Collective Neutrino-Antineutrino Pairing Oscillations

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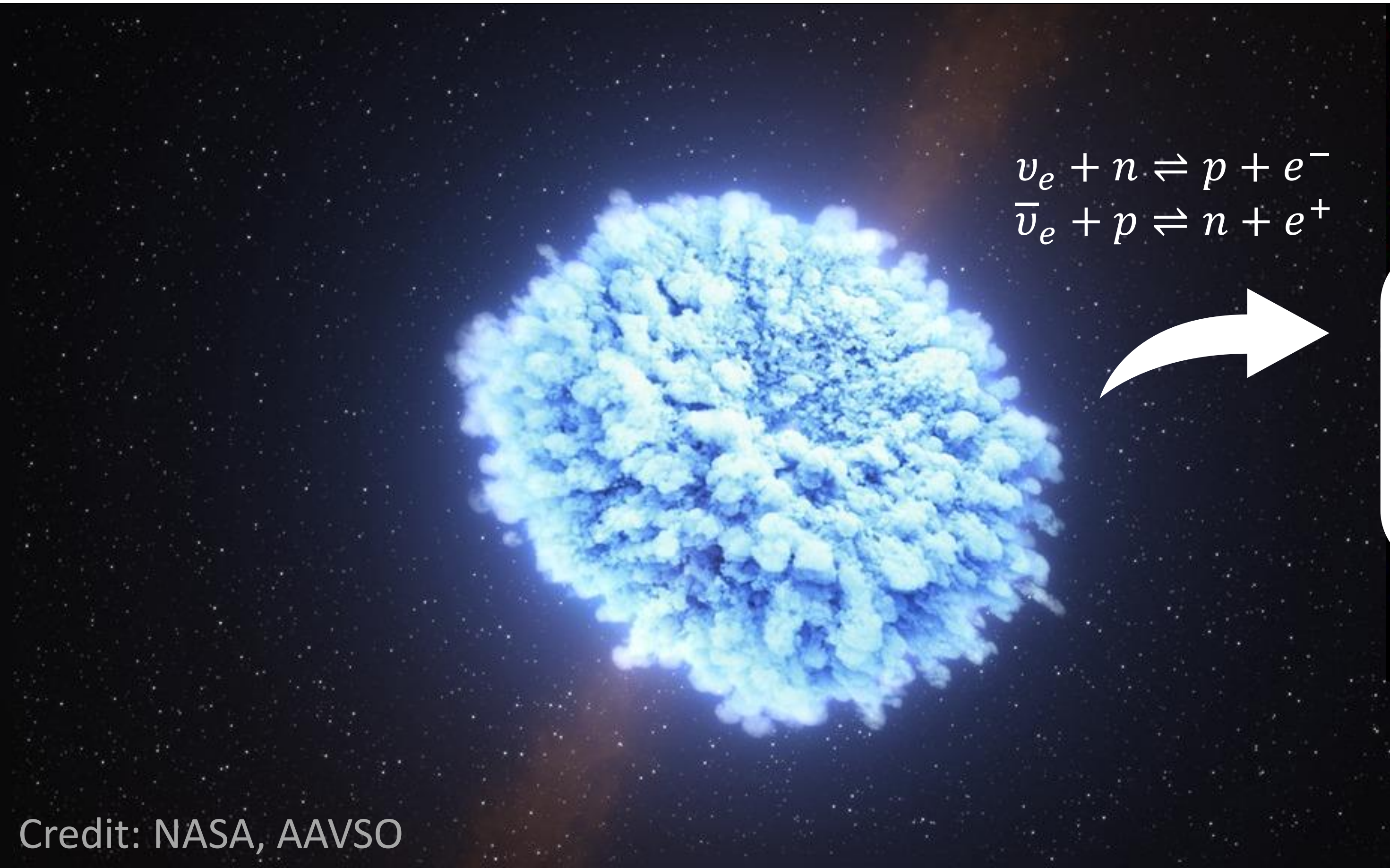
Outline

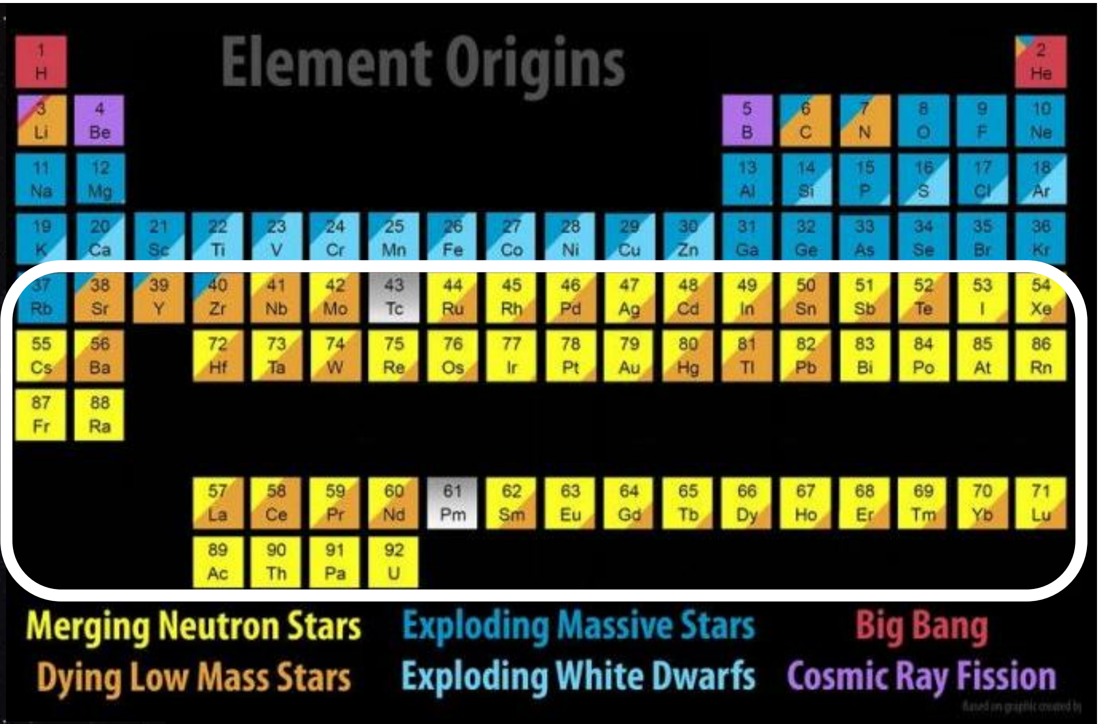
- Introduction
- Generalized Mean-Field
- Instability Analysis
- Simulation Result
- Summary

