Parton physics from a heavy-quark operator product expansion

Dynamical lattice QCD calculation of moments of the pion and kaon light-cone distribution amplitudes

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THE HOPE COLLABORATION

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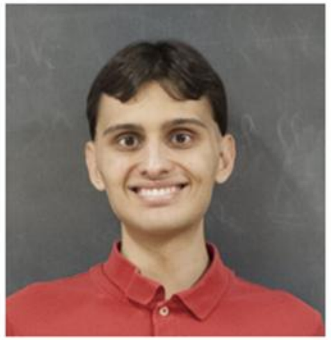

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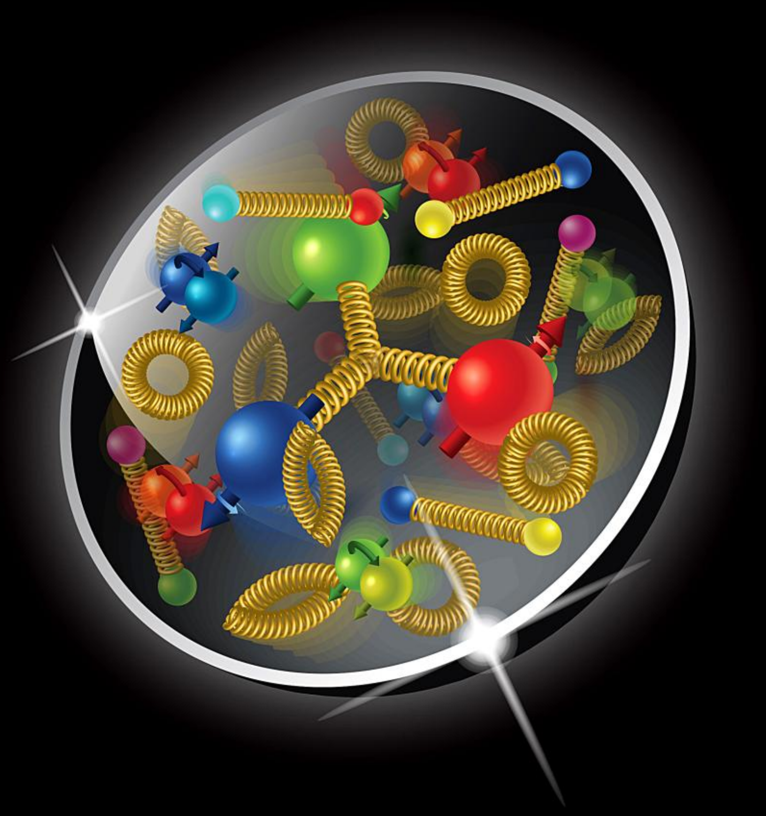


- **More than 99% of the visible mass in the Universe comes from hadrons.**

- **Hadron mass and structure emerge mainly from *strong-interaction* dynamics among quarks and gluons.**

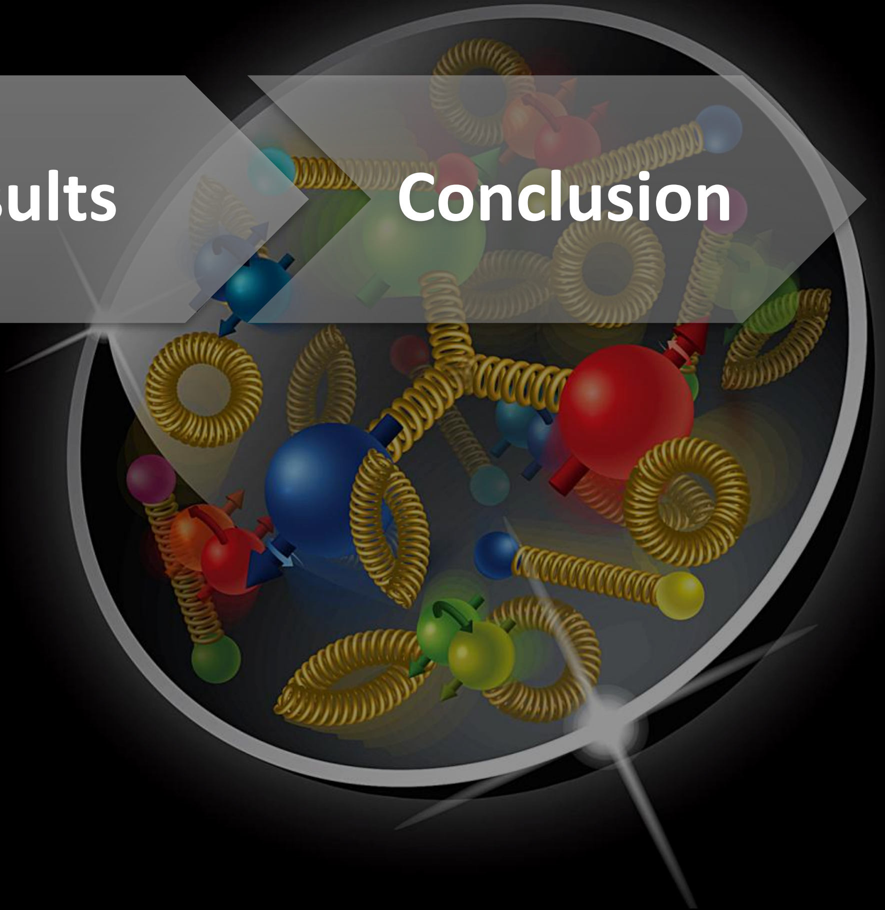
- **The goal of this work is to determine hadron structure (e.g. the moments of Kaon LCDA) from first-principles *lattice QCD*.**

Outline

- Heavy-Quark Operator Product Expansion (HOPE Method)

Introduction → **Method** → **Results** → **Conclusion**

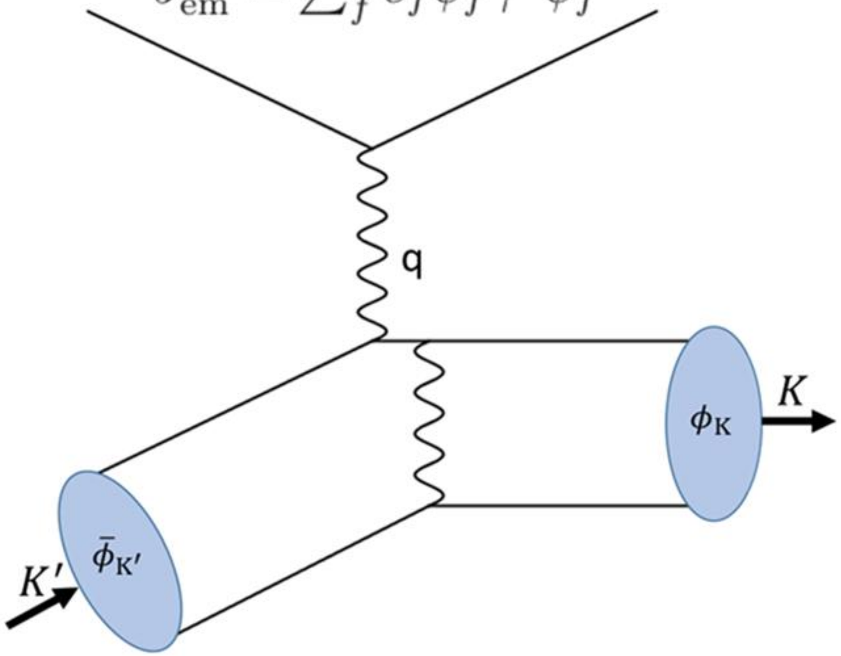
- ◆Light-Cone Distribution Amplitudes
- ◆Light-Cone OPE & Limitations on Lattice



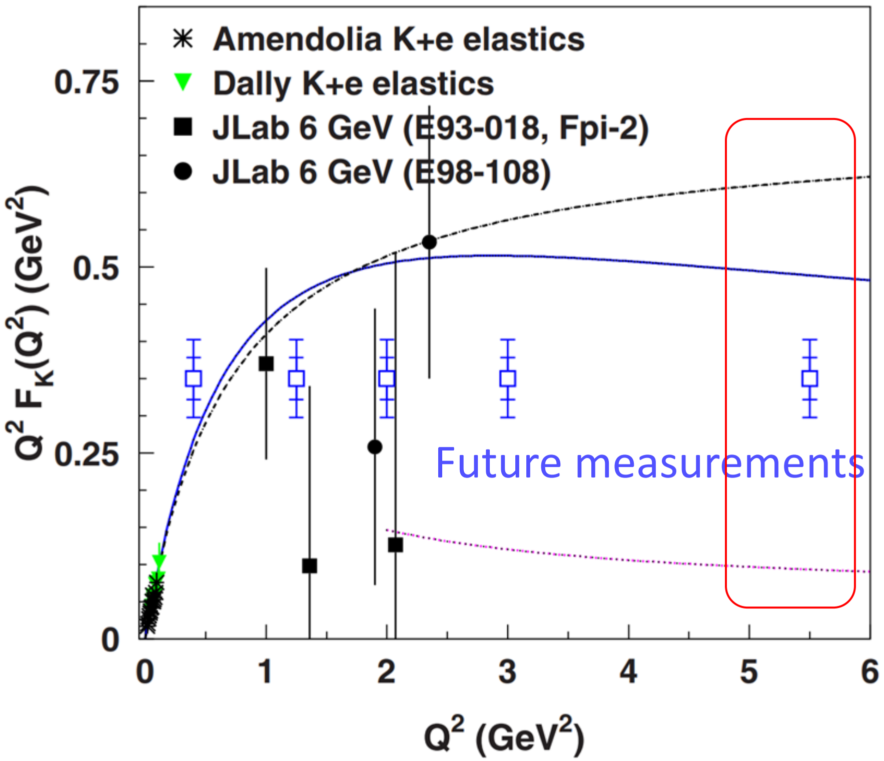
Kaon Electromagnetic Form Factors

$$\langle K^+(P')|J^\mu_{\rm em}|K^+(P)\rangle = F_K(Q^2)(P^\mu + P'^\mu)$$

$$J^\mu_{\rm em} = \sum_f e_f \bar{\psi}_f \gamma^\mu \psi_f$$



elastic electron-kaon scattering



Monopole parameterization fixed from low-Q^2 data

Dyson–Schwinger equation

Leading-twist pQCD with Conformal-limit PDA

Marco Carmignotto PhysRevC.97.025204

- Model-dependent. We still do not fully understand this!!
- High Q^2 experimental data is very hard to get.

