Constraints on the Stochastic Gravitational-Wave Background from the CMB to Gravitational-Wave Detectors

Yuki Inoue

National Central University and Academia Sinica

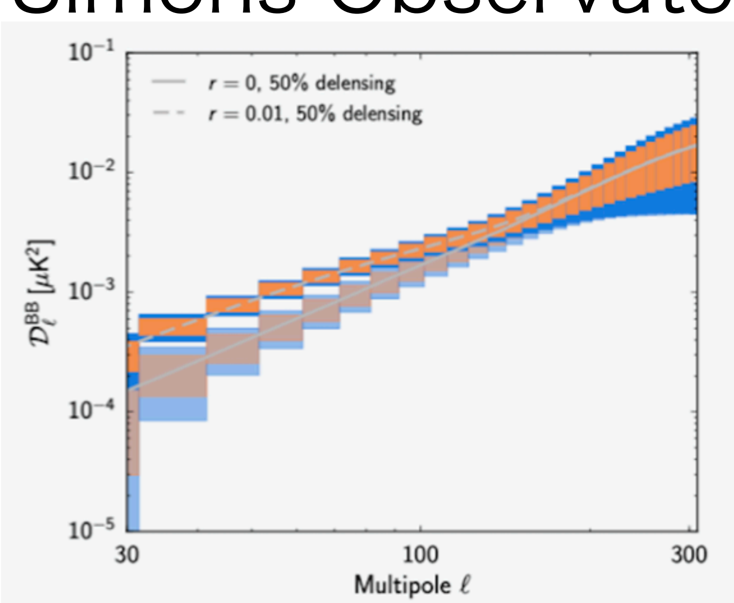
Outline

- Introduction
- Science
- Constraint of stochastic background at low frequency
- Test of Cosmological model (Tensor tilt)
- Summary

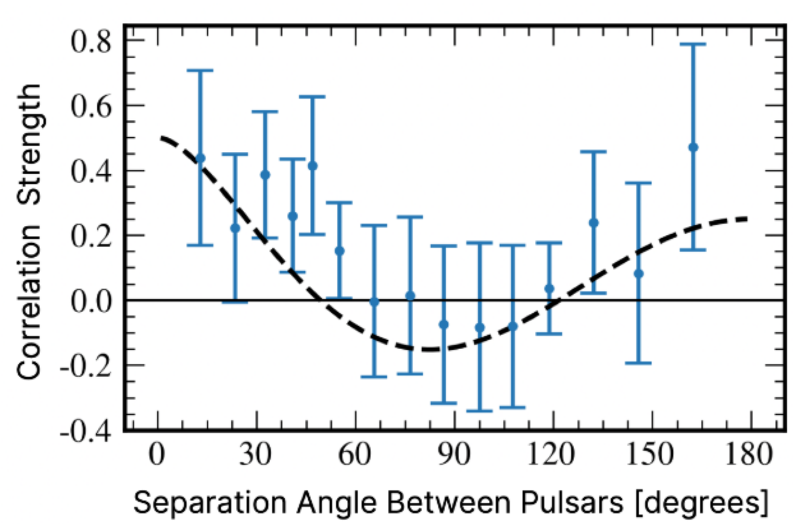
Introduction

- Analysis approach for Exploring the landscape of Stochastic Gravitational-Wave Background
- Simons Observatory, Pulser Timing Array and LIGO will provide gorgeous data set within 5 year.
- We still have a chance to give a constraint of Ω_{GW} with the latest data.
- Now is the time for preparation period to establish the current constraint.

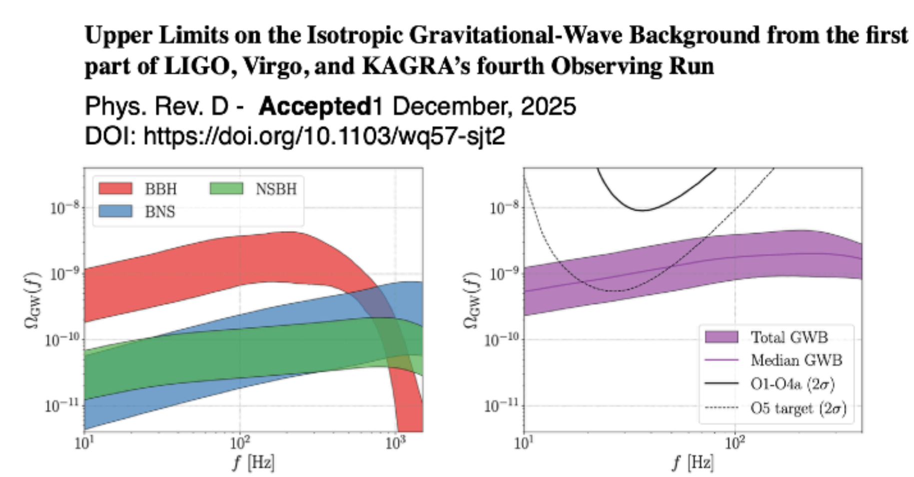
Simons Observatory



Pulser Timing Array

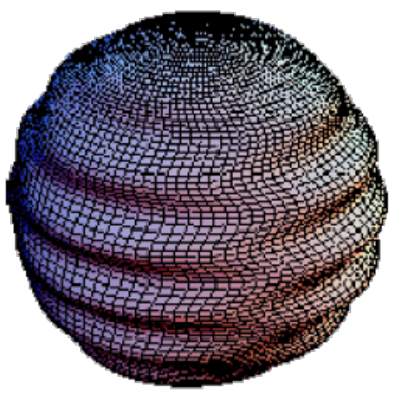


LIGO-Virgo-KAGRA



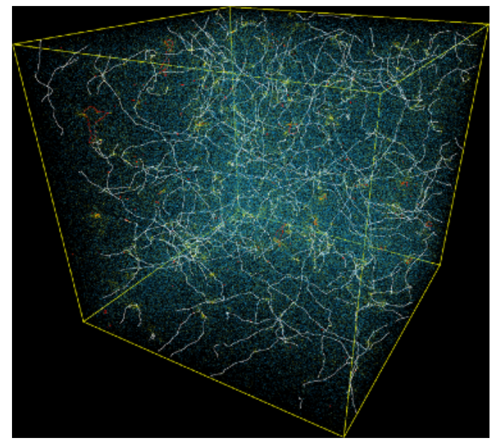
Stochastic background source

Primordial



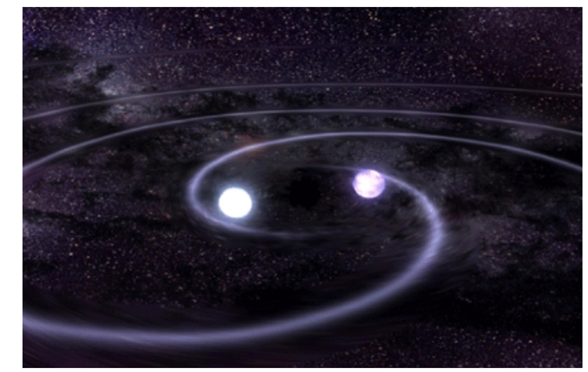
- Initial fluctuation with Inflation

Phase transition

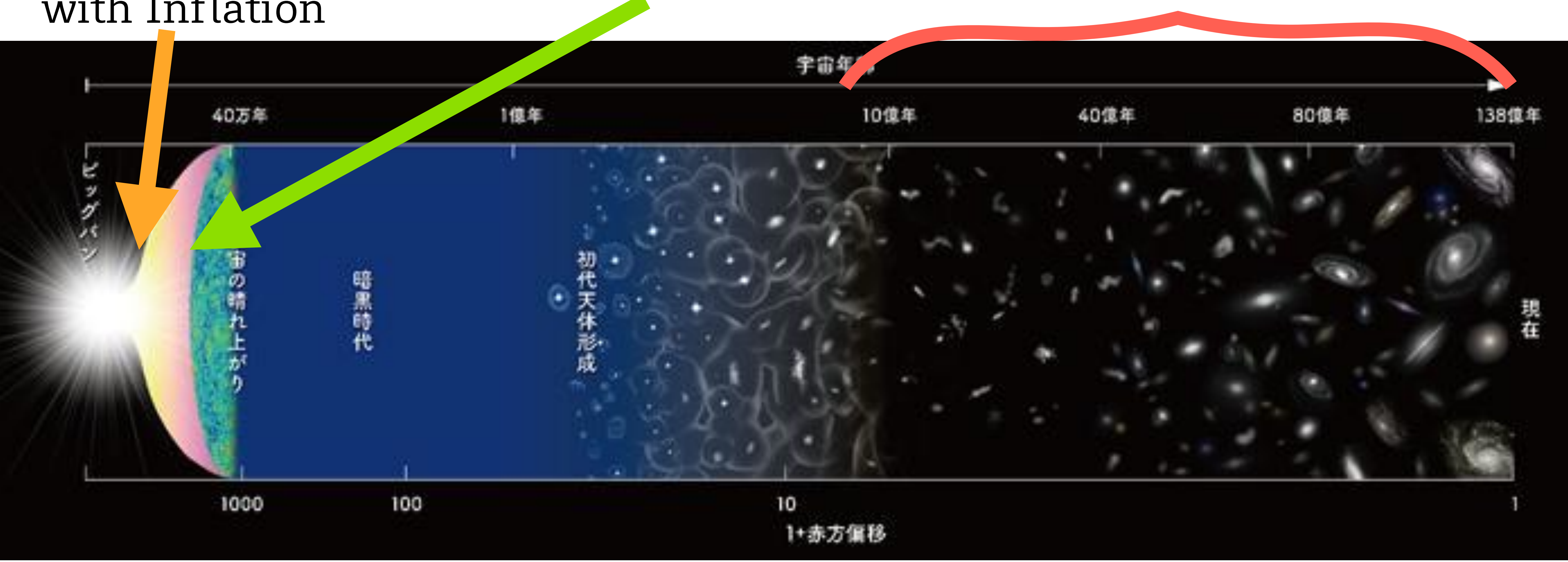


- Cosmic string

Astronomical



- BBH and BNS



Our Goals

1.Expanding Experimental Sensitivity into Unexplored Frequency Regimes

➡ Session 2: CHRONOS

2. Exploring the landscape of Stochastic Gravitational-Wave Background with Ω_{GW} Constraints

3. Development of an Analysis Framework Combining Data from the CMB to Gravitational-Wave Experiments across 20 Orders of Magnitude

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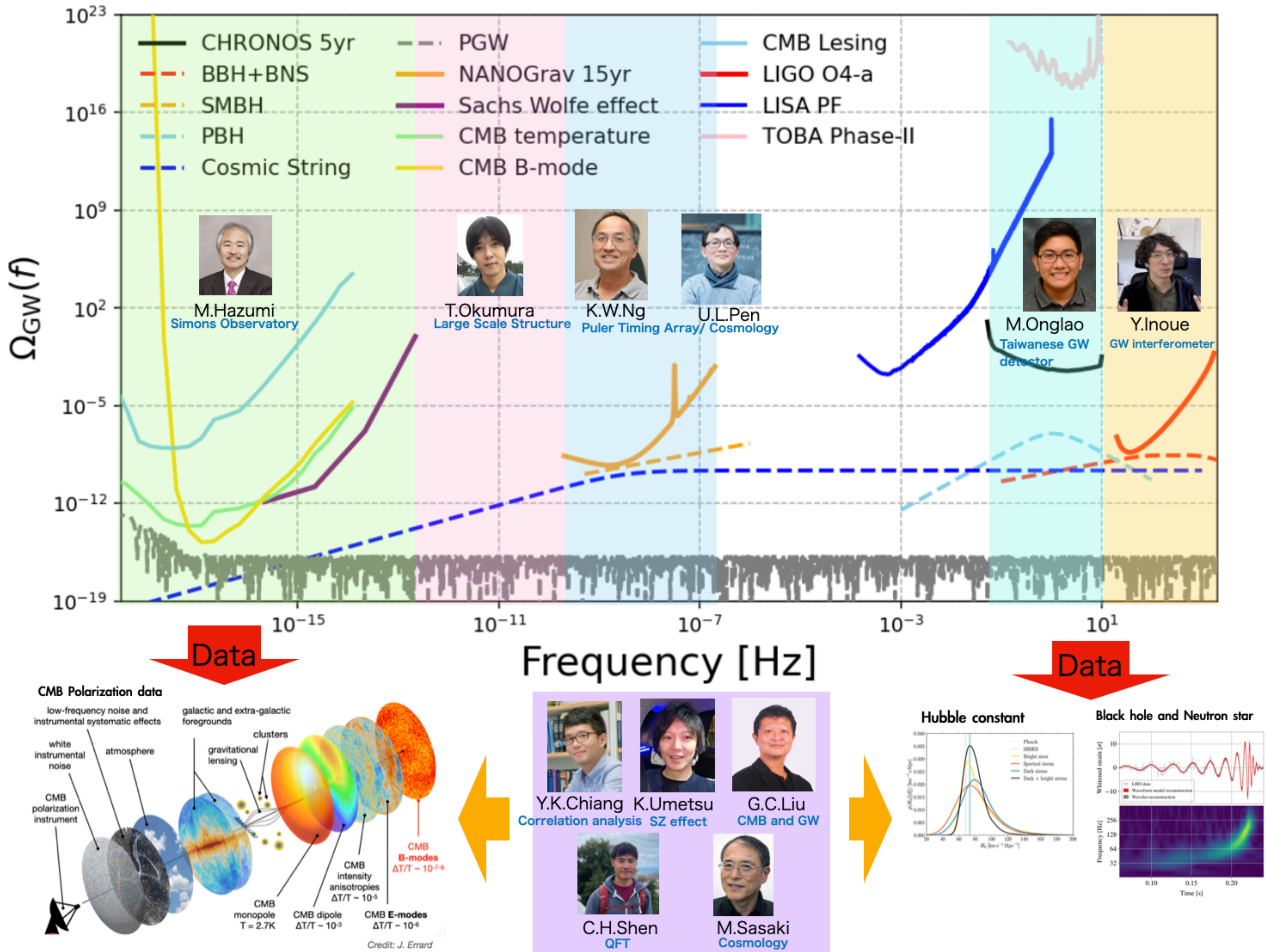
Team Taiwan



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BBH+BNS
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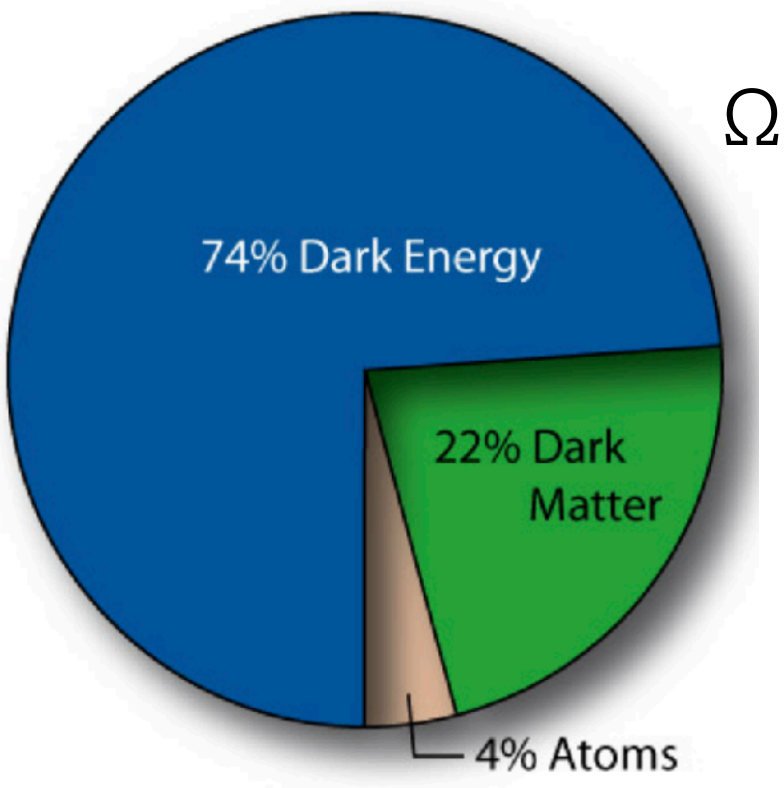
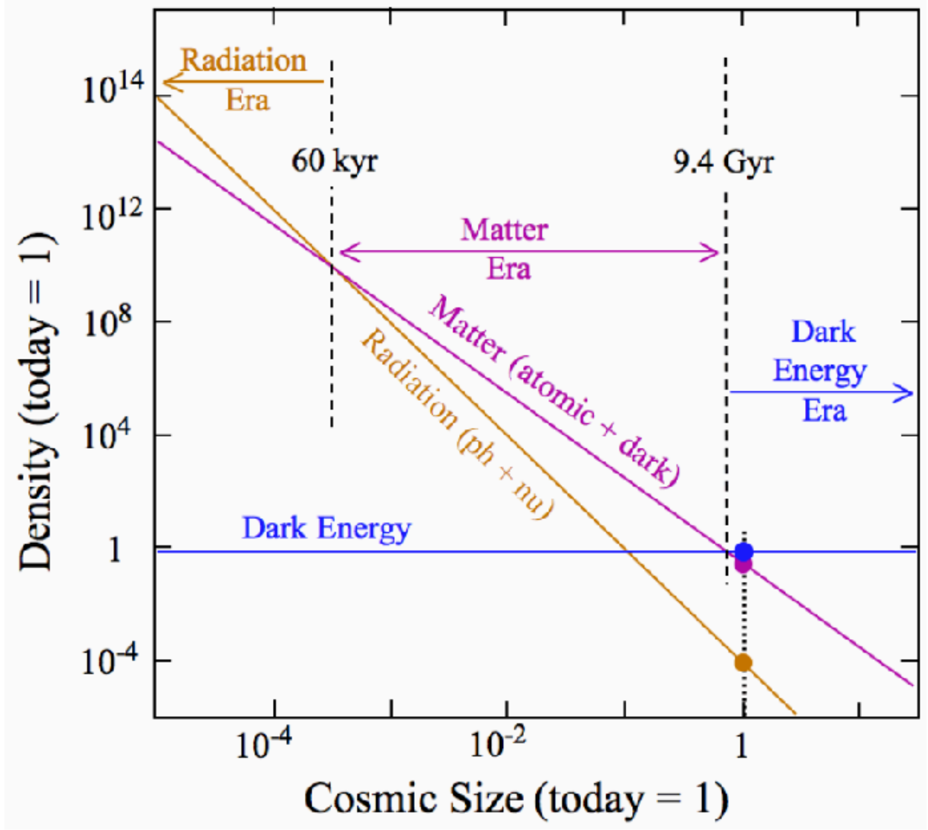
PBH
Cosmic String
PGW
NANOGrav 15yr
Sachs Wolfe effect
CMB temperature
CMB B-mode
CMB Lesing
LIGO O4-a
LISA PF
TOBA Phase-II
$\Omega_{GW}(f)$
10^{23}
10^{16}
10^{9}
10^{2}
10^{-5}
10^{-12}
10^{-19}
10^{-15}
10^{-11}
10^{-7}
10^{-3}
10^{1}
Frequency [Hz]
M.Hazumi
Simons Observatory
T.Okumura
Large Scale Structure
K.W.Ng
U.L.Pen
Puler Timing Array/ Cosmology
M.Onglao
Taiwanese GW detector
Y.Inoue
GW interferometer
Data
Data
CMB Polarization data
low-frequency noise and instrumental systematic effects
galactic and extra-galactic foregrounds
clusters
white instrumental noise
atmosphere
gravitational lensing
CMB polarization instrument
CMB B-modes ΔT/T ~ 10-7-8
CMB intensity anisotropies ΔT/T ~ 10-5
CMB monopole T = 2.7K
CMB dipole ΔT/T ~ 10-3
CMB E-modes ΔT/T ~ 10-6
Credit: J. Errard
Y.K.Chiang
Correlation analysis
K.Umetsu
SZ effect
G.C.Liu
CMB and GW
C.H.Shen
QFT
M.Sasaki
Cosmology
Hubble constant
Black hole and Neutron star

