 09, 096009]

$$T >> m_e$$

$$\sqrt{|eB|} >> m_e$$

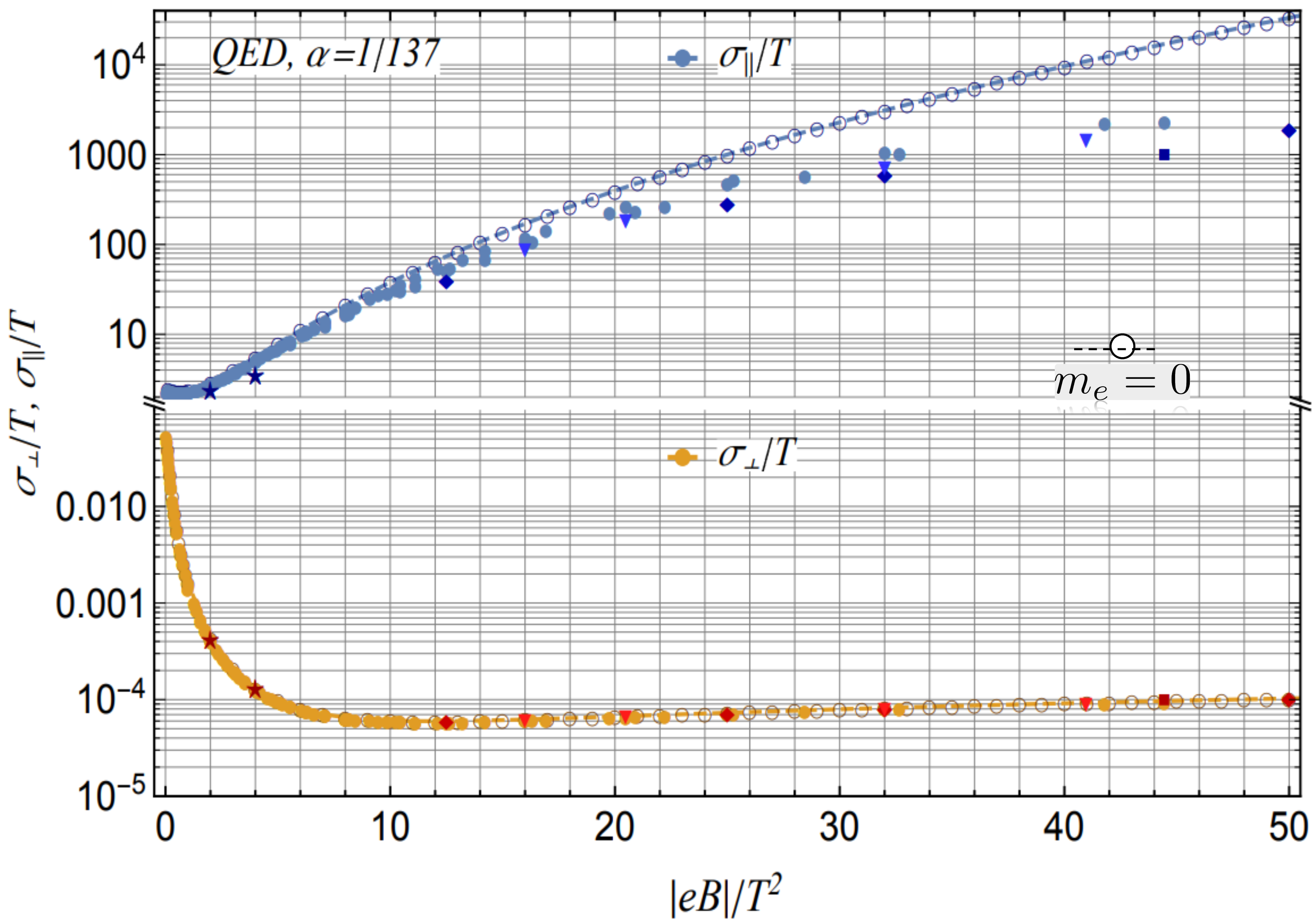
σ/T ~ universal functions of the dimensionless ratio $|eB|/T^2$

$$\sigma_{\|}/T = \tilde{\sigma}_{\|}(|eB|/T^2)$$

$$\sigma_{\perp}/T = \tilde{\sigma}_{\perp}(|eB|/T^2)$$

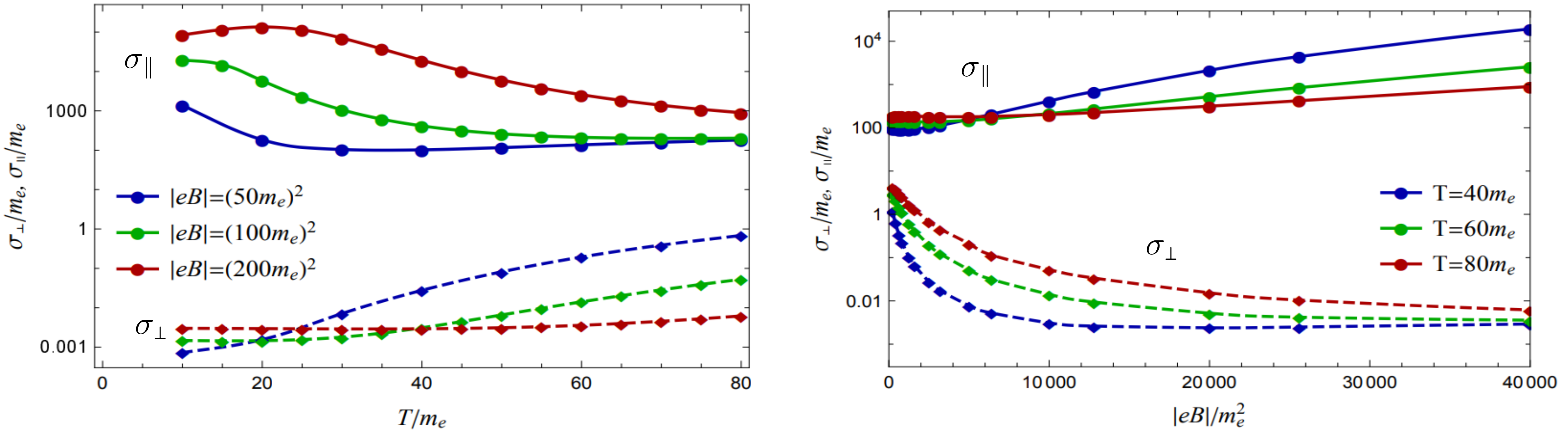
$$\sigma_{\|} \propto 1/\Gamma_n(p_z)$$

$$\sigma_{\perp} \propto \Gamma_n(p_z)$$



$$15m_e \le T \le 80m_e \quad \text{and} \quad (15m_e)^2 \le |eB| \le (200m_e)^2$$