Theoretically interesting and experimentally difficult, so nonperturbative theory is essential!!

Kaon Electromagnetic Form Factors

$$\langle K^+(P')|J^\mu_{\rm em}|K^+(P)\rangle = F_K(Q^2)(P^\mu + P'^\mu)$$

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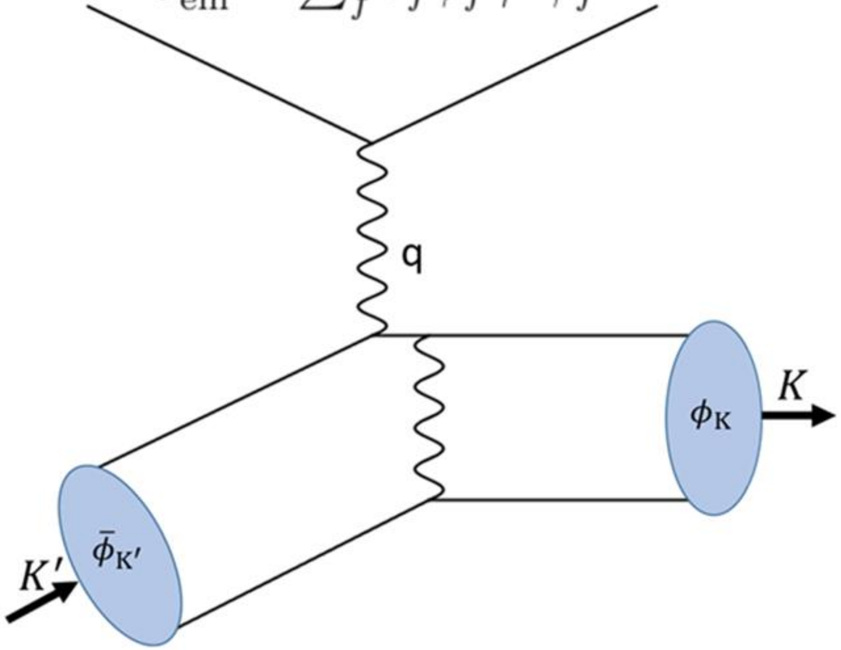
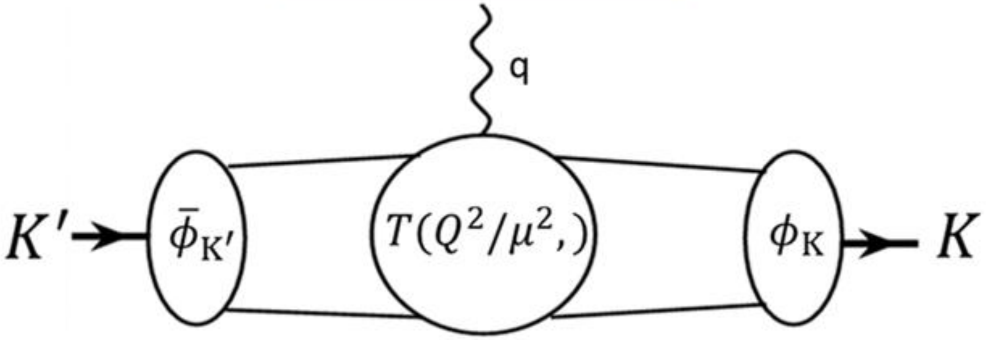


elastic electron-kaon scattering

QCD Factorization

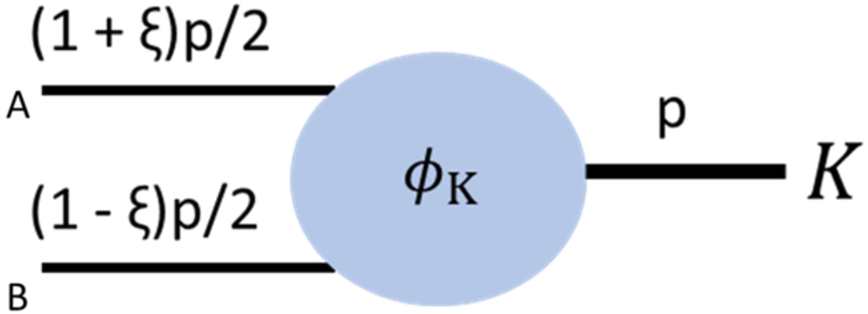


$$F_K(Q^2) = \int dxdy \bar{\phi}_K(x,Q^2) T(x,y,Q^2) \phi_K(y,Q^2)$$

- Hard scattering kernel (T) calculable in perturbative QCD.
- Light-Cone Distribution Amplitudes (ϕ) encoding the non-perturbative structure.

→ Lattice QCD

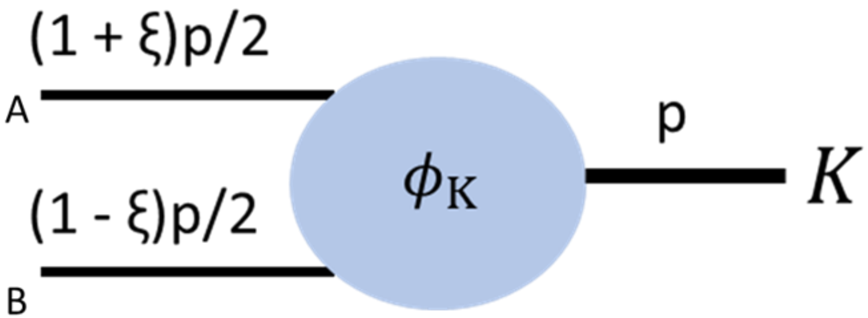
Light-Cone Distribution Amplitudes (LCDAs)



$$\langle\Omega|\, \bar{\psi}_A(z)\gamma_\mu\, \gamma_5\, W[z,-z]\psi_B(-z)|\mathrm{K}^+(p)\rangle$$
$$= i f_{\mathrm{K}}\, p_\mu \int_{-1}^{1} d\xi\, e^{-i\xi\, p\cdot z}\, \phi_{\mathrm{K}}(\xi,\mu^2)$$

- f_K is the pseudoscalar kaon decay constant and W is light-like Wilson line

Light-Cone Distribution Amplitudes (LCDAs)



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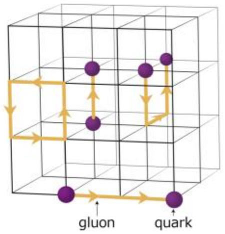
Direct calculation of light-cone objects is impossible on a Euclidean lattice



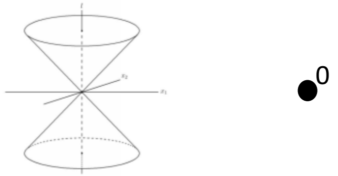
Lattice QCD

✕ Directly

LCDAs

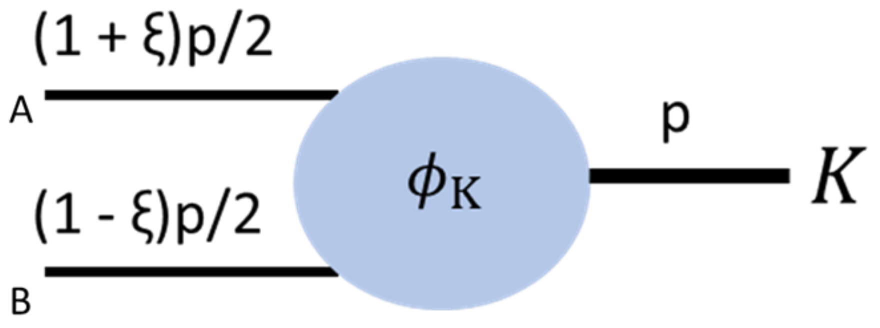
Wick rotation $t \to -i\tau \Rightarrow$ Minkowski $\to$ Euclidean

light cone, defined by $z^2 = 0$



There is no notion of a light-cone in Euclidean space.

Light-Cone Distribution Amplitudes (LCDAs)



$$\langle\Omega|\, \bar{\psi}_A(z)\gamma_\mu\, \gamma_5\, W[z,-z]\psi_B(-z)|\mathrm{K}^+(p)\rangle$$
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- f_K is the pseudoscalar kaon decay constant and W is light-like Wilson line

Light-Cone OPE

$$\bar{\psi}(z)\gamma^\mu\gamma_5 W[z,-z]\psi(-z) = \sum_{n=0}^{\infty} \frac{(-i)^n}{n!} z_{\mu_1}\cdots z_{\mu_n}\, \bar{\psi}(0)\gamma^{\{\mu}\gamma_5(i\overleftrightarrow{D}^{\mu_1})\cdots(i\overleftrightarrow{D}^{\mu_n\}})\psi(0)\Big|_{\text{traceless}}$$

$$O^{\mu_0\mu_1..\mu_n}$$

$$\langle 0|\big[\, \bar{\psi}_A\, \gamma^{\{\mu_0}\gamma_5\, (i\overleftrightarrow{D}^{\mu_1}) \ldots (i\overleftrightarrow{D}^{\mu_n\}})\psi_B \;-\; trace\big]|K^+(p)\rangle$$

local & twist-two

twist = dim – spin

$$= \; f_K\, \langle\xi^n\rangle\, [\, p^{\mu_0}\, p^{\mu_1} \ldots p^{\mu_n} \;-\; trace]$$

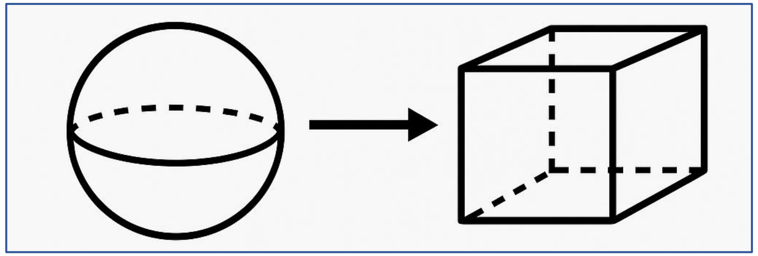
Mellin moments

$$\langle\xi^n\rangle = \int_{-1}^{1} d\xi\; \xi^n \phi_{\mathrm{K}}(\xi,\mu^2)$$

