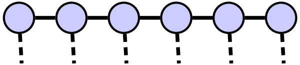
Hadron structure from tensor networks: a Hamiltonian approach

Manuel Schneider
`manuel.schneider@nycu.edu.tw`



NATIONAL
YANG MING CHIAO TUNG
UNIVERSITY

理 論 與 計 算 物 理 中 心
Center for Theoretical and Computational Physics

[arXiv:2504.07508]
[arXiv:2409.16996]

Taiwan Nuclear Physics Retreat 2026
@ Jiaoxi, Yilan
22 April 2026

Collaborators

Mari Carmen Bañuls

Krzysztof Cichy

C.-J. David Lin

Outline

Outline

Parton Distribution Functions

- PDF: probability of constituent with momentum fraction ξ

[EIC]

Parton Distribution Functions

- PDF: probability of constituent with momentum fraction ξ
- factorization at large energy $Q^2 = -q^2$, e.g. DIS:

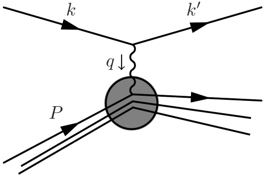
$$\sigma(\xi, Q^2) = \hat{\sigma}(\xi, Q^2) \otimes f(\xi)$$

experiment

perturbative

non-perturbative

[EIC]



Deep Inelastic Scattering
[Schwartz 2014]

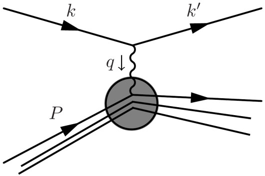
Parton Distribution Functions

- PDF: probability of constituent with momentum fraction ξ
- factorization at large energy $Q^2 = -q^2$, e.g. DIS:

$$\underset{\nearrow\ \text{experiment}}{\sigma(\xi, Q^2)} = \underset{\uparrow \atop \text{perturbative}}{\hat{\sigma}(\xi, Q^2)} \otimes \underset{\nwarrow\ \text{non-perturbative}}{f(\xi)}$$

$$\boxed{f_\psi(\xi) = \int dz^- e^{-i\xi P^+ z^-} \left\langle P \middle| \bar{\psi}(z^-)\gamma^+ W(z^- \leftarrow 0)\psi(0) \middle| P \right\rangle}$$

[EIC]



Deep Inelastic Scattering
[Schwartz 2014]

Parton Distribution Functions

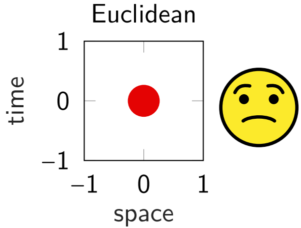
- PDF: probability of constituent with momentum fraction ξ
- factorization at large energy $Q^2 = -q^2$, e.g. DIS:

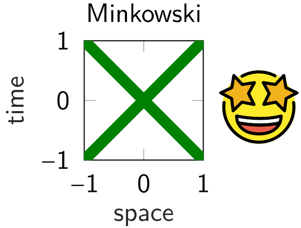
$$\sigma(\xi, Q^2) = \hat{\sigma}(\xi, Q^2) \otimes f(\xi)$$

experiment perturbative non-perturbative

$$\boxed{f_\Psi(\xi) = \int dz^- e^{-i\xi P^+ z^-} \langle P | \bar{\psi}(z^-)\gamma^+ W(z^- \leftarrow 0)\psi(0) | P \rangle}$$

- z^-: lightcone coordinate
- lattice QCD in Euclidean space: lightcone → point
- Hamiltonian formalism: lightcone in Minkowski space
  → tensor network states / quantum devices





Outline

Tensor Network States

- generic state scales exponentially

$$|\psi\rangle = \sum_{s_1, s_2, \dots, s_N} \Psi^{s_1 s_2 \dots s_N} |s_1\rangle \otimes |s_2\rangle \otimes \dots \otimes |s_N\rangle$$

Tensor Network States

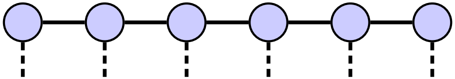
- generic state scales exponentially

$$|\psi\rangle = \sum_{s_1, s_2, \ldots, s_N} \Psi^{s_1 s_2 \ldots s_N} |s_1\rangle \otimes |s_2\rangle \otimes \ldots \otimes |s_N\rangle$$

- tensor network state as ansatz

$$\Psi^{s_1 s_2 \ldots s_N} = \sum_{\{i_x\}} A^{1,s_1}_{i_1} \cdot A^{2,s_2}_{i_1,i_2} \cdot A^{3,s_3}_{i_2,i_3} \ldots A^{N,s_N}_{i_{N-1}}$$

