# Introduction to HNLi’s group

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Research fields: 1. New Physics, 2. QCD

# New physics

- Postdoc: Shu-Yu Ho (AS)
- Topic: dark matter candidate in 4$^{th}$ generation model

- Collaborators: Song-Ming Wang (AS), Yun-Ju Lin (AS), Kai-Feng Chen NTU)
- Topic: search for 4$^{th}$ generation quark b' (TeV-scale mass) and charged lepton tau' (EW-scale mass) at LHC

# QCD

- Heavy flavor and CP violation
- Perturbative QCD
- Dispersive approach to nonperturbative QCD

- First two topics are standard, so focus on last one

# Conventional QCD approaches

- QCD observables involve nonpert dynamics. How to handle it?
- Factorization theorem: absorb nonpert dynamics into universal PDFs; break down at high powers eventually
- QCD sum rules: simple but hard to control uncertainties from assumption of quark-hadron duality, determination of stability window in Borel mass; less predictive power
- Lattice QCD, 1$^{st}$ principle but tedious numerics; inapplicable to complicated processes (D -> pi pi,…)
- Effective theories (chiral pert theory,…), models (chiral quark model,…)

# Our strategy---dispersive approach

- Adopt dispersion relation like sum rules---based only on analyticity of physical observables (rigorousness)
- OPE in Euclidean region calculable to high orders and powers with universal condensates (no breakdown, systematical improvement of precision)
- Handle dispersion relation as inverse problem---solve integral equation directly to get unknown spectral functions with OPE inputs (mature mathematical tools available)
- Have shown uniqueness of solution
- Higher predictive power without strong assumptions

# Contour integration

- Two-current correlator

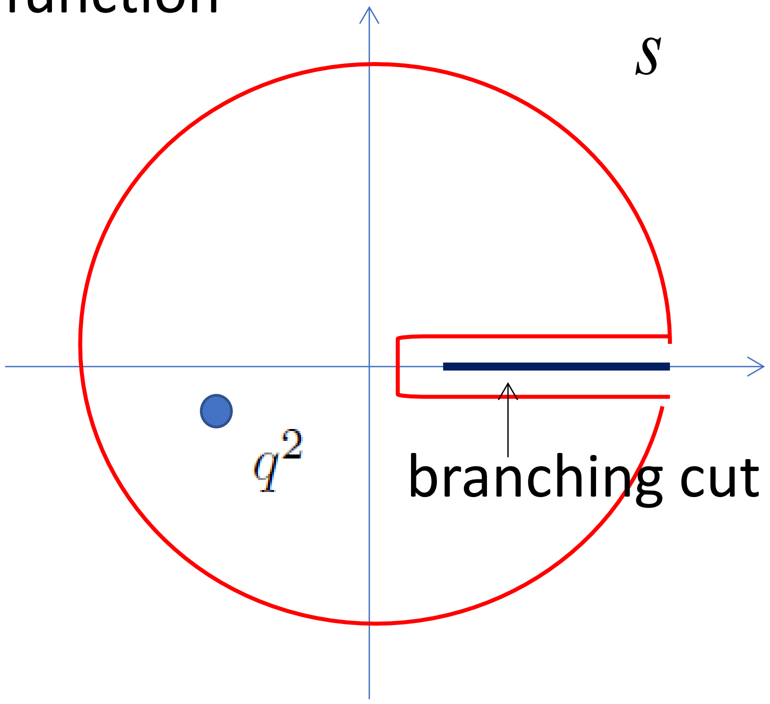
$$J_\mu = (\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d)/\sqrt{2}.$$

$$\Pi_{\mu\nu}(q^2) = i\int d^4x e^{iq\cdot x}\langle 0|T[J_\mu(x)J_\nu(0)]|0\rangle$$
$$= (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2)$$

vacuum polarization function

- Identity from contour integration

$$\Pi(q^2) = \frac{1}{2\pi i}\oint ds \frac{\Pi(s)}{s-q^2}$$



# Dispersion relation

- Operator product expansion for LHS

$$\Pi^{\mathrm{OPE}}(q^2) = \frac{1}{2\pi i}\oint ds \frac{\Pi^{\mathrm{pert}}(s)}{s-q^2} + \frac{1}{12\pi}\frac{\langle \alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q \bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa \alpha_s \langle \bar{q}q\rangle^2}{(q^2)^3}$$