Compute Local matrix elements directly

Limitations of Traditional OPE on Lattice:

The lattice regularization H(4) breaks SO(4)



Continuum Lattice

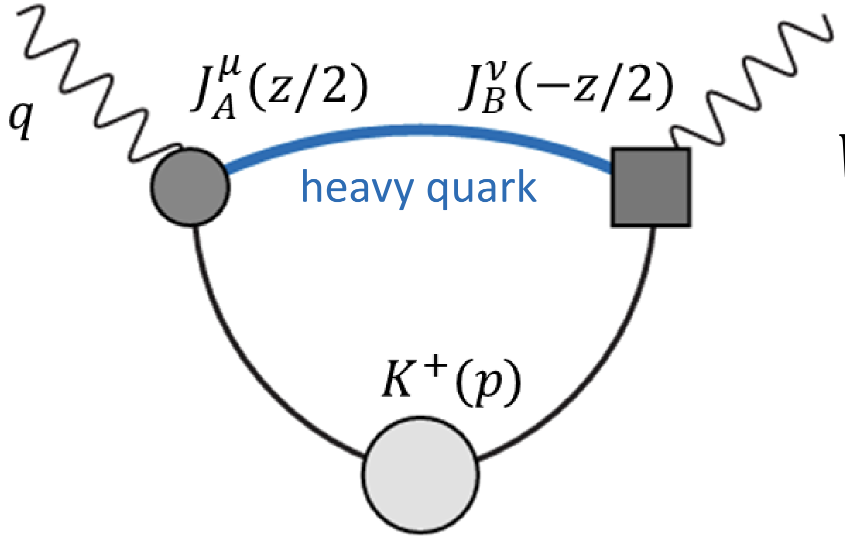
twist = dim – ~~spin~~

For large **Mellin moments (≥second moment)**
mix with lower dimension operators and **the mixing coefficients contain power divergences.**

$$\mathcal{O}_i = \sum_j C_{ij}^{\text{latt}}(a)\mathcal{O}_j^{\text{latt}}$$

→ Traditional OPE infeasible beyond the first few Mellin moments

HOPE Method



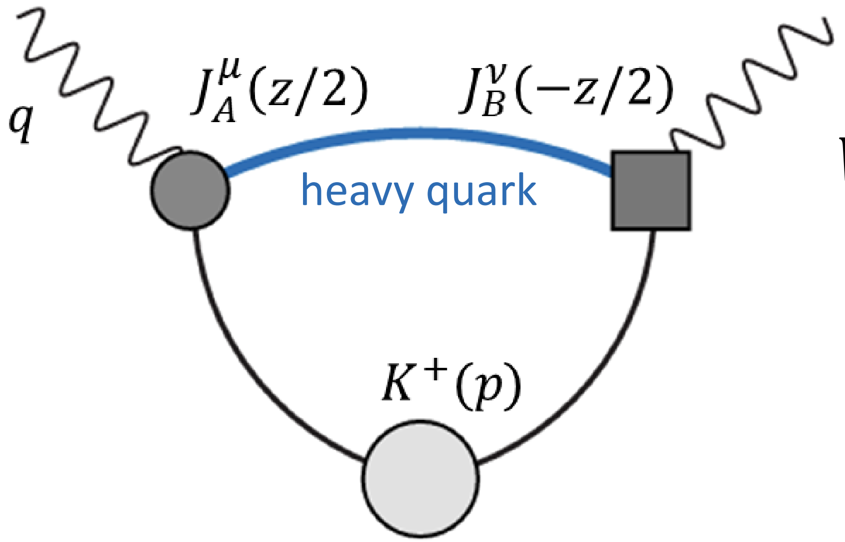
◆ Hadronic Tensor

$$V^{\mu\nu}(\mathrm{p},\mathrm{q}) = \int d^4z\, e^{i\mathrm{q}\cdot z} \langle\Omega|T\{J_A^\mu(z/2) J_B^\nu(-z/2)\}|K^+(p)\rangle$$

$$J_A^\mu(z) = \bar{\Psi}(z)\gamma^\mu\gamma_5\psi_A(z) + \bar{\psi}_A(z)\gamma^\mu\gamma_5\Psi(z)$$

Ψ is the fictitious valence heavy quark

HOPE Method



◆ Hadronic Tensor

$$V^{\mu\nu}(\mathrm{p},\mathrm{q}) = \int d^4z\, e^{i\mathrm{q}\cdot z} \langle \Omega | T\{ J_A^\mu(z/2) J_B^\nu(-z/2)\} | K^+(p)\rangle$$

$$J_A^\mu(z) = \bar{\Psi}(z)\,\gamma^\mu\,\gamma_5\,\psi_A(z) + \bar{\psi}_A(z)\,\gamma^\mu\,\gamma_5\,\Psi(z)$$

Ψ is the fictitious valence heavy quark

$$V^{[\mu\nu]}(\mathrm{q},\mathrm{p}) = \frac{-2\,i\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{q}_\rho\,\mathrm{p}_\sigma}{\tilde{Q}^2} f_K \sum_{n=0}^{\infty} C_W^{(n)}(\tilde{Q}^2, \mu^2, m_\Psi) \langle \xi^n \rangle \left(\frac{\tilde{\omega}}{2}\right)^n$$

LQCD calculations:
extract hadronic tensor

Fitting

QCD perturbation theory:
One-loop Wilson coefficients

Fit parameters:
- $\langle \xi^n \rangle$ – Mellin moments
- f_K – kaon decay constant
- m_Ψ – heavy quark mass

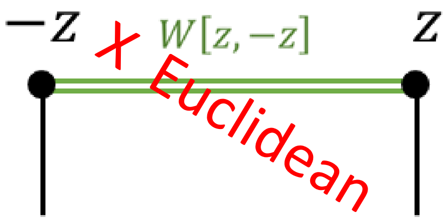
$$\tilde{\omega} = (2\,\mathrm{q}\cdot\mathrm{p})/\tilde{Q}^2$$

$$\tilde{Q}^2 = -q^2 + m_\Psi^2$$

W. Detmold and C.-J. D. Lin, (2006), Phys. Rev. D 73, 014501

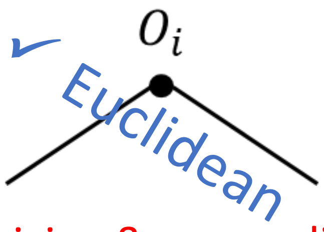
HOPE Method

LCDA



Light-Cone OPE

$$\sum_{i=0}^{\infty} C_W^{(i)}(z)$$



☐ operators mixing & power divergence

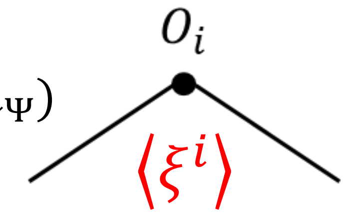
HOPE

computed $V^{\mu\nu}(\mathrm{p},\mathrm{q})$ on the lattice.

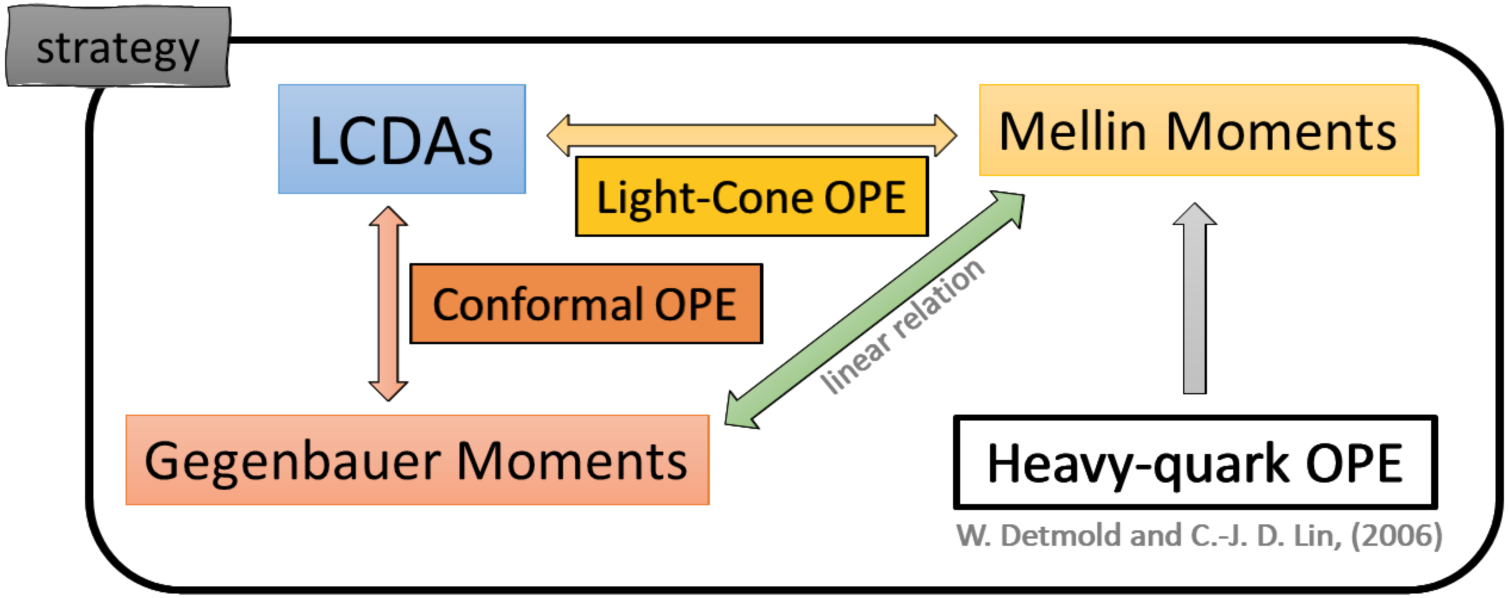
Short-distance & heavy scale

$\Lambda_{\mathrm{QCD}} \ll m_\Psi$

$$\sum_{i=0}^{\infty} C_{HOPE}^{(i)}(z, m_\Psi)$$



HOPE avoids power-divergent lattice operator mixing



LCDA(x)

Quasi-DA / LaMET

Xiangdong Ji,
Phys.Rev.Lett. 110 (2013)

Pseudo-DA

Anatoly Radyushkin,
Phys.Lett. B767 (2017)

Moments

Braun–Müller approach

Two currents separated by space-like distance
V. M. Braun and D. Müller,
Eur. Phys. J. C 55, 349 (2008).