Introduction

The flavor evolution of neutrino plays a vital role in shaping the **dynamics** and **element compositions** during explosive astrophysical events, such as core-collapse supernovae and binary neutron star mergers.



$$\nu_e + n \rightleftharpoons p + e^-$$
$$\bar{\nu}_e + p \rightleftharpoons n + e^+$$



Credit: NASA, AAVSO

Introduction

Collective Flavor Oscillations

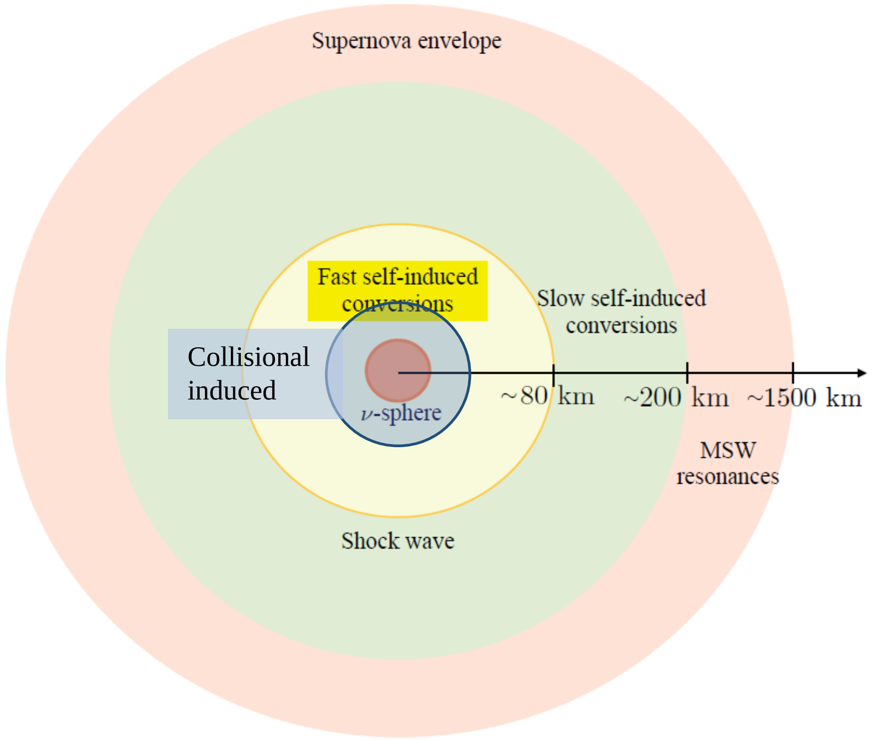
- Collisional instability
- Fast mode
- Slow mode

People used to deal with neutrinos and antineutrino separately.

$$\rho = \begin{pmatrix} \rho_{ee} & \rho_{ex} \\ \rho_{xe} & \rho_{xx} \end{pmatrix}$$

EOM

$$i\dot{\rho} = [h, \rho] + \mathcal{C}, \qquad i\dot{\bar{\rho}} = [\bar{h}, \bar{\rho}] + \bar{\mathcal{C}}.$$



Irene Tamborra and Shashank Shalgar 2021

Generalized Mean-Field

Density Matrix Approach

Flavor evolution of neutrinos and antineutrinos, in a background can be determined using one-body density matrices:

$$\rho_{ij}(q) = \left\langle a_j^\dagger(q) a_i(q) \right\rangle$$

$$\bar{\rho}_{ij}(q) = \left\langle b_i^\dagger(q) b_j(q) \right\rangle$$

Cristina Volpe realized the pairing densities should also be considered (2015):

$$\kappa_{ij}(q) = \left\langle b_j(q) a_i(-q) \right\rangle$$

$$\kappa_{ij}^\dagger(q) = \left\langle a_j^\dagger(q) b_i^\dagger(-q) \right\rangle$$

Cristina Volpe 2015

Generalized Mean-Field

Density Matrix Approach

Equations of motion for the neutrino and pairing density matrices can be obtained from the Ehrenfest theorem :

$$i\dot{\rho}_{ij}(t,q) = \langle [a_j^\dagger(t,q)a_i(t,q), H_{eff}(t)] \rangle$$

$$i\dot{\kappa}_{ij}(t,q) = \langle [b_j(t,q)a_i(t,-q), H_{eff}(t)] \rangle$$

Generalized Equations of motion :

$$i\dot{\mathcal{R}}(q) = [\mathcal{H}(q), \mathcal{R}(q)],$$

$$\mathcal{R}(q) = \begin{pmatrix} \rho(\mathrm{q}) & \kappa(\mathrm{q}) \\ \kappa^\dagger(\mathrm{q}) & 1-\bar{\rho}(\mathrm{q}) \end{pmatrix}.$$

Cristina Volpe 2015

We need to consider the model at least 2D in space

$$\mathcal{H}(q) = \begin{pmatrix} \Gamma^{\nu\nu}(\mathrm{q}) & \Gamma^{\nu\bar{\nu}}(\mathrm{q}) \\ \Gamma^{\bar{\nu}\nu}(\mathrm{q}) & \Gamma^{\overline{\nu\nu}}(\mathrm{q}) \end{pmatrix}$$

$$\Gamma^{\nu\bar{\nu}}(\mathrm{q}) = -\hat{\epsilon}_q^* \cdot \sqrt{2}\, G_F \int \frac{dq'^3}{(2\pi)^3} [\rho(q') + \bar{\rho}(-q')]\, \hat{q}'$$

$$+ [other\ terms\ including\ \kappa]$$

Note:

$$\hat{\epsilon}_p \cdot \hat{\epsilon}_p = 0, \quad \hat{\epsilon}_p \cdot \hat{\epsilon}_p^* = 2$$

$$e.g. \quad \hat{\epsilon}_p = \hat{p}_\theta - i\hat{p}_\phi$$

$$\begin{aligned} \hat{\epsilon}_q^* \cdot \hat{q}' = & \cos(\theta)\sin(\theta')\cos(\phi' - \phi) \\ & - \cos(\theta')\sin(\theta) \\ & + i\sin(\theta')\sin(\phi' - \phi) \end{aligned}$$

