Prospects of GPD Measurements at EIC

TIDC Workshop at NCKU
August 18, 2022

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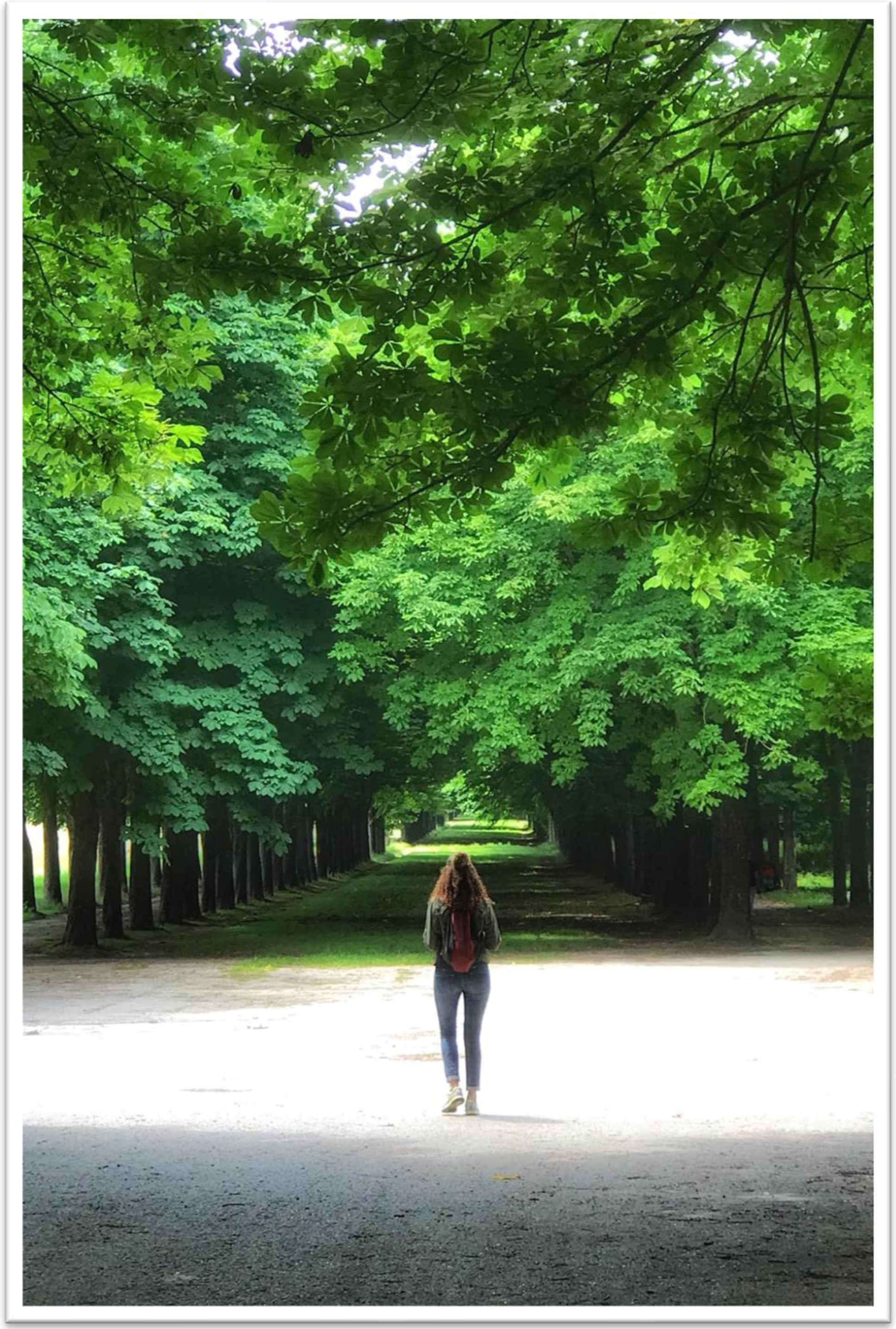
Outline & Confession

1. **Review**
2. **Outlook**

In this presentation, I blatantly took a lot of materials from:

- EIC white paper
- EIC yellow report
- Great talks presented by Andrey Kim, Dariah Sokhan, Stepan Stepanyan, Nicole d'Hose and many others

Trodded paths & where we are



Generalized Parton Distributions (GPDs)

Unpolarized

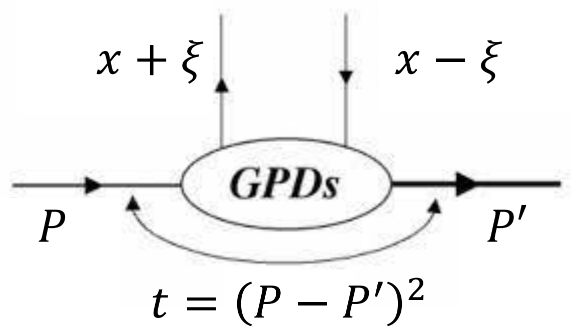
$H_f(x,0,0) = q_f(x)$

$\tilde{H}_f(x,0,0) = \Delta q_f(x)$

Polarized

$\xi = t = 0$

PDFs



First Moment

Form Factors

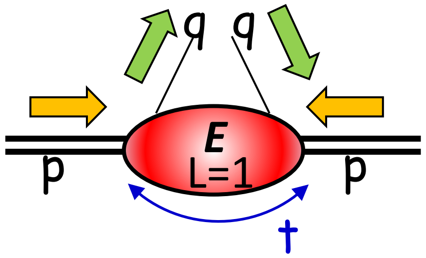
$\int H_f(x,\xi,t)dx = F_1^f(t)$ ***Dirac FF***

$\int E_f(x,\xi,t)dx = F_2^f(t)$ ***Pauli FF***

- **GPDs embody both PDFs and FFs**

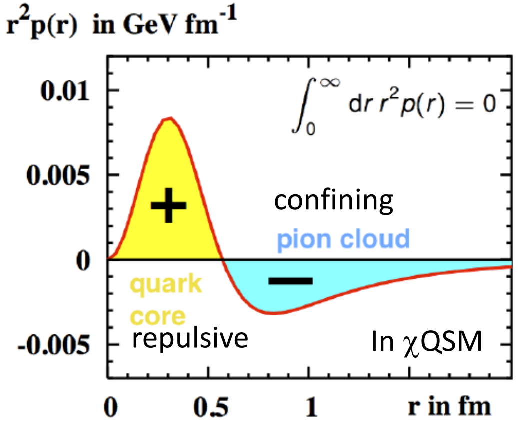
Provides information on the interesting properties of the nucleon.

- **Mapping the transverse plane distribution of parton**
- **Pressure distribution inside nucleon**
- **Angular momentum of parton**



$$J_q = \frac{1}{2}\int_{-1}^{1} dx\, x[H^q(x,\xi,0) + E^q(x,\xi,0)]$$

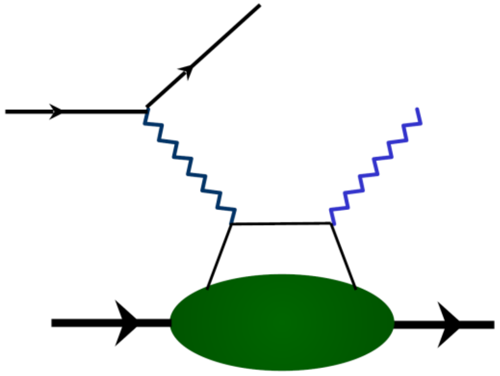
Ji's Sum Rule



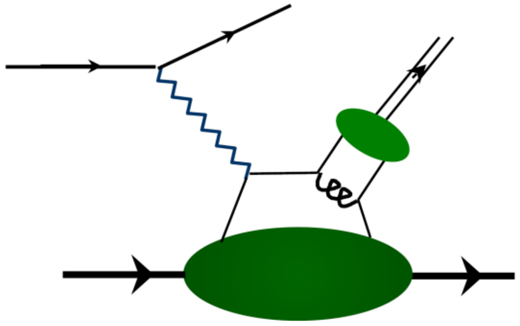
M. Polyakov, P. Schweitzer, Int.J.Mod.Phys. A33 (2018)

Exclusive Process

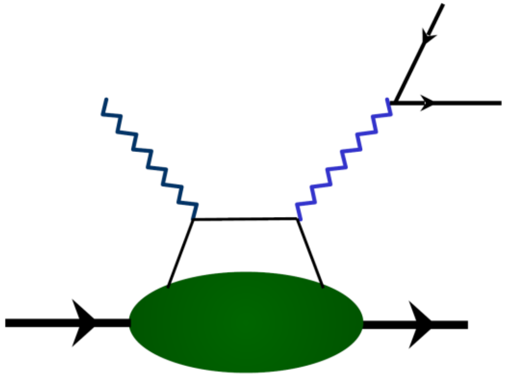
- Use exclusive processes, where all final state particles are "detected", to access the muliti-variable dependence of GPDs, and constrain the GPD parameterization with measurements in various phase space.
- Processes:
    - Deeply Virtual Compton Scattering (DVCS)
    - Deeply Virtual Meson Production (DVMP)
    - Time-like Compton Scattering (TCS)
    - Double DVCS (DDVCS)
    - …



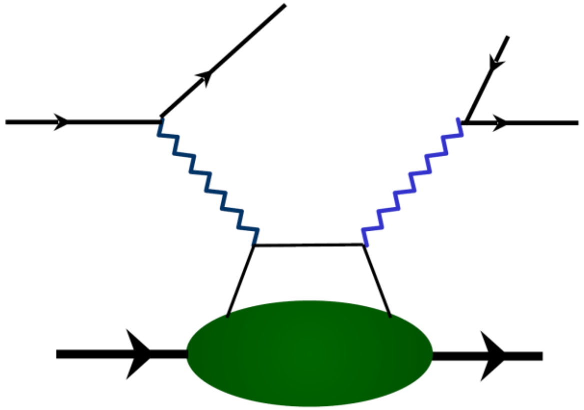
DVCS



DVMP

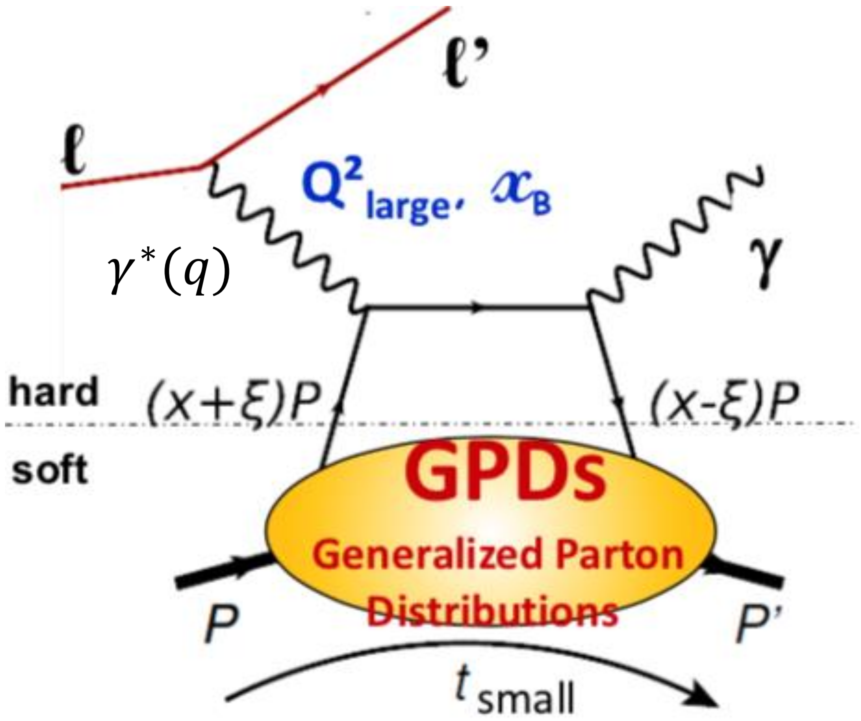


TCS



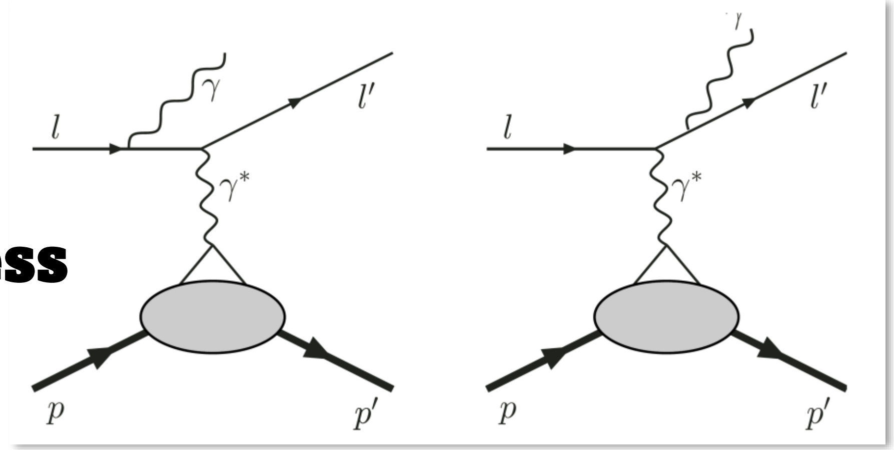
DDVCS

Deeply Virtual Compton Scattering



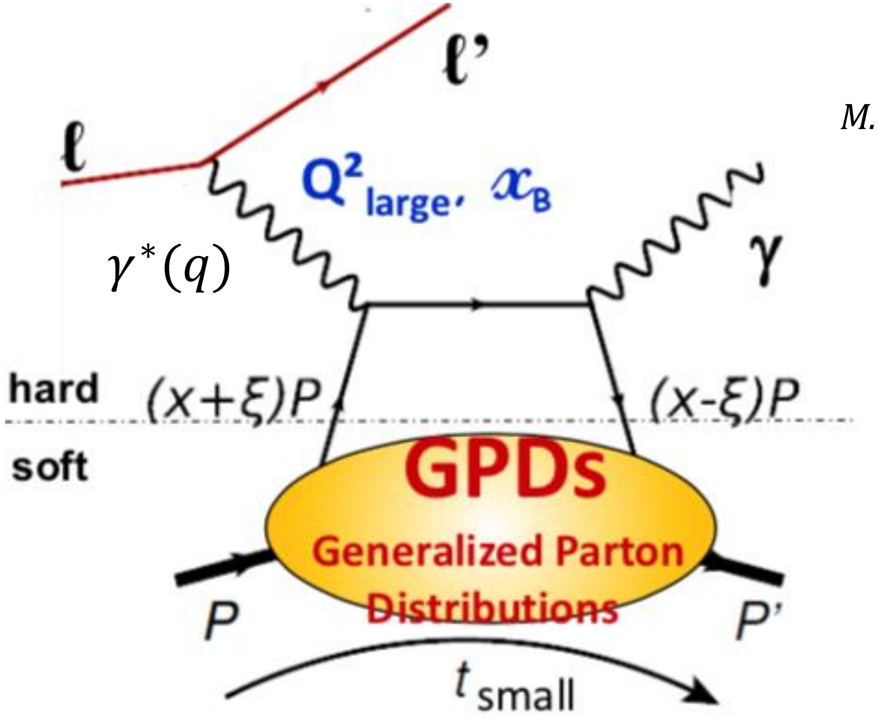
DVCS: $l + p \to l' + p' + \gamma$

BH Process



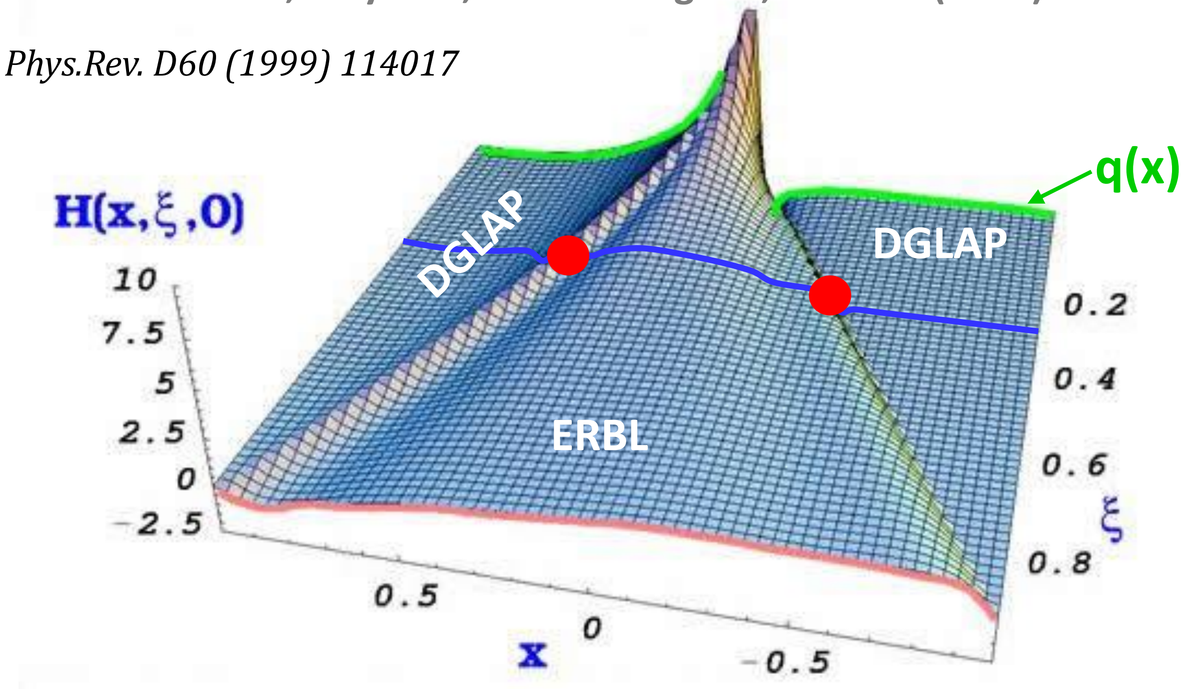
- DVCS is regarded as the golden channel and gives access to four chiral-even GPDs $H, \tilde{H}, E, \tilde{E}(x, \xi, t)$. Its interference with the well-understood Bethe-Heitler process gives access to more info.

Compton Form Factors (CFFs)



M. Polyakov , C. Weiss, Phys.Rev. D60 (1999) 114017

From Goeke, Polyakov, Vanderhaeghen, PNPP47 (2001)



CFF GPD REAL part Imaginary part

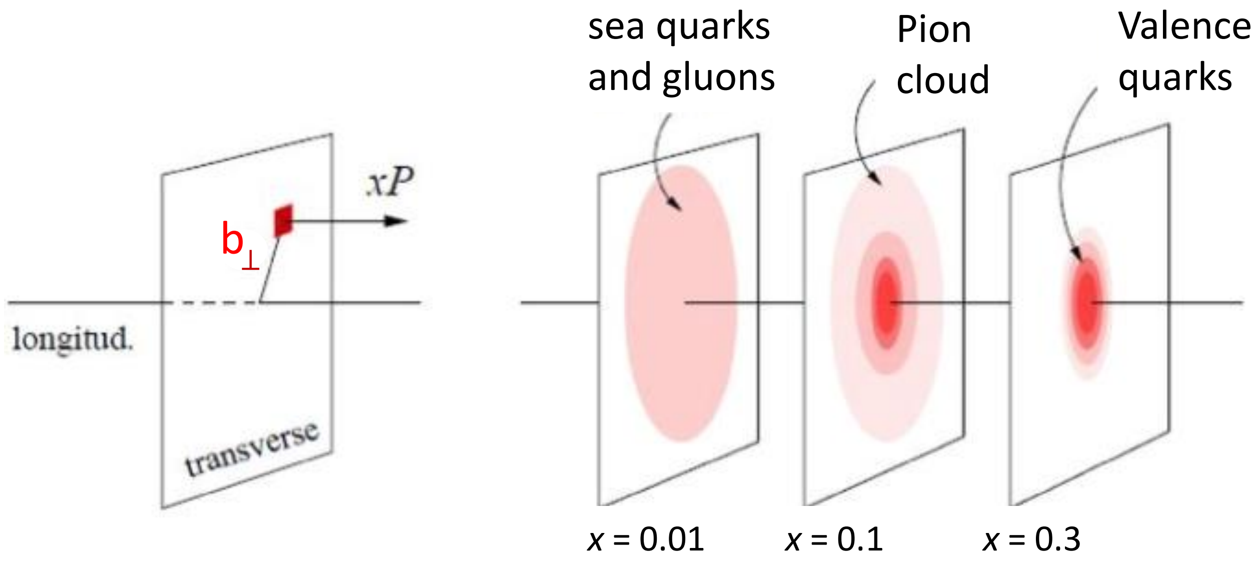
$$\mathcal{H}(\xi,t) = \int_{-1}^{+1} dx \, \frac{\mathbf{H}(x,\xi,t)}{x-\xi+i\varepsilon} + \ldots = \mathcal{P}\int_{-1}^{+1} dx \, \frac{\mathbf{H}(x,\xi,t)}{x-\xi} - i\pi \, \mathbf{H}(x=\pm\xi,\xi,t) + \ldots$$

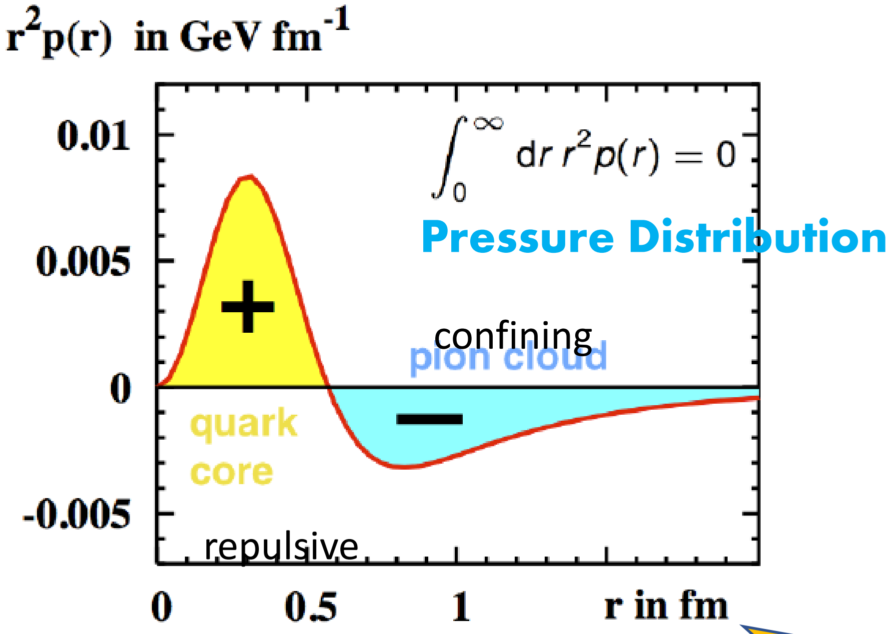
$$Re\,\mathcal{H}(\xi,t) = \mathcal{P}\int dx \, \frac{Im\,\mathcal{H}(x,t)}{x-\xi} + \Delta(t)$$

Transverse Imaging and Pressure Distribution

M. Polyakov, P. Schweitzer, Int.J.Mod.Phys. A33 (2018)

Mapping in the transverse plance





FT of H(x, ξ=0,t)

CFF

GPD

REAL part

Imaginary part

$$\mathcal{H}(\xi,t) = \int_{-1}^{+1} dx \frac{\mathbf{H}(x,\xi,t)}{x-\xi+i\varepsilon} + \ldots = \mathcal{P}\int_{-1}^{+1} dx \frac{\mathbf{H}(x,\xi,t)}{x-\xi} - i\pi\, \mathbf{H}(x=\pm\xi,\xi,t) + \ldots$$

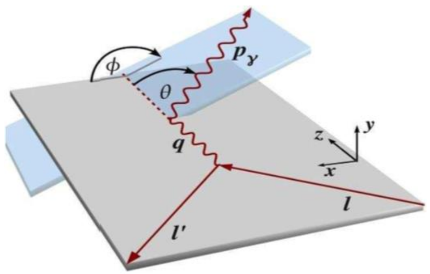
$$Re\,\mathcal{H}(\xi,t) = \mathcal{P}\int dx \frac{Im\,\mathcal{H}(x,t)}{x-\xi} + \Delta(t)$$

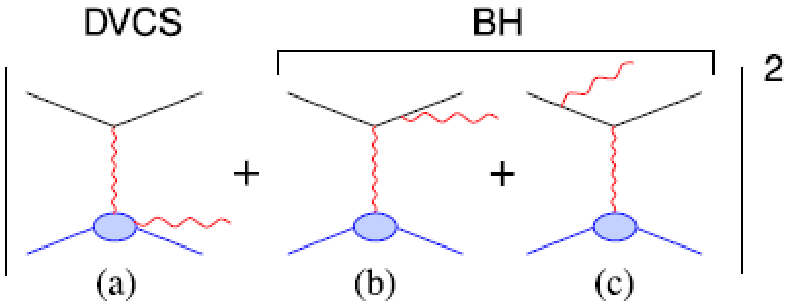
$d_1(t)$

D-term

Polarized Beam & Unpolarized Target

- Experimental access by cross-sections and spin asymmetries





$$\frac{d^4\sigma(\ell p \to \ell p \gamma)}{dx_B dQ^2 d|t| d\phi} = \underbrace{d\sigma^{BH}}_{\text{Well known}} + \left(d\sigma^{DVCS}_{unpol} + P_\ell\, d\sigma^{DVCS}_{pol}\right) + \left(e_\ell \mathrm{Re}\, I + e_\ell P_\ell \mathrm{Im}\, I\right)$$

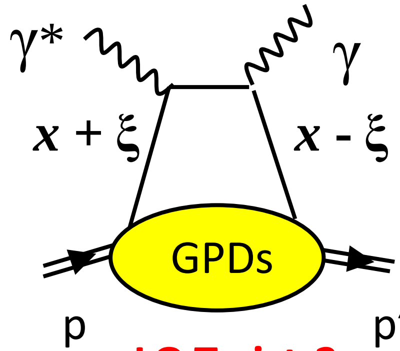
$$d\sigma^{BH} \propto c_0^{BH} + c_1^{BH}\cos\phi + c_2^{BH}\cos 2\phi$$

$$d\sigma^{DVCS}_{unpol} \propto c_0^{DVCS} + c_1^{DVCS}\cos\phi + c_2^{DVCS}\cos 2\phi$$

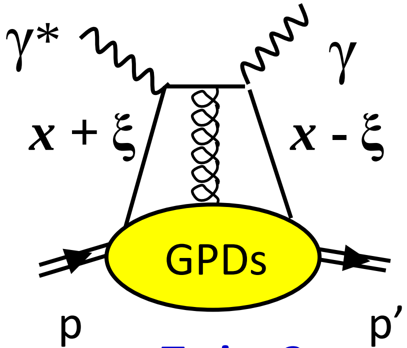
$$d\sigma^{DVCS}_{pol} \propto s_1^{DVCS}\sin\phi$$

$$\mathrm{Re}\, I \propto c_0^I + c_1^I\cos\phi + c_2^I\cos 2\phi + c_3^I\cos 3\phi$$

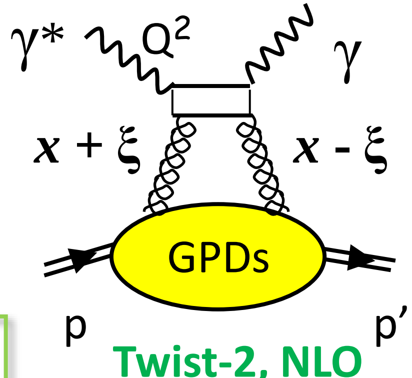
$$\mathrm{Im}\, I \propto s_1^I\sin\phi + s_2^I\sin 2\phi$$



LO,Twist-2



Twist-3



Twist-2, NLO

Change P_ℓ → $s_1^I = \mathrm{Im}\, \mathcal{F}$

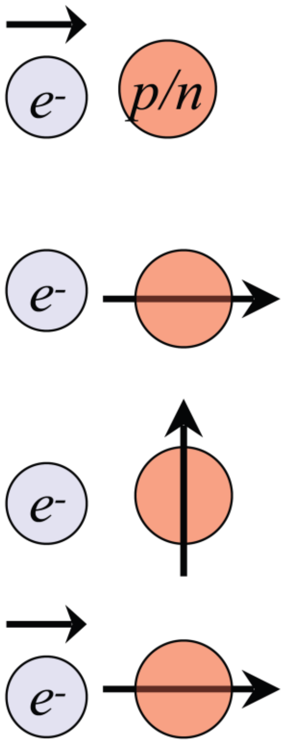
Change e_ℓ, P_ℓ → $c_1^I = \mathrm{Re}\, \mathcal{F}$

$\mathcal{F} = F_1\mathcal{H} + \xi(F_1+F_2)\widetilde{\mathcal{H}} + t/4m^2 F_2 \mathcal{E}$

Sensitivity to CFFs

➢ The target polarization can be explored as well.