Density parameter



$\Omega i = \rho_i / \rho_{cr}$



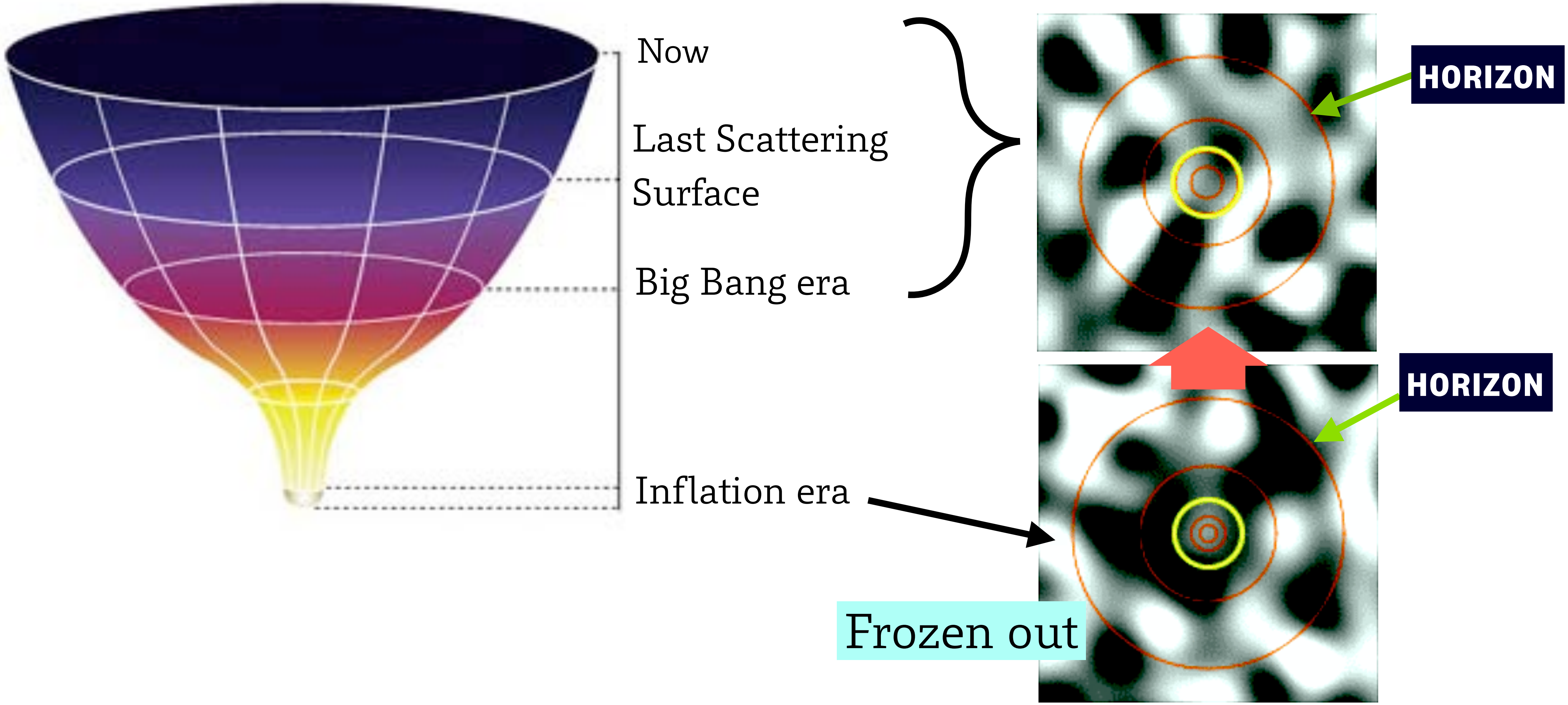
Friedmann equation: $H^2(t) = H_0^2 \left[\Omega_{\gamma,0} a^{-4} + \Omega_{m,0} a^{-3} + \Omega_{\kappa} a^{-2} + \Omega_{\Lambda} \right]$

radiation matter curvature dark energy

Fundamental questions: What's the energy density of GW, Ω_{GW}?

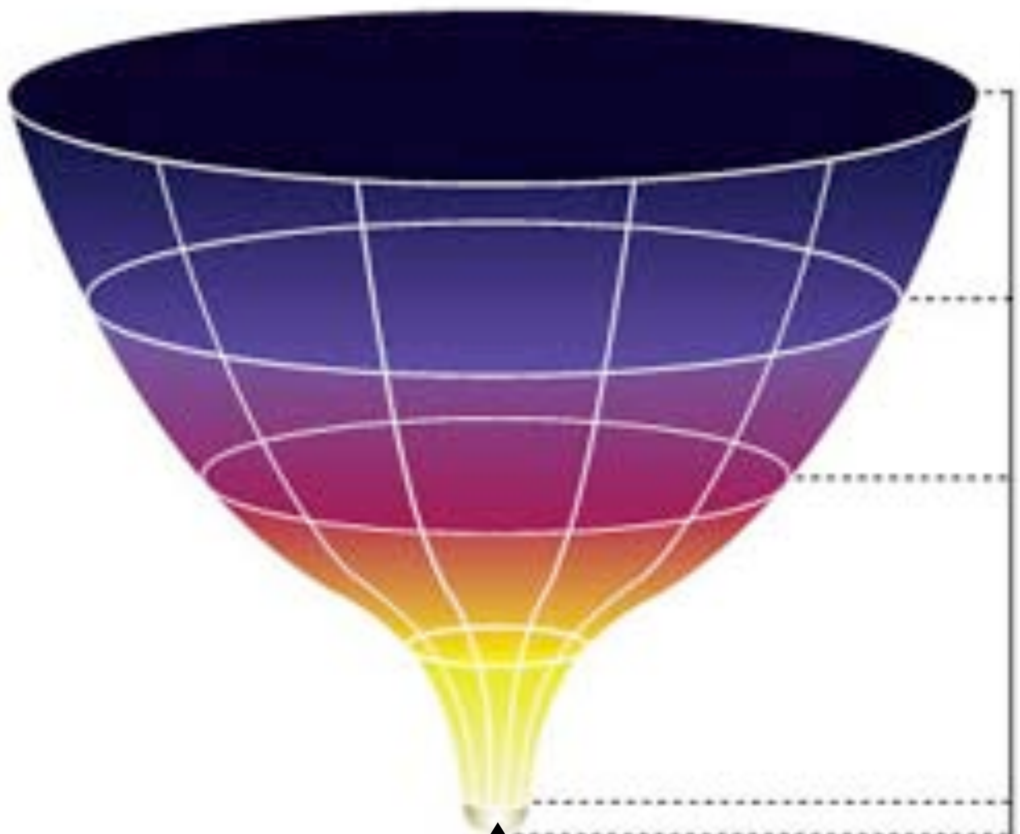
Primordial Gravitational wave

Primordial GW come back to observable universe



How to count up the Energy density of GW in our universe?

How to calculate energy density?

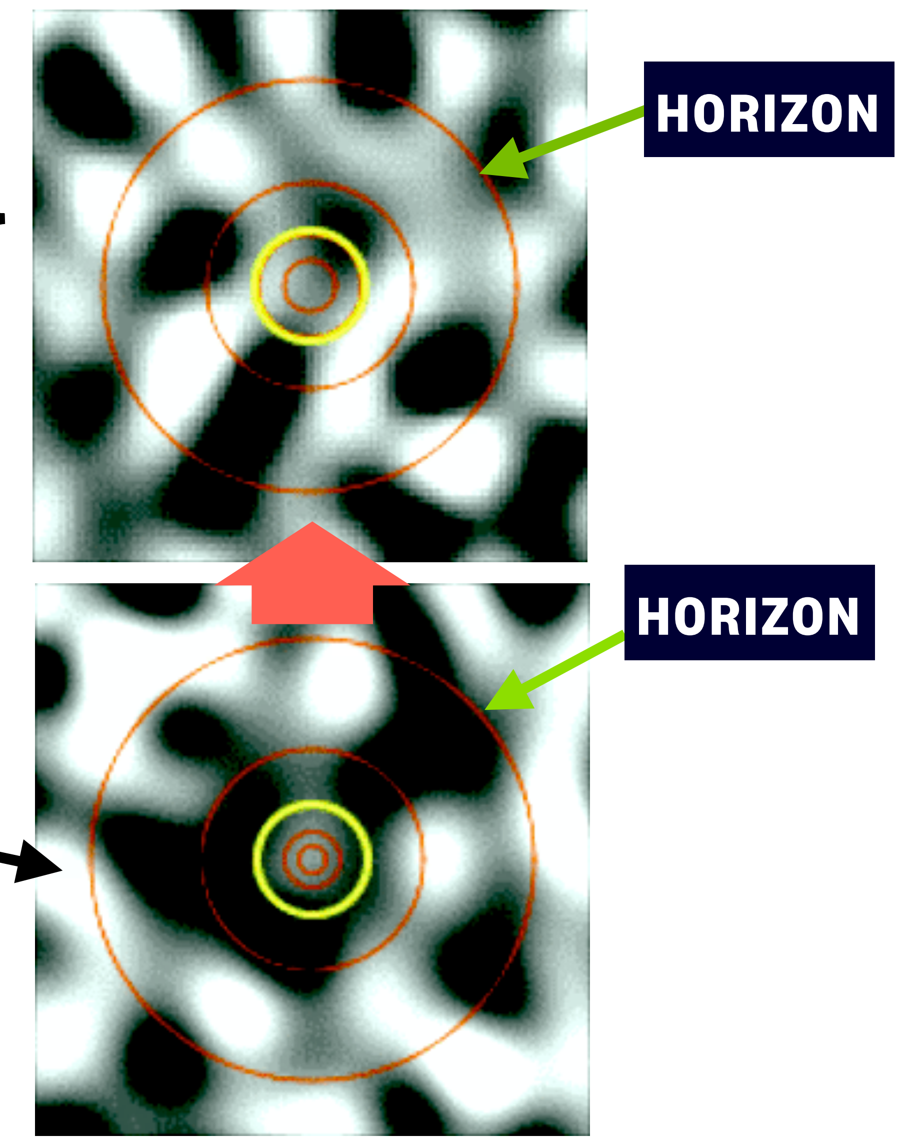


Now

Last Scattering Surface

Big Bang era

Inflation era

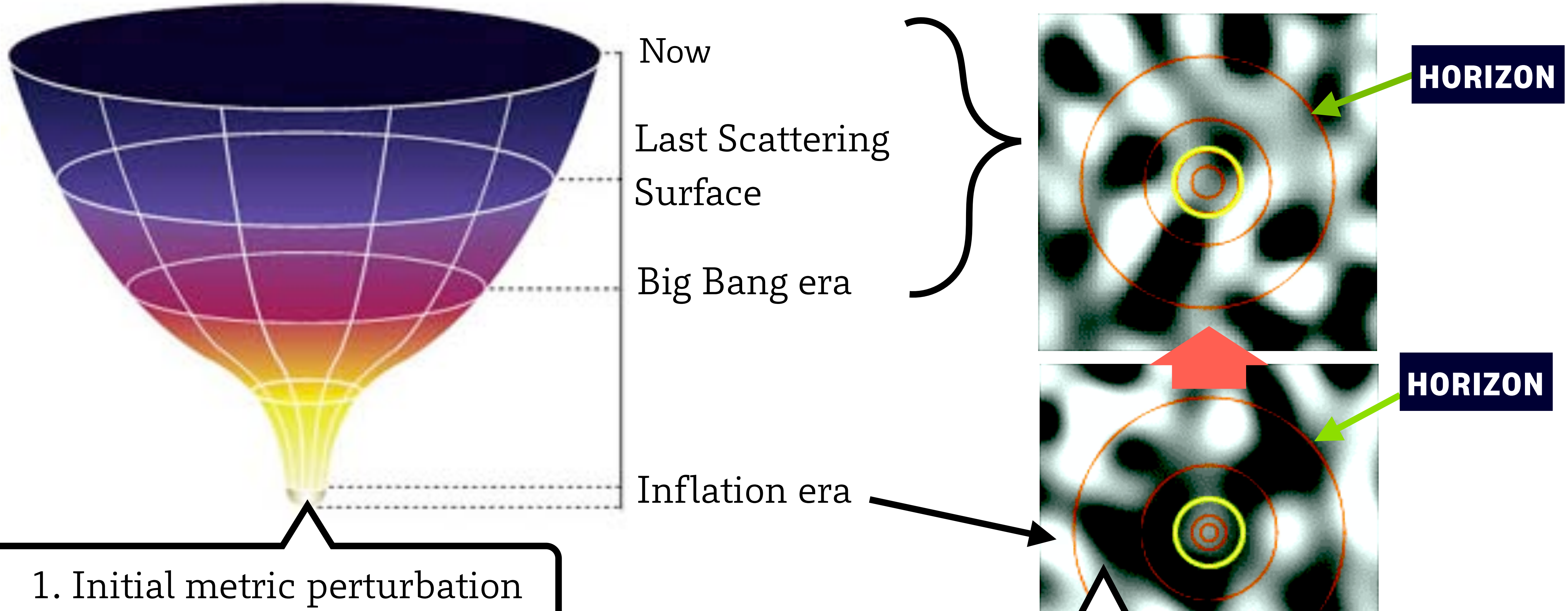


1. Initial metric perturbation from vacuum fluctuation

$$\mathcal{P}_{\mathcal{R}}^{vac}(k) = A_S\,(k/k_0)^{n_S - 1}$$

$$\mathcal{P}_{T}^{vac}(k) = A_T\left(\frac{k}{k_0}\right)^{n_T}$$

How to calculate energy density?



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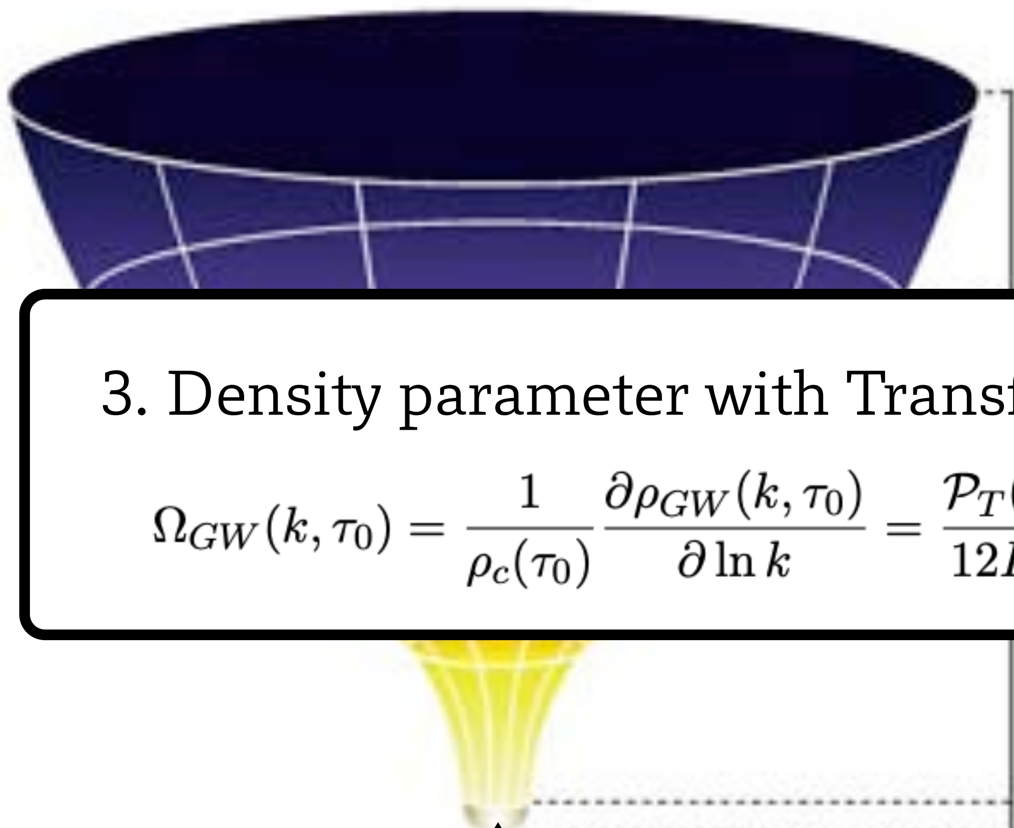
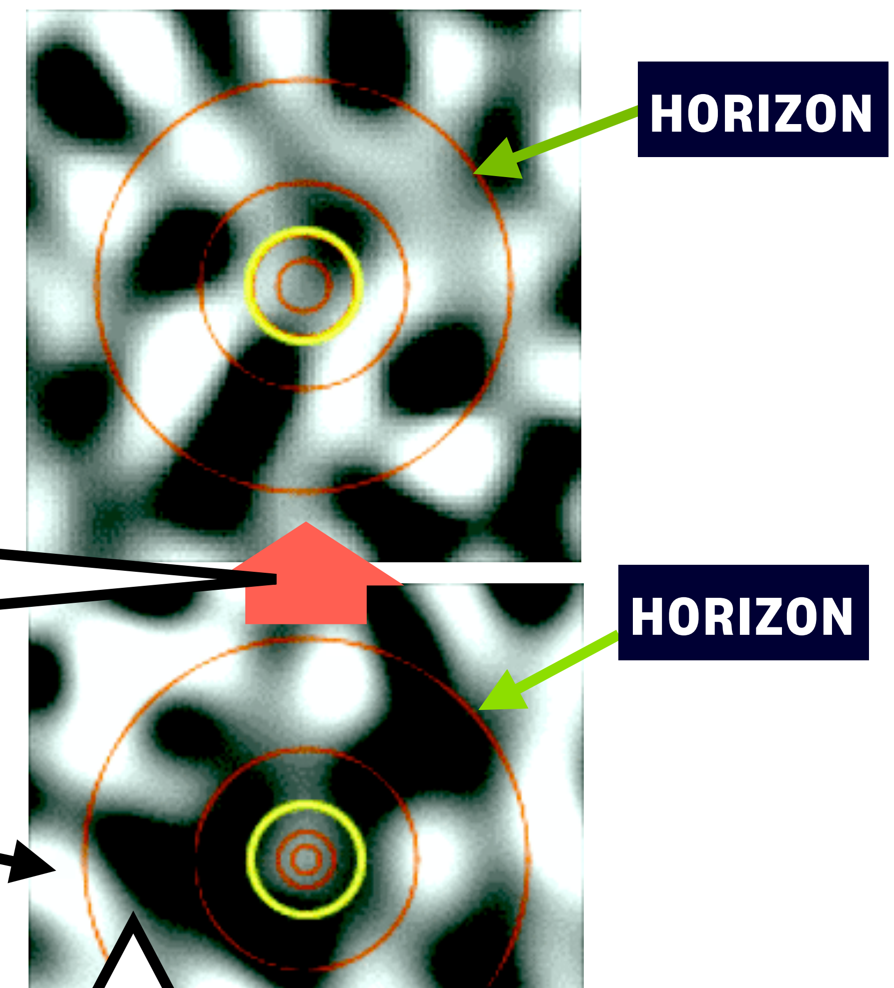
2. Inflationary expansion: inflaton

slow roll approximation

$$\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \qquad \frac{p}{\rho} \simeq \frac{\frac{1}{2}\dot{\phi}^2 - V}{\frac{1}{2}\dot{\phi}^2 + V} \simeq -1$$

How to calculate energy density?