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$$+ [other\ terms\ including\ \kappa]$$

We define the coupling strength as follow :

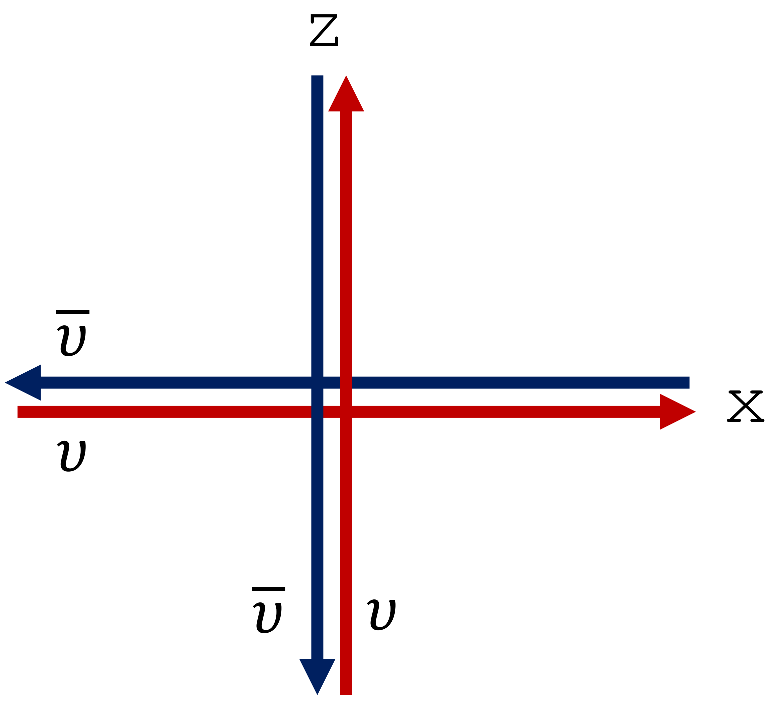
$$\mu = \frac{\sqrt{2}}{(2\pi)^3} G_F\,(MeV)^3 \approx\ 10^{-14}\, MeV;$$

To speed up the calculation we set

$$\mu =\ 10^{-6}\, MeV.$$

Model Setting

Single-Energy, Two Pair



We consider the case of single flavor and set the initial value:

$$\rho_{\vec{x}} = 1, \quad \rho_{\vec{z}} = 0, \quad \bar{\rho}_{-\vec{x}} = 1, \quad \bar{\rho}_{-\vec{z}} = 0,$$

$$\kappa_{\vec{x}} = 0, \quad \kappa_{\vec{z}} = 0.$$

$$i\dot{\mathcal{R}}(q) = [\mathcal{H}(q), \mathcal{R}(q)],$$

$$\mathcal{R}(q) = \begin{pmatrix} \rho(\mathrm{q}) & \kappa(\mathrm{q}) \\ \kappa^{\dagger}(\mathrm{q}) & 1 - \bar{\rho}(\mathrm{q}) \end{pmatrix}.$$

Linearized Instability Analysis

Single-Energy, Two Pair

Keep the first-order terms after expanding the EOM, and then we will have the following time derivative equation:

$$\dot{y} = \mathrm{A\,y + c}, \qquad y(t) = e^{\mathrm{A}t} y_0 + \mathrm{c\,t}.$$

Where $\mathbf{y} = (Re[\kappa_x], Re[\kappa_z], Im[\kappa_x], Im[\kappa_z])^T$

Thus, we can have the growth rate of the pairing instability by solving the eigenvalues of the matrix $\mathbf{A}$.

Linearized Instability Analysis

Single-Energy, Two Pair

Keep the first-order terms after expanding the EOM, and then we will have the following time derivative equation:

CONSTANT

$$\dot{y} = \mathrm{A}\,\mathrm{y} + \mathrm{c}, \qquad y(t) = e^{\mathrm{A}t} y_0 + \mathrm{c}\,\mathrm{t}.$$

Where $\mathrm{y} = (Re[\kappa_x], Re[\kappa_z], Im[\kappa_x], Im[\kappa_z])^T$

Thus, we can have the growth rate of the pairing instability by solving the eigenvalues of the matrix $\mathbf{A}$.

Linearized Instability Analysis

Single-Energy, Two Pair

The eigenvalues λ of matrix $\mathbf{A}$ are:

$$\lambda = \pm 2\mathrm{E}\sqrt{\gamma} \qquad \text{where } \gamma = \left(-1 \pm 2\,\mu_0/\mathrm{E}\,\sqrt{bb'}\right)$$

and $b = (1 - \rho_x - \bar{\rho}_{-x})$, $b' = (1 - \rho_z - \bar{\rho}_{-z})$.

Note:

E is neutrino energy;

$\mu_0 \equiv \mu \cdot \frac{4\pi \mathrm{E}^2 \Delta \mathrm{E}}{(\mathrm{Me}V)^3}$;

μ is coupling strength.

When $E \gg \mu_0$, we derived the criteria:

When $bb' < 0$ the instability exists.

Linearized Instability Analysis

Single-Energy, Two Pair

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and $b = (1 - \rho_x - \bar{\rho}_{-x}),\;\; b' = (1 - \rho_z - \bar{\rho}_{-z})$.

The growth rate $\mathbf{Re}[\lambda]$ is:

$$\mathrm{Re}[\lambda] = 2\mu_0\sqrt{|bb'|}$$

Note:

E is neutrino energy;