Beam, target polarisation



For example:

Beam, target polarisation	Cross section	Proton / Neutron
LU	$\Delta\sigma_{LU} \sim \sin\phi\, \Im(F_1 H + \xi G_M \tilde{H} - \frac{t}{4M^2} F_2 E)\, d\phi$	$Im\{\mathbf{H_p}, \tilde{H}_p, E_p\}$ $Im\{H_n, \tilde{H}_n, \mathbf{E_n}\}$
UL	$\Delta\sigma_{UL} \sim \sin\phi\, \Im(F_1 \tilde{H} + \xi G_M (H + \frac{x_B}{2} E) - \xi \frac{t}{4M^2} F_2 \tilde{E} + \ldots)\, d\phi$	$Im\{\mathbf{H_p}, \mathbf{\tilde{H}_p}\}$ $Im\{\mathbf{H_n}, E_n, \tilde{E}_n\}$
UT	$\Delta\sigma_{UT} \sim \cos\phi\, \Im(\frac{t}{4M^2}(F_2 H - F_1 E) + \ldots) d\phi$	$Im\{\mathbf{H_p}, \mathbf{E_p}\}$ $Im\{\mathbf{H_n}\}$
LL	$\Delta\sigma_{LL} \sim (A + B\cos\phi)\, \Re(F_1 \tilde{H} + \xi G_M (H + \frac{x_B}{2} E) + \ldots) d\phi$	$Re\{\mathbf{H_p}, \mathbf{\tilde{H}_p}\}$ $Re\{\mathbf{H_n}, E_n, \tilde{E}_n\}$

Sensitivity to CFFs

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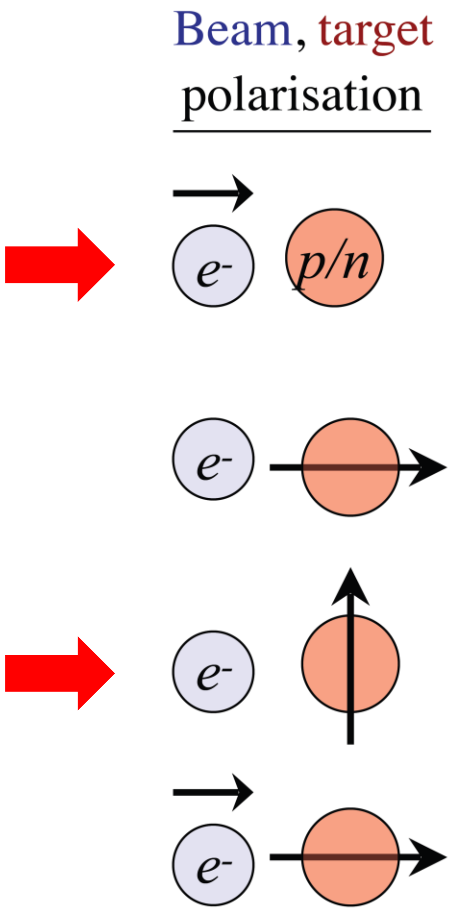

For example:

Beam, target polarisation			Proton / Neutron
LU	$\Delta\sigma_{LU} \sim \sin\phi\, \Im(F_1 H + \xi G_M \tilde{H} - \frac{t}{4M^2} F_2 E)\, d\phi$	→	$Im\{H_p, \tilde{H}_p, E_p\}$ $Im\{H_n, \tilde{H}_n, \mathbf{E_n}\}$
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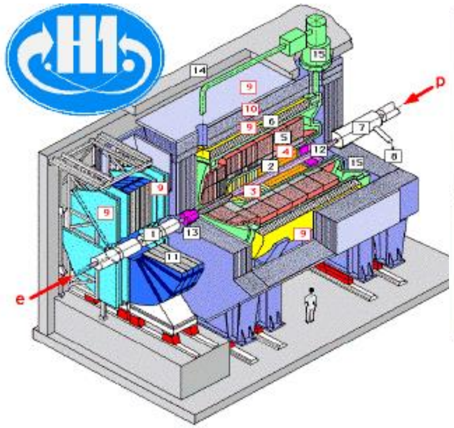
UT	$\Delta\sigma_{UT} \sim \cos\phi\, \Im(\frac{t}{4M^2}(F_2 H - F_1 E) + \ldots) d\phi$	→	$Im\{H_p, \mathbf{E_p}\}$ $Im\{H_n\}$
LL	$\Delta\sigma_{LL} \sim (A + B\cos\phi)\, \Re(F_1 \tilde{H} + \xi G_M (H + \frac{x_B}{2} E) + \ldots) d\phi$	→	$Re\{H_p, \tilde{H}_p\}$ $Re\{H_n, E_n, \tilde{E}_n\}$

- **Neutron target: flavor decomposition & access to *E***

The Past and Present Experiments



e-p Collider forward fast proton

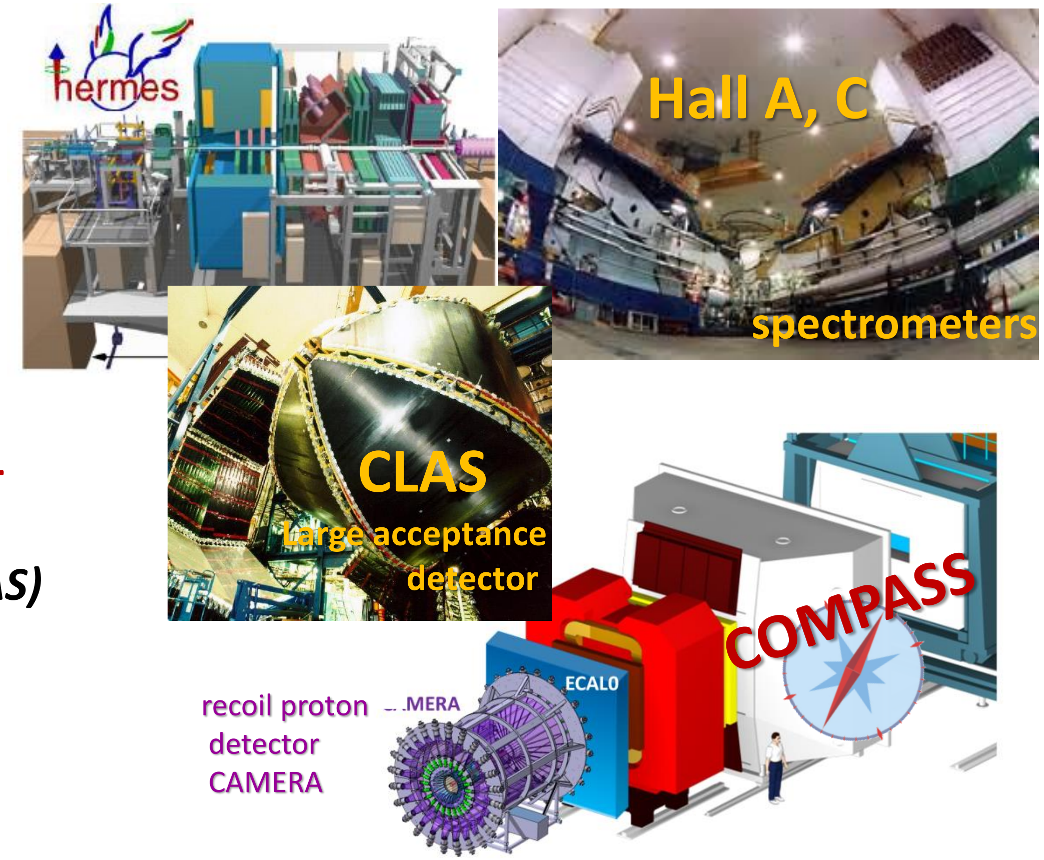
- **HERA**: **H1** and **ZEUS**
  Polarised **27 GeV** e-/e+
  Unpolarized **920 GeV** proton
  ~ ***Full event reconstruction***

Fixed target mode slow recoil proton

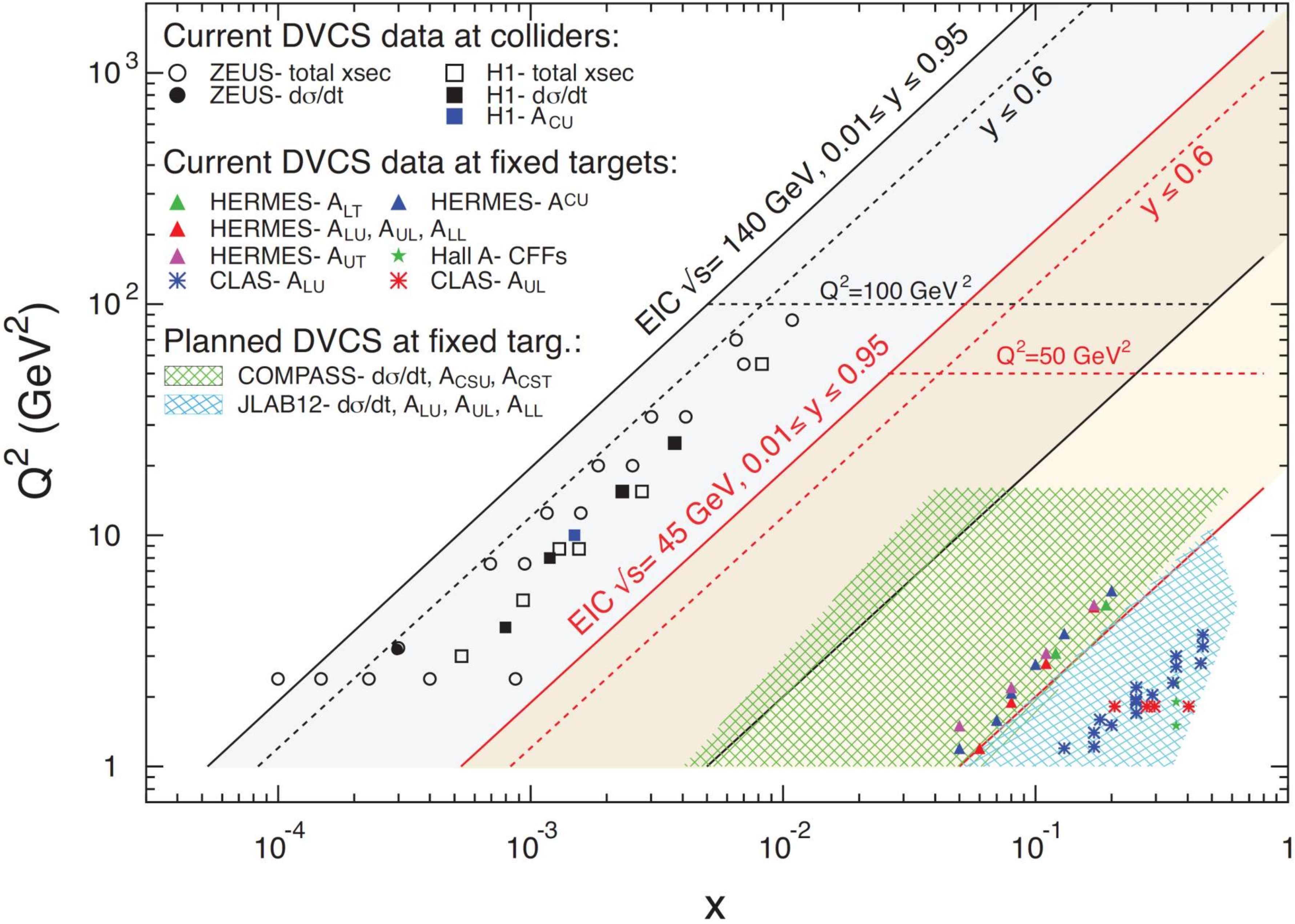
- **HERMES:** Polarised **27** GeV e-/e+
  Long., Trans. polarised p, d target
  Missing mass technique, 2006-09 with recoil detector

- **Jlab: Hall A, C, CLAS** High Luminosity Polar. **6** & **12 GeV e-**
  Long., (Trans.) polarised p, d target
  Missing mass technique (A,C) and complete detection (CLAS)

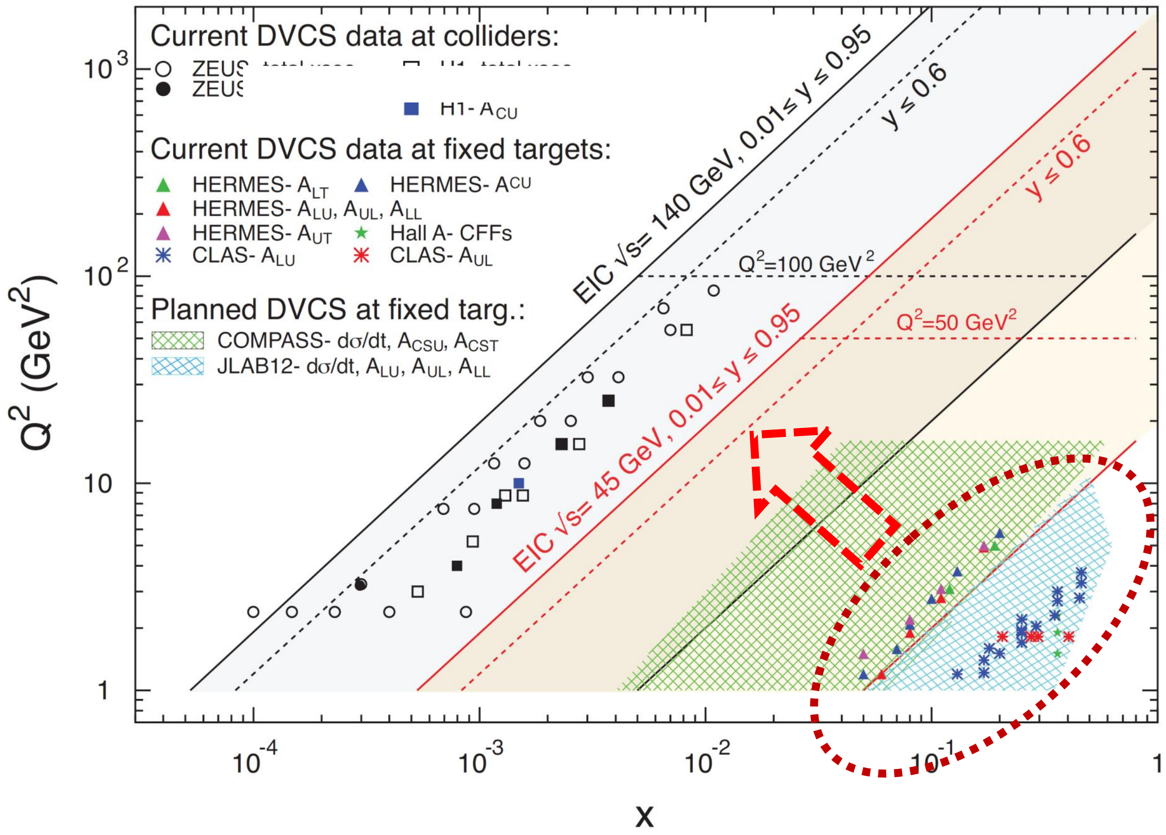
- **COMPASS @ CERN:** Polarised **160 GeV μ+/μ-**
  p target, (Trans.) polarised ***target with recoil p detection***



Landscape – Global Programs of DVCS

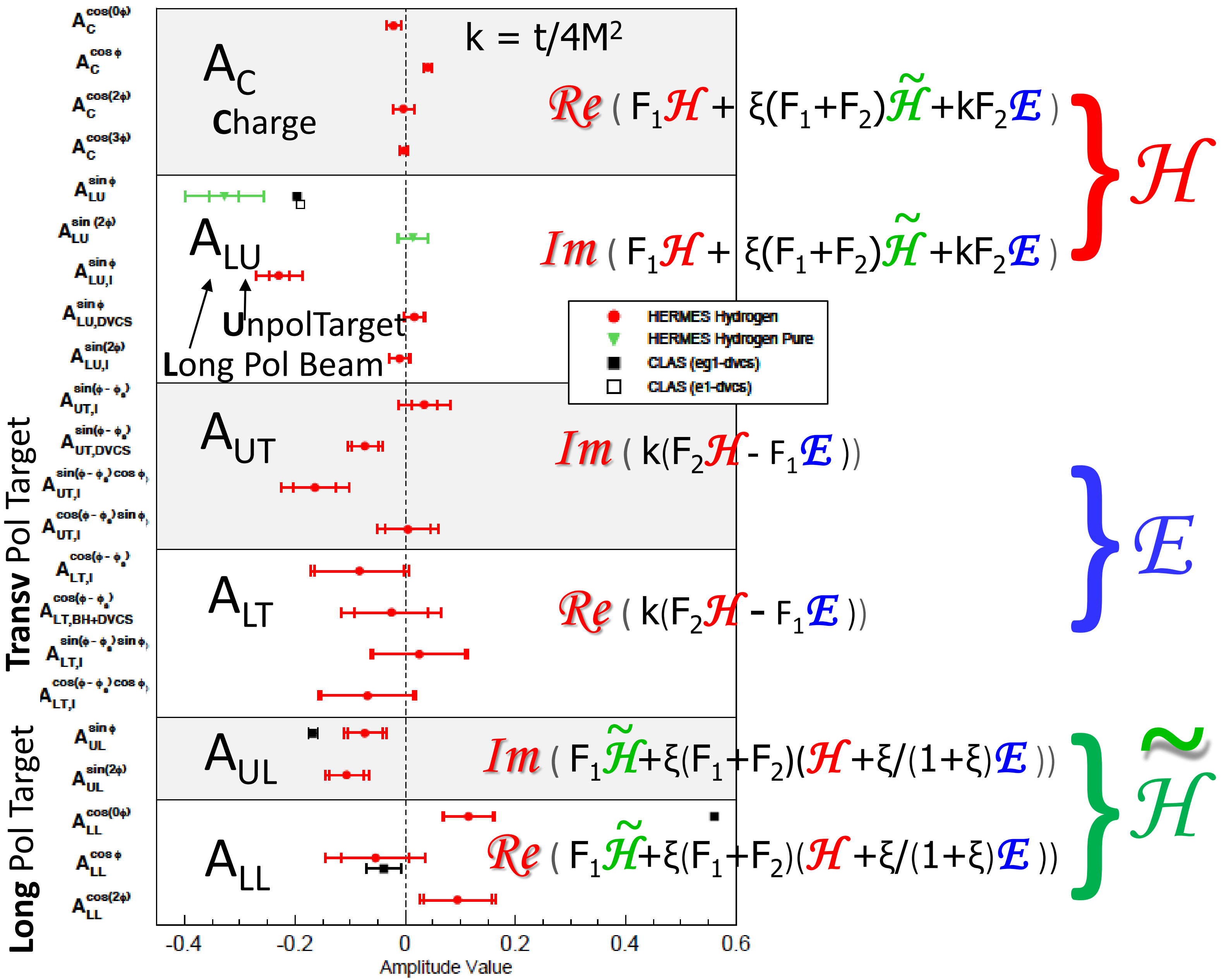


DVCS Measurements



Starting with lower energy – intermediate to high x

A complete set of DVCS asymmetries at Hermes



HERMES provided a complete set of observables

2001: 1st DVCS publication as CLAS & H1

2007: end of data taking

2012: still important publications

JHEP 07 (2012) 032 A_C A_{LU}

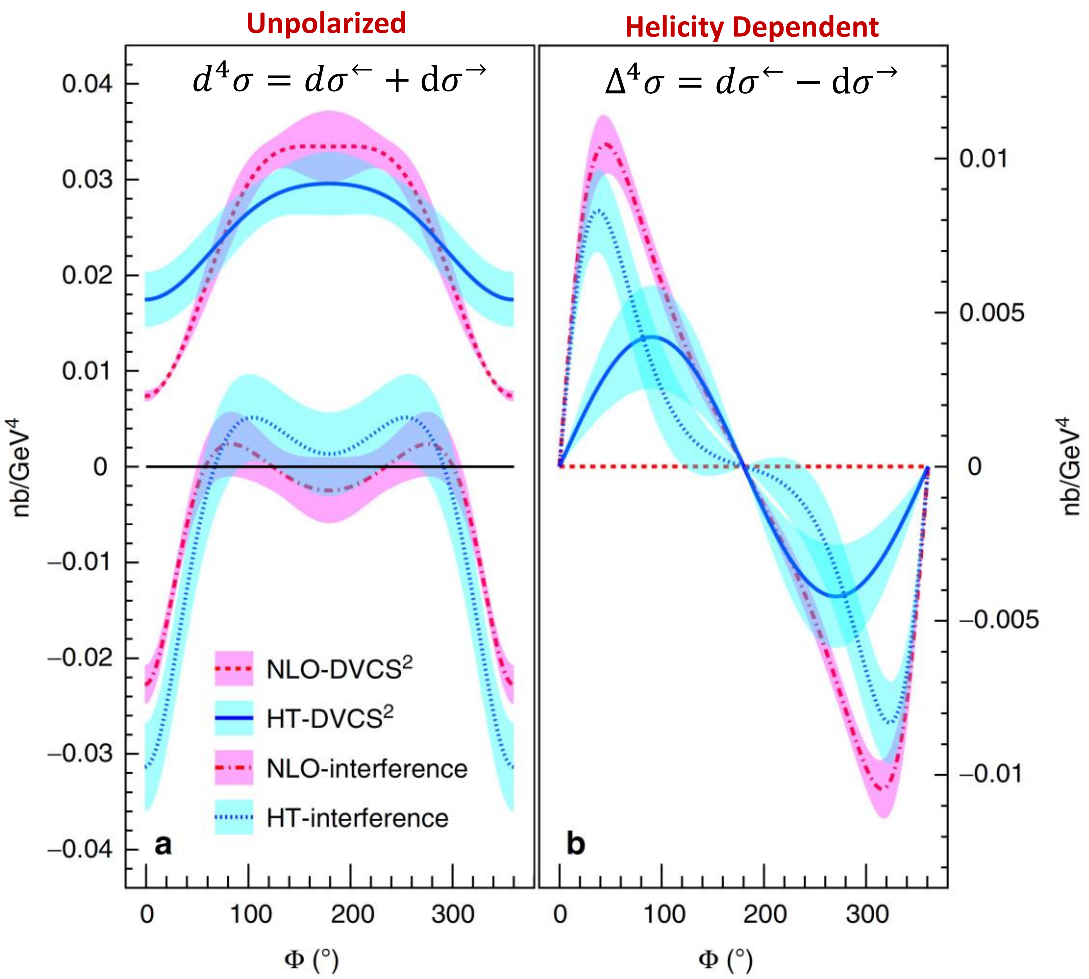
JHEP10(2012) 042 A_{LU}

with recoil detection (2006-7)

- Electron & positron beams on proton
- Beam energy of 27.6 GeV
- Luminosity $\leq 10^{31}\ cm^{-2}s^{-1}$
- Most data within:
  $0.05 \leq x_B \leq 0.2$
  $2\ GeV^2 \leq Q^2 \leq 6\ GeV^2$

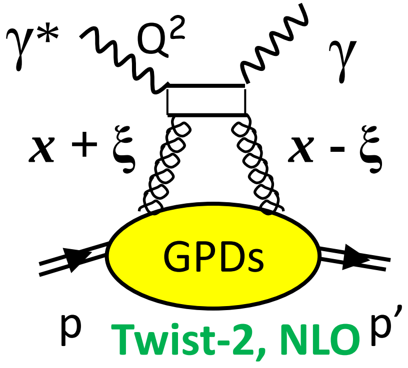
Beam Spin Sum and Diff of DVCS at JLab Hall A

- After the pioneering E00-110 in 2004 at Hall-A, the E07-007 experiment in 2010
- High precision cross-section measurement in a small kinematic region: Generalized Rosenbluth separation of the DVCS^2 (scales as E_e^2) and the BH-DVCS interference (scales as E_e^3) terms. NLO and/or higher-twist improve model agreement



$\overleftrightarrow{e}\, p \rightarrow e\, \gamma\, p$

- E_e: 4.5 & 5.6 GeV
- Q^2: 1.5, 1.9, 2.3 GeV^2 at fixed x_B: 0.36

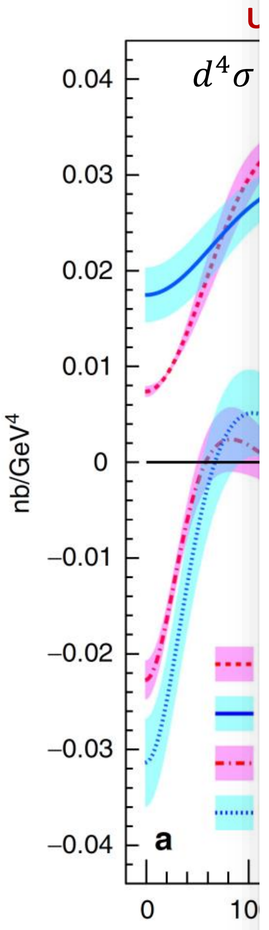


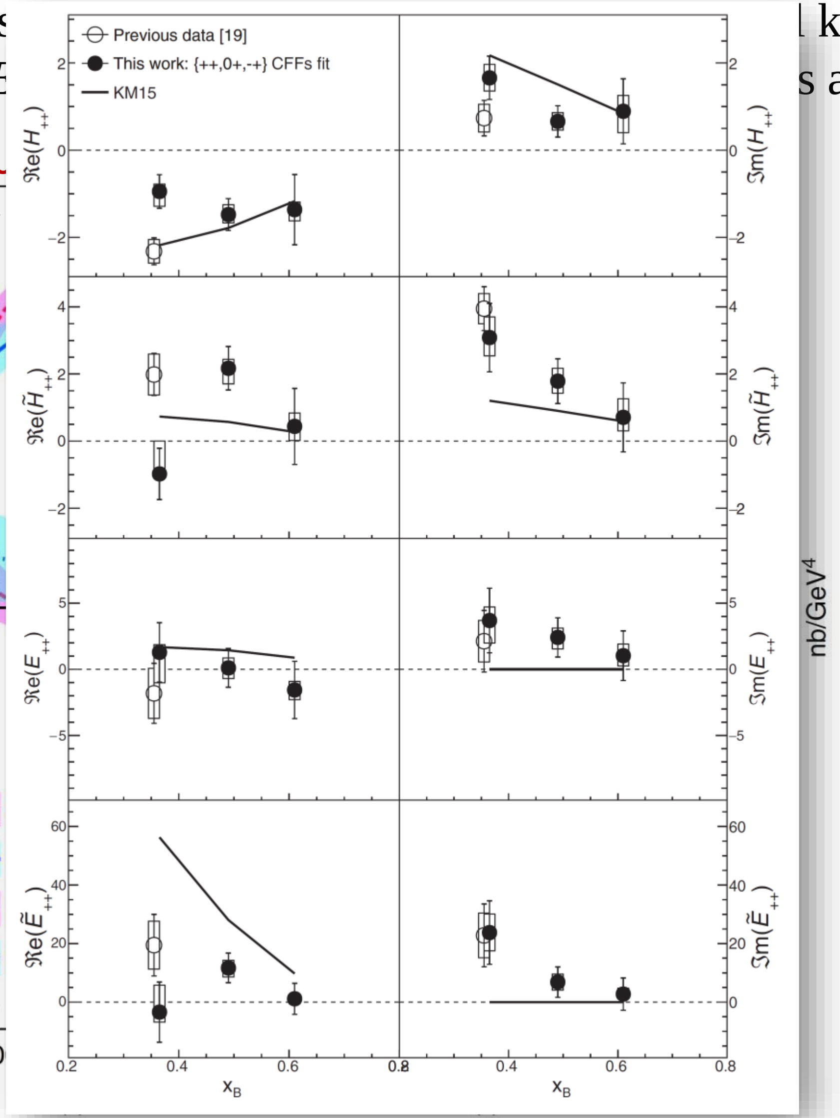
- Two scenarios: higher-twist or next-to-leading order
- Significant differences between pure DVCS and interference contributions.
- Sensitivity to gluons.
- Separation of HT and NLO effects requires scans across wider ranges of Q^2 and beam energy → JLab 12

Defurne et al., Nature Communications 8 (2017) 1408

Beam Spin Sum and Diff of DVCS at JLab Hall A

- After the pioneering E00-110 in 2004 at Hall-A, the E07-007 experiment in 2010
- High precis … kinematic region: Generalized Rosenbluth separation of the DVCS[2] (scales as E … s as E_e^3) terms. NLO and/or higher-twist improve model agreement





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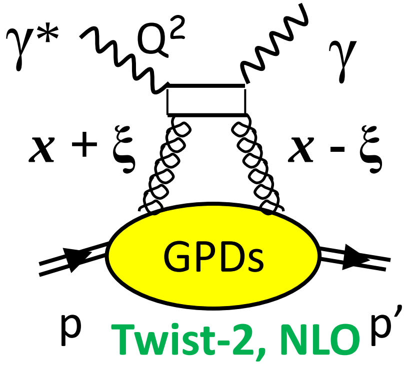

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- First experimental extraction of all four helicity-conserving CFFs

F. Georges et al. (JLab Hall A Collaboration), Phys. Rev. Lett. 128, 252002 (June 2022)

Defurne et al., Nature Communications 8 (2017) 1408

Nucleon Tomography in the Valence Domain with CLAS Data

Fit of 8 CFFs at **L.O.** and **L.T.**

($\mathrm{Im}\mathcal{H}$, $\mathrm{Re}\mathcal{H}$, $\mathrm{Im}\mathcal{E}$, $\mathrm{Re}\mathcal{E}$, $\mathrm{Im}\tilde{\mathcal{H}}$, $\mathrm{Re}\tilde{\mathcal{H}}$, $\mathrm{Im}\tilde{\mathcal{E}}$, $\mathrm{Re}\tilde{\mathcal{E}}$)