T and B-dependence



transverse (dashed lines) and longitudinal (solid lines)

T- dependence:

$\sigma_\parallel \propto 1/\Gamma_n(p_z)$ tends to decrease with temperature (like metals)

$\sigma_\perp \propto \Gamma_n(p_z)$ tends to increase with temperature (like semiconductors)

B- dependence:

$\sigma_\parallel$ increases and $\sigma_\perp$ decreases with B

[Ghosh, Shovkovy, Eur. Phys.J.C 84, 1179 (2024)]

Transport Mechanism

- Longitudinal conductivity:

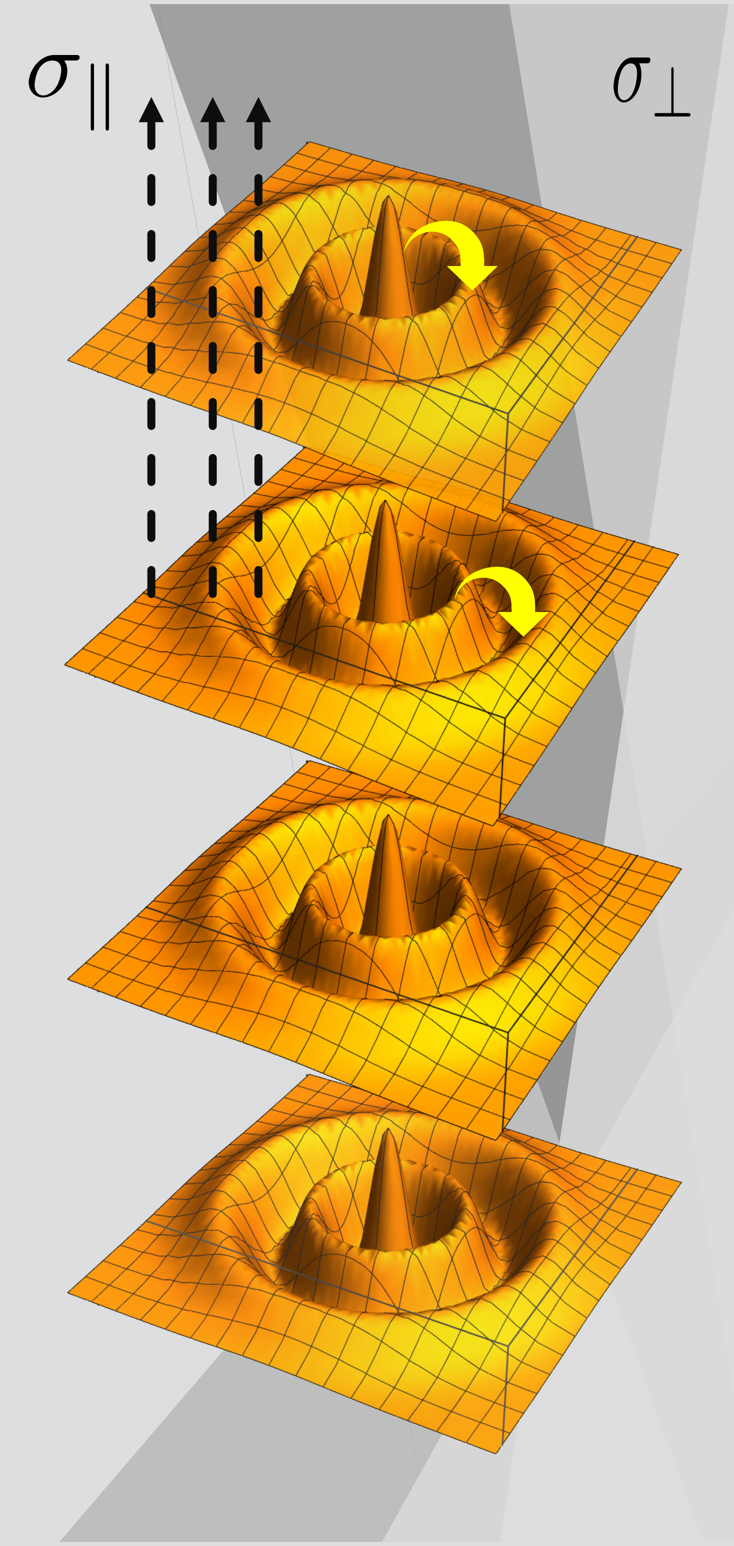
$$\sigma_{\|} \propto \alpha \sum_{n=0}^{\infty} \int dp_z \frac{F_{\|}(E_n, p_z)}{\Gamma_n(p_z)}$$

- Conductivity is hindered by transitions or scattering events.
- damping rates are determined by scattering and transitions to other LLs
- ---Individual LLs contribute like independent species
  - Transverse conductivity:

$$\sigma_{\perp} \propto \alpha \sum_{n=0}^{\infty} \int dp_z \Gamma_n(p_z) F_{\perp}(E_n, E_{n+1}, p_z)$$