- RHS contains unknown spectral density
- Equality of two sides gives dispersion relation
- Contributions from big circles cancel, and those from branch cuts remain

arbitrary radius helps identify physical resonances

$$\frac{1}{\pi}\int_0^R ds \frac{\mathrm{Im}\Pi(s)}{s-q^2} = \frac{1}{\pi}\int_0^R ds \frac{\mathrm{Im}\Pi^{\mathrm{pert}}(s)}{s-q^2} + \frac{1}{12\pi}\frac{\langle \alpha_s G^2\rangle}{(q^2)^2} + 2\frac{\langle m_q \bar{q}q\rangle}{(q^2)^2} + \frac{224\pi}{81}\frac{\kappa \alpha_s \langle \bar{q}q\rangle^2}{(q^2)^3}$$

Fredholm equation of the 1st kind

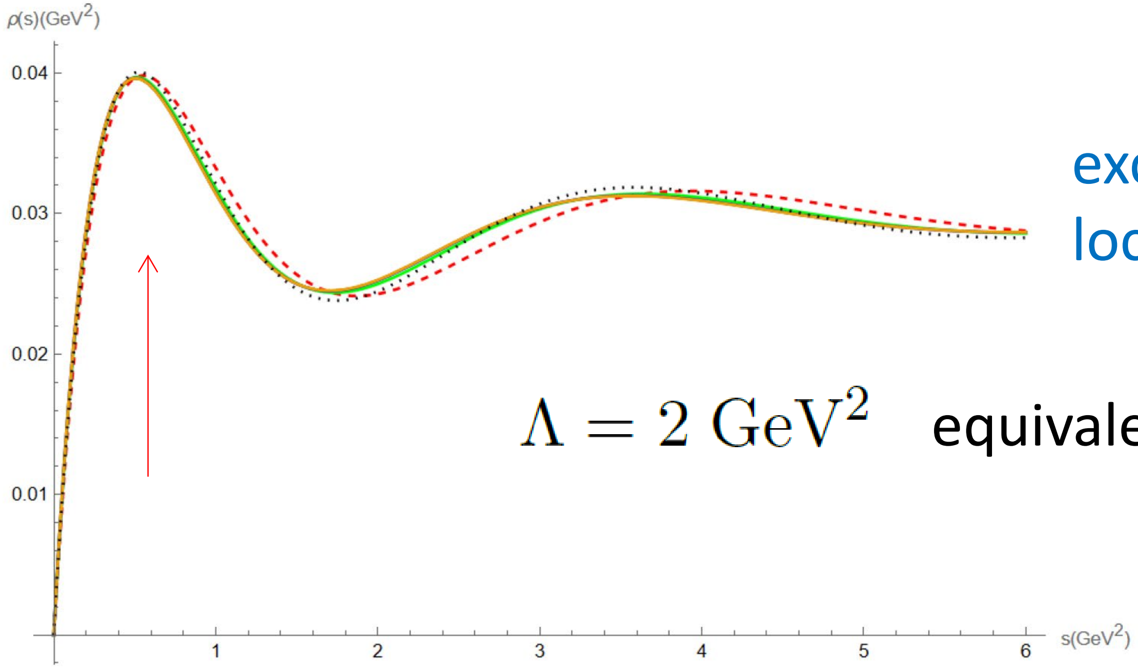
# rho meson peak emerges

- Take condensate inputs, and solve integral equation directly

$$\langle \alpha_s G^2 \rangle = 0.028\ \mathrm{GeV}^4, \quad \langle m_q \bar{q} q \rangle = 0.007 \times (-0.215)^3\ \mathrm{GeV}^4,$$

$$\alpha_s \langle \bar{q} q \rangle^2 = 6.64 \times 10^{-5}\ \mathrm{GeV}^6 \quad \alpha_s = 0.5, \quad \kappa = 2,$$

difficult due to ill-posedness




excited-state contribution violates local quark-hadron duality

$\Lambda = 2\ \mathrm{GeV}^2$ equivalent to big circle radius R

# rho meson properties

- Vary $\Lambda$ (i.e., R), find peak location
- Stable solution in $\Lambda$ with tiny error