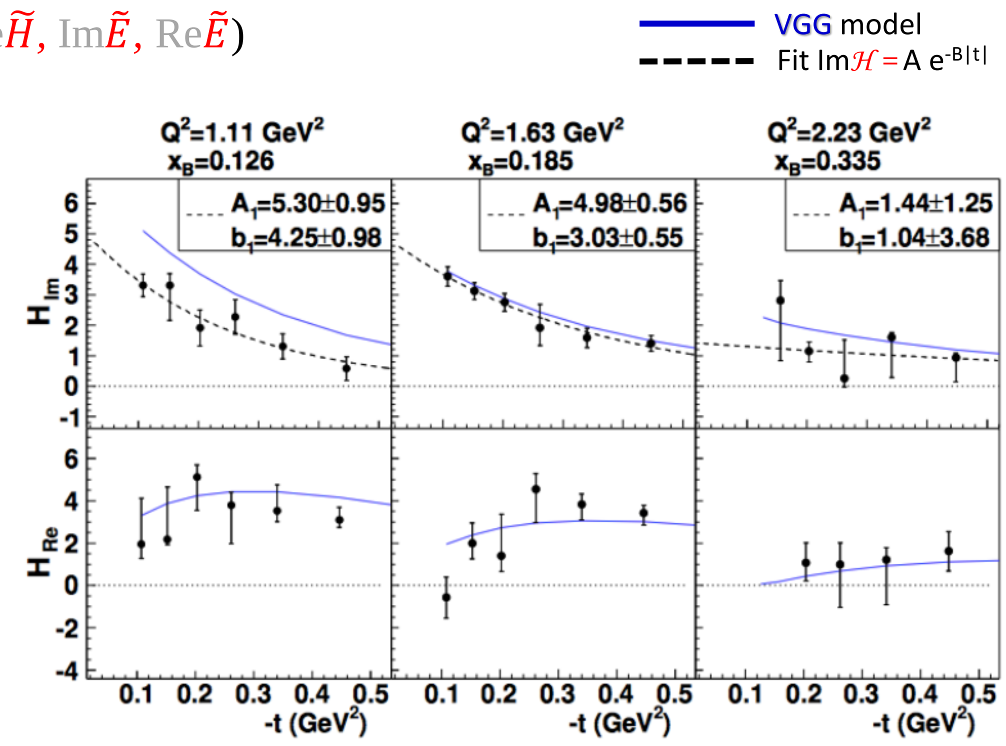
Better Constrained

- Wide kinematic coverage
- Carried out measurements with logitudinally polarized target as well

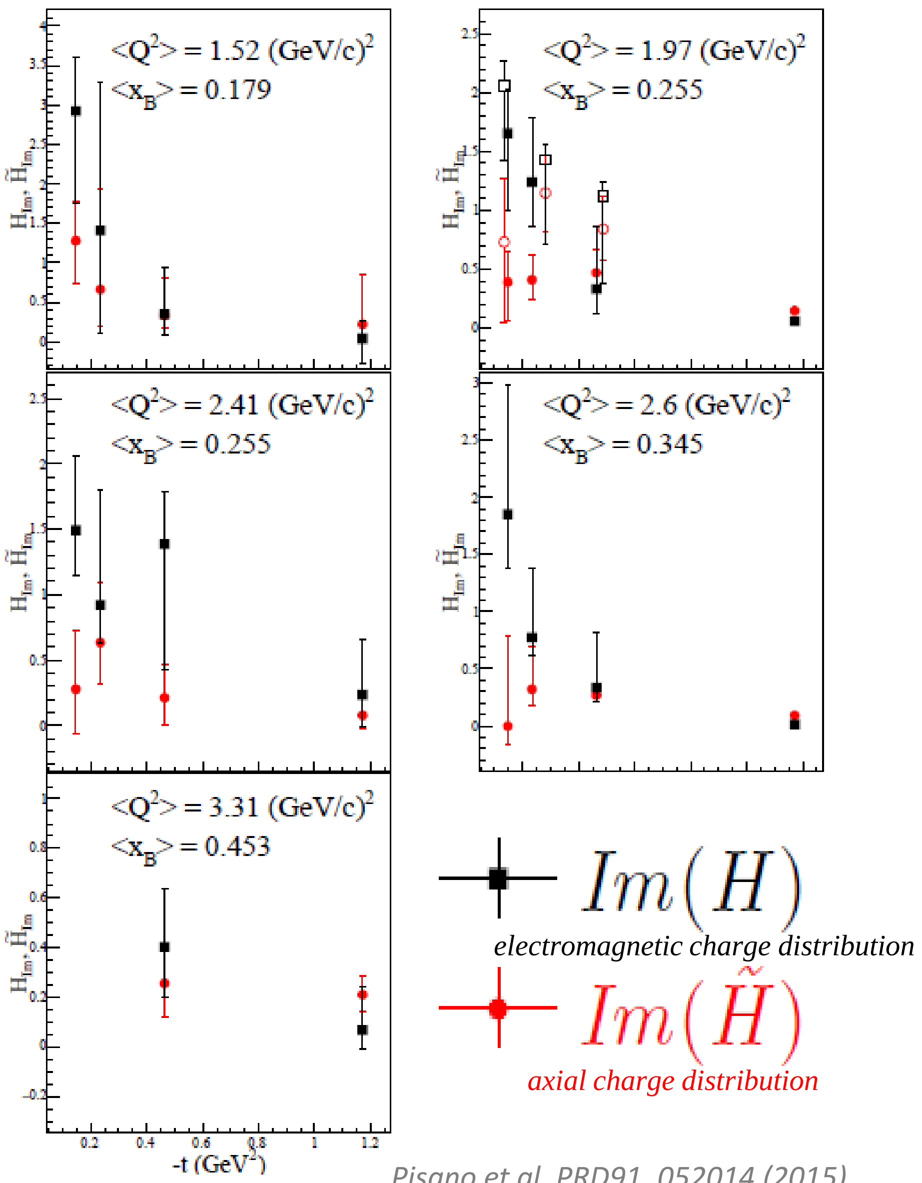
$\overleftrightarrow{e}\, p \rightarrow e\, \gamma\, p$

- Valence quarks at centre
- Sea quarks spread out towards the periphery.

$\overleftrightarrow{e}\, \vec{p} \rightarrow e\, \gamma\, p$



H.-S. Jo et al. (CLAS) PRL115, 212003 (2015) 212003
N. Hirlinger Saylor et al (CLAS) PRC 98 (2018) 045203



- Simultaneous fit to BSA, TSA & DSA → Information on relative distribution of quark momenta (PDFs) and quark helicity, $\Delta q(x)$

$$H(x,0,0) = q(x) \qquad \tilde{H}(x,0,0) = \Delta q(x)$$

$$\int_{-1}^{+1} H dx = F_1 \qquad \int_{-1}^{+1} \tilde{H}\, dx = G_A$$

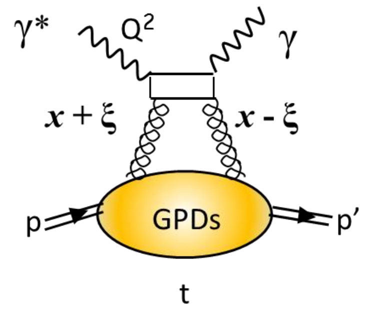
- Indication that axial charge is more concentrated than electromgnatic charge

Pisano et al. PRD91, 052014 (2015)
Seder et al. PRL114, 032001 (2015)

Nucleon Tomography in the Gluon Domain at HERA

$d\sigma^{DVCS}/d|t| \propto e^{-B|t|}$

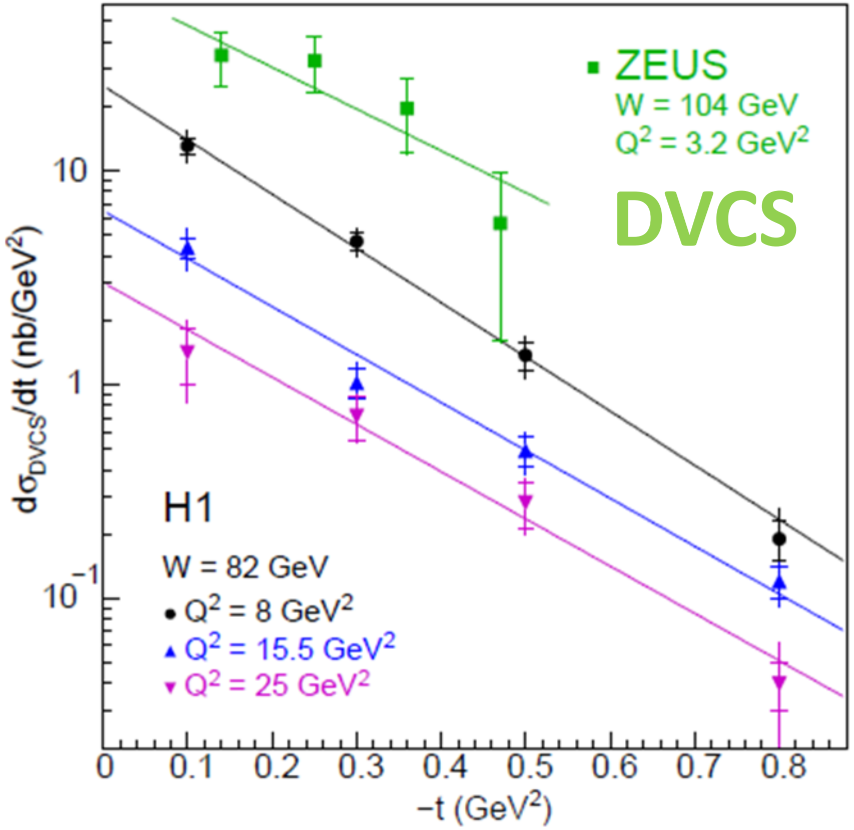
Dominance of $Im\mathcal{H}$

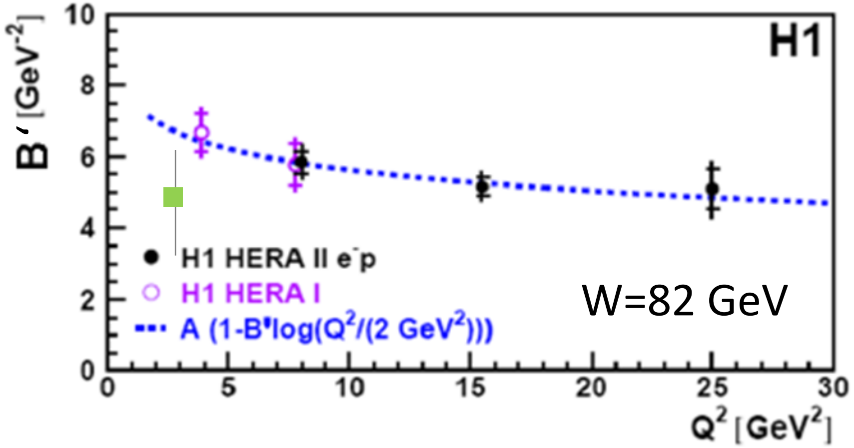


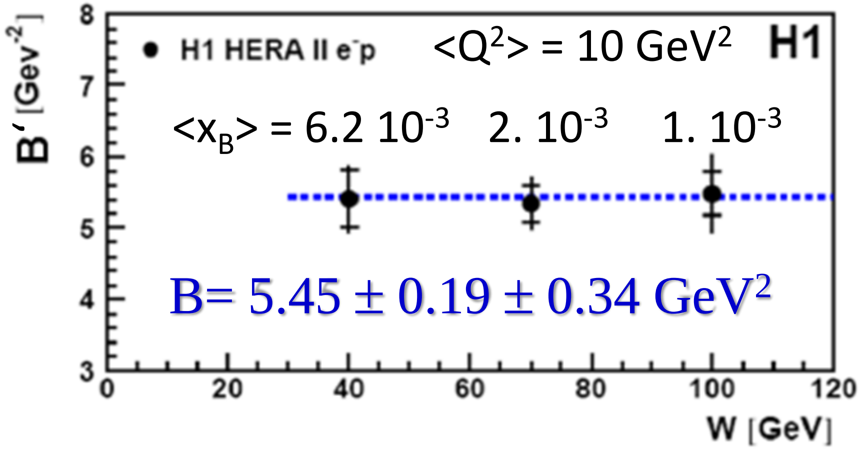
ZEUS-H1
Data collected
1995-2007

B' related to the transversed size of the scattering object

Aaron et al., H1 Coll, PLB659 (2008)





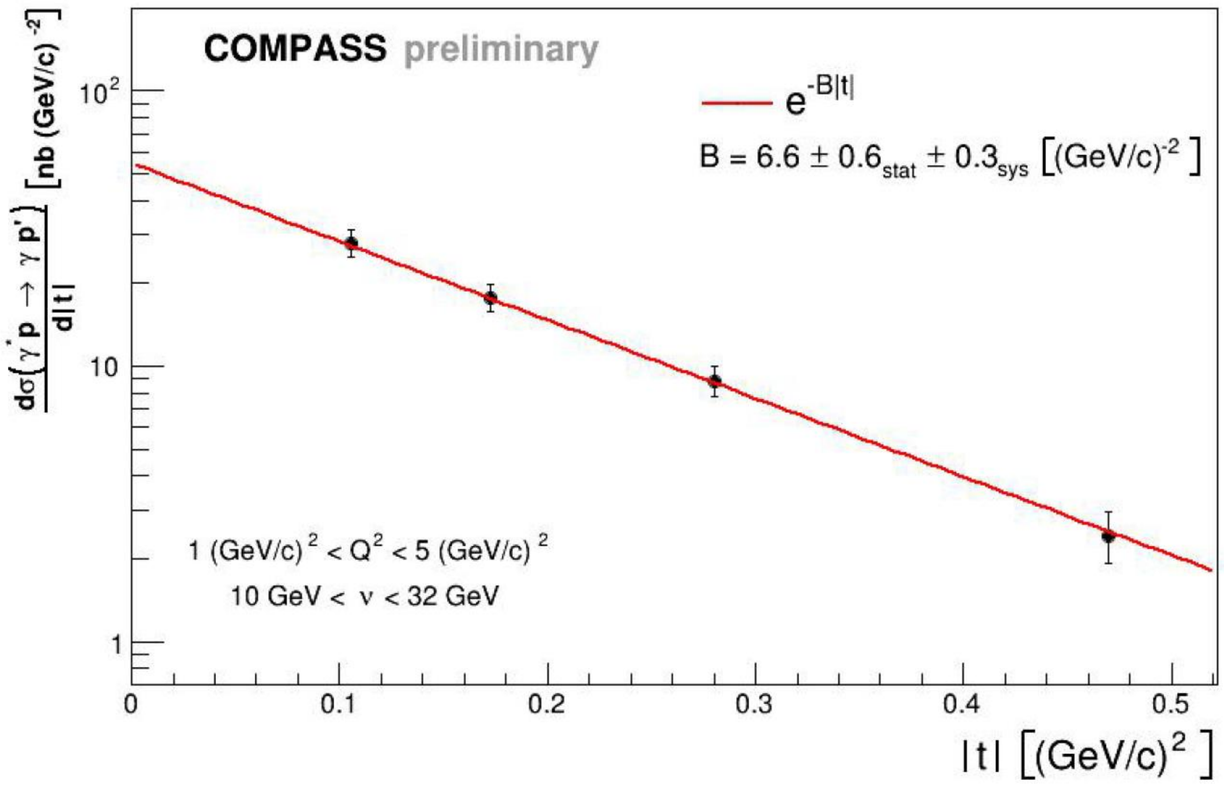


$< r_\perp^2 (x_B) > \approx 2\,B'(x_B)$

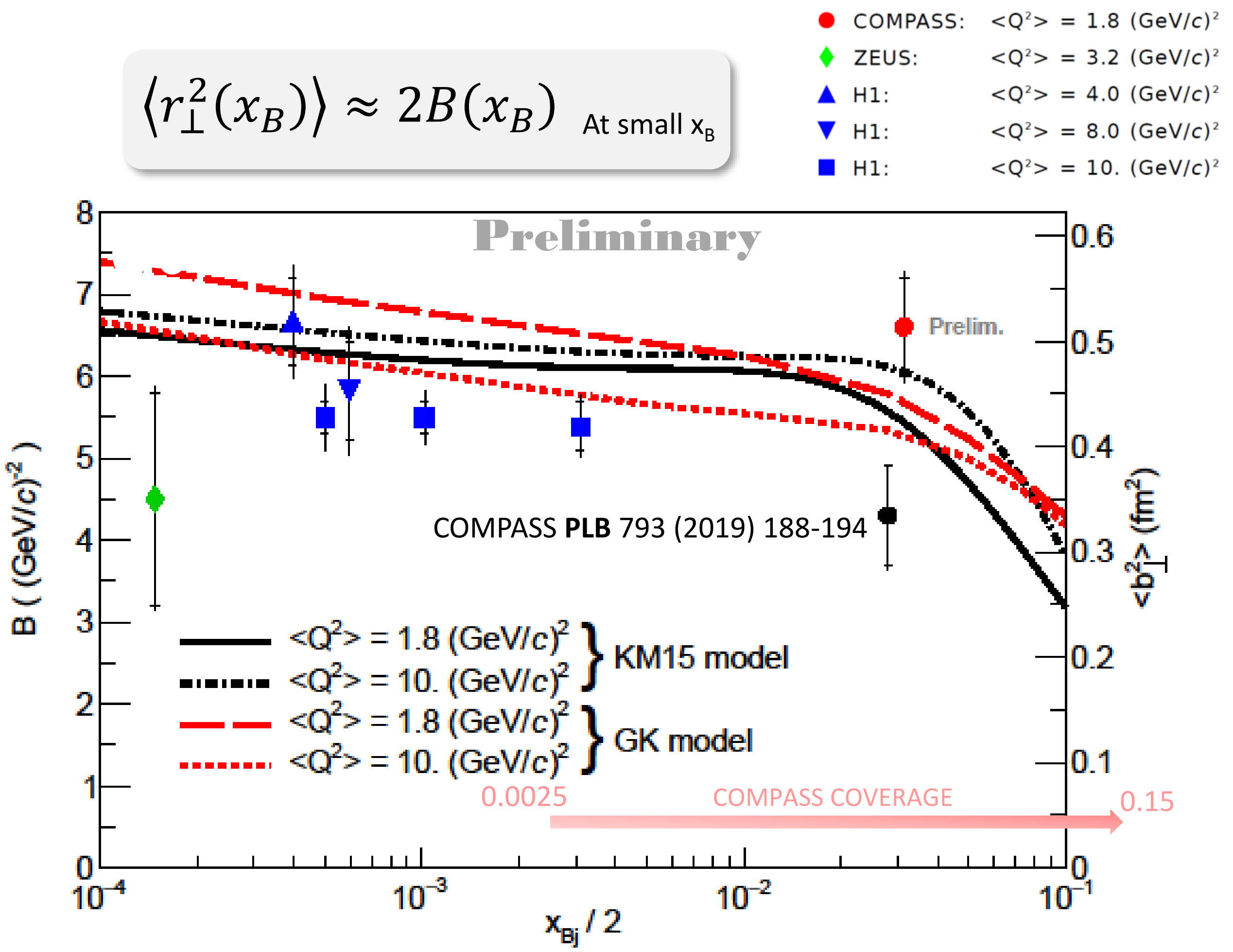
$\sqrt{< r_\perp^2 >} = 0.65 \pm 0.02$ fm

Nucleon Tomography of COMPASS Preliminary Result

$d\sigma^{DVCS}/d|t| \propto e^{-B|t|}$



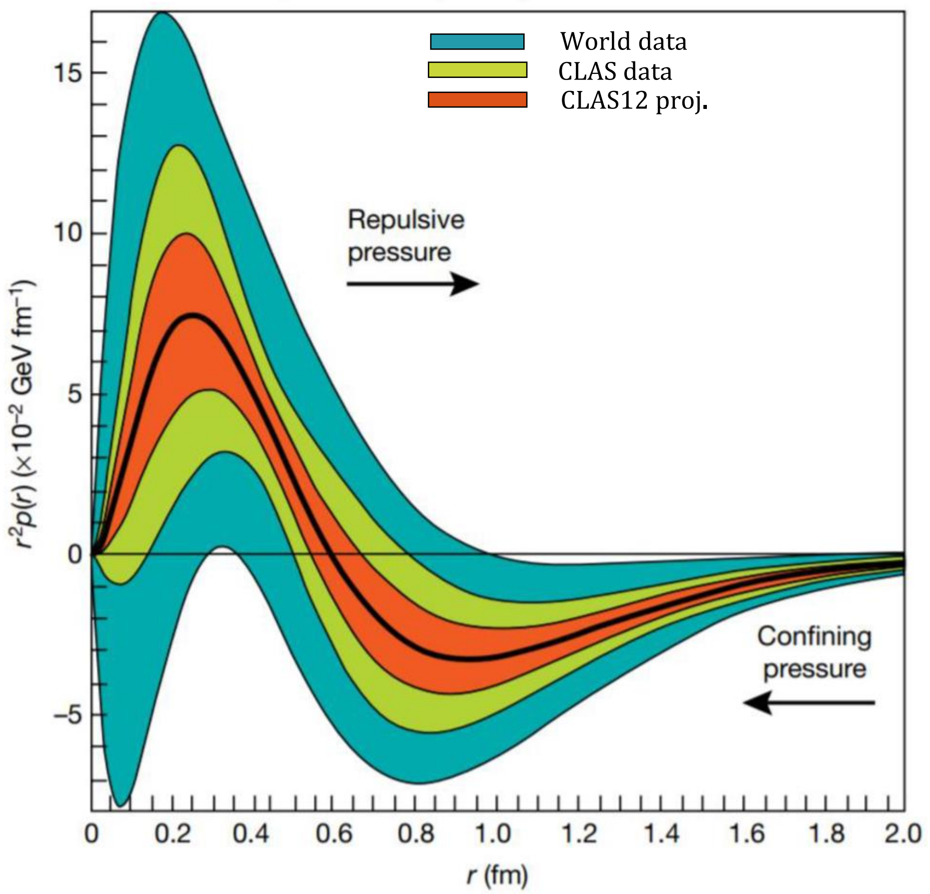
$\langle r_\perp^2(x_B)\rangle \approx 2B(x_B)$ At small x_B



➢ The transverse-size evolution as a function of x_B → Expect at least 3 x_B bins from full 2016-17 data

GPDs and Pressure Distribution

V. D. Burkert, L. Elouadrhiri, F. X. Girod
Nature **557**, 396-399 (2018)



- **Repulsive** pressure near center
  $p(r=0) = 10^{35}$ Pa
- **Confining** pressure at r > 0.6 fm

➢ **With all the data from beam spin sum and difference of CLAS at 6 GeV**

$\overleftrightarrow{e}\,p \rightarrow e\,\gamma\,p$

$$\int xH(x,\xi,t)\mathrm{d}x = M_2(t) + \boxed{\frac{4}{5}\xi^2 d_1(t)}$$

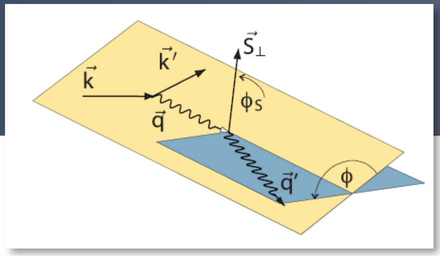
$$d_1(t) \propto \int \frac{j_0(r\sqrt{-t})}{2t} p(r)\mathrm{d}^3 r$$

$M_2(t)$: Mass/energy distribution inside the nucleon
$d_1(t)$: Forces and pressure distribution

Bessel Integral relates *$d_1(t)$* to the radial pressure p(r).

Atmospheric pressure: 10^5 Pa
Pressure in the center of neutron stars ≤ 10^{34} Pa

GPDs and Nucleon Spin



$\ell\, d \rightarrow \ell\, n\, \gamma\, (p)$

$$\Delta\sigma_{LU}^{\sin\phi} = Im\,(F_{1n}\mathcal{H} + \xi(F_{1n}+F_{2n})\widetilde{\mathcal{H}} + t/4m^2 F_{2n}\mathcal{E})$$

$\vec{\ell}\, p^{\uparrow} \rightarrow \ell\, p\, \gamma$

$$\Delta\sigma_{UT}^{\sin(\phi-\phi s)\cos\phi} = -\,t/4m^2\ Im\,(F_{2p}\mathcal{H} - F_{1p}\mathcal{E})$$

$$\Delta\sigma_{LT}^{\sin(\phi-\phi s)\cos\phi} = -\,t/4m^2\ Re\,(F_{2p}\mathcal{H} - F_{1p}\mathcal{E})$$

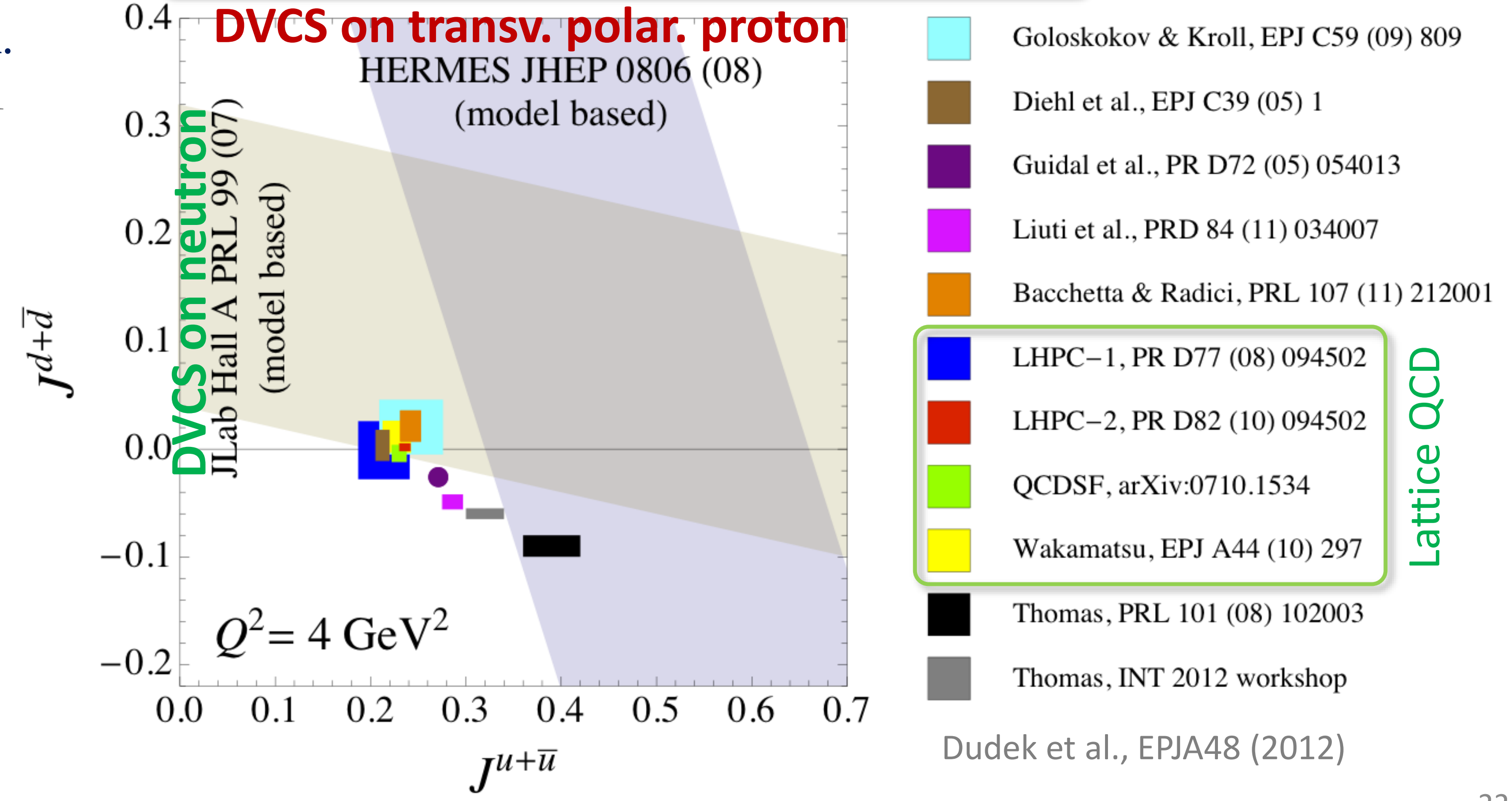
- First experimental constraint on E^q from neutron DVCS beam spin asymmetry at Hall A.