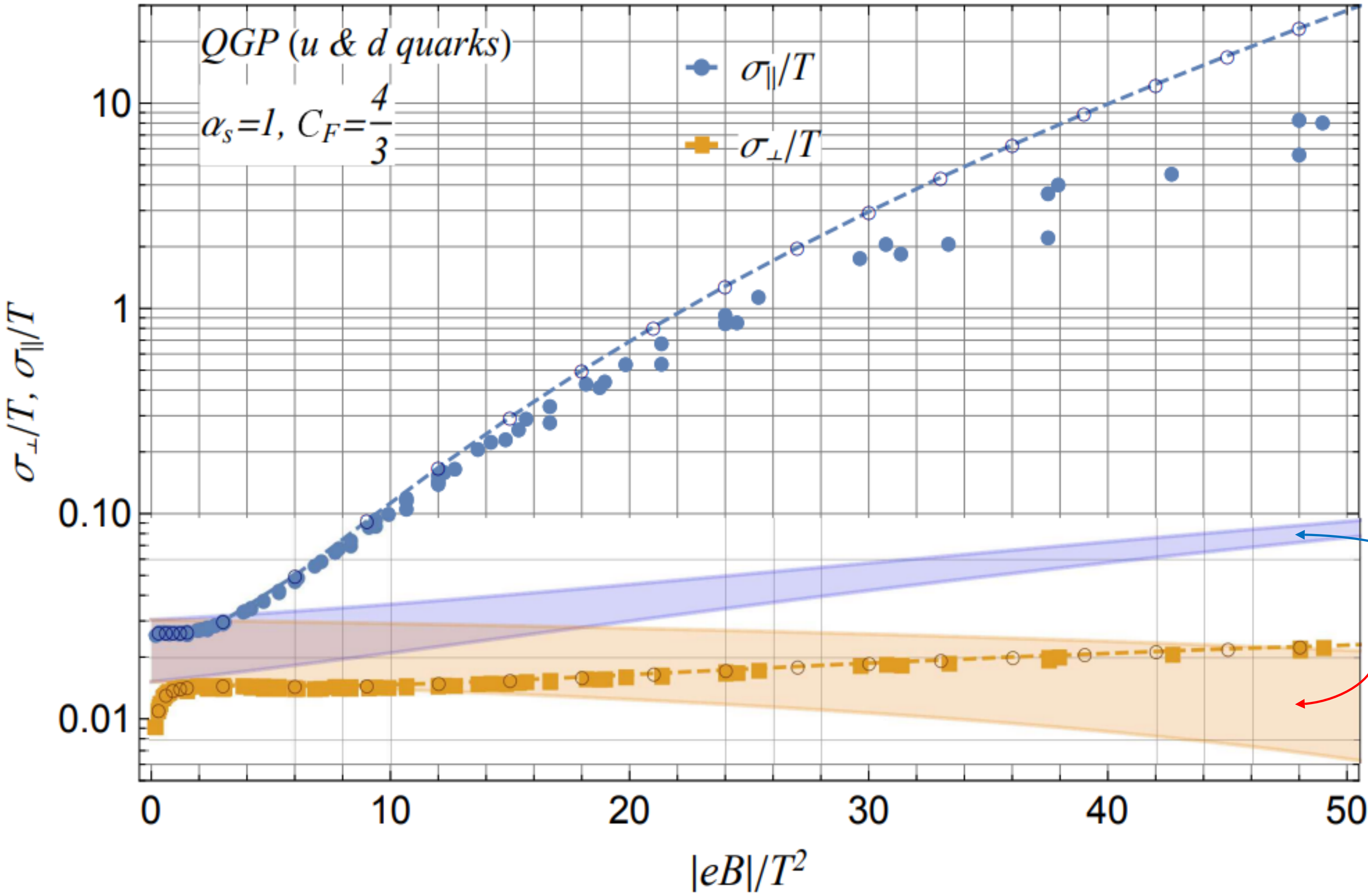
- Conductivity is driven by transitions (hopping) between LLs

- At least, transitions between 0th and 1st LLs are required



Conductivity of QCD plasma

[Ghosh, Shovkovy, Eur. Phys.J.C 84, 1179 (2024)]



$m = 0$

$$\frac{\Delta\,\sigma_{\parallel}}{TC_{\text{em}}} = \frac{3\,|eB|}{4\pi^2 T^2} C_\tau(|eB|)$$

$C_\tau(4\ \text{GeV}^2) \approx 0.134$

$C_\tau(9\ \text{GeV}^2) \approx 0.142$

[Almirante et al. arXiv:2406.18504]

Lattice

$2 \to 2$

Processes?

α_s?

