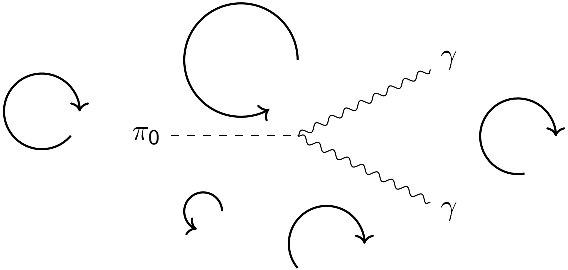
Vorticity induced-effects from WZW terms



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based on arXiv:2510.25459 [hep-ph] with Naoki Yamamoto and Di-Lun Yang

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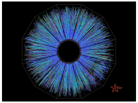
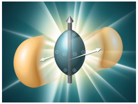
Wess-Zumino-Witten terms

Quantum Anomaly: when a conserved current of the classical theory is not conserved in the quantum theory, or

$$\partial_\mu j^\mu_{\text{classical}} = 0 \quad \Rightarrow \quad \partial_\mu j^\mu_{\text{quantum}} \neq 0 \, .$$

- 't Hooft anomaly matching condition: anomaly should manifest in the effective field theory [G. 't Hooft, NATO SS B 59, (1980)]
- Chiral anomaly in QCD $\Rightarrow$ anomalous hadronic interactions described by Wess-Zumino-Witten (WZW) effective action [J. Wess and B. Zumino, PLB 37 (1971); E. Witten, NPB 223 (1983)]
- Anomalous effects, e.g., chiral magnetic and chiral vortical [K. Fukushima et al., PRD 78, (2008); D. Kharzeev and A. Zhitnitsky, NPA 797, (2007)]

Images: Brookhaven National Laboratory

Vorticity as an axial-vector field

From the two facts:

1. Four-vorticity $\omega^\mu = \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha u_\beta / 2$ can be regarded as a fictitious spacetime torsion (u^ν is the four-velocity) [A. Shitade and T. Kimura, PRB 90 (2014); Z. V. Khaidukov and M. A. Zubkov, JETP Lett. 108 (2018)]
2. Massless Dirac fermions in a spacetime with torsion $\Leftrightarrow$ massless Dirac fermions coupled to an axial-vector field in flat spacetime [I. L. Shapiro, P. Rept. 357 (2002)]

one concludes that *vorticity can be introduced as an emergent axial gauge field* [N. Yamamoto and D. L. Yang, PRD 103 (2021)]

$$A^\mu \equiv \frac{1}{2}\omega^\mu \, .$$

We may also introduce a chemical potential μ_f for flavour f via

$$\bar{\psi}_f \gamma^0 \mu_f \psi_f \quad \Rightarrow \quad \bar{\psi}_f \not{V}_f \psi_f \, ,$$

with fermions ψ_f and vector field V_f^μ [D. T. Son and Ariel R. Zhitnitsky, PRD 70 (2004)].

Goal

Re-derive the WZW terms in external vector, axial-vector, and pseudoscalar fields to determine the anomalous effects induced by vorticity.

How?

Using a derivative expansion of the fermionic determinant found from a linear sigma model [I. J. R. Aitchison and C. M. Fraser, PRD 31 (1985)].

Linear sigma model

We start from

$$\mathcal{L} = \bar{\psi}\left(\mathrm{i}\partial\!\!\!/ - V\!\!\!/ + \gamma^5 A\!\!\!/ - m\mathrm{e}^{\mathrm{i}\theta\gamma^5}\right)\psi\,,$$

where m is the fermion mass and θ is a pseudoscalar field. For an Abelian A^μ, we have the *local* axial symmetry

$$\psi \to \mathrm{e}^{-\mathrm{i}\lambda\gamma^5}\psi\,, \qquad A_\mu \to A_\mu + \partial_\mu\lambda\,, \qquad \theta \to \theta + 2\lambda\,.$$

We will also consider a vector-gauge symmetry under which ψ, V^μ and θ transform.