$=$



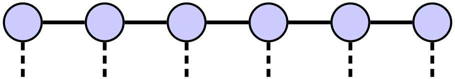
Tensor Network States

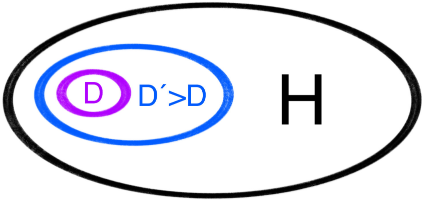
- generic state scales exponentially
- tensor network state as ansatz
- truncation to bond dimenion D
- polynomial resource scaling

$$|\psi\rangle = \sum_{s_1,s_2,\ldots,s_N} \Psi^{s_1 s_2 \ldots s_N} \left|s_1\right\rangle \otimes \left|s_2\right\rangle \otimes \ldots \otimes \left|s_N\right\rangle$$

$$\Psi^{s_1 s_2 \ldots s_N} \approx \sum_{\{i_x\}=1}^{D} A^{1,s_1}_{i_1} \cdot A^{2,s_2}_{i_1,i_2} \cdot A^{3,s_3}_{i_2,i_3} \ldots A^{N,s_N}_{i_{N-1}}$$

=





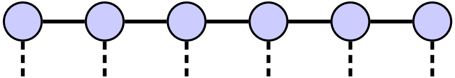
Tensor Network States

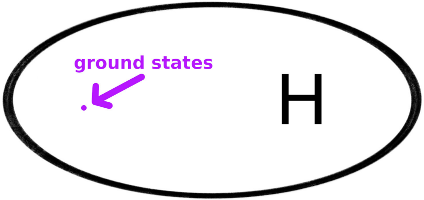
- generic state scales exponentially
- tensor network state as ansatz
- truncation to bond dimenion D
- polynomial resource scaling
- good for ground states and low excited states [Hastings 2007]
- no sampling, fundamentally different systematics compared to Monte Carlo

$$|\psi\rangle = \sum_{s_1,s_2,\dots,s_N} \Psi^{s_1 s_2 \dots s_N} |s_1\rangle \otimes |s_2\rangle \otimes \dots \otimes |s_N\rangle$$

$$\Psi^{s_1 s_2 \dots s_N} \approx \sum_{\{i_x\}=1}^{D} A^{1,s_1}_{i_1} \cdot A^{2,s_2}_{i_1,i_2} \cdot A^{3,s_3}_{i_2,i_3} \dots A^{N,s_N}_{i_{N-1}}$$

$$=$$





Outline

Schwinger model [Hamer et al. 1997]

- quantum electrodynamics in 1+1 dimensions, $U(1)$ symmetry
- fermion couples to gauge boson → partons
- bound states → hadrons [Bañuls et al. 2013]
- scattering → PDF [Dai et al. 1995]
- Lagrange density:

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - g\not{A} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

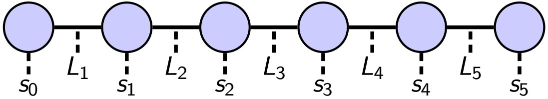
Schwinger model [Hamer et al. 1997]

- quantum electrodynamics in 1+1 dimensions, $U(1)$ symmetry
- fermion couples to gauge boson → partons
- bound states → hadrons [Bañuls et al. 2013]
- scattering → PDF [Dai et al. 1995]
- Lagrange density:

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - g\not{A} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- for TN/QC: transform action into spin-model Hamiltonian:



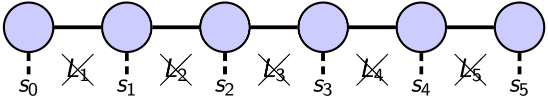
Schwinger model [Hamer et al. 1997]

- quantum electrodynamics in 1+1 dimensions, $U(1)$ symmetry
- fermion couples to gauge boson → partons
- bound states → hadrons [Bañuls et al. 2013]
- scattering → PDF [Dai et al. 1995]
- Lagrange density:

$$\mathcal{L} = \bar{\Psi}(i\not{\partial} - g\not{A} - m)\Psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - A_0\rho$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

- for TN/QC: transform action into spin-model Hamiltonian:



$$\boxed{H = x\sum_{n=0}^{N-2}\left[\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right] + \frac{\mu}{2}\sum_{n=0}^{N-1}\left[1 + (-1)^n\sigma_n^z\right] + \sum_{n=0}^{N-2}\left[\frac{1}{2}\sum_{k=0}^{n}\left((-1)^k + \sigma_k^z + 2q_k\right)\right]^2}$$

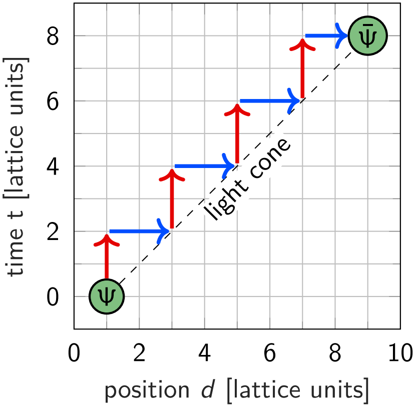
$$\left(x = \frac{1}{a^2g^2}, \mu = \frac{2m}{ag^2}\right)$$