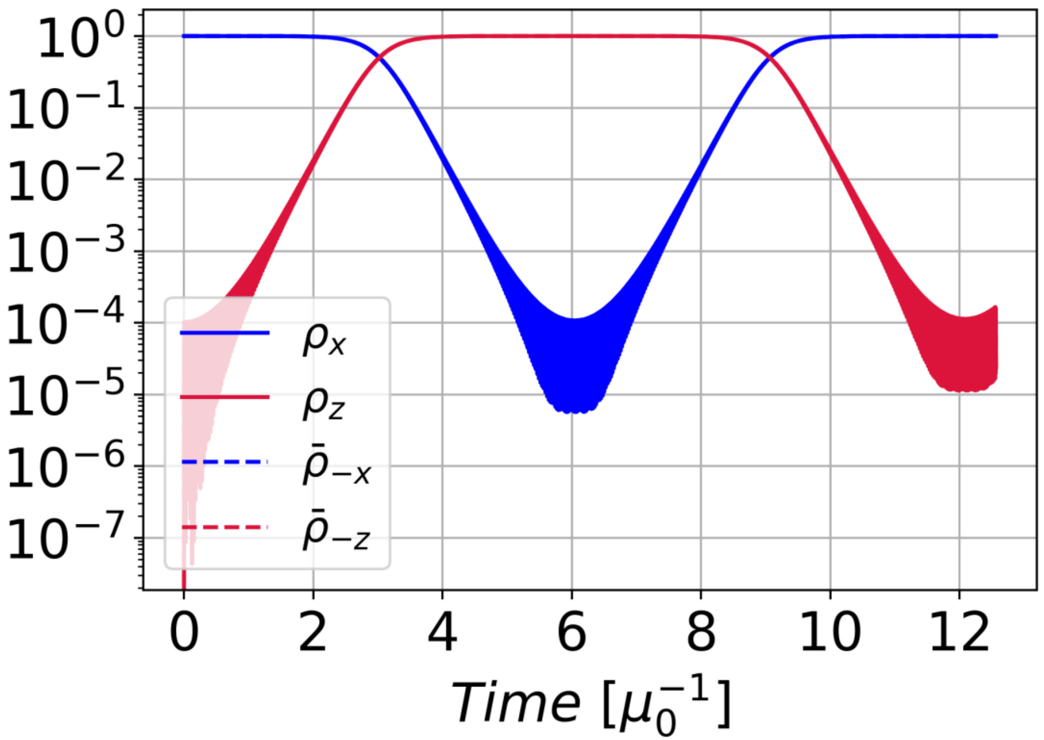
$\mu_0 \equiv \mu \cdot \frac{4\pi \mathrm{E}^2 \Delta \mathrm{E}}{(\mathrm{M}eV)^3}$;

μ is coupling strength.

