

# Thermalization of $SU(2)$ Lattice Gauge Fields on Quantum Computers

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# Thermalization: The Puzzle

- ☞ **Thermalization:** Macroscopic observables become **time-independent**
- ☞ **Classical systems:** Thermalization arises from **collisions and randomness**.
- ☞ **Quantum systems:** Time evolution is **unitary** and **reversible**  $\Rightarrow$  information is preserved (entropy is zero)
- ☞ **Key question:** How can an **isolated quantum system** appear thermal?
- ☞ **Example (Heavy-ion collisions):** Single-hadron energy spectra look **thermal** at very early times

# Eigenstate Thermalization Hypothesis (ETH)

- 👉 **Answer:** Thermal behavior emerges from **entanglement growth**.
- 👉 **Eigenstate Thermalization Hypothesis (ETH):** Local observables in energy eigenstates match thermal values.

$$\langle E_n | \mathcal{O}_{local} | E_n \rangle \approx \mathcal{O}_{thermal}(E)$$

## 👉 **Physical picture:**

- Total system: **pure quantum state**
- Subsystem: appears **thermal** due to entanglement
- Local observables  $\rightarrow$  thermal values

👉 **Implication:** Thermalization does **not** require classical collisions or true equilibrium.

👉 **Motivation:** Real-time dynamics of non-Abelian gauge theories are **hard for classical computation**.

👉 **This work:** Growth of Rényi-2 entropy  $\Rightarrow$  probe of thermalization

# 2+1D square plaquette chain in SU(2) LGT

- 👉 Study Rényi-2 entropy of 2+1D chain of square plaquettes in SU(2) lattice gauge theory
- 👉 Each plaquette forms a **closed loop of gauge links**
- 👉 Each link characterized by SU(2) representation (**truncated Hilbert Space**):

$$j = 0, \frac{1}{2} \quad (j_{\max} = \frac{1}{2})$$

- 👉 **Spin mapping:**

$$j = 0 \rightarrow |\downarrow\rangle, \quad j = \frac{1}{2} \rightarrow |\uparrow\rangle$$

- 👉 **Plaquettes chain  $\Rightarrow$  Effective 1D spin chain**



# Hamiltonian of 1D spin chain

☞ Hamiltonian:

$$\mathcal{H}_{\text{sc}} = J \sum_{i=0}^{N-2} Z_i Z_{i+1} + h_z \sum_{i=0}^{N-1} Z_i + \frac{h_x}{16} \sum_{i=0}^{N-1} X_i - \frac{3h_x}{16} \sum_{i=0}^{N-2} [Z_i X_{i+1} + X_i Z_{i+1} - 3Z_i X_{i+1} Z_{i+2}], \quad (1)$$

$$\mathcal{H}_{\text{sc}} \equiv H_{ZZ} + H_Z + H_X + H_{ZX} + H_{XZ} + H_{ZXZ}. \quad (2)$$

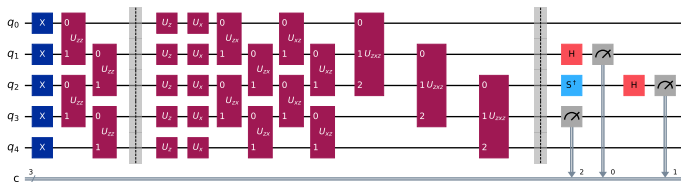
☞ We begin with strong-coupling vacuum state

$$|\psi(t=0)\rangle = |\downarrow\downarrow\downarrow\downarrow\rangle \equiv |11111\rangle \quad (3)$$

☞ Pure state: No entanglement at beginning.

$$|\psi(t)\rangle = \left( \prod_{n=1}^{t/\delta t} e^{-i \mathcal{H}_{\text{sc}} \delta t} \right) |11111\rangle \quad (4)$$

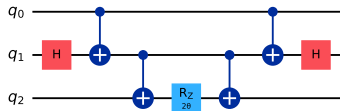
# Quantum Circuit for Time Evolution



$U_Z = R_Z(\theta, i)$ , and  
 $U_X = R_X(\theta, i)$ ,

$U_{ZZ} = R_{ZZ}(\theta, i, i + 1)$ ,  
 $U_{ZX} = R_{ZX}(\theta, i, i + 1)$ , and  
 $U_{XZ} = R_{ZX}(\theta, i + 1, i)$

$U_{ZXZ} = e^{-i\theta Z_i X_{i+1} Z_{i+2}}$



# Reduced Density Matrix and Rényi-2 Entropy of a Subsystem

☞ Reduced density matrix of a subsystem  $A$  of size  $k$ :

$$\rho_A = \frac{1}{2^k} \sum_{P_A \in \{I, X, Y, Z\}^k} \langle \psi(t) | P_A \otimes I_{\bar{A}} | \psi(t) \rangle P_A, \quad (5)$$

