Color glass condensate for EIC

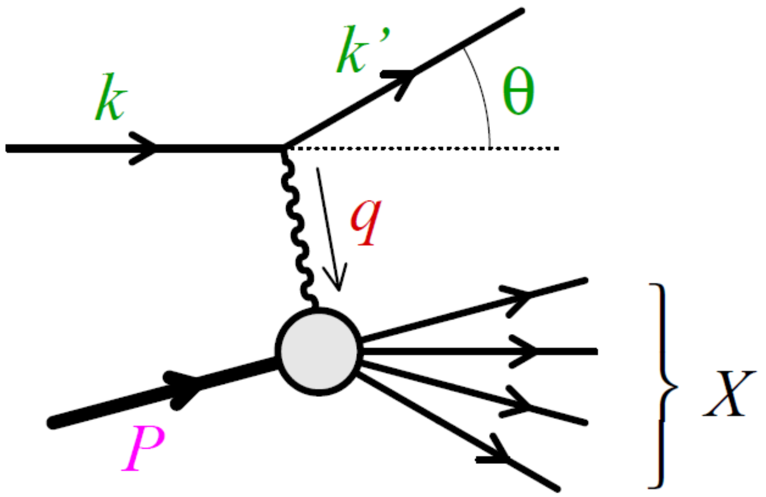
Hsiang-nan Li
Academia Sinica

Presented at NCKU, Aug. 18, 2022

Refs.: 1002.0333; Yellow Report

Deeply inelastic scattering (DIS)

- Kinematics



$$s \equiv (P+k)^2$$
$$Q^2 \equiv -q^2$$
$$x \equiv Q^2/2P\cdot q$$ Bjorken variable
$$xy \equiv Q^2/s$$

energy transfer ↗

- Factorization theorem for structure function

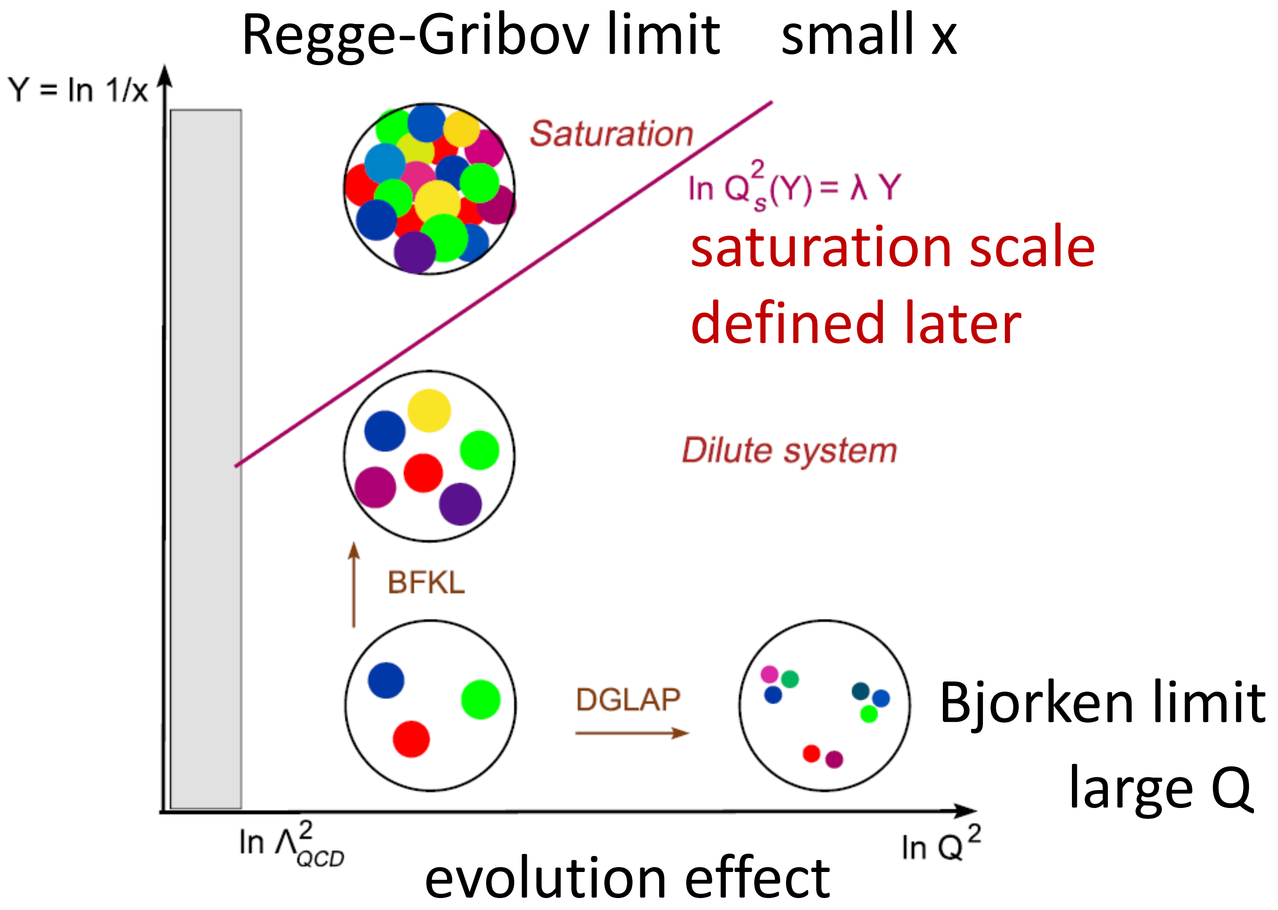
parton distribution function, probability for parton carrying momentum fraction