3. Density parameter with Transfer function

$$\Omega_{GW}(k,\tau_0) = \frac{1}{\rho_c(\tau_0)} \frac{\partial \rho_{GW}(k,\tau_0)}{\partial \ln k} = \frac{\mathcal{P}_T(k)}{12H_0^2} \cdot \left[\mathcal{T}'(k,\tau_0)\right]^2$$

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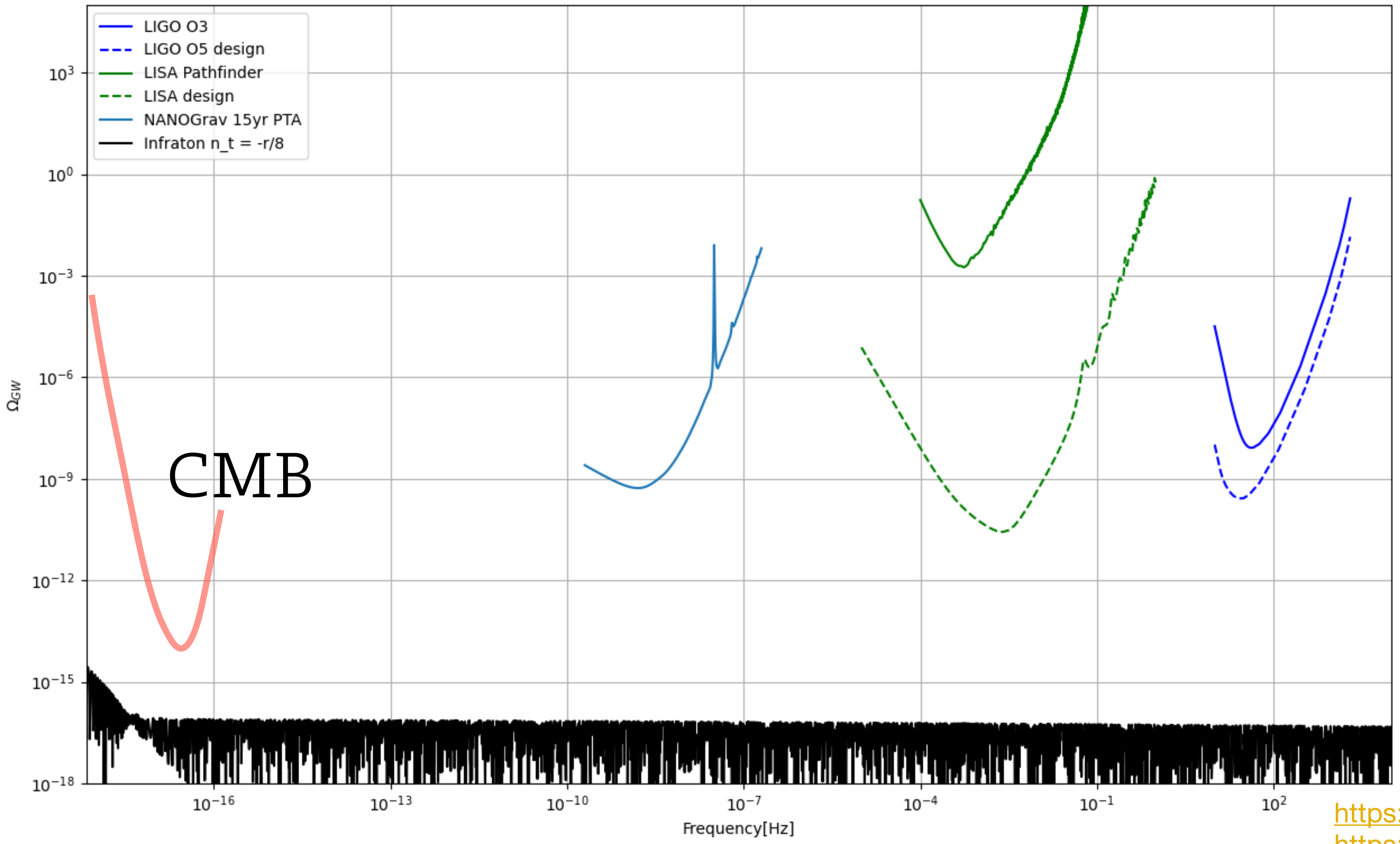
Density Parameter: Ω_{GW}

Transfer function for matter and radiation dominant:

$$\Omega_{GW}(k,\tau_0) = \frac{\mathcal{P}_T(k)}{12H_0^2}k^2 \cdot \begin{cases} \frac{\tau_*^2}{\tau_0^2}\left[A(k)j_2(k\tau_0)+B(k)y_2(k\tau_0)\right]^2, & \text{if } k > k_*, \\ \left[\frac{3j_2(k\tau_0)}{k\tau_0}\right]^2, & \text{if } k < k_*, \end{cases}$$

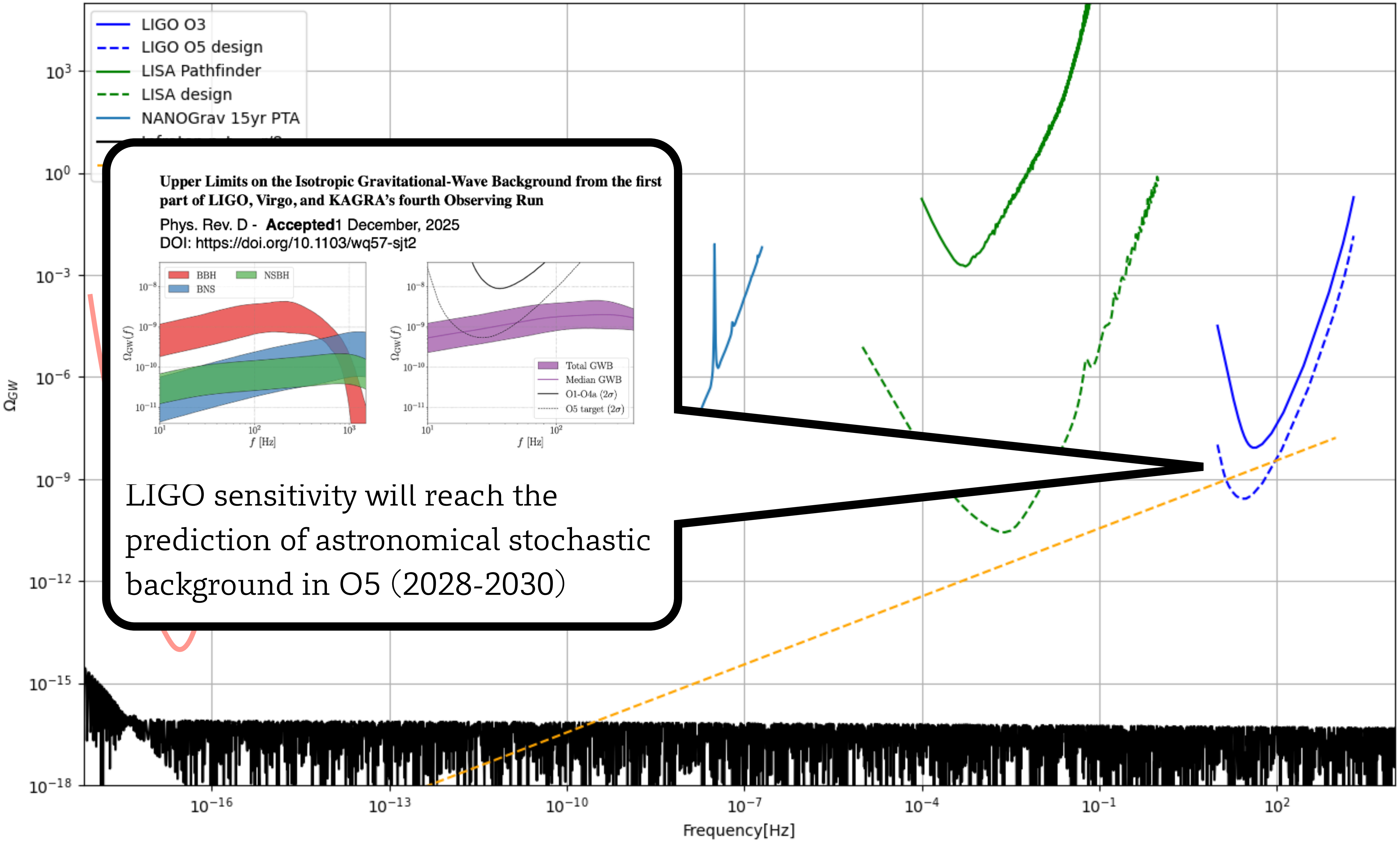
Radiation dominant $\propto f^0$

Matter dominant $\propto f^{-2}$



https://arxiv.org/pdf/2106.12843
https://arxiv.org/pdf/1904.02115

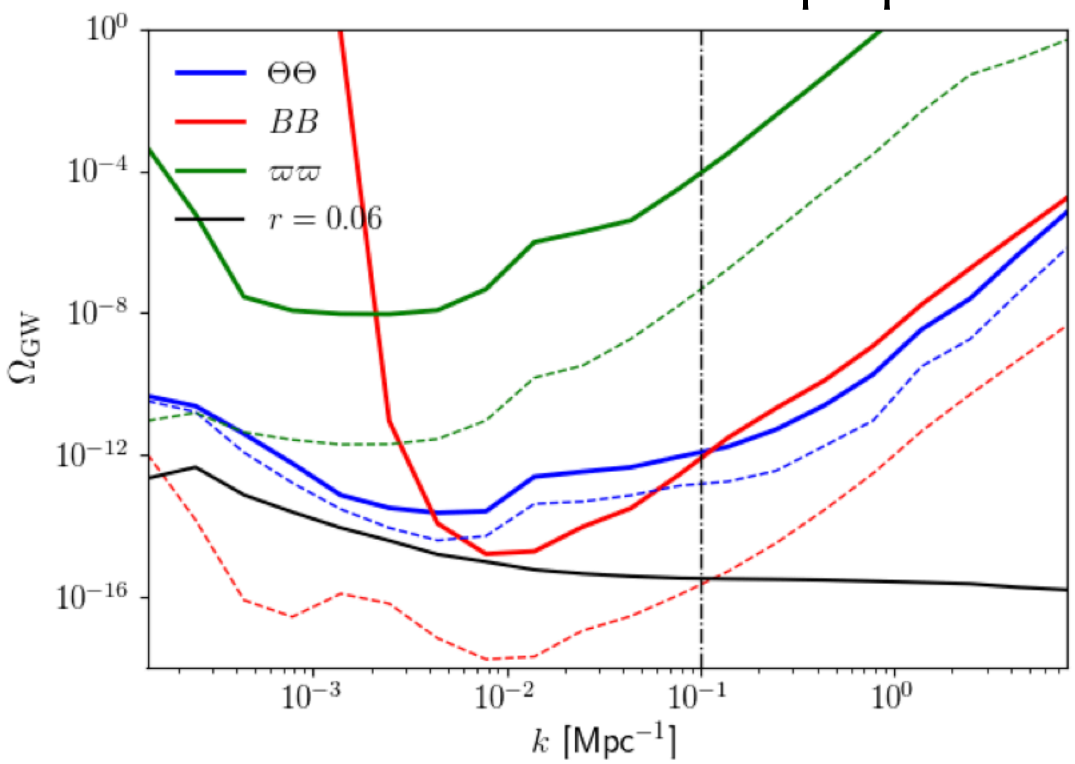
Expected next challenge of LIGO!



Introduction of Namikawa et al.

Namikawa et al., Phys. Rev. D **100**, 021303(R) – **Published 19 July, 2019**
DOI: https://doi.org/10.1103/PhysRevD.100.021303

Conclusion of this paper



→Upper limit of ΩGW with Planck and BK result

Principle (with example of TT)

$$\Omega_{\rm tot} = \Omega_\Lambda + \Omega_{\rm r} + \Omega_{\rm m} + \dots$$

$$\Omega_\gamma + \Omega_{\rm gw} + \dots \qquad \Omega_{\rm dm} + \Omega_{\rm b} + \dots$$

$\Omega_{\rm gw} > 0$

non-zero.

But, not measured.

Frequency dependency

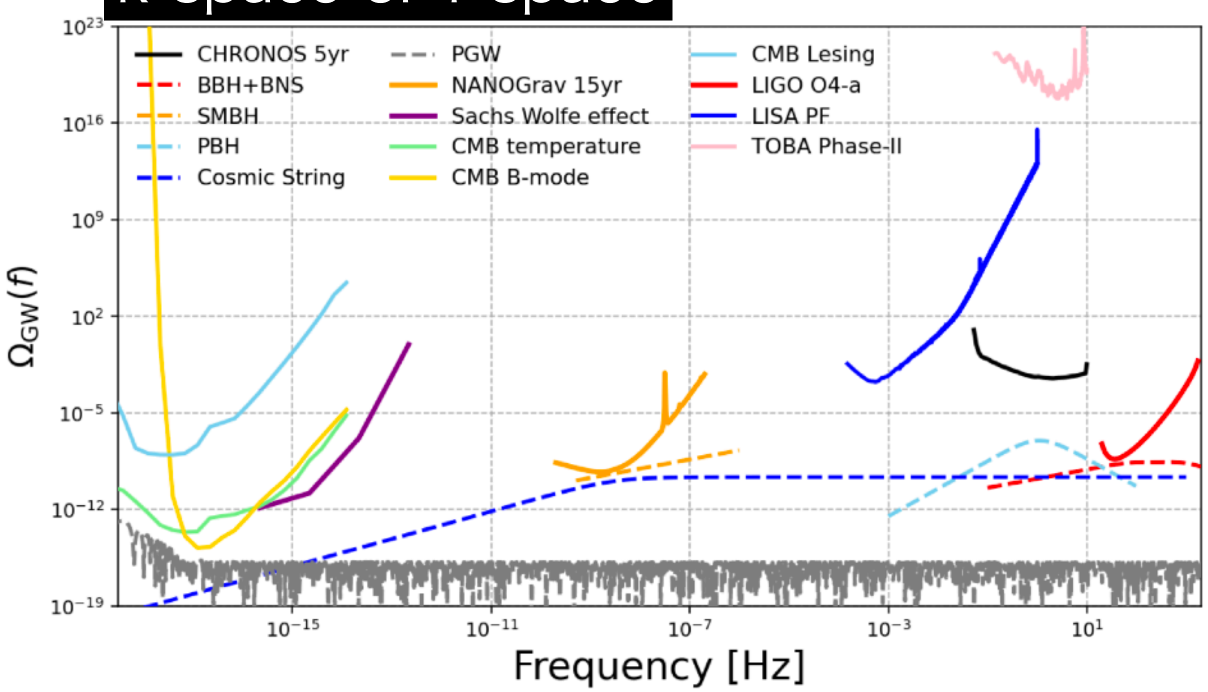
$$\Omega_{\rm GW}(f) = \frac{1}{\rho_c}\frac{d\rho_{\rm GW}}{d\ln f},$$

Low frequency region

$$\Omega_{\rm GW}(k) = \Omega_{\rm GW}^{\rm CMB}\left(\frac{k}{k_*}\right)^{n_t}\left[\frac{1}{2}\left(\frac{k_{\rm eq}}{k}\right)^2 + \frac{16}{9}\right].$$

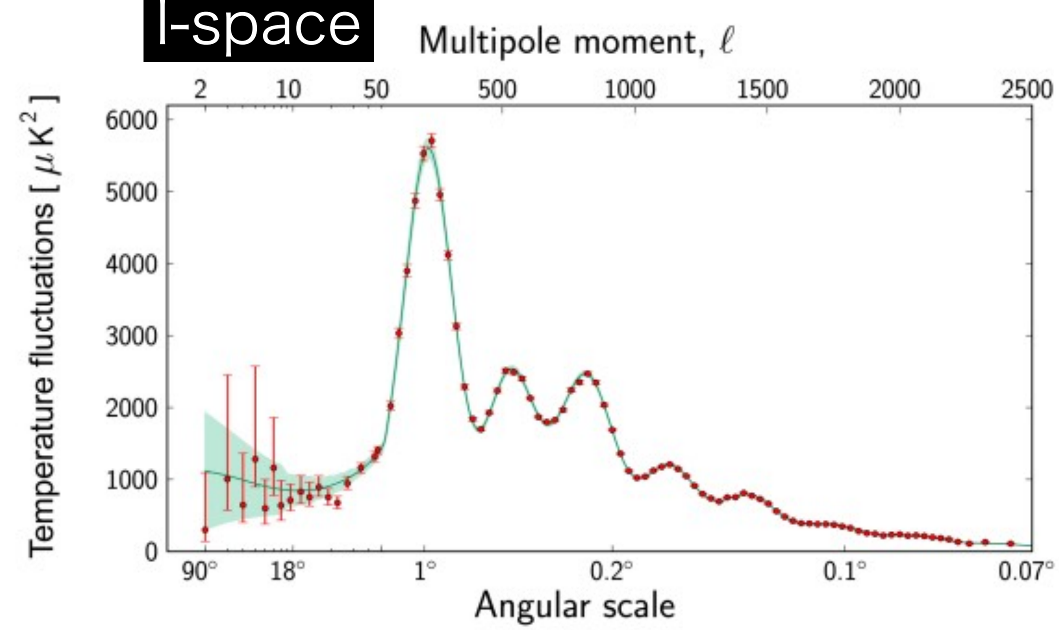
$$k = 2\pi f a_0.$$

k-space or f-space



How to connect?