PDF in the Schwinger model

- matrix elements:

$$\mathcal{M} = \left\langle P \middle| \bar{\Psi}(z^-)\gamma^+ W(z^- \leftarrow 0)\Psi(0) \middle| P \right\rangle$$

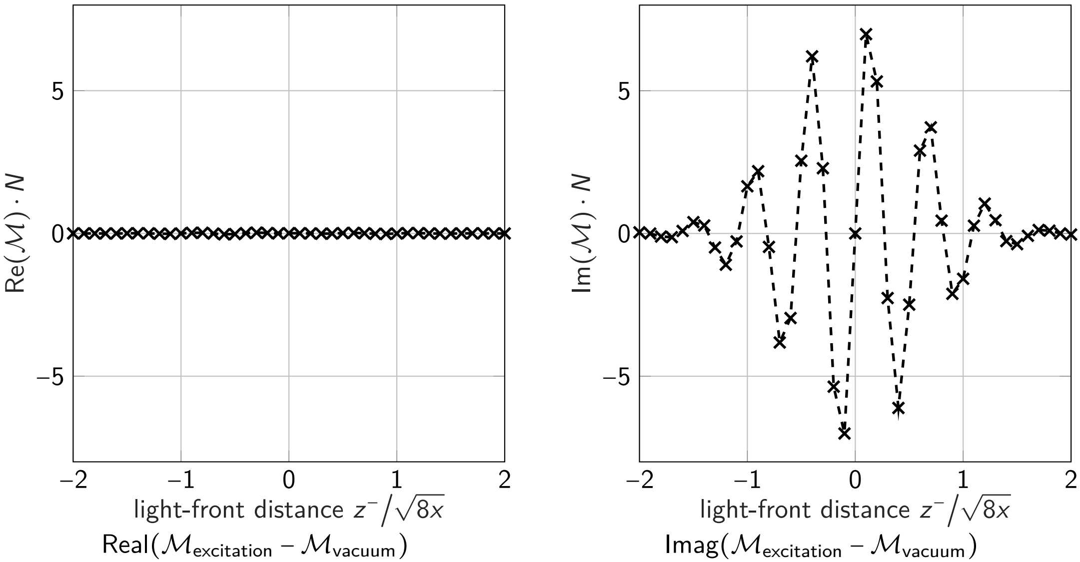
- lightcone
  → small time- and space-like steps
- spatial evolution:
  change electric field along the path
- trotterized time evolution



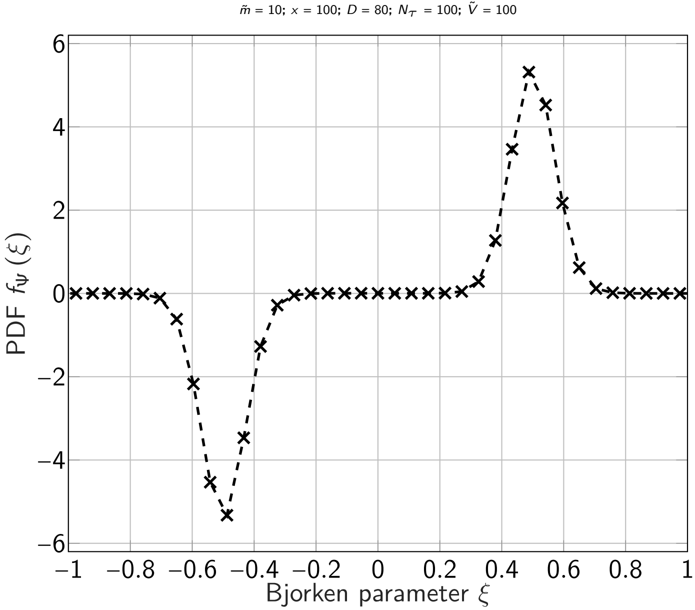
Outline

Results: Matrix elements

$\tilde{m} = 10;\ x = 100;\ D = 80;\ N_T = 100;\ \tilde{V} = 100$



Results: PDF



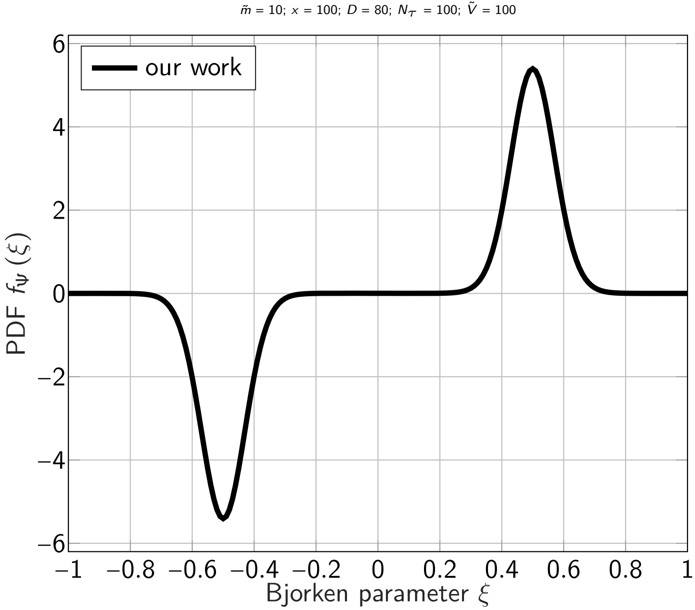
observations:

- $\xi > 0$: $f_\psi \approx$ symmetric around $\xi = 0.5$
- antifermion PDF from negative ξ:

$$f_{\overline{\psi}}(\xi) = -f_\psi(-\xi)$$

- observed symmetry
  $\rightarrow f_{\overline{\psi}}(\xi) = f_\psi(\xi)$
  $\Rightarrow$ meson ✓

Results: PDF



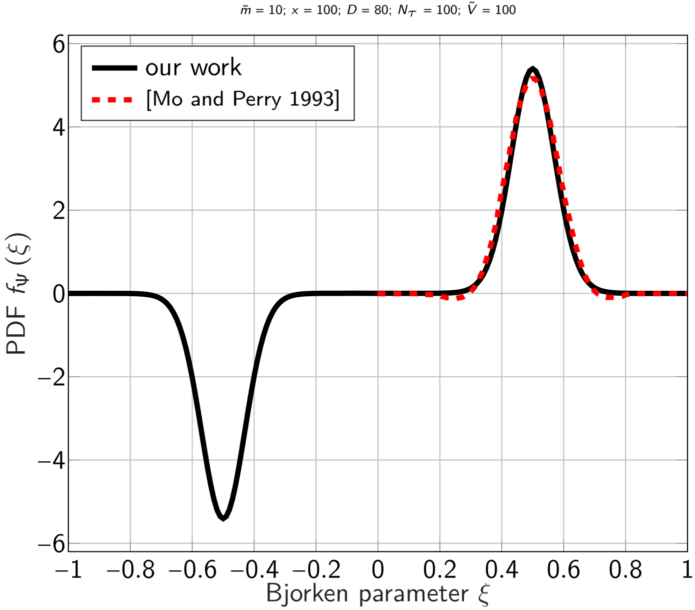
observations:

- $\xi > 0$: $f_\psi \approx$ symmetric around $\xi = 0.5$
- antifermion PDF from negative ξ:

$$f_{\overline{\psi}}(\xi) = -f_\psi(-\xi)$$

- observed symmetry
  $\rightarrow f_{\overline{\psi}}(\xi) = f_\psi(\xi)$
  $\Rightarrow$ meson ✓

Results: PDF



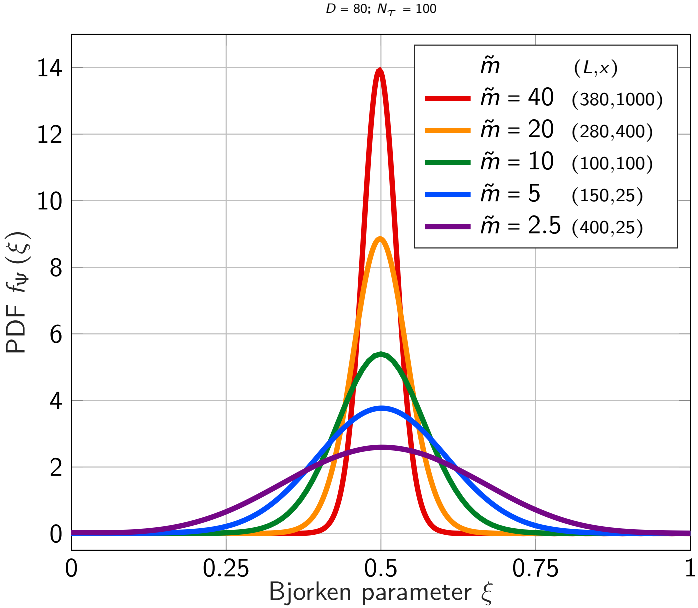
observations:

- $\xi > 0$: $f_\psi \approx$ symmetric around $\xi = 0.5$
- antifermion PDF from negative ξ:

$$f_{\overline{\psi}}(\xi) = -f_\psi(-\xi)$$

- observed symmetry
  $\rightarrow f_{\overline{\psi}}(\xi) = f_\psi(\xi)$
  $\Rightarrow$ meson ✓