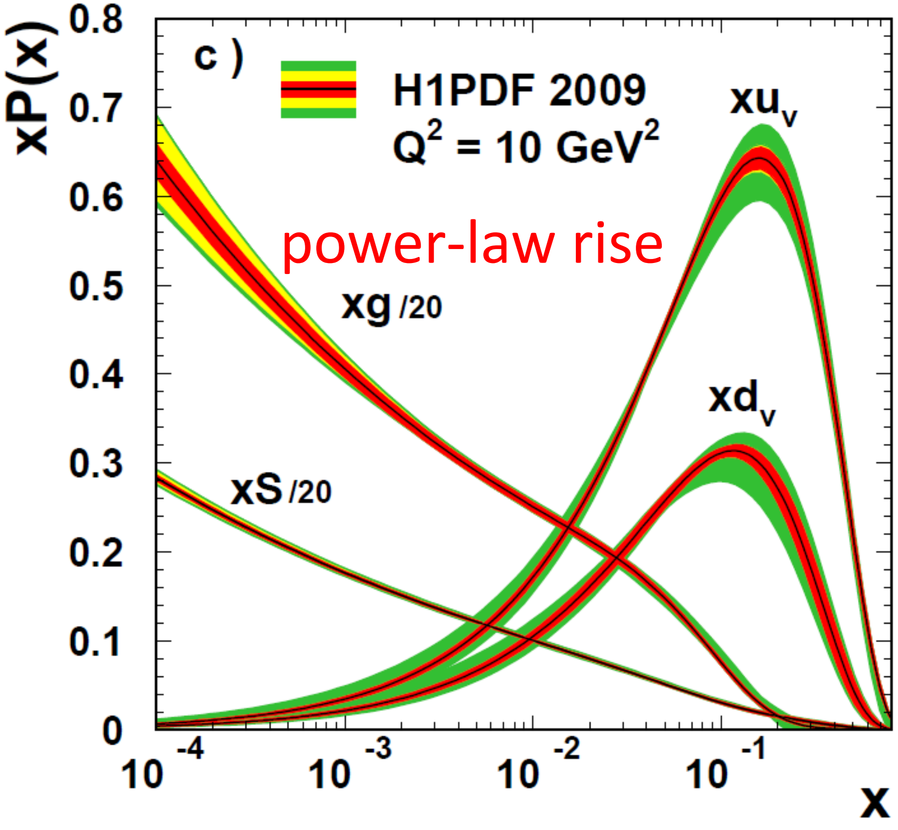
$$F(x) = \Sigma_f \int_x^1 (d\xi/\xi) H_f(x/\xi) \phi_{f/N}(\xi)$$

hard kernel

Parton distribution function (PDF)

Chung-Wen's talk





higher Q, higher resolution, see smaller partons

smaller x requires more radiation, see more partons → x dependence of gluon PDF can be predicted

BFKL equation

- Soft gluon effect organized by the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation

color factor

$$dP_{\mathrm{Brem}} \simeq \frac{\alpha_s C_R}{\pi^2} \frac{d^2 k_\perp}{k_\perp^2} \frac{dx}{x}$$

collinear, soft divergences

$$\frac{\alpha_s C_R}{\pi} \int_{\Lambda^2_{\mathrm{QCD}}}^{Q^2} \frac{dk_\perp^2}{k_\perp^2} = \frac{\alpha_s C_R}{\pi} \ln\left(\frac{Q^2}{\Lambda^2_{\mathrm{QCD}}}\right)$$

at small Bjorken variable, Q not large, it is not large log

$$\alpha_s^n \int_x^1 \frac{dx_n}{x_n} \int_{x_n}^1 \frac{dx_{n-1}}{x_{n-1}} \cdots \int_{x_2}^1 \frac{dx_1}{x_1} = \frac{1}{n!}\left(\alpha_s \ln\frac{1}{x}\right)^n$$

Small Bjorken variable, large log

power-law rise at low x

- Summed to all orders by BFKL: $N_g \sim \exp(-\alpha \ln x) = \exp(ln x^{-\alpha}) = x^{-\alpha}$