  M. Mazouz *et al*, PRL 99 (2007) 242501

- Provides constraints on orbital angular momentum of quarks

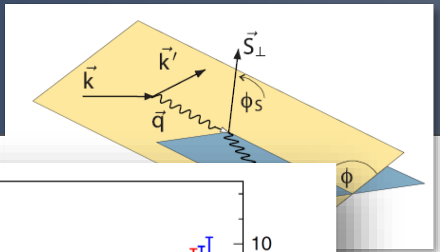
$$J_q = \frac{1}{2}\Sigma_q + L_q = \frac{1}{2}\int_{-1}^{1} dx\, x[H^q(x,\xi,0) + E^q(x,\xi,0)]$$

Model dependent extraction of J^u and J^d



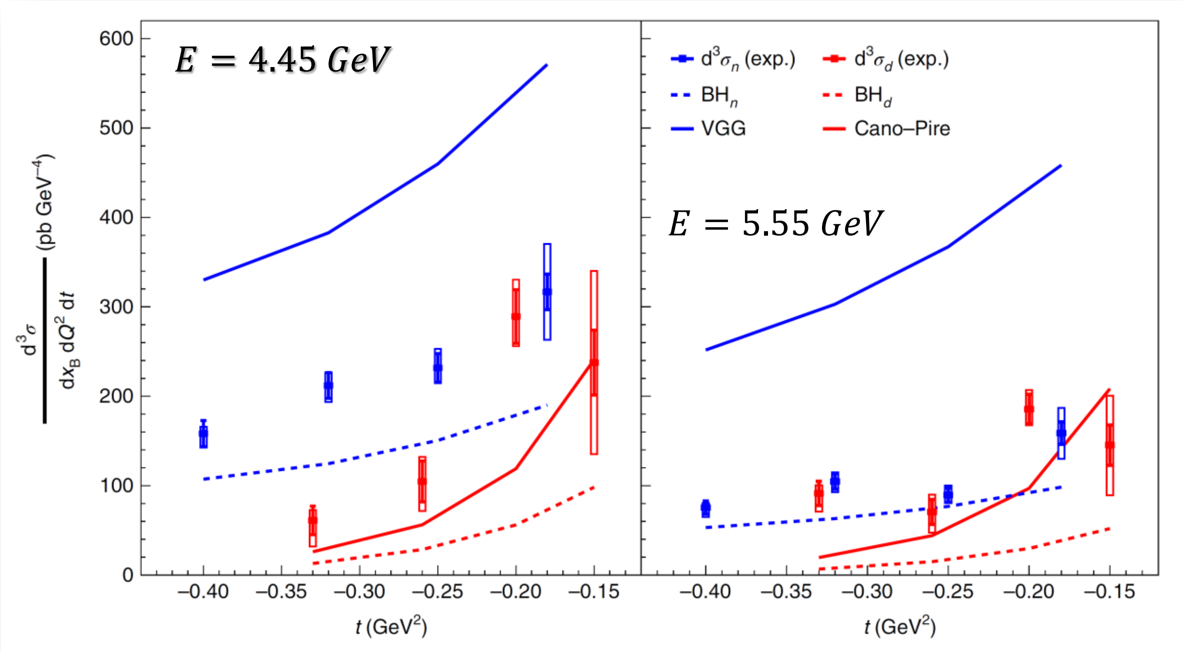
Dudek et al., EPJA48 (2012)

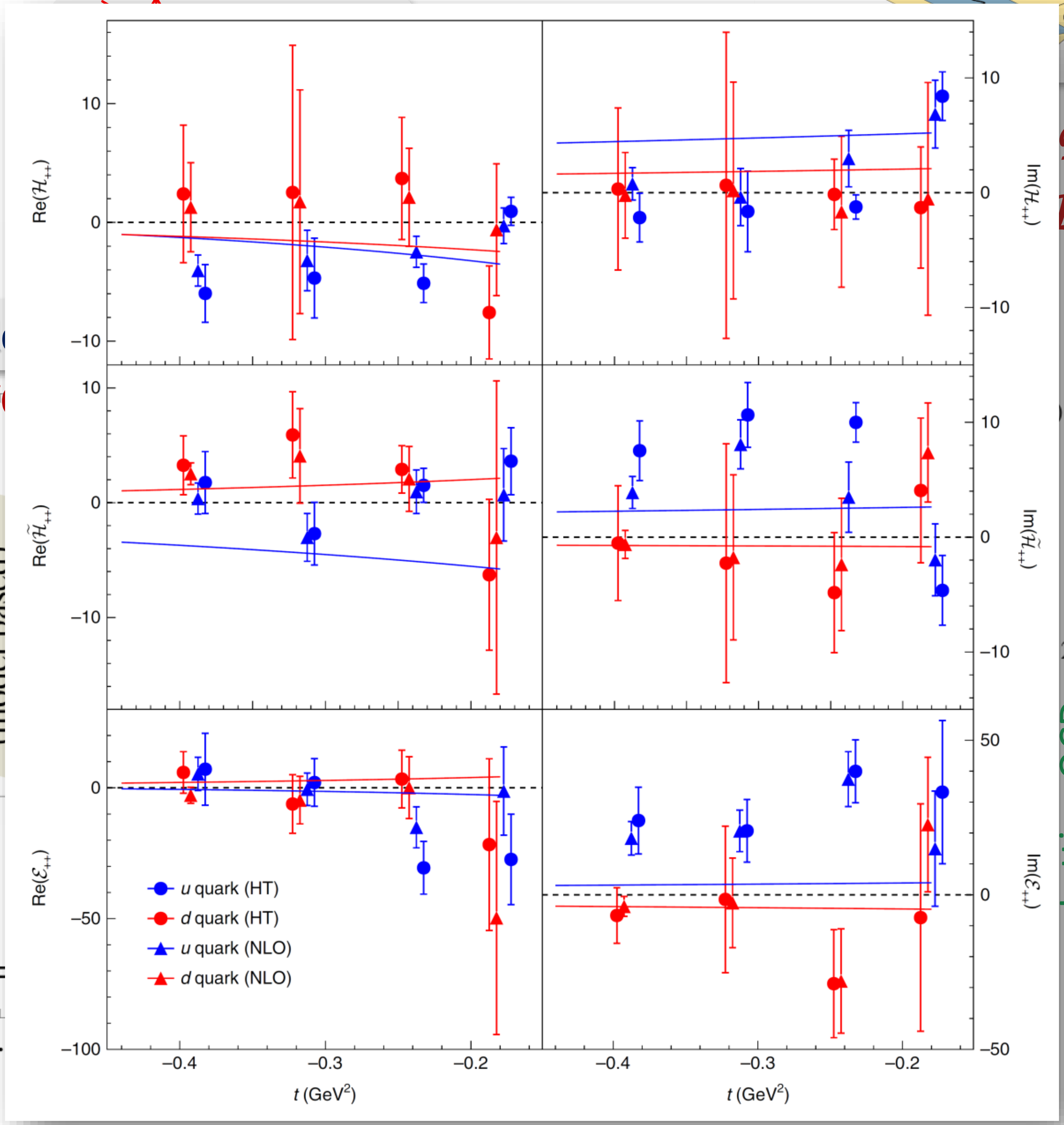
GPDs and Nucleon Spin



ℓ d → ℓ n γ (p)

$$\Delta\sigma_{LU}^{\sin\phi} = Im\,(F_{1n}\mathcal{H} + \xi(F_{1n}+F_{2n})\widetilde{\mathcal{H}} + t/4m^2\, F_{2n}\mathcal{E})$$

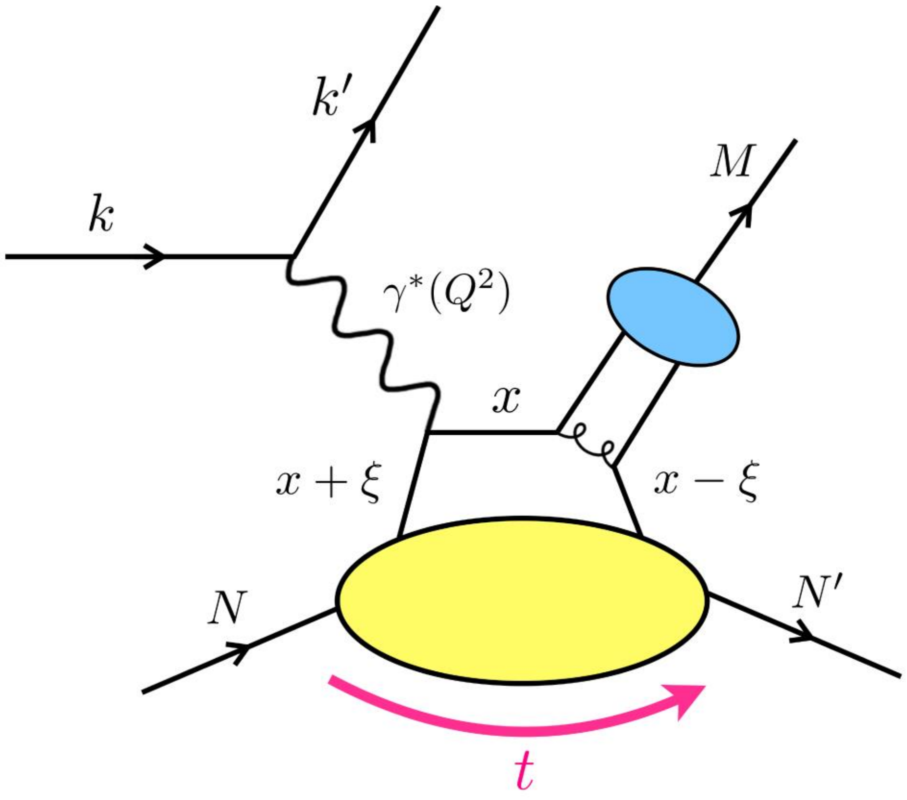




- Recent input from Result of neutron-DVCS at Hall A E08-025 (done on 2010)
- with $E_e = 4.5$ & 5.5 GeV on LD_2 target. $< Q^2 > = 1.75$ GeV², $< x_B > = 0.36$

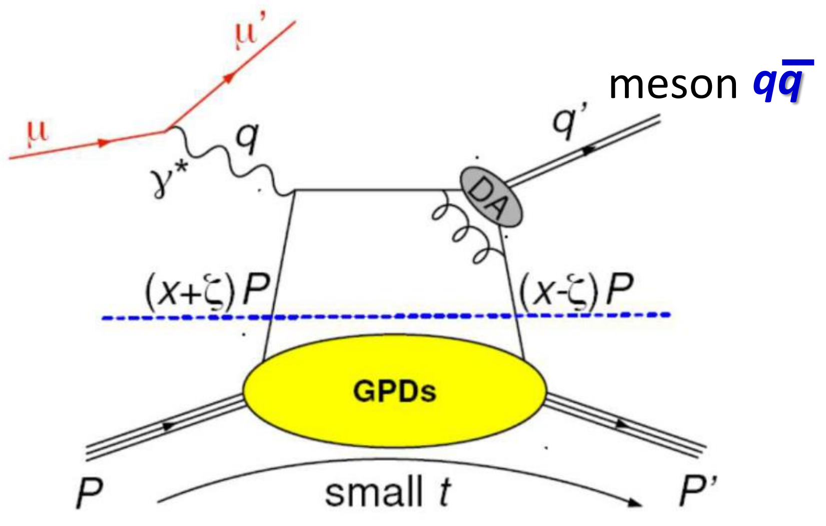
M. Benali *et al.*, Nature Phys. 16(2), 191 **(2020)**

DVMP

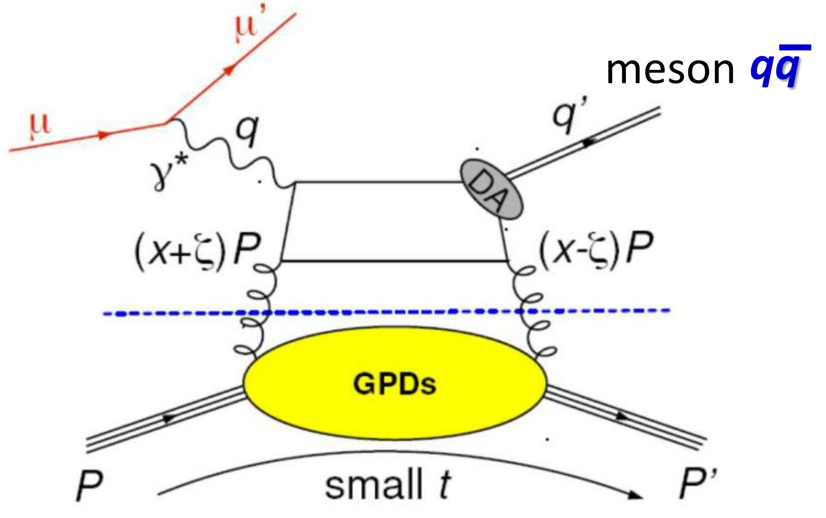


Deeply Virtual Meson Production (DVMP)

quark contribution



gluon contribution



4 chiral-even GPDs: helicity of parton unchanged

$H^q(x, \xi, t)$ $E^q(x, \xi, t)$

$\tilde{H}^q(x, \xi, t)$ $\tilde{E}^q(x, \xi, t)$

+ 4 chiral-odd or transversity GPDs: helicity of parton changed

$H_T^q(x, \xi, t)$ $E_T^q(x, \xi, t)$

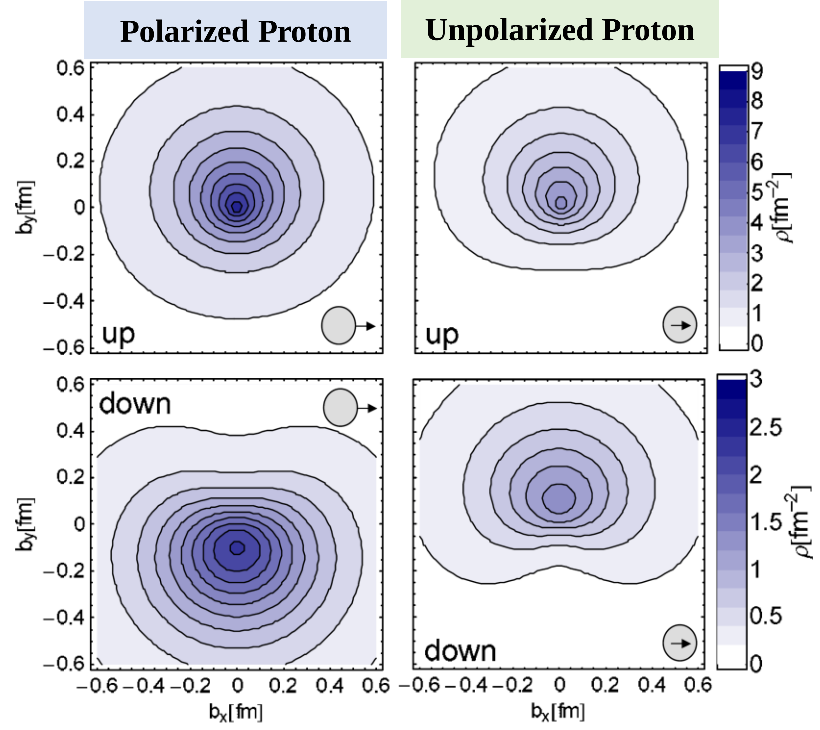
$\tilde{H}_T^q(x, \xi, t)$ $\tilde{E}_T^q(x, \xi, t)$

$$\bar{E}_T^q = 2\,\tilde{H}_T^q + E_T^q$$

- Universality of GPDs, quark flavor filter
- Ability to probe the chiral-odd GPDs.
- Additional non-perturbative term from meson wave function → more difficult for GPD extraction
- In addition to nuclear structure, provide insights into reaction mechanism

What Can We Learn from Chiral-GPDs

		Quark polarization		
		U	L	T
Nucleon polarization	U	H		$\bar{E}_T$
	L		$\tilde{H}$	$\tilde{E}_T$
	T	E	$\tilde{E}$	H_T, $\tilde{H}_T$



Described by E Described by $\bar{E}_T = 2\tilde{H}_T + E_T$

Lattice Calculation: Gockeler *PRL* 222001 (2007)

- $\bar{E}_T$ is related to the distortion of the polarized quark distribution in the transverse plane for an unpolarized nucleon
- Chiral-odd GPDs H_T
  - Generalization of transversity distribution $h_1(x)$ → related to the transverse spin structure
  - Tensor charge

GPDs parametrization:

H_T

→ tensor charge: T.Ledwig, A.Silva, H.C. Kim
$\int dx H_T(x, \xi, t)$

→ transversity PDF: M.Anselmino
$H_T(x, \xi = 0, t = 0) = h_1$

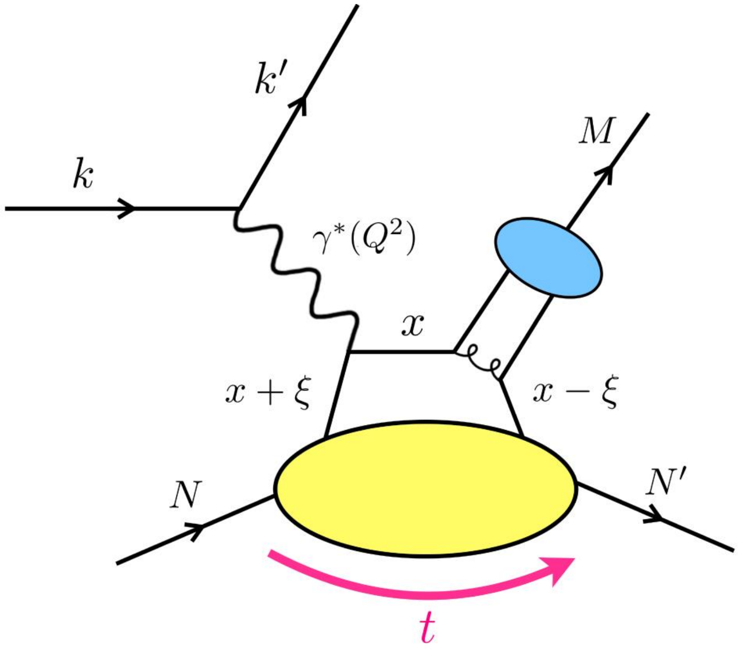
DVMP Structure Functions with Longitudinally Polarized Beam & Target

$$\frac{2\pi}{\Gamma}\frac{d^4\sigma}{dQ^2 dx_B dt d\phi} = \sigma_T + \epsilon\sigma_L + \epsilon\sigma_{TT}\cos 2\phi + \sqrt{\epsilon(1+\epsilon)}\sigma_{LT}\cos\phi$$ → Unpolarized

$$+P_b\sqrt{\epsilon(1-\epsilon)}\sigma_{LT'}\sin\phi$$ → Longitudinally polarized beam

$$+P_{tg}\left(\sqrt{\epsilon(1+\epsilon)}\sigma_{UL}^{\sin\phi}\sin\phi + \epsilon\sigma_{UL}^{\sin 2\phi}\sin 2\phi\right)$$ → Longitudinally polarized target

$$+P_b P_{tg}\left(\sqrt{1-\epsilon^2}\sigma_{LL} + \sqrt{\epsilon(1-\epsilon)}\,\sigma_{LL}^{\cos\phi}\cos\phi\right)$$ → Longitudinally polarized beam and target



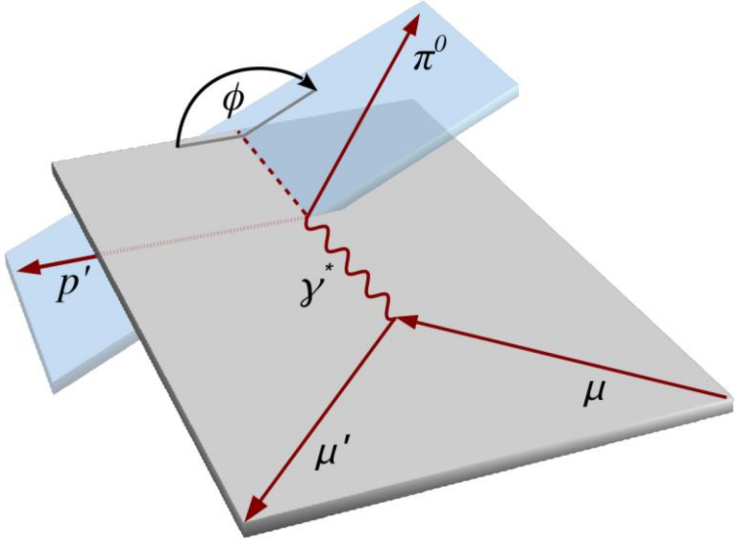
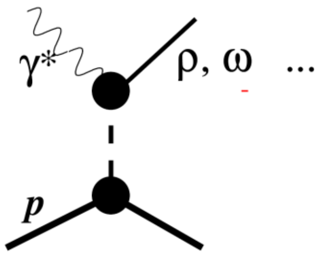


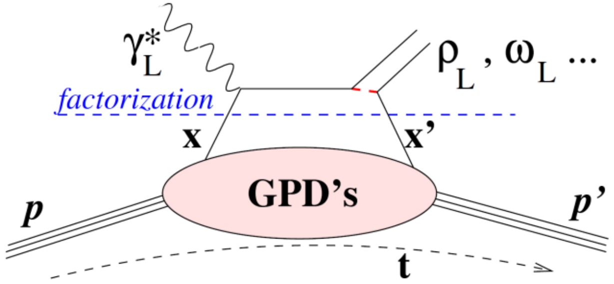
Fig: M.G. Alexeev et al. *Phys.Lett.B* 805 (2020)

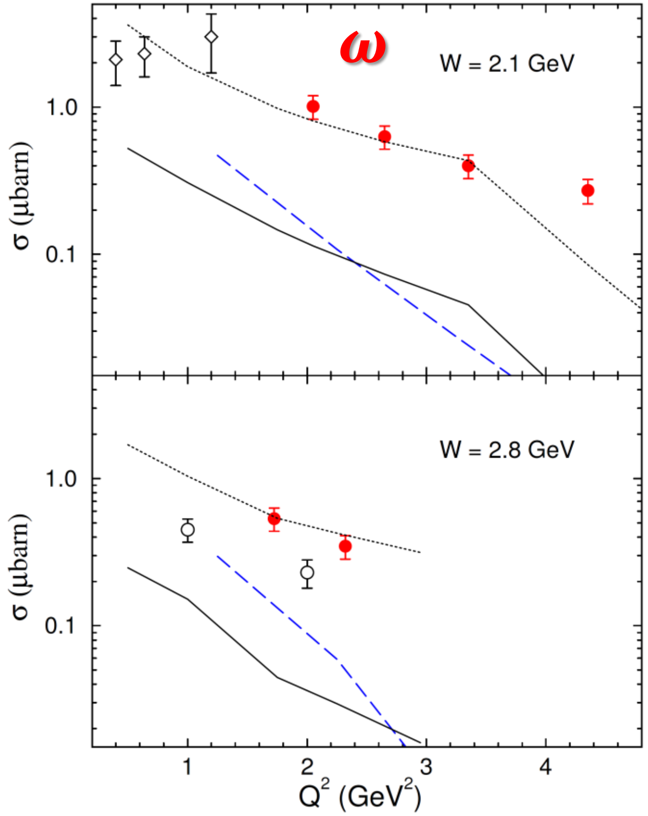
ϵ : degree of longitudinal polarization
P_b: initial lepton polarization
P_{tg}: initial target polarization

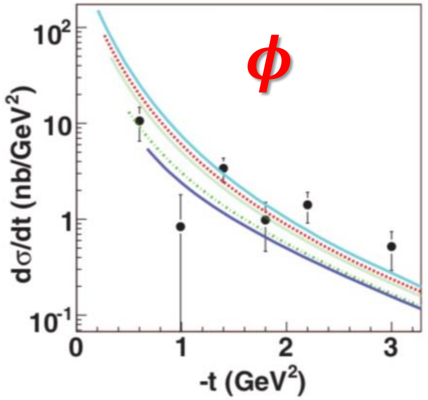
Vector Meson Production at CLAS

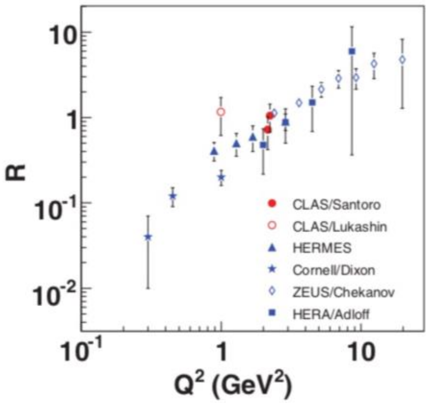
- Pilot analysis of exclusive ω-electroproduction published in EPJ A **24**, 445 (2005), followed by analyses of ϕ, Phys. Rev. C **78**, 025210 (2008), and ρ^0, EPJ A **39**, 5-31 (2009)
- Test two hypotheses → t-channel Regge trajectory exchange on the hadronic level and the handbag diagram approach on the partonic level
- Regge Model favored by data in CLAS kinematics.

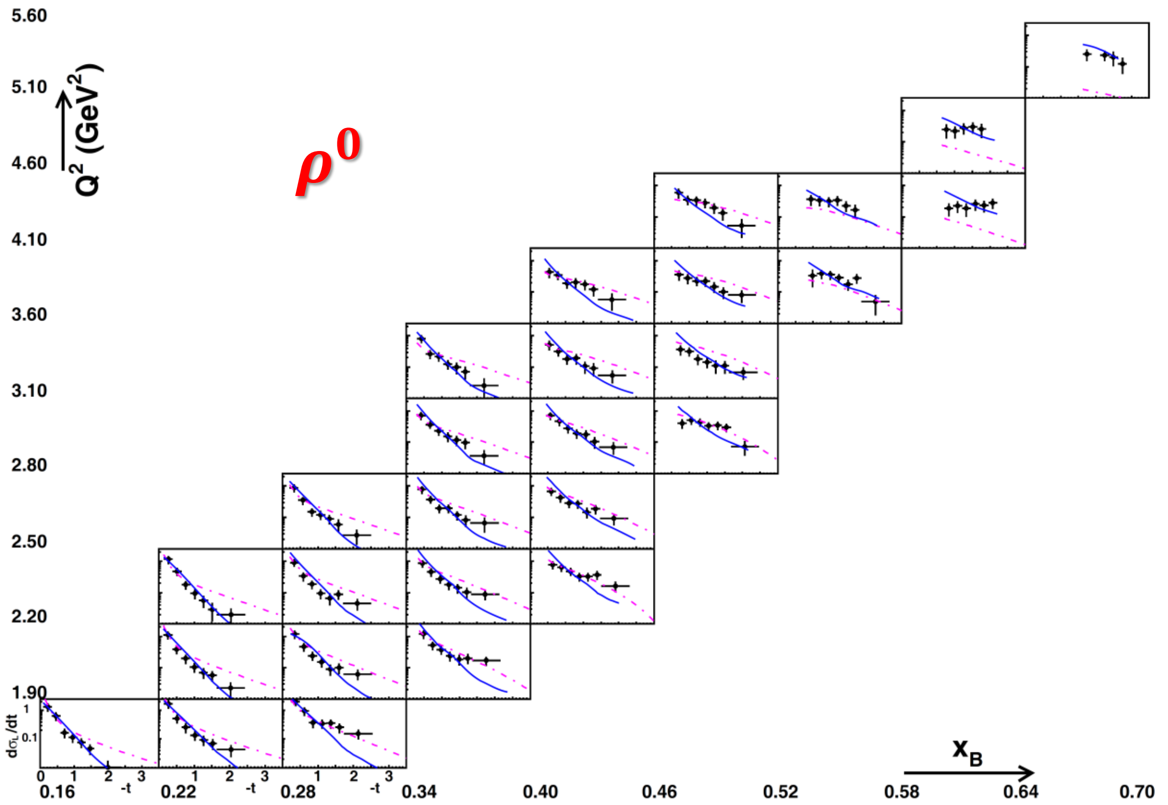




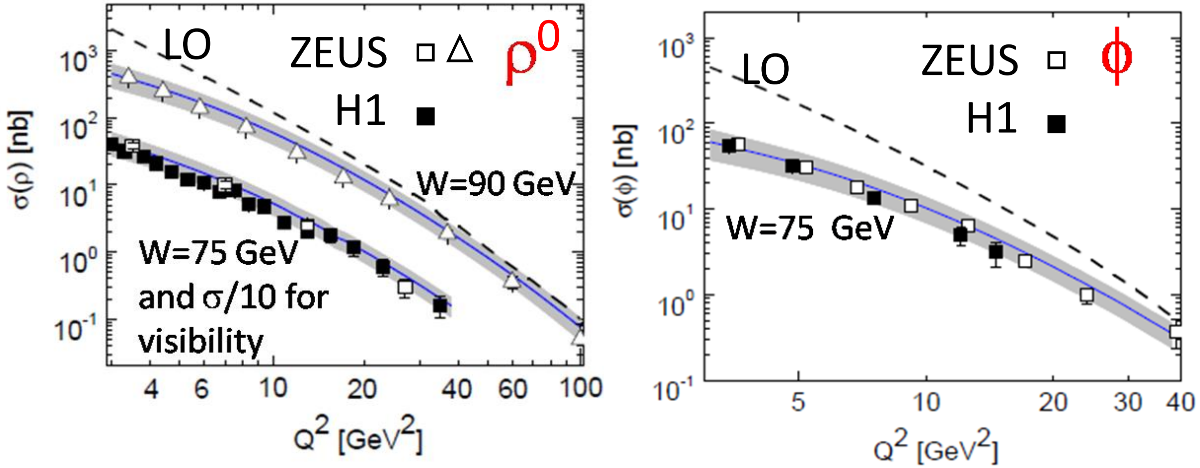


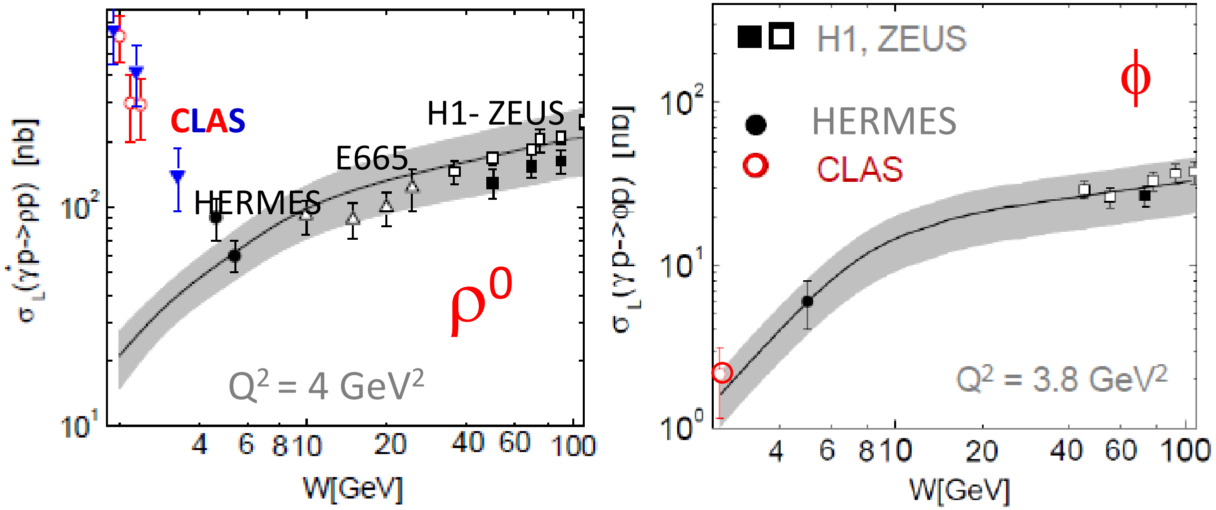






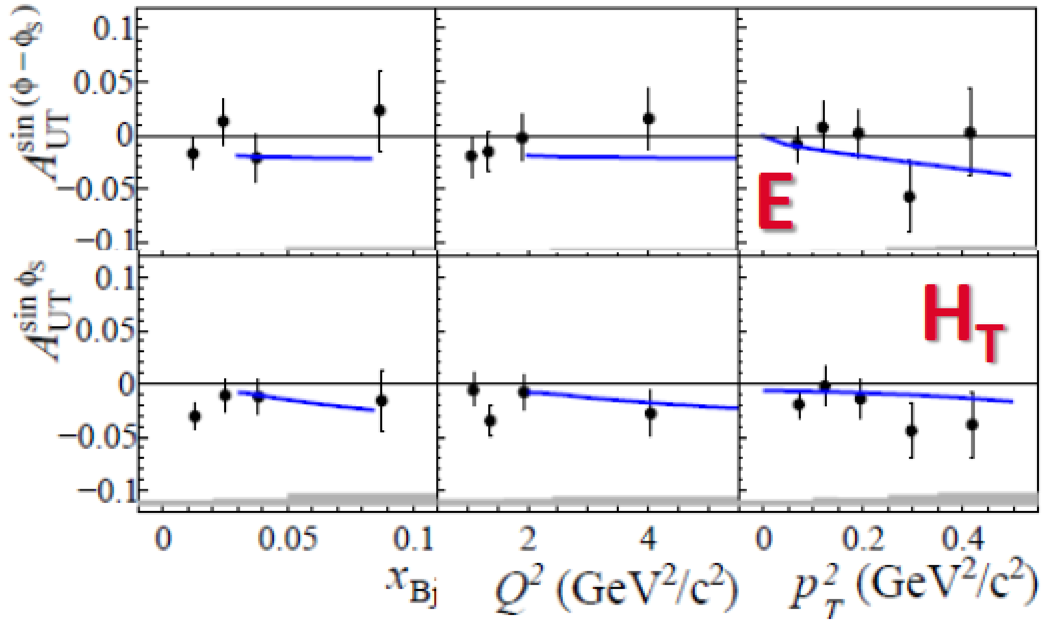
GPDs with Vector Meson Production





ρ^0 ($\rightarrow \pi^+\pi^-$) production at COMPASS with Transversely Polarized Target