- Reconstructing the full x-dependence is difficult near the end points.
- Moments provide observables that do not require reconstructing the full x-dependence.

Second and Fourth Mellin Moments of the Pion

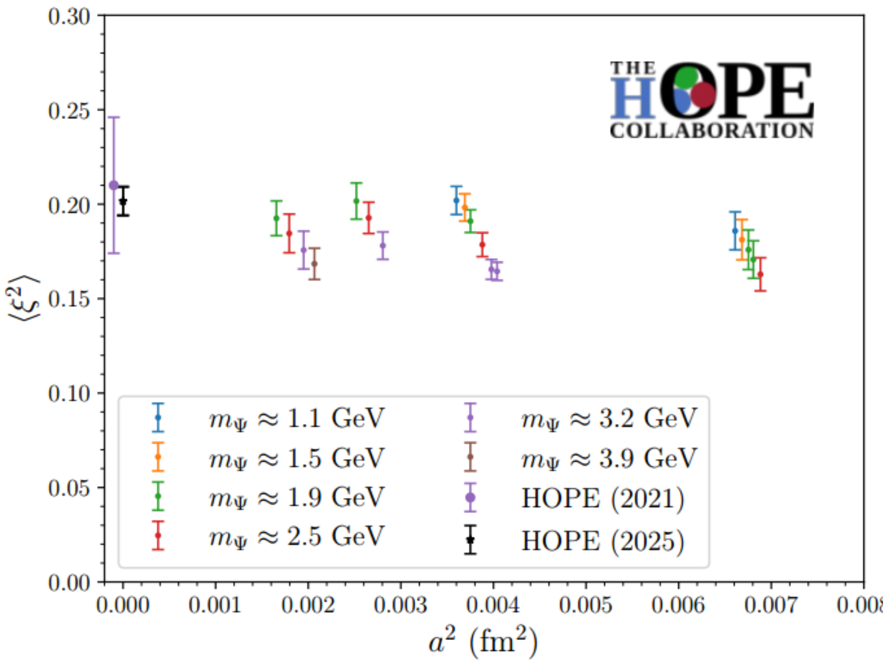
Our recent computation of the 2nd and 4th Mellin moments in the **quenched approximation**.

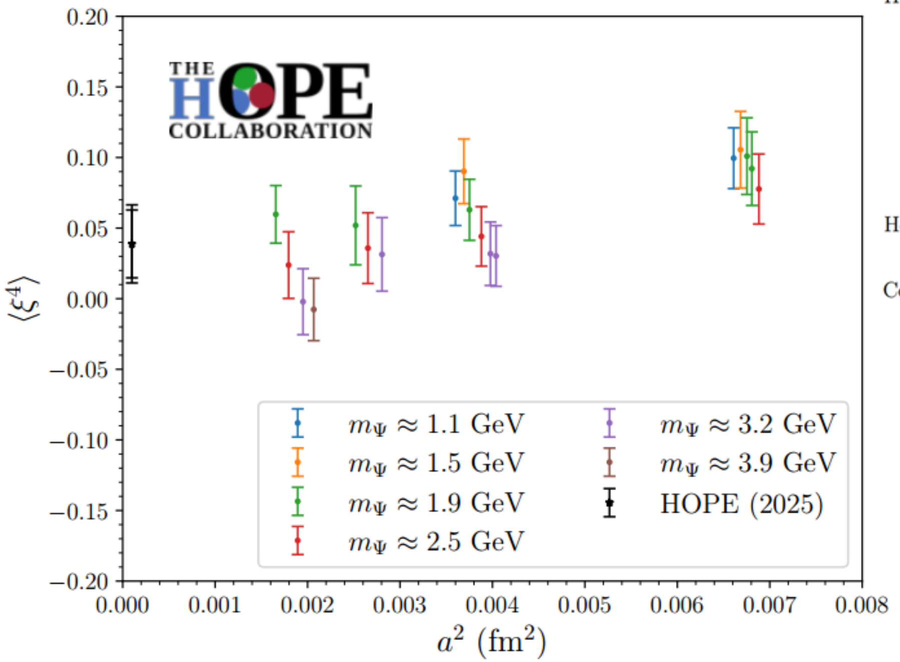
Continuum and Twist-2 Extrapolation

$$\langle \xi^n \rangle (a, m_\Psi) = \langle \xi^n \rangle + \frac{A_n}{m_\Psi} + B_n a^2 + C_n a^2 m_\Psi + D_n a^2 m_\Psi^2$$

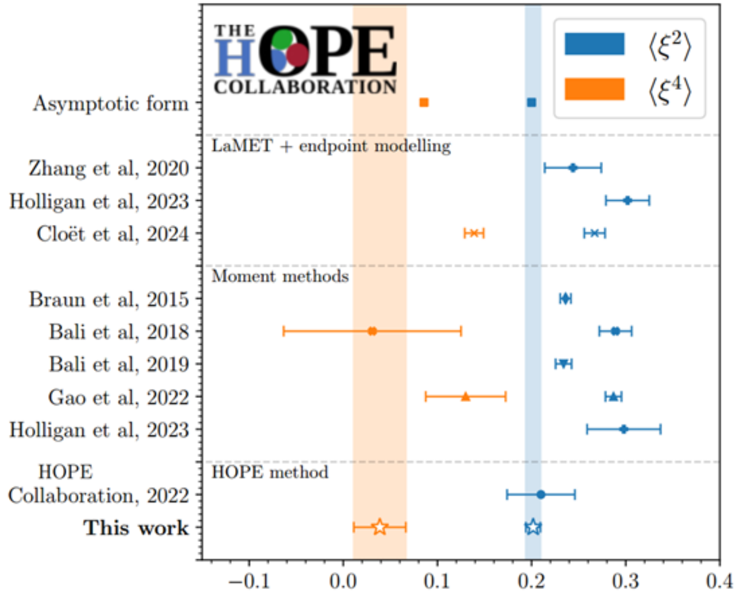
$$\langle \xi^2 \rangle\, (\mu = 2\ \text{GeV}) = 0.202(8)(9),$$

$$\langle \xi^4 \rangle\, (\mu = 2\ \text{GeV}) = 0.039(28)(11).$$





HOPE Collaboration, (2025)



NEXT

Dynamical lattice QCD calculation

HOPE Method for Kaon LCDA

$$V^{[\mu\,\nu]}(\mathrm{q},\mathrm{p}) \;= \frac{-2\,i\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{q}_\rho\,\mathrm{p}_\sigma}{\tilde{Q}^2} f_K \sum_{n=0}^{\infty} C_W^{(n)}(\tilde{Q}^2,\mu^2,m_\Psi)\langle\xi^n\rangle\left(\frac{\tilde{\omega}}{2}\right)^n$$

$$\tilde{\omega} = (2\,\mathrm{q}\cdot p)/\tilde{Q}^2$$
$$\tilde{Q}^2 = -\mathrm{q}^2 + m_\Psi^2$$

◆ Separated by **even** and **odd** Mellin moments

- Antisymmetric (q → −q)

$$V_{even}^{[\mu\,\nu]}(\mathrm{q},\mathrm{p}) = \frac{1}{2}\left(V^{[\mu\,\nu]}(\mathrm{q},\mathrm{p}) - V^{[\mu\,\nu]}(-\mathrm{q},\mathrm{p})\right)$$

– Even Mellin Moments

- Symmetric (q → −q)

$$V_{odd}^{[\mu\,\nu]}(\mathrm{q},\mathrm{p}) = \frac{1}{2}\left(V^{[\mu\,\nu]}(\mathrm{q},\mathrm{p}) + V^{[\mu\,\nu]}(-\mathrm{q},\mathrm{p})\right)$$

– Odd Mellin Moments

HOPE Method for Kaon LCDA

$$V^{[\mu\,\nu]}(\mathrm{q},\mathrm{p}) = \frac{-2\,i\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{q}_\rho\,\mathrm{p}_\sigma}{\tilde{Q}^2} f_K \sum_{n=0}^{\infty} C_W^{(n)}(\tilde{Q}^2,\mu^2,m_\Psi)\langle\xi^n\rangle\left(\frac{\tilde{\omega}}{2}\right)^n$$

$$\tilde{\omega} = (2\,\mathrm{q}\cdot p)/\tilde{Q}^2$$
$$\tilde{Q}^2 = -\mathrm{q}^2 + m_\Psi^2$$

↑ Complex

Special kinematics chosen

$$i\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{q}_\rho\,\mathrm{p}_\sigma = i(\mathrm{q}_0\mathrm{p}_3 - \mathrm{p}_0\mathrm{q}_3) = -\mathrm{q}_4\mathrm{p}_3 - iE_k\mathrm{q}_3$$

chose $p_e(0,0,1)$, $p_m(1/2,0,-1)$ $\rightarrow$ $p_3 = 0$

$$i\,\epsilon^{\mu\nu\rho\sigma}\,\mathrm{q}_\rho\,\mathrm{p}_\sigma = -iE_k\mathrm{q}_3$$ pure imaginary

Fitting parameters in each channel:

	Imag part	Real part
$V_{even}^{[\mu\,\nu]}(\mathbf{q},\mathbf{p})$	f_k 、 m_Ψ	$\langle\xi^2\rangle$ (leading order)
$V_{odd}^{[\mu\,\nu]}(\mathbf{q},\mathbf{p})$	$\langle\xi^1\rangle$ 、 $\langle\xi^3\rangle$	$\langle\xi^1\rangle$ 、 $\langle\xi^3\rangle$

$$\Lambda_{\mathrm{QCD}} \ll m_\Psi \sim \sqrt{q^2} \ll \frac{1}{\hat{a}}$$

$|\tilde{\omega}| < 1$ large p → large $|\tilde{\omega}|$ →sensitivity to higher moments

Fourier transform of HOPE

$$V^{[\mu\,\nu]}(\mathrm{p},\mathrm{q}) = \int_{-\infty}^{\infty} d\tau\, e^{i\tau q_4}\, \mathrm{R}^{[\mu\,\nu]}(\tau; p, q)$$