~ Pomeron intercept

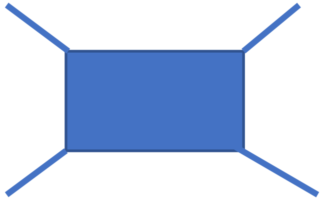
Saturation

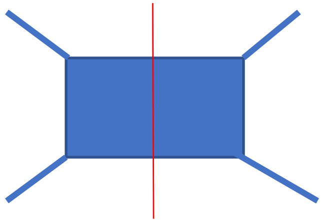
- BFKL predicts power-law rise at small x, ie., power-law rise with center-of-mass energy S, $x \sim Q^2/S$ for finite y
- Violate unitarity (Froissart) bound
- This bound can be understood naively via dispersion relation

2 to 2 box diagram finite contribution →

$$\Pi(q^2) = \frac{1}{\pi}\int_{s_i}^{\infty} ds \frac{\mathrm{Im}\Pi(s)}{s - q^2 - i\epsilon}$$

← scattering cross section if power-law rise, integral diverges





- Gluon PDF must saturate at x -> 0, as unitarity limit approached
- Modify BFKL into non-linear equation by including recombination
- Or, go for NLO BFKL, but gluon PDF becomes negative as x->0

Two limits of DIS

- Bjorken limit: $\text{fixed } x \text{ with } Q^2, s \to \infty.$ dilute partons
- Theoretical tool: factorization theorem for hard and collinear dynamics (fast parton with finite Bjorken variable)
- Hard probe interacts with a single parton, and PDF describes probability of this parton carrying some momentum
- Regge-Gribov limit: $\text{fixed } Q^2, x \to 0 \text{ and } s \to \infty.$ dense partons due to saturation
- Factorization theorem not suitable, because interactions of hard probe with multiple partons become important (strong correlation)
- Use different framework---color glass condensate, which turns nonperturbative collective nature into perturbative one

Color glass condensate (CGC)

- Both fast and slow gluons exist (with low Bjorken variable)
- Fast gluons with $k^+ > \Lambda^+$ are frozen due to time dilation
- Slow gluons with $k^+ < \Lambda^+$ are dynamical
- The former form color sources described by color configuration ρ and distributed randomly from event to event
- Stochastic nature of color sources justifies the term "color glass"
- "condensate" refers to large occupation number of saturated gluons
- Hard probe interacts with ρ , which organizes multi-parton effects

universal distribution of ρ

new factorization formula

$$\langle \mathcal{O} \rangle_{\Lambda^+} \equiv \int [D\rho]\, W_{\Lambda^+}[\rho]\, \mathcal{O}[\rho]$$

like PDF

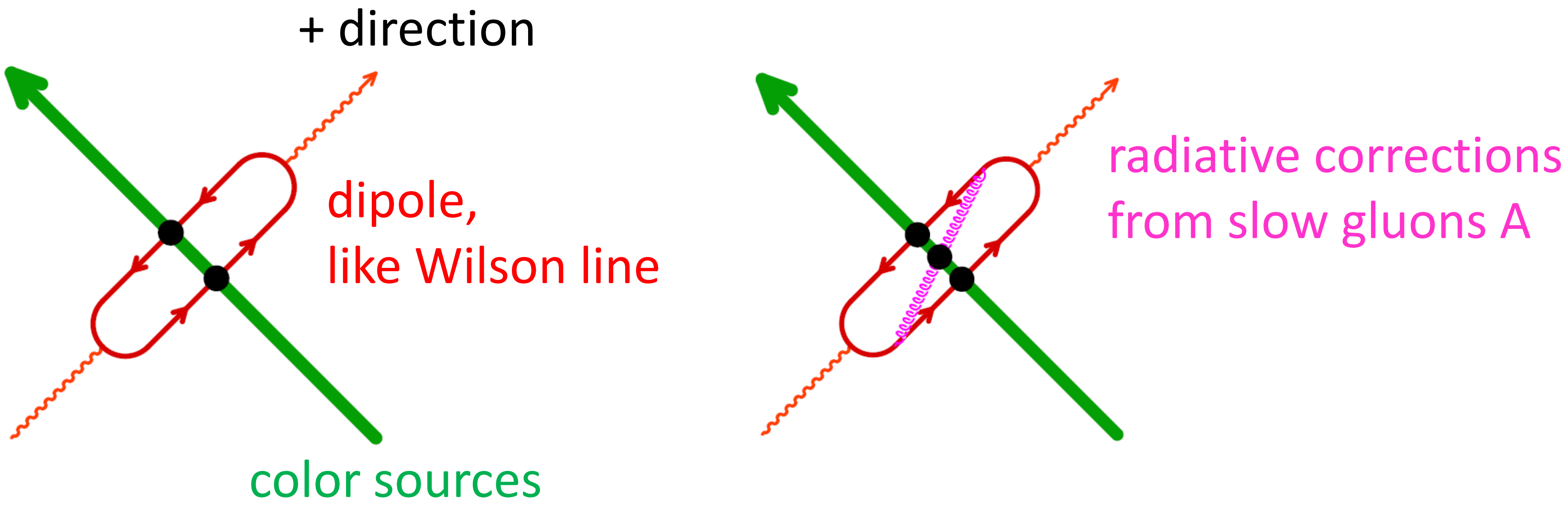
expectation value of operator O (hard probe) with particular configuration---like hard kernel, perturbation holds