COMPASS, NPB 865 (2012) 1-20, PLB731 (2014) 19



GK Model by Goloskokov, Kroll, constrained by DVMP at small x_B (or large W)

- leading-twist longitudinal $\gamma_L^*\, p \rightarrow M\, p$ and transv. polar. $\gamma_T^*\, p \rightarrow M\, p$
- quark and gluon contributions (GPDs **H, E, H_T**) and beyond leading twist

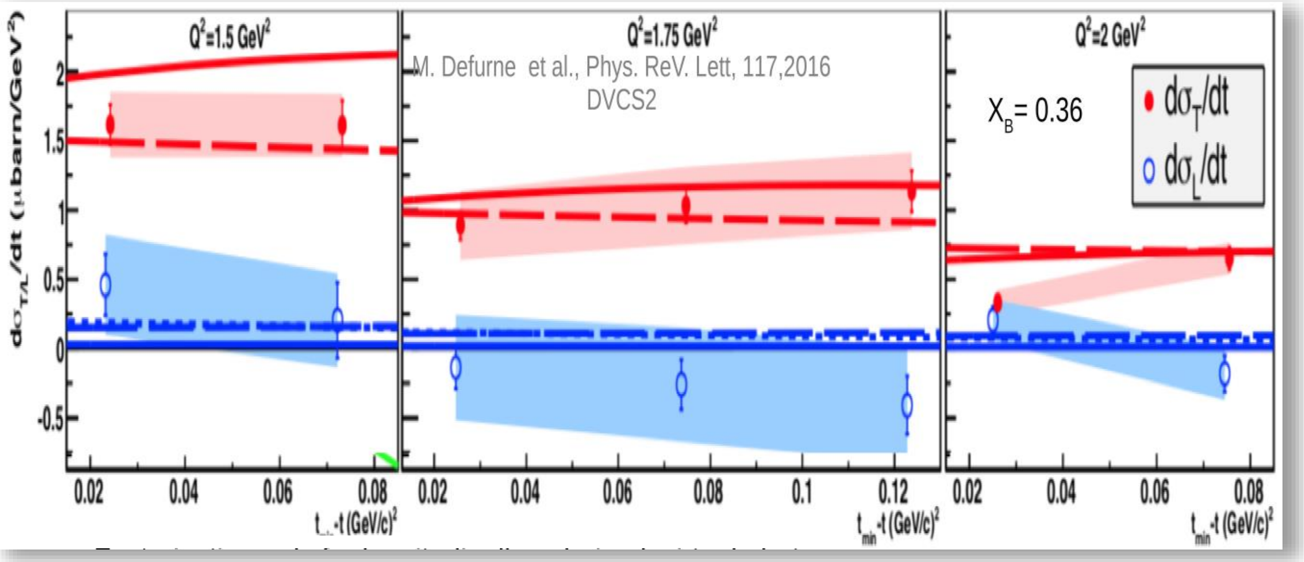
Exclusive π^0 Production

$\ell\, p \rightarrow \ell\, \pi^0\, p$

$$\frac{d^4\sigma}{dQ^2 dx_B dt d\phi} = \frac{1}{2\pi}\Gamma_\gamma(Q^2, x_B, E)\left[\frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt} + \sqrt{2\epsilon(1+\epsilon)}\frac{d\sigma_{TL}}{dt}\cos(\phi) + \epsilon\frac{d\sigma_{TT}}{dt}\cos(2\phi) + h\sqrt{2\epsilon(1-\epsilon)}\frac{d\sigma_{TL'}}{dt}\sin(\phi)\right]$$

- $\frac{d\sigma_L}{dt} = \frac{4\pi\alpha}{k'}\frac{1}{Q^6}\left\{(1-\xi^2)\left|\langle\tilde{H}\rangle\right|^2 - 2\xi^2\mathrm{Re}\left[\langle\tilde{H}\rangle^*\langle\tilde{E}\rangle\right] - \frac{t'}{4m^2}\xi^2\left|\langle\tilde{E}\rangle\right|^2\right\}$
- $\frac{d\sigma_T}{dt} = \frac{4\pi\alpha}{2k'}\frac{\mu_\pi^2}{Q^8}\left[(1-\xi^2)\left|\langle H_T\rangle\right|^2 - \frac{t'}{8m^2}\left|\langle\bar{E}_T\rangle\right|^2\right]$
- $\frac{\sigma_{LT}}{dt} = \frac{4\pi\alpha}{\sqrt{2}k'}\frac{\mu_\pi}{Q^7}\xi\sqrt{1-\xi^2}\frac{\sqrt{-t'}}{2m}\mathrm{Re}\left[\langle H_T\rangle^*\langle\tilde{E}\rangle\right]$
- $\frac{\sigma_{TT}}{dt} = \frac{4\pi\alpha}{k'}\frac{\mu_\pi^2}{Q^8}\frac{t'}{16m^2}\left|\langle\bar{E}_T\rangle\right|^2$

$$\bar{E}_T = 2\tilde{H}_T + E_T$$



- Significant transverse contribution:
  Coupling between chiral-odd (quark helicity flip) GPDs to the twist-3 pion amplitude.

—— S. V. Goloskokov and P. Kroll, Eur. Phys. J. C65:137 (2010)   - - - G. R. Goldstein, J. O. Hernandez, S. Liuti, Phys. Rev. D84 (2011)

GPDs and Hard Exclusive π^0 Production

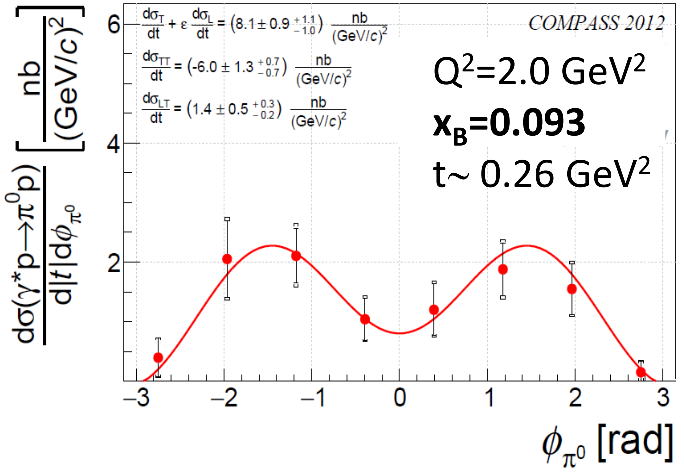
$\ell\, p \rightarrow \ell\, \pi^0\, p$

$$\frac{d^2\sigma}{dt\,d\phi_\pi} = \frac{1}{2\pi}\left[\left(\epsilon\frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt}\right) + \epsilon\cos 2\phi_\pi \frac{d\sigma_{TT}}{dt} + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_\pi \frac{d\sigma_{LT}}{dt}\right]$$

$$\left|\langle\tilde{H}\rangle\right|^2 - \frac{t'}{4m^2}\left|\langle\tilde{E}\rangle\right|^2 \qquad \left|\langle H_T\rangle\right|^2 - \frac{t'}{8m^2}\left|\langle\bar{E}_T\rangle\right|^2 \qquad \frac{t'}{16m^2}\left|\langle\bar{E}_T\rangle\right|^2 \qquad \frac{\sqrt{-t'}}{2m}\,\mathrm{Re}\left[\langle H_T\rangle^*\langle\tilde{E}\rangle\right]$$

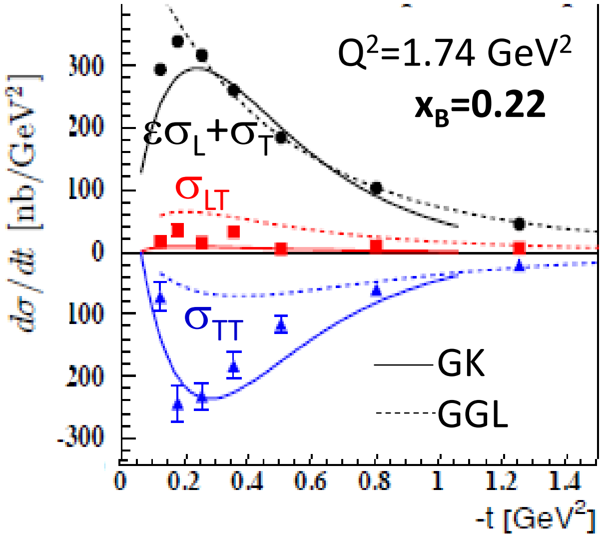
COMPASS 4 weeks 2012 pilot run

COMPASS, PLB 805 (2020) 135454



JLab 6 GeV CLAS

Bedlinskiy et al, PRL109 (2012), PRC90 (2014)



JLab 6 GeV Hall-A

Different beam energies

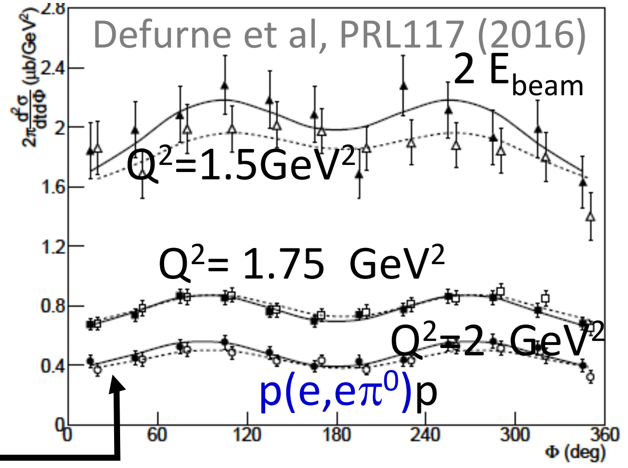
➔ L/T separation

LH2 target → proton

LD2 target → neutron+deuteron

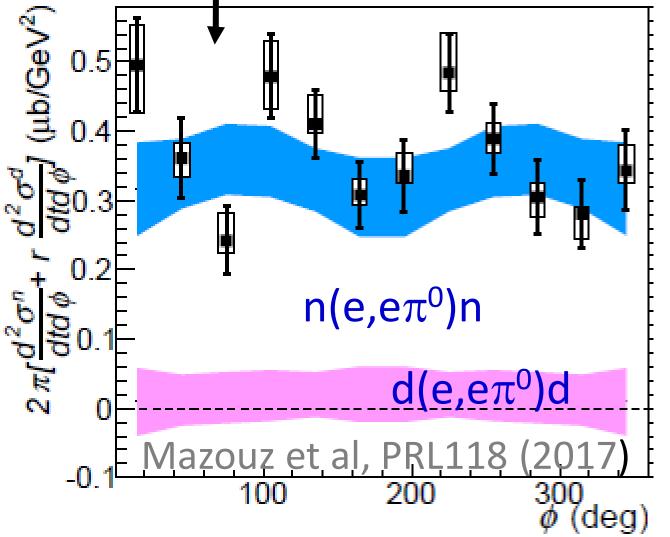
x_B=0.36

t~ 0.18 GeV^2



D(e,eπ⁰)X - p(e,eπ⁰)p = n(e,eπ⁰)n + d(e,eπ⁰)d

Q^2=1.75 GeV^2



➢ Flavor decomposition

H_T^u and H_T^d

$\bar{E}_T^u$ and $\bar{E}_T^d$

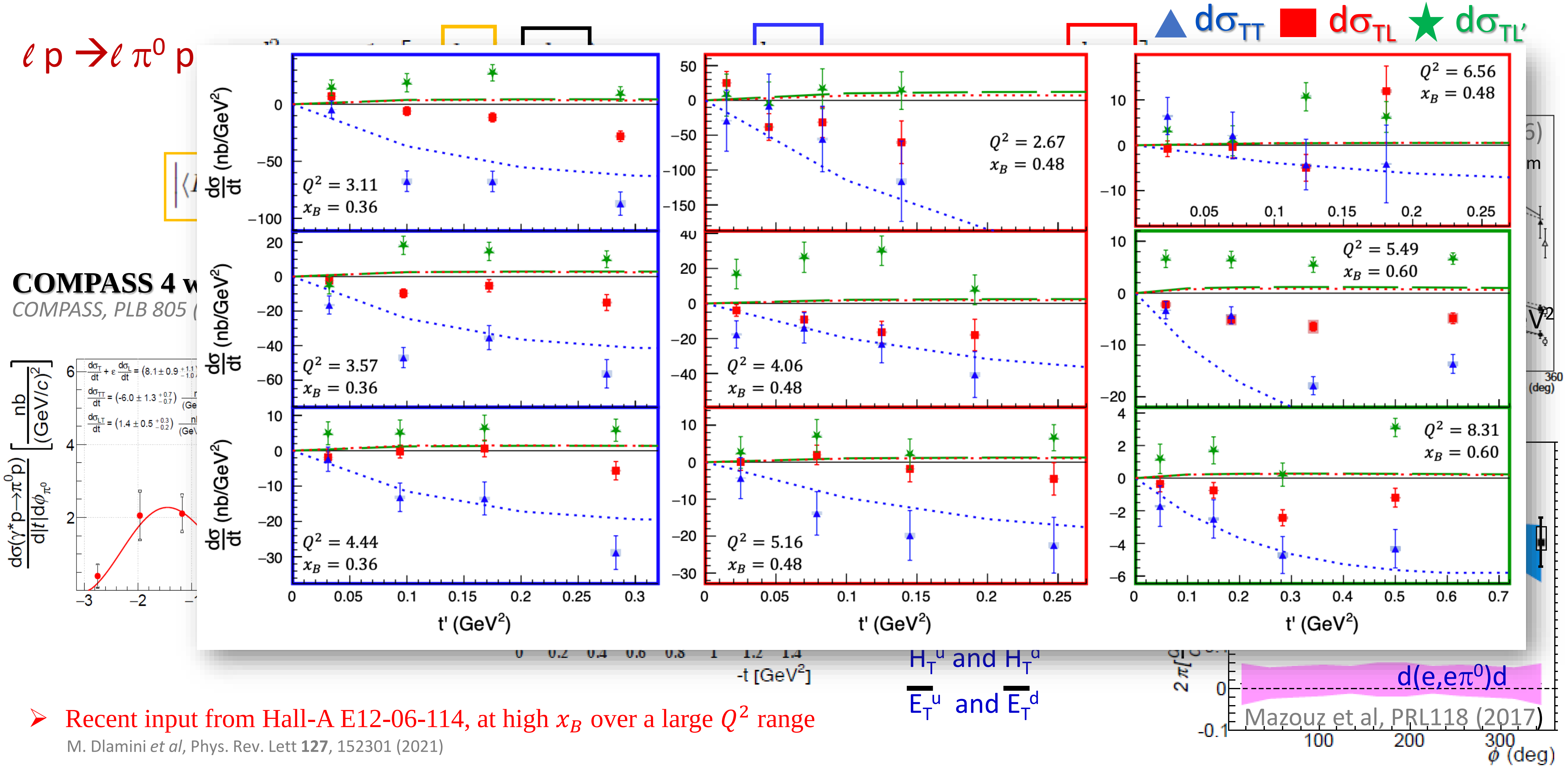
➢ Provide constraints on H_T and $\bar{E}_T$

GPDs and Hard Exclusive π^0 Production

ℓ p $\rightarrow \ell$ π^0 p

COMPASS 4 w

COMPASS, PLB 805 (



➢ Recent input from Hall-A E12-06-114, at high x_B over a large Q^2 range

M. Dlamini *et al*, Phys. Rev. Lett **127**, 152301 (2021)

GPDs and Hard Exclusive π^0 Production

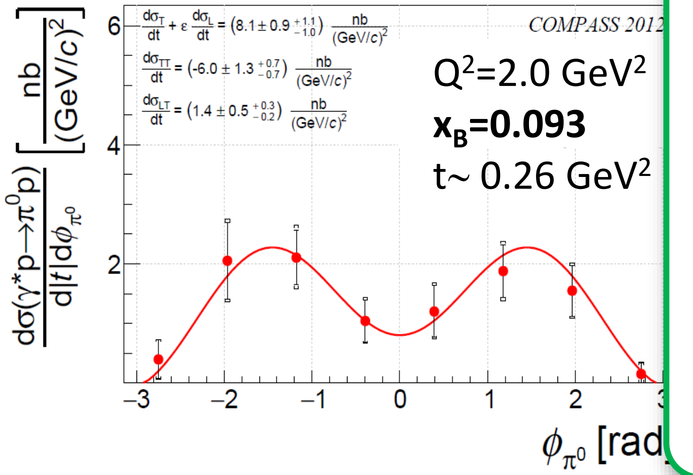
$\ell\, p \rightarrow \ell\, \pi^0\, p$

$$\frac{d^2\sigma}{dt\,d\phi_\pi} = \frac{1}{2\pi} \ldots$$

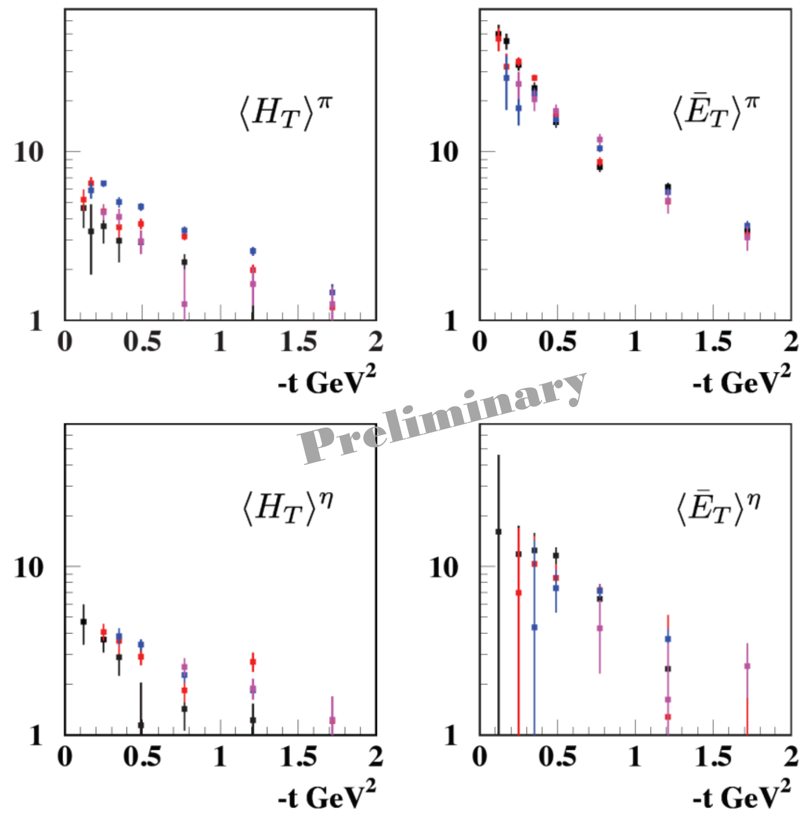
$$\left|\langle \tilde{H} \rangle\right|^2 - \frac{t'}{4m^2} \left|\langle \tilde{E} \rangle\right|^2$$

COMPASS 4 weeks 2012 pilot r...

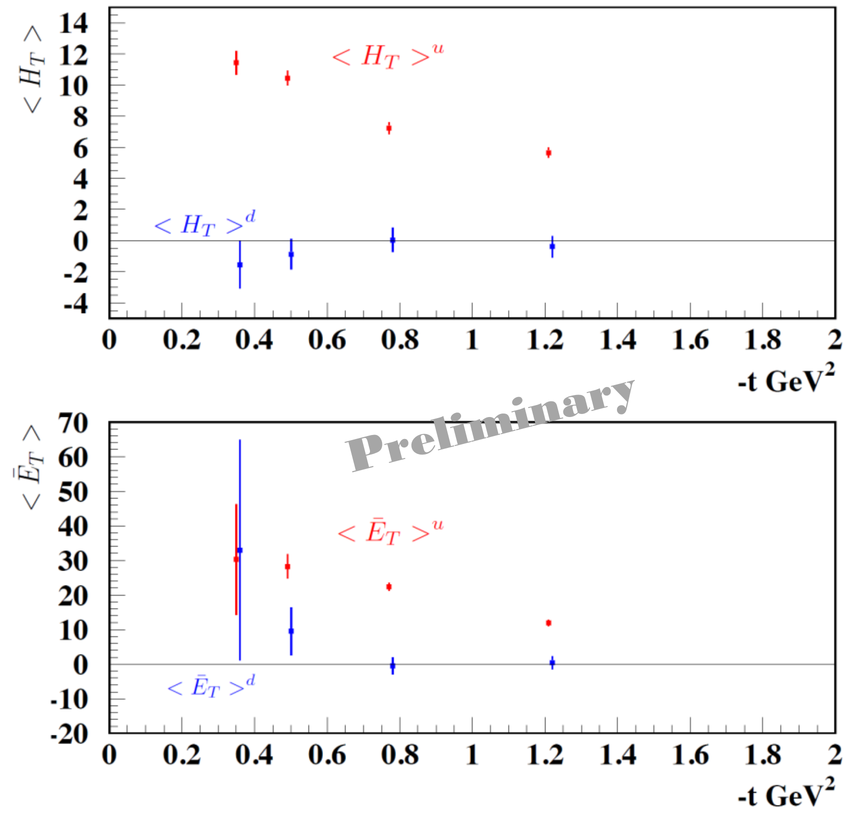
COMPASS, PLB 805 (2020) 135454



➢ Generalized Form Factors



➢ Quark Flavor Decomposition



Preliminary attempts using π^0 & η data from CLAS

Valery Kubarovsky, arXiv:1601.04367v2

016)

beam

GeV^2

-t [GeV^2]

H_T^u and H_T^d

$\bar{E}_T^u$ and $\bar{E}_T^d$

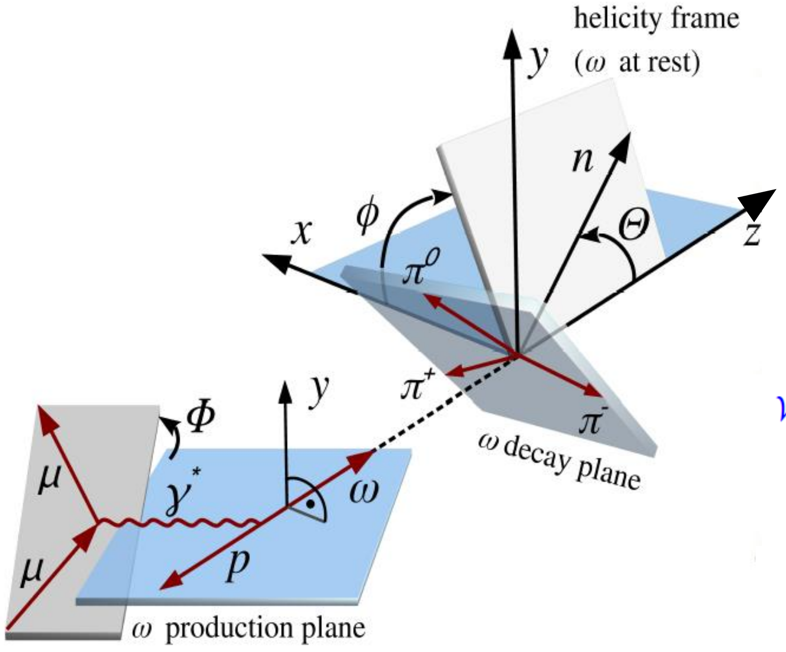
d(e,eπ^0)d

Mazouz et al, PRL118 (2017)

➢ Preliminary attempts using π^0 & η data from CLAS

Vector Meson Production: Spin Density Matrix Elements

Experimental angular distributions



$$\frac{d\sigma}{d\phi\, d\Phi\, d\Theta\, dQ^2\, dx_B\, dt} = \Gamma(Q^2, x_B, E)\frac{1}{2\pi}\left\{\frac{d\sigma_T}{dt} + \epsilon\frac{d\sigma_L}{dt}\right\}\mathcal{W}^{U+L}(\Phi, \phi, \cos\Theta)$$

$$\mathcal{W}^{U+L}(\Phi, \phi, \cos\Theta) = \mathcal{W}^{U}(\Phi, \phi, \cos\Theta) + P_b\mathcal{W}^{L}(\Phi, \phi, \cos\Theta)$$

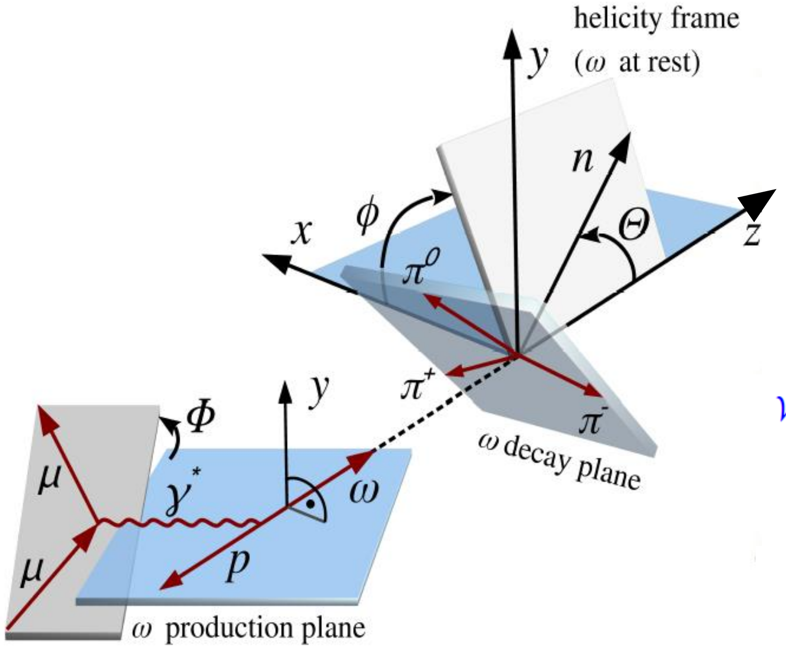
15 unpolarized SDMEs in $\mathcal{W}^U$ and 8 polarized in $\mathcal{W}^L$

$$\mathcal{W}^U(\Phi,\phi,\cos\Theta) = \frac{3}{8\pi^2}\Bigg[\frac{1}{2}(1-r^{04}_{00}) + \frac{1}{2}(3r^{04}_{00}-1)\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^{04}_{10}\}\sin 2\Theta\cos\phi - r^{04}_{1-1}\sin^2\Theta\cos 2\phi$$
$$-\epsilon\cos 2\Phi\left(r^1_{11}\sin^2\Theta + r^1_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^1_{10}\}\sin 2\Theta\cos\phi - r^1_{1-1}\sin^2\Theta\cos 2\phi\right)$$
$$-\epsilon\sin 2\Phi\left(\sqrt{2}\mathrm{Im}\{r^2_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^2_{1-1}\}\sin^2\Theta\sin 2\phi\right)$$
$$+\sqrt{2\epsilon(1+\epsilon)}\cos\Phi\left(r^5_{11}\sin^2\Theta + r^5_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^5_{10}\}\sin 2\Theta\cos\phi - r^5_{1-1}\sin^2\Theta\cos 2\phi\right)$$
$$+\sqrt{2\epsilon(1+\epsilon)}\sin\Phi\left(\sqrt{2}\mathrm{Im}\{r^6_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^6_{1-1}\}\sin^2\Theta\sin 2\phi\right)\Bigg],$$

$$\mathcal{W}^L(\Phi,\phi,\cos\Theta) = \frac{3}{8\pi^2}\Bigg[\sqrt{1-\epsilon^2}\left(\sqrt{2}\mathrm{Im}\{r^3_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^3_{1-1}\}\sin^2\Theta\sin 2\phi\right)$$
$$+\sqrt{2\epsilon(1-\epsilon)}\cos\Phi\left(\sqrt{2}\mathrm{Im}\{r^7_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^7_{1-1}\}\sin^2\Theta\sin 2\phi\right)$$
$$+\sqrt{2\epsilon(1-\epsilon)}\sin\Phi\left(r^8_{11}\sin^2\Theta + r^8_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^8_{10}\}\sin 2\Theta\cos\phi - r^8_{1-1}\sin^2\Theta\cos 2\phi\right)\Bigg]$$