Results: PDF



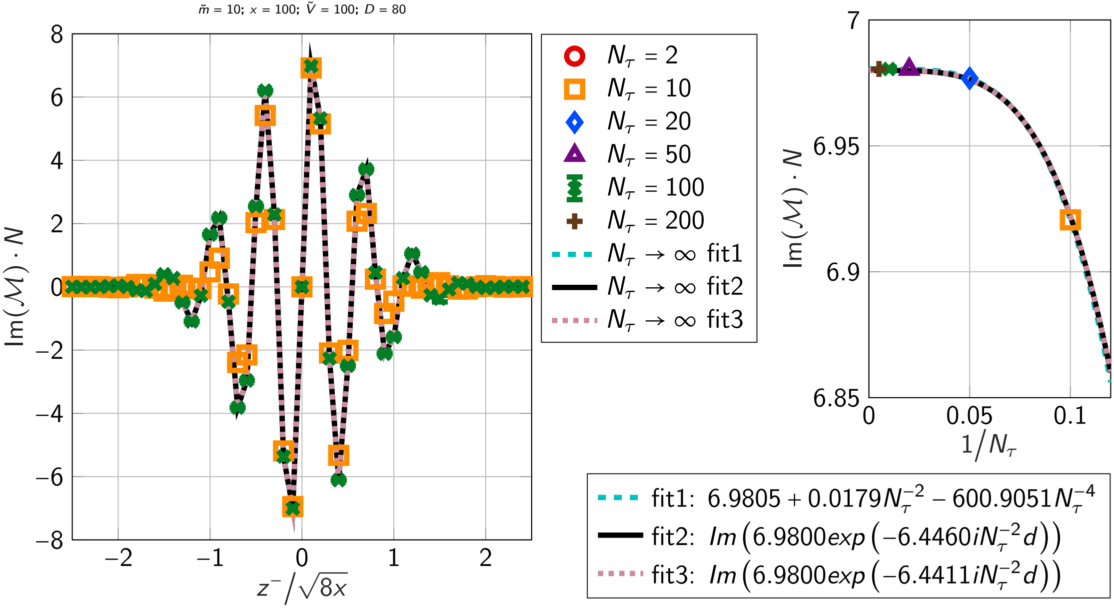
observations:

- $\xi > 0$: $f_\psi \approx$ symmetric around $\xi = 0.5$
- antifermion PDF from negative ξ:

$$f_{\overline{\psi}}(\xi) = -f_\psi(-\xi)$$

- observed symmetry
  $\rightarrow f_{\overline{\psi}}(\xi) = f_\psi(\xi)$
  $\Rightarrow$ meson ✓
- peak broadens with decreasing fermion mass ✓