Saturation scale

- As gluons dense enough, gluon recombination becomes important, and saturation reached
- Limit on gluon transverse dimension determines saturation scale Q_s
- Given occupation number n (number of gluons per unit transverse phase space and per unit rapidity), area of proton is filled up by gluons with transverse momenta $k_\perp \leq Q_s$ $(b_\perp \sim 1/Q_s)$
- Q_s increases (smaller parton) as x decreases (more partons), so saturation maintains, $Q_s^2(Y) \simeq Q_0^2 \, \mathrm{e}^{\lambda_s Y}$, $Y \equiv \ln \frac{1}{x}$
- Hard probe interaction with CGC, characterized by Q_s, becomes perturbative as stated before, and factorization works
- Q_s depends on number of nucleons A in e+A collision (at EIC), Lorentz contraction and nucleon overlap, n ~ $A^{1/3}$, $Q_0^2 \propto A^{1/3}$

DIS cross section with CGC at small x

- Virtual photon fluctuates into quark–antiquark pair
- The dipole scatters off static color sources (CGC)



- Leading order: dipole interacts with classical potential produced by color source, described by Wilson lines

BK and JIMWLK equations

- Gluon recombination introduces additional non-linear term to BFKL, giving Balitsky-Kovchegov (BK) equation

9509348; 9901281

emission probability proportional to n

order-unity coefficients

$$\frac{\partial n}{\partial Y} \simeq \omega \alpha_s n + \chi \alpha_s \partial_t^2 n - \beta \alpha_s^2 n^2$$

BFKL

crucial as $n \sim \mathcal{O}(1/\alpha_s)$

$t \equiv \ln(k_\perp^2)$ from expansion of nonlocal (k-dependent) emission vertex up to 2nd derivative in t; variable change to eliminate 1st derivative

- BK equation describes when saturation is reached as x descreases
- Evolution of PDF in energy governed by DGLAP
- Evolution of $W_{\Lambda^+}[\rho]$ in Λ^+ governed by Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner (JIMWLK) equation

already in saturation

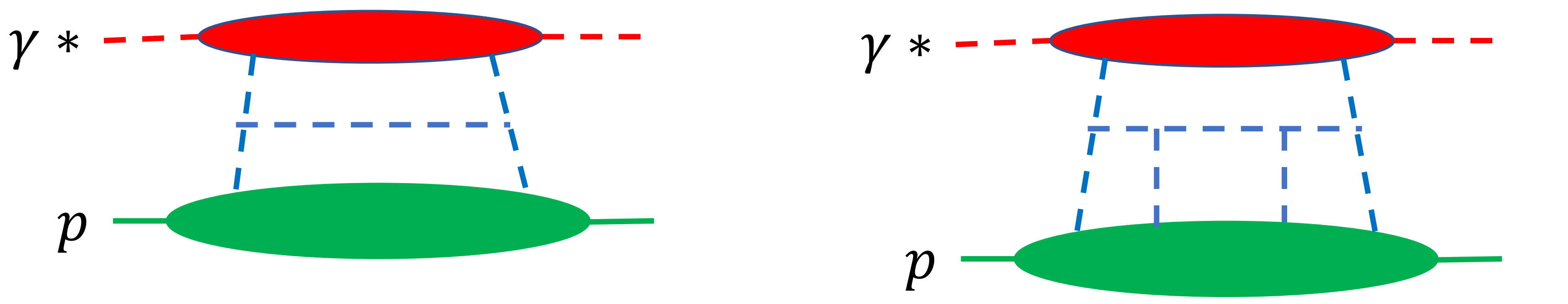
$$\frac{\partial W_{\Lambda^+}[\rho]}{\partial \ln(\Lambda^+)} = -\mathcal{H}\left[\rho, \frac{\delta}{\delta\rho}\right] W_{\Lambda^+}[\rho]$$

9701284; 0011241

JIMWLK Hamiltonian, formulated in terms of Wilson lines

BK vs JIMWLK

- Solving JIMWLK equation is difficult
- Simplified by adopting BK equation, i.e., gluon TMD (BFKL+ recombination) for saturation



- large Nc approximation (gluon ladder only, no complicated gluon crossing)