l-space



Narrow band spectrum

$$\Pi(k;k_c) = \begin{cases} \frac{1}{2\Delta \ln k}, & k_c(1-\epsilon) \le k \le k_c(1+\epsilon), \\ 0, & \text{otherwise}, \end{cases}$$

$$P_t(k;k_c,A_t) = A_t\,\Pi(k;k_c).$$

Namikawa et al., Phys. Rev. D **100**, 021303(R) – **Published 19 July, 2019**
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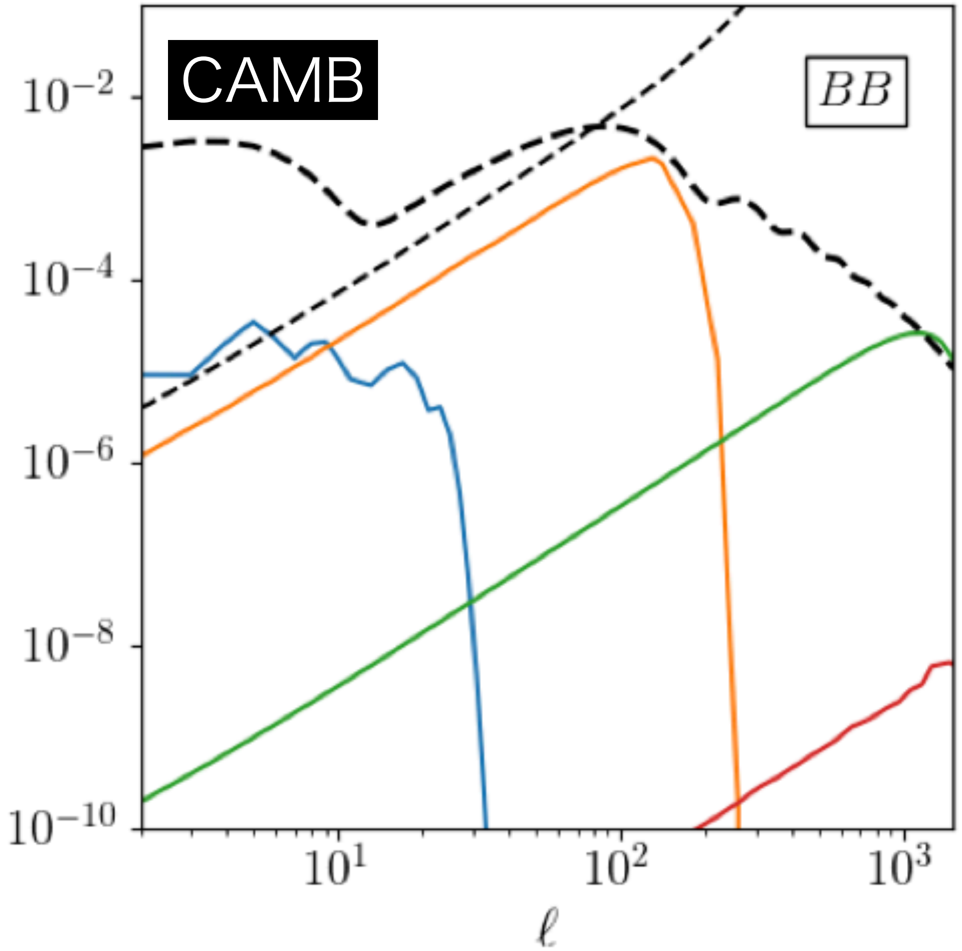


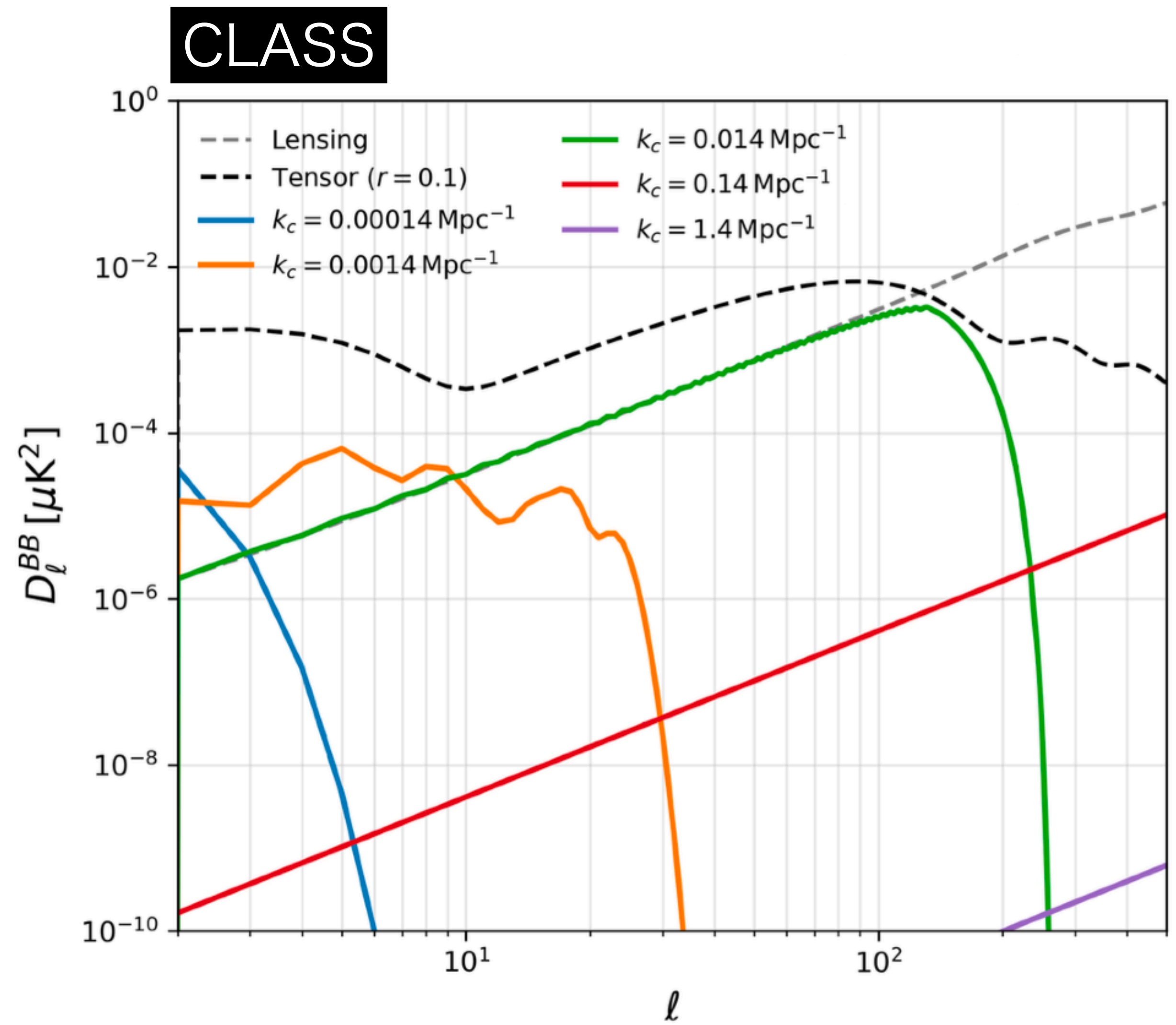


Figure 1: CMB B-mode spectra D_ℓ^{BB} for narrow-band primordial tensor perturbations with different central wavenumbers k_c. Colored lines show the resulting spectra, while the black and gray dashed lines represent the scale-invariant tensor spectrum ($r = 0.1$) and lensing contribution, respectively. Each narrow-band spectrum produces a localized peak in multipole space.

Likelihood BK18 dataset

$$\mathbf{d}^{\mathrm{th}} = \mathbf{d}^{\Lambda\mathrm{CDM}} + \mathbf{d}^{\mathrm{FG}}. \qquad \mathbf{\Delta d} = \mathbf{d}^{\mathrm{obs}} - \mathbf{d}^{\mathrm{th}}.$$

{WMAP, Planck, BICEP, KECK} {95GHz, 150GHz, 220GHz}

Result (Inoue et al. in preparation)

$$-2\ln\mathcal{L}(A_t|k_c) = (\mathbf{\Delta d}-\mathbf{d}^{\mathrm{narrow}})^{\top}\mathbf{C}^{-1}(\mathbf{\Delta d}-\mathbf{d}^{\mathrm{narrow}}),$$

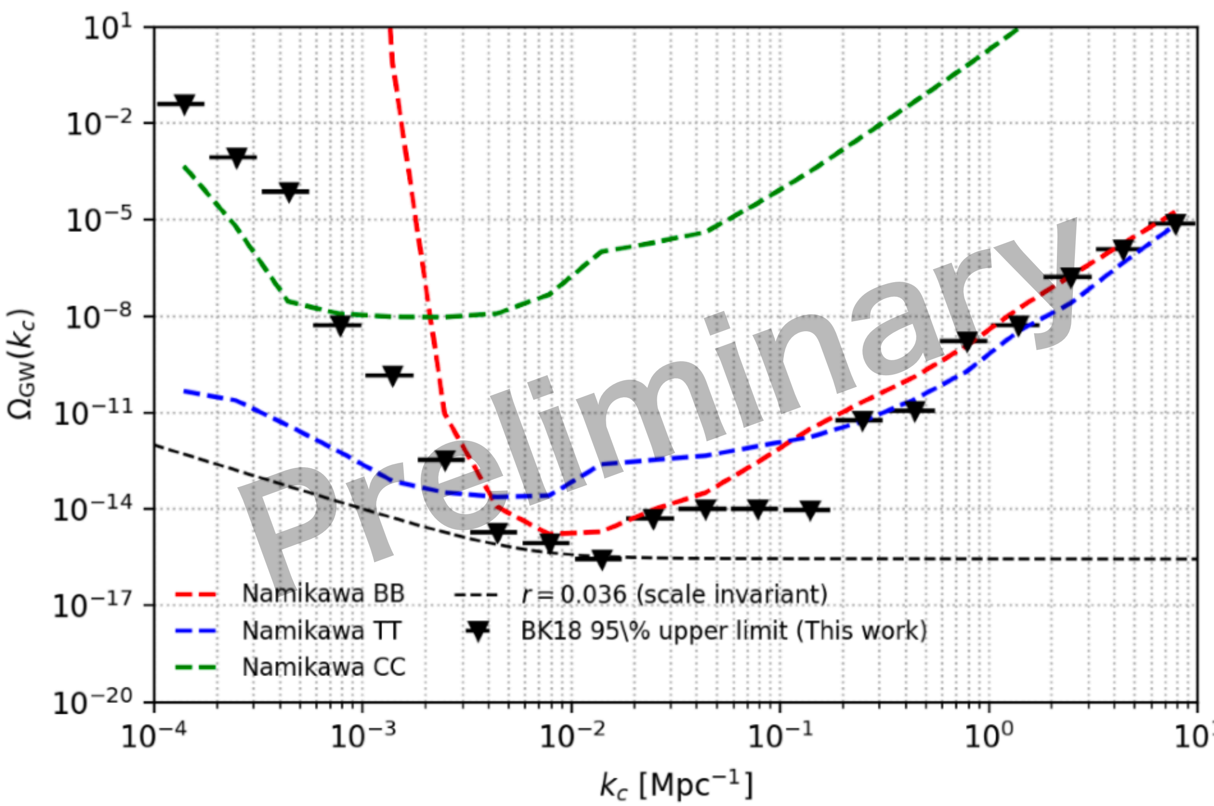


Figure 2: Constraints on $\Omega_{\mathrm{GW},95}(k_c)$ from BK18 B-mode data. Black triangles denote the 95% upper limits obtained in this work, compared with results from Namikawa et al. (dashed lines) and the scale-invariant case ($r = 0.036$, black dashed).

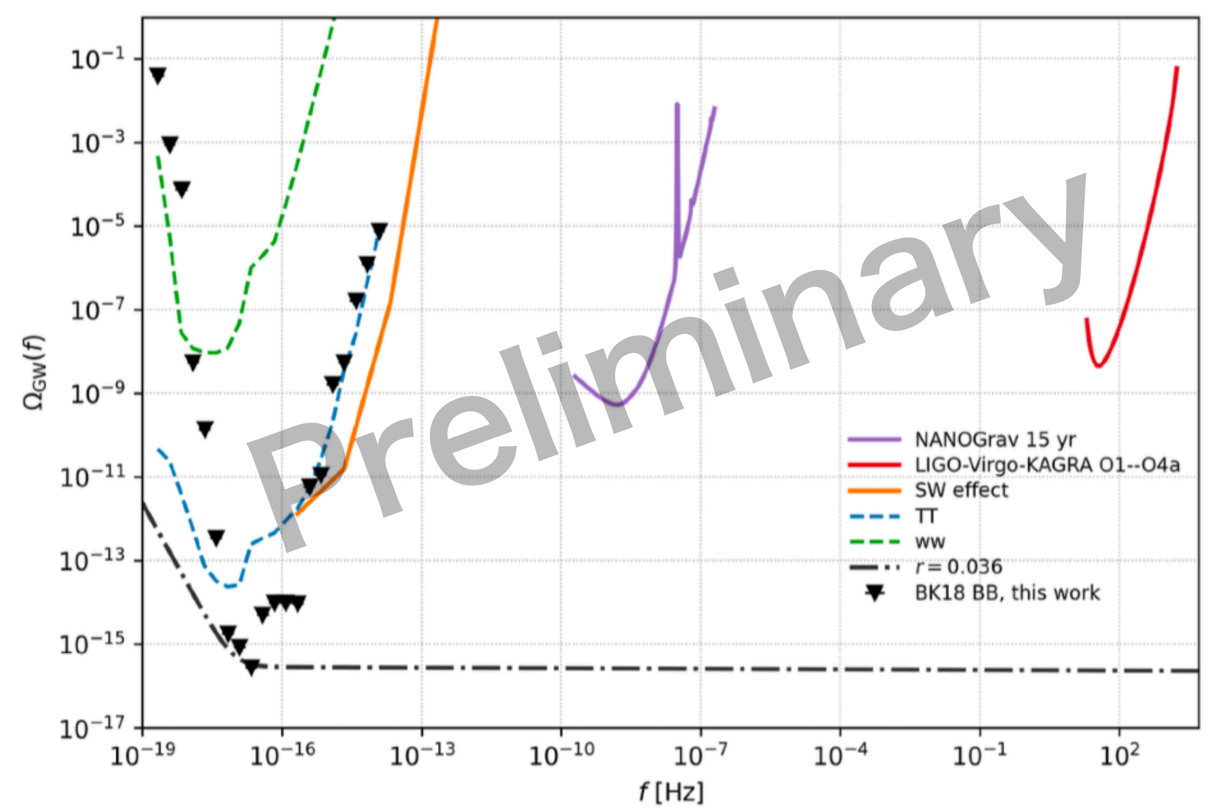


Figure 3: Constraints on $\Omega_{\mathrm{GW},90\mathrm{th}5}(f)$ over a wide frequency range. Black triangles show the BK18 limits obtained in this work. Dashed curves denote CMB-based constraints from TT, $\omega\omega$, and SW analyses (temperature, cross-correlation, and Sachs–Wolfe, respectively), together with results from NANOGrav 15 year and LIGO–Virgo from O1 to O4.

Similar study

Redshift-space fluctuations in stochastic gravitational wave background

Kin-Wang Ng[1,2]
[1] *Institute of Physics, Academia Sinica, Taipei 11529, Taiwan*
[2] *Institute of Astronomy and Astrophysics, Academia Sinica, Taipei 11529, Taiwan*
(Dated: July 22, 2022)

B. A narrow power spectrum

For the narrow power spectrum (10), we adopt the subhorizon-mode solution (11). The induced CMB anisotropy power spectrum is then given by

$$C_l \simeq \frac{1}{2\pi}(l+2)(l+1)l(l-1) \times \Delta \ln k_* \left| \int_{\eta_1}^{\eta_0} d\eta \frac{dh(k_*\eta)}{d\eta} \frac{j_l[k_*(\eta_0-\eta)]}{k_*^2(\eta_0-\eta)^2} \right|^2, \quad (20)$$

where $\eta_1 = \max(\eta_{\rm dec}, \eta_*)$. Assuming that the spectrum spans a range of $\Delta \ln k_* \simeq 1$ and that the universe was in a matter-dominated epoch with $a(\eta) = (\eta/\eta_0)^2$, we obtain

$$C_l \simeq \frac{M_*^2}{2\pi M_p^2}\left(\frac{\eta_*}{\eta_0}\right)^4 (l+2)(l+1)l(l-1) \times \left| \int_0^{x_1} dx \frac{1 - x/x_0 - i2/x_0}{(1-x/x_0)^3} e^{ix} \frac{j_l(x)}{x^2} \right|^2, \quad (21)$$

where $x = k_*(\eta_0 - \eta)$, $x_0 = k_*\eta_0$, and $x_1 = k_*(\eta_0 - \eta_1)$.