HOPE formula

Construct ratio from correlators

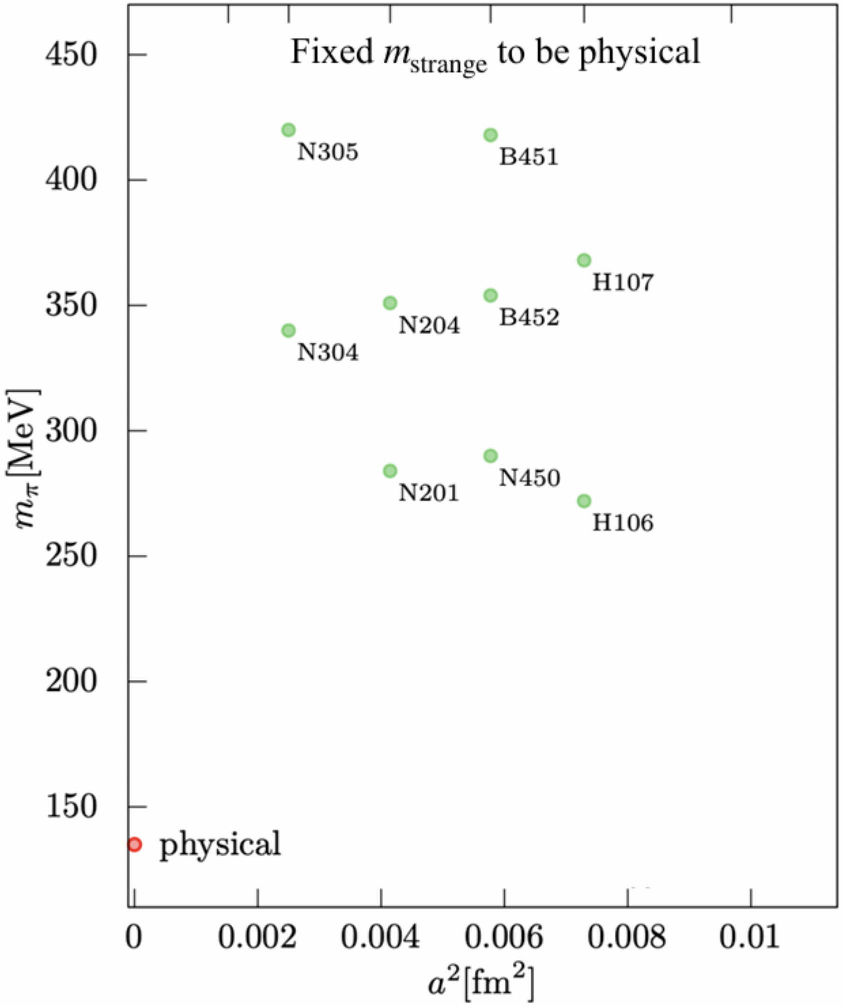
Inverse Fourier transform of HOPE formula

$$R^{[\mu\,\nu]}(\tau, p, q)$$
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} dq_4 e^{-i\tau q_4} \frac{-2\, i\, \epsilon^{\mu\nu\rho\sigma}\, \mathrm{q}_\rho\, \mathrm{p}_\sigma}{\tilde{Q}^2} f_K \sum_{n=0}^{\infty} C_W^{(n)}(\tilde{Q}^2, \mu^2, m_\Psi)\langle \xi^n \rangle \left(\frac{\widetilde{\omega}}{2}\right)^n$$

→ numerical Fourier transform

Resource

the Fugaku project id: hp220312, hp230466.

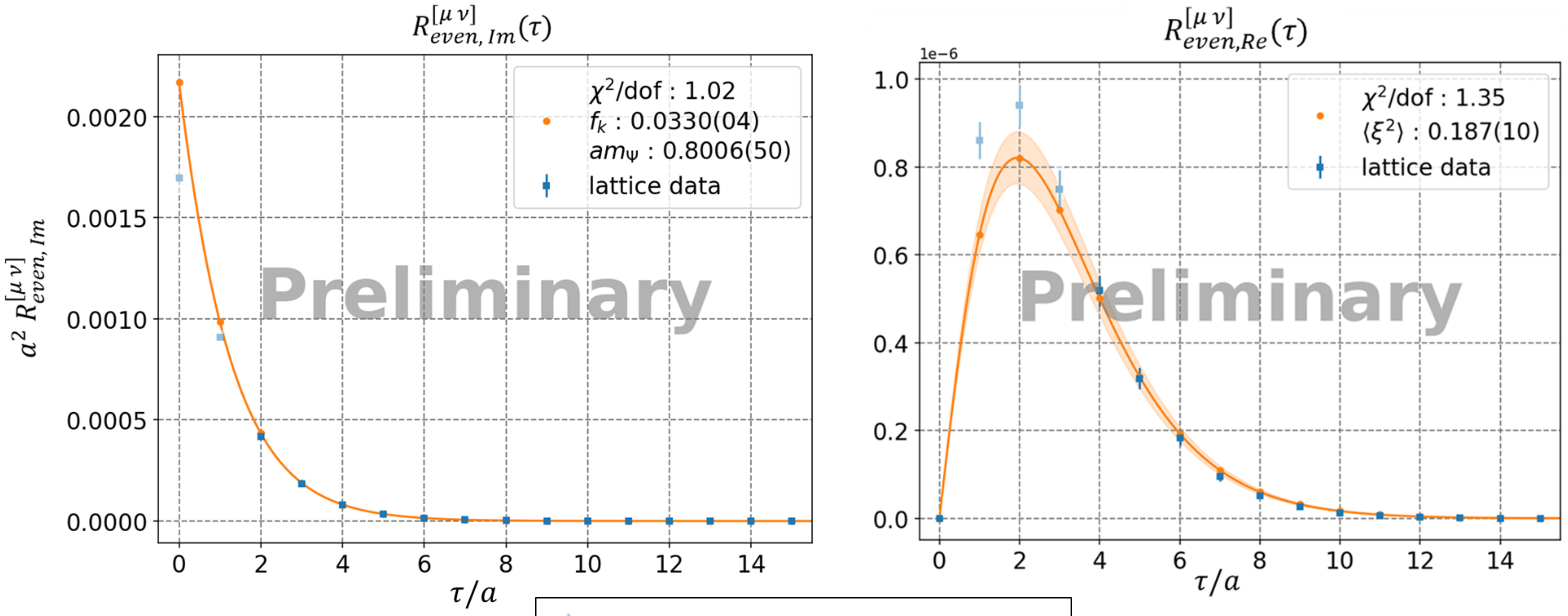


Planned our calculations on these CLS ensembles

Thanks to CLS and R-CCS

Result

$(L/a)^3$ *T/a	(a) (fm)	Configurations Used	Sources/Config	M_π(MeV)	M_k(MeV)
32^3 *64	0.0750	400	16	355.8(1.2)	551.1(0.9)



data show significant lattice artifacts

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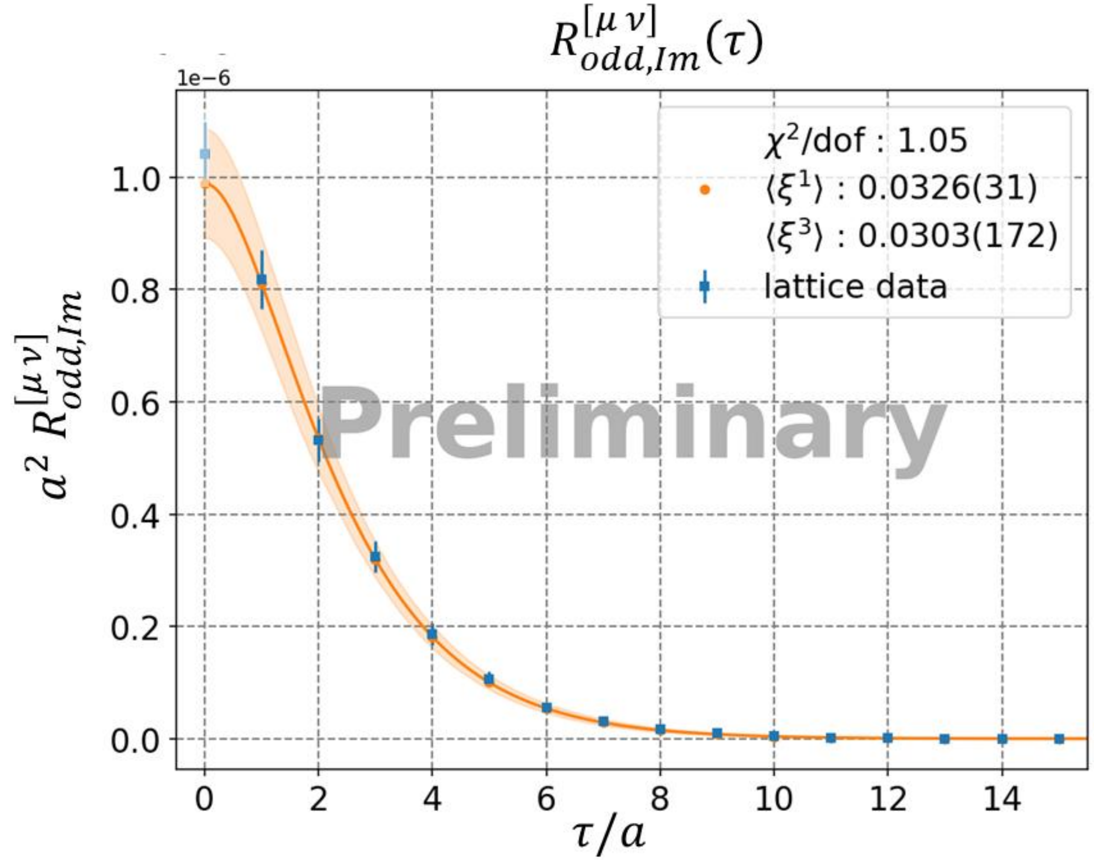
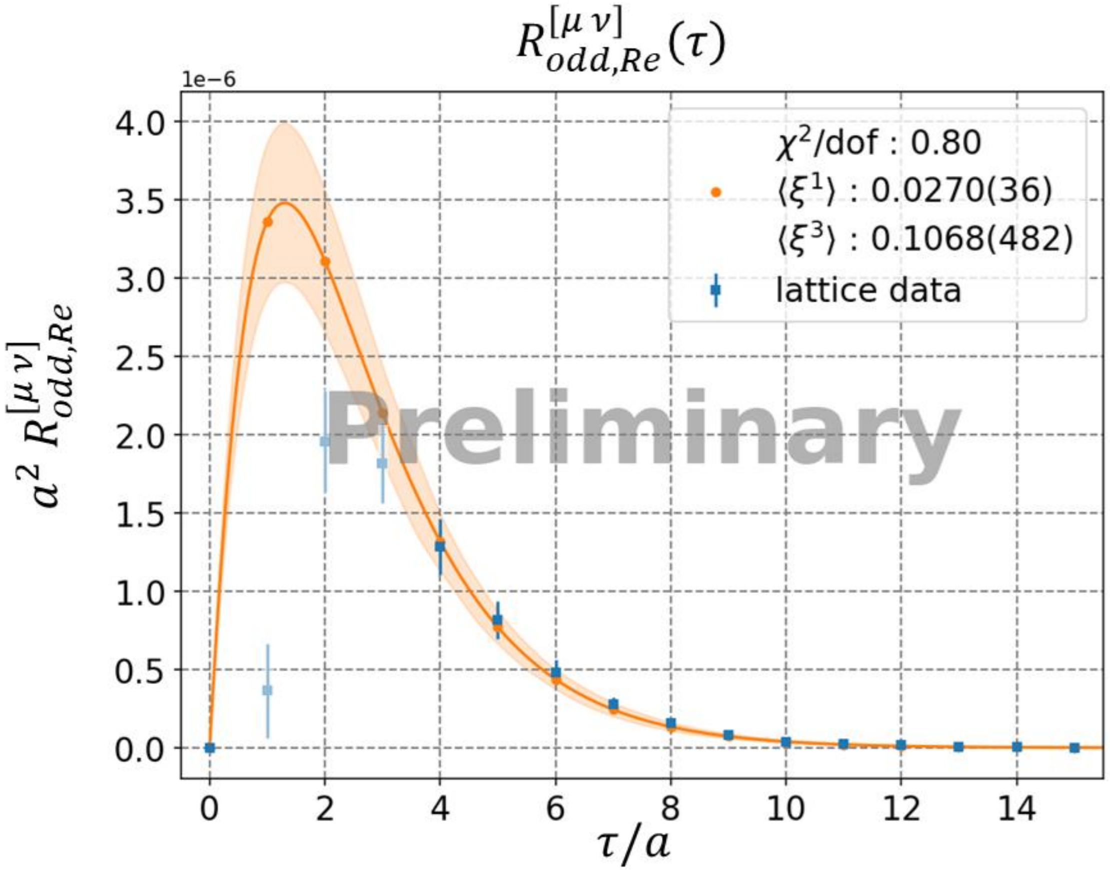