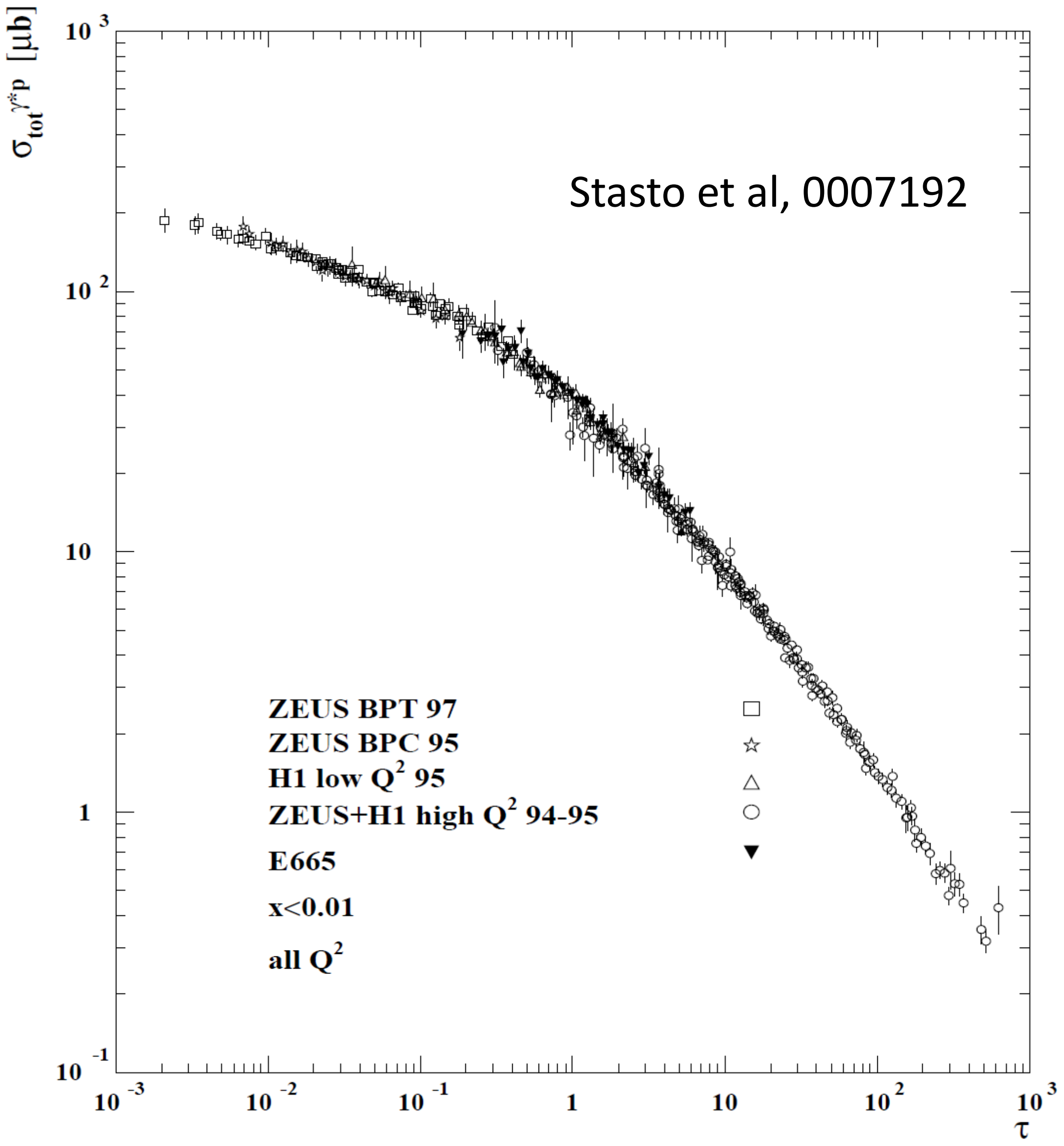
Geometric scaling

- Geometrical scaling discovered in low-x e + p collisions
- DIS inclusive cross section at small x is function of scaling variable

$$\tau \equiv Q^2/Q_s^2(x) \quad Q_s^2 = Q_0^2(x_0/x)^\lambda$$

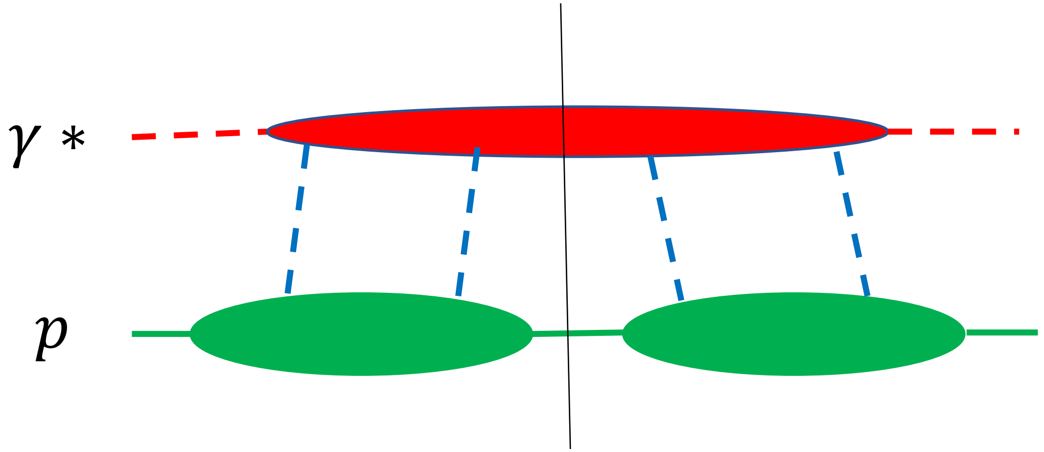
$$Q_0^2 = 1 \text{ GeV}^2,\ x_0 = 3 \cdot 10^{-4} \qquad \lambda \approx 0.3$$