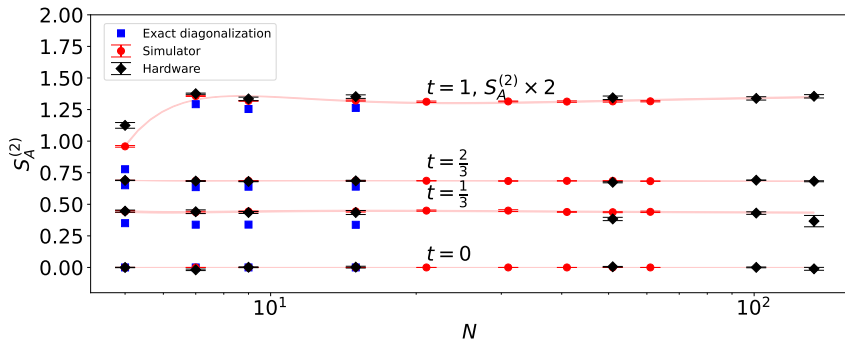
☞ The Rényi-2 entropy:

$$S_A^2 = -\log(\text{Tr}[\rho_A^2]) = -\log\left(\frac{1}{2^k} \sum_{P_A \in \{I, X, Y, Z\}^k} \langle \psi(t) | P_A | \psi(t) \rangle^2\right), \quad (6)$$

☞  $\langle P_A \rangle$  by changing measurement axis as  $X \rightarrow Z = HXH$  and  $Y \rightarrow Z = HS^\dagger YSH$ ,

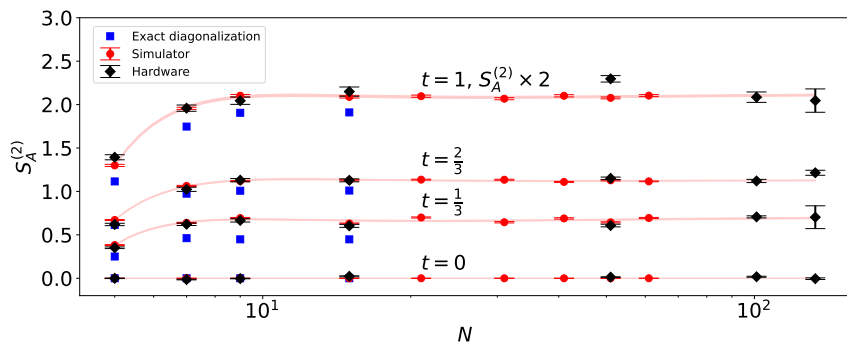
☞ The Rényi-2 entropy is obtained for subsystem size  $N_A = 1, 2$ , and  $3$  for various system size  $N$

# Results For $N_A = 1$



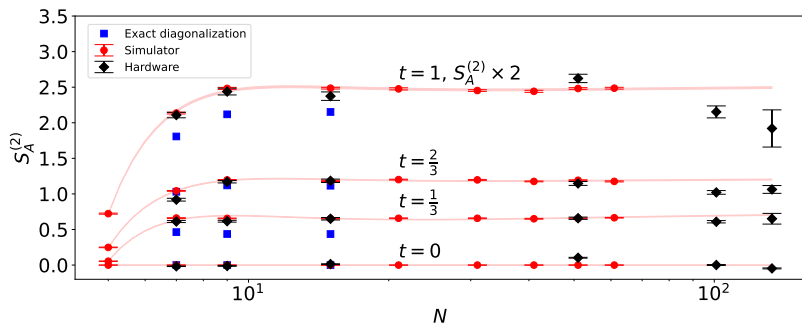
- 👉 Classical simulation (20K shots) and quantum hardware (4K–8K)
- 👉 Classical data fitted with  $S_A^{(2)} = a + \frac{b}{N} + \frac{c}{N^2} + \frac{d}{N^3}$
- 👉 Good agreement between quantum hardware and classical results

# Result For $N_A = 2$



👉 Similar behavior observed as in  $N_A = 1$ .

# Result For $N_A = 3$



- 👉  $N \geq 101$ : deviate from classical simulation.
- 👉  $N = 133$ : **Larger error bars**, reason is **break down of linear connectivity** between nearest neighbors  $\rightarrow$  larger 2-qubits gate depth
- 👉 **Error mitigation** techniques used:
  - 1. Dynamical Decoupling (**DD**) 2. Pauli Twirling (**PT**) 3. Operator Decoherence Renormalization (**ODR**)