Derivative expansion of fermion determinants (1/2)

- We want to obtain the effective action Γ and extract the effective Lagrangian $\mathcal{L}_{\text{eff}}$ via

$$\Gamma = \int \mathrm{d}^4 x \, \mathcal{L}_{\text{eff}}$$

- From the partition function,

$$\mathcal{Z} = \int D\psi D\bar{\psi} \, \exp\left(\mathrm{i} \int \mathrm{d}^4 x \mathcal{L}\right) = \mathrm{e}^{\mathrm{i}\Gamma} \,,$$

$$\Rightarrow \Gamma = -\mathrm{i} \, \mathrm{tr} \ln \left[\not{p} - \not{V} + \gamma^5 \not{A} - m \mathrm{e}^{\mathrm{i}\theta\gamma^5} \right] ,$$

where p^μ is the momentum operator

Derivative expansion of fermion determinants (2/2)

- Expand around $V_\mu = A_\mu = \theta = 0$

$$\Gamma = -\mathrm{i}\,\mathrm{tr}\ln\left(\not{p} - m\right) + \mathrm{i}\,\mathrm{tr}\,\frac{1}{\not{p} - m}\tilde{M} + \frac{\mathrm{i}}{2}\,\mathrm{tr}\,\frac{1}{\not{p} - m}\tilde{M}\frac{1}{\not{p} - m}\tilde{M} + \ldots\,,$$

where $\tilde{M} = \not{V} - \gamma^5\not{A} + m(\mathrm{e}^{\mathrm{i}\theta\gamma^5} - 1)$

- Write functional trace as momentum and position space integral, then use commutation relations

$$[p^\mu, \phi] = \mathrm{i}\partial^\mu\phi\,, \qquad [p^2, \phi] = (\partial^2 + 2\mathrm{i}p\cdot\partial)\phi\,,$$

and

$$\left[\frac{1}{p^2 - m^2}, \phi\right] = -\frac{1}{(p^2 - m^2)^2}\left[p^2, \phi\right] - \frac{1}{(p^2 - m^2)^3}\left[p^2, \left[p^2, \phi\right]\right] + \ldots\,,$$

(where $\phi(x)$ is some field operator) to shift all momentum operators to one side and integrate over the momentum

What do we get?

To fourth order,

$$
\begin{aligned}
\Gamma_{\text{WZW}} =& \frac{\mathrm{i}N_c}{48\pi^2}\epsilon^{\mu\nu\alpha\beta}\int \mathrm{d}^4x\,\mathrm{Tr}_f\Big(-2\mathrm{i}\partial_\mu\theta\,(3\partial_\nu V_\alpha V_\beta + \partial_\nu A_\alpha A_\beta) \\
& - \partial_\mu\theta\partial_\nu\theta\partial_\alpha\theta V_\beta + 6(\partial_\mu\theta V_\nu V_\alpha V_\beta + \theta V_\nu \partial_\mu V_\alpha V_\beta) \\
& + 2\theta\partial_\mu V_\nu A_\alpha A_\beta - 2\theta V_\nu A_\alpha \partial_\mu A_\beta + 2\partial_\mu\theta A_\nu V_\alpha A_\beta \\
& - 6\theta A_\nu \partial_\mu V_\alpha A_\beta - 2\theta\partial_\mu A_\nu A_\alpha V_\beta + 2\theta A_\nu A_\alpha \partial_\mu V_\beta \\
& + 2[\partial_\mu\theta,\theta]\{\partial_\nu V_\alpha, A_\beta\} + 2(\partial_\mu\theta\partial_\nu V_\alpha\theta - \theta\partial_\nu V_\alpha\partial_\mu\theta)A_\beta \\
& + \theta\theta[\partial_\nu V_\alpha, \partial_\mu A_\beta]\Big) + \dots
\end{aligned}
$$

which agrees with previous results, where N_c is the number of colours, Tr_f is the trace over flavour space [Ö . Kaymakcalan et al., PRD 30,(1984); J. L. Mañes, Nucl. Phys. B 250, 369 (1985); N. K. Pak and P. Rossi, Nucl. Phys. B 250 (1985) and many others . . .].

Abelian case and the axial anomaly

- Specialising to $V_\mu = eQV^Q_\mu$, with V^Q_μ and A_μ Abelian,

$$\Gamma = \frac{N_c}{8\pi^2}\epsilon^{\mu\nu\alpha\beta}\int \mathrm{d}^4x\,\mathrm{Tr}_f\left\{\partial_\mu\theta(e^2Q^2\partial_\nu V^Q_\alpha V^Q_\beta + \frac{1}{3}\partial_\nu A_\alpha A_\beta)\right.$$
$$\left. -\frac{\mathrm{i}e}{3}Q\partial_\mu\theta\partial_\nu\theta\partial_\alpha\theta V^Q_\beta + \frac{\mathrm{i}e}{3}Q[\partial_\mu\theta,\theta]\partial_\nu V^Q_\alpha A_\beta\right\},$$

  where Q is a charge matrix

- We reproduce the anomaly;

$$\partial_\mu j^\mu_V = 0\,,$$
$$\partial_\mu j^\mu_A = \frac{N_c}{4\pi^2}\epsilon^{\mu\nu\alpha\beta}\left(\frac{e^2}{4}F^V_{\mu\nu}F^V_{\alpha\beta}\,\mathrm{Tr}_f\,Q^2 + \frac{N_f}{12}F^A_{\mu\nu}F^A_{\alpha\beta}\right),$$

  where $j^\mu_{V/A}$ is the vector/axial current with field strength tensor $F^{V/A}_{\mu\nu}$ and N_f is the number of flavours [W. A. Bardeen, Phys. Rev. 184 (1969)]