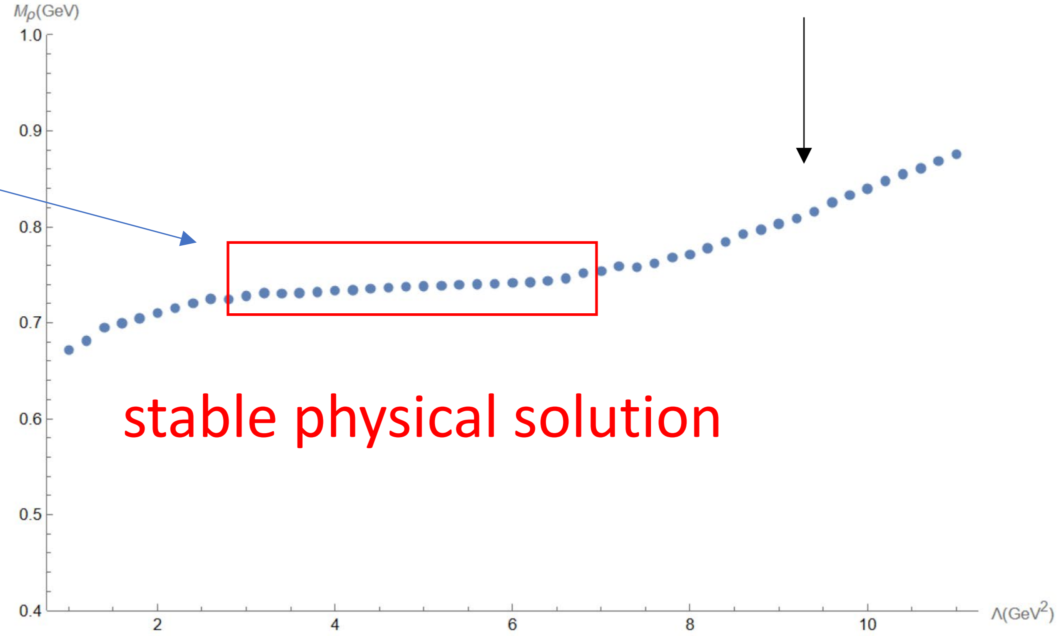
$$M_\rho = (0.737 \pm 0.004)\ \mathrm{GeV}$$

- Errors from condensates 5%

- Decay constant squared:
  area below 1st peak

$$f_\rho \approx 0.218\ \mathrm{GeV}$$

scaling (unphysical) region
peak locations drift with $\Lambda$



# Correlator for proton

Zeng, Li, Xu, 2603.15989

- Two-point correlator

$$\Pi(k) = i\int d^4x e^{ik\cdot x}\langle 0|T[\eta(x)\bar{\eta}(0)]|0\rangle = A(k^2)\not{k} + B(k^2)$$

$$\eta(x) = \epsilon^{abc}[u^{aT}(x)C\gamma^\mu u^b(x)]\gamma_5\gamma_\mu d^c(x)$$ baryonic interpolating current

- At hadron level, correlator describes proton propagator

pole residue

$$\Pi(k) = \lambda_P^2 \frac{\not{k} + m_P}{m_P^2 - k^2} + \ldots \quad \Rightarrow \quad A(k^2) = \frac{\lambda_P^2}{M_P^2 - k^2}, \quad B(k^2) = \frac{\lambda_P^2 M_P}{M_P^2 - k^2}$$