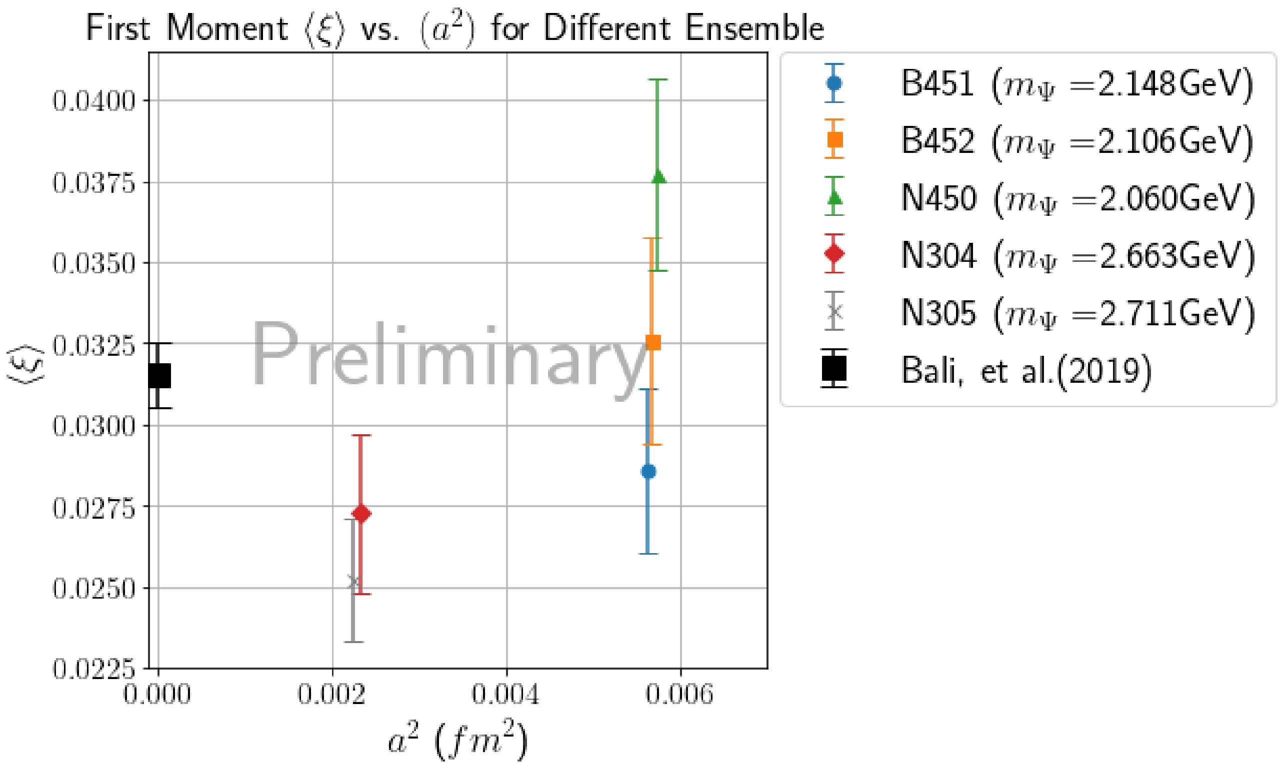
data show significant lattice artifacts

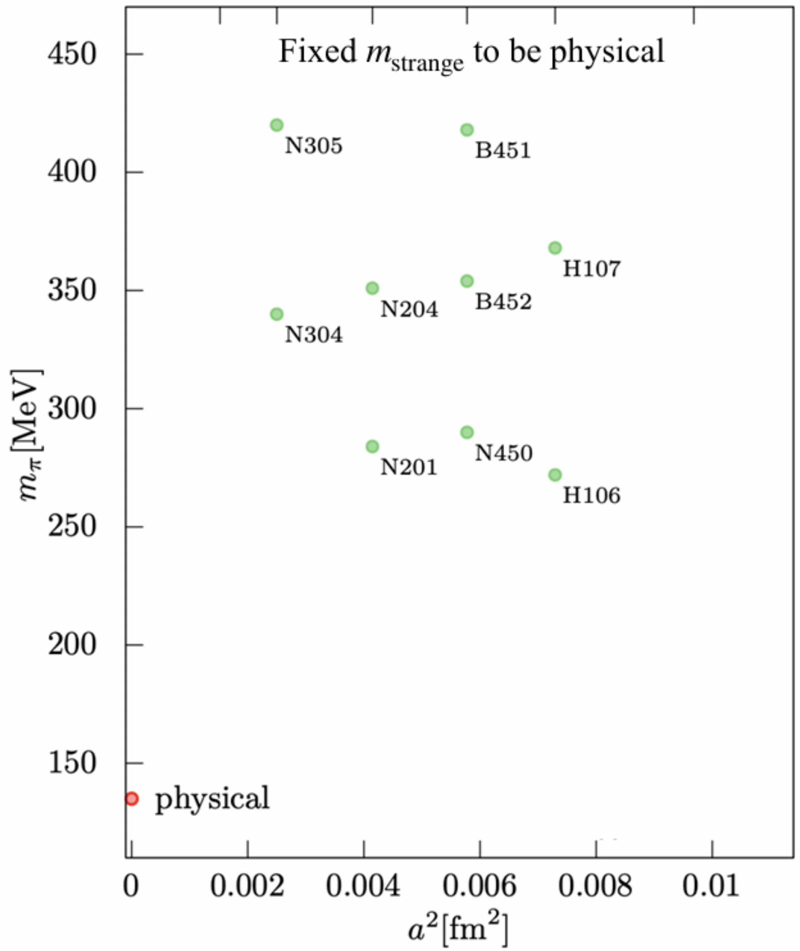
Continuum and Twist-2 Extrapolation

$$\langle \xi^n \rangle(a, m_\Psi) = \langle \xi^n \rangle + \frac{A_n}{m_\Psi} + B_n a^2 + C_n a^2 m_\Psi + D_n a^2 m_\Psi^2$$