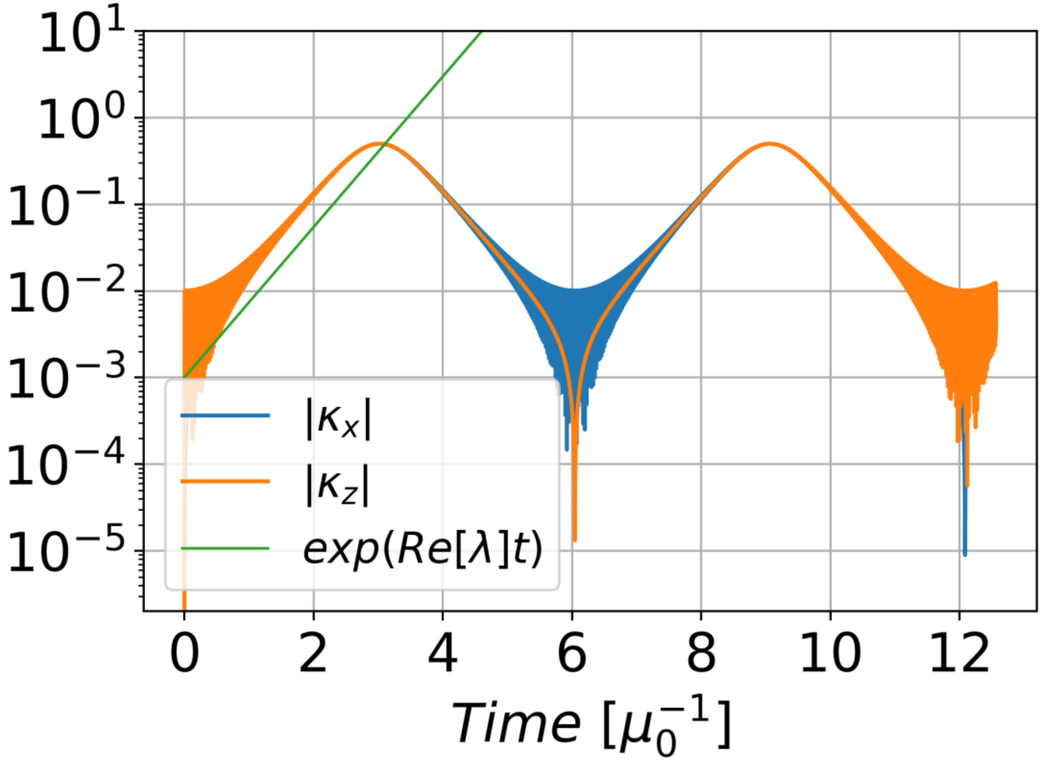
Simulation

Single-Energy, Two Pair

Time evolution of ρ and $\bar{\rho}$

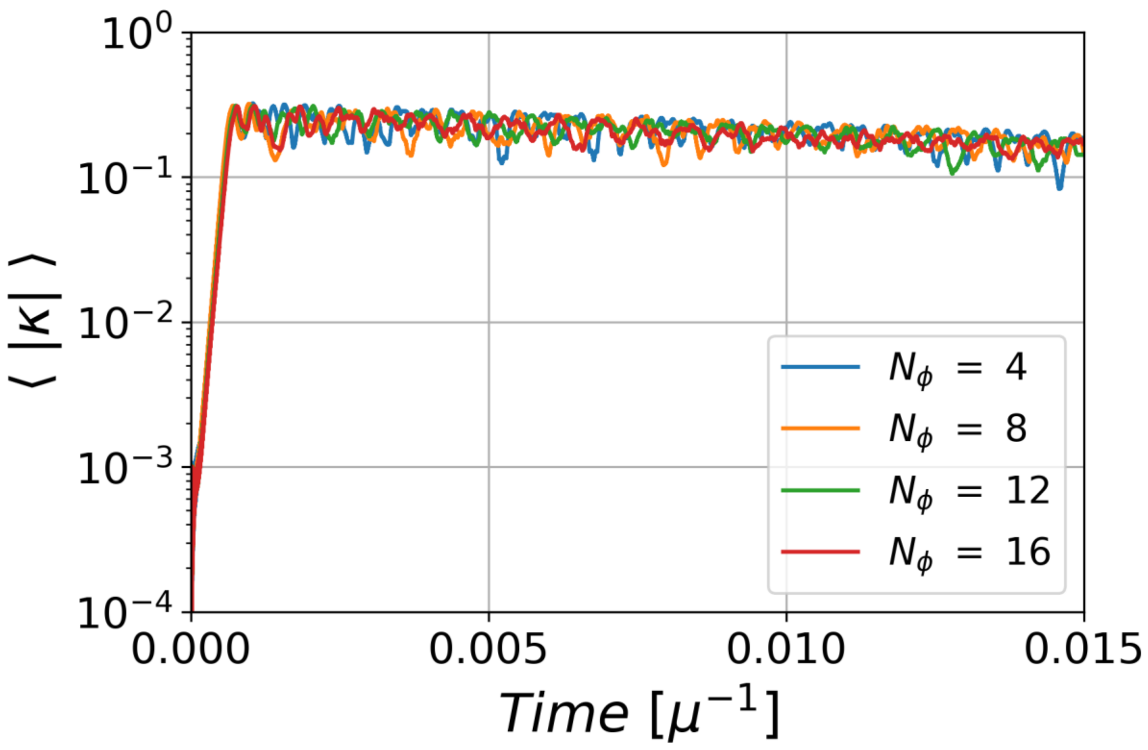


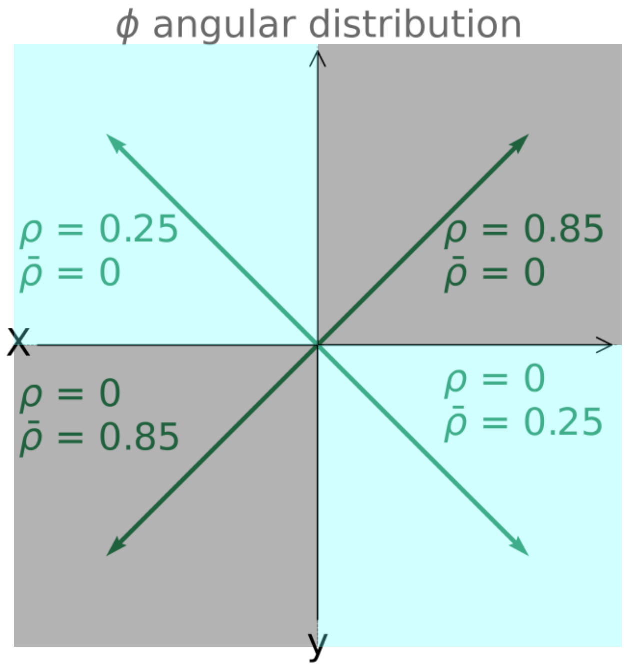
Time evolution of $|\kappa|$



Simulation

Single-Energy Bin & Multi-Angular Pairs





$Energy : 10\ MeV$

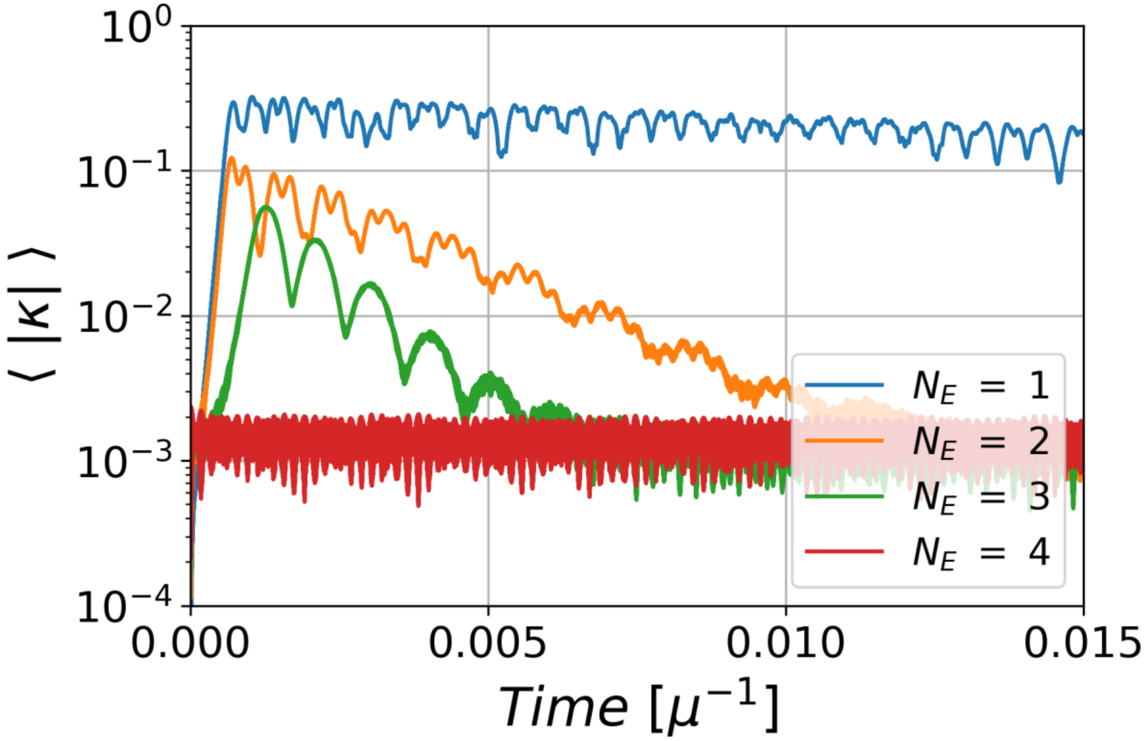
$N_E = 1$

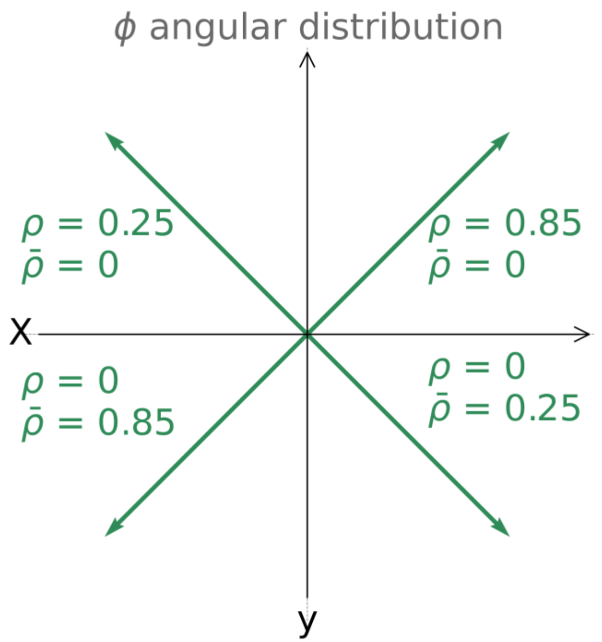
$N_\theta = 1$

$N_\phi = 4, 8, 12, 16$

Simulation

Multi-Energy Bins & 4 Angular Pairs