even dimensions

- OPEs

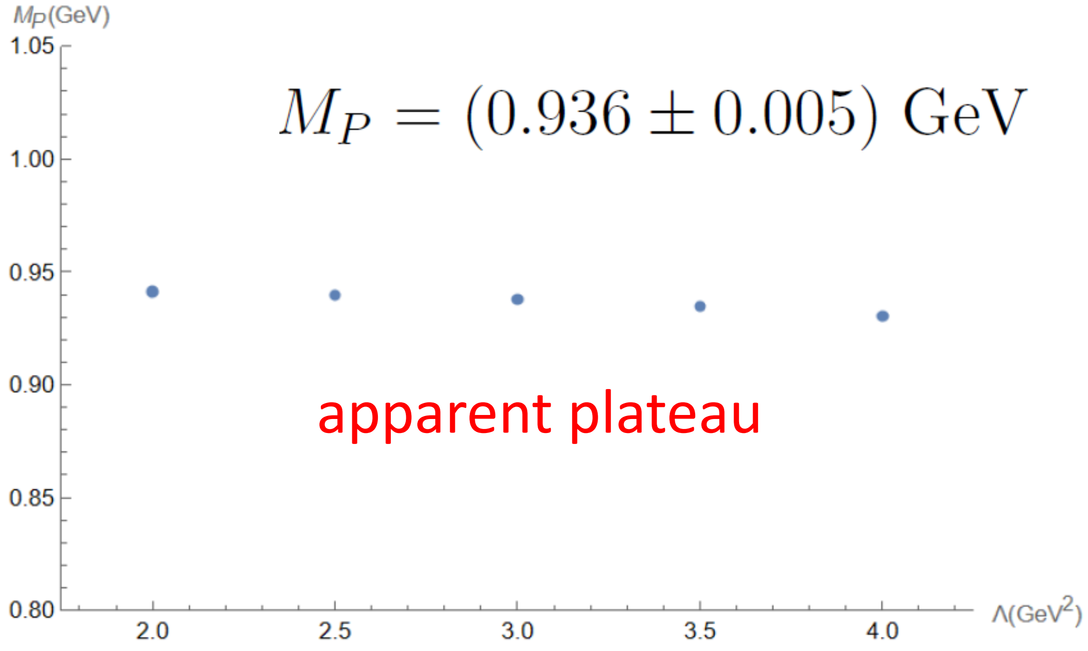
$$\Pi_A(k^2) = \frac{k^4}{64\pi^4}\ln\left(\frac{-k^2}{\mu^2}\right) + \frac{\langle\alpha_s G^2\rangle}{32\pi^3}\ln\left(\frac{-k^2}{\mu^2}\right) - \frac{2}{3k^2}\kappa\langle\bar{q}q\rangle^2$$ chiral-even

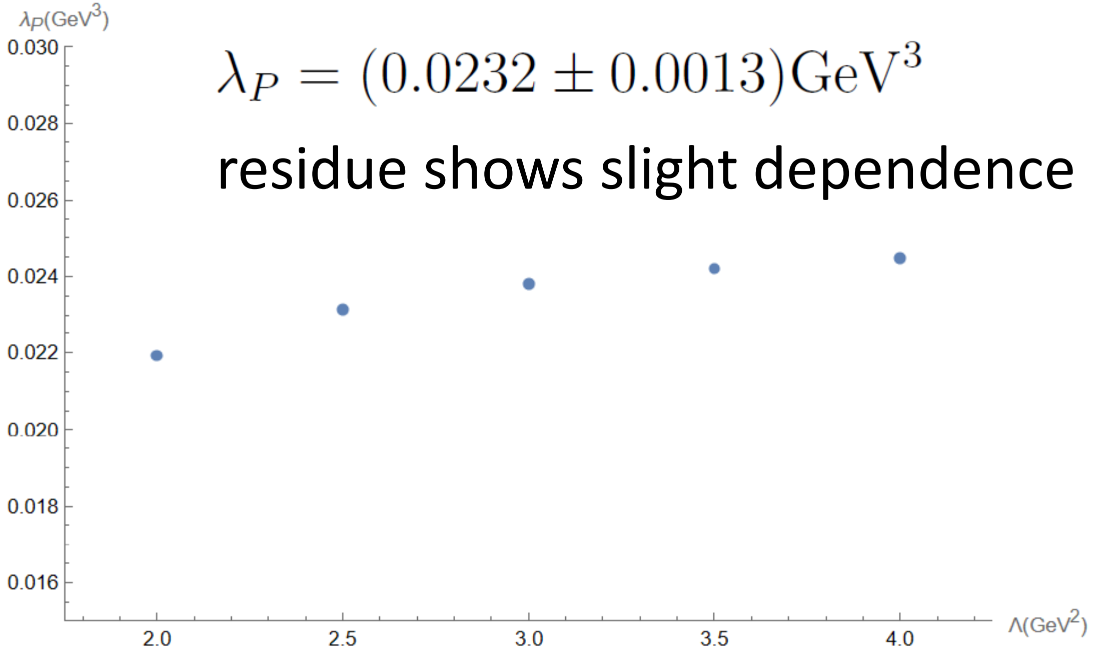
$$\Pi_B(k^2) = -\left(\frac{k^2}{4\pi^2}\langle\bar{q}q\rangle + \frac{1}{8\pi^2}\langle\bar{q}g\sigma\cdot Gq\rangle\right)\ln\left(\frac{-k^2}{\mu^2}\right),$$ chiral-odd

quark-gluon condensate

# Proton mass and pole residue

- Intersections insensitive to $\Lambda$, physical solution established for proton