- All data points within the range of x < 0.01 and $Q^2 < 450\ GeV^2$ fall on single curve
- Hint existence of saturation scale



- Dipole size $\sim 1/Q$, parton size $\sim 1/Q_s$, the two scales in the process

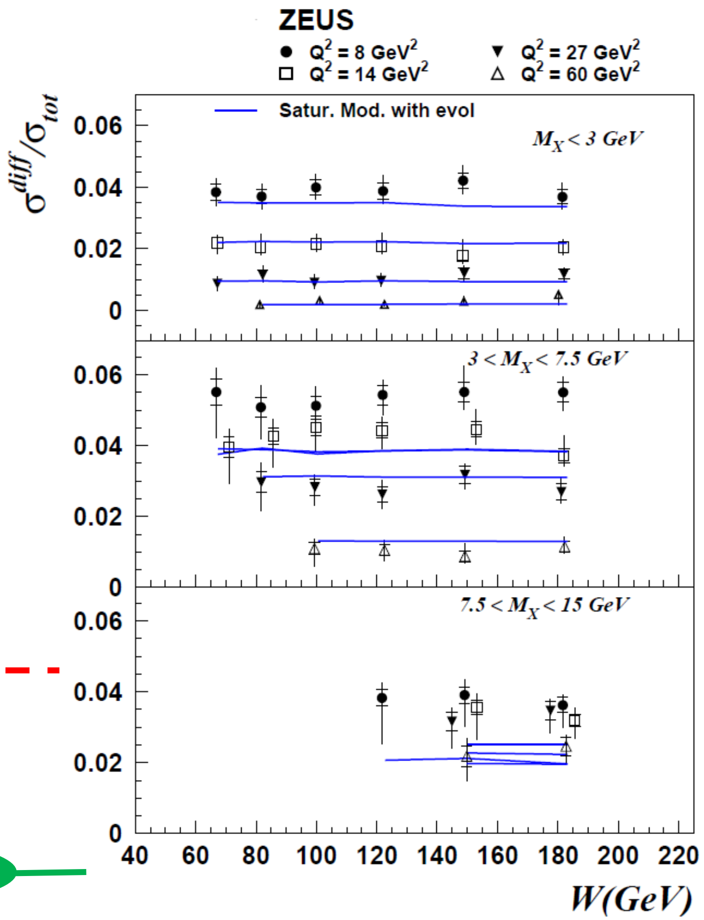
DIS Diffraction

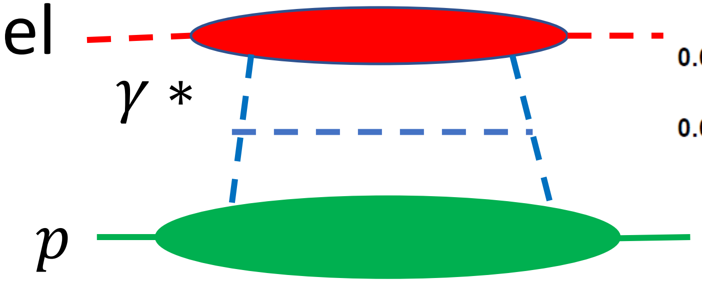


- Diagram for e+p -> p+X; t-channel
- DIS total cross section at finite x involves s-channel (box diagram)
- Cross section ratio $\sigma_{\rm diff}/\sigma_{\rm tot}$ depends on s
- If CGC exists at small x (Regge-Gribov limit), total cross section comes from t-channel, and depends only on $Q^2/Q_s^2(x)$

Bartels et al, 2002

- $\sigma_{\rm diff}/\sigma_{\rm tot}$ indep of energy, confirmed by ZEUS
- Weaker evidence than geometric scaling; pomeron exchange also t-channel