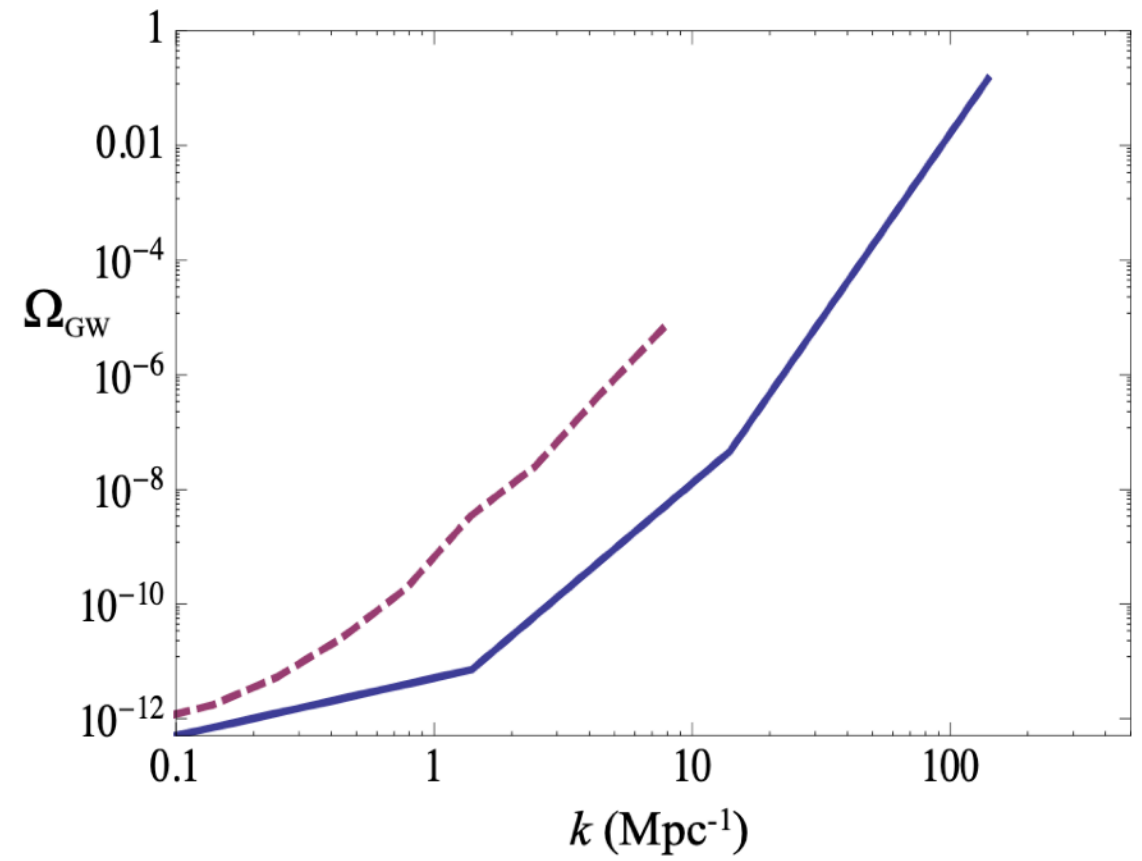
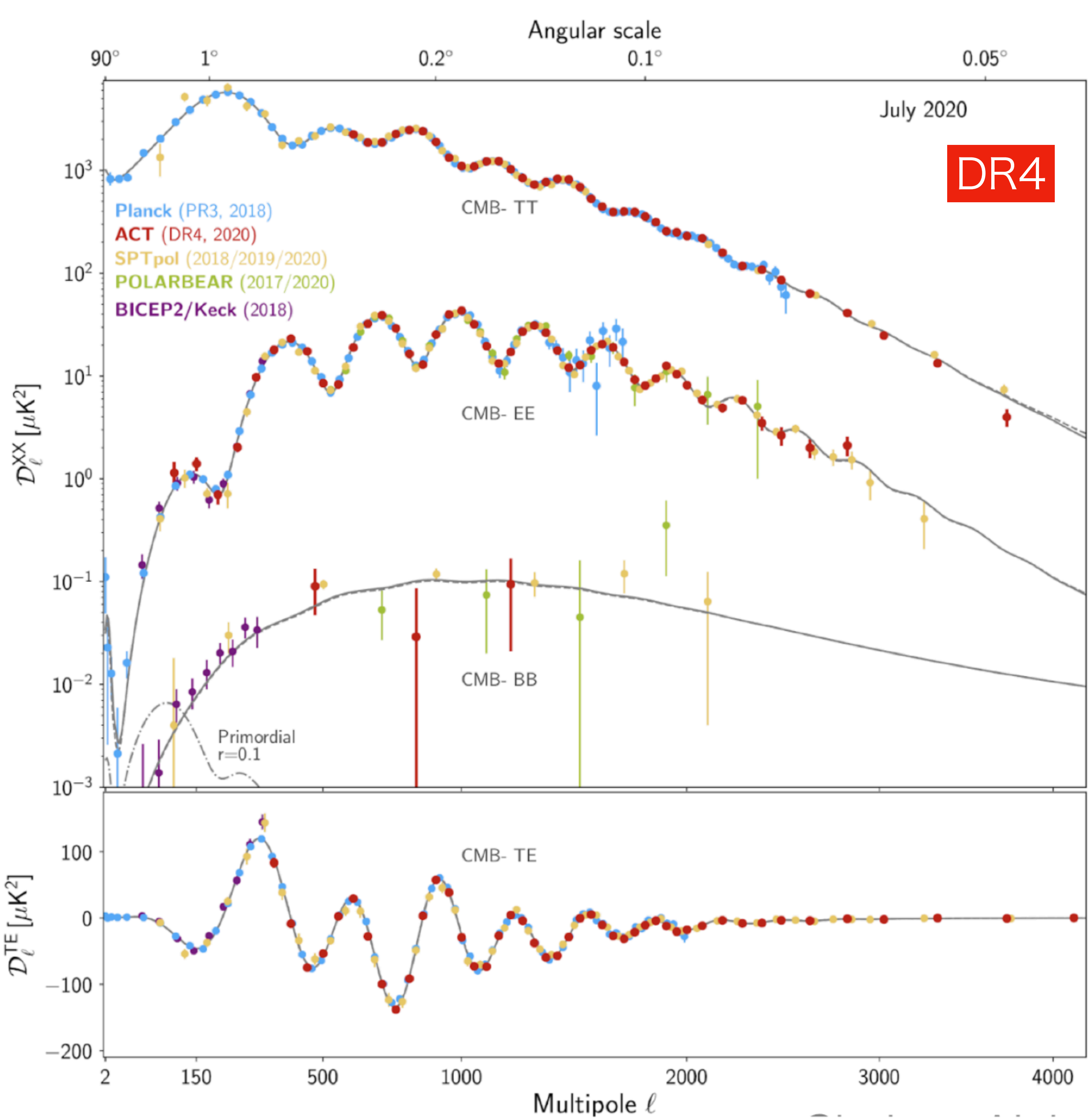
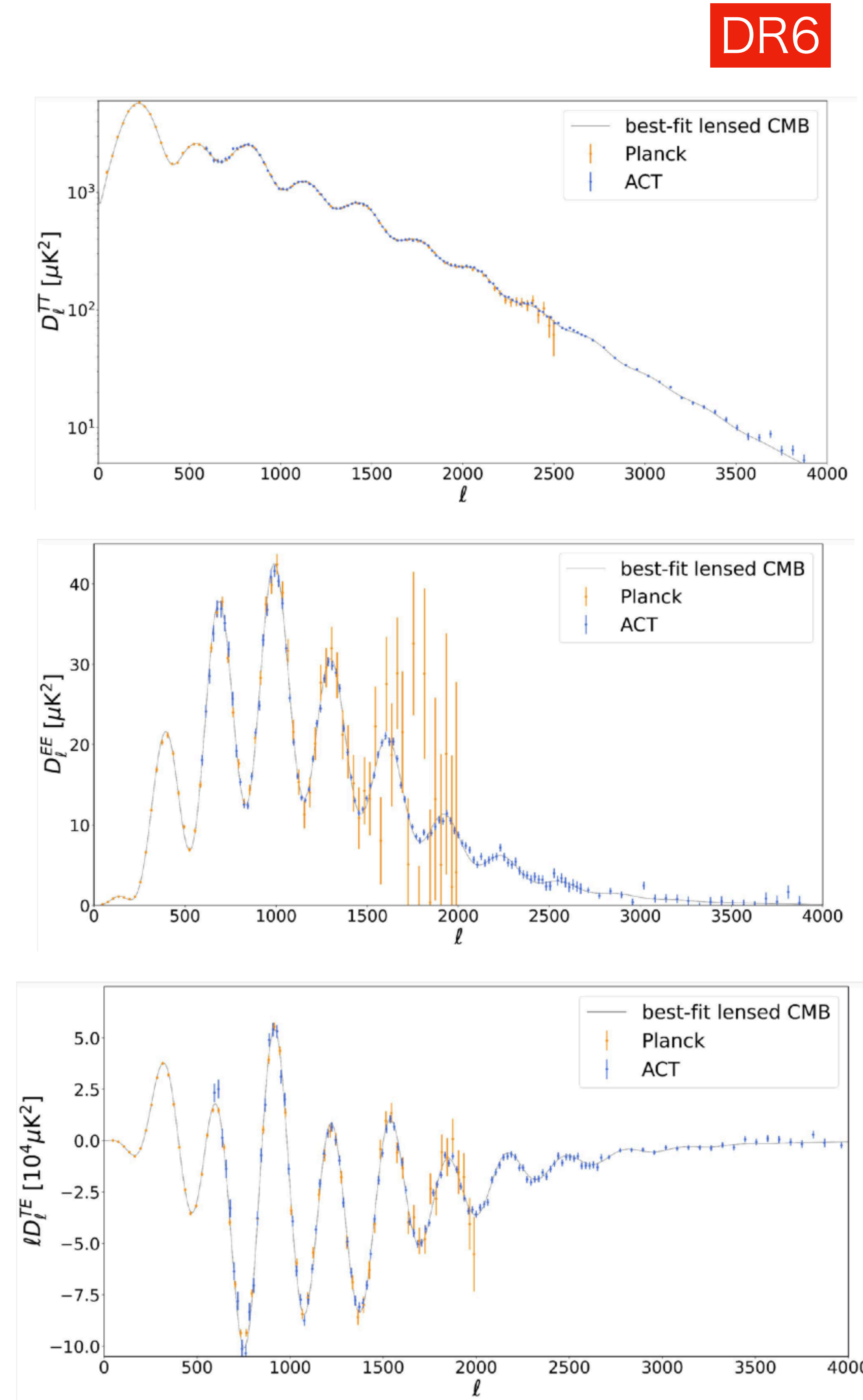


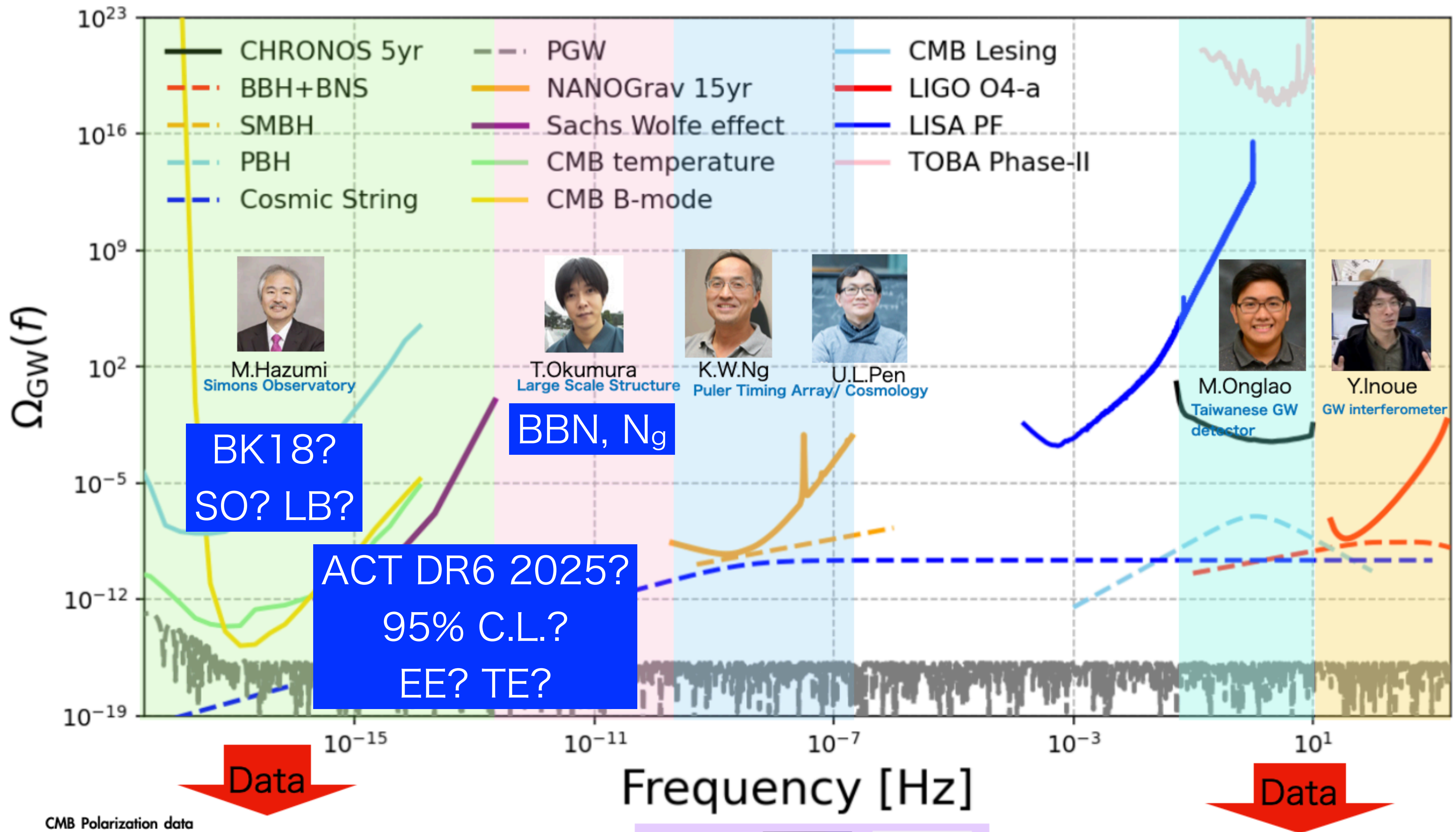
FIG. 2. Solid line is the upper bound on the spectral energy density of the SGWB derived from Planck, ACT, and SPT small-scale CMB temperature anisotropy measurements in this work. The dashed line, drawn from the blue solid curve in Figure 4 of Ref. [9], is the upper bound derived from a likelihood analysis using the CMB temperature anisotropy data made by Planck.

Example: ACT DR6 Data set





Team Taiwan



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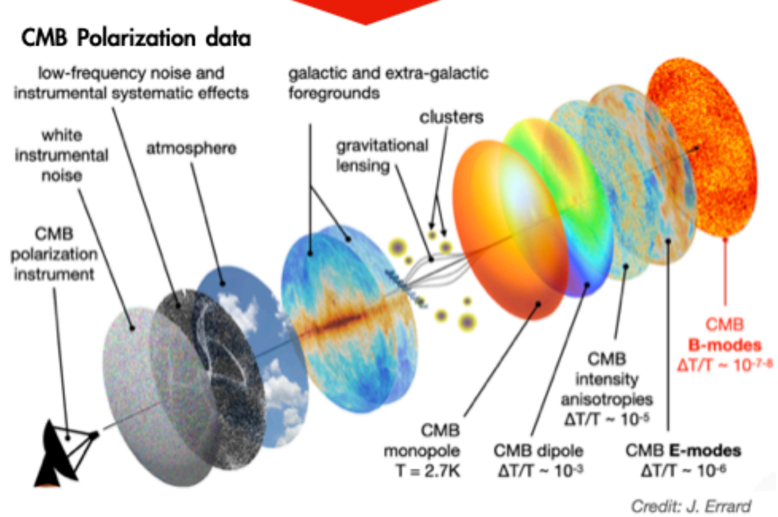
Puler Timing Array/ Cosmology
M.Onglao
Taiwanese GW detector
Y.Inoue
GW interferometer
BK18?
SO? LB?
BBN, N_g
ACT DR6 2025?
95% C.L.?
EE? TE?
Data
Data



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low-frequency noise and instrumental systematic effects
galactic and extra-galactic foregrounds
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white instrumental noise
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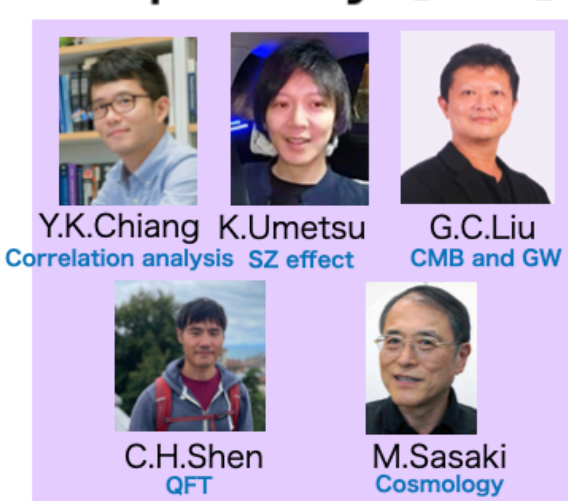
Credit: J. Errard