Results: N_τ-dependence

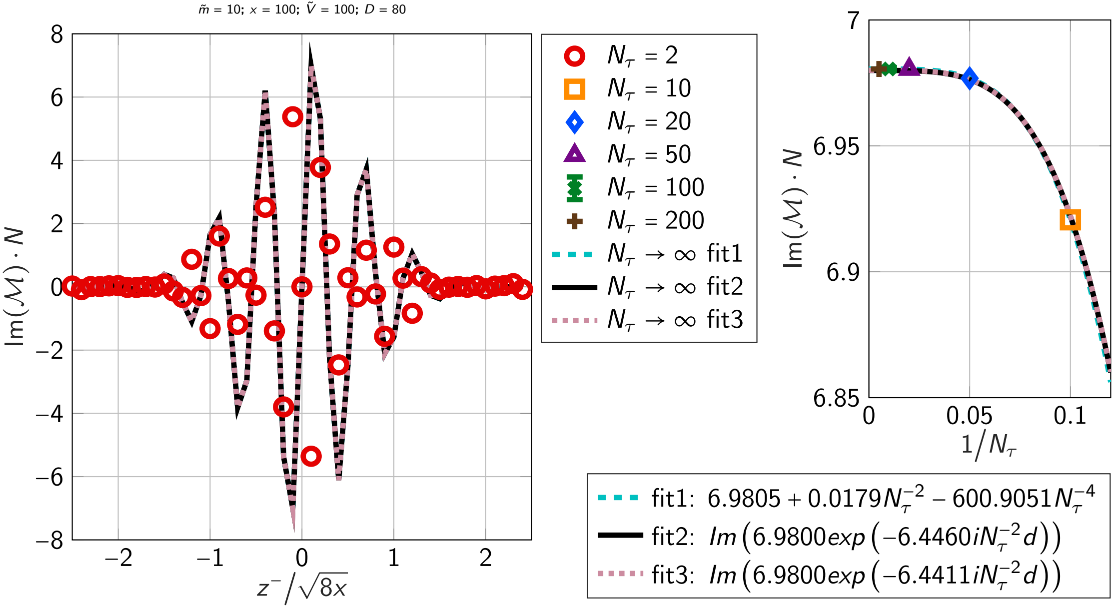


fit1: $6.9805 + 0.0179 N_\tau^{-2} - 600.9051 N_\tau^{-4}$

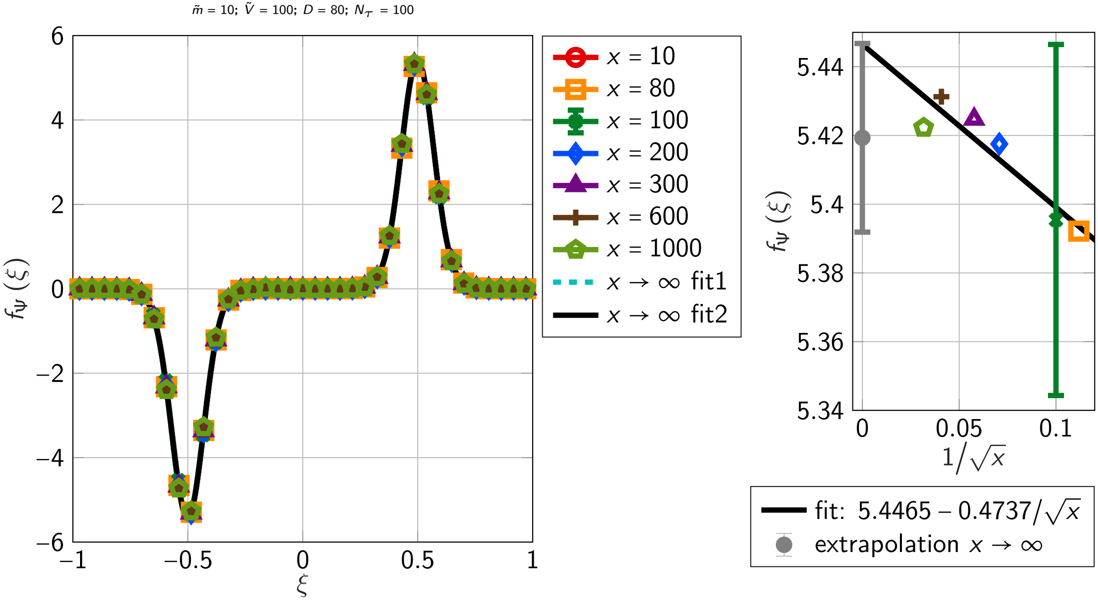
fit2: $Im\left(6.9800 exp\left(-6.4460 i N_\tau^{-2} d\right)\right)$