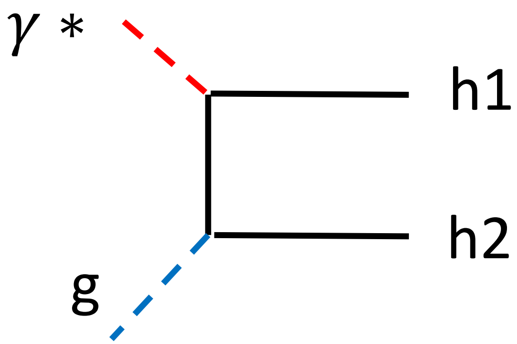


CGC vs nuclear shadowing

- Amplitude for fixed impact parameter is proportional to number of overlapping nucleons $\sim A^{1/3}$. Nuclear transverse area $\sim A^{2/3}$ leads to diffractive (or elastic) cross section proportional to $\sigma^D \sim A^{4/3}$

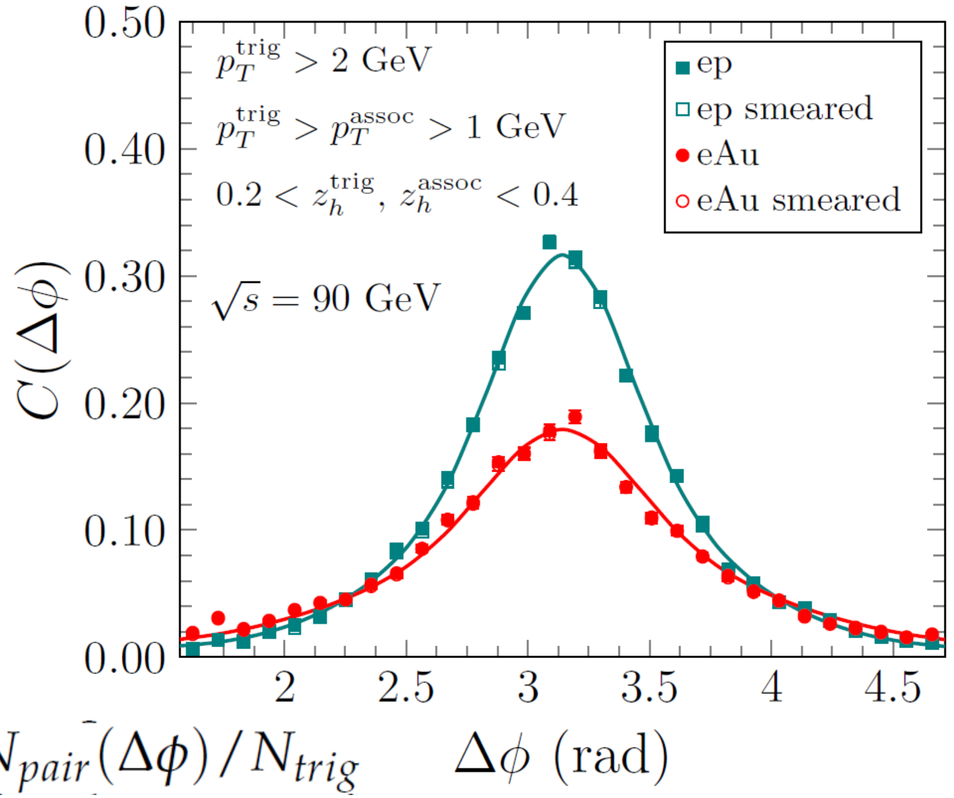
  ↑ $\sigma^2_{\text{dipole}}(x, \boldsymbol{r}_\perp)$ see diffraction diagram on previous page
- Diffraction cross section increases with A in CGC
- Diffractive events made up 10–15% of total e+p cross section at HERA, but > 20% of total cross section at EIC (model estimate)
- If incoherent interaction (no saturation), nuclear shadowing reduces cross section; stronger suppression at larger A
- Nuclear suppression vs enhancement provides sensitive probe of coherence and saturation

Dijet or dihadron correlation



- Saturation can be studied via dijet or dihedron correlation in e+A -> e' + h1 + h2 + X, or with dijet
- Away-side peak (azimuthal angle $\Delta\Phi = \pi$ between two hadrons) from hard scattering
- Saturation grants gluons in color configuration various kT < Qs
- Saturation smears peak, enhanced by A
- Also indirect evidence
- Rely on precise knowledge of parton shower effects (generate kT), which also cause decorrelation