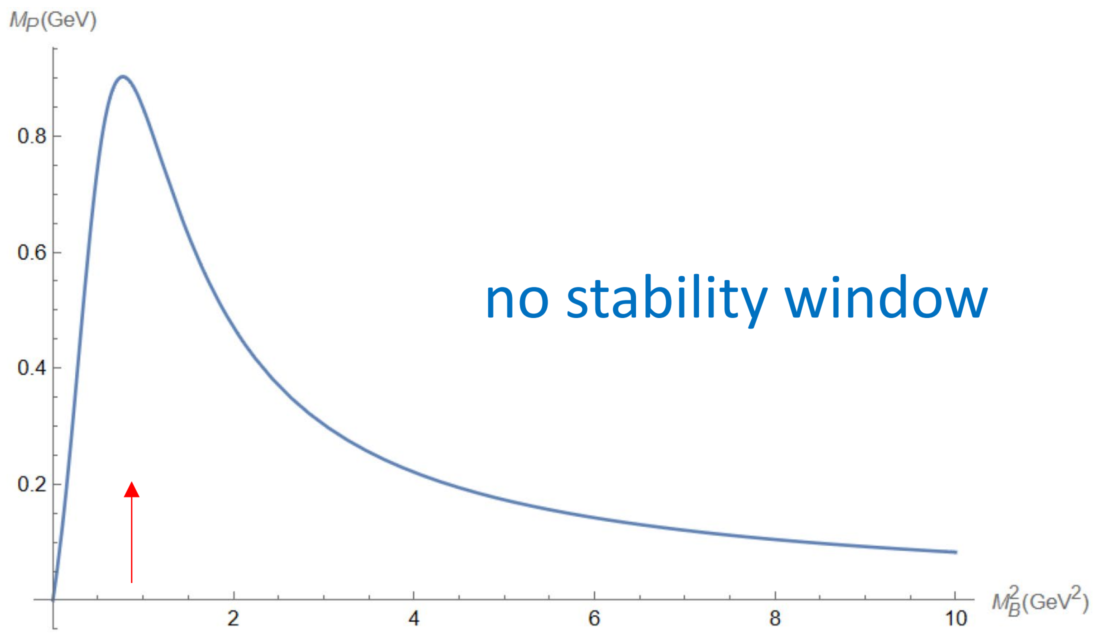

- Imposing chiral-even, odd dispersive constraints simultaneously specifies physical resonance

# Comparison to conventional QSR

- Apply  Borel transformation to chiral-even, odd dispersion relations, take their ratio to cancel pole residue
- Proton mass as function of Borel mass

Ioffe, 1981

$$M_P(M_B^2) = \frac{-\frac{\langle \bar{q}q\rangle}{4\pi^2}M_B^4 - \frac{\langle \bar{q}g\sigma\cdot Gq\rangle}{8\pi^2}M_B^2}{\frac{M_B^6}{32\pi^4} + \frac{2}{3}\kappa\langle\bar{q}q\rangle^2}$$




no stability window

- Supposed to be independent of $M_B$
- Need to assume accidental equality of $M_B$ and $M_P$ to get appropriate result

# Future projects

- Excited states (rho’, rho’’, excited scalar, pseudoscalar glueballs done)
- Baryon light-cone distribution amplitudes (pion LCDAs done)
- Form factors (3-point correlators)
- Heavy-light mesons (B, Bc, …)
- B meson semileptonic, radiative decays
- Multi-particle states (tetraquarks,…)
- Compton scattering (4-point correlators)
- Structure functions (Imaginary part of Compton scattering)
- Extract PDFs, GPD, GDA,…
- Eventually, an analytical nonperturbative formalism