fit3: $Im\left(6.9800 exp\left(-6.4411 i N_\tau^{-2} d\right)\right)$

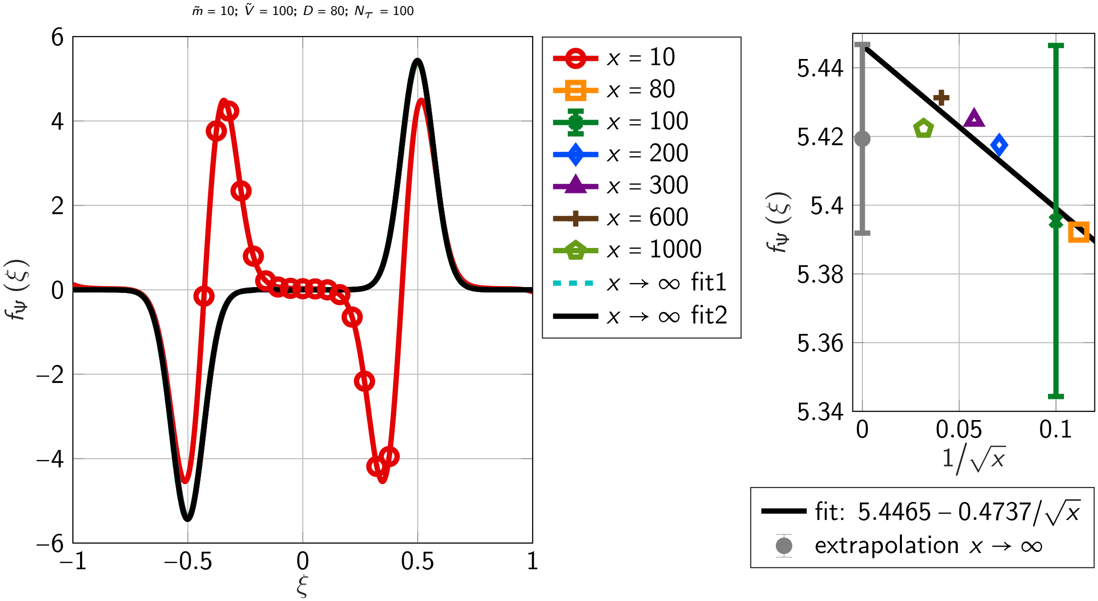
Results: N_τ-dependence – too small



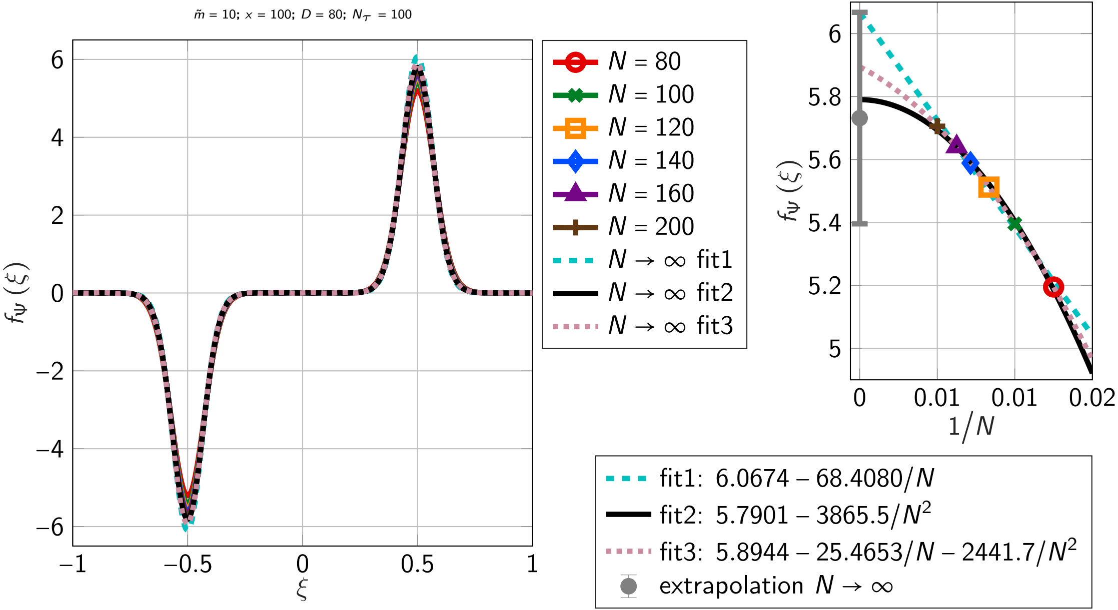
Results: x-dependence



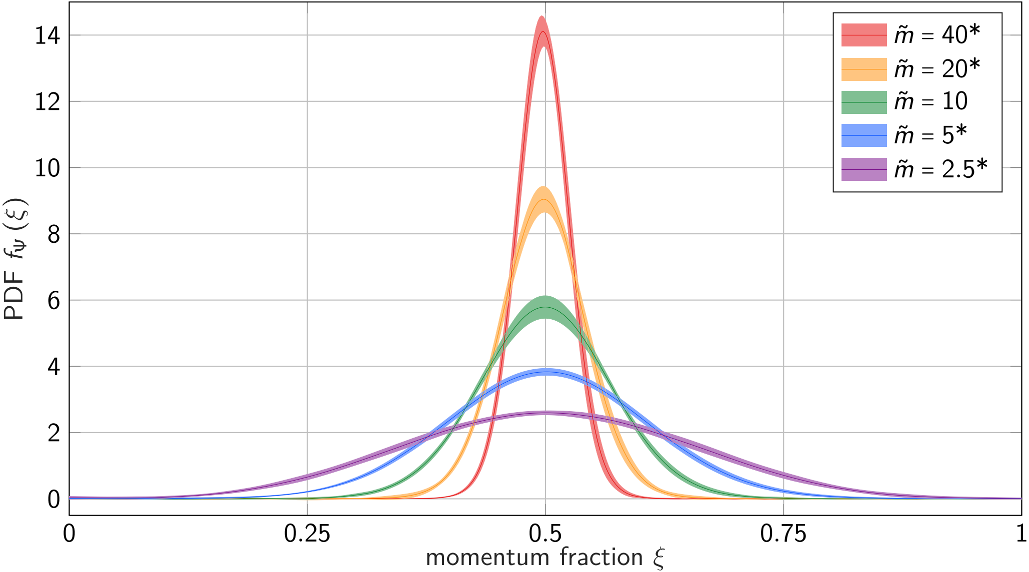
Results: x-dependence – too small



Results: *N*-dependence

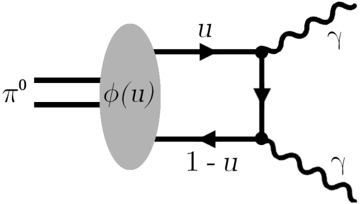


Results: PDF [*preliminary]



Lightcone Distribution Amplitude (LCDA) [preliminary]