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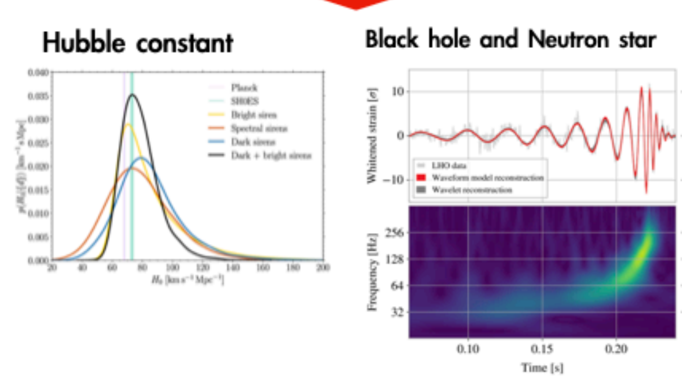
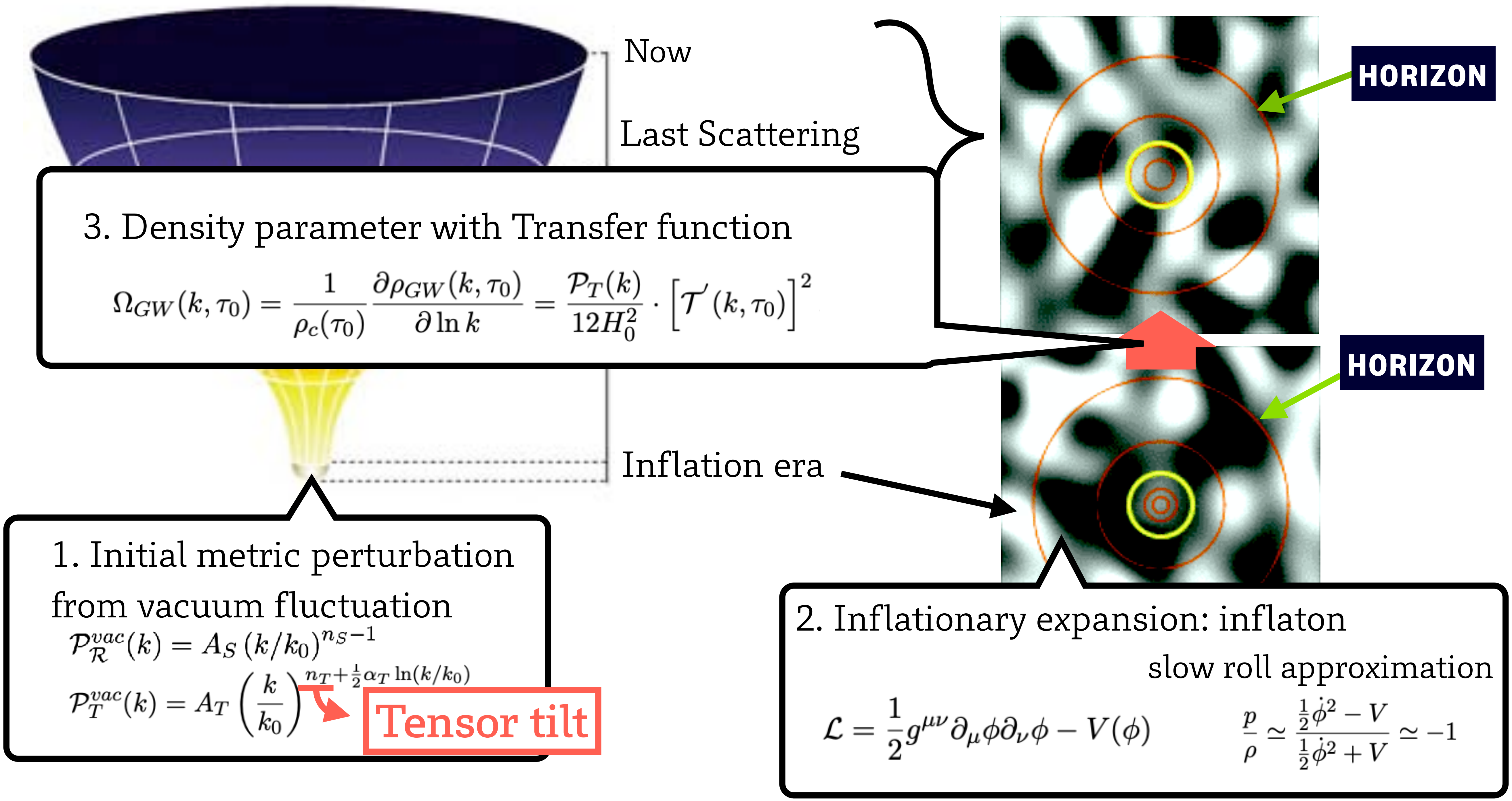


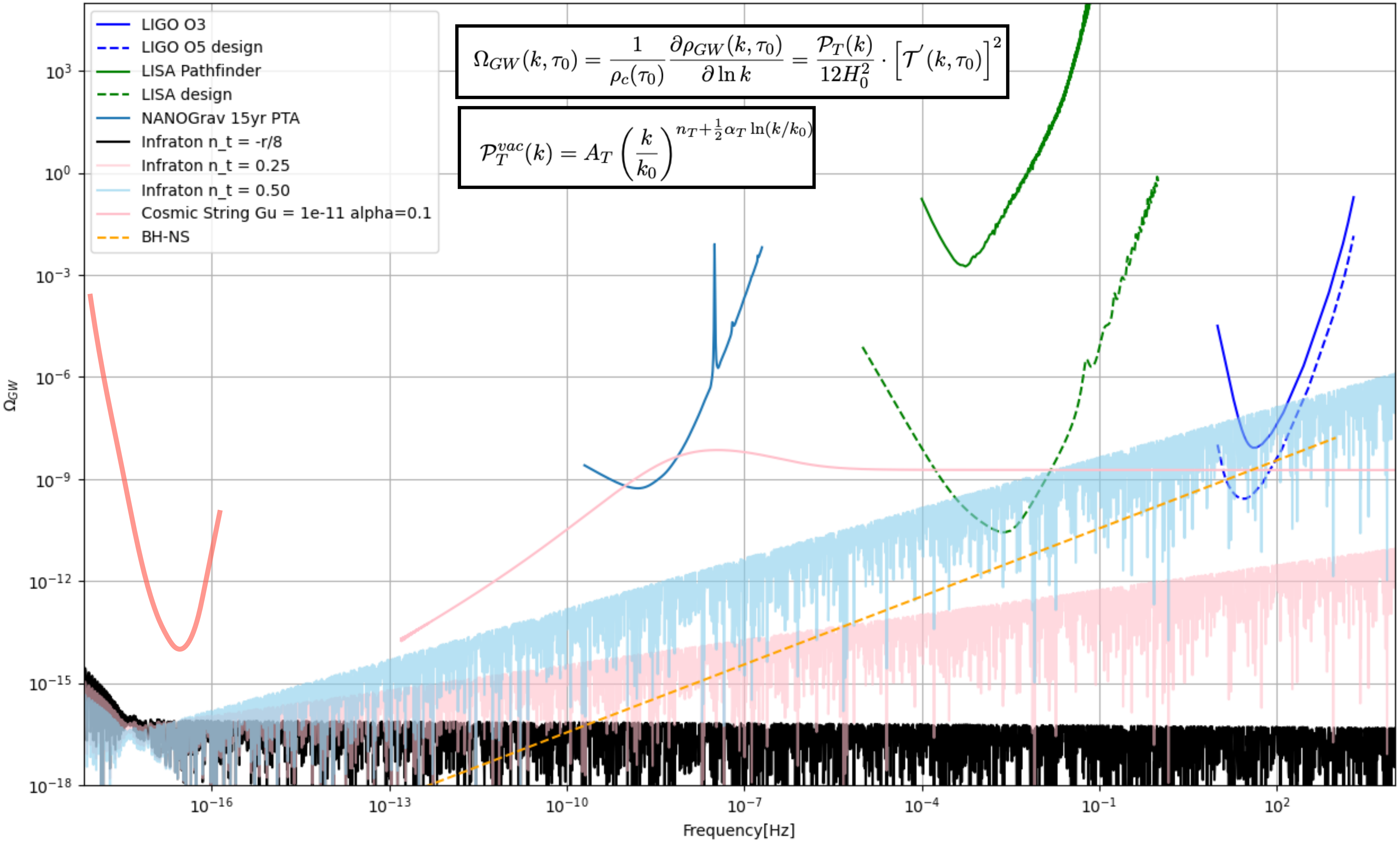
Hubble constant
Black hole and Neutron star

Consistency check of inflation

Test of model



Tensor tilt



https://journals.aps.org/prx/pdf/10.1103/PhysRevX.6.011035

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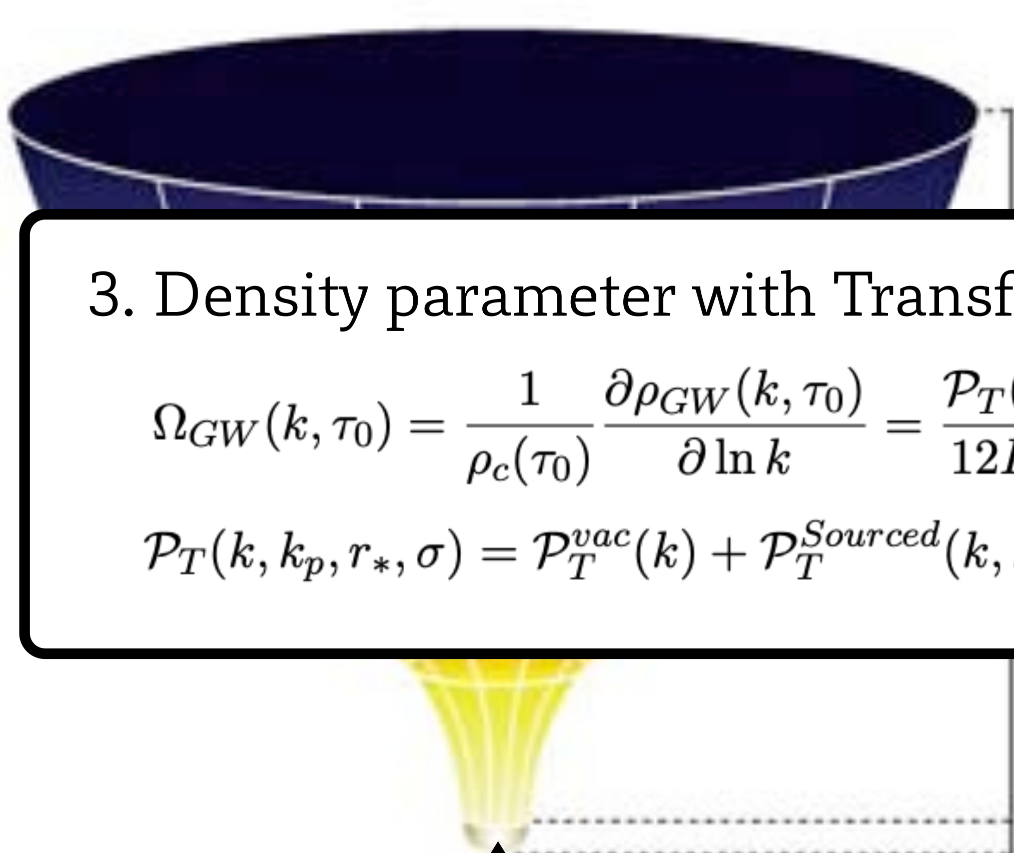

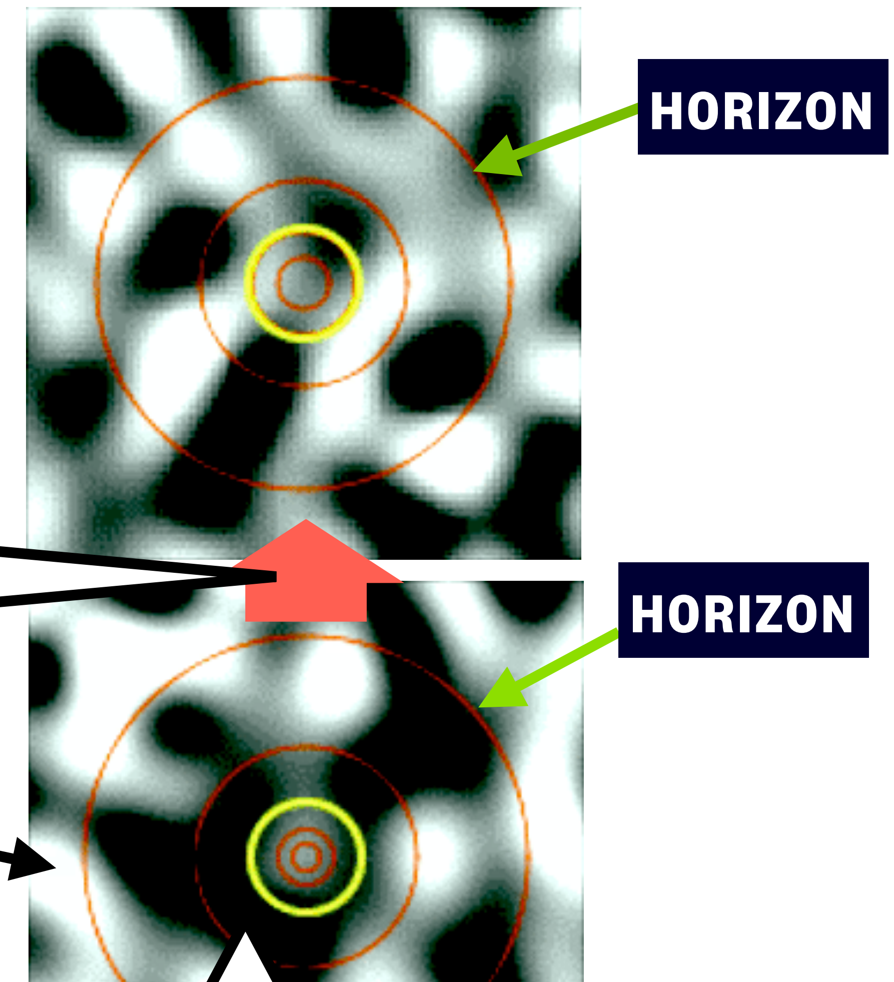

3. Density parameter with Transfer function

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$$\mathcal{P}_T(k,k_p,r_*,\sigma) = \mathcal{P}_T^{vac}(k) + \mathcal{P}_T^{Sourced}(k,k_p,r_*,\sigma),$$

1. Initial metric perturbation from vacuum fluctuation

$$\mathcal{P}_{\mathcal{R}}^{vac}(k) = A_S\,(k/k_0)^{n_S-1}$$

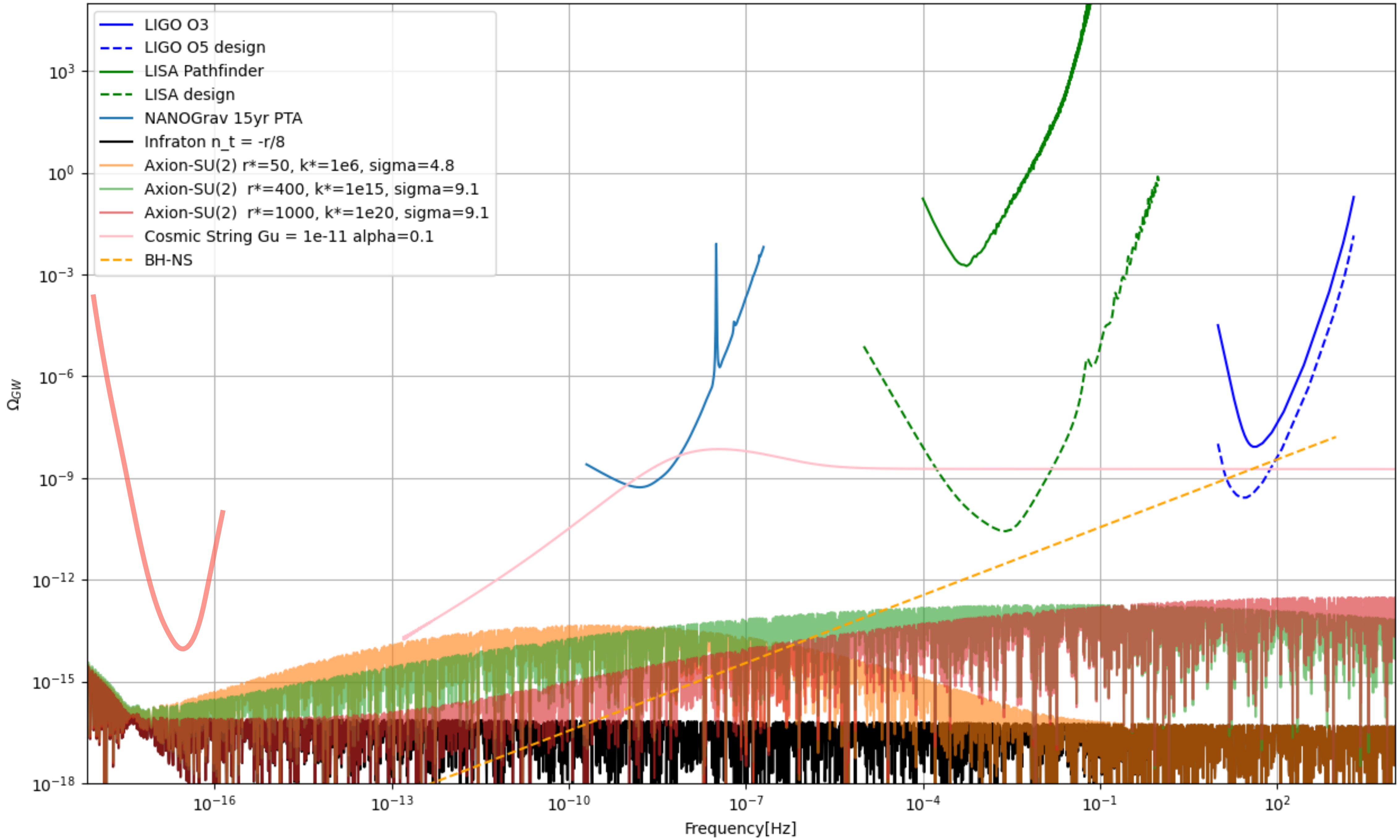
$$\mathcal{P}_T^{vac}(k) = A_T \left(\frac{k}{k_0}\right)^{n_T+\frac{1}{2}\alpha_T \ln(k/k_0)}$$

2. Inflationary expansion: inflaton + Axion

$$\mathcal{L} = \mathcal{L}_{inflaton} + \frac{1}{2}\left(\partial_\mu \chi\right)^2 - \mu^4\left[1+\cos\left(\frac{\chi}{f}\right)\right] - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \frac{\lambda}{4f}\chi F_{\mu\nu}\tilde{F}^{a\mu\nu},$$