$Energy: \ 0 \sim 20 \ MeV$

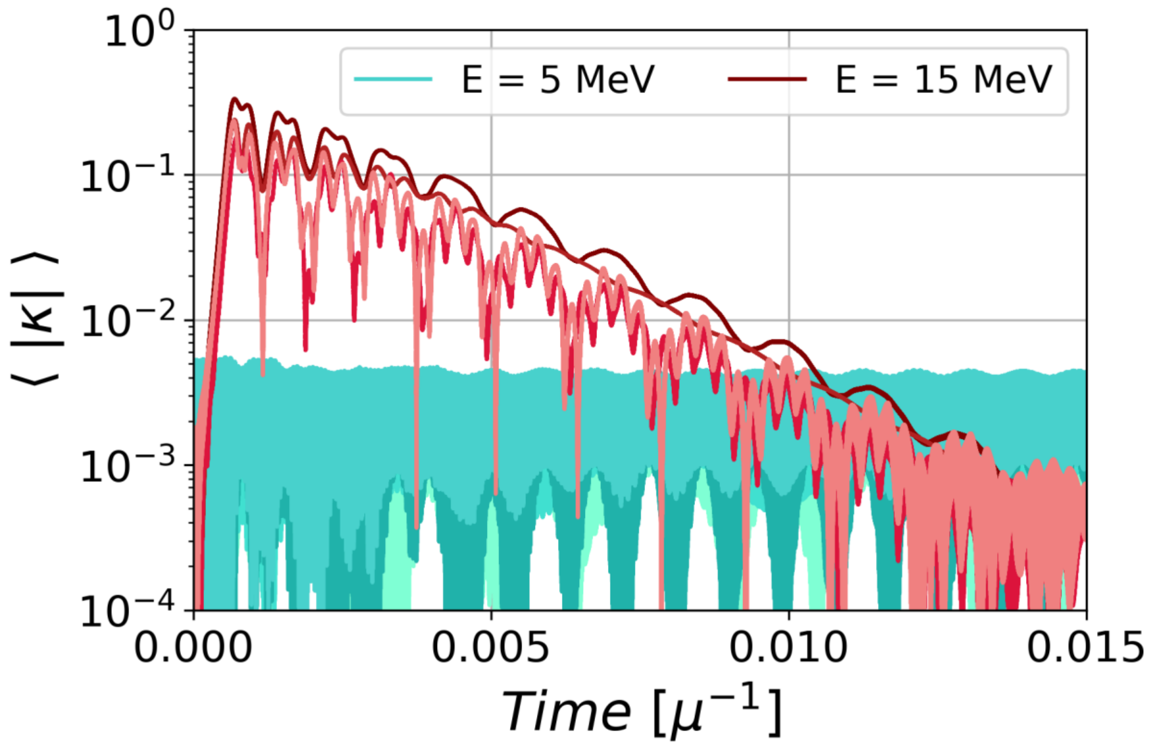
$N_E = 1, 2, 3, 4$

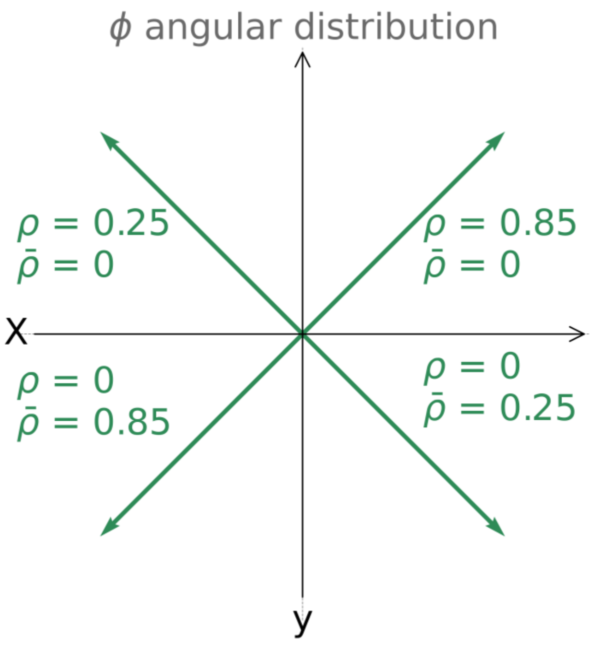
$N_\theta = 1$

$N_\phi = 4$

Simulation

2-Energy Bins & 4 Angular Pairs





$Energy : 5, 15 MeV$

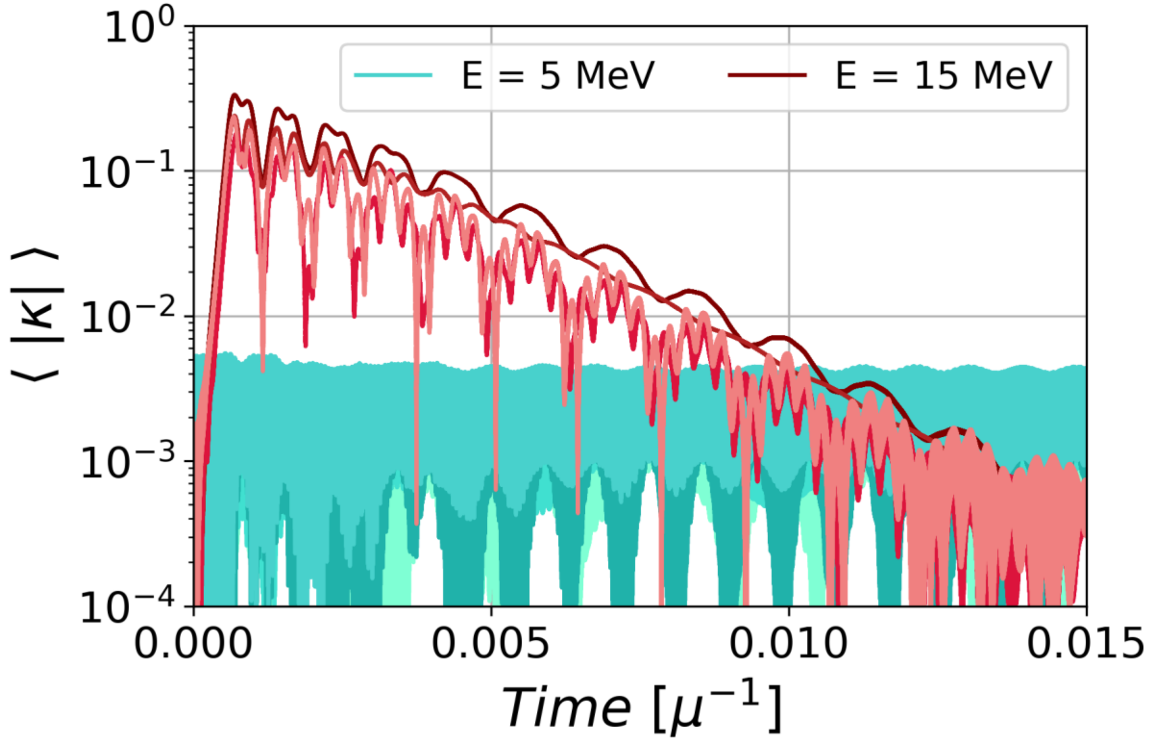
$N_E = 2$

$N_\theta = 1$

$N_\phi = 4$

Simulation

2-Energy Bins & 4 Angular Pairs

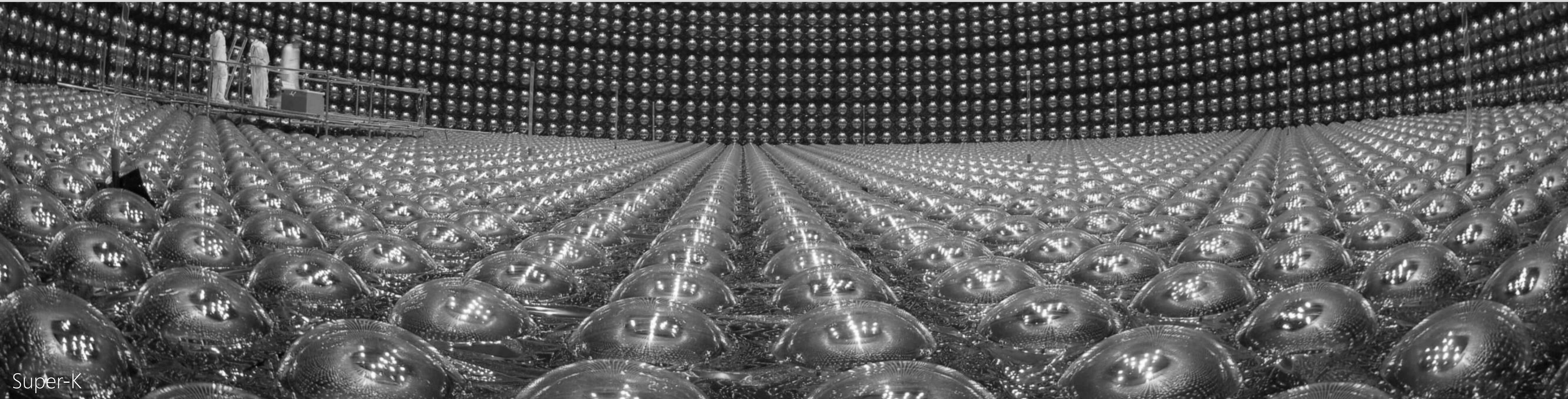


Pairing instability is only confined in a certain energy bin.

Summary