LCDA: decay or hadronization

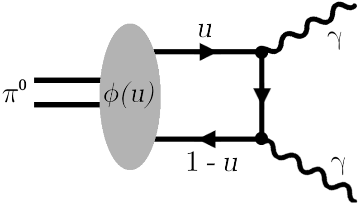


$$\pi^0 \to q(u)\bar{q}(1-u) \to \gamma\gamma$$

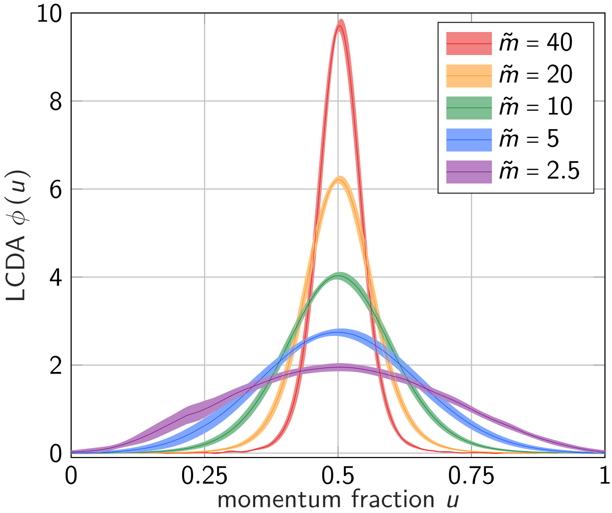
$$\boxed{if\phi(u) = \int dz^- e^{iuP^+z^-} \left\langle 0 \middle| \bar{\psi}(z^-)\gamma^+\gamma_5 W(z^- \leftarrow 0)\psi(0) \middle| P \right\rangle}$$

Lightcone Distribution Amplitude (LCDA) [preliminary]

LCDA: decay or hadronization



$$\pi^0 \to q(u)\bar{q}(1-u) \to \gamma\gamma$$



$$\boxed{if\phi(u) = \int dz^- e^{iuP^+z^-} \left\langle 0 \middle| \bar{\psi}(z^-)\gamma^+\gamma_5 W(z^- \leftarrow 0)\psi(0) \middle| P \right\rangle}$$

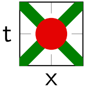
Outline

1. Motivation & Goal: Parton Distribution Functions
2. Method: Tensor Network States
3. Application: Schwinger Model
4. Results
5. Summary & Outlook

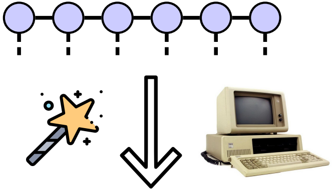
Summary

Summary:

- PDF → universal structure of hadrons
- Euclidean space: lightcone → point



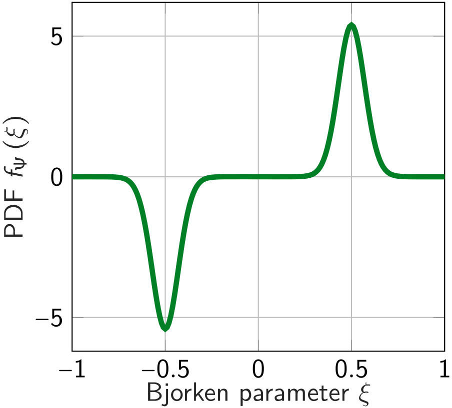
- ⇒ use tensor network states / quantum devices
- Schwinger model:
  fermion- and anti-fermion-PDF for the vector meson



Outlook:

- further lightcone observables
- same analysis for QCD 🙂

[arXiv:2504.07508]
[arXiv:2409.16996]



[1] M. C. Bañuls, K. Cichy, C.-J. D. Lin, and M. Schneider, "Parton distribution functions in the schwinger model from tensor network states," Phys. Rev. D **113**, L011502 (2026) `doi:10.1103/lcyh-rxf3`.

[2] M. Schneider, M. C. Bañuls, K. Cichy, and C.-J. D. Lin, "Parton Distribution Functions in the Schwinger Model with Tensor Networks," in Proceedings of the 41st international symposium on lattice field theory — pos(lattice2024), Vol. 466 (2025), p. 024, `doi:10.22323/1.466.0024`.

[3] M. D. Schwartz, *Quantum Field Theory and the Standard Model*, (Cambridge University Press, Mar. 2014), ISBN: 978-1-107-03473-0, 978-1-107-03473-0, `doi:10.1017/9781139540940`.

[4] M. B. Hastings, "An area law for one-dimensional quantum systems," Journal of Statistical Mechanics: Theory & Exp. **2007**, 08024 (2007) `doi:10.1088/1742-5468/2007/08/P08024`.

[5] M. C. Bañuls, K. Cichy, J. I. Cirac, and K. Jansen, "The mass spectrum of the schwinger model with matrix product states," JHEP **2013**, 158 (2013) `doi:10.1007/JHEP11(2013)158`.

[6] J. Dai, J. Hughes, and J. Liu, "Perturbative analysis of the massless schwinger model," Phys. Rev. D **51**, 5209–5215 (1995) `doi:10.1103/PhysRevD.51.5209`.

[7] C. J. Hamer, Z. Weihong, and J. Oitmaa, "Series expansions for the massive schwinger model in hamiltonian lattice theory," Phys. Rev. D **56**, 55–67 (1997) `doi:10.1103/PhysRevD.56.55`.

[8] Y. Mo and R. J. Perry, "Basis function calculations for the massive schwinger model in the light-front tamm-dancoff approximation," Journal of Computational Physics **108**, 159–174 (1993) `doi:10.1006/jcph.1993.1171`.

[9] Further image sources, EIC, www.computerhistory.org/timeline/1981, https://openmoji.org, www.flaticon.com/free-icons/search.