Vector Meson Production: Spin Density Matrix Elements

Experimental angular distributions



$$r^1_{00}\sigma_0 \sim \left|\bar{E}_T\right|^2$$

$$r^5_{00}\sigma_0 \sim \mathrm{Re}\left[\langle\bar{E}_T\rangle\langle H\rangle + \langle H_T\rangle\langle E\rangle\right]$$

$$r^8_{00}\sigma_0 \sim \mathrm{Im}\left[\langle\bar{E}_T\rangle\langle H\rangle + \langle H_T\rangle\langle E\rangle\right]$$

$$\begin{aligned}\mathcal{W}^U(\Phi,\phi,\cos\Theta) = \frac{3}{8\pi^2}\Bigg[&\frac{1}{2}(1-r^{04}_{00}) + \frac{1}{2}(3r^{04}_{00}-1)\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^{04}_{10}\}\sin 2\Theta\cos\phi - r^{04}_{1-1}\sin^2\Theta\cos 2\phi \\ &-\epsilon\cos 2\Phi\Big(r^1_{11}\sin^2\Theta + r^1_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^1_{10}\}\sin 2\Theta\cos\phi - r^1_{1-1}\sin^2\Theta\cos 2\phi\Big) \\ &-\epsilon\sin 2\Phi\Big(\sqrt{2}\mathrm{Im}\{r^2_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^2_{1-1}\}\sin^2\Theta\sin 2\phi\Big) \\ &+\sqrt{2\epsilon(1+\epsilon)}\cos\Phi\Big(r^5_{11}\sin^2\Theta + r^5_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^5_{10}\}\sin 2\Theta\cos\phi - r^5_{1-1}\sin^2\Theta\cos 2\phi\Big) \\ &+\sqrt{2\epsilon(1+\epsilon)}\sin\Phi\Big(\sqrt{2}\mathrm{Im}\{r^6_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^6_{1-1}\}\sin^2\Theta\sin 2\phi\Big)\Bigg],\end{aligned}$$

$$\begin{aligned}\mathcal{W}^L(\Phi,\phi,\cos\Theta) = \frac{3}{8\pi^2}\Bigg[&\sqrt{1-\epsilon^2}\Big(\sqrt{2}\mathrm{Im}\{r^3_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^3_{1-1}\}\sin^2\Theta\sin 2\phi\Big) \\ &+\sqrt{2\epsilon(1-\epsilon)}\cos\Phi\Big(\sqrt{2}\mathrm{Im}\{r^7_{10}\}\sin 2\Theta\sin\phi + \mathrm{Im}\{r^7_{1-1}\}\sin^2\Theta\sin 2\phi\Big) \\ &+\sqrt{2\epsilon(1-\epsilon)}\sin\Phi\Big(r^8_{11}\sin^2\Theta + r^8_{00}\cos^2\Theta - \sqrt{2}\mathrm{Re}\{r^8_{10}\}\sin 2\Theta\cos\phi - r^8_{1-1}\sin^2\Theta\cos 2\phi\Big)\Bigg]\end{aligned}$$

2012 COMPASS Exclusive ω Prod. On Unpolarized Proton

SCHC ($\lambda_\gamma = \lambda_V$)

(S-Channel Helicity Conservation)

SCHC implies:

- $r^1_{1-1} + \mathrm{Im}\, r^2_{1-1} = 0$

= - 0.010 ± 0.032 ± 0.047 OK

- $\mathrm{Re}\, r^5_{10} + \mathrm{Im}^6_{10} = 0$

= 0.014 ± 0.011 ± 0.013 OK

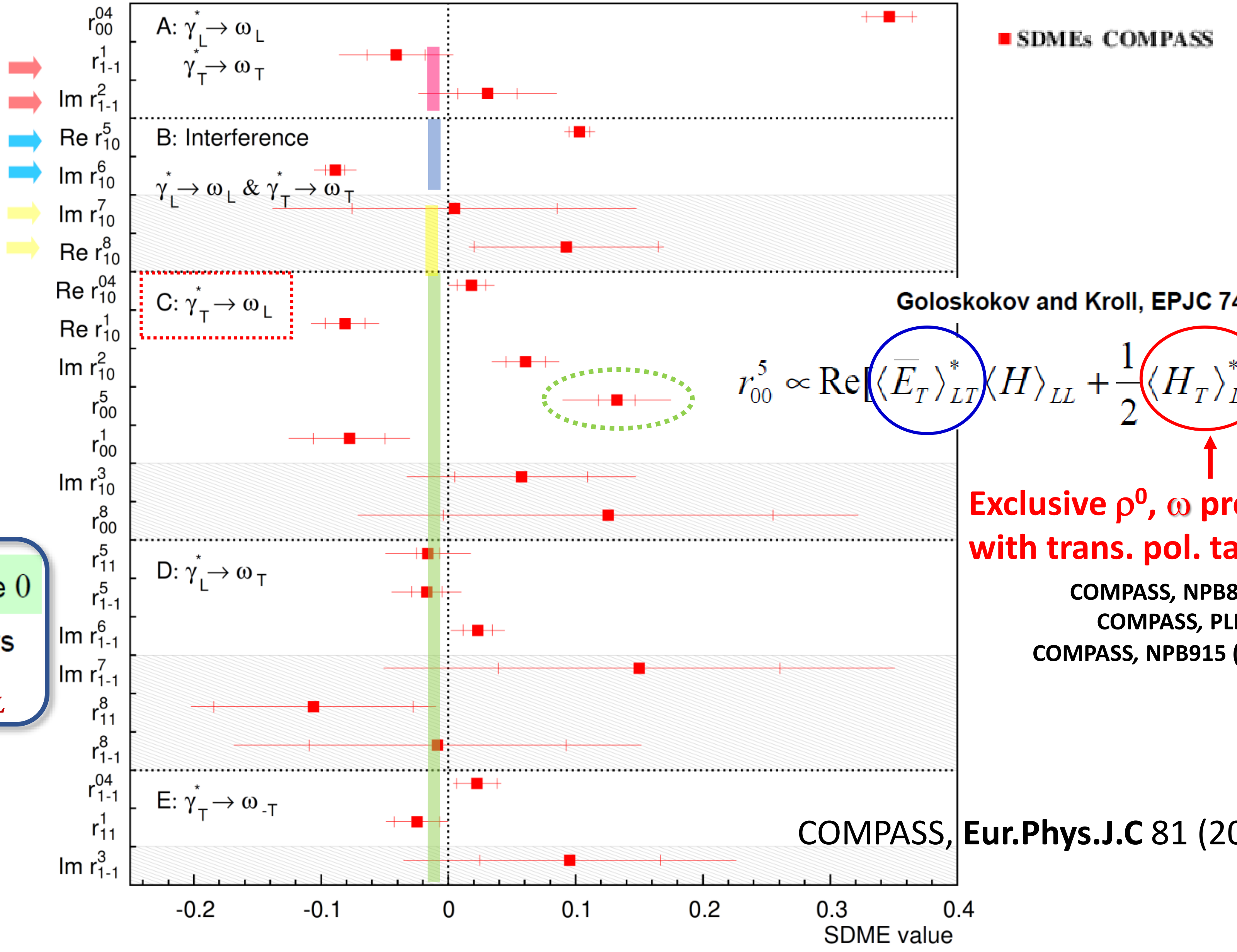
- $\mathrm{Im}\, r^7_{10} - \mathrm{Re}\, r^8_{10} = 0$

= - 0.088 ± 0.110 ± 0.196 OK

- all elements of classes C, D, E should be 0

for $\gamma^*_L \to \omega_T$ and $\gamma^*_T \to \omega_{-T}$ OK within errors

NOT OBSERVED for transitions $\gamma^*_T \to \omega_L$



Goloskokov and Kroll, EPJC 74 (2014) 2725

$$r^5_{00} \propto \mathrm{Re}[\langle \bar{E}_T \rangle^*_{LT} \langle H \rangle_{LL} + \frac{1}{2} \langle H_T \rangle^*_{LT} \langle E \rangle_{LL}]$$

Exclusive ρ^0, ω production with trans. pol. target

COMPASS, NPB865 (2012) 1-20
COMPASS, PLB731 (2014) 19
COMPASS, NPB915 (2017) 454-475

COMPASS, **Eur.Phys.J.C** 81 (2021) 126

2012 COMPASS Exclusive ρ^0 Prod. On Unpolarized Proton

SCHC ($\lambda_\gamma = \lambda_V$)
(S-Channel Helicity Conservation)

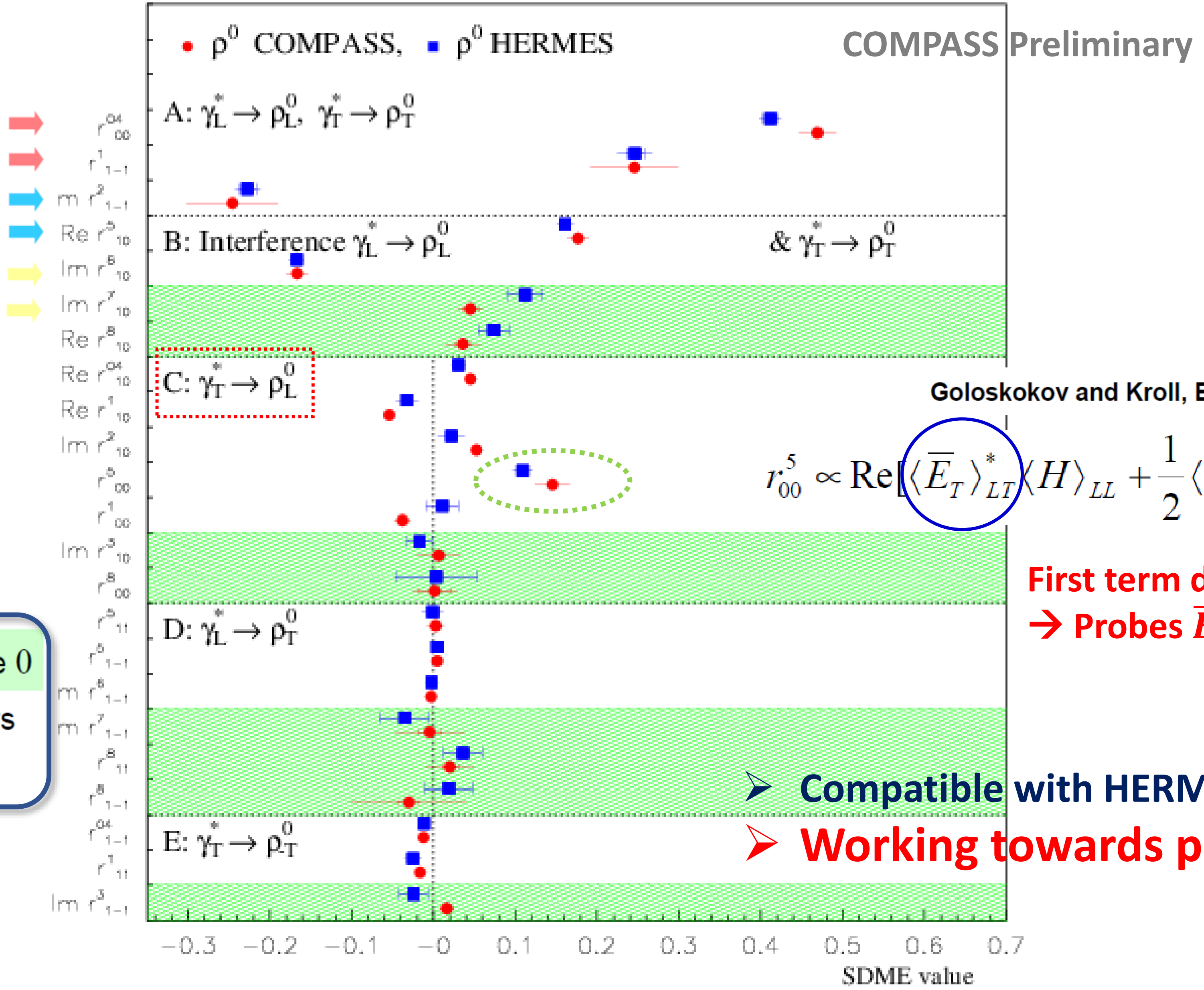
SCHC implies:

- $r^1_{1-1} + \mathrm{Im}\, r^2_{1-1} = 0$ **OK**
- $\mathrm{Re}\, r^5_{10} + \mathrm{Im}^6_{10} = 0$ **OK**
- $\mathrm{Im}\, r^7_{10} - \mathrm{Re}\, r^8_{10} = 0$ ✓**OK**

- all elements of classes C, D, E should be 0

for $\gamma^*_L \to \rho^0_T$ and $\gamma^*_T \to \rho^0_{-T}$ OK within errors

NOT OBSERVED for transitions $\gamma^*_T \to \rho^0_L$



Goloskokov and Kroll, EPJC 74 (2014) 2725

$$r^5_{00} \propto \mathrm{Re}[\langle \overline{E}_T \rangle^*_{LT} \langle H \rangle_{LL} + \frac{1}{2} \langle H_T \rangle^*_{LT} \langle E \rangle_{LL}]$$

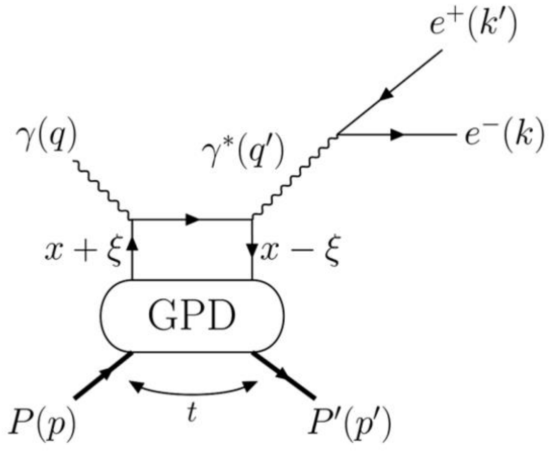
First term dominates
→ Probes $\overline{E}_T$

- **Compatible with HERMES**
- **Working towards publication!**

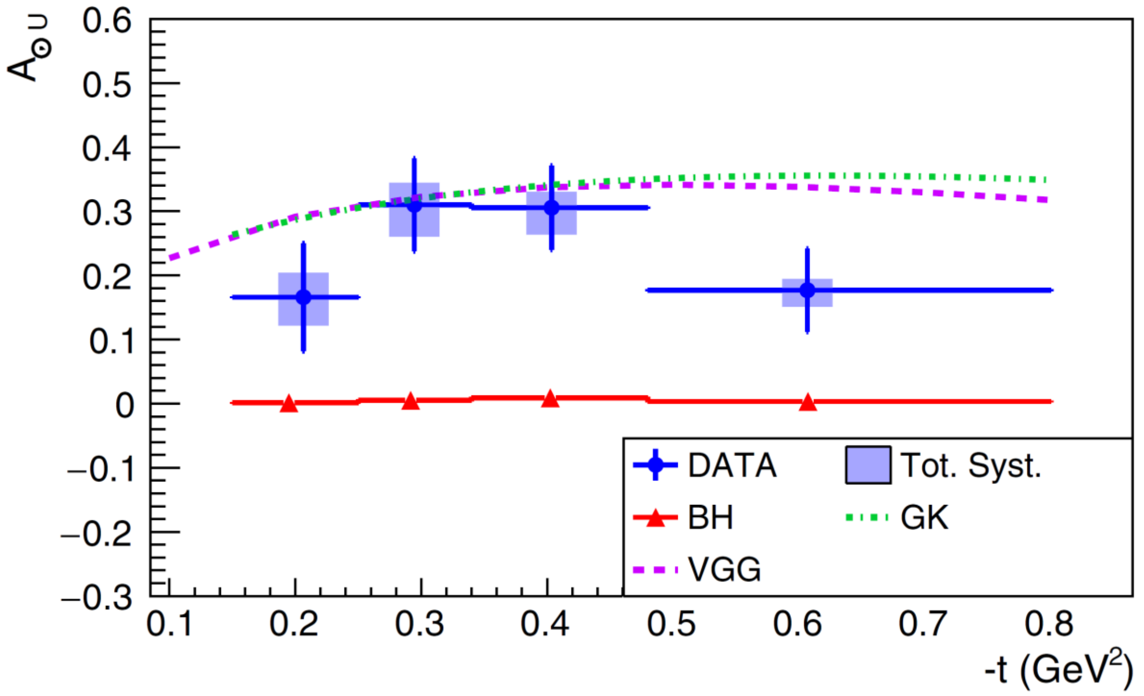
Other COMPTON Scatterings

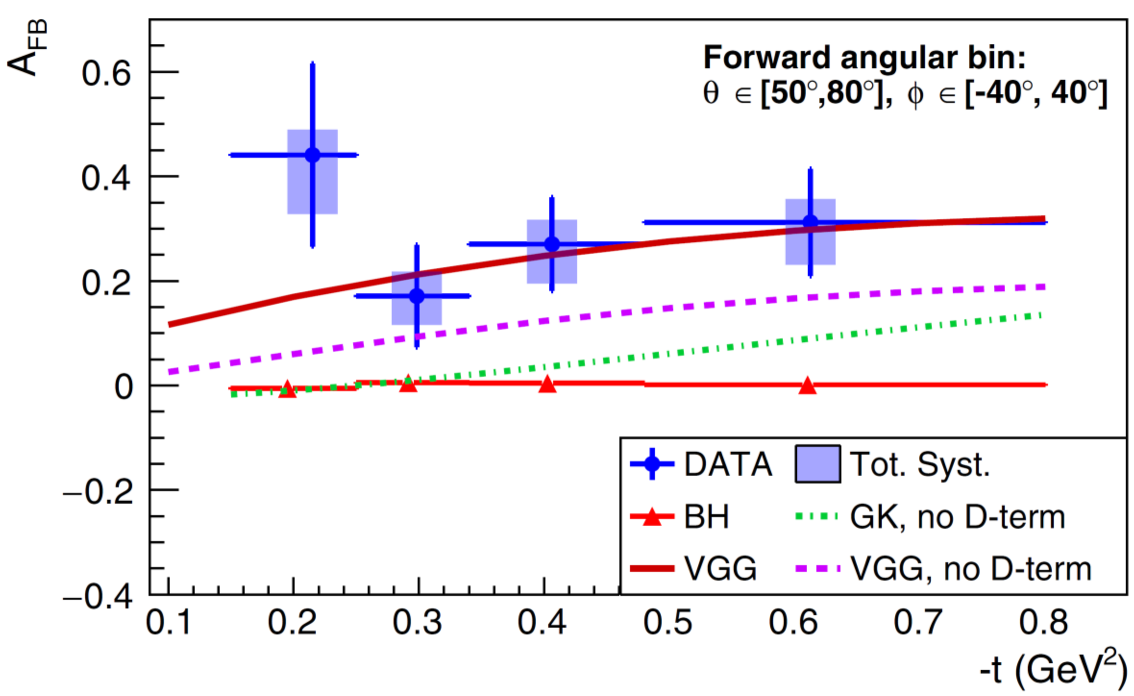
Timelike Compton Scattering (TCS)

- First ever Timelike Compton Scattering Measurement at CLAS
  Phys. Rev. Lett. 127, 262501 (2021)
- Photon polarization asymmetry $A_{\odot U} \sim sin\phi \cdot \text{Im}\widetilde{M}^{--}$ → GPD universality
- Forward backward asymmetry $A_{FB} \sim cos\phi \cdot \text{Re}\widetilde{M}^{--}$ → Access D-term

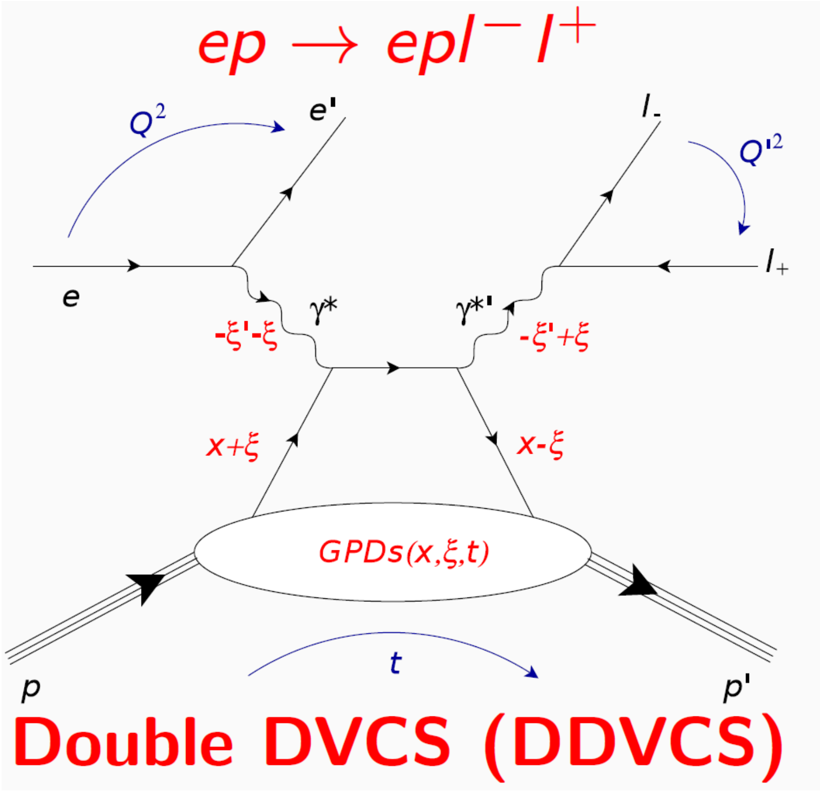


$$\tilde{M}^{--} = \left[F_1\mathcal{H} - \xi(F_1+F_2)\tilde{\mathcal{H}} - \frac{t}{4m_p^2}F_2\mathcal{E} \right]$$



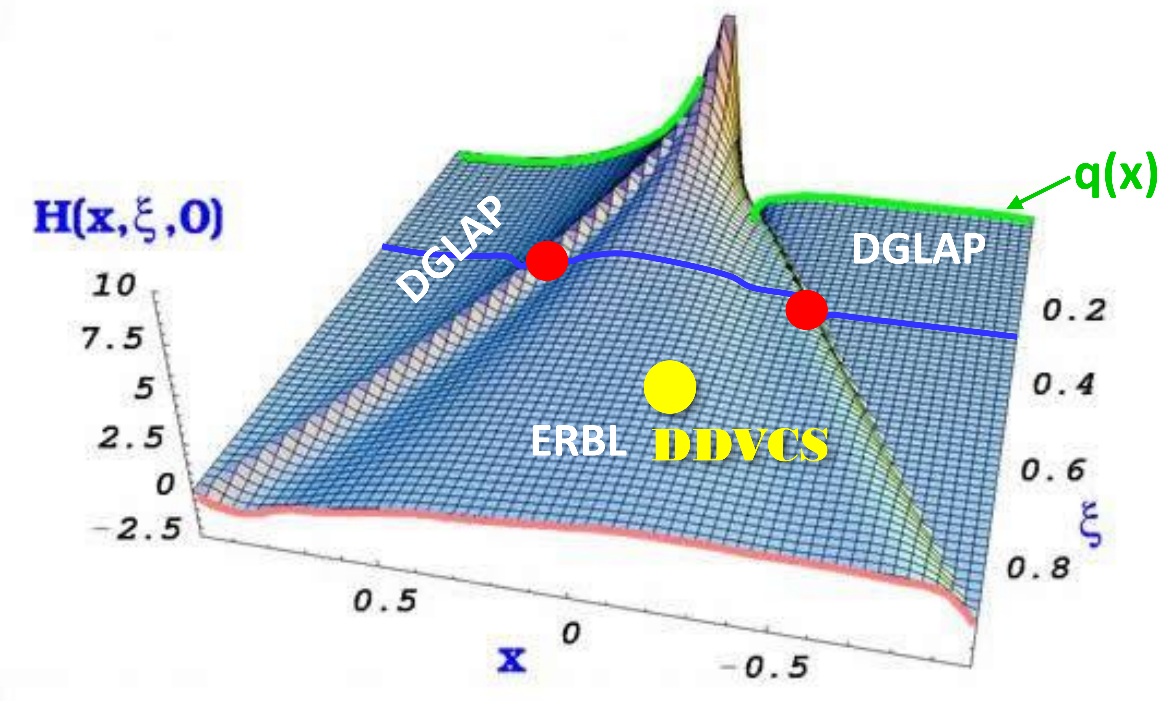


Double DVCS (DDVCS)



- Both space-like and time-like photons can set the hard scale

$$\mathcal{H}(\xi', \xi, t) = \sum_q e_q^2 \left\{ \mathcal{P} \int_{-1}^{1} dx\, H^q(x, \xi, t) \left[\frac{1}{x - \xi'} + \frac{1}{x + \xi'} \right] - i\pi \left[H^q(\xi', \xi, t) - H^q(-\xi', \xi, t) \right] \right\}$$



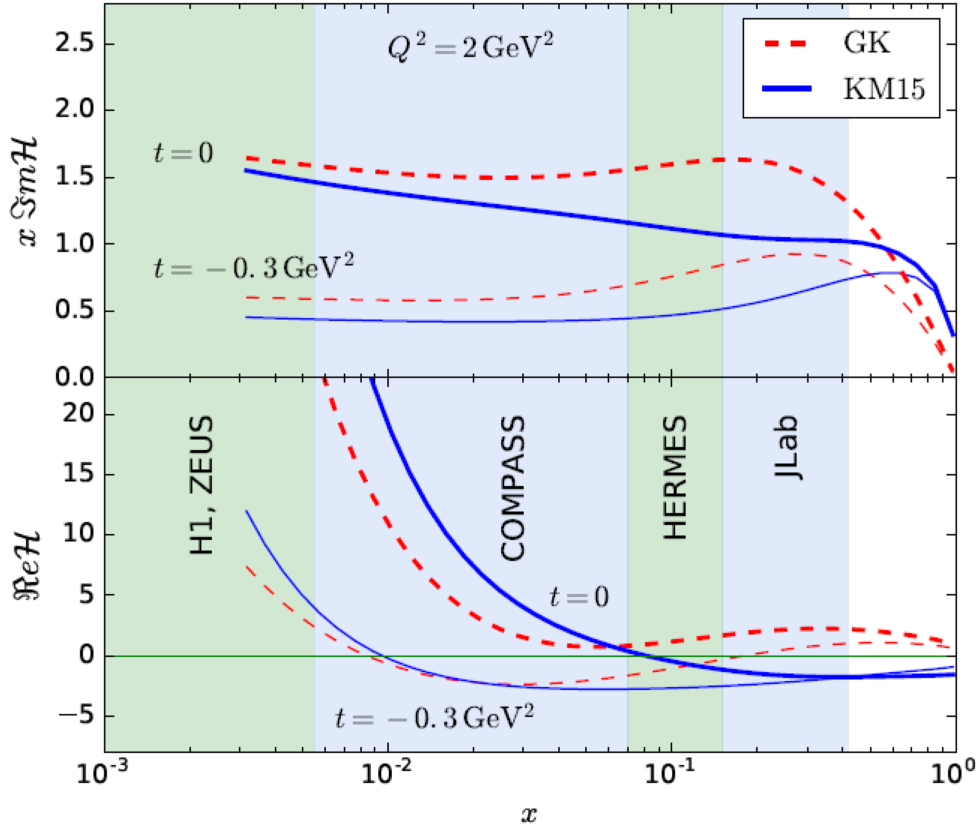
- Double DVCS gives access to phase space where $x \neq \xi$
- VGG model: order of about 0.1 pb, about 100 to 1000 times smaller than DVCS
- **Interference term enhanced by BH**


The view ahead
– What do we expect?