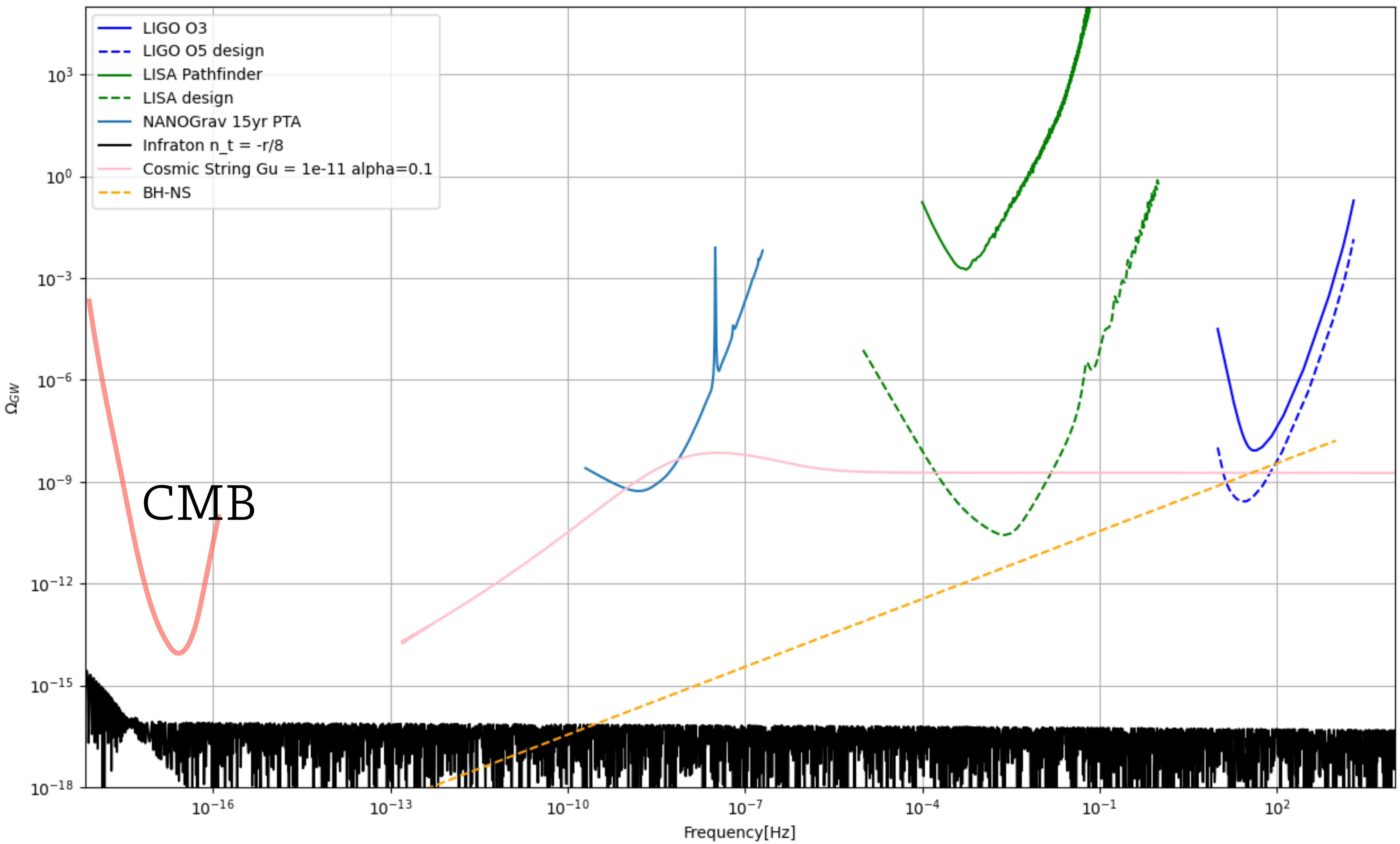
$$\mathcal{P}_T^{L,\,Sourced}(k) = r_*\mathcal{P}_{\mathcal{R}}(k)\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{k}{k_p}\right)\right]$$

SU(2) Axion



https://arxiv.org/pdf/2007.04241

Cosmic string (Collaboration is welcome)



PhysRevD.101.103508

Simulation Setup

- Boltzmann code: CLASS
- MCMC: cobaya
- Likelihood: $\mathcal{L}_{\rm total} = \mathcal{L}_{\rm CMB}\,\mathcal{L}_{\rm GW}.$
  - likelihood:
  - planck 2018 high l TT,TE,EE
  - planck 2018 low l.TT
  - planck 2018 low l EE
  - bicep keck 2018
  - $\Omega_{\rm GW}$ Likelihood
- Parameter: $\Theta = \{\Omega_b, \Omega_c, h, \tau, n_s, A_s, r, n_t\}$

GW Likelihood

$$\Omega_{\rm sens}(f) = \frac{2\pi^2}{3H_0^2} f^3 S_h(f).$$

$$\sigma_i = \frac{\Omega_{\rm sens}(f_i)}{\sqrt{2T\,\Delta f_{{\rm bin},i}}}.$$

$$\chi^2 = \sum_i \frac{\Omega_{\rm GW}^{\rm th}(f_i)^2}{\sigma_i^2}.$$

Model

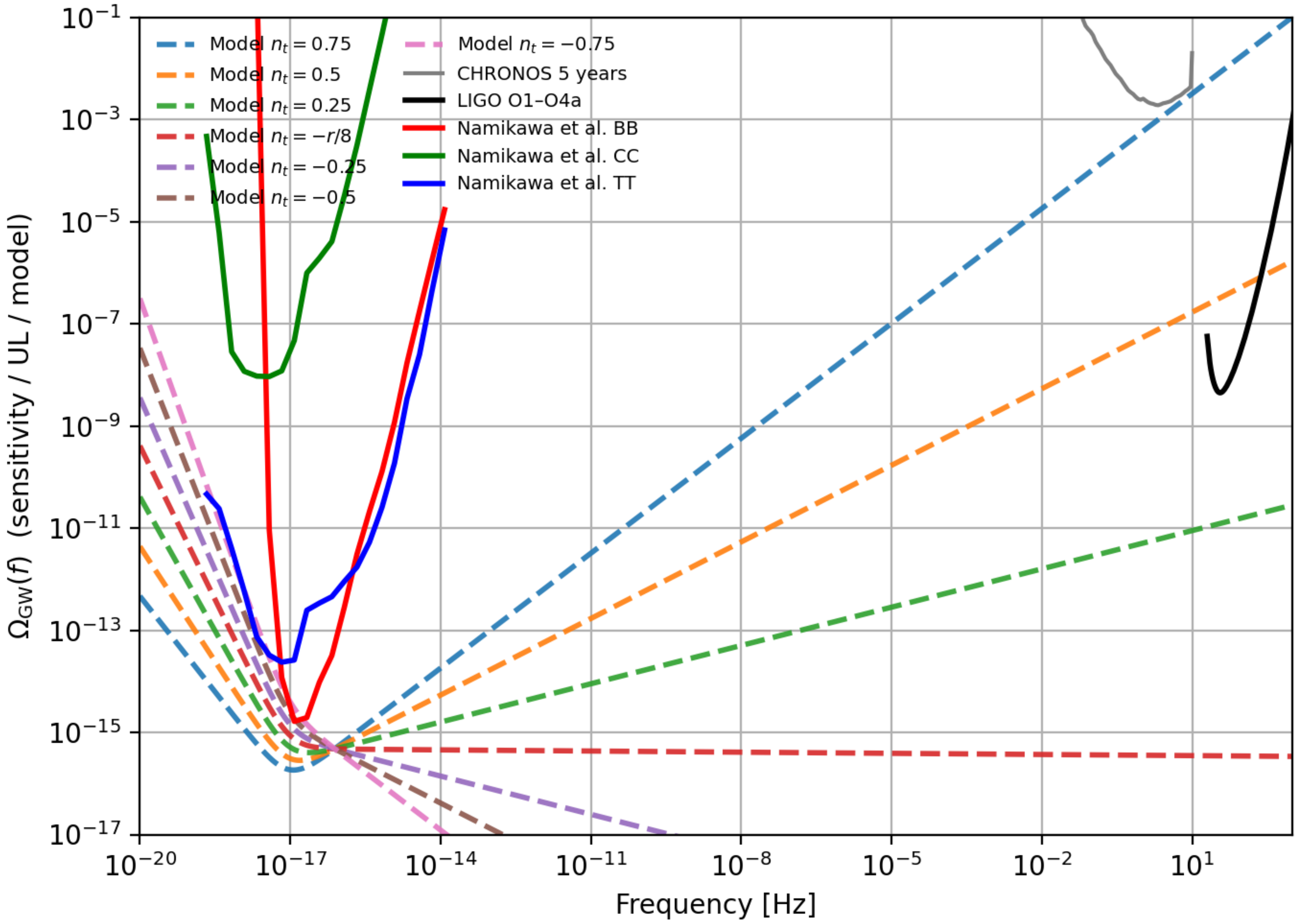
$$\Omega_{\rm GW}(k) = \Omega_{\rm GW}^{\rm CMB}\left(\frac{k}{k_*}\right)^{n_t}\left[\frac{1}{2}\left(\frac{k_{\rm eq}}{k}\right)^2 + \frac{16}{9}\right].$$

$$\Omega_{\rm GW}^{\rm CMB} = \frac{3}{128} r A_s \Omega_r.$$

Giacomo Gallonic et al. (2024)

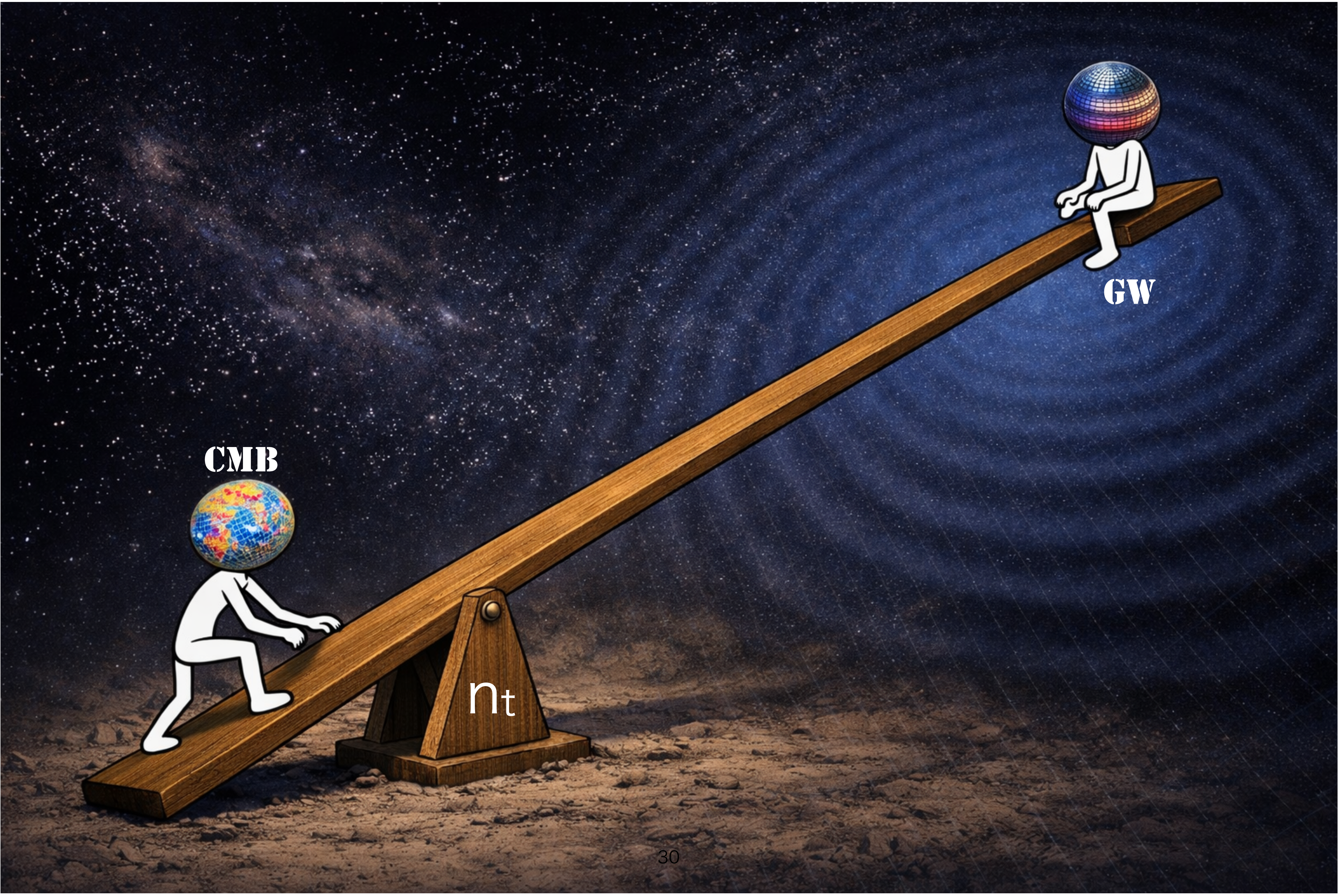
https://journals.aps.org/prd/pdf/10.1103/PhysRevD.110.063511

Tensor tilt parameter



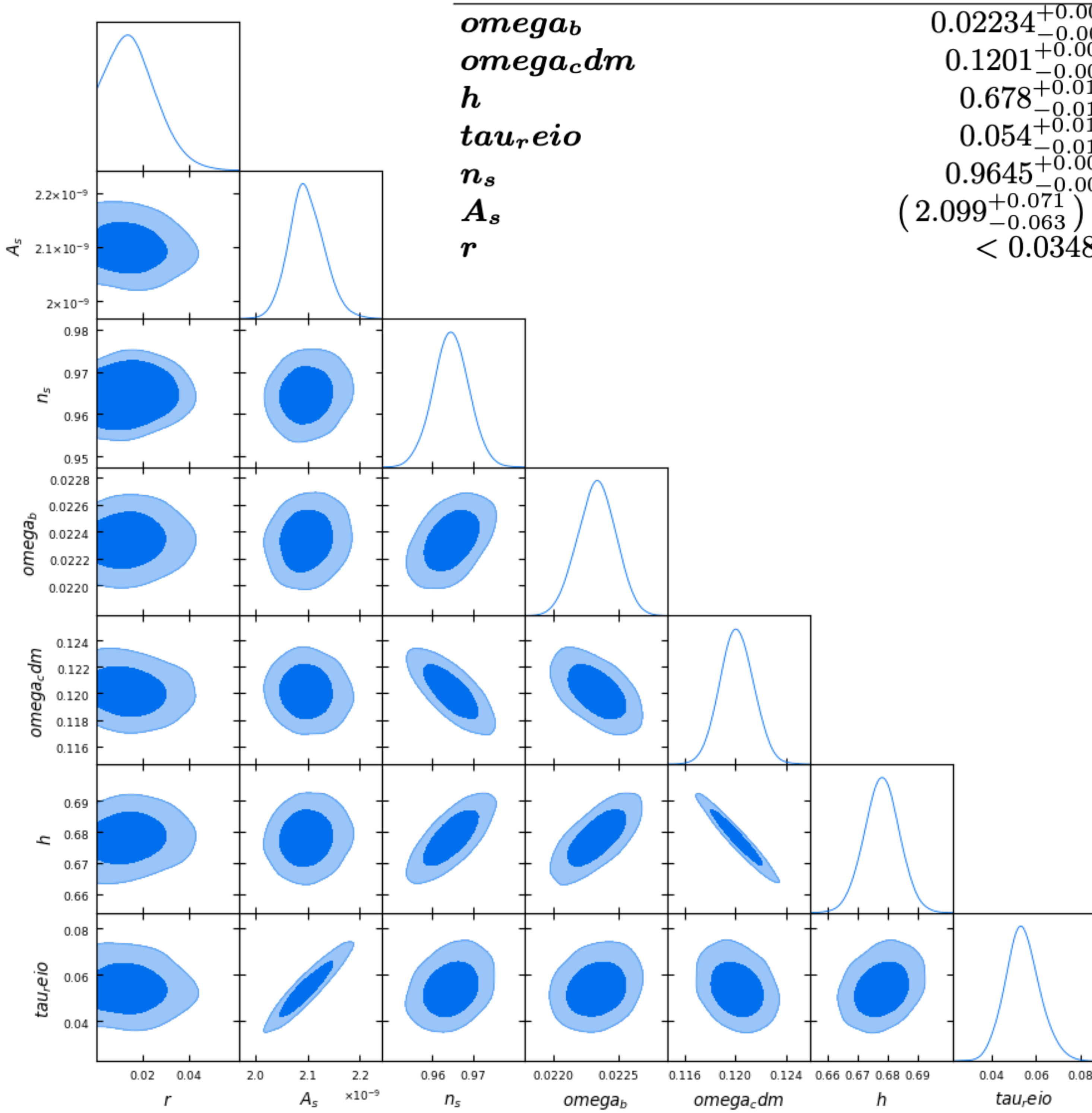
https://journals.aps.org/prx/pdf/10.1103/PhysRevX.6.011035