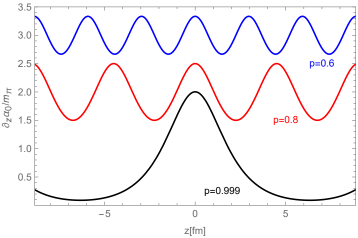
Neutral pions at large magnetic field

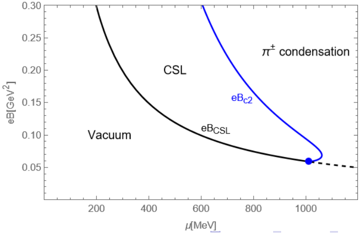
- Focus on pions $\rightarrow N_f = 2,\ N_c = 3$
- When $A_\mu = 0$, we can extract

$$\mathcal{L}_{\text{baryon}} = \frac{e}{4\pi^2 f_\pi}\mu_B \nabla\pi_0 \cdot \boldsymbol{B}\,,$$
$$\mathcal{L}_{\text{isospin}} = \frac{e}{8\pi^2 f_\pi}\mu_I \nabla\pi_0 \cdot \boldsymbol{B}\,,$$

  where $\mu_{B/I}$ is the baryon/isospin chemical potential and f_π is the pion decay constant

- Responsible for Chiral Soliton Lattice (CSL) [T. Brauner and N. Yamamoto, JHEP 4 (2017); M. S. Grønli and T. Brauner, Eur. Phys. J. C 82 (2022)]





Anomalous quantities for charged pions

When $A_\mu = \omega_\mu/2$, we can extract

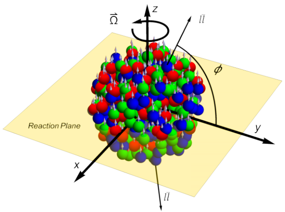
$$\mathcal{L}_{\pi\pi V\omega} = -\frac{\mathrm{i}e}{16\pi^2 f_\pi^2}\epsilon^{\mu\nu\alpha\beta}(\pi^+\partial_\mu\pi^- - \pi^-\partial_\mu\pi^+)F^V_{\nu\alpha}\omega_\beta\,.$$

Assuming constant vorticity, we obtain the anomalous charge and angular momentum density;

$$n = -\frac{i}{4\pi^2 f_\pi^2}(\nabla\pi^+ \times \nabla\pi^-)\cdot\boldsymbol{\omega}\,,$$
$$\boldsymbol{J} = -\frac{e n_I}{8\pi^2 f_\pi^2}\boldsymbol{B}\,,$$

respectively, where $n_I = j_I^0$ and j_I^μ is the non-anomalous isospin current.

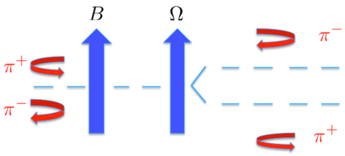
Charged pions in electromagnetic fields and vorticity



[M. Wei et al., PRD 105, (2022)]

- Vorticity-modified photon-pion coupling
- Could modify the current-current correlators associated with photon and dilepton production

- Rotation causes split in charged pion Landau Levels ⇒ charged pion condensation
- Anomalous magneto-vortical contribution?



[Y. Liu and I. Zahed, PRL 120 (2018)]

Summary and Outlook

- Re-derived the WZW effective action from a linear sigma model using a derivative expansion of the fermion determinant
- New anomalous terms involving vorticity from axial-vector field correspondence $\Rightarrow$ anomalous charge and angular momentum density
- Phenomenological implications e.g. modifying the photon and dilepton production in a pion gas at finite temperature and/or density [B. Singh et al., PRD 100 (2019); M. Wei et al., PRD 105 (2022); L. Dong and S. Lin, EPJ. A 58 (2022)]
- No explicit chiral vortical or helical magnetic effect [X. G. Huang et al. JHEP 02 (2018), N. Yamamoto and D. L. Yang, PRD 103 (2021)]
- Anomalous effects in superfluid nuclear/quark matter? [D. T. Son and A. R. Zhitnitsky, PRD **70** (2004)]