where $\langle \xi^n \rangle, A_n, B_n. C_n$ and D_n are the fit parameters

$$\Lambda_{\rm QCD}, m_\pi \ll m_\Psi \lesssim a^{-1}\,.$$





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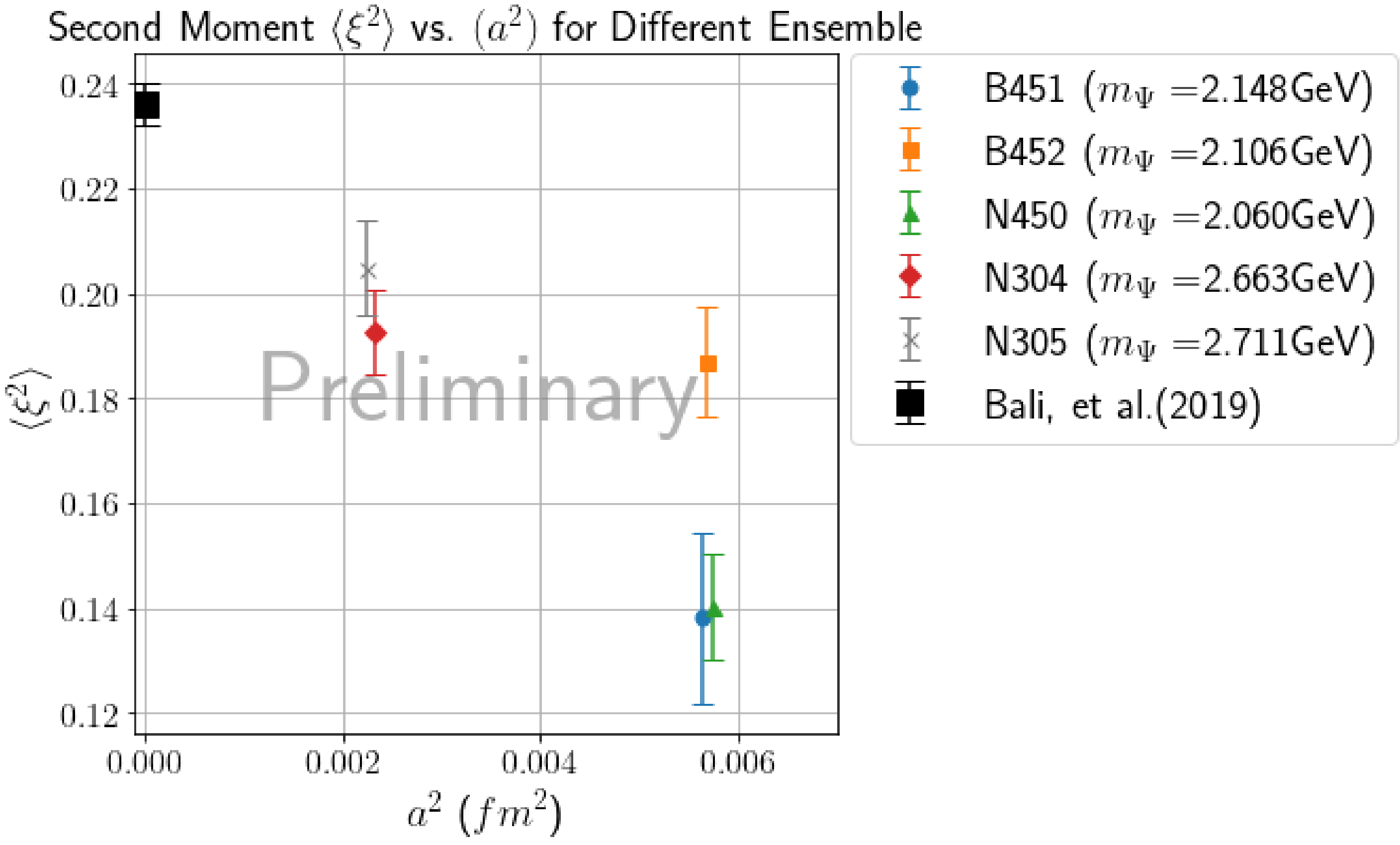
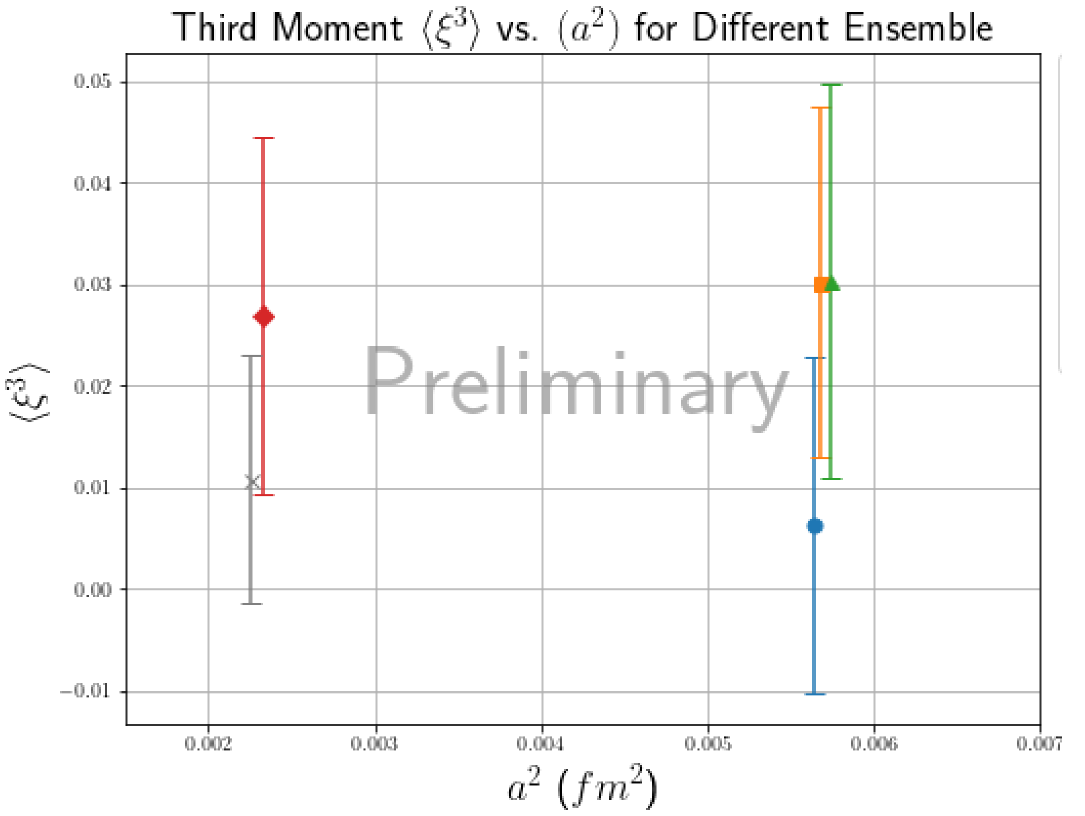
Continuum and Twist-2 Extrapolation





Summary

Summary

- We use the **Heavy-Quark Operator Product Expansion (HOPE)** to access the **moments of kaon LCDAs** from **Lattice QCD** calculations.

✓ Second and Fourth Moments of the Pion **(quenched)**

First, Second and Third Moments of the Kaon **(dynamical)**

Future Work

- Incorporate **more lattice data (different heavy quark mass)** to improve **Continuum and Twist-2 Extrapolation** and **Chiral extrapolation.**

Effect of Moments on the Shape of the LCDA

◆ Gegenbauer OPE:

$$\phi_K(\xi,\mu^2) = \frac{3}{4}(1-\xi^2)\sum_{n=0}^{\infty}\phi_n(\mu^2)\, C_n^{3/2}(\xi)$$

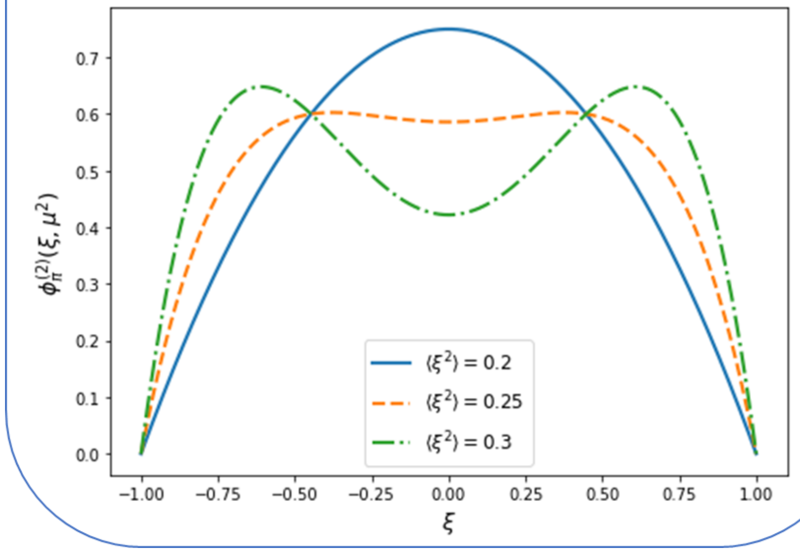
Gegenbauer moments *Gegenbauer polynomials*

$$\phi_0 = \langle\xi^0\rangle = 1, \quad \phi_1 = \frac{5}{3}\langle\xi\rangle, \quad \phi_2 = \frac{7}{12}\left(5\langle\xi^2\rangle - \langle\xi^0\rangle\right)$$

Pion LCDA

$$\phi_\pi^{(2)}(\xi,\mu^2) = \frac{3}{4}(1-\xi^2)\sum_{n=0,even}^{2}\phi_n(\mu^2)\, C_n^{3/2}(\xi)$$

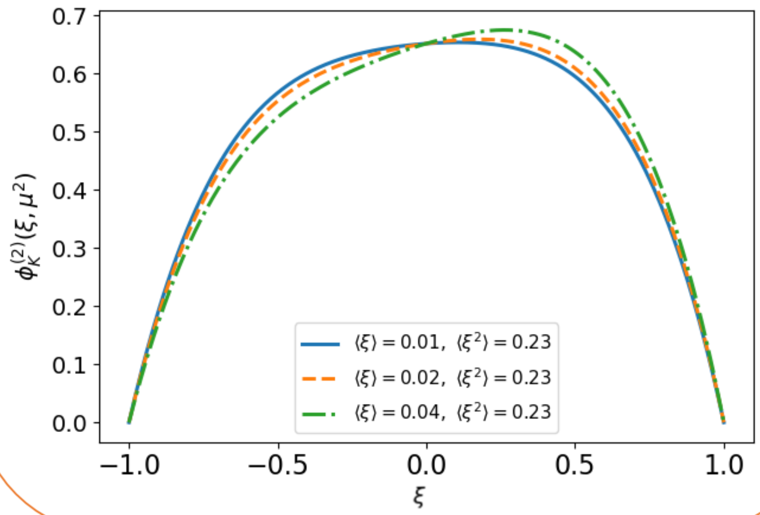
single-humped or double-humped structure