Global Analysis

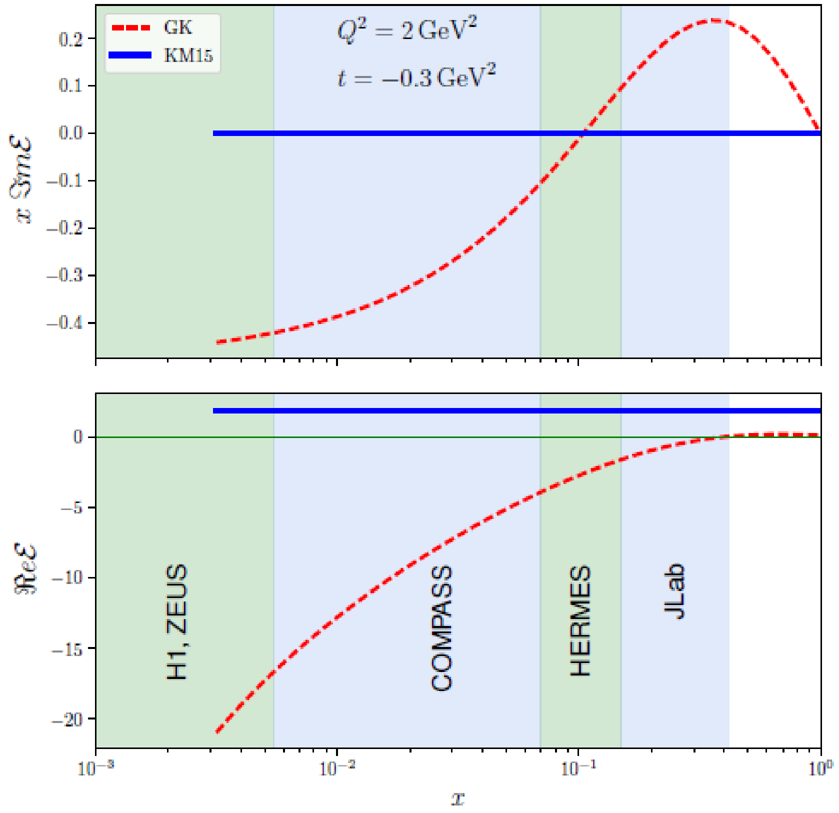
Figures made by D. Mueller and K. Kumericki

KM15 K Kumericki and D Mueller arXiv:1512.09014v1
GK S.V. Goloskokov, P. Kroll, EPJC53 (2008), EPJA47 (2011)



Im $\mathcal{H}$ is better known

Re $\mathcal{H}$ linked to the ***d term*** is still not so well constrained



Im $\mathcal{E}$ is fairly unknown

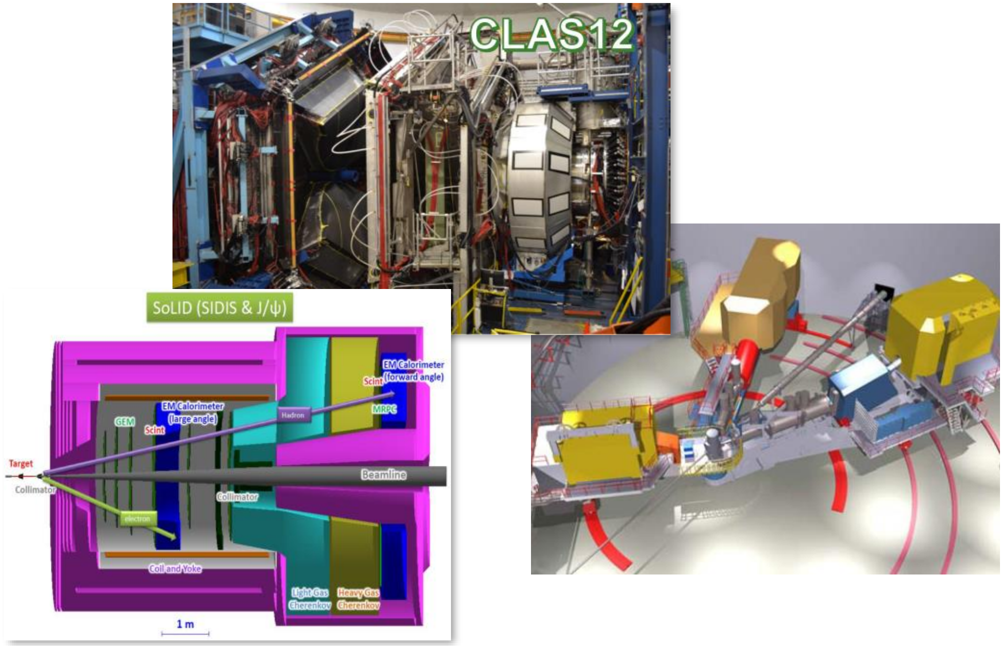
Re $\mathcal{E}$ is fairly unknown

- ➢ Very little is known for chiral-odd GPDs as well.
- ➢ **We need more experimental inputs!**

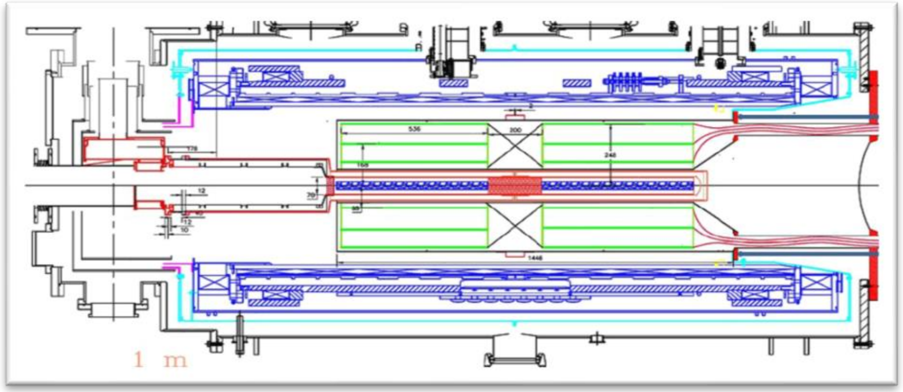
  - Various processes and precise data mapping, with high granularity and phase space wider than what has been covered, are required to fully constrain the entire set of GPDs

Near Future

- Expect fruitful measurements coming from JLab-12
  - Released: DVCS & π^0 at Hall A, TCS at CLAS
  - DVCS, nuclear DVCS, DVMP, TCS, even DDVCS?

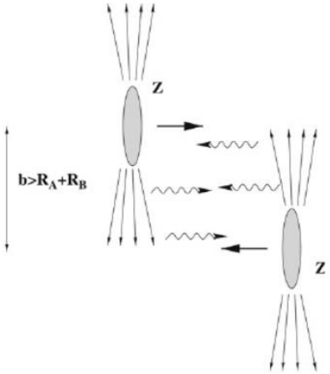


- COMPASS/AMBER
  - DVCS → ReH with charge-spin asymmetry
  - DVMP of $\pi^0, \omega, \rho, J/\psi$
  - Transversely polarized target in AMBER?



Silicone proton recoil detector between target & polarizing magnet

- Other possibilities at J-Parc, RHIC, FAIR, or LHC
  - Exclusive Drell-Yan
  - Ultra-peripheral collisions for TCS or exclusive J/ψ production for GPD E of the gluon



The Ideal Experiment

➢ High & variable beam energy

- Large kinematic domain & hard regime → large Q^2 span for evolution
- **Polarized** beams → various spin asymmetries
- Variable energy for:
  - Energy separation for $DVCS^2$ and DVCS-BH interferences
  - L/T separation for pseudo scalar meson production
- Availability of **positive** and **negative** leptons → real part of CFFs

➢ H_2, D_2, and nuclear beams

➢ High luminosity

- Small cross section
- Multi-dimentional binning for fully differential analysis (x_B, Q^2, t, ϕ)

➢ Hermetic detectors

- Ensure exclusivity

Does not exist (yet)

The Ideal Experiment – Challenges @ EIC

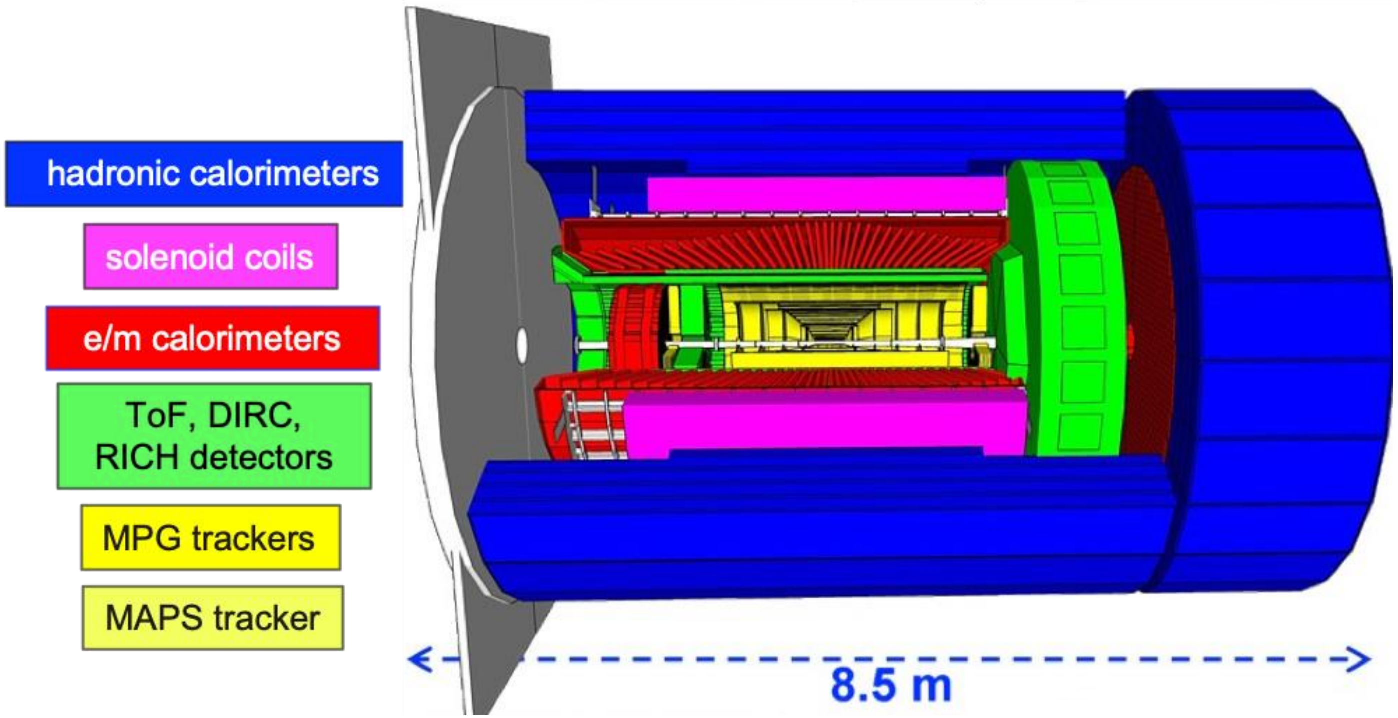
- **High & variable beam energy**
  - Lar
  - **Pol**
  - Var
    - ○
    - ○
  - Ava
- **H_2, I**
- **Hig**
  - Sm
  - Mu
- **Her**
  - Ens

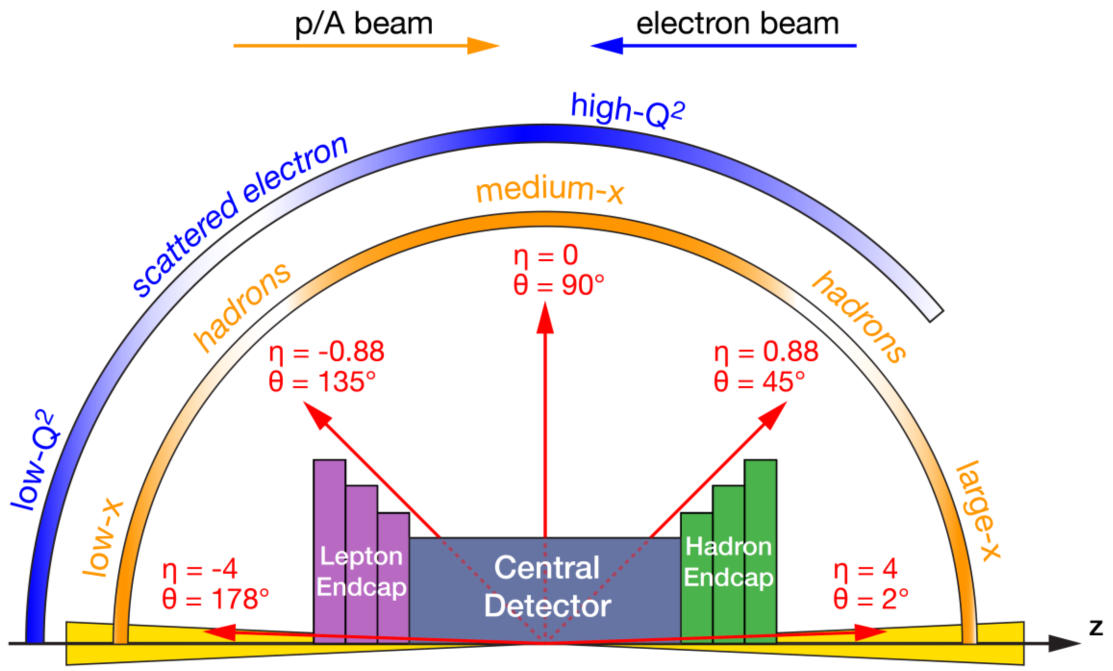
> - **Beam polarization:**
>   - For asymmetry measurements, statistical uncertainties inversely proportional to the degree of polarization achieved. → High polarison required.
>   - Longitudinal for e^- , transverse & longitudinal for polarizable nuclei → aim for ~70% polarization for both beams
> - **Luminosity: one of the most demanding aspects of EIC**
>   - Luminosity 10^{33} to 10^{34} $cm^{-1}s^{-1}$. with 10^{33} $cm^{-1}s^{-1}$, 10 fb^{-1} integrated luminosity achieved with 30 weeks of operation. GPD would ask for 100 fb^{-1} → needs higher luminosity of 10^{34} $cm^{-1}s^{-1}$.
> - $0.03 < |t| < 1.6$ GeV^2 → careful design of the interaction & hadron beam parameters

Does not exist (yet)

The Electron Ion Collider

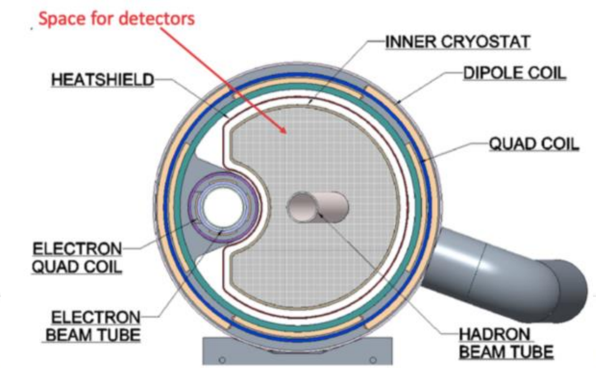
Detector 1 → ePIC





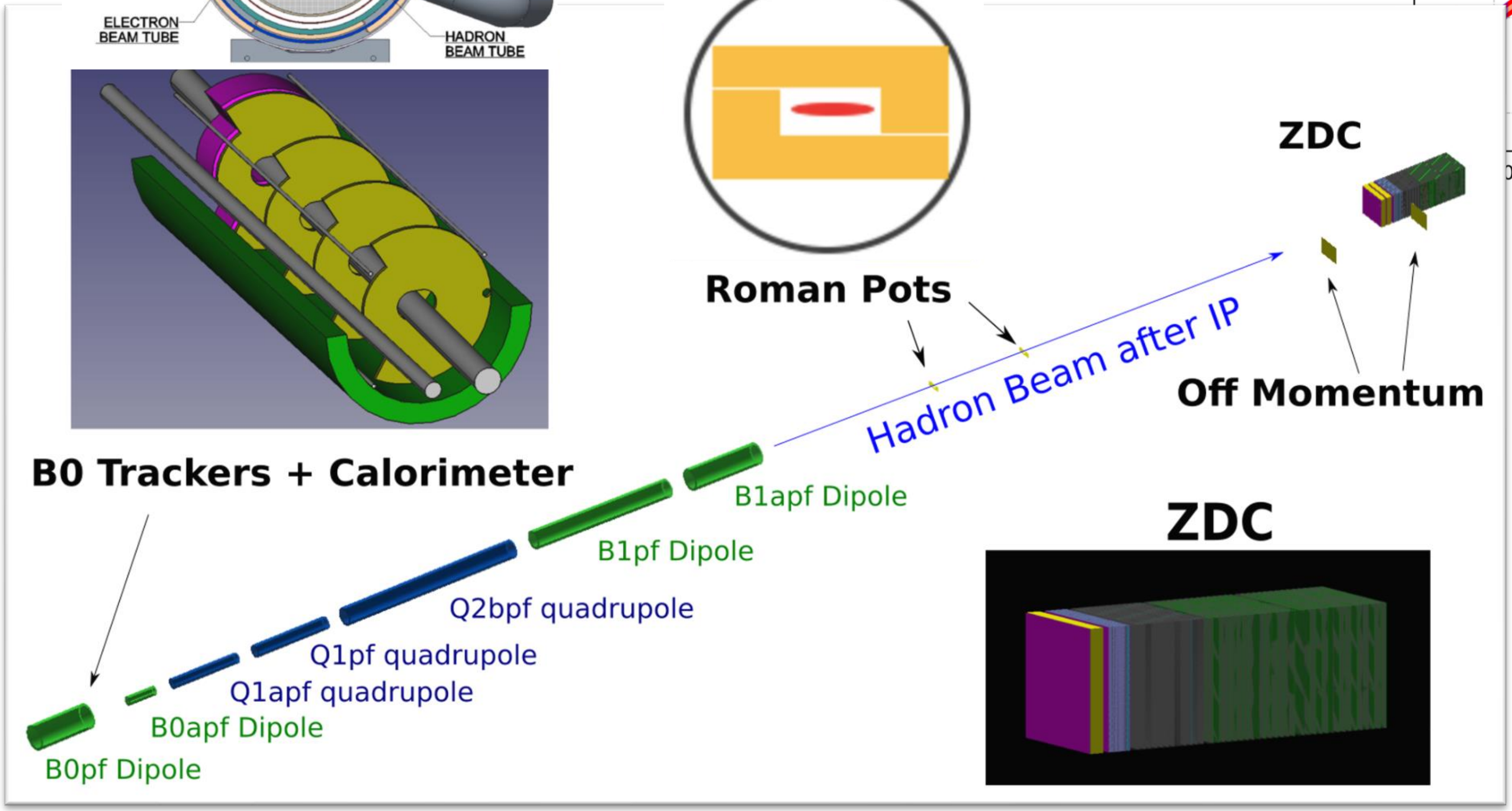
- ➢ Auxiliary detectors needed to tag particles with very small scattering angles both in the outgoing lepton and hadron beam direction.

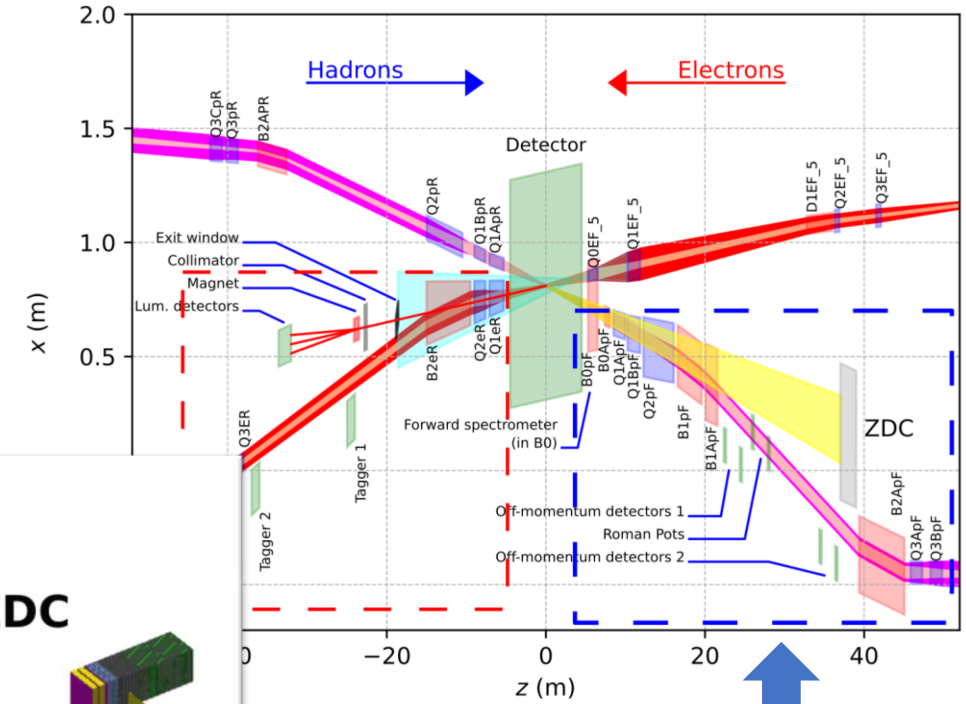
Far-Forward Detectors



B0 Spectrometer Configuration

Hadron beam pipe & Roman Pots in cross-section

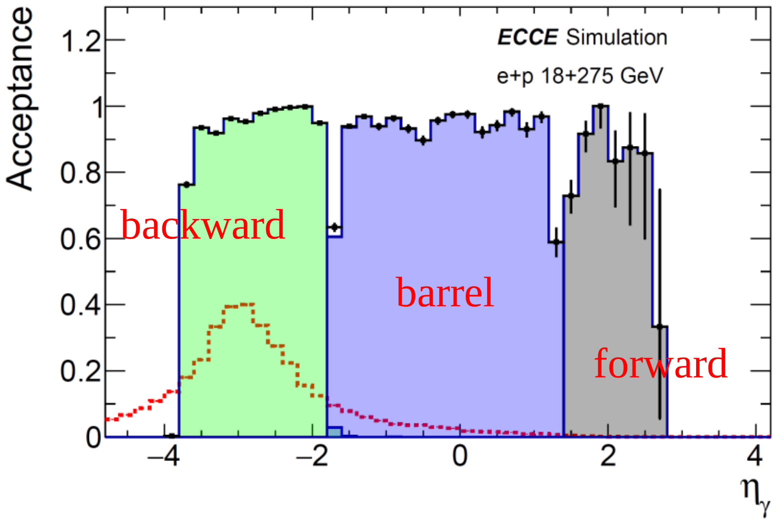


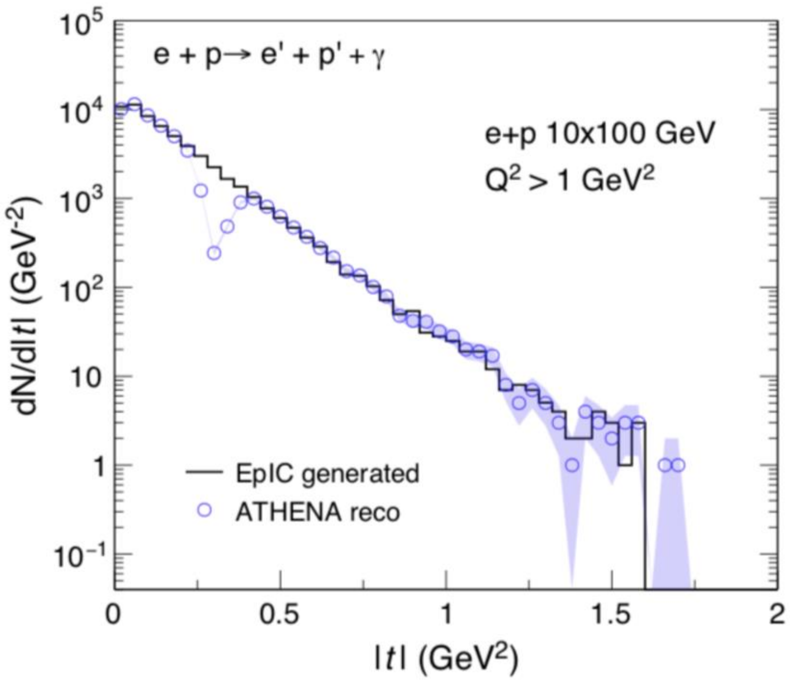


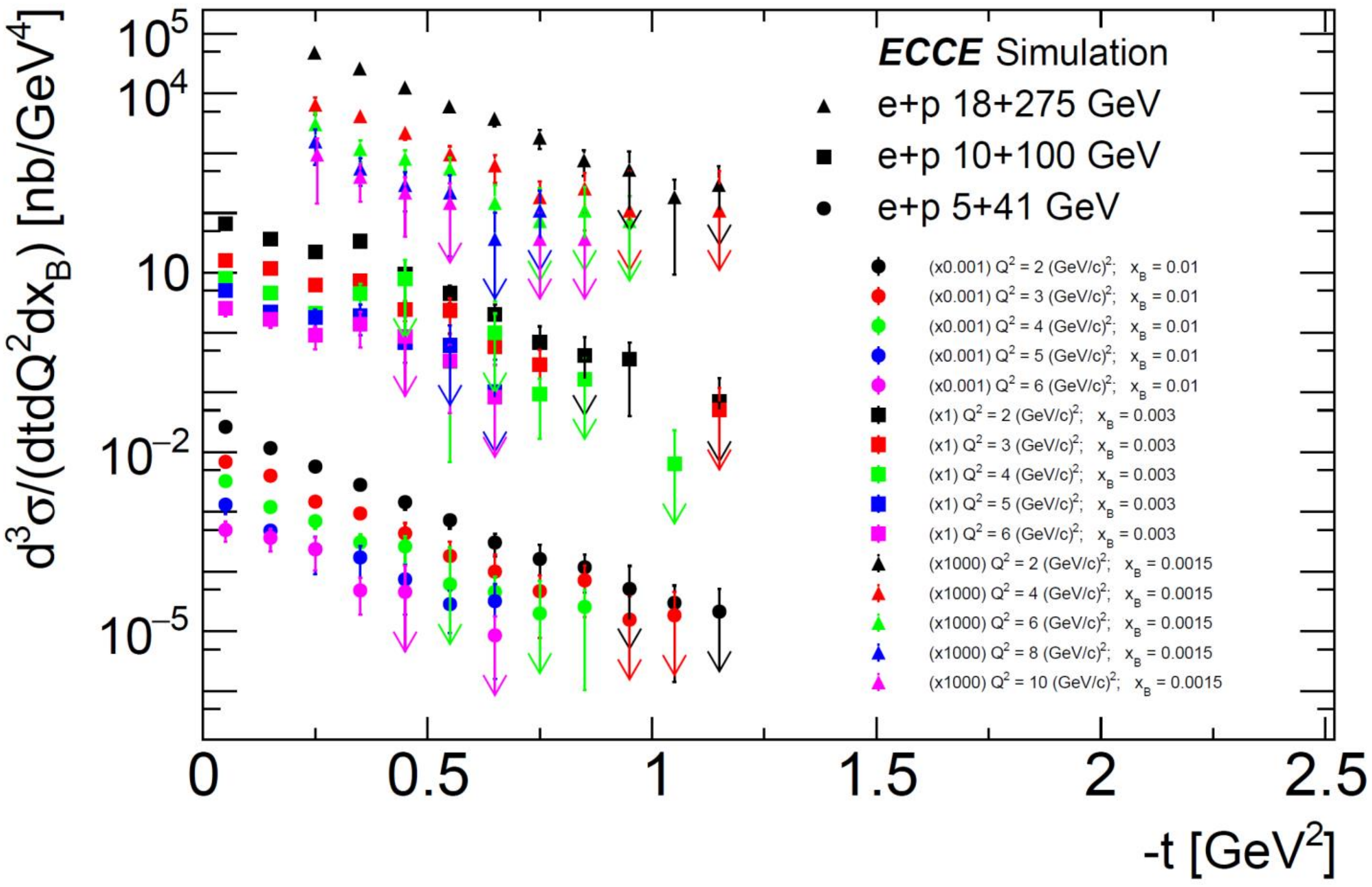
- Forward detection particularly crucial in exclusive measurements - proton/ion measurement required
- B0 and/or Roman Pots are the critical forward detection regions
  - Roman Pot: lowest values of t
  - B0: for higher values

DVCS Simulation

Plots: I. Korover (MIT), Kong Tu (BNL)





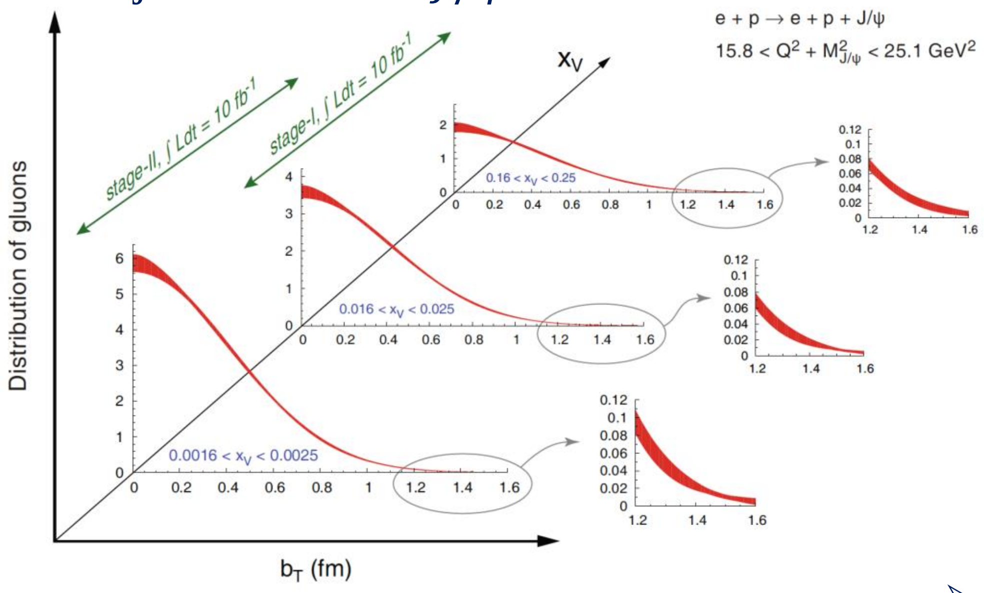


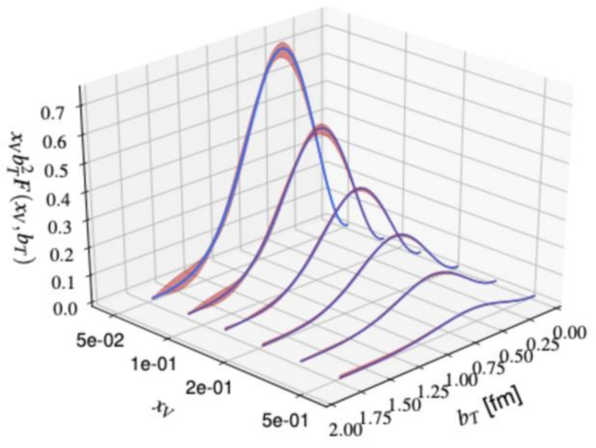
- Practically hermetic coverage for photons
- Wide range of t
- Multi-dimensional binning possible

Outlook

EIC yellow report

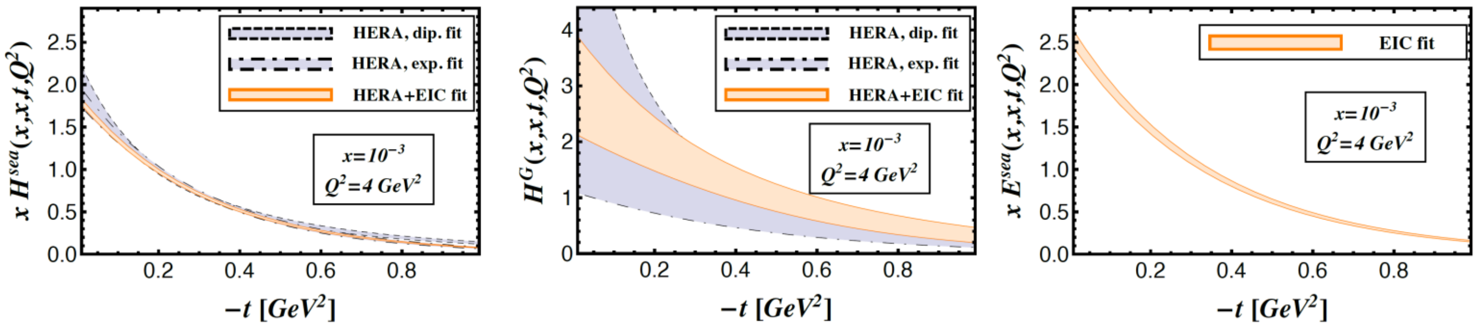
Projcted IPD from J/ψ





Projcted IPD from Υ

GPD H - sea quarks | GPD H - gluon | GPD E – sea quarks



- Compton Scatterings – DVCS, TCS, DDVCS
  - GPD mapping, consistensy of factorisation & universality test
- DVMP
  - Flavor separation, chiral-odd GPDs
  - heavy mesons (J/ψ, Υ) → mechanism of saturation by gluon distribution from high to low x_B
- New methods: diffractive process, charged-current processes of meson production…

Summary

- A lot of interesting properties of proton can be revealed by GPDs and EIC can offer us unprecedented opportunity for a precise determination of GPDs.