# Qubits Layout of ibm\_aachen

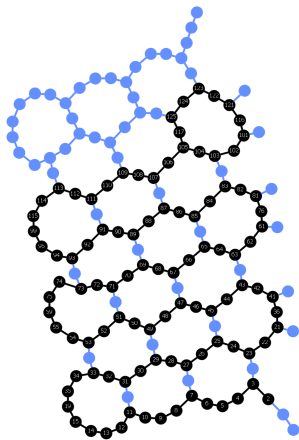


Figure:  $N=101$

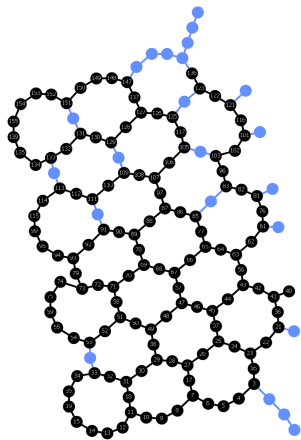


Figure:  $N=133$

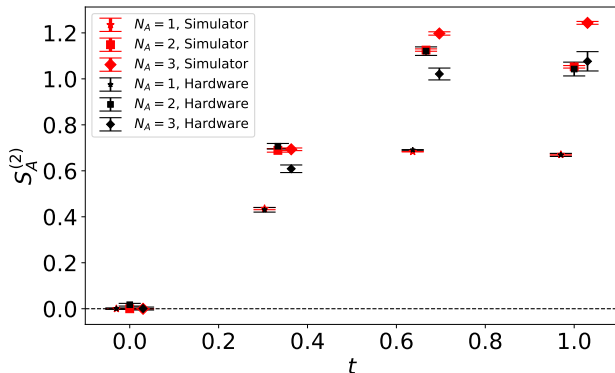
👉 Breakdown of linear connectivity between the nearest qubits for  $N = 133$

# Summary and Next Goal

- Studied Thermalization via time evolution of Rényi-2 entropy.
  - The thermalization begin to emerge for  $t \geq 2/3$ .
  - Good agreement between **classical simulation** and **quantum hardware**.
  - Statistical noise increases for larger subsystems ( $N_A \geq 3$ )  $\Rightarrow$  requires more measurement shots.
- 👉 **Next goal:** Compute  $S_A^{(2)}$  for larger subsystems ( $N_A = 8$ ) using **classical shadow techniques**

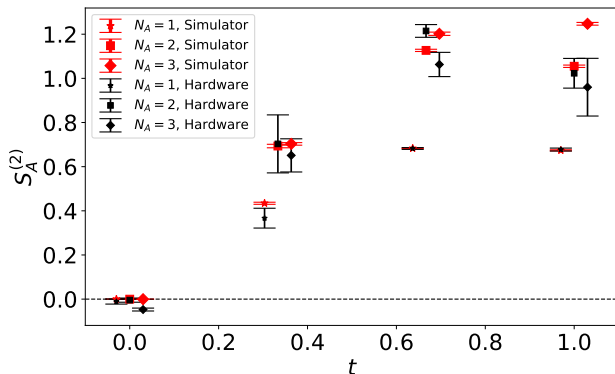
Thank you for your attention ...

# Results for $N=101$



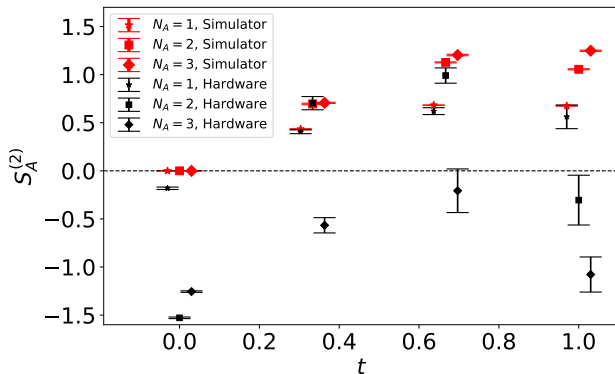
- 👉  $S_A^{(2)}$  measured at  $t = 0, 1/3, 2/3,$  and  $1$
- 👉 Initial product state:  $S_A^{(2)}(t = 0) = 0$
- 👉  $S_A^{(2)}$  increases and approaches a thermal value after a steady growth ( $t \gtrsim 2/3$ )
- 👉 Statistical uncertainty  $\lesssim 2.3\%, 3\%, 4\%$  for  $N_A = 1, 2, 3$

# Results for $N=133$



- Similar behavior observed as for  $N = 101$
- Good agreement with classical simulation within error bars
- Increased hardware connectivity overhead  $\rightarrow$  larger two-qubit gate depth  $\rightarrow$  higher errors
- Statistical uncertainty  $\lesssim 12.3\%$ ,  $18.5\%$ ,  $13.5\%$  for  $N_A = 1, 2, 3$

# Results for $N=151$



- 👉 Large deviations observed for  $N_A > 1$  (only  $N_A = 1$  remains reliable)
- 👉 Scaling to larger systems ( $\sim 156$  qubits) requires more advanced error mitigation

# Gate depth and Gates Counts

$N$	Total gate depth	2-qubit gate depth	2-qubit gate number
5	96	28	36
7	97	31	54
9	97	45	74
15	97	58	132
51	99	64	480
101	99	63	964
133	366	300	1707
151	582	502	2703

# Anti Flatness

- 👉  $\mathcal{F}_A = \text{Tr}(\rho_A^3) - [\text{Tr}(\rho_A^2)]^2$
- 👉  $\mathcal{F}_A$  attend a peak at  $t = \frac{1}{3}$ , indicate high quantumness
- 👉  $\mathcal{F}_A = 0$  : spectrum is flat
- 👉 For  $t \gtrsim \frac{2}{3}$ ,  $\mathcal{F}_A$  saturates to a nonzero value
- 👉 Good agreement between quantum hardware and classical simulation within error bars