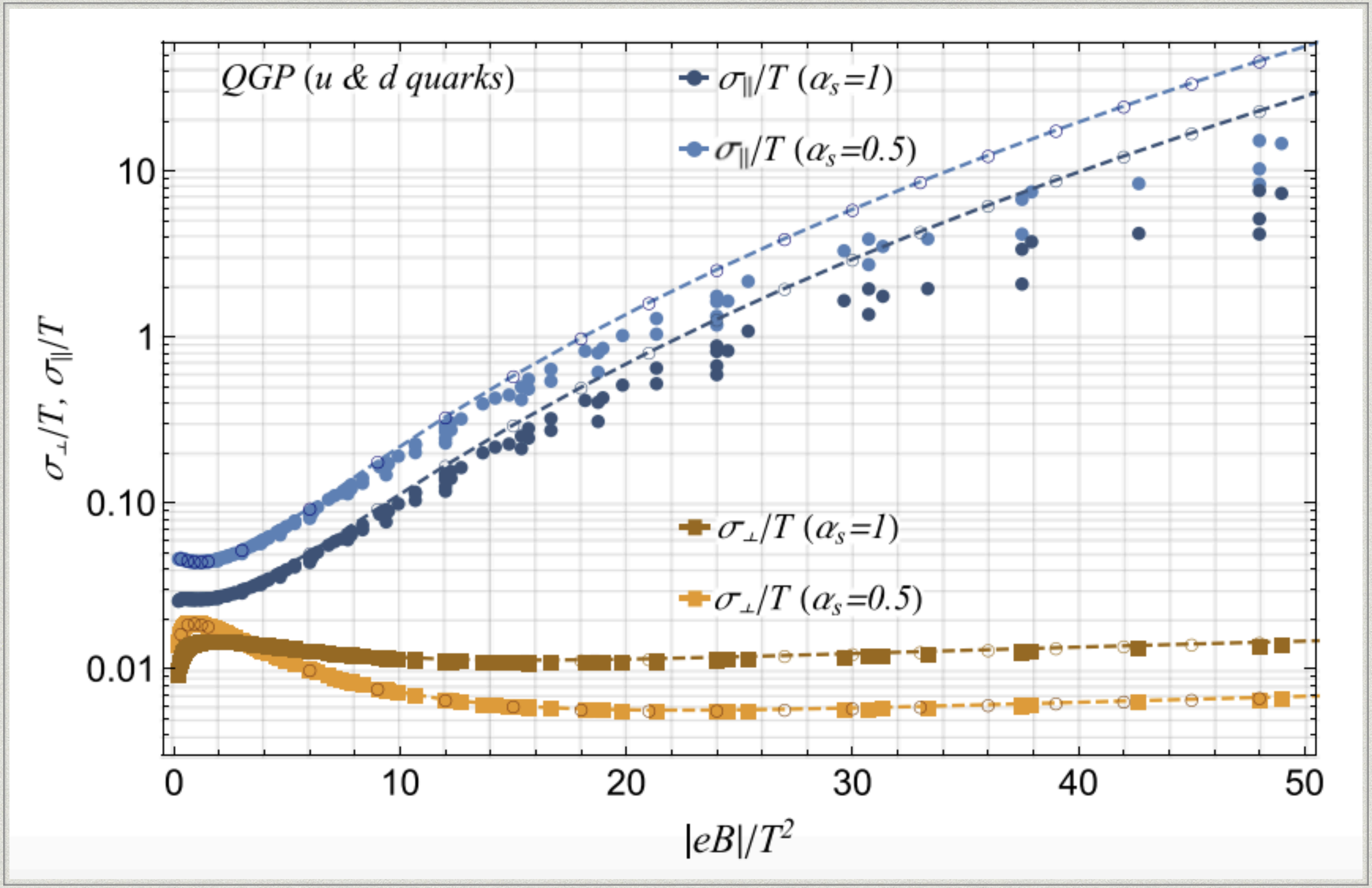
$B = 0$ (Lattice)

$\sigma \approx 1.1\,\text{MeV}@T = 200\,\text{MeV to } 5.6\,\text{MeV}@T = 350\,\text{MeV}$

[Aarts et al. JHEP 1502, 186 (2015)]

Conductivity of QCD plasma

[Ghosh, Shovkovy, Eur. Phys.J.C 84, 1179 (2024)]



Summary: Conductivity @ $B \neq 0$

- Conductivity is calculated for a plasma in a strong magnetic field from first principles within a gauge theory (QED/QCD).

- The transverse conductivity is suppressed, while the longitudinal conductivity is enhanced by a strong magnetic field. ---Anisotropic nature …Different mechanisms

- Transverse conduction critically relies on quantum transitions between Landau levels, effectively lifting charge trapping in localized Landau orbits.

- The damping rates are determined by $1 \leftrightarrow 2$ processes.

- The results are relevant for understanding a wide range of extreme astrophysical environments, such as those found in neutron stars, supernovae, and heavy-ion collisions.

Future direction:

- How other sub-leading processes contribute to the conductivity?
- Behavior at finite chemical potential?


Thank
you

Electrical Conductivity

[Ghosh, Shovkovy, Phys.Rev.D 110 (2024) 09, 096009] & [Ghosh, Shovkovy, Eur. Phys.J.C 84, 1179 (2024)]

$$\sigma_{ij} = \lim_{\Omega\to 0} \frac{\mathrm{Im}\left[\Pi_{ij}(\Omega + i0; \mathbf{0})\right]}{\Omega} = -\frac{\alpha}{8\pi T} \sum_{f=1}^{N_f} q_f^2 \int \frac{dk_0 d^3\boldsymbol{k}}{\cosh^2 \frac{k_0-\mu_f}{2T}} \mathrm{tr}\left[\gamma^i A_{\boldsymbol{k}}^f(k_0)\gamma^j A_{\boldsymbol{k}}^f(k_0)\right].$$

Current-current correlation

Fermion spectral function

$$A_{\boldsymbol{k}}^f(k_0) = \frac{i e^{-k_\perp^2 \ell_f^2}}{\pi} \sum_{\lambda=\pm} \sum_{n=0}^{\infty} \frac{(-1)^n}{E_{f,n}} \Big\{ \left[E_{f,n}\gamma^0 - \lambda k_z \gamma^3 + \lambda m_f\right] \times \left[\mathcal{P}_+ L_n\left(2k_\perp^2\ell_f^2\right) - \mathcal{P}_- L_{n-1}\left(2k_\perp^2\ell_f^2\right)\right] + 2\lambda(\boldsymbol{k}_\perp \cdot \boldsymbol{\gamma}_\perp) L_{n-1}^1\left(2k_\perp^2\ell_f^2\right) \Big\} \frac{\Gamma_{f,n}}{\left(k_0 - \lambda E_{f,n}\right)^2 + \Gamma_{f,n}^2}$$