trigger hadron & associated hadron



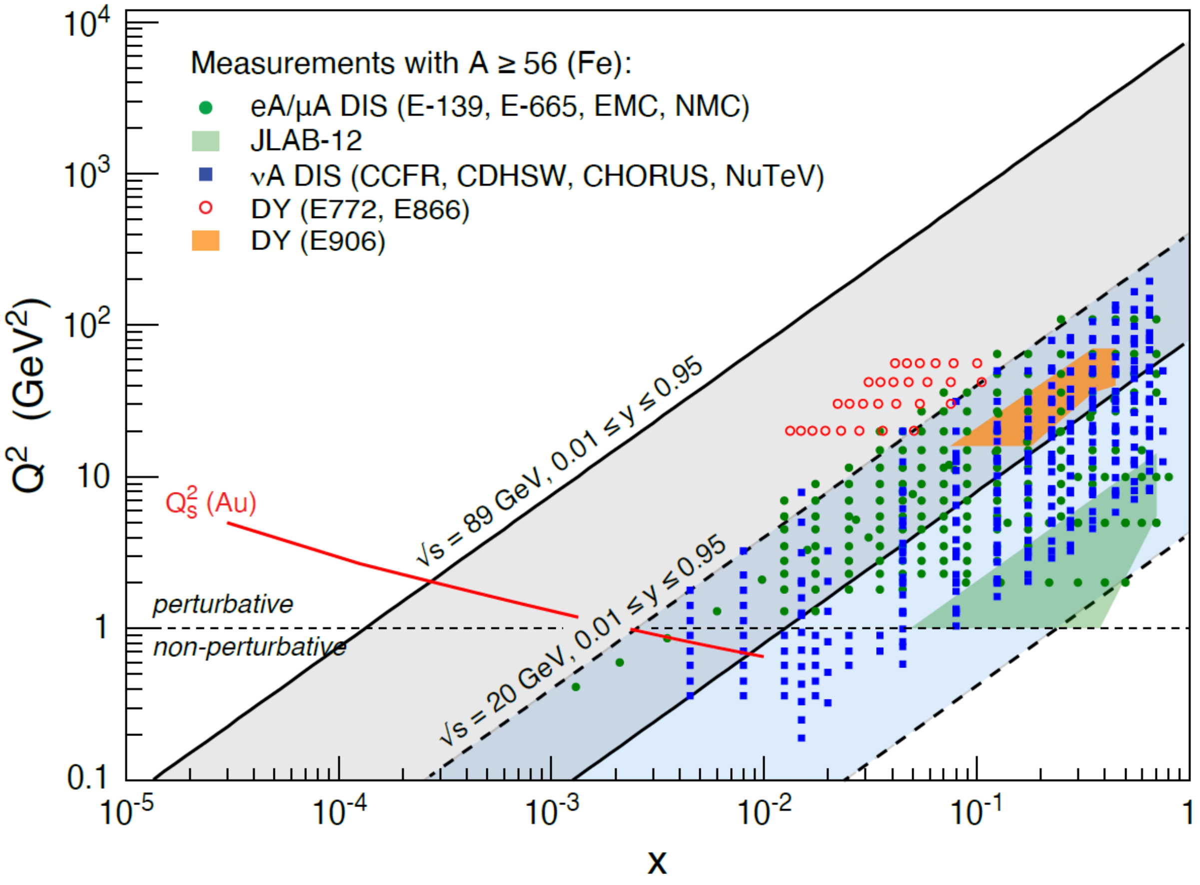
$C(\Delta\phi) = N_{pair}(\Delta\phi)/N_{trig}$

Stasto et al, 1805.05712

Probe CGC at EIC

Chia-Ming's talk

- Wider ranges in x, Q at EIC
- Measure saturation scale
- Check $Q^2_{s,A}(x) = Q^2_{s,p}(x) A^{1/3}$
- Study diffraction (energy indep; nuclear enhancement)
- Proportional to (gluon TMD)^2, sensitive to nonlinear dynamics
- Study decorrelation (suppression of away-side correlation) of dijets and dihadrons
- Unambiguously establish CGC



kinematic coverage of EIC for DIS on nuclei compared to those of previous experiments

Supplement

- DIS at small x

$$\sigma_{\gamma^* T} = \int_0^1 \mathrm{d}z \int \mathrm{d}^2\boldsymbol{r}_\perp |\psi(z,\boldsymbol{r}_\perp)|^2\ \sigma_{\mathrm{dipole}}(x,\boldsymbol{r}_\perp)$$

wave function of $q\bar{q}$ component

- Dipole cross section

$$\sigma^{\mathrm{LO}}_{\mathrm{dipole}}(x,\boldsymbol{r}_\perp) = 2\int d^2\mathbf{b} \int [D\rho] W_{\Lambda_0^-}[\rho]\ \boldsymbol{T}_{\mathrm{LO}}(\boldsymbol{b}+\frac{\boldsymbol{r}_\perp}{2},\boldsymbol{b}-\frac{\boldsymbol{r}_\perp}{2})$$

- Forward scattering amplitude

$$\boldsymbol{T}_{\mathrm{LO}}(\boldsymbol{x}_\perp,\boldsymbol{y}_\perp) = 1 - \frac{1}{N_c}\mathrm{tr}\left(U(\boldsymbol{x}_\perp)U^\dagger(\boldsymbol{y}_\perp)\right)$$

- Wilson line

$$U(\boldsymbol{x}_\perp) = \mathrm{T}\exp ig\int^{1/xP^-} dz^+\,\mathcal{A}^-(z^+,\boldsymbol{x}_\perp)$$

↑ solved from classical YM equation with the color source