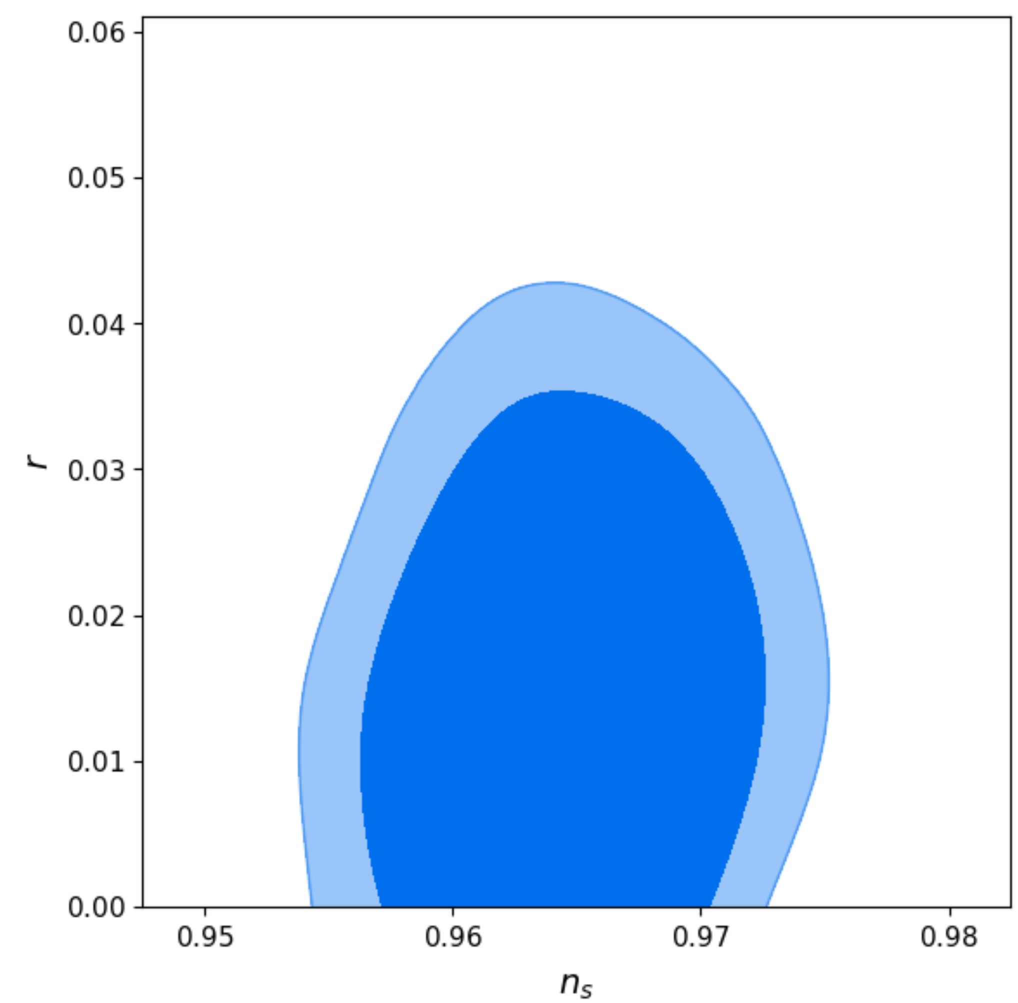
Cosmic Arm-Lever Effect



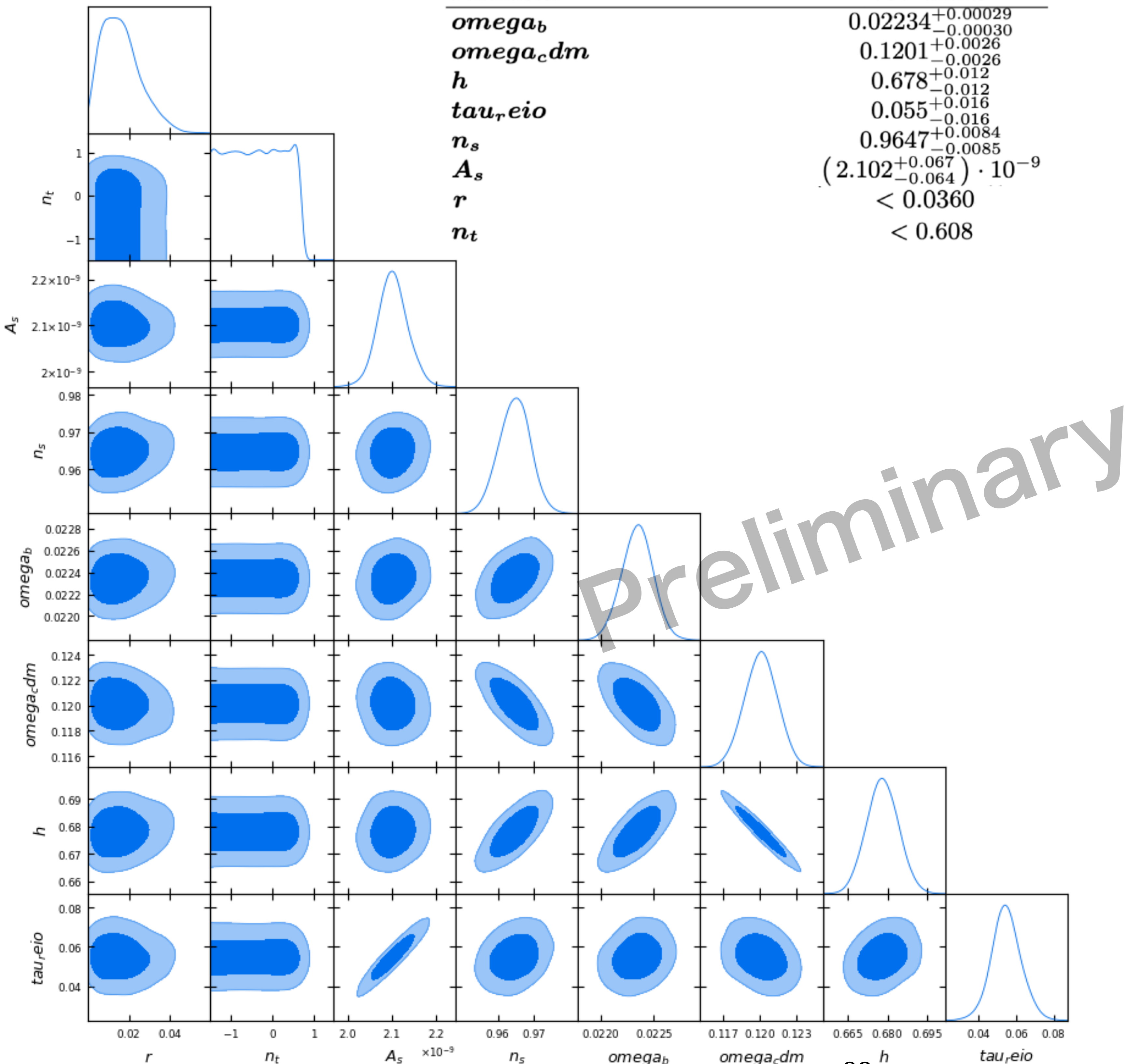
BICEP+Keck+Planck18

Parameter	95% limits
$\boldsymbol{omega_b}$	$0.02234^{+0.00028}_{-0.00030}$
$\boldsymbol{omega_c dm}$	$0.1201^{+0.0026}_{-0.0026}$
$\boldsymbol{h}$	$0.678^{+0.011}_{-0.012}$
$\boldsymbol{tau_r eio}$	$0.054^{+0.017}_{-0.015}$
$\boldsymbol{n_s}$	$0.9645^{+0.0087}_{-0.0088}$
$\boldsymbol{A_s}$	$\left(2.099^{+0.071}_{-0.063}\right) \cdot 10^{-9}$
$\boldsymbol{r}$	< 0.0348

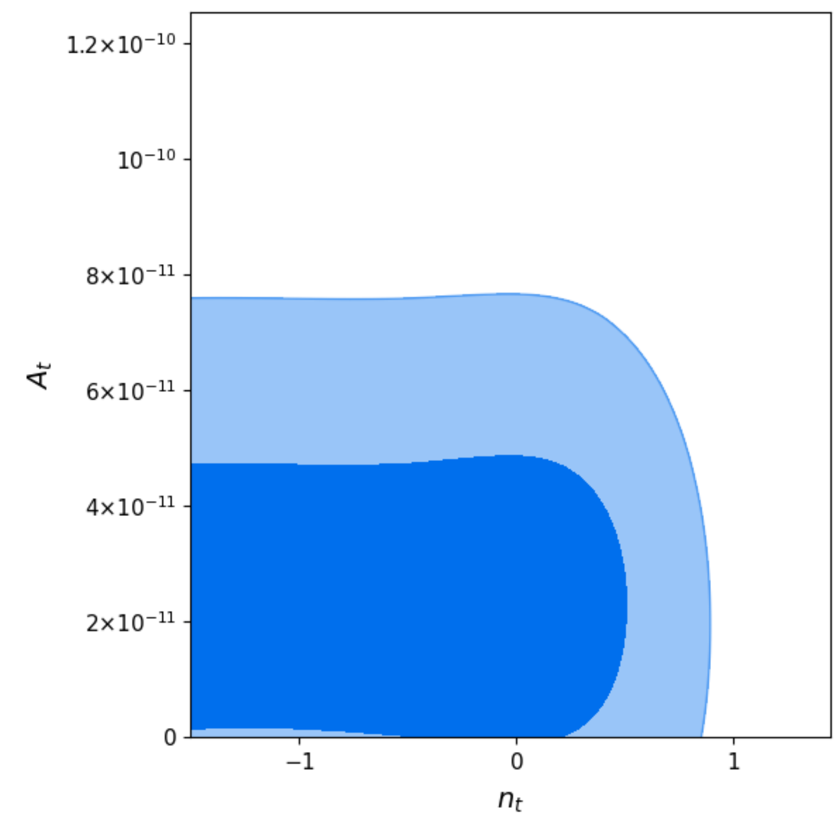


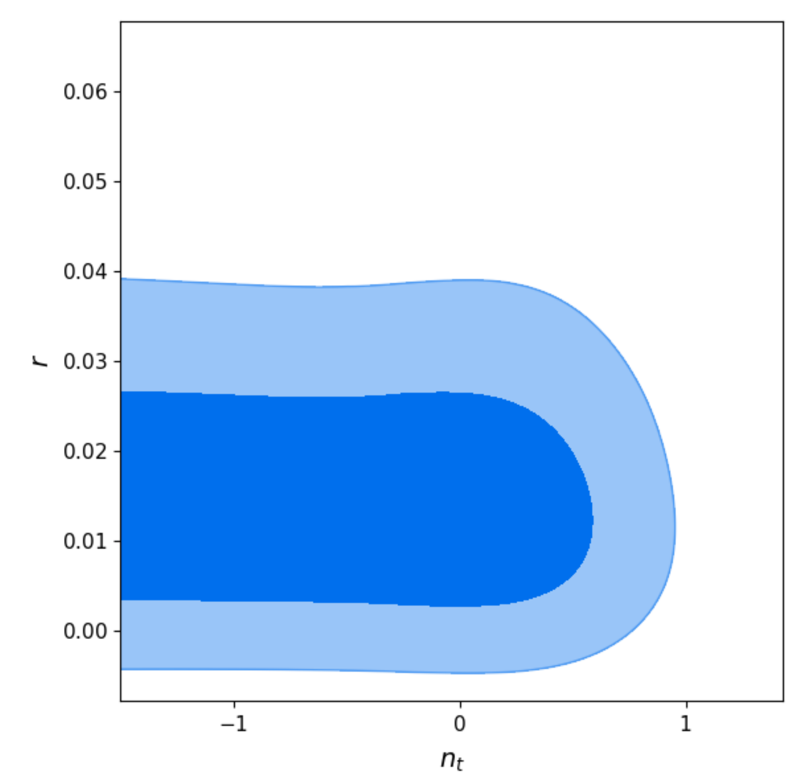


BICEP+Keck+Planck18+CHRONOS

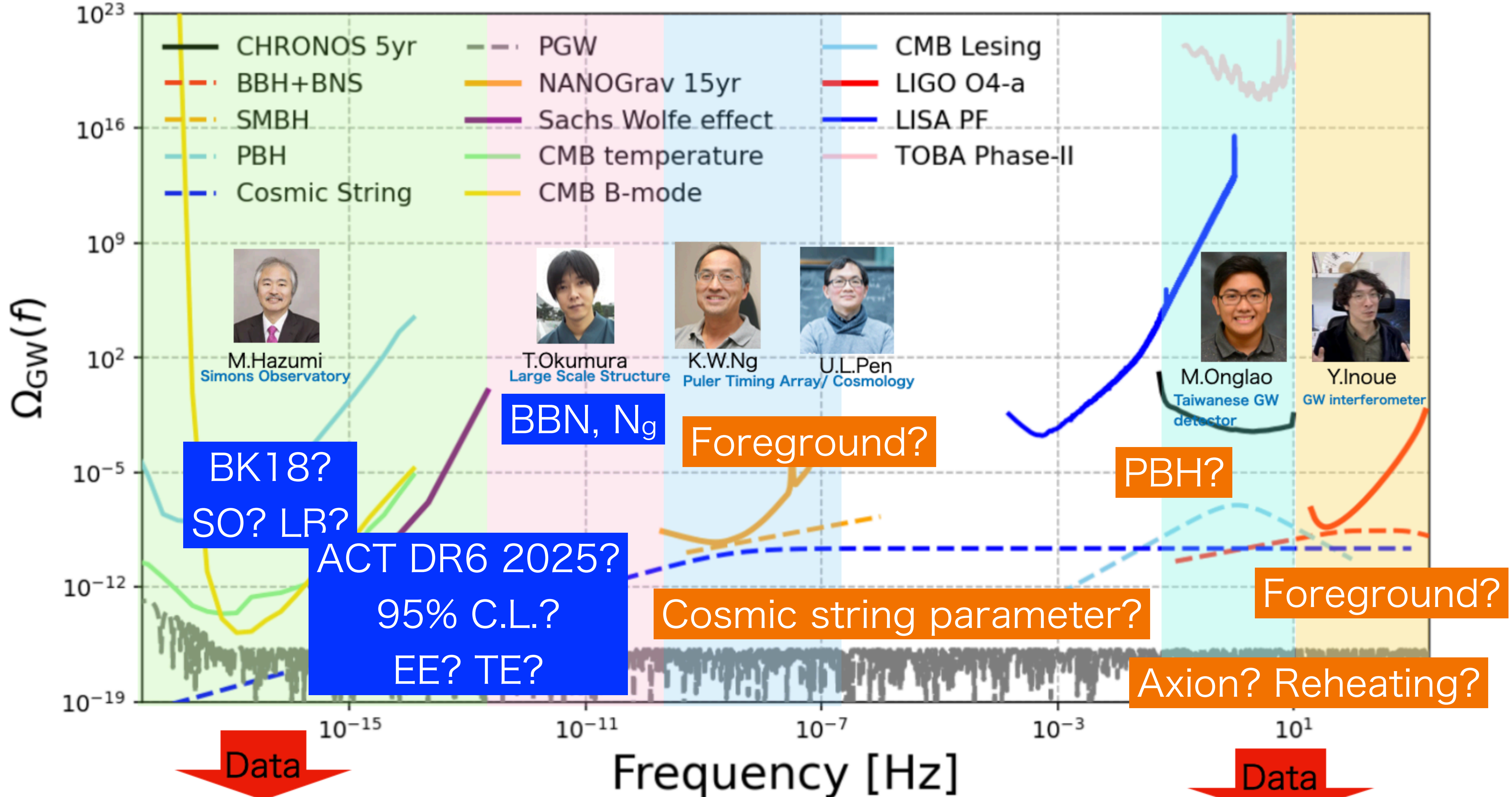


Parameter	95% limits
$\boldsymbol{omega_b}$	$0.02234^{+0.00029}_{-0.00030}$
$\boldsymbol{omega_c dm}$	$0.1201^{+0.0026}_{-0.0026}$
$\boldsymbol{h}$	$0.678^{+0.012}_{-0.012}$
$\boldsymbol{tau_r eio}$	$0.055^{+0.016}_{-0.016}$
$\boldsymbol{n_s}$	$0.9647^{+0.0084}_{-0.0085}$
$\boldsymbol{A_s}$	$(2.102^{+0.067}_{-0.064}) \cdot 10^{-9}$
$\boldsymbol{r}$	< 0.0360
$\boldsymbol{n_t}$	< 0.608

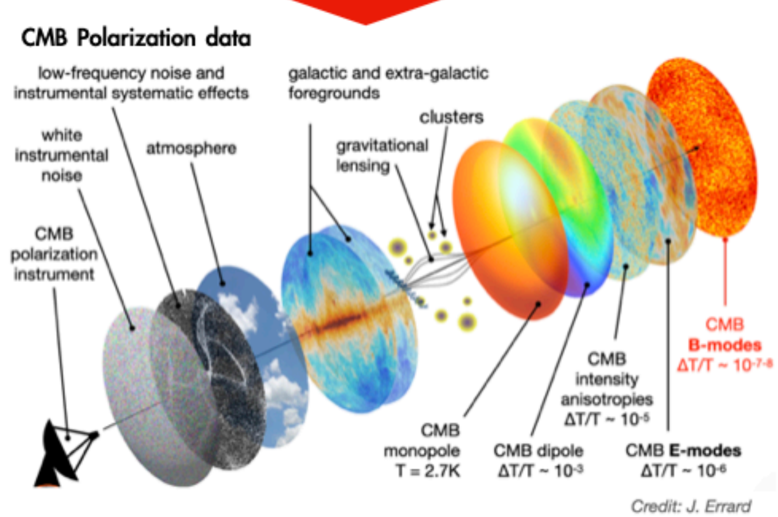




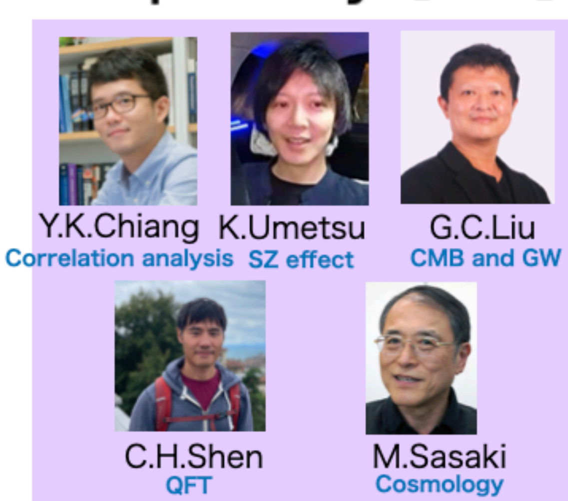
Team Taiwan



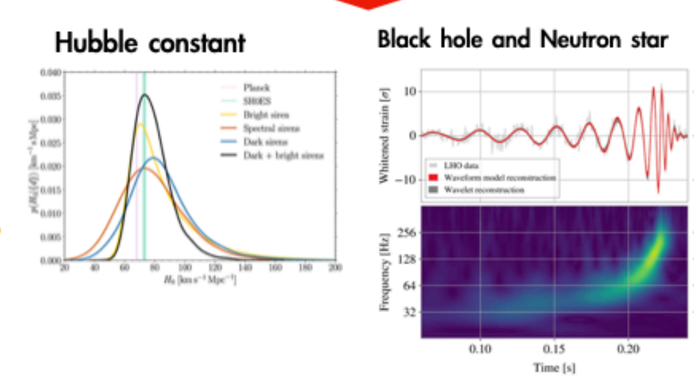
10^{23}
10^{16}
10^{9}
10^{2}
10^{-5}
10^{-12}
10^{-19}
$\Omega_{GW}(f)$
CHRONOS 5yr
BBH+BNS
SMBH
PBH
Cosmic String
PGW
NANOGrav 15yr
Sachs Wolfe effect
CMB temperature
CMB B-mode
CMB Lesing
LIGO O4-a
LISA PF
TOBA Phase-II
M.Hazumi
Simons Observatory
T.Okumura
Large Scale Structure
K.W.Ng
U.L.Pen
Puler Timing Array/ Cosmology
M.Onglao
Taiwanese GW detector
Y.Inoue
GW interferometer
BBN, N_g
Foreground?
PBH?
BK18?
SO? LB?
ACT DR6 2025?
95% C.L.?
EE? TE?
Cosmic string parameter?
Foreground?
Axion? Reheating?
10^{-15}
10^{-11}
10^{-7}
10^{-3}
10^{1}
Frequency [Hz]
Data
Data



CMB Polarization data
low-frequency noise and instrumental systematic effects
galactic and extra-galactic foregrounds
clusters
white instrumental noise
atmosphere
gravitational lensing
CMB polarization instrument
CMB B-modes ΔT/T ~ $10^{-7\text{-}8}$
CMB intensity anisotropies ΔT/T ~ 10^{-5}
CMB monopole T = 2.7K
CMB dipole ΔT/T ~ 10^{-3}
CMB E-modes ΔT/T ~ 10^{-6}
Credit: J. Errard



Y.K.Chiang
Correlation analysis
K.Umetsu
SZ effect
G.C.Liu
CMB and GW
C.H.Shen
QFT
M.Sasaki
Cosmology



Hubble constant
Black hole and Neutron star

Summary

- Introduced two approaches for Analysis approach of Ω_{GW}.
- Using BK18 data set, we updated the constraint of Ω_{GW} at low frequency region
- Introduced the approach of n_t parameter test with Cosmological Standard model
- We still have a chance to give a constraint of Ω_{GW} with the latest data.
- Now is the time for preparation period to establish the current constraint.