$$\mathcal{P}_\pm = \left(1 \pm s_\perp i\gamma^1\gamma^2\right)/2$$

Damping rates of charge carriers in their quantized Landau level states

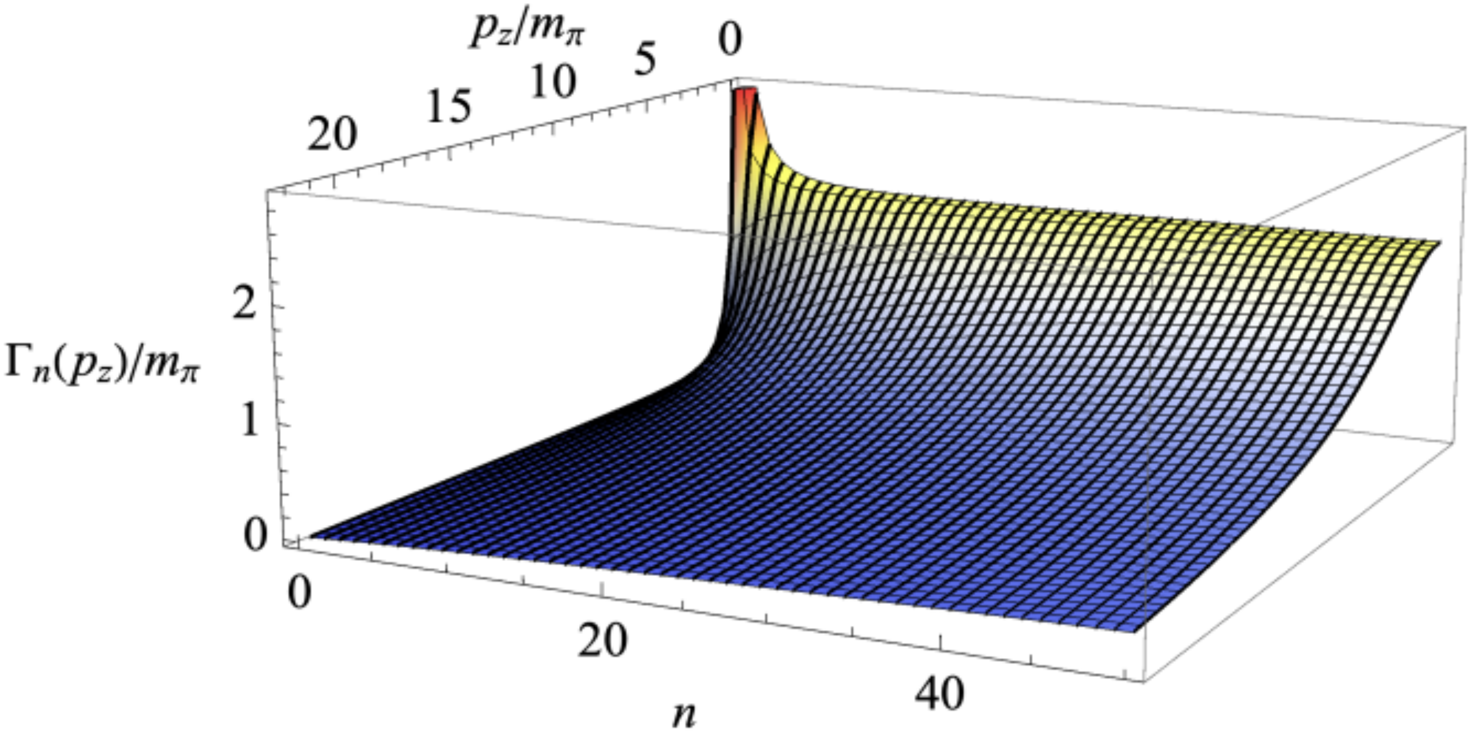
EXPRESSIONS FOR TRANSVERSE AND LONGITUDINAL CONDUCTIVITIES

$$\sigma_{\perp} = \frac{\alpha}{\pi^2 T} \sum_{f=1}^{N_f} \frac{q_f^2}{\ell_f^2} \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dk_0 dk_z}{\cosh^2 \frac{k_0-\mu_f}{2T}} \times \frac{\Gamma_{f,n+1}\Gamma_{f,n}\left[\left(k_0^2 + E_{f,n}^2 + \Gamma_{f,n}^2\right)\left(k_0^2 + E_{f,n+1}^2 + \Gamma_{f,n+1}^2\right) - 4k_0^2\left(k_z^2 + m_f^2\right)\right]}{\left[\left(E_{f,n}^2 + \Gamma_{f,n}^2 - k_0^2\right)^2 + 4k_0^2\Gamma_{f,n}^2\right]\left[\left(E_{f,n+1}^2 + \Gamma_{f,n+1}^2 - k_0^2\right)^2 + 4k_0^2\Gamma_{f,n+1}^2\right]},$$

$$\sigma_{\parallel} = \frac{\alpha}{2\pi^2 T} \sum_{f=1}^{N_f} \frac{q_f^2}{\ell_f^2} \sum_{n=0}^{\infty} \beta_n \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dk_0 dk_z}{\cosh^2 \frac{k_0-\mu_f}{2T}} \frac{\Gamma_{f,n}^2\left[\left(E_{f,n}^2 + \Gamma_{f,n}^2 - k_0^2\right)^2 + 4k_0^2\left(2k_z^2 + \Gamma_{f,n}^2\right)\right]}{\left[\left(E_{f,n}^2 + \Gamma_{f,n}^2 - k_0^2\right)^2 + 4k_0^2\Gamma_{f,n}^2\right]^2}$$

Damping rate:

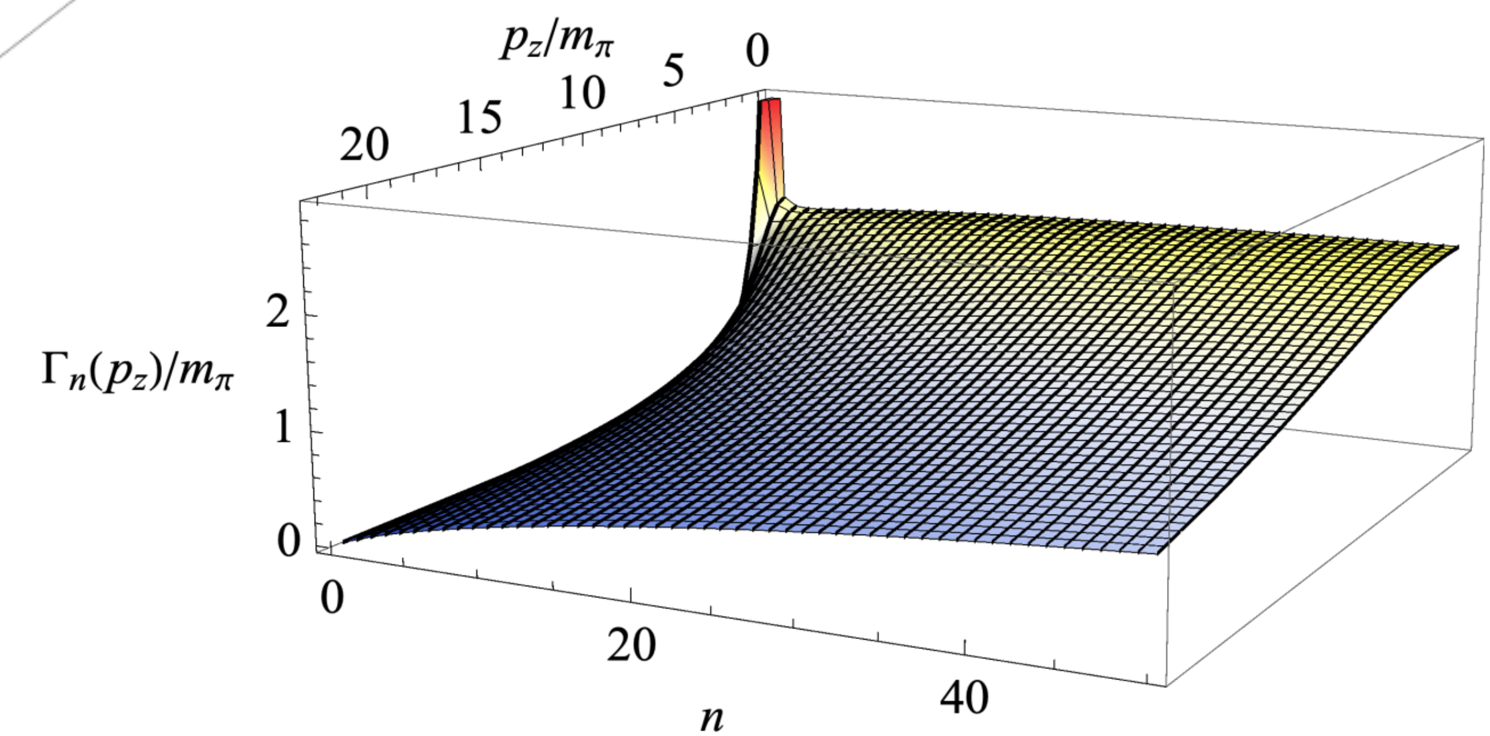
$n'_{\max} = 2n_{\max} = 100$



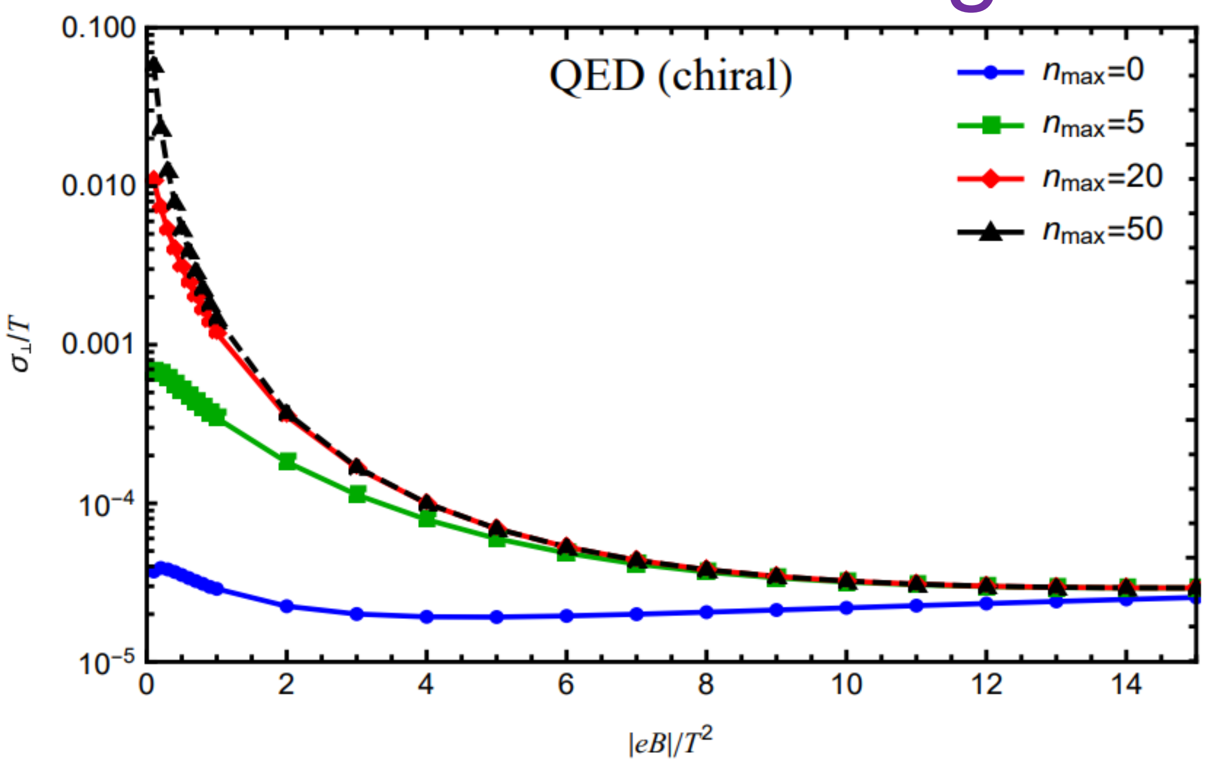
$|qB| = (75\,\mathrm{MeV})^2$

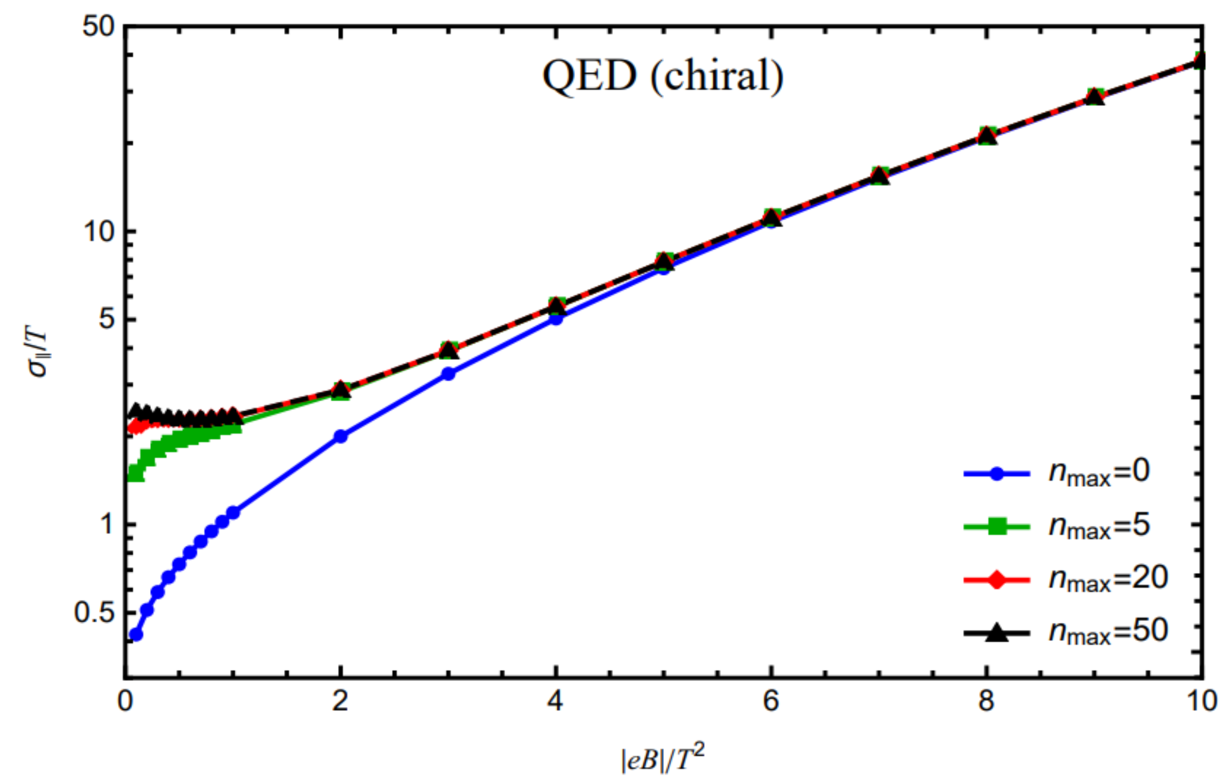
T=400 MeV

$|qB| = (200\,\mathrm{MeV})^2$



Landau-Level Sum Convergence



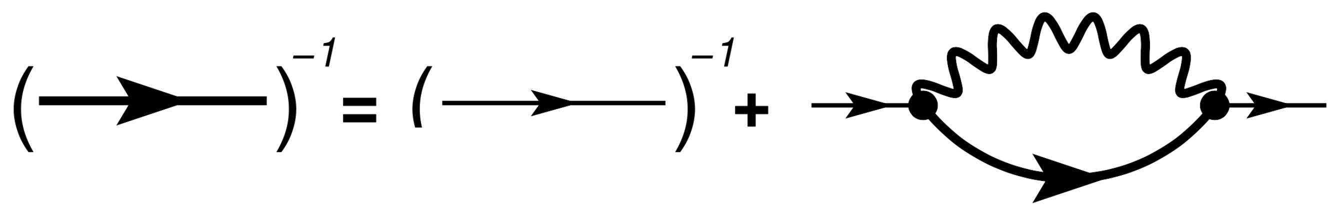


- A significant number of Landau levels must be included across a broad range of parameters.

$\sigma_\parallel$: Requires $n_{\max} \gtrsim 10T^2/|eB|$

$\sigma_\perp$: Requires $n_{\max} \gtrsim 30T^2/|eB|$

2. Damping rates from the poles of the propagator:



Quark self-energy

$$\bar{G}^{-1} = \bar{S}^{-1} + i\Sigma$$

$$\begin{aligned}\bar{\Sigma}(p_\parallel, \boldsymbol{p}_\perp) &= -2e^{-p_\perp^2\ell^2}\sum_{n=0}^{\infty}(-1)^n\left[\delta v_{\parallel,n}(p_\parallel\cdot\gamma_\parallel) + i\gamma^1\gamma^2(p_\parallel\cdot\gamma_\parallel)\tilde{v}_n - \delta m_n - i\gamma^1\gamma^2\tilde{m}_n\right]\left[\mathcal{P}_+L_n(2p_\perp^2\ell^2) - \mathcal{P}_-L_{n-1}(2p_\perp^2\ell^2)\right] \\ &- 4e^{-p_\perp^2\ell^2}\sum_{n=0}^{\infty}(-1)^n\delta v_{\perp,n}(\boldsymbol{\gamma}_\perp\cdot\boldsymbol{p}_\perp)L_{n-1}^1(2p_\perp^2\ell^2)\end{aligned}$$