- *Pairing correlation* could introduce a new instability which can trigger *collective pairing conversions*.
- This *pairing instability* is not sensitive to multiple angular distribution.
- Increasing energy bins can highly suppress this *pairing instability.*

THANK YOU



Appendix

$$a_p^\dagger \propto \int dx^3 e^{-ipx}\,\psi^\dagger(x)\;;\; a_p \propto \int dx^3 e^{ipx}\,\psi(x)$$

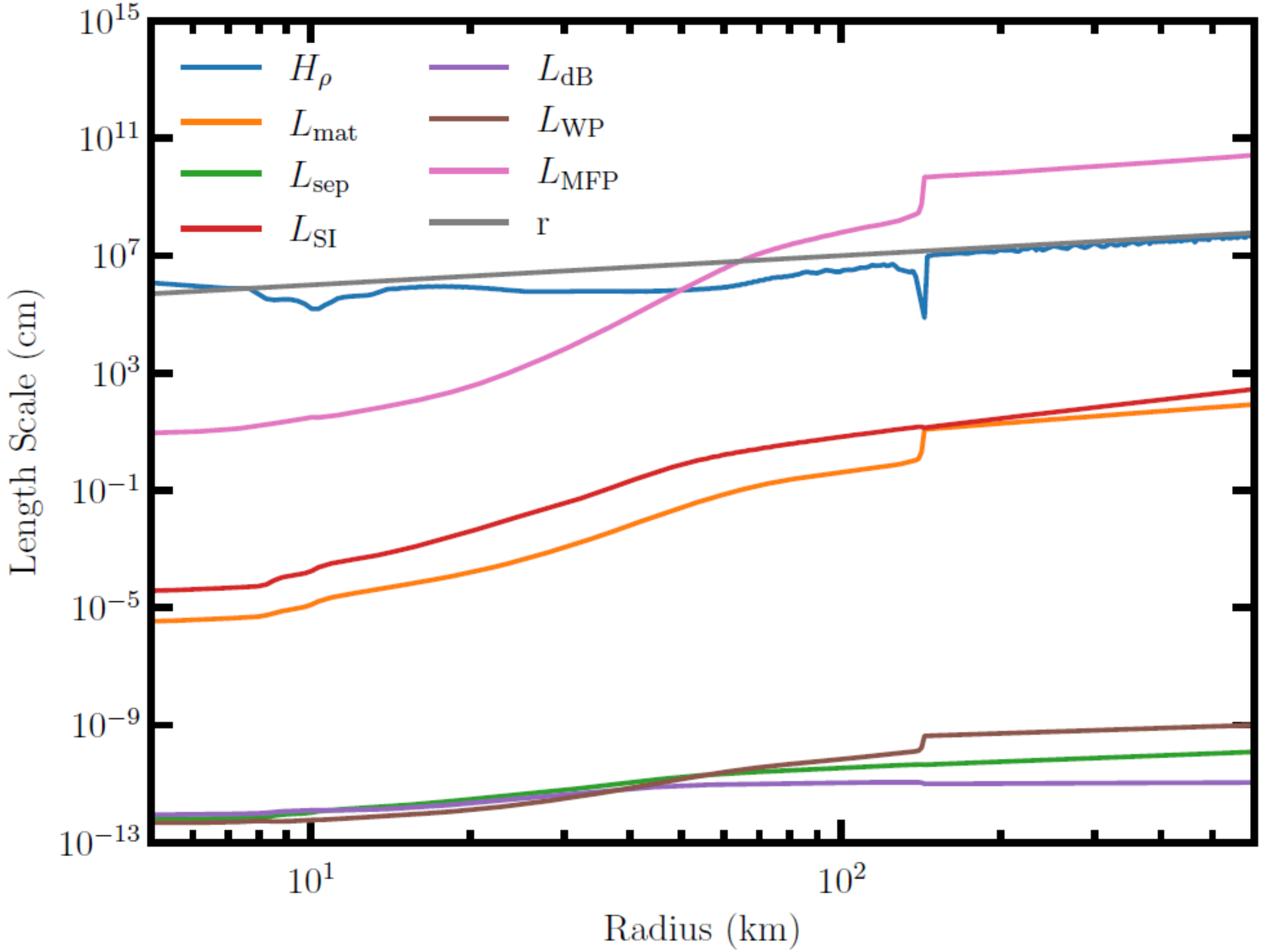
Two point function take the form of $G(x-y)$