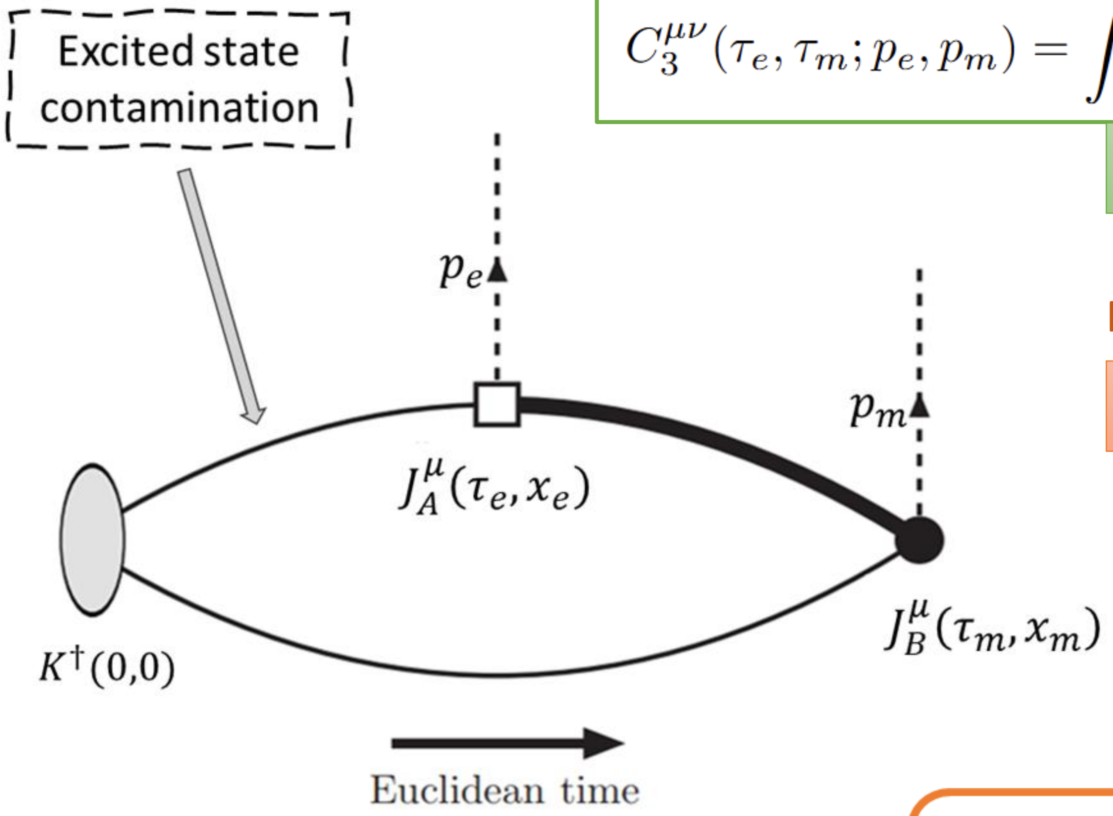
Kaon LCDA

$$\phi_K^{(2)}(\xi,\mu^2) = \frac{3}{4}(1-\xi^2)\sum_{n=0}^{2}\phi_n(\mu^2)\, C_n^{3/2}(\xi)$$

LCDA asymmetry is clearly visible when $\langle\xi\rangle>0$.



HOPE Method



$$p = p_e + p_m$$
$$q = (q_e - q_m)/2$$
$$\tau = \tau_e - \tau_m$$

3pt-function

$$C_3^{\mu\nu}(\tau_e, \tau_m; p_e, p_m) = \int d^3x_e\, d^3x_m\, e^{ip_e\cdot x_e} e^{ip_m\cdot x_m} \langle 0|\mathcal{T}\left[J_A^\mu(\tau_e, x_e) J_B^\nu(\tau_m, x_m) \mathcal{O}_K^\dagger(0)\right]|0\rangle$$

$$\langle 0|T\{J_A^\mu(\tau_e, p_e) J_B^\nu(\tau_m, p_m) K^+(0,p)\}|0\rangle$$

Fourier transform of Hadronic Tensor

$$R^{\mu\nu}(\tau, p, q) = \langle 0|T\{J_A^\mu(\tau_e, p_e) J_B^\nu(\tau_m, p_m)\}|K^+(p)\rangle$$

2pt-function

$$C_2(\tau, p) = \int d^3x\, e^{ip\cdot x} \langle 0|\mathcal{O}_K(\tau, x)\, \mathcal{O}_K^\dagger(0,0)|0\rangle$$

$$R^{\mu\nu}(\tau; p, q) \sim C_3^{\mu\nu}(\tau_e, \tau_m; p_e, p_m) \frac{2\, E_K(p)}{Z_K(p)} e^{-E_K(p)(\tau_e+\tau_m)/2}$$

$$G_3^{\mu\nu}(x,y) = \langle \Omega | \mathcal{T}\{ J_A^\mu(x) J_A^\nu(y) \mathcal{O}_\pi^\dagger(0) \} | \Omega \rangle$$

$$J_A^\mu = Z_A^{(0)}(1+\tilde{b}_A a \tilde{m}_{ij}) \left[\overline{\psi}\gamma_\mu\gamma_5\Psi + ac_A\partial_\mu\overline{\psi}\gamma_5\Psi - a\frac{c'_A}{4}\left(\overline{\psi}\gamma_\mu\gamma_5(\overrightarrow{D}+m_\Psi)\Psi - \overline{\psi}(\overleftarrow{D}+m_\psi)\gamma_\mu\gamma_5\Psi\right) + (\psi \leftrightarrow \Psi)\right]$$

$$\begin{aligned} G_3^{\mu\nu}(x,y) = Z^2(a) Z_\pi \langle \Omega | T \Big\{ \Big(& \overline{\Psi}^{(0)}(x)\gamma^\mu\gamma^5\psi^{(0)}(x) + \overline{\psi}^{(0)}(x)\gamma^\mu\gamma^5\Psi^{(0)}(x) \\ & + ac_A\partial^\mu\left\{\overline{\Psi}^{(0)}(x)\gamma^5\psi^{(0)}(x)\right\} + ac_A\partial^\mu\left\{\overline{\psi}^{(0)}(x)\gamma^5\Psi^{(0)}(x)\right\}\Big) \\ & \times \Big(\overline{\Psi}^{(0)}(y)\gamma^\nu\gamma^5\psi^{(0)}(y) + \overline{\psi}^{(0)}(y)\gamma^\nu\gamma^5\Psi^{(0)}(y) \\ & + ac_A\partial^\nu\left\{\overline{\Psi}^{(0)}(y)\gamma^5\psi^{(0)}(y)\right\} + ac_A\partial^\nu\left\{\overline{\psi}^{(0)}(y)\gamma^5\Psi^{(0)}(y)\right\}\Big) \\ & \times [\overline{\psi}^{(0)}(0)\gamma^5\psi^{(0)}(0)]^\dagger \Big\} |\Omega\rangle + \mathcal{O}(a^2)\,, \end{aligned}$$

$$ac_A\, ip_2^\nu \int d^4x\, d^4y\, e^{-ip_1\cdot x} e^{-ip_2\cdot y} \langle \Omega | T\{ (\bar{\Psi}\gamma^\mu\gamma^5\psi)(x)(\bar{\psi}\gamma^5\Psi)(y)[\bar{\psi}\gamma^5\psi(0)]^\dagger\} |\Omega\rangle.$$

$$\begin{aligned} \frac{ac'_A}{4} \sum_{\mathbf{x}_e,\mathbf{x}_m,x} & \left[D_u^{-1}(0|x_m)_{da}^{\eta\alpha}\right]\left[D_d^{-1}(x_e|0)_{cd}^{\varepsilon\zeta}\right]\left[D_\Psi^{-1}(x|x_e)_{bc}^{\gamma\delta}\right]\left[D_\Psi(x_m|x)_{ab}^{\beta\gamma}\right] \\ & \left(\gamma^\nu\gamma^5\right)^{\alpha\beta}\left(\gamma^\mu\gamma^5\right)^{\delta\varepsilon}\left(\gamma^5\right)^{\zeta\eta} e^{i\mathbf{p}_e\cdot\mathbf{x}_e} e^{i\mathbf{p}_m\cdot\mathbf{x}_m}\,. \end{aligned}$$

$$\sum_x D_\Psi(x_m|x)_{ab}^{\beta\gamma} D_\Psi^{-1}(x|x_e)_{bc}^{\gamma\delta} = \delta^{(4)}(x_m - x_e)\delta^{\beta\delta}\delta_{ac}$$

$$|\pi(\mathbf{p})\rangle \to |u(\frac{1}{2}(1+x_0)p,\uparrow)\bar{d}(\frac{1}{2}(1-x_0)p,\downarrow)\rangle$$

$$V_q^{\mu\nu(0)}(q,p) = \bar{v}(\frac{1}{2}(1-x_0)p,\downarrow)\left[\gamma^\mu\gamma_5\frac{i}{\not{q}+\frac{x_0\not{p}}{2}-m_\Psi}\gamma^\nu\gamma_5+\gamma^\nu\gamma_5\frac{i}{-\not{q}+\frac{x_0\not{p}}{2}-m_\Psi}\gamma^\mu\gamma_5\right]u(\frac{1}{2}(1+x_0)p,\uparrow)$$

$$\begin{aligned} V_q^{[\mu\nu](0)}(q,p,m_\Psi,x_0) &= \epsilon^{\mu\nu\rho\sigma}q_\rho\bar{v}\gamma_\sigma\gamma_5 u\left[\frac{1}{q^2+x_0 p\cdot q-m_\Psi^2}+\frac{1}{q^2-x_0 p\cdot q-m_\Psi^2}\right] \\ &= -\frac{2\epsilon^{\mu\nu\rho\sigma}q_\rho}{\tilde{Q}^2}\bar{v}\gamma_\sigma\gamma_5 u\sum_{\substack{n=0,\\ \text{even}}}^{\infty}\mathcal{F}_n^{(0)}(\tilde{\omega})\phi_n^{(0)}(x_0)\,, \end{aligned}$$

$$\mathcal{F}_n^{(0)}(\tilde{\omega}) = \frac{3}{2}\frac{\sqrt{\pi}(n+1)(n+2)n!}{2^{n+2}\Gamma\left(n+\frac{5}{2}\right)}\left(\frac{\tilde{\omega}}{2}\right)^n {}_2F_1\left(\frac{n+1}{2},\frac{n+2}{2};n+\frac{5}{2};\frac{\tilde{\omega}^2}{4}\right).$$

$$\mathcal{F}_n(\tilde{Q}^2,\mu^2,\tilde{\omega},\tau) = \mathcal{F}_n^{(0)}(\tilde{\omega})\left[1+\frac{\alpha_s C_F}{4\pi}\gamma_n^{(0)}\ln\frac{\mu^2}{\tilde{Q}^2}\right]+\frac{\alpha_s C_F}{4\pi}\mathcal{R}_n^{(1)}(\tilde{Q}^2,\mu^2,\tau,\tilde{\omega})+\mathcal{O}(\alpha_s^2) \qquad \phi_n(x_0,\epsilon) = \phi_n^{(0)}(x_0)\left[1+\frac{\alpha_s C_F}{4\pi}\frac{\gamma_n^{(0)}}{\epsilon'}+\mathcal{O}(\alpha_s^2)\right]$$

Beyond leading logarithm, the conformal symmetry is broken, so the Gegenbauer moments start to mix.