splitting of the parallel velocities and masses of the two spin states

Poles:

$$\det[\bar{G}^{-1}] = 0$$

The Fourier transform of the translation invariant part of the propagator

$$\bar{G}(p_\parallel, \boldsymbol{p}_\perp) = ie^{-p_\perp^2\ell^2}\sum_{n=0}^{\infty}(-1)^n D_n(p_\parallel, \boldsymbol{p}_\perp)\frac{1}{\mathcal{M}_n - 2nv_{\perp,n}^2|qB|},$$

where the nth Landau level contribution is determined by

$$\begin{aligned} D_n(p_\parallel, \boldsymbol{p}_\perp) &= 2\left[v_{\parallel,n}(p_\parallel \cdot \gamma_\parallel) - i\gamma^1\gamma^2\tilde{v}_n(p_\parallel \cdot \gamma_\parallel) + m_n - i\gamma^1\gamma^2\tilde{m}_n\right]\left[\mathcal{P}_+ L_n\left(2p_\perp^2\ell^2\right) - \mathcal{P}_- L_{n-1}\left(2p_\perp^2\ell^2\right)\right] \\ &+ 4v_{\perp,n}\left(\boldsymbol{p}_\perp \cdot \boldsymbol{\gamma}_\perp\right)L_{n-1}^1\left(2p_\perp^2\ell^2\right). \end{aligned}$$