$$\langle a_p^\dagger a_q \rangle = \int dR^3 e^{-i(p-q)R} \int dr^3 e^{-i(p+q)r}\, f(r)$$

Where R = x+y; r = x-y.

Length Scales

1D CCSN simulation at the time of maximum shock radius



Mean-Field Theory

Density Matrix Approach

Equations of motion for the neutrino density matrix can be obtained from the Ehrenfest theorem:

$$i\dot{\rho}_{ij}(t,q) = \left\langle [a_j^\dagger(t,q)a_i(t,q), H_{eff}(t)] \right\rangle$$

Equations of motion:

$$i\dot{\rho}(t) = \Gamma^{\nu\nu}(t)\cdot\rho(t) - \rho(t)\cdot\Gamma^{\nu\nu}(t) + \Gamma^{\nu\bar{\nu}}(t)\cdot\kappa^\dagger(t) - \kappa(t)\cdot\Gamma^{\bar{\nu}\nu}(t),$$
$$i\dot{\bar{\rho}}(t) = \Gamma^{\bar{\nu}\bar{\nu}}(t)\cdot\bar{\rho}(t) - \bar{\rho}(t)\cdot\Gamma^{\bar{\nu}\bar{\nu}}(t) - \Gamma^{\bar{\nu}\nu}(t)\cdot\kappa(t) + \kappa^\dagger(t)\cdot\Gamma^{\nu\bar{\nu}}(t),$$
$$i\dot{\kappa}(t) = \Gamma^{\nu\nu}(t)\cdot\kappa(t) - \kappa(t)\cdot\Gamma^{\bar{\nu}\bar{\nu}}(t) - \Gamma^{\nu\bar{\nu}}(t)\cdot\bar{\rho}(t) - \rho(t)\cdot\Gamma^{\nu\bar{\nu}}(t) + \Gamma^{\nu\bar{\nu}}(t)$$

Mean-Field Theory

Density Matrix Approach

Equations of motion for the neutrino density matrix can be obtained from the Ehrenfest theorem:

$$i\dot{\rho}_{ij}(t,q) = \left\langle [a_j^\dagger(t,q) a_i(t,q), H_{eff}(t)] \right\rangle$$

Equations of motion:

$$i\dot{\rho}(t) = \Gamma^{\nu\nu}(t)\cdot\rho(t) - \rho(t)\cdot\Gamma^{\nu\nu}(t) + \Gamma^{\nu\bar{\nu}}(t)\cdot\kappa^\dagger(t) - \kappa(t)\cdot\Gamma^{\bar{\nu}\nu}(t),$$

$$i\dot{\bar{\rho}}(t) = \Gamma^{\bar{\nu}\bar{\nu}}(t)\cdot\bar{\rho}(t) - \bar{\rho}(t)\cdot\Gamma^{\bar{\nu}\bar{\nu}}(t) - \Gamma^{\bar{\nu}\nu}(t)\cdot\kappa(t) + \kappa^\dagger(t)\cdot\Gamma^{\nu\bar{\nu}}(t),$$

$$i\dot{\kappa}(t) = \Gamma^{\nu\nu}(t)\cdot\kappa(t) - \kappa(t)\cdot\Gamma^{\bar{\nu}\bar{\nu}}(t) - \Gamma^{\nu\bar{\nu}}(t)\cdot\bar{\rho}(t) - \rho(t)\cdot\Gamma^{\nu\bar{\nu}}(t) + \Gamma^{\nu\bar{\nu}}(t)$$

Self-Interaction Hamiltonian

$$\Gamma_{ij}^{\text{self}}(t,\vec{x}) = \frac{G_F}{\sqrt{2}}\gamma_\mu(1-\gamma_5)\Big[T_{ij}^\mu(t,\vec{x}) + \delta_{ij}T_{kk}^\mu(t,\vec{x})\Big]$$

Where $T_{ij}^\mu(t,\vec{x}) = \frac{1}{2}\langle\bar{\psi}_j(t,\vec{x})\gamma^\mu(1-\gamma_5)\psi_i(t,\vec{x})\rangle$

$$\bar{u}(\vec{p})\gamma^\mu(1-\gamma_5)u(\vec{p}) = \bar{v}(\vec{p})\gamma^\mu(1-\gamma_5)v(\vec{p}) = 2n^\mu(\hat{p})$$

$$\bar{v}(-\vec{p})\gamma^\mu(1-\gamma_5)u(\vec{p}) = 2\epsilon^\mu(\hat{p})$$

Here $n^\mu(\hat{p}) = \begin{pmatrix} 1 \\ \hat{p} \end{pmatrix}$ and $\epsilon^\mu(\hat{p}) = \begin{pmatrix} 0 \\ \hat{\epsilon}_p \end{pmatrix}$