- **Let’s build it.**

Backup Slides

GPD Models

- VGG model (Vanderhaeghen, Guichon, Guidal 1999):
  - Based on double distributions
  - Includes a D-term to restore full polynomiality
  - Includes a Regge inspired and a factorized t-ansatz
  - Skewness depending on free parameters b_{val} and b_{sea}
  - Includes twist-3 contributions

VGG Vanderhaeghen, Guichon, Guidal
PRL80(1998),PRD60(1999), PPNP47(2001), PRD72(2005)
1rst model of GPDs
improved regularly

- Dual model: (Guzey, Teckentrup 2006)
  - GPDs based on an infinite sum of t-channel resonances
  - Includes a Regge inspired and a factorized t-ansatz
  - Does not include twist-3

KM10a – – – (KM10 ....) Kumericki, Mueller, NPB (2010) 841
Flexible parametrization of the GPDs based on both a Mellin-Barnes representation and dispersion integral which entangle skewness and t dependences
Global fit on the world data ranging from H1, ZEUS to HERMES, JLab

KMS12 Kroll, Moutarde, Sabatié, EPJC73 (2013)
using the **GK** model
Goloskokov, Kroll, EPJC42,50,53,59,65,74
for GPD adjusted on
the hard exclusive meson production at small x_B
"universality" of GPDs

Goloskokov-Kroll Model for Pseud-mson Production

Eur. Phys. J. A (2011) **47**: 112
DOI 10.1140/epja/i2011-11112-6

THE EUROPEAN PHYSICAL JOURNAL A

Regular Article – Theoretical Physics

Transversity in hard exclusive electroproduction of pseudoscalar mesons

S.V. Goloskokov[1,a] and P. Kroll[2,3,b]

- **UNPOLARIZED STRUCTURE FUNCTIONS:**

$$\sigma_L \sim \left\{(1-\xi^2)\left|\langle\tilde{H}\rangle\right|^2 - 2\xi^2 \mathrm{Re}\left[\langle\tilde{H}\rangle^*\langle\tilde{E}\rangle\right] - \frac{t'}{4m^2}\xi^2\left|\langle\tilde{E}\rangle\right|^2\right\}$$

$$\sigma_T \sim \left[(1-\xi^2)\,|\langle H_T\rangle|^2 - \frac{t'}{8m^2}\left|\langle E_T\rangle\right|^2\right]$$

$$\sigma_{TT} \sim \left|\langle\bar{E}_T\rangle\right|^2$$

- **POLARIZED OBSERVABLES:**

$$A_{LU}^{\sin\phi}\sigma_0 \sim \mathrm{Im}\left[\langle H_T\rangle^*\langle\tilde{E}\rangle\right]$$

$$A_{UL}^{\sin\phi}\sigma_0 \sim \mathrm{Im}\left[\langle\bar{E}_T\rangle^*\langle\tilde{H}\rangle + \xi\langle H_T\rangle^*\langle\tilde{E}\rangle\right]$$

$$A_{LL}^{\cos 0\phi}\sigma_0 \sim |\langle H_T\rangle|^2$$

$$A_{LL}^{\cos\phi}\sigma_0 \sim \mathrm{Re}\left[\langle\bar{E}_T\rangle^*\langle\tilde{H}\rangle + \xi\langle H_T\rangle^*\langle\tilde{E}\rangle\right]$$

$$\bar{E}_T = 2\tilde{H}_T + E_T$$

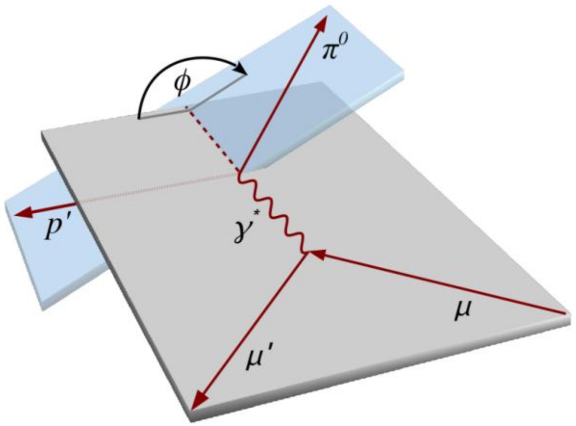
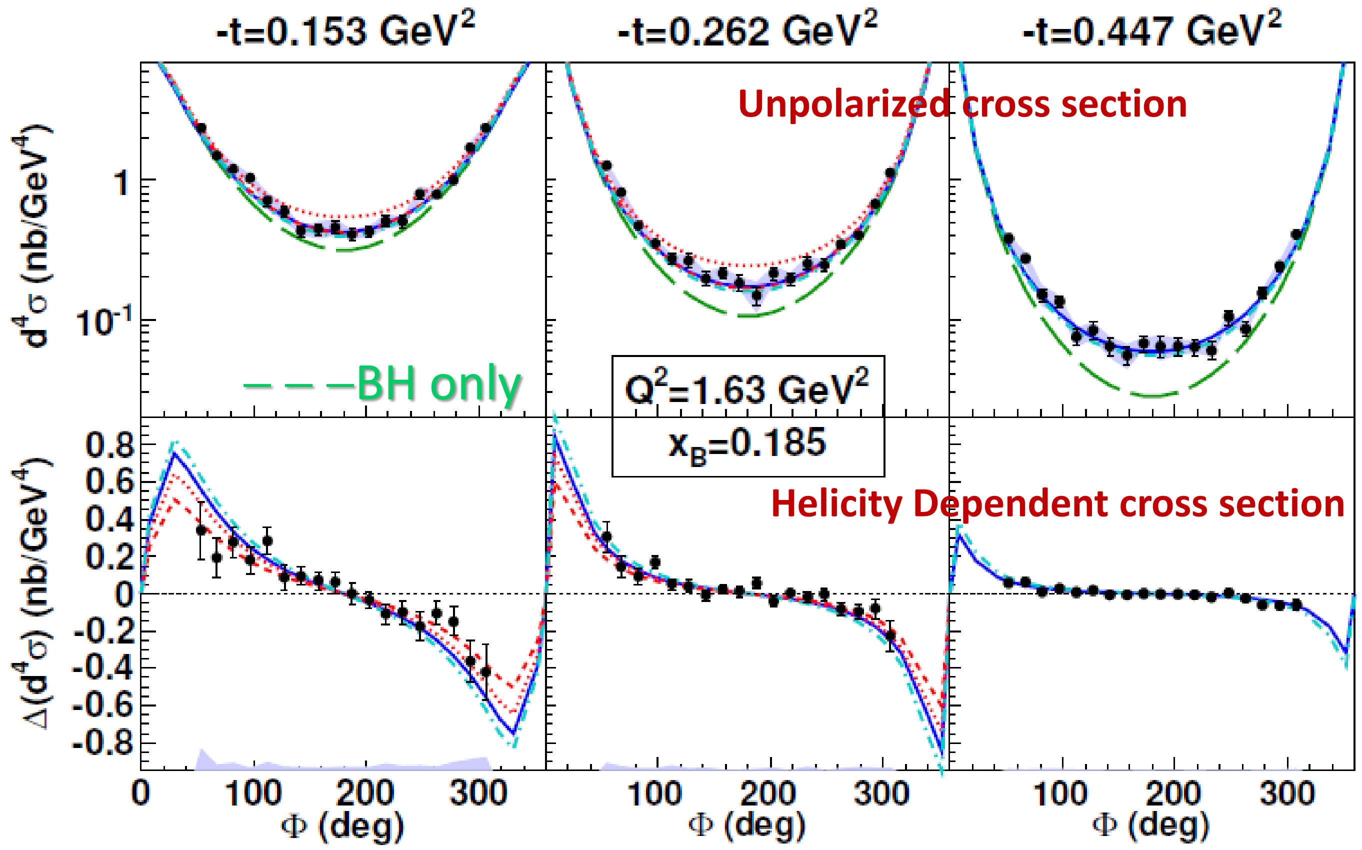


Fig: M.G. Alexeev et al. *Phys.Lett.B* 805 (2020)

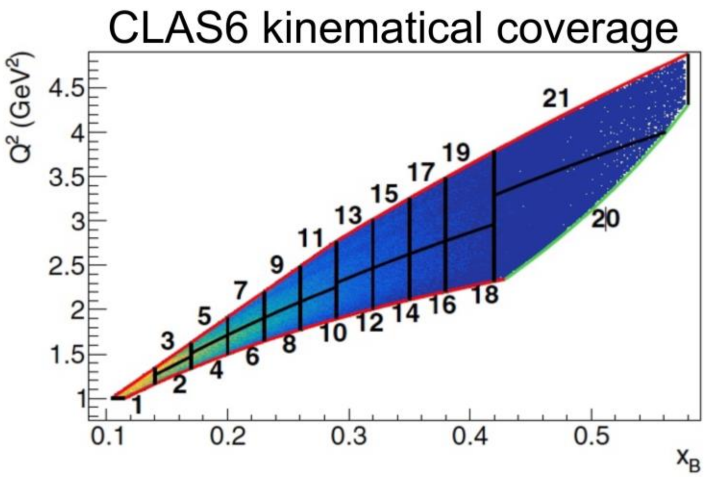
$< F >$: Generalized Form Factor, convolution of hard subprocess with GPD F

Beam Spin Sum and Diff of DVCS at CLAS

- Wide kinematic range → 21 bins in (x_B,Q^2) or 110 bins (x_B,Q^2,t) with 3 months data taken in 2005
  → CFF constraints



$\overleftrightarrow{e}\, p \rightarrow e\, \gamma\, p$



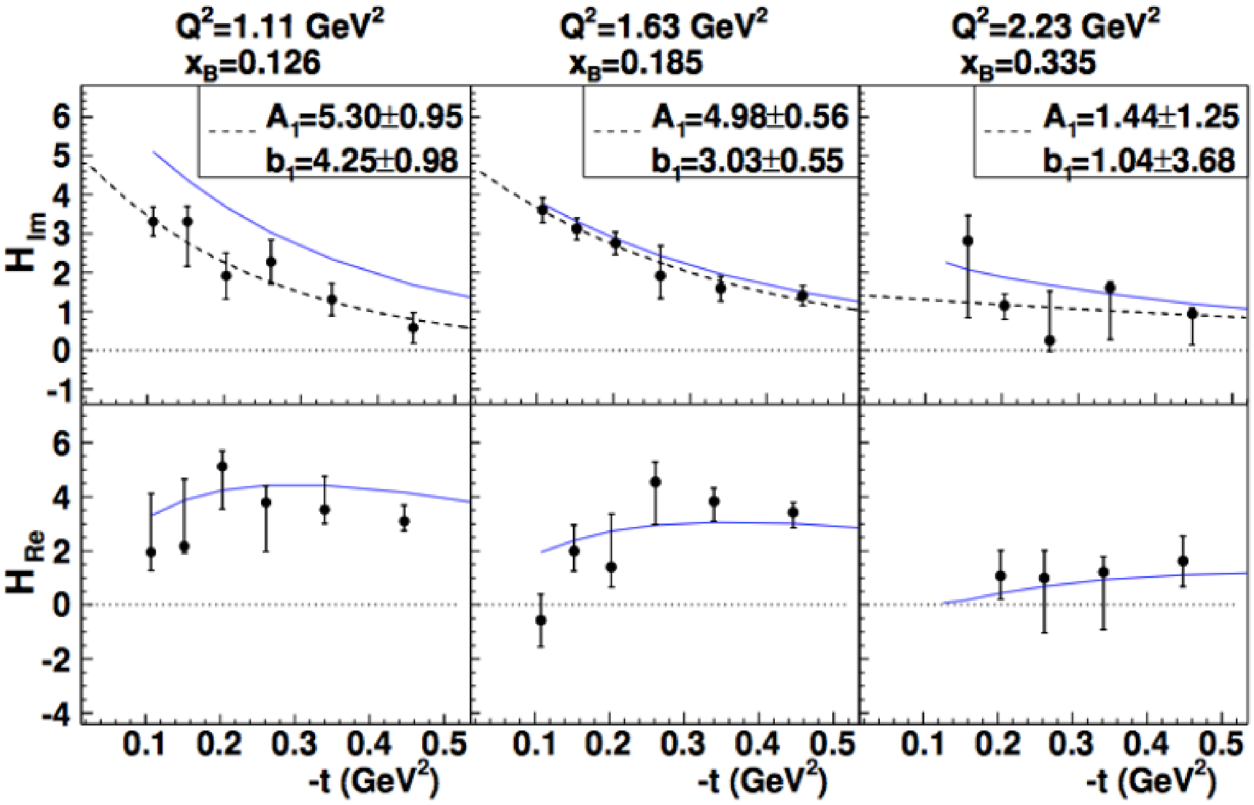
- BH only
- VGG (*Vanderhaeghen, Guichon, Guidal) - H only*
- KM10 (Kumericki, Mueller) *includes strong* $\tilde{H}$
- KM10a *(sets* $\tilde{H}$ *to zero)*
- KMS *(Kroll, Moutarde, Sabatié, tuned on low* x_B *meson-production data)*

H.-S. Jo et al. (CLAS) PRL115, 212003 (2015) 212003

Nucleon Tomography in the Valence Domain

- Wide kinematic range → 21 bins in (x_B,Q^2) or 110 bins (x_B,Q^2,t) with 3 months data taken in 2005
  → **Nucleon tomography in the valence domain**

VGG model
Fit Im$\mathcal{H}$ = A $e^{-B|t|}$



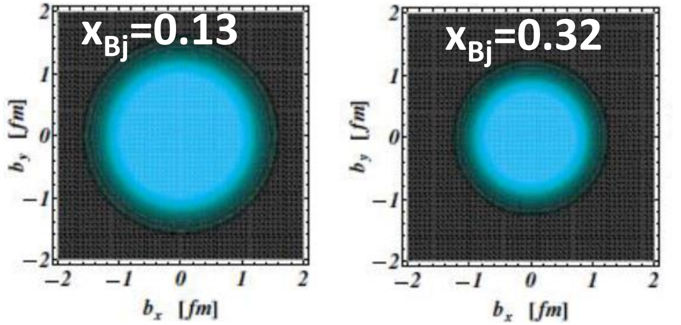
Fit of 8 CFFs at **L.O.** and **L.T.**
(ImH, ReH, ImE, ReE, Im$\tilde{H}$, Re$\tilde{H}$, Im$\tilde{E}$, Re$\tilde{E}$)

- Dominance of H in unpolarized cross-section.
- H_{Im} slope B give information on the trasverse extension of the partons → becomes flatter at higher x_B

↓

Valence quarks at centre
Sea quarks spread out towards the periphery.

Guidal, Moutarde, Vanderhaeghen, PNPP 76 (2013)



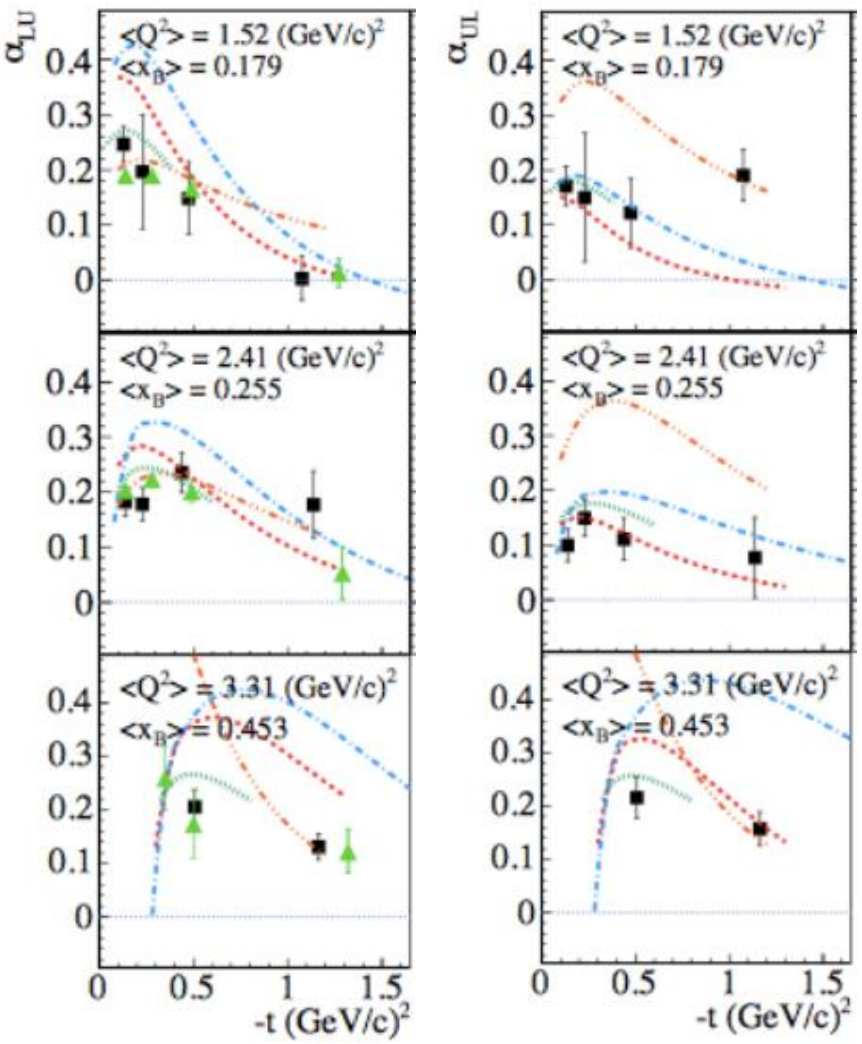
H.-S. Jo et al. (CLAS) PRL115, 212003 (2015) 212003
N. Hirlinger Saylor et al (CLAS) PRC 98 (2018) 045203

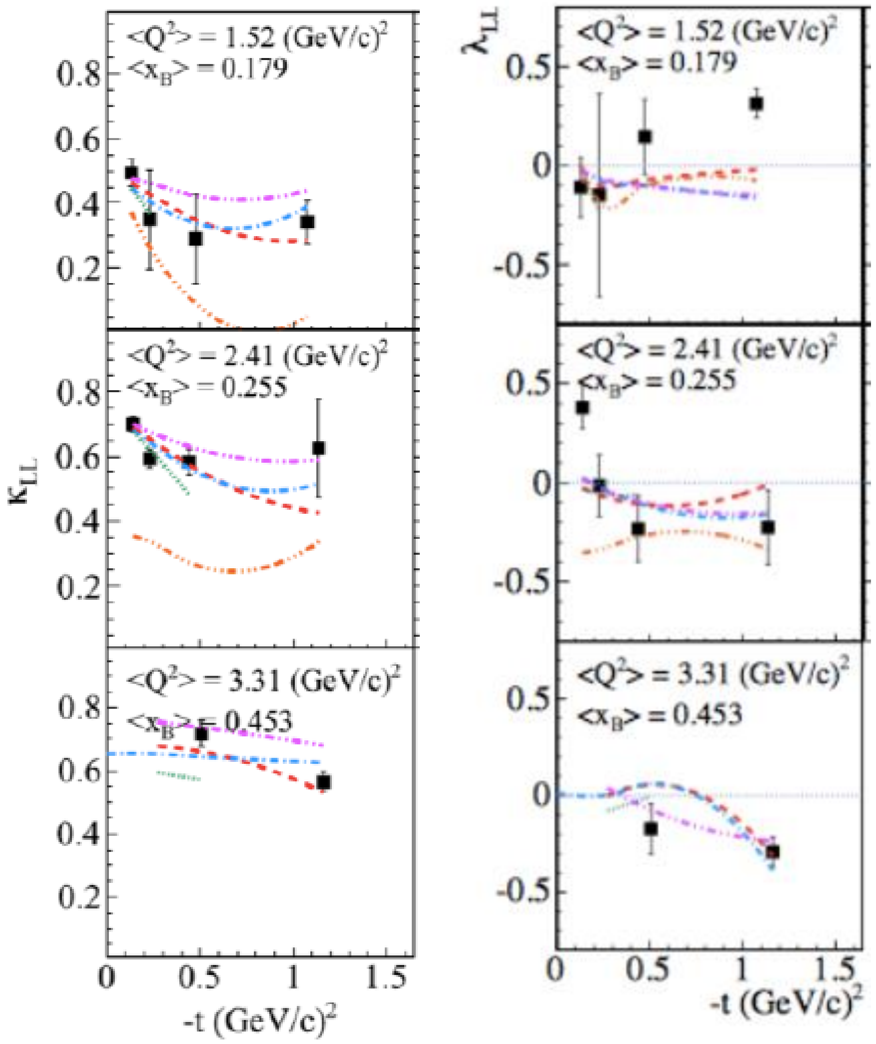
Beam- and Target-spin asymmetries at CLAS

$$A_{LU\,(UL)} = \frac{\alpha_{LU(UL)}\sin\phi}{1+\beta\cos\phi} \qquad A_{LL} = \frac{\kappa_{LL}+\lambda_{LL}\cos\phi}{1+\beta\cos\phi}$$

$\overleftrightarrow{e}\ \vec{p} \rightarrow e\,\gamma\,p$

Longitudinally polarized NH3 target on 2009





- LU consistent with previous data
- UL for prarameterizing the t-dependence of H & $\widetilde{H}$
- Dominance of BH in A_{LL}

- Simultaneous fit to BSA, TSA and DSA.

CLAS A_{LU} on H_2
data
VGG Vanderhaeghen, Guichon, Guidal
KMM Kumericki, Mueller, Murray
GK Goloskokov, Kroll
GGL Goldstein, Gozalez, Luiti
BH

Seder et al. PRL114, 032001 (2015) Pisano et al. PRD91, 052014 (2015)

Nucleon Tomography in the Valence Domain

Fit of 8 CFFs at L.O and L.T.

Dupré, Guidal, Nicolai, Vanderhaeghen, PRD95, 011501(R)(2017) Eur.Phys.J. A53 (2017)

$s_1^I = Im\, F_1\mathcal{H}$ is the best constrained

$$\rho^q(x, \mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} H_-^q(x, 0, -\mathbf{\Delta}_\perp{}^2).$$

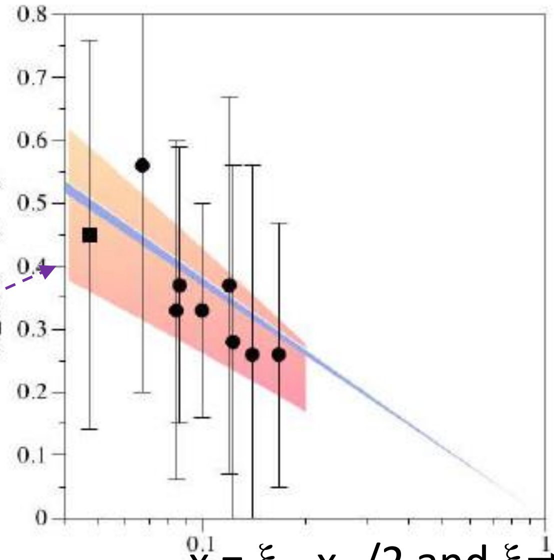
$$\langle b_\perp^2\rangle^q(x) = -4\frac{\partial}{\partial\mathbf{\Delta}_\perp^2}\ln H_-^q(x, 0, -\mathbf{\Delta}_\perp{}^2)\Big|_{\mathbf{\Delta}_\perp=0}.$$

$< b_\perp{}^2 > \approx 4\,B$

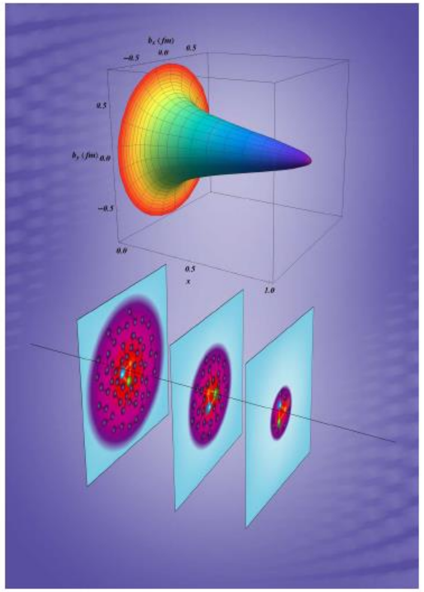
— Fit $A\, e^{-B|t|}$

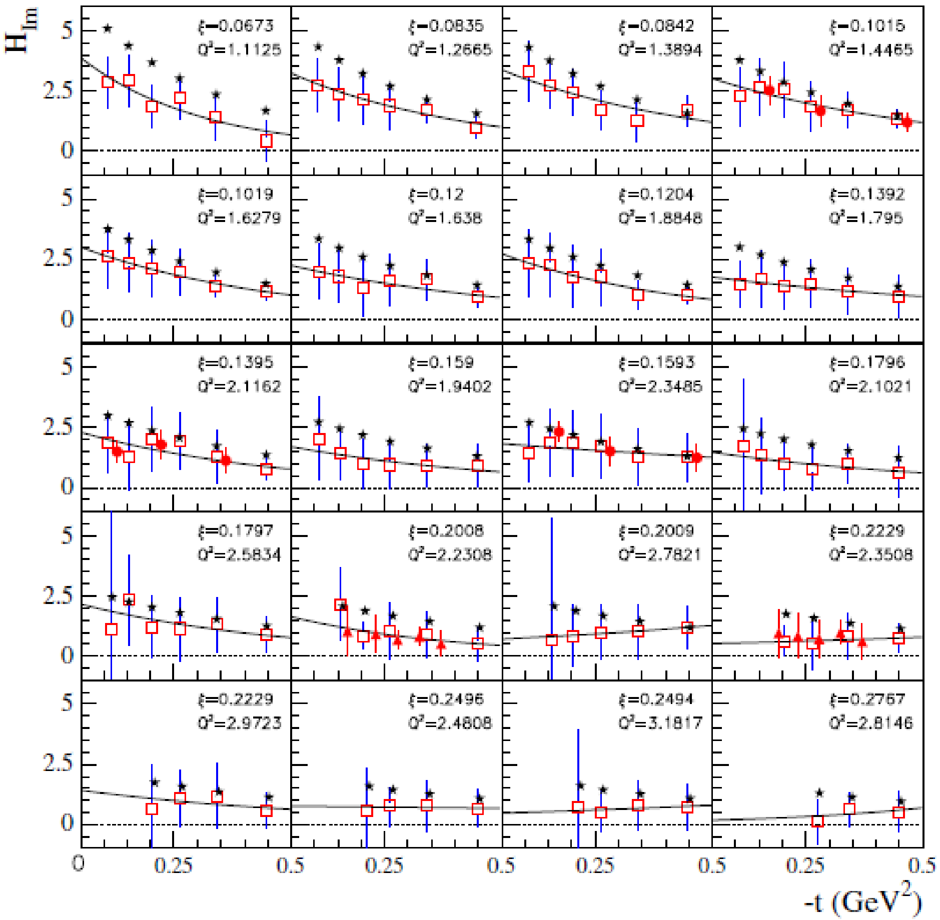
HERMES

+ 8 data from JLab



$x = \xi \sim x_{Bj}/2$ and $\xi \to 0$





☐ CLAS σ and Δσ

▲ HallA σ and Δσ

● CLAS A_{UL} and A_{LL}

★ VGG model