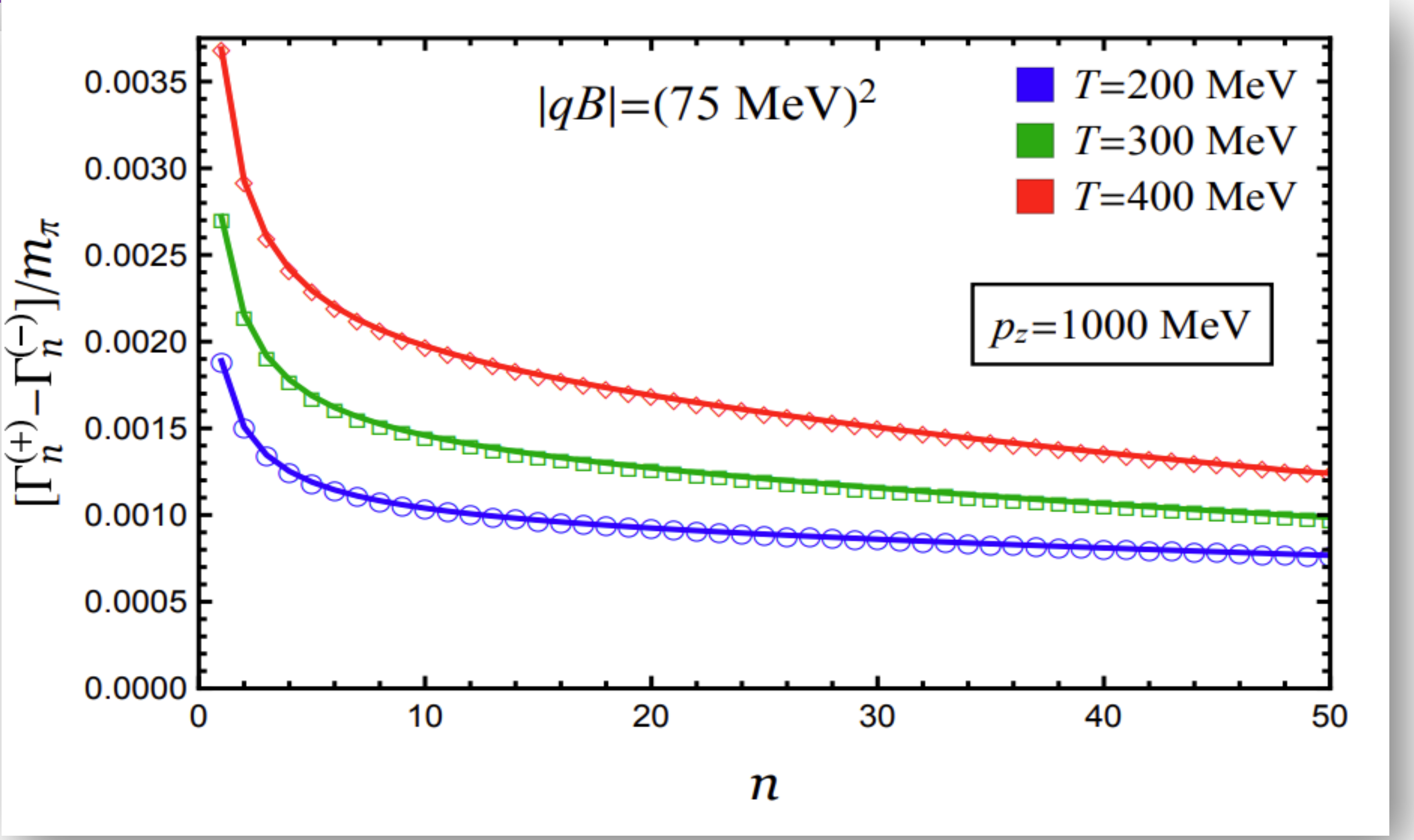
$$\frac{1}{\mathcal{M}_n - 2nv_{\perp,n}^2|qB|} = \frac{1}{U_n}\left[\left(v_{\parallel,n}^2 - \tilde{v}_n^2\right)p_\parallel^2 - 2nv_{\perp,n}^2|qB| - m_n^2 + \tilde{m}_n^2 - 2i\gamma^1\gamma^2(m_n\tilde{v}_n - \tilde{m}_n v_{\parallel,n})(p_\parallel \cdot \gamma_\parallel)\right],$$

where

$$U_n = \left[\left(v_{\parallel,n}^2 - \tilde{v}_n^2\right)p_\parallel^2 - 2nv_{\perp,n}^2|qB| - m_n^2 + \tilde{m}_n^2\right]^2 - 4p_\parallel^2\left(m_n\tilde{v}_n - \tilde{m}_n v_{\parallel,n}\right)^2.$$

- $v_\parallel$: velocity along $B \to$ modifies motion in z-direction
- $v_\perp$: transverse velocity $\to$ Landau level dynamics
- $\tilde{v}$: spin-dependent velocity $\to v_\parallel \pm \tilde{v}$
- m: effective mass (interaction corrected)
- $\tilde{m}$: spin splitting $\to m \pm \tilde{m}$

Two spin-split Landau-level states : different damping rates



$$\Gamma_n^{(\pm)}$$

$$|\Gamma_n^{(+)} - \Gamma_n^{(-)}| \ll \Gamma_n^{(\mathrm{ave})}$$

$$\Gamma_n^{(\mathrm{ave})} \equiv (\Gamma_n^{(+)} + \Gamma_n^{